Networks of game Go

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Part I : Introduction

Networks

->Recent field: study of complex networks, tools and models have been created;

->Many networks are scale-free with power-law distribution of links difference between directed and non directed networks

->Important examples from recent technological developments: internet, World Wide Web, social networks...

->Can be applied also to less recent objects
in particular, study of human behavior: languages, friendships…

Networks for games

-> Network theory never applied to games

-> Games are nevertheless a very ancient activity, with a mathematical theory attached to the more complex ones

-> Games represent a privileged approach to human decision-making

->Can be very difficult to modelize or simulate

The game of go

→Game of go: very ancient Asian game, probably originated in China in Antiquity (image on the left from VIIIth century)

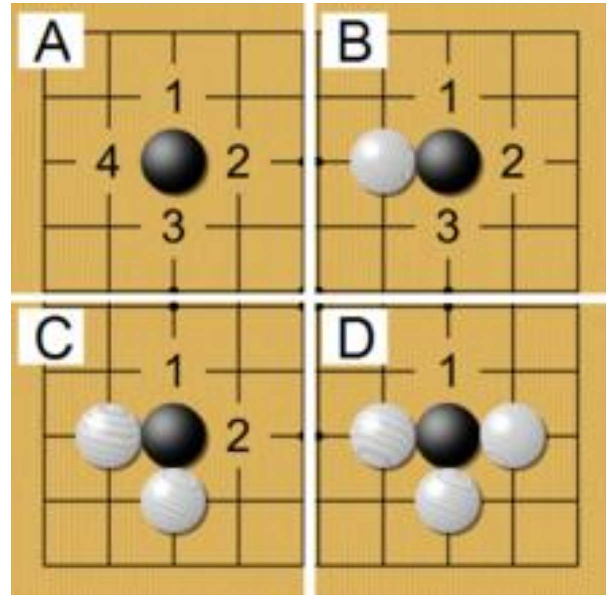
-> Go is the Japanese name; Weiqi in Chinese, Baduk in Korean

The game of go

-> Go is a very popular game played by many parts of the population (ex. right) on a board called Goban (see below)

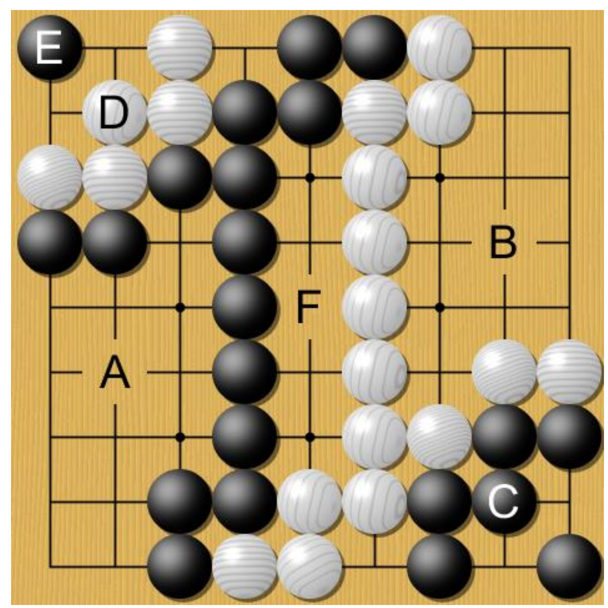
Rules of go

->White and black stones alternatively put at intersections of 19 x19 lines



->Stones without liberties are removed

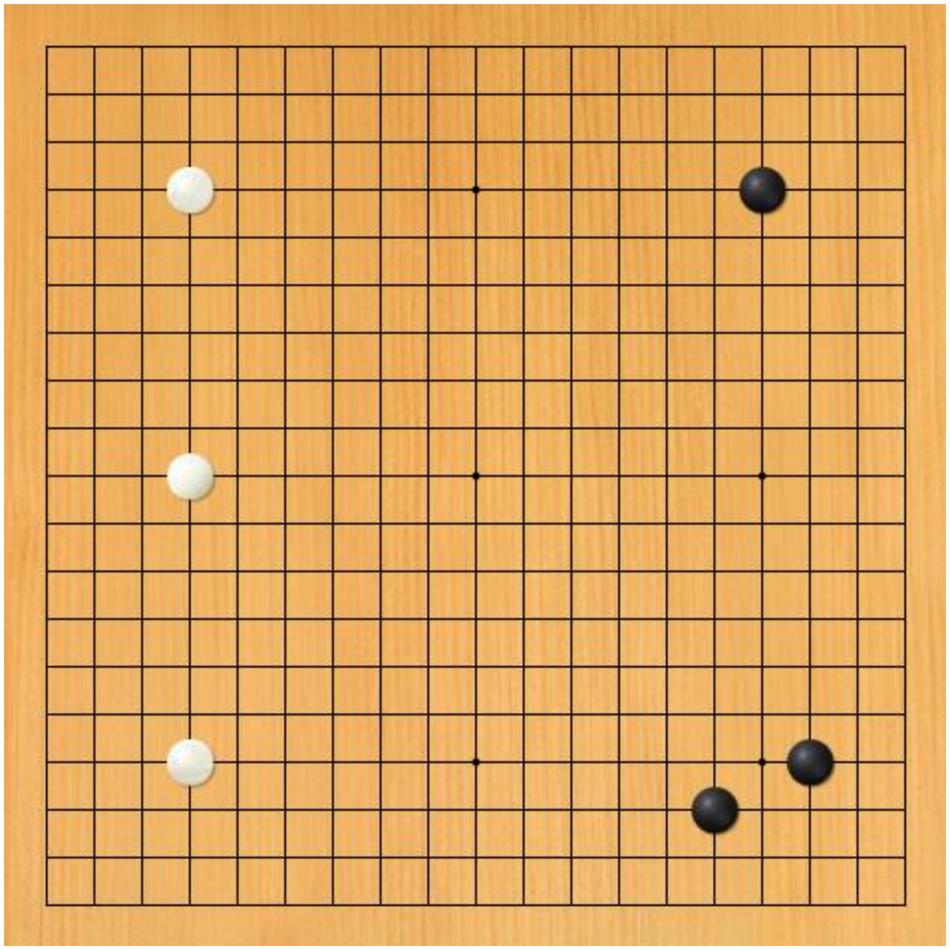
->A chain with only one liberty is said in atari

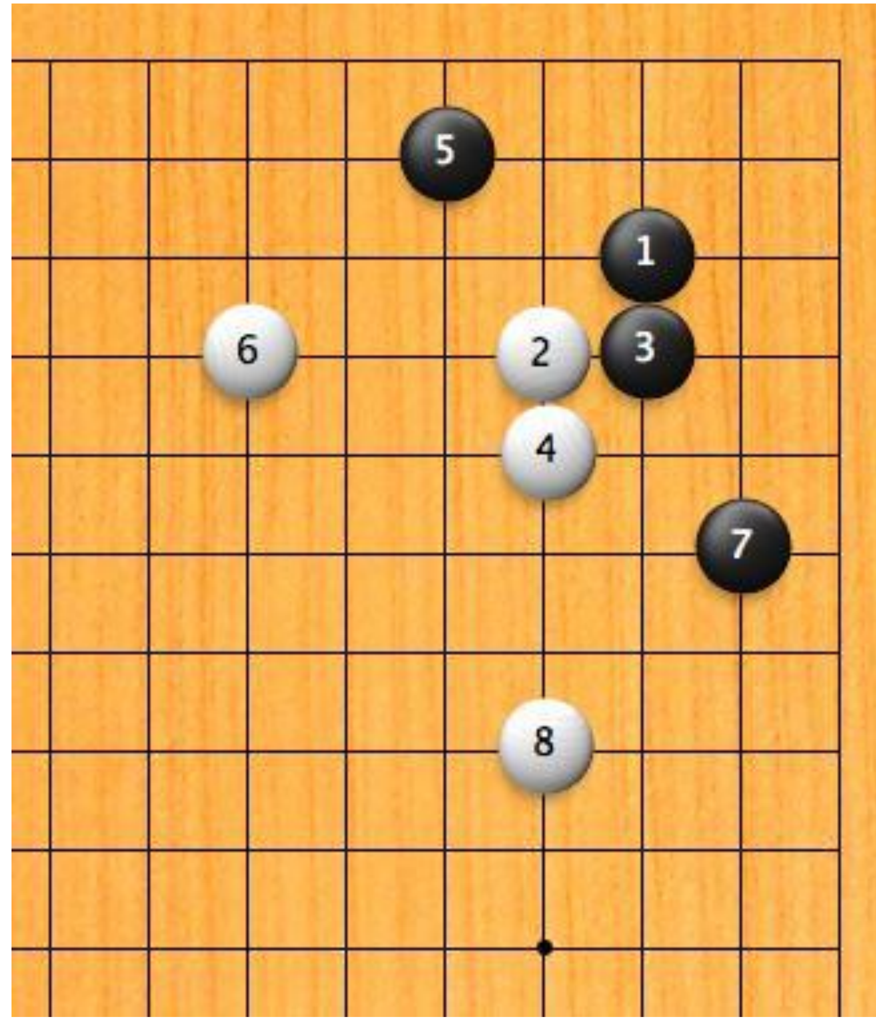


->Handicap stones can be placed

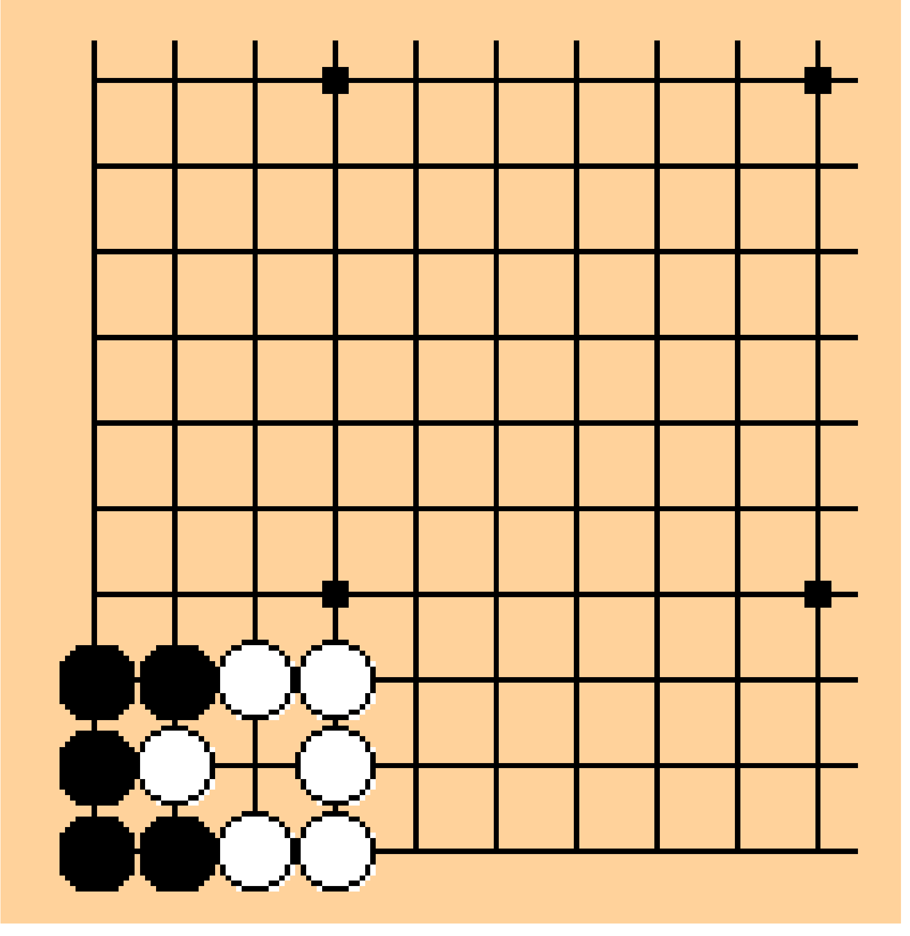
->Aim of the game: construct protected territories

