Comment on the Phys. Rev. Lett. by Frahm and Shepelyansky

The paper [1] is a follow-up on the series of papers by Shepelyansky and co-authors (see [2,3] and quite a few other papers extensively cited in [1]). In those previous papers it was argued that a system of coupled nonlinear CLASSICAL oscillators equilibrates to a distribution which exhibits quantum features, and which was designated by a somewhat strange term ``quantum Gibbs distribution'' (QG). It was stressed in [3] that the QG ``is drastically different from usually expected energy equipartition''. This purported QG distribution was propagated by Shepelyansky as a puzzle or a "paradox" of significant conceptual importance in statistical mechanics, see his note [4] which contains a somewhat pompous statement: ``But for me the main SURPRISE is that we obtain QUANTUM distributions for CLASSICAL nonlinear modes which is DRASTICALLY different from energy equipartition expected by all historical figures of statistical physics starting from Boltzmann.'' (All capitalized words are in the original text).

It has been pointed out in [5] (see footnote [63] in [5]) that the work [3] is based on an incorrect assumption and that the correct result is the classical Rayleigh-Jeans (RJ) distribution rather than the mysterious QG. A detailed criticism of [3] appears in the note [6] written in response to [4] and sent to the same addressees as [4].

In his most recent work [1] Shepelyansky finally agrees that the correct distribution is the RJ which he erroneously keeps calling "equipartition", although the RJ obviously contains the mode frequency. The authors of [1] consider couplings $J_{i,l}$, between pairs of oscillators (i,l), taken from the standard random matrix ensemble (GOE). These couplings are long-range, unlike the more often assumed nearest neighbor couplings. The authors present their model as a novel ``generic model of many-body oscillator systems with nonlinear interactions'', inspired by quantum mechanics of many-body systems. This model, however, is a special case of the model introduced and studied in [5]. Indeed, if in Eq. (15) of [5] one sets $J_0 = 0$ -and this is the case studied in Sec.III.D of [5]- one obtains essentially the GOE random matrix for $J_{i,l}$. (Strictly speaking, the resulting $J_{i,l}$ has zero diagonal elements but this difference from GOE is of no significance in the large$-N$ limit, which is the limit of interest.) The work [5] has been communicated to Shepelyansky, so it is strange, to say the least, that it is not cited in [1].

Furthermore, the RJ distribution has been derived in [5] for a generic model, i.e. with arbitrary couplings $J_{i,l}$ (short or long-range, with or without disorder). Of course, this distribution has been also derived previously for various models [7-12], as indicated in [5]. Thus, the RJ distribution for a system of linear modes in equilibrium, coupled by weak nonlinearity, is a well-established fact. Another question, of course, is whether this equilibrium state can be reached, in a reasonable time, starting from some initial non-equilibrium state of the system (see for example [13-15]). It has been shown in [5] that in some cases the equilibrium is quickly reached while in some other cases the system "gets stuck" in a spin-glass-like state.

In conclusion, the paper [1] is disconnected from the most relevant literature on the subject and is trying to give the impression that its main result is actually new. It creates an unnecessary mystery around a subject that has been entirely clear for a long time. Moreover, it reproduces models and results from other works without giving any reference. The good news, though, is that the "Shepelyansky paradox" is finally demolished by its own creator (in agreement with earlier work by other authors), so that Statistical Mechanics remains safe and sound.

With best wishes,
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References

[1]Klaus M. Frahm and Dima L. Shepelyansky, Physical Review Letters 131, 077201 (2023) (arXiv:2212.11955v2)
[2] M. Mulansky, K. Ahnert, A. Pikovsky, and D. L. Shepelyansky, Phys. Rev. E 80, 056212 (2009).
[3] L. Ermann and D. L. Shepelyansky, New J. Phys. 15, 123004 (2013).
[4] D. L. Shepelyansky, ``Dynamical thermalization conjecture (DTC) paradox'', (May 2019) http://www.quantware.ups-tlse.fr/dima/myrefs/myunp007.pdf, distributed by Shepelyansky among some members of statistical mechanics community.
[5] A. Ramos, L. Fernández-Alcázar, T. Kottos, and B. Shapiro. Phys. Rev. X 10, 031024 (2020)
[6] B. Shapiro, ``There is no paradox'', (January 2020) http://www.quantware.ups-tlse.fr/dima/myrefs/myunp007shapiro1.pdf
[7] F. Wu, A. Hassan, and D. Christodoulides, Nat. Photonics 13, 776 (2019).
[8] M. Parto, F. Wu, P. Jung, K. Makris, and D. Christodoulides, Opt. Lett. 44, 3936 (2019).
[9] P. Buonsante, R. Franzosi, and A. Smerzi, Phys. Rev. E 95, 052135 (2017).
[10] P. Aschieri, J. Garnier, C. Michel, V. Doya, and A. Picozzi, Phys. Rev. A 83, 033838 (2011).
[11] S. Dyachenko, A. Newell, A. Pushkarev, and V. Zakharov, Physica(Amsterdam) 57D, 96 (1992).
[12] A. Picozzi, Opt. Express 15, 9063 (2007).
[13] D. M. Basko, Phys. Rev. E 89, 022921 (2014)
[14] C. Shi, T. Kottos, B. Shapiro, Physical Review Research 3, 033219 (2021).
[15]A. Y. Ramos, C. Shi, L. J. Fernandez-Alcazar, D. N. Christodoulides, T. Kottos, Comm. Phys. 6, 189 (2023)