Reduced Google Matrix

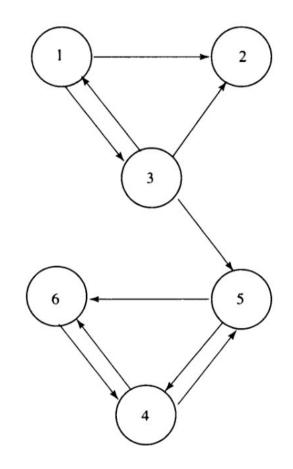
Samer EL ZANT

University of Toulouse INP-IRIT

17 May 2016

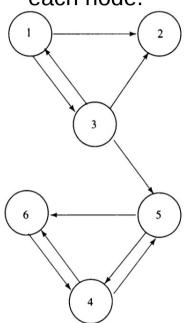
Introduction

- A network is composed of many nodes.
- Each node has his own score.
- This score is calculated by taking into consideration many factors (popularity-content-areacookies...).
- One of these factors is based on the link analysis which is the popularity or importance score.
- A node is important if it is pointed by other important pages.



Page Rank

 Page Rank method is used to calculate the rank vector or the importance of each node.

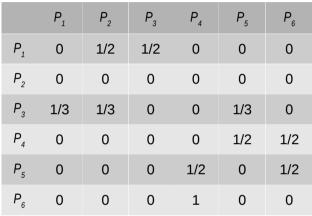


- *B_{pi}*:set of pages pointing into *P_i*.
- |P_j|:number of out-links from page P_j.
- r_{k+1}(P_i):page-rank of page P_i at iteration k+1.

$$\begin{split} r(P_i) &= \sum_{P_j \in B_{P_i}} \frac{r(P_j)}{|P_j|} \quad r_{k+1}(P_i) = \sum_{P_j \in B_{P_i}} \frac{r_k(P_j)}{|P_j|} \\ \hline r(P_i) & r_{0} & r_{0} & r_{1} & r_{1} & r_{2} & r_{2} \\ r(P_i) & r_{0} & r_{0} & r_{1} & r_{1} & r_{2} & r_{2} \\ r(P_i) & r_{0} & r_{0} & r_{1} & r_{1} & r_{2} & r_{2} \\ r(P_i) & r_{0} & r_{0} & r_{0} & r_{1} & r_{1} & r_{1} & r_{2} & r_{2} \\ r(P_i) & r_{0} & r_{0} & r_{0} & r_{1} & r_{1} & r_{1} & r_{1} & r_{2} & r_{2} \\ r(P_i) & r_{0} & r_{0}$$

Google Matrix

• Using matrices we compute a page rank vector at each iteration $\pi^{(k+1)T} = \pi^{(k)T} \mathbf{H}$



- H matrix
- H_{ij}=1/|P_i| if there is a link going from I to j and zero otherwise.
- sub-stochastic

	P_1	P_2	P ₃	P_4	P_{5}	$P_{_6}$
<i>P</i> ₁	0	1/2	1/2	0	0	0
P_2	1/6	1/6	1/6	1/6	1/6	1/6
$P_{_3}$	1/3	1/3	0	0	1/3	0
P_4	0	0	0	0	1/2	1/2
P ₅	0	0	0	1/2	0	1/2
P ₆	0	0	0	1	0	0

- S matrix
- Prevent rank sink π¹³ = [0,0,0,2/3,1/3,1/5]
- stochastic $\mathbf{S} = \mathbf{H} + \mathbf{a}(1/n \mathbf{e}^T)$

	P ₁	P ₂	$P_{_3}$	P_4	P_{5}	P ₆
P_1	1/60	7/15	7/15	1/60	1/60	1/60
P_2	1/6	1/6	1/6	1/6	1/6	1/6
P ₃	19/60	19/60	1/60	1/60	19/60	1/60
P_4	1/60	1/60	1/60	1/60	7/15	7/15
P_{5}	1/60	1/60	1/60	7/15	1/60	7/15
$P_{_6}$	1/60	1/60	1/60	11/12	1/60	1/60

- G matrix
- Google Matrix
- To guarantee convergence.
- By using α , we take into consideration the probability that a surfer follows hyper-link or a new node.

$$\mathbf{G} = \alpha \mathbf{S} + (1 - \alpha) 1/n \, \mathbf{e} \mathbf{e}^T$$

$$\boldsymbol{\pi}^{(k+1)T} = \boldsymbol{\pi}^{(k)T}\mathbf{G}$$

Reduced Google Matrix

- Describes the properties and interactions of a certain subsets of selected nodes belonging to a much larger network.
- Example: top 40 French politicians in French wikipedia. Number of nodes of frwiki = N = 1,352,825. number of selected node = $N_r = 40$.

Name	Order/ index	Name	Order/ Index	Name	Order/ Index	Name	Order/ Index
NicolasSarkozy	1	JackLang	11	JeanFrancoisCope	21	BriceHortefeux	31
FrancoisHollande	2	AlainJuppe	12	NathalieKosciuskoMorizet	22	RamaYade	32
JeanMarieLePen	3	JeanLouisBorloo	13	ArnaudMontebourg	23	PierreMoscovici	33
SegoleneRoyal	4	BertrandDelanoe	14	ClaudeBartolone	24	ManuelValls	34
JeanPierreRaffarin	5	JeanLucMelenchon	15	RachidaDati	25	ClaudeGueant	35
DominiquedeVillepin	6	MarineLePen	16	OlivierBesancenot	26	HerveMorin	36
FrancoisFillon	7	ChristineLagarde	17	NicolasDupontAignan	27	CecileDuflot	37
FrancoisBayrou	8	MartineAubry	18	EvaJoly	28	MichelSapin	38
LaurentFabius	9	DanielCohnBendit	19	ChristianeTaubira	29	HenriGuaino	39
DominiqueStraussKahn	10	ValeriePecresse	20	ElisabethGuigou	30	FlorianPhilippot	40

Reduced Google Matrix Dima and Klaus algorithm

- N_r is the number of nodes in the reduced network.
- N is the number of nodes in the whole network.
- N_s=N-N_r
- We can write $G = \begin{pmatrix} G_{rr} & G_{rs} \\ G_{sr} & G_{ss} \end{pmatrix}$
- GP = P
- The page rank vector $P = \begin{pmatrix} P_r \\ P_s \end{pmatrix}$
- As a result we obtain: $G_R P_r = P_r$, $G_R = G_{rr} + G_{rs} (\mathbf{1} G_{ss})^{-1} G_{sr}$
- G_R is the reduced Google matrix of size $N_r^*N_r$

Reduced Google Matrix Dima and Klaus algorithm

• To avoid the problem of slow convergence in G_R a slight modification can be done.

$$G_{\text{mod}} = \begin{pmatrix} \mathbf{1} & (1-\eta)U_{rs} \\ 0 & \eta \mathbf{1} \end{pmatrix} \times \begin{pmatrix} G_{rr} & G_{rs} \\ G_{sr} & G_{ss} \end{pmatrix}$$

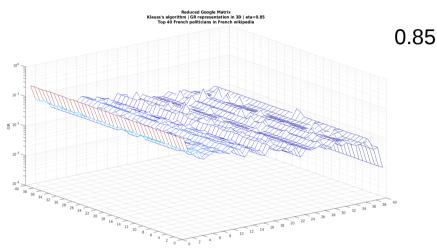
- η is an additional damping factor (0.5< η <1).
- $U_{rs} = (1/N_r)E_rE_s^T$
- E^T=(1,1,1,....,1,1,1) of size N.

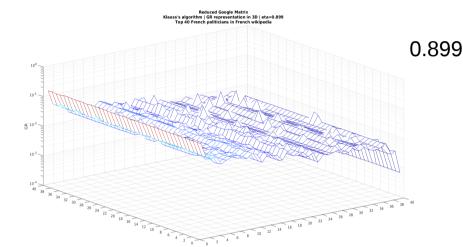
 $G_{\text{Rmod}} = G_{rr} + (1 - \eta)U_{rs}G_{sr} + \eta[G_{rs} + (1 - \eta)U_{rs}G_{ss}](1 - \eta G_{ss})^{-1}G_{sr}$

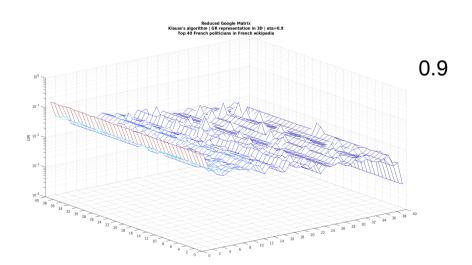
$$(\mathbf{1} - G_{ss})^{-1} = \sum_{l=0}^{\infty} G_{ss}^{l}$$

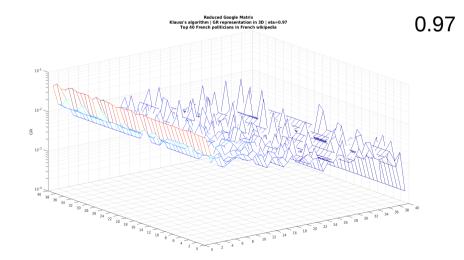
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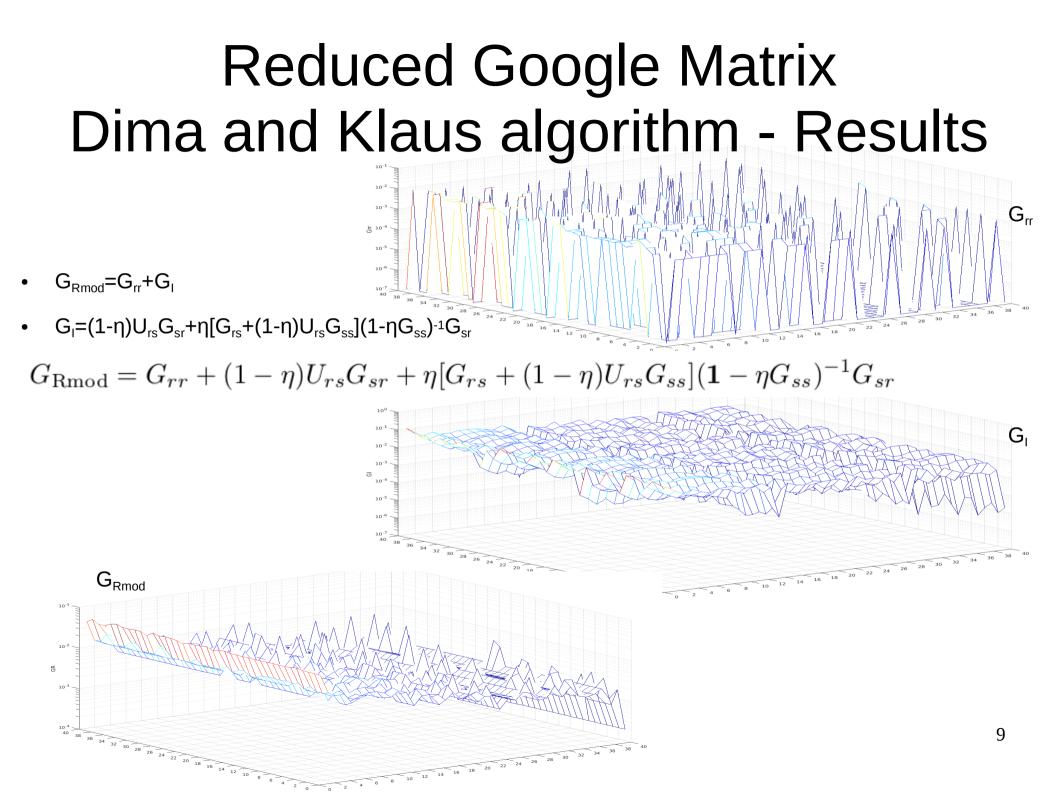
Reduced Google Matrix Dima and Klaus algorithm - Results











Reduced Google Matrix Dima and Klaus algorithm - Results

I FlorianPhilippot 0.0003348118 40 0.0008723755 40 0.0008608463 40 0.0008611847 40 0.0007874833 40 0																	
arr 0.0832713.0 2 0.0932685.3 2 0.0322685.3 2 0.0 0 0 0 arr 0.09553575.3 0.09567477 4 0.05583475 1 1 1 1 1 1 0 0 arr 0.05583575 1 0.0558479 1 0.0558479 1 0.0558479 1 0.0558479 1 0 0.0558479 1 0 0.0558479 1 0 0.0558479 1 0.055847073 7 0.05467074 1 0 0.055847073 1 0.041141411 1 0.041243471 1 <th< th=""><th>Name</th><th>frwiki_norm</th><th>pg_order</th><th>Eta_0.85</th><th>Eta_85</th><th>Eta_0.9</th><th>Eta_9</th><th>Eta_0.899</th><th>Eta_899</th><th>Eta_0.97</th><th>Eta_97</th><th>fr_85</th><th>fr_9</th><th>fr_899</th><th>fr_97</th><th>85_97</th><th>9_97</th></th<>	Name	frwiki_norm	pg_order	Eta_0.85	Eta_85	Eta_0.9	Eta_9	Eta_0.899	Eta_899	Eta_0.97	Eta_97	fr_85	fr_9	fr_899	fr_97	85_97	9_97
Operator Operator A Operator	8 NicolasSarkozy	0.193771494	1	0.1929990298	1	0.1917553425	1	0.1917918248	1	0.1838686995	1	0	0	0	0	0	0
SegmentPayl 0.05699778 4 0.0509778 3 0.05099778 4 0.05199978 1 1 1 1 </td <td>6 FrancoisHollande</td> <td>0.0663077616</td> <td>2</td> <td>0.0639258053</td> <td>2</td> <td>0.0639188775</td> <td>2</td> <td>0.0639190617</td> <td>2</td> <td>0.063906395</td> <td>2</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td>	6 FrancoisHollande	0.0663077616	2	0.0639258053	2	0.0639188775	2	0.0639190617	2	0.063906395	2	0	0	0	0	0	0
Description Description <thdescription< th=""> <thdescription< th=""></thdescription<></thdescription<>	8 JeanMarieLePen	0.0555352675	3	0.0508677407	4	0.0505696593	4	0.0505784552	4	0.0485939435	4	1	1	1	1	0	0
Domingue/Wiley/D 0.023207563 6 0.035450203 7 0.0354702130 7 0.0354702130 7 0.0354702130 7 0.0354702130 7 0.0354702130 7 0.0354702130 7 0.0354702130 8 0.041344597 5 2 <th2< th=""> 1<</th2<>	3 SegoleneRoyal	0.0549899728	4	0.0530477781	3	0.0532011062	3	0.0531966035	3	0.0541834108	3	1	1	1	1	0	0
December Deltary Cr31 P Deltary Cr31 S Deltary Cr31	7 JeanPierreRaffarin	0.0431100728	5	0.0276338479	13	0.0276440214	14	0.0276437251	14	0.0277033643	14	8	9	9	9	1	0
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Jumer Fahas D. 0247/35/1 J D. 0244/35/4 JU D. 0244/35/4 JU D. 0244/35/1 JU D. 0234/05/1 JU D. 0244/05/1 JU D. 0214/05/1 JU JU JU JU JU JU	2 FrancoisFillon	0.0418714731	7	0.041064781	5	0.0411440093	5	0.0411416911	5	0.0416348587	5	2	2	2	2	0	0
Demomspatianas/Ham 0.047713749 10 0.02884292 11 0.02884092 11 0.039971089 11 0.039971089 11 0.039971089 11 0.039971089 11 0.039971089 11 0.039971089 11 0.039971089 11 0.039971089 10 0.038971071 11 0.039971089 10 0.038971071 11 0.017972442 12 0.03971089 14 0.018098577 14 0.018098577 14 0.018098577 14 0.028705759 14 0.02820571 15 0.028026571 15 0.028026571 15 0.028010519 16 0.022110555 15 0.0260571 0.028010519 16 0.022110555 15 0.022410555 15 0.02485189 21 0.018091102 0 0.01907102 13 0.02889377 12 6 6 6 0 0 0.027107569 14 0.027107567 13 0.02849171 10 0.018491787 12 0.018491899 12 0.018498199 12 14	8 FrancoisBayrou	0.037068743	8	0.0348237858	8	0.0349333024	8	0.0349300998	8	0.0356115109	8	0	0	0	0	0	0
backLang 0.031928182 11 0.0391170222 6 0.0390972257 6 0.039072577 6 0.039072577 6 0.039072577 6 0.039072577 6 0.039072577 7 2 3 5 5	5 LaurentFabius	0.0352170941	9	0.0344733076	9	0.0344942584	10	0.0344935971	10	0.0347050148	10	0	1	1	1	1	0
Daminuppe D0331147 12 D034004002 D0 0.034004003 9 D0358967625 7 2 3 3 3 2 SharkLassBotione 0.025975599 14 0.01769571 19 0.01769577 18 0 5 5 6 1 0 SharkLassBotione 0.022970756 15 0.026405767 15 0.02440576 15 0.02440576 15 0.02440576 15 0.02460576 15 0.02460576 15 0.02450469 15 0	9 DominiqueStraussKahn	0.0347213746	10	0.0285822134	12	0.0288540928	11	0.0288461033	11	0.0306013503	11	2	1	1	1	1	0
Dest. Dest. <th< td=""><td>BJackLang</td><td>0.0331928182</td><td>11</td><td>0.0391170232</td><td>6</td><td>0.0390972257</td><td>6</td><td>0.0390978088</td><td>6</td><td>0.0389695952</td><td>6</td><td>5</td><td>5</td><td>5</td><td>5</td><td>0</td><td>0</td></th<>	BJackLang	0.0331928182	11	0.0391170232	6	0.0390972257	6	0.0390978088	6	0.0389695952	6	5	5	5	5	0	0
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Darkuz/Melenchon 0.0229707059 15 0.023940676 15 0.0240643172 15 0.0240647267 15 0.0240540999 15 0	5 JeanLouisBorloo	0.0254923511	13	0.0177592751	19	0.017805316	18	0.0178039574	18	0.0181095857	18	6	5	5	5	1	0
MarineLePen 0.0191072018 16 0.022012036 16 0.022015656 16 0.022922175 16 0 0	9 BertrandDelanoe	0.0237575599	14	0.0288258571	11	0.0287734542	12	0.0287749688	12	0.0284789401	13	3	2	2	1	2	1
ChristineLagarde 0.013421153 17 0.0134271788 23 0.013467015 23 0.013467015 23 0.01346819 23 0.013467015 23 0.01346819 23 0.013467015 23 0.01346819 23 0.013467015 23 0.027677692 13 0.027677692 13 0.027677692 13 0.027677692 12 4 5 6 2 1 DanielColmBendit 0.01399352 10 0.0151191045 21 0.015081499 22 1 1 1 2 1 1 2 1 1 2 1 1 2 1 1 2 1 1 2 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	7 JeanLucMelenchon	0.0229707059	15	0.0239460576	15	0.0240683172	15	0.0240647267	15	0.0248504999	15	0	0	0	0	0	0
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DanielCohnBendit 0.017995352 19 0.017627235 19 0.0176649 19 0.018694676 20 1 0 0 1 2 11 ValentePacresse 0.0153641952 20 0.015191045 21 0.01504577 17 0.0167460577 17 0.01973454 17 4 4 4 4 0 0 DamFrancisCope 0.014448619 21 0.016805377 17 0.018760577 17 0.018760577 19 3 3 4 1 1 AmauMontebourg 0.0118055277 23 0.016871678 26 0.017049528 20 0.017099252 20 0.018306101 19 8 3 4 1 1 ClaudeBartone 0.0111906433 24 0.00167941217 37 0.0078917415 36 13 13 13 12 1 1 1 1 1 1 1 1 1 1 1 1 1 1	7 ChristineLagarde	0.0182841153	17	0.0134271798	23	0.0134616946	23	0.0134607015	23	0.0136483819	23	6	6	6	6	0	0
DanielCohnBendit 0.017995352 19 0.017627235 19 0.0176649 19 0.018694676 20 1 0 0 1 2 11 ValentePacresse 0.0153641952 20 0.015191045 21 0.01504577 17 0.0167460577 17 0.01973454 17 4 4 4 4 0 0 DamFrancisCope 0.014448619 21 0.016805377 17 0.018760577 17 0.018760577 19 3 3 4 1 1 AmauMontebourg 0.0118055277 23 0.016871678 26 0.017049528 20 0.017099252 20 0.018306101 19 8 3 4 1 1 ClaudeBartone 0.0111906433 24 0.00167941217 37 0.0078917415 36 13 13 13 12 1 1 1 1 1 1 1 1 1 1 1 1 1 1		0.0174884028	18	0.0275979108	14	0.0277626474	13	0.0277577692	13	0.0288839637	12	4	5	5	6	2	1
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JeanFrancoisCope 0.0141488119 21 0.0185537971 17 0.0187464977 17 0.019734654 17 4 4 4 4 0 0 NamaleKoscuskoMorzet 0.011838329 22 0.0117446455 26 0.011820452 26 0.0122916426 25 4 4 4 3 1 1 SAmauMontebourg 0.011835277 23 0.00748578 20 0.0170445588 20 0.0170399252 20 0.008015501 19 3 3 4 1 1 CalcadeBartone 0.0114906493 24 0.0074862016 37 0.0748588 37 0.01997705 25 0.013112315 26 0 0 0 1 1 1 1 0 1 1 1 1 1 1 0 0.0197705 25 0.013112315 26 0 0 0 1 1 1 1 1 1 0 0 0 1 1 1 0 0.01312015 31 0.010433344 20 0.014814817	2 ValeriePecresse	0.0153641952	20	0.0151191045	21		21	0.0150405347	21	0.0144939269		1	1	1	2	1	1
2hathaleKosciuskoMorizet 0.0118838329 22 0.0117446485 26 0.011822461 26 0.012916426 25 4 4 4 3 1 ShmauMontebourg 0.0116355277 23 0.010687578 20 0.017045528 20 0.01039522 20 0.0180691501 19 3 3 4 1 1 ClaudeBattolone 0.0114906493 24 0.007425016 37 0.00748588 37 0.007448121 37 0.007917415 36 13 13 12 1 <t< td=""><td>8 JeanFrancoisCope</td><td>0.0141486119</td><td>21</td><td>0.0185537971</td><td>17</td><td>0.0187504537</td><td>17</td><td>0.0187446977</td><td>17</td><td>0.0199734654</td><td>17</td><td>4</td><td>4</td><td>4</td><td>4</td><td>0</td><td>0</td></t<>	8 JeanFrancoisCope	0.0141486119	21	0.0185537971	17	0.0187504537	17	0.0187446977	17	0.0199734654	17	4	4	4	4	0	0
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a ClaudeBartolone 0.0114906493 24 0.0074262016 37 0.007485888 37 0.0074841217 37 0.0078917415 36 13 13 12 1 1 ClaudeBartolone 0.0111763099 25 0.010477246 25 0.011974444 25 0.0104780254 27 0.008816562 31 1	5 ArnaudMontebourg	0.0116355277	23		20	0.0170445528	20	0.0170399252	20	0.0180691501		3	3	3	4	1	1
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Nicolas DupontAignam 0.0098456554 27 0.0104533848 29 0.0104522053 28 0.010416279 28 2 1	7 OlivierBesancenot	0.0111050871	26		27	0.0104749625	27		27	0.0098165562		1	1	1	5	4	4
EvaJoly 0.0096459119 28 0.0141364919 22 0.014196269 22 0.0145929345 21 6 6 6 7 1 1 3 ChristianeTaubira 0.0091570795 29 0.008792094 33 0.0086754519 34 0.0086788752 34 0.0079332504 35 4 5 5 6 2 1 4 ElisabethGuigou 0.009108003 30 0.0096610899 31 0.0097517209 31 0.0097496297 31 0.012324093 29 1 1 1 2 2 5 BriceHortefeux 0.008100505 31 0.0128026652 24 0.0128080261 24 0.0128107662 24 7 7 7 7 7 7 0 0 2	3 NicolasDupontAignan		27		29	0.0104522053	28		28	0.0104166279		2	1	1	1	1	0
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I FlorianPhilippot 0.0003348118 40 0.0008723755 40 0.0008608463 40 0.0008611847 40 0.0007874833 40 0												0		~		0	0 1
102 102 102 122 42 30												0	0	0	0	0	
	an isinan mippor	0.000000010110	TV	0.0000720100	40	0.00000000000		0.000011047	-10	0.0001014000		102	102	102	122	42	30
												2.55					0.75