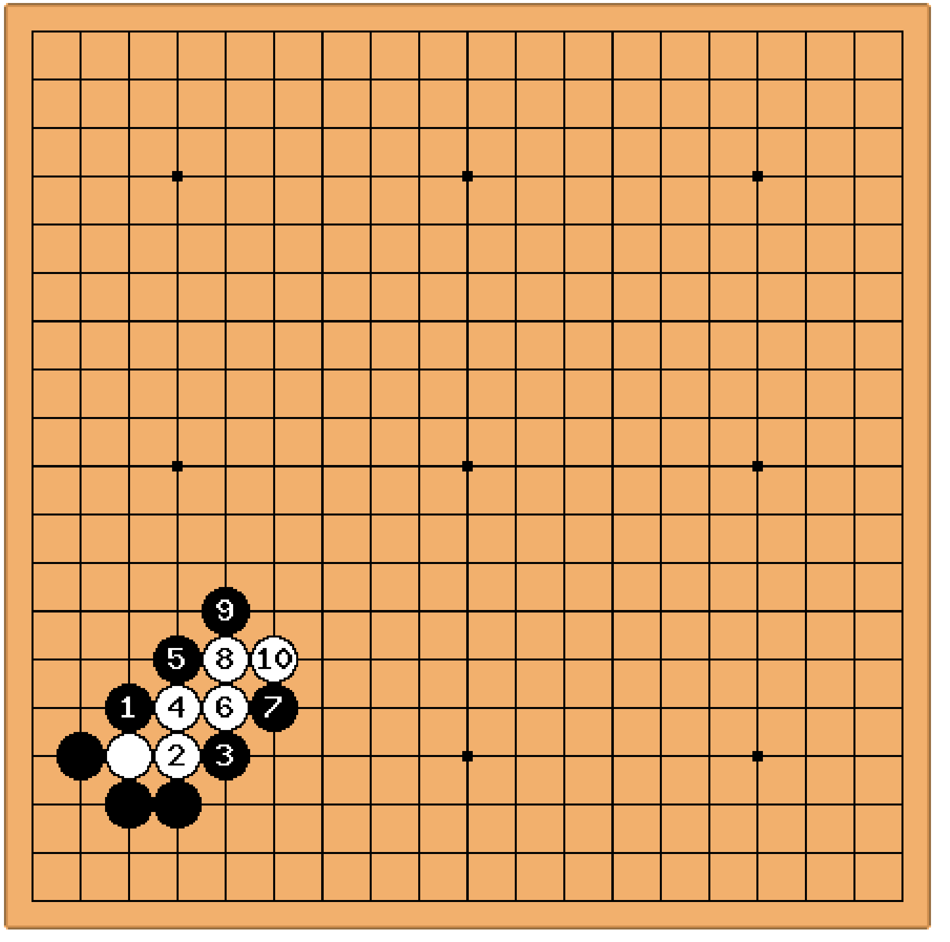
Beginnings: Fuseki and Joseki



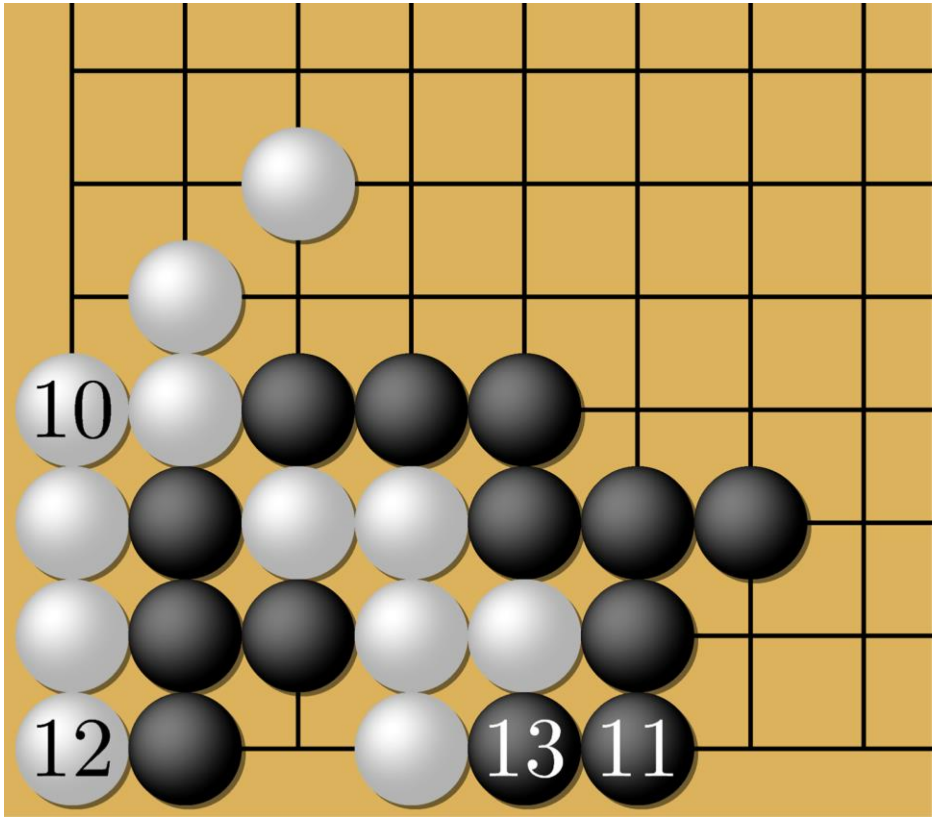


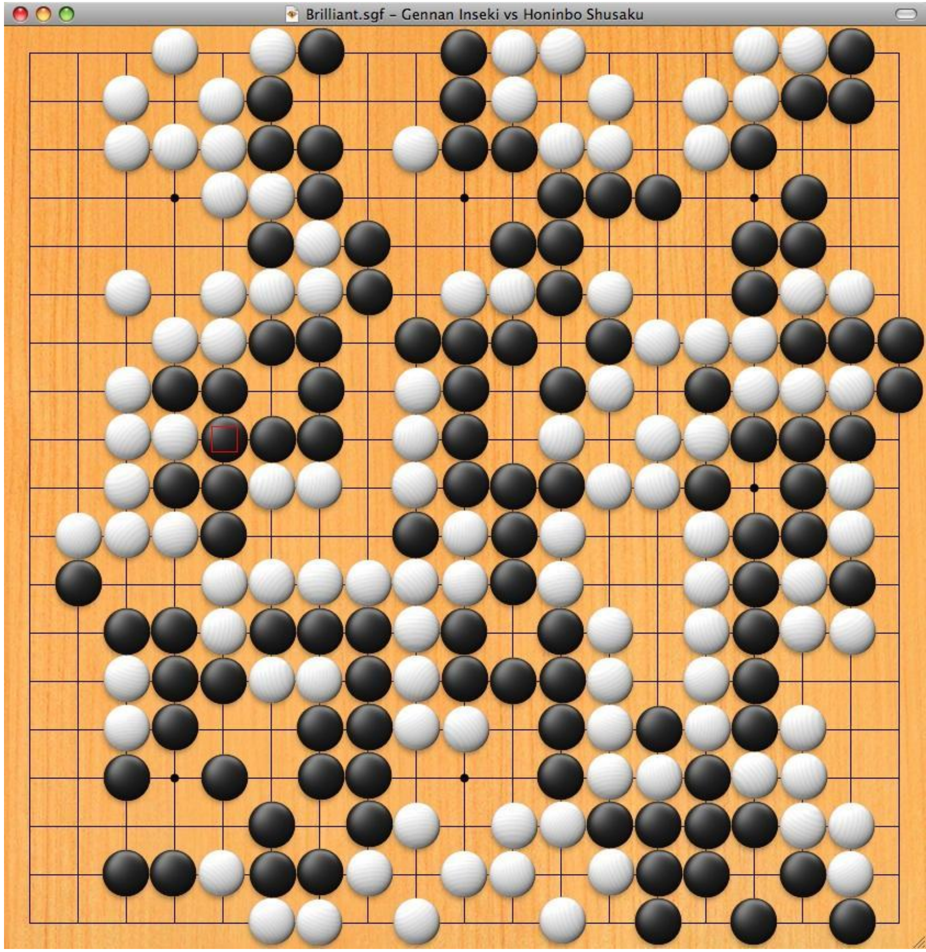
During the game-Ko and ladders





Endgames-life and death





Player rankings

→There are nine levels (dans) of professionals (top players) followed by nine levels of amateurs
->A handicap stone can compensate for roughly one dan: like in golfing, players of different levels can play evenly thanks to handicaps
->There are regular tournaments of go since very long times

Computer simulations

-->While Deep Blue famously beat the world chess champion Kasparov in 1997, We had to wait March 2016 for Go with Alphago a computer program wich has beaten one of best go player. Why is this game so difficult to simulate?

