Resonant Tunneling in a Dissipative Environment: Quantum Critical Behavior

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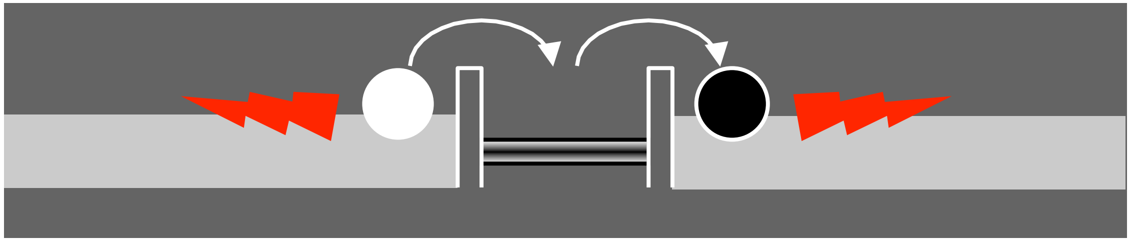
U.S. DEPARTMENT OF ENERGY

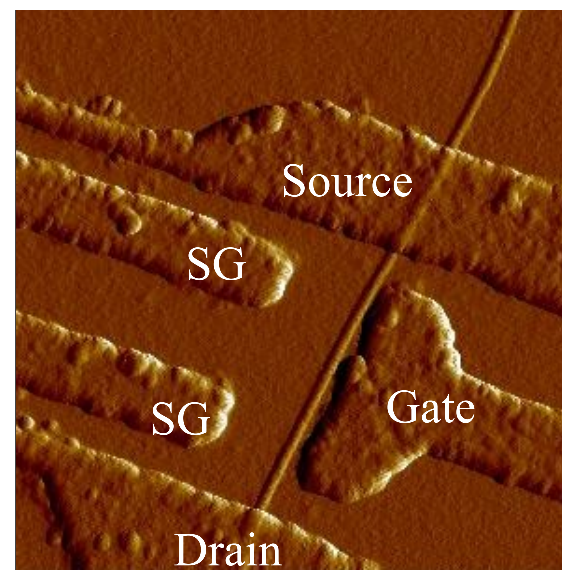


1. Quantum mechanics + dissipation: open system

- resonant tunneling in a dissipative electromagnetic environment
- dissipation + symmetry of coupling → competition → QPT

2. Quantum phase transition (QPT)

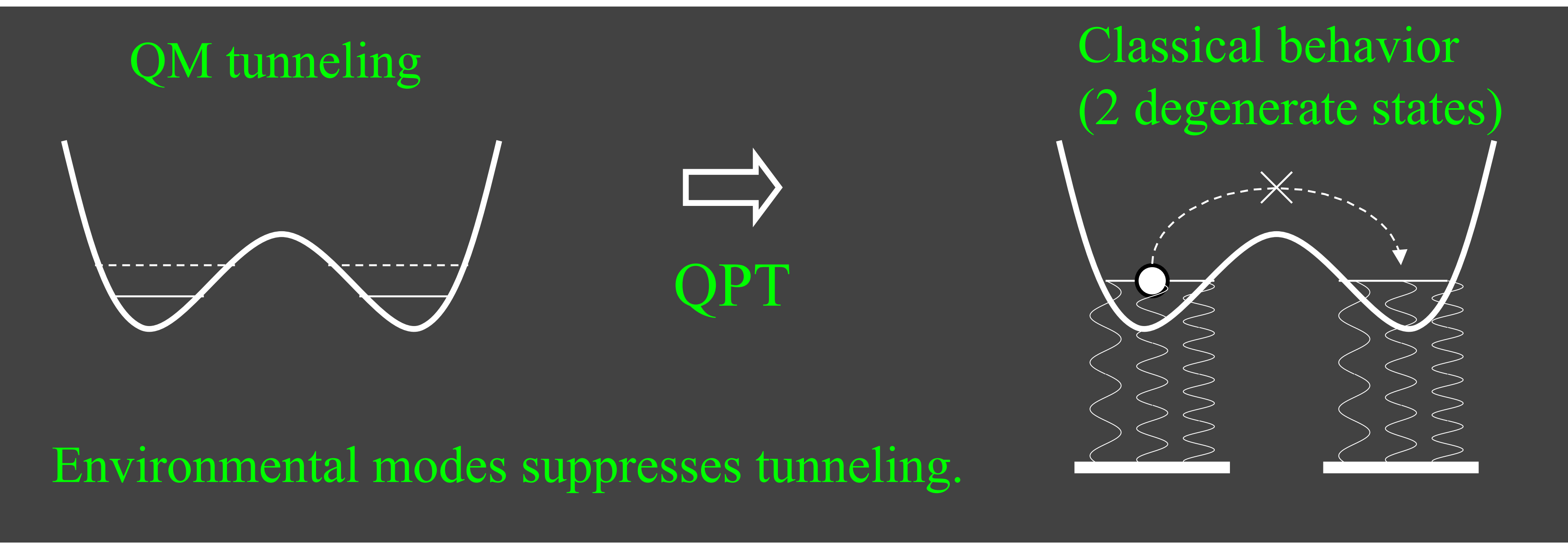
- change in ground state upon varying a parameter
- exotic state of matter at the critical point
- non-equilibrium properties??



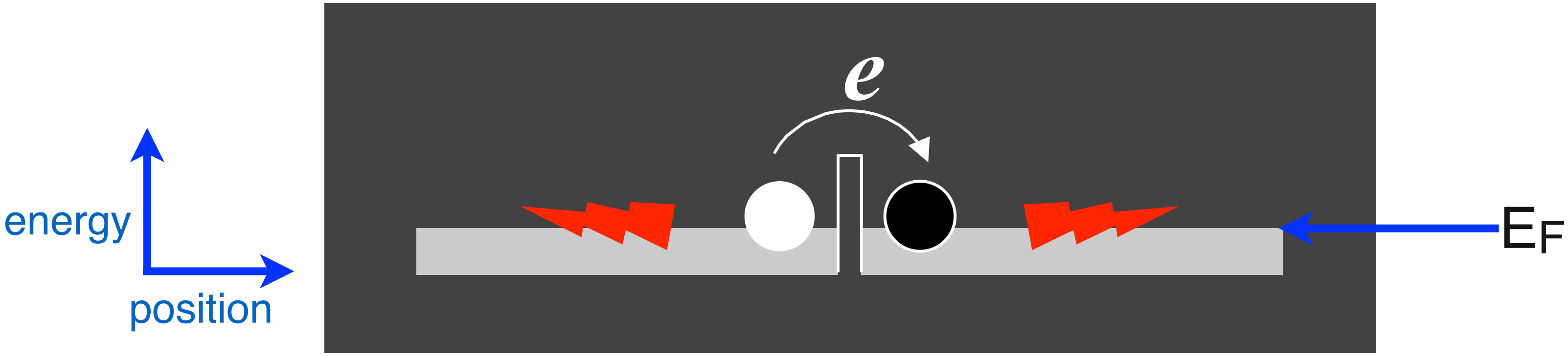
Quantum Mechanics + Environment

Tunneling with dissipation:

- environment as a collection of oscillators-- a “bosonic bath”
  [Feynman & Vernon, 1963]
- spin-boson model: 2 states + bosonic environment
  [Leggett, Dorsey, Fisher, Garg & Zwerger, RMP 1987]



In Quantum Transport Expt?: “Environmental Coulomb Blockade”



After tunneling event, spreading of charge inhibited by environment

→ Coulomb interaction leads to a charging energy

→ blocks (suppresses) tunneling of electron

[Reviews: Devoret,Esteve,Urbina LesHouches 95, Ingold&Nazarov 92,
Flensberg PhysicaScripta 91]

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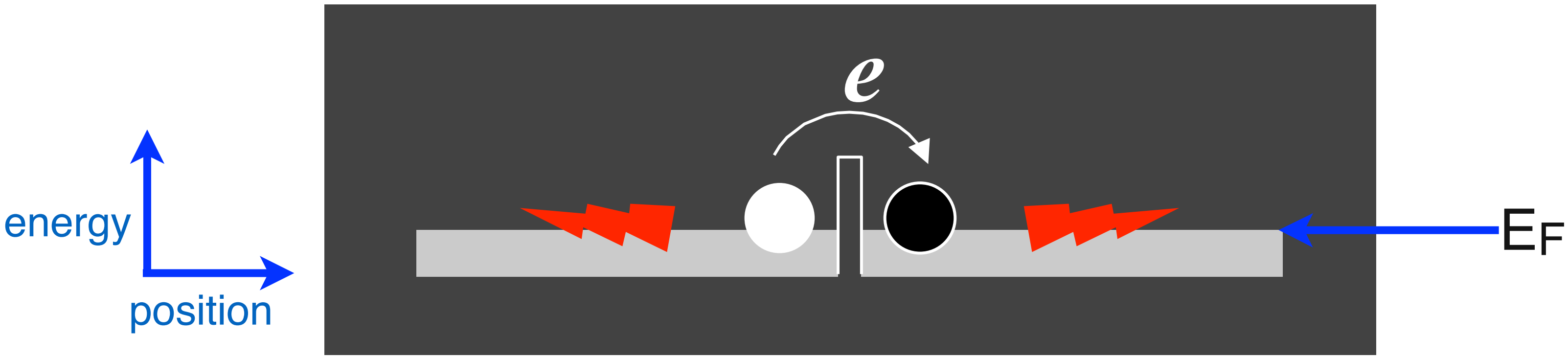

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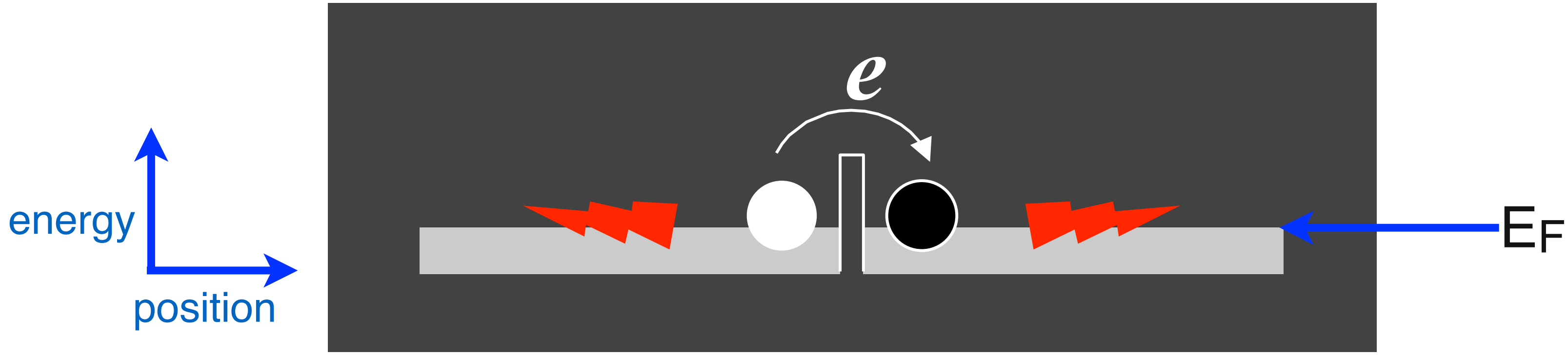

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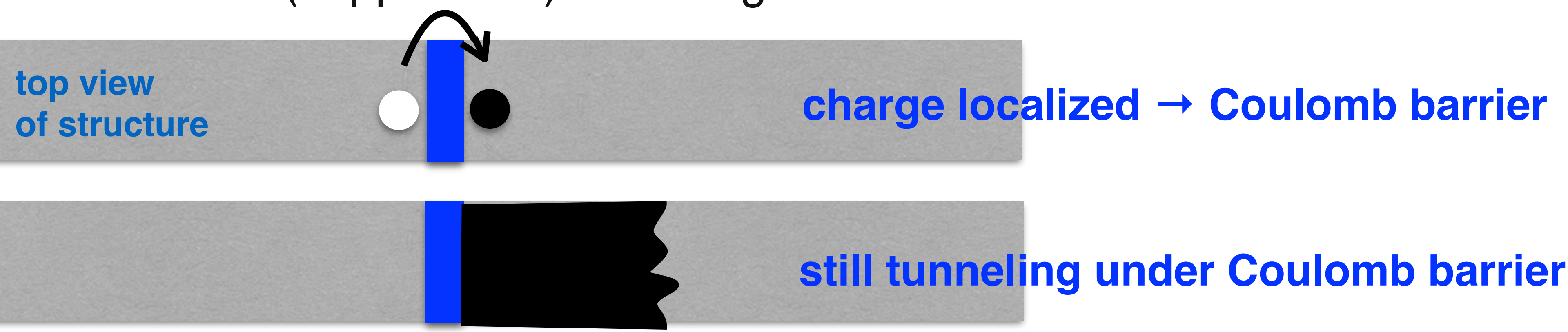

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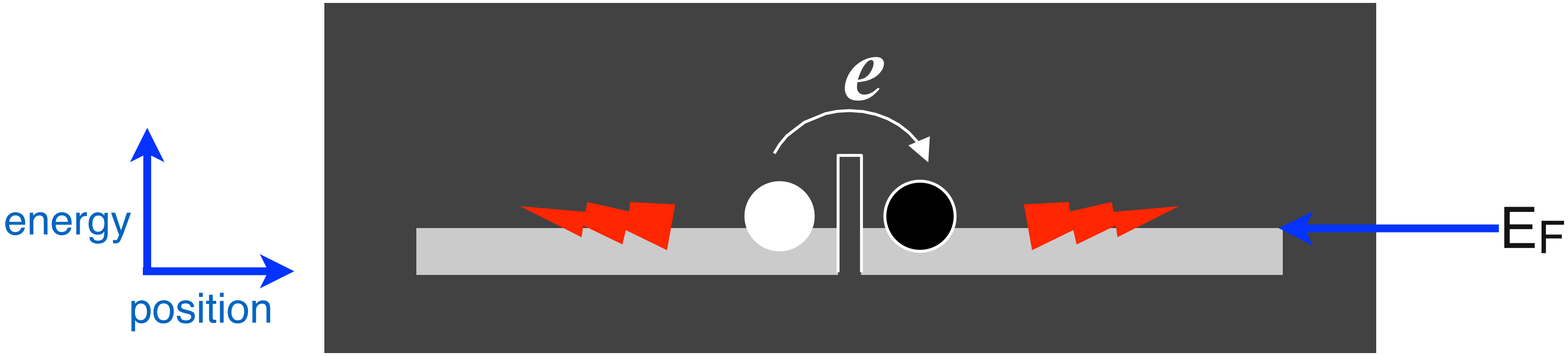

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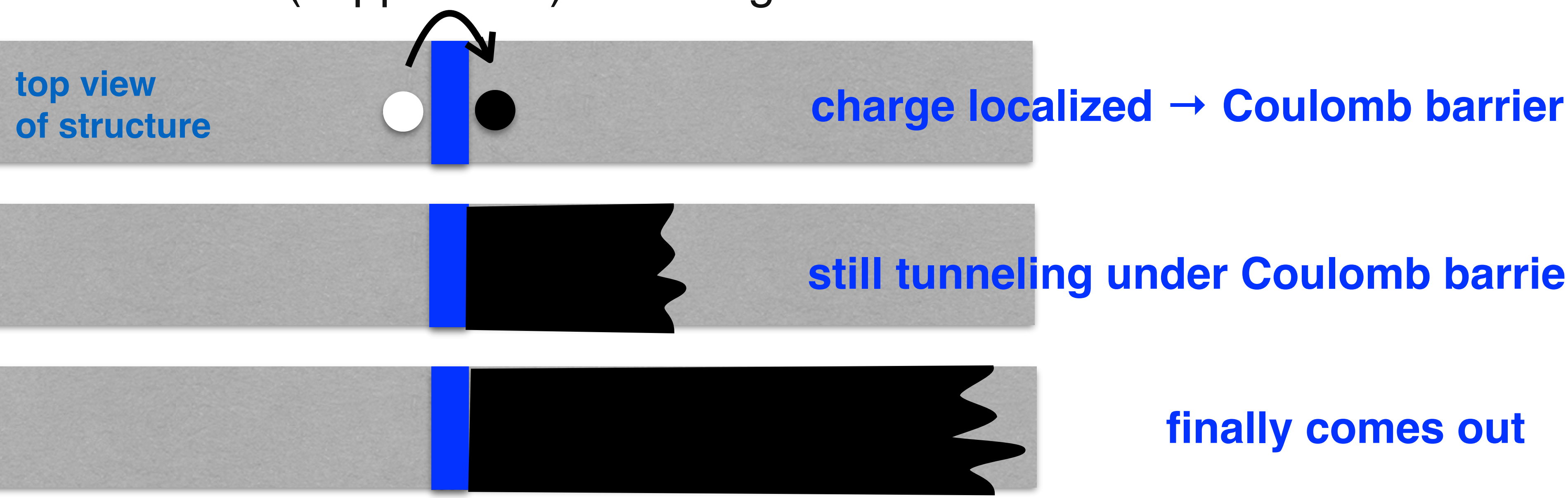

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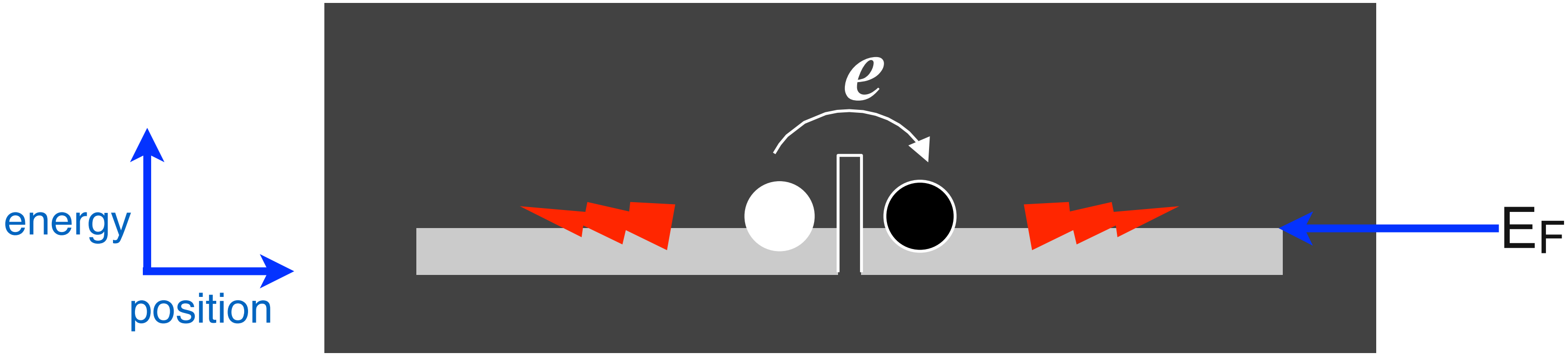

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→ Coulomb interaction leads to a charging energy

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measured observable: (differential) conductance, $G \equiv \frac{dI}{dV}$

$$G \propto V^{2r} \text{ with } r \equiv \frac{e^2}{h} R_{\text{leads}} \quad (\approx 0.75 \text{ here})$$

$$\left(I \propto V^{2r+1}\right)$$

[Reviews: Devoret,Esteve,Urbina LesHouches 95, Ingold&Nazarov 92, Flensberg PhysicaScripta 91]

Outline

1. Experiment

- carbon nanotube q.dot
- dirty metal leads $\rightarrow R \sim \frac{h}{e^2}$
- B=6T $\Rightarrow$ “spinless”

2. Theory of approach to quantum critical point

- map to interacting 1D model-- a Luttinger liquid
- power laws from scaling at strong and weak coupling
- amazing consistency with experiment!

3. Model of quantum critical system/state

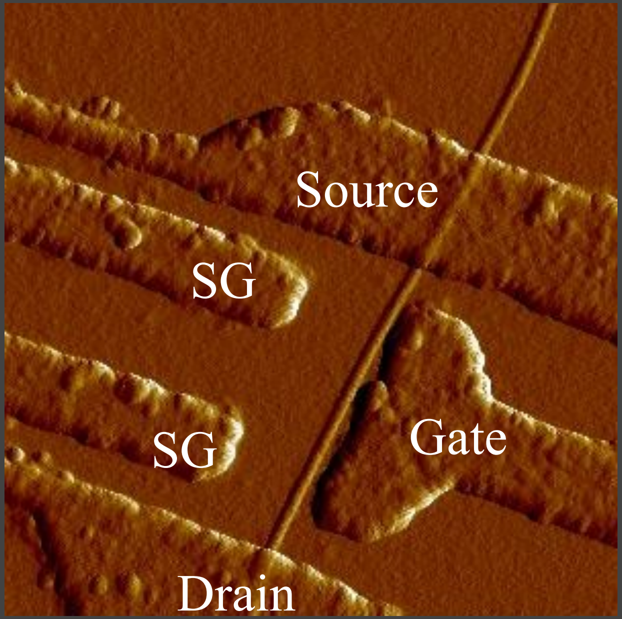
- introduce Majorana fermion representation
- QCP described by a decoupled zero-mode Majorana
- indirect experimental signature of Majorana: linear T dependence

Experimental System: Carbon Nanotube Quantum Dot

Gleb Finkelstein group: H. Mebrahtu, I. Borzenets, Y. Bomze, A. Smirnov, GF