Reduced Google Matrix Leo's algorithm

 Leo's proposal defined a reduced google matrix in dimension r+1 as follows

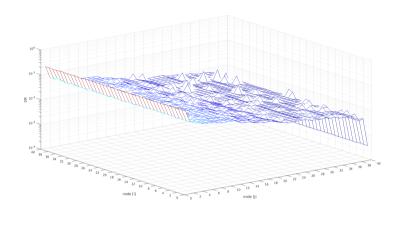
$$\tilde{G}\tilde{P} = \begin{pmatrix} \tilde{G}_{rr} & \tilde{G}_{r1} \\ \tilde{G}_{1r} & \tilde{G}_{11} \end{pmatrix} \begin{pmatrix} \tilde{P}_r \\ \tilde{P}_1 \end{pmatrix} = \begin{pmatrix} \tilde{P}_r \\ \tilde{P}_1 \end{pmatrix}$$

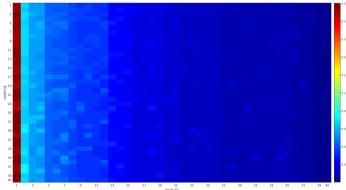
$$\tilde{G}_{1r} = 1 - \sum_{rows} G_{rr} = \sum_{rows} G_{sr} \qquad \tilde{P}_r = P_r \qquad \tilde{G}_{rr} = G_{rr} \qquad \tilde{P}_1 = \sum P_s = 1 - \sum P_r$$

$$\tilde{G}_{r1} = \frac{1}{\tilde{P}_1} G_{rs} P_s$$

$$G_{R}^{E} = G_{rr} + \frac{1}{1 - \tilde{G}_{11}} \tilde{G}_{r1} \tilde{G}_{1r}$$

Reduced Google Matrix Leo's algorithm - Results





Order\Index	Name	Order\Index	Name
1	NicolasSarkozy	21	JeanFrancoisCope
2	FrancoisHollande	22	NathalieKosciuskoMorizet
3	JeanMarieLePen	23	ArnaudMontebourg
4	SegoleneRoyal	24	ClaudeBartolone
5	JeanPierreRaffarin	25	RachidaDati
6	DominiquedeVillepin	26	OlivierBesancenot
7	FrancoisFillon	27	NicolasDupontAignan
8	FrancoisBayrou	28	EvaJoly
9	LaurentFabius	29	ChristianeTaubira
10	DominiqueStraussKahn	30	ElisabethGuigou
11	JackLang	31	BriceHortefeux
12	AlainJuppe	32	RamaYade
13	JeanLouisBorloo	33	PierreMoscovici
14	BertrandDelanoe	34	ManuelValls
15	JeanLucMelenchon	35	ClaudeGueant
16	MarineLePen	36	HerveMorin
17	ChristineLagarde	37	CecileDuflot
18	MartineAubry	38	MichelSapin
19	DanielCohnBendit	39	HenriGuaino
20	ValeriePecresse	40	FlorianPhilippot

- It has the same order of pagerank vector for whole network.
- The problem with this proposal is that the pagerank of whole network has full influence on the results.

Conclusion

 Klaus's algorithm of reduced Google matrix analyzes effective interactions between a subset of selected nodes belonging to a much larger network taking into consideration the indirect links via whole network.

Thank you