->Total number of legal positions 10^{171}, vs "only" 10^{50} for chess

-> Not easy to assign positional advantage to a move

-> Alphago uses Monte Carlo Go: play random games starting from one move and see the outcome until a value can be assigned to the move, and deep Learning techniques by neural networks

Databases

->We used databases of expert and amateur games in order to construct networks from the different sequences of moves, and study the properties of these networks

http://ww

->Whole available record, from 1941 onwards, of the most important historical professional Japanese go tournaments: Kisei (143 games), Meijin (259 games), Honinbo (305 games), Judan (158 games)

Contains also 135 000 amateur games played online

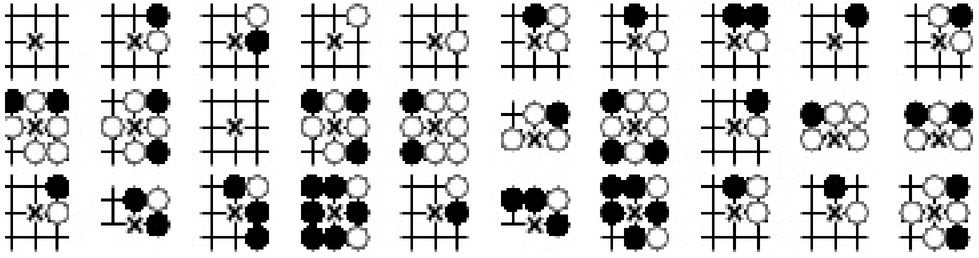
->Level of players is known, mutually assessed according to games played

->We compare databases from human players to networks constructed from computer-generated games (program Gnugo)

Vertices of the network I

->"plaquette" : square of 3 x3 intersections

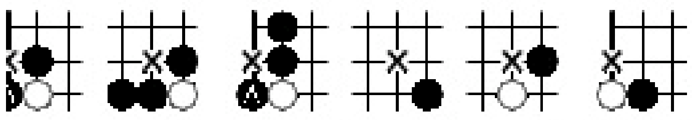
->We identify plaquettes related by symmetry
->We identify plaquettes with colors swapped
->1107 nonequivalent plaquettes with empty centers
->vertices of our network



Vertices of the networks II

->"plaquette'' : square of 3 x3 intersections + atari status of nearest-neighbors

->We still identify plaquettes related by symmetry
->Because of rules restrictions, only 2051 legal nonequivalent plaquettes with empty centers

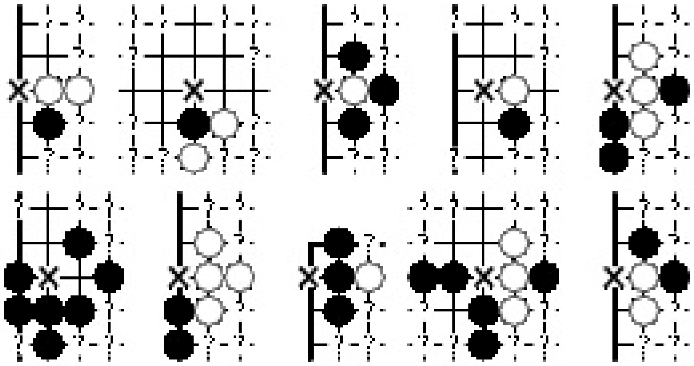


Vertices of the networks III

->"plaquette" : diamond of 3 x3 +4 intersections

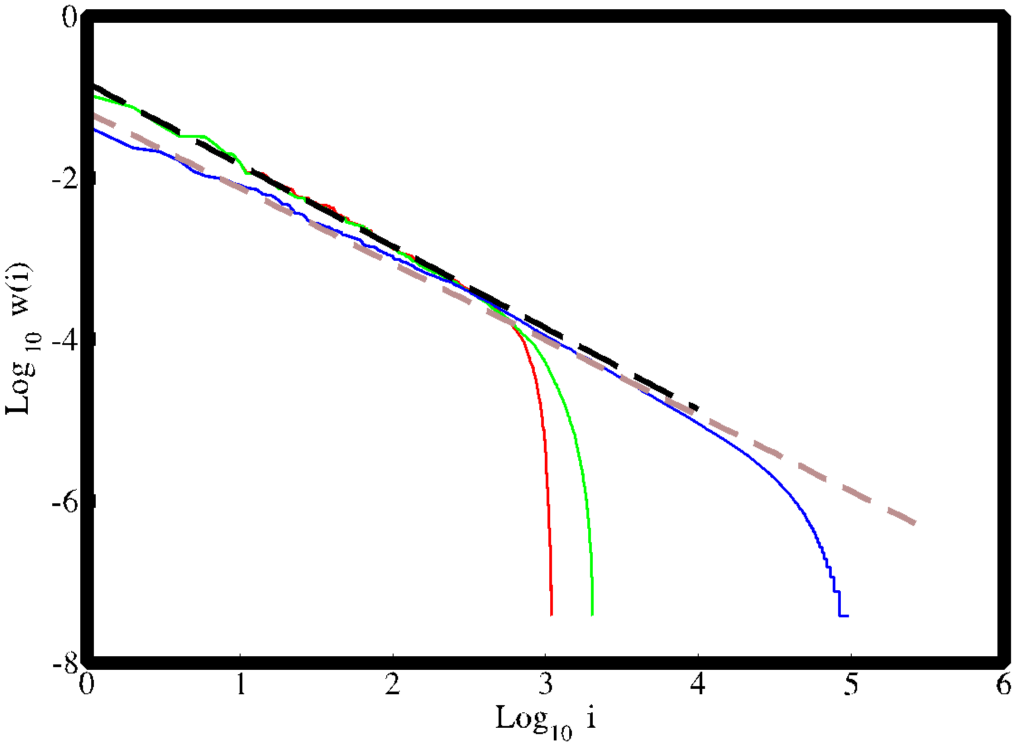
->We still identify plaquettes related by symmetry

->193995 nonequivalent plaquettes with empty centers
(96771 actually never used in the database)



Zipf's law

->Zipf's law: empirical law observed in many natural distributions (word frequency, city sizes...)
->If items are ranked according to their frequency, predicts a power-law decay of the frequency vs the rank.
->integrated distribution of three network nodes clearly follows a Zipf's law, with exponent close to 1



Normalized integrated frequency distribution of three types of nodes. Thick dashed line is y=-x.

Links of the network

->we connect vertices corresponding to moves a and b if b follows a in a game at a distance $< d$.
->Each choice of d defines a different network. The choice of d determines the distance beyond which two moves are considered nonrelated.
->Sequences of moves follow Zipf's law (cf languages) Exponent decreases as longer sequences reflect individual strategies
->move sequences are well hierarchized by d=5
->amateur database departs from all professional ones, playing more often at shorter distances

Sizes of the three networks

-> Total number of links including degeneracies is 26 116 006, the same for all networks

->Network I: 1107 nodes, 558190 links without degeneracies

->Network II: 2051 nodes, 852578 links without degeneracies

->Network III: 193995 nodes, 7405395 links without degeneracies

->Very dense networks, especially the smallest ones

-> Very different from e.g. the World Wide Web

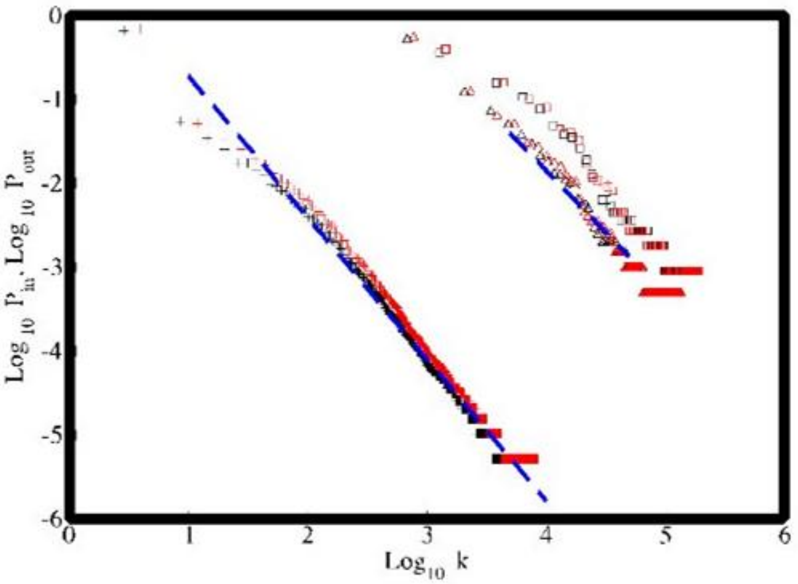
Part II : Networks from human games

Link distribution

->Tails of link distributions very close to power-law for all three networks

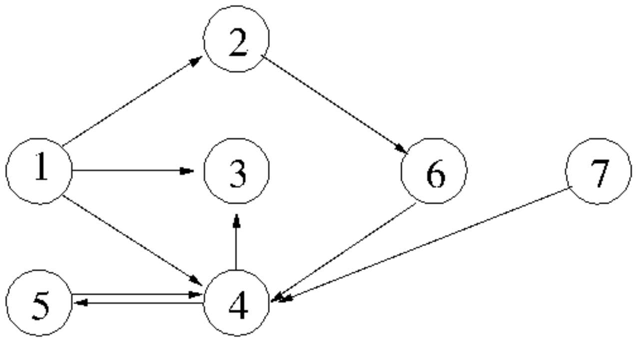
->network displays the scale-free property

->symmetry between ingoing and outgoing links is a peculiarity of this network



Normalized integrated distribution of links for the three networks

Matrix for directed networks



Weighted adjacency matrix

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Google algorithm

Ranking pages $\{1, \ldots, N\}$ according to their importance.
Idea:

- The importance of a page i depends on the importance of the pages j pointing on it
- If a page has many outgoing links the importance it transmits is inversely proportional to the number of pages it points to.

PageRank p_i should thus verify

$$p_i = \sum_{j \to i} \frac{p_j}{n_j}.$$

n_j= number of outgoing links of page j.

With the (stochastic) matrix H introduced above,

$$\mathbf{p} = H\mathbf{p}$$

.

Computation of PageRank

$\mathbf{p} = H\mathbf{p} \;\Rightarrow$ **p**= stationary vector of H:
can be computed by iteration of H.

To remove convergence problems:

Replace columns of 0 (dangling nodes) by $\frac{1}{N}$: $H \rightarrow$ matrix S

In our example, $H = \begin{pmatrix} 0 & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{7} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{7} & 0 & 1 & 1 & 1 \\ 0 & 0 & \frac{1}{7} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 \end{pmatrix}$.

To remove degeneracies of the eigenvalue 1, replace S by

$$G = \alpha S + (1 - \alpha)\frac{1}{N}\,\mathrm{ee}^T$$

PageRank and CheiRank

- The PageRank algorithm gives the PageRank vector, with amplitudes p_i, with $0 \leq p_i \leq 1$
- All webpages can then be ordered according to their PageRank value
- The PageRank value of a webpage can be understood as the average time a random surfer will spend there
- It ranks websites according to the number of links pointing to them which come from high-PageRank sites.

->PageRank is associated to the largest eigenvalue of the matrix G. It is based on ingoing links

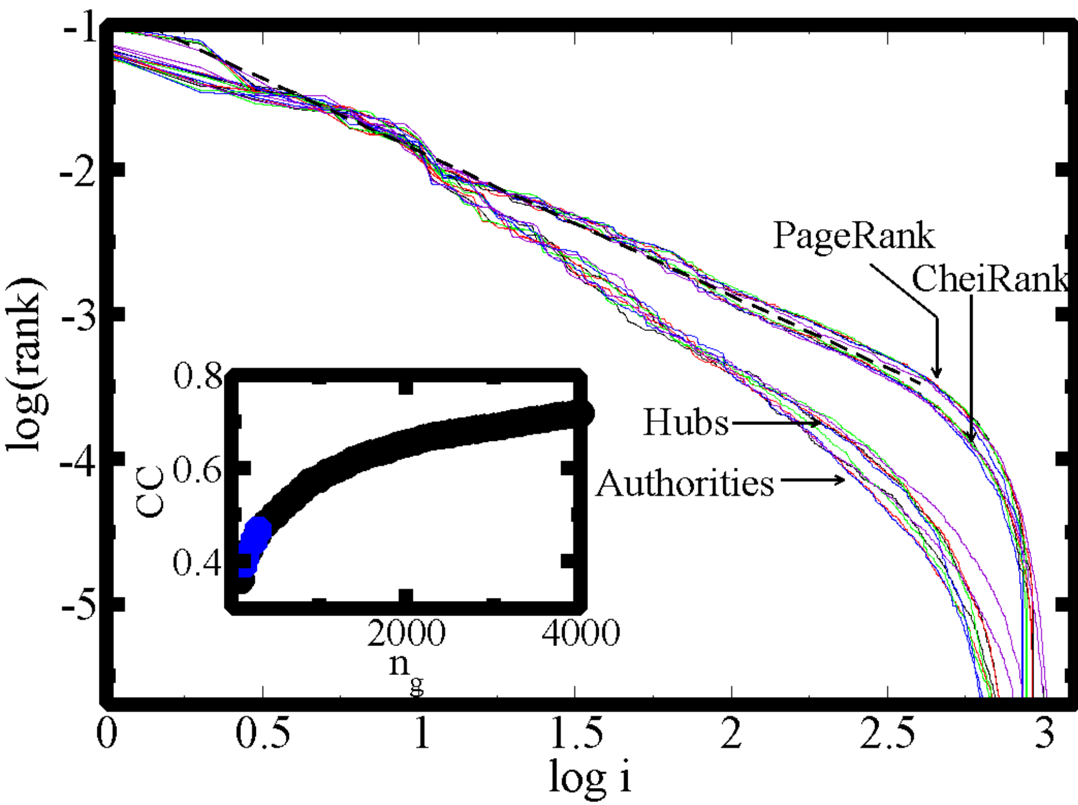
->CheiRank corresponds to the PageRank of the network obtained by inverting all links. It can be associated to a new matrix G*, and is based on outgoing links

Ranking vectors: network I

->PageRank: ingoing links
->CheiRank: outgoing links
->HITS algorithm:Authorities (ingoing links) and Hubs (outgoing links)

->Ranking vectors follow an algebraic law

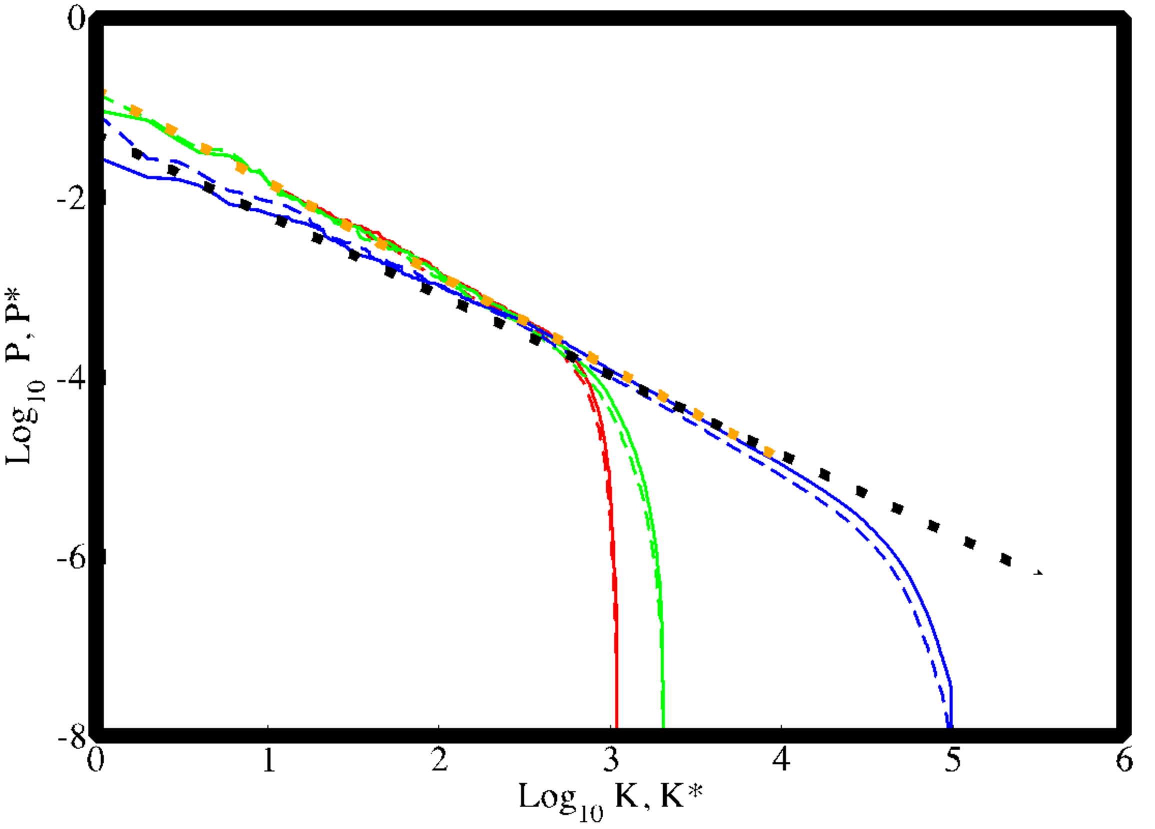
->Symmetry between distributions of ranking vectors based on ingoing links and outgoing links.



Ranking vectors: other networks

->Still symmetry between distributions of ranking vectors based on ingoing links and outgoing links.

->Power law different for the largest network



->Ranking vectors of G and G* for the three networks
red: size 1107, green: size 2051, blue: size 193995.

Ranking vectors: correlations

->Strong correlations between PageRank and CheiRank

->Strong correlation between moves which open many possibilities of new moves and moves that can follow many other moves.

->However, the symmetry is far from exact.

->Correlation less strong for largest network

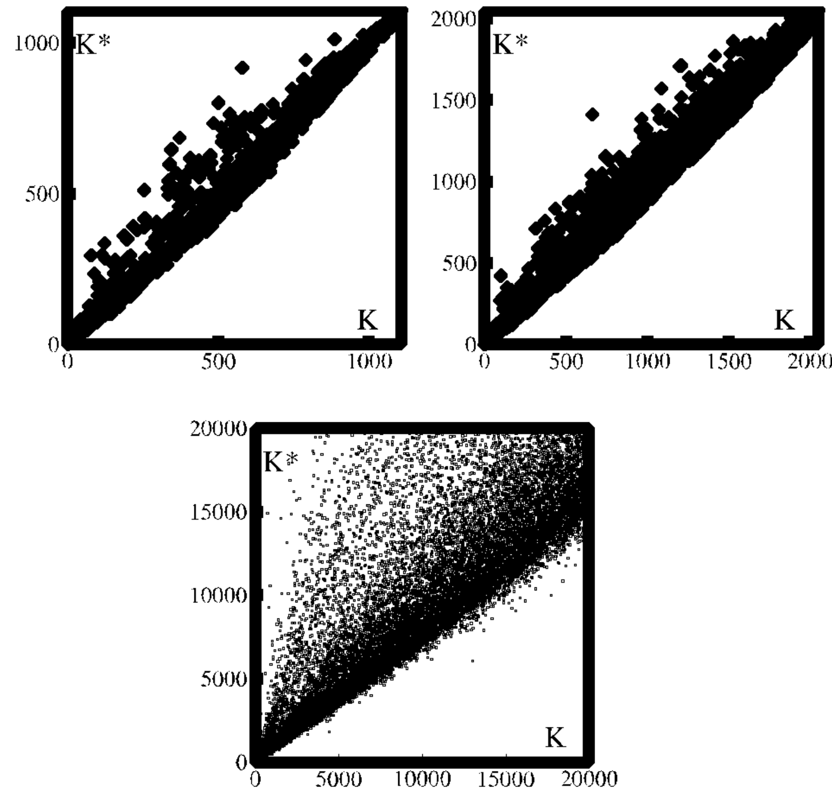
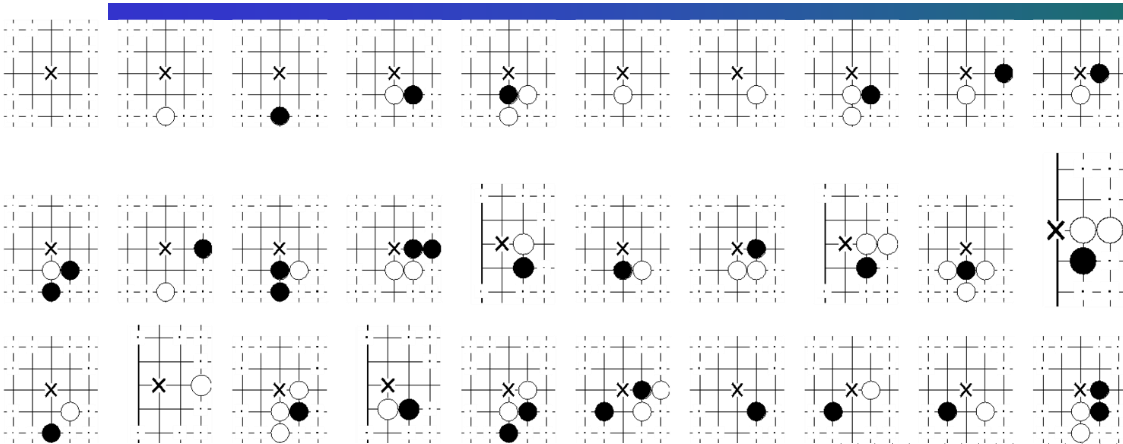


Figure: K* vs K where K (resp. K*) is the rank of a vertex when ordered according to PageRank vector (resp CheiRank) for the three networks (sizes 1107, 2051, 193995)

Ranking vectors vs most common moves



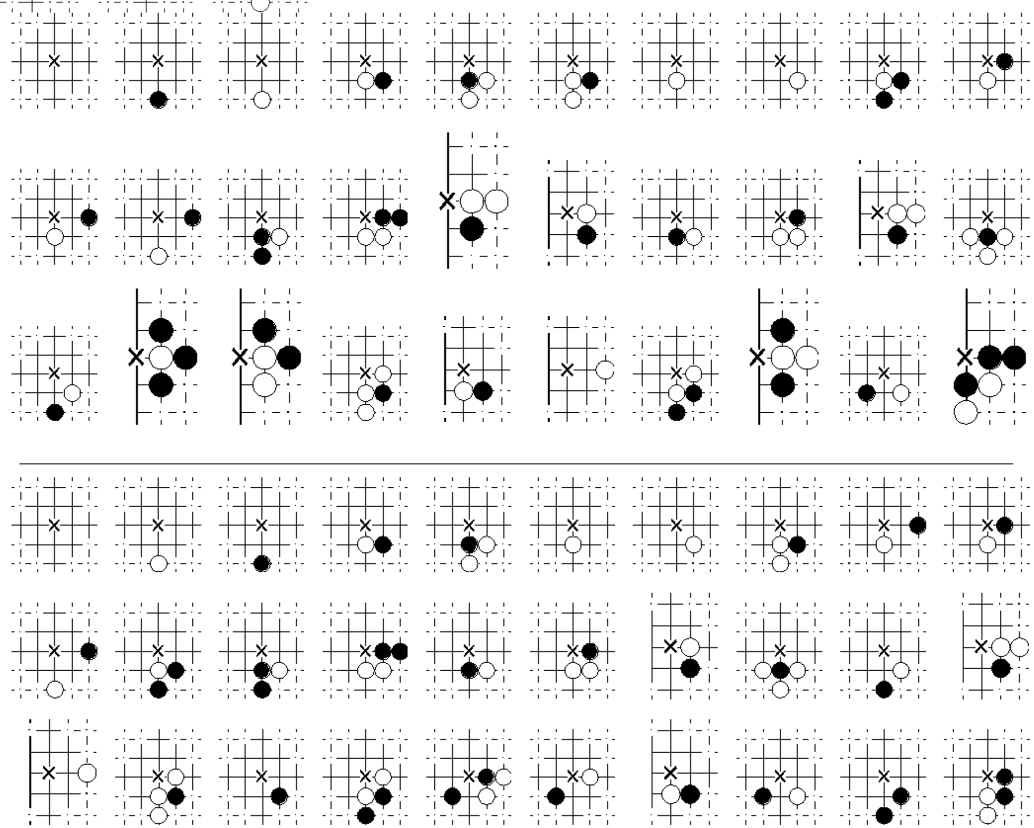


Figure: Top 30 most common moves of network III; right: top 30 PageRank and CheiRank for same network

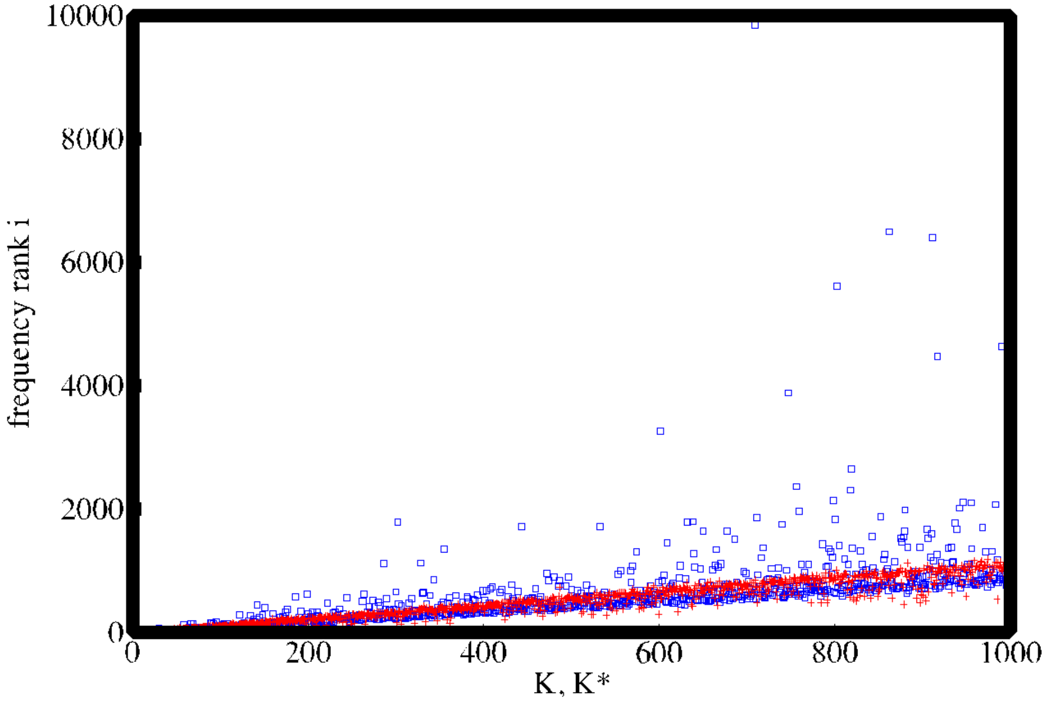
Ranking vectors vs most common moves

->There are correlations between PageRank, CheiRank, and most common moves

->However, there are also many differences, which mark the importance of specific moves in the network even if they are not that common

->Genuinely new information, which can be obtained only from the network approach

Figure: frequency rank vs PageRank (blue) and CheiRank (red) for network III



Ranking vectors vs most common moves

-> In the World Wide Web, frequency count corresponds to ranking by e. g. indegree

->PageRank takes into account indegree but weighted by importance of nodes from where the links are coming

-> Here PageRank underlines moves to which converge many well-trodden paths in the database

->CheiRank does the same in the reverse direction, highlighting moves which open many such paths

-> Could be used to bias or calibrate the Monte Carlo Go

Spectrum of the Google matrix

->For second and third networks, still gap between the first eigenvalue and next ones

->Radius of the bulk of eigenvalues changes with size of network

->More structure in the networks with larger plaquettes which disambiguate the different game paths and should make more visible the communities of moves

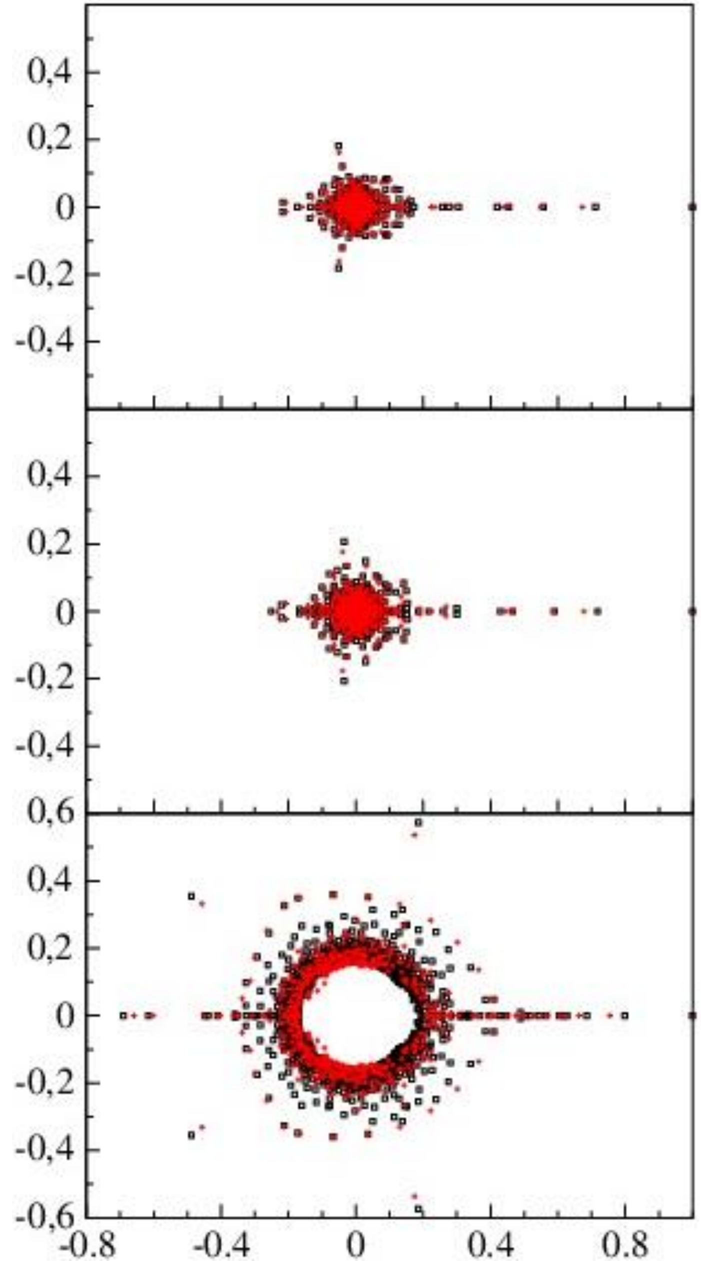


Figure: Eigenvalues of G in the complex plane for the networks with 1107, 2051 and 193995 nodes

What is the meaning of eigenvectors of the Google matrix ?

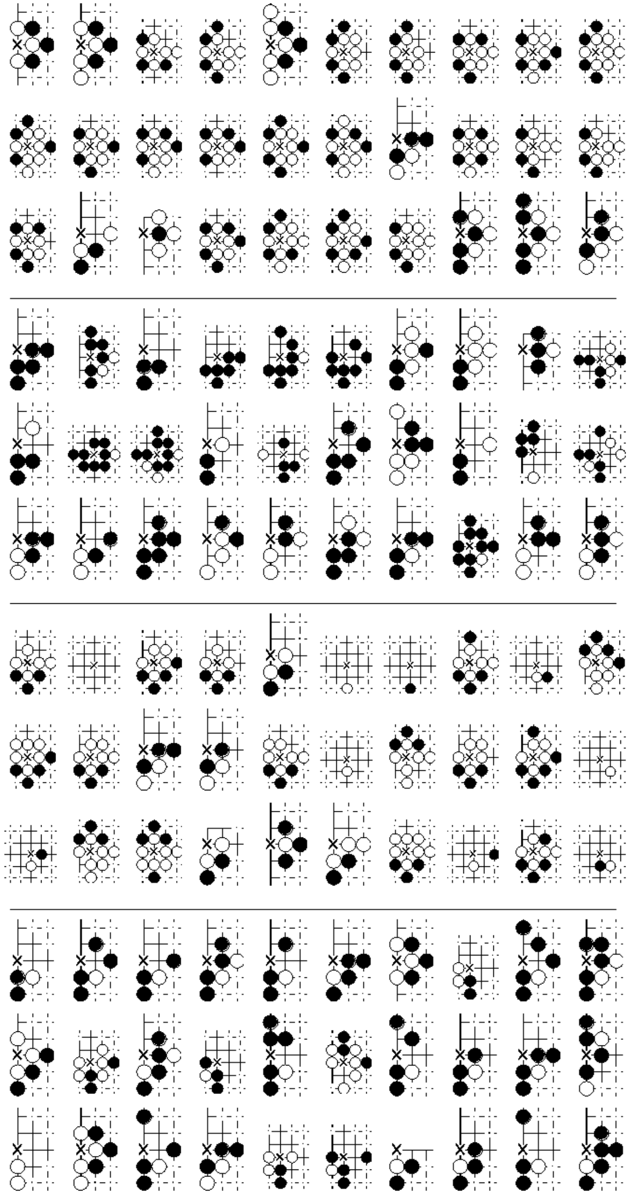
->Next to leading eigenvalues are important, may indicate the presence of communities of moves with common features

->Indeed, eigenvectors of G for large eigenvalues correspond to parts of the network where the random surfer gets stopped for some time before going elsewhere

-> Correspond to sets of moves which are more linked together than with the rest of the network

-> Should indicate communities of moves which tend to be played together

Eigenvectors for network III

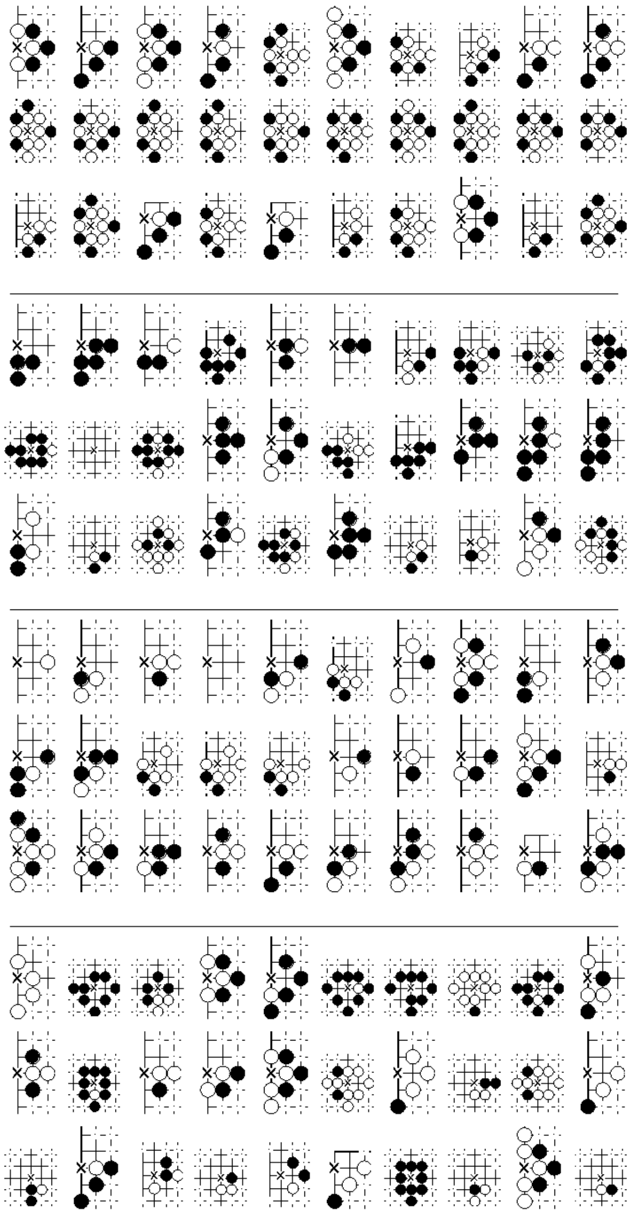


Top 30 moves

7th, 11th, 13th and 21th eigenvectors of G (left)

7th, 11th, 13th and 21th eigenvectors of G* (right)

Impression: different groups mixed in the same eigenvector



Networks for different game phases

->Eigenvectors are different from those of full game network, showing specific communities

->Bias toward more empty plaquettes for beginnings, more filled plaquettes towards the end

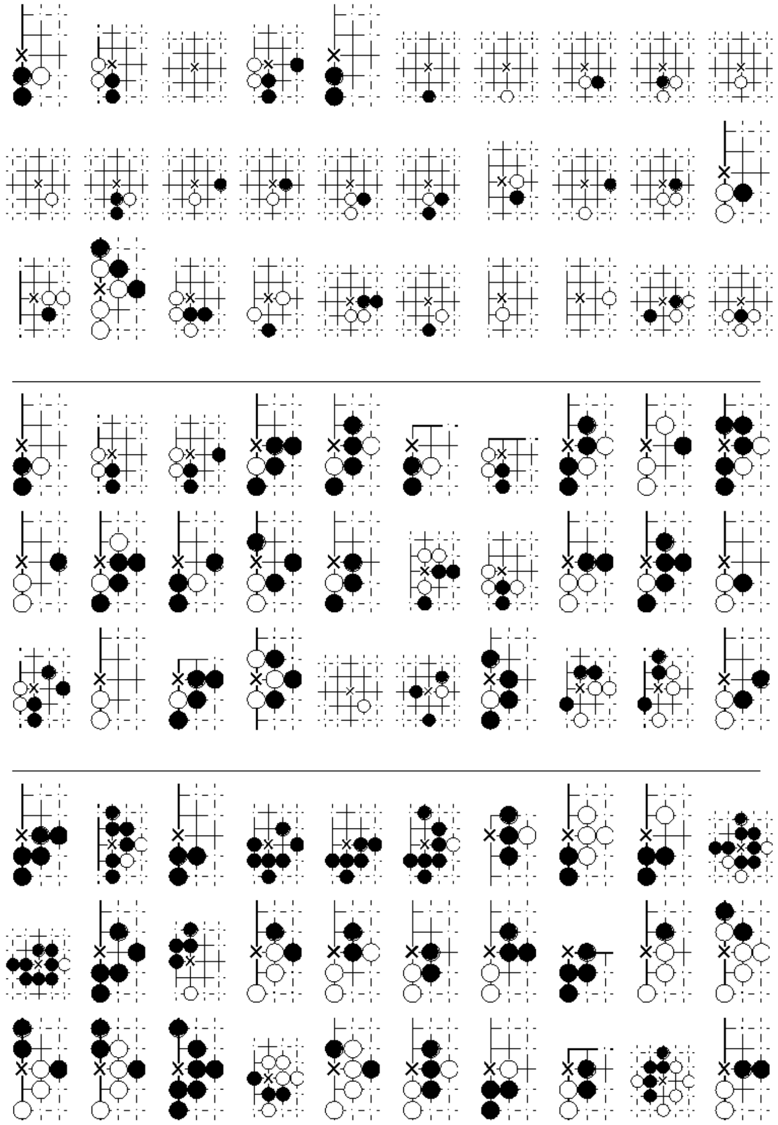


Figure: fourth eigenvector of G for 50 first moves (top), middle 50 (middle) and last 50 (bottom)

Part III : Networks from Computer-generated games

Databases

Gnugo 19X19:
7000 Independent Games
72 hours = 1000 Games

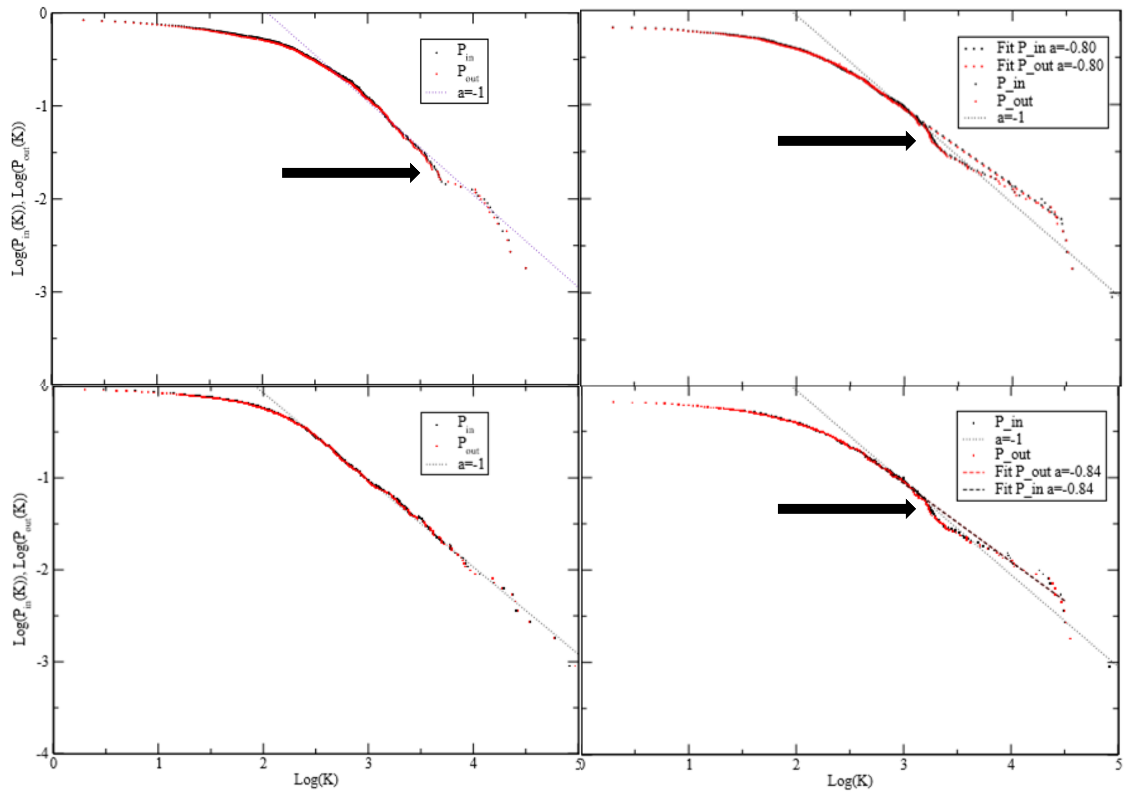
U-go 19x19:
18 000 amateurs Games

Gnugo 9X9:
20 000 Normal Games
20 000 Games with Monte-Carlo
10 hours = 1000 Games

Link distribution

Gnugo 19X19 4000 parties

Gnugo Monte-Carlo 9X9 20 000 parties



U-go 19X19 4000 parties amateurs

Gnugo 9X9 20 000 parties

19X19 human and Gnugo seem to be the same, a=-1

9X9 different from 19X19 a=-0.8 -> more filled plaquettes played

PageRank/CheiRank: Network I

Figure: Top PageRank of 19X19 Gnugo and human Networks and PageRank of 9X9 without and with Monte-Carlo option

Bottom CheiRank of 19X19 Gnugo and human Networks and PageRank of 9X9 without and with Monte-Carlo option

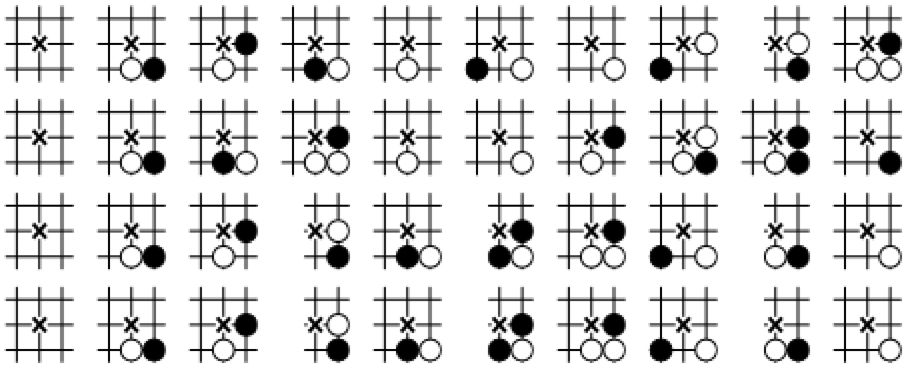


FIG. 1 – Top 10 des PageRank, de haut en bas *Gnugo* 19 × 19 (parties ordinateurs), parties amateurs 19 × 19 *U-go*(parties humaines), *Gnugo* 9 × 9 sans et avec option Monte-Carlo

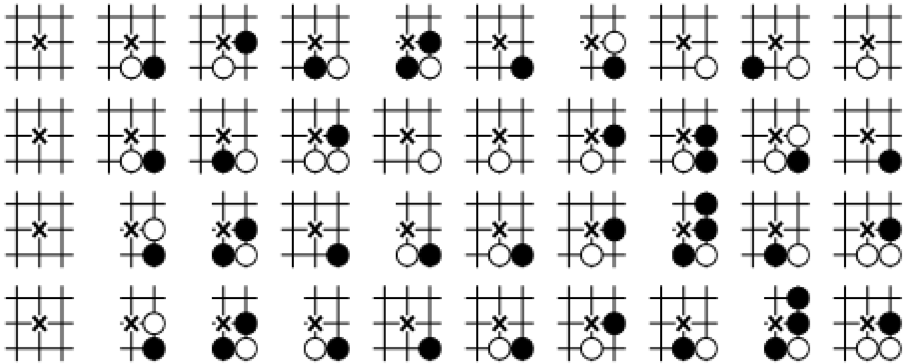


FIG. 4 – Top 10 des CheiRank, de haut en bas *Gnugo* 19 × 19 (parties ordinateurs), parties amateurs 19 × 19 *U-go*(parties humaines), *Gnugo* 9 × 9 sans et avec option Monte-Carlo

Ohter Ranking Vector: Network I

-> Difference between Monte-Carlo Gnugo and not Monte-Carlo Gnugo starting from 4th Right Eigenvector

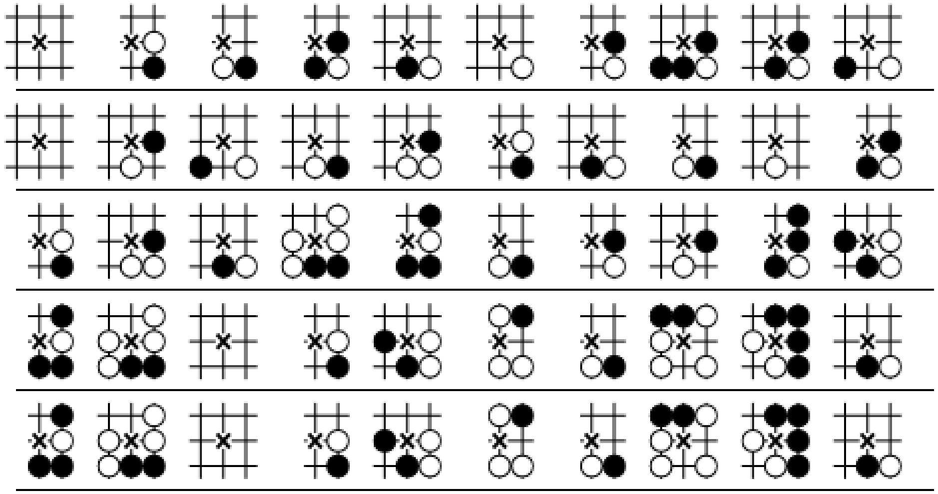


FIG. 9 – Les top 10 des plaquettes de 20 000 parties *Gnugo* 9 × 9 avec 5 autres valeurs propres, de haut en bas $\lambda_2, \lambda_3, \lambda_4, \lambda_5$ et λ_6

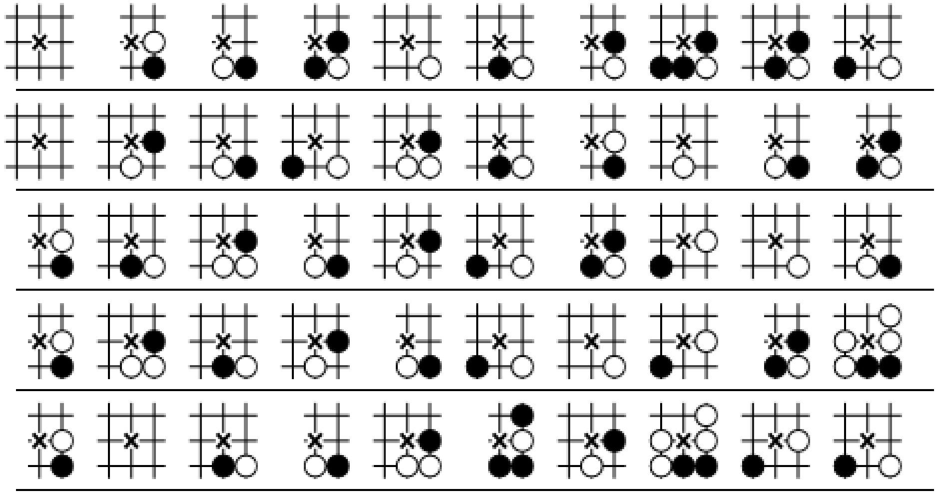


FIG. 10 – Les top 10 des plaquettes de 20 000 parties *Gnugo* 9 × 9 Monte-Carlo avec 5 autres valeurs propres, de haut en bas $\lambda_2, \lambda_3, \lambda_4, \lambda_5$ et λ_6

Spectrum for Gnugo

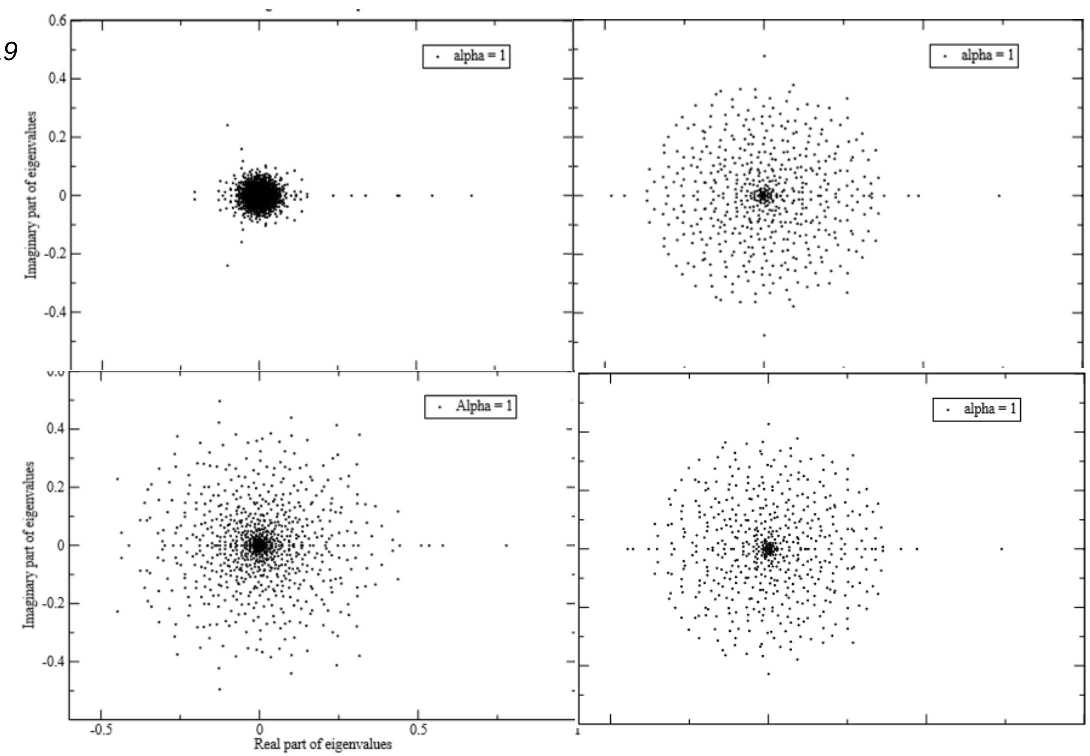
->For Gnugo Network, still gap between the first eigenvalue and next ones

->Radius of the bulk of eigenvalues changes with Computer-generated games wich is more exploded

->More structure in every spectrum from Gnugo databases

Gnugo 20 000 Games 9x9 No MC

U-go 4000 Games 19x19 (amateurs)



Gnugo 4000 Games 19x19

Gnugo 20 000 Games 9x9 MC

Histogram of Spectrum

->Density for Eigenvalues inside bulk decreases faster with human than computer-generated games

-0.65 vs -0.11

$$\frac{\#\lambda_{r_{i-1}<||\lambda||\leq r_i}}{2\pi r_i}$$

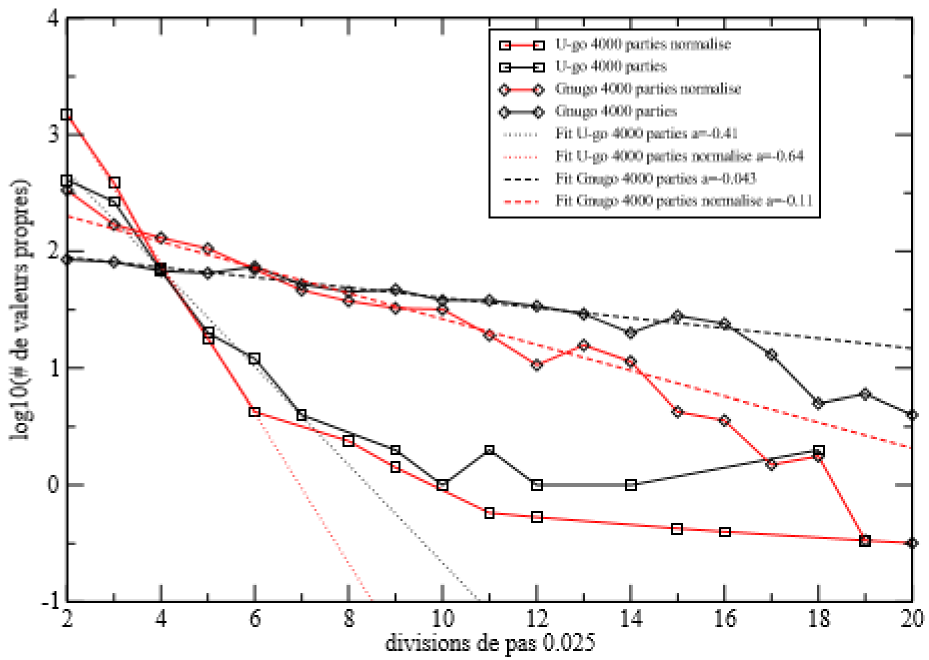


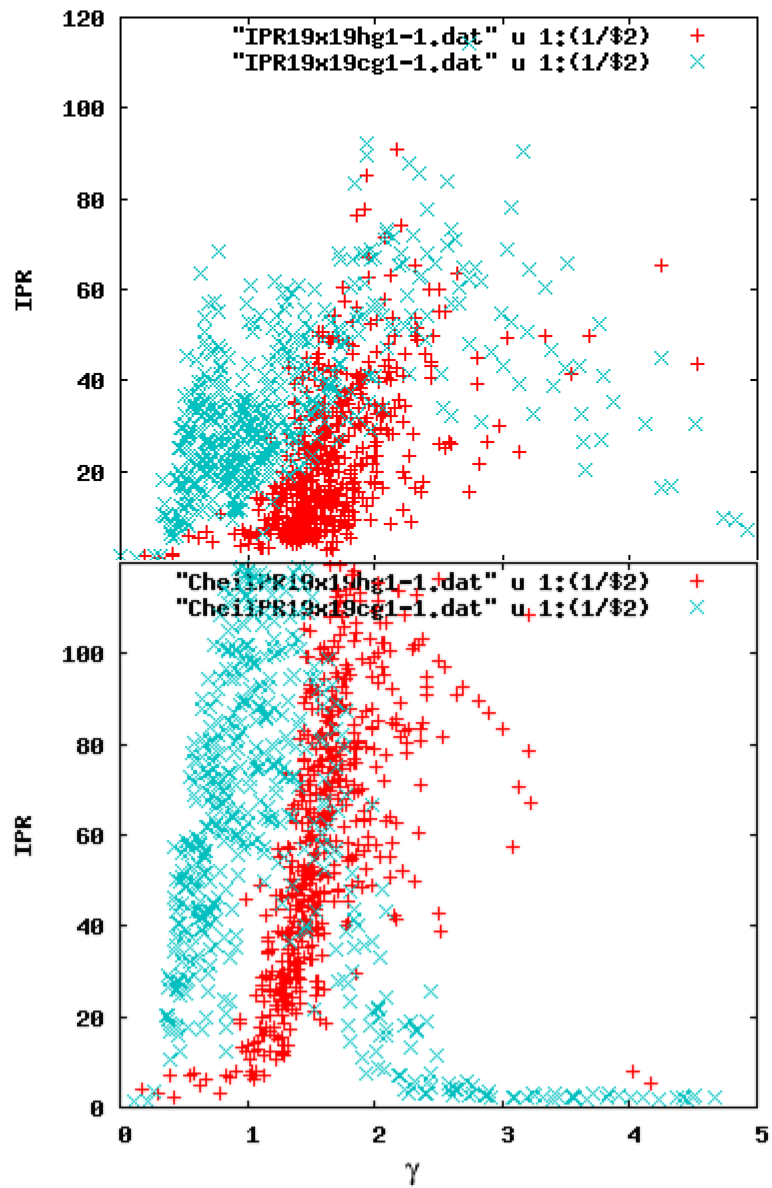
Figure 32: Plot des courbes normalisées et non normalisées pour 4000 parties *Gnugo/U-go* 19 × 19 on peut voir les pentes des courbes normalisées autour de -0.11 pour *Gnugo* et -0.65 pour *U-go*

Figure: radius from 0.05 to 0.5
square: Human
diamond: Gnugo
Red/black: Normalized/Not Normalized

Inverse Participation Ration

->Difference beetween Gnugo and Human

-> Red dots cloud (Human) shifted to the right



$$\frac{(\sum_{i=1}^{n} |\psi_i|^2)^2}{\sum_{i=1}^{n} |\psi_i|^4}$$

$$|\lambda| = e^{-2\gamma}$$

Go turing test

->What if we could distinguish human from computer players?

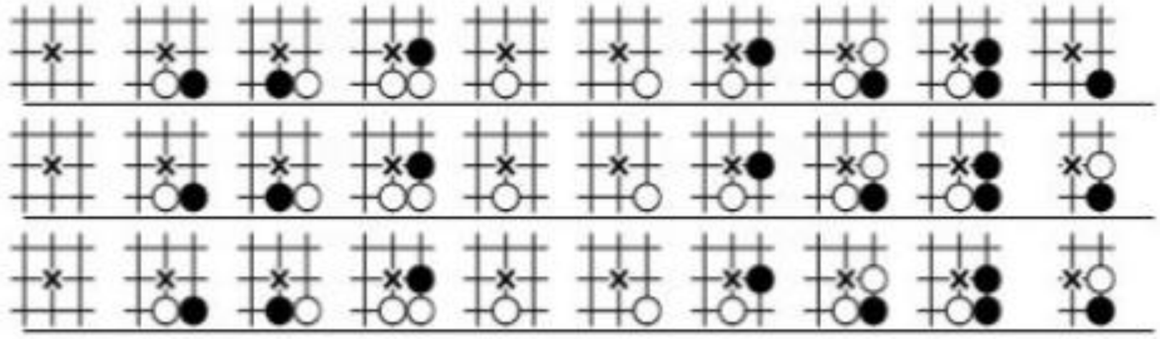


Fig. 17 – En haut le premier groupe, au milieu le second et en bas le troisième groupe de 4000 parties U-go

->We used 3x4000 games

Conclusion

->We have studied the game of go, one of the most ancient and complex board games, from a complex network perspective.

->Ranking vectors highlight specific moves which are pivotal but may not be the most common

->Preliminary results: Networks built from human games and computer-generated games show some clear differences at various levels

->Computer seems to play differently from humans

->Can we construct estimators which will allow to distinguish human from computer at go? (go Turing test)

Thank you for your attention!!

Networks for different levels of play

->The presence of handicaps means that the winner may not be the best player

-> However, the level of players is known (number of dans)

-> One can construct networks for 1d vs 1d and compare with 9d vs 9d. We look at

$$r_j = \sum_{i \leftarrow j} |k_i - k_i'| / \sum_i k_i$$

which quantifies the difference in outgoing links between two networks

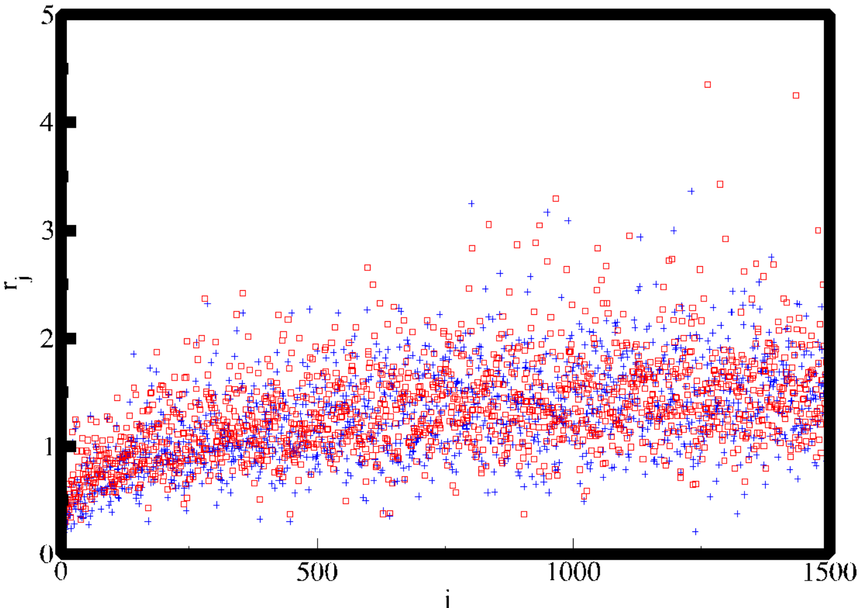


Figure: red is for 1d/1d vs 9d/9d, blue for 6d/6d Network with 193995 vertices.

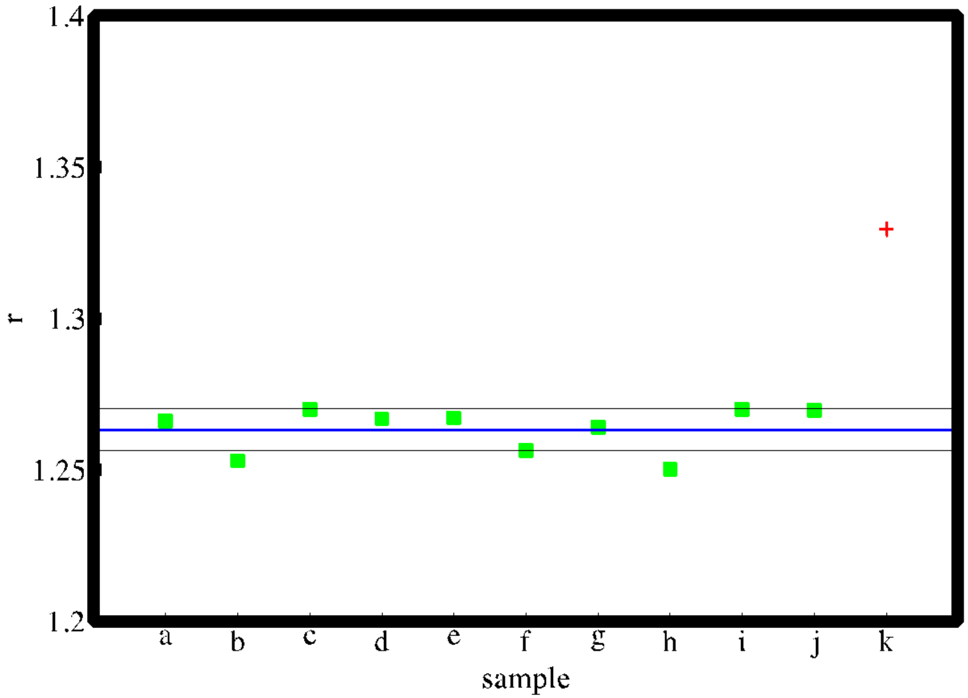
Is this difference significant?

Networks for different levels of play

-> We compared different samples of 6d/6d to the 1d/9d and computed $r = \langle r_j \rangle$ in each case

-> Result: statistically significant difference between 1d/9d and the 6d/6d samples

->Differences can be seen between the networks built from moves of players of different levels



Networks for different game phases

->One can separate the games into beginning, middle, and end
->The three networks are different, with markedly different spectra and eigenvectors

Figure: spectrum for all moves (black), 50 first moves (red), middle 50 (green) and last 50 (blue), Network with 193995 vertices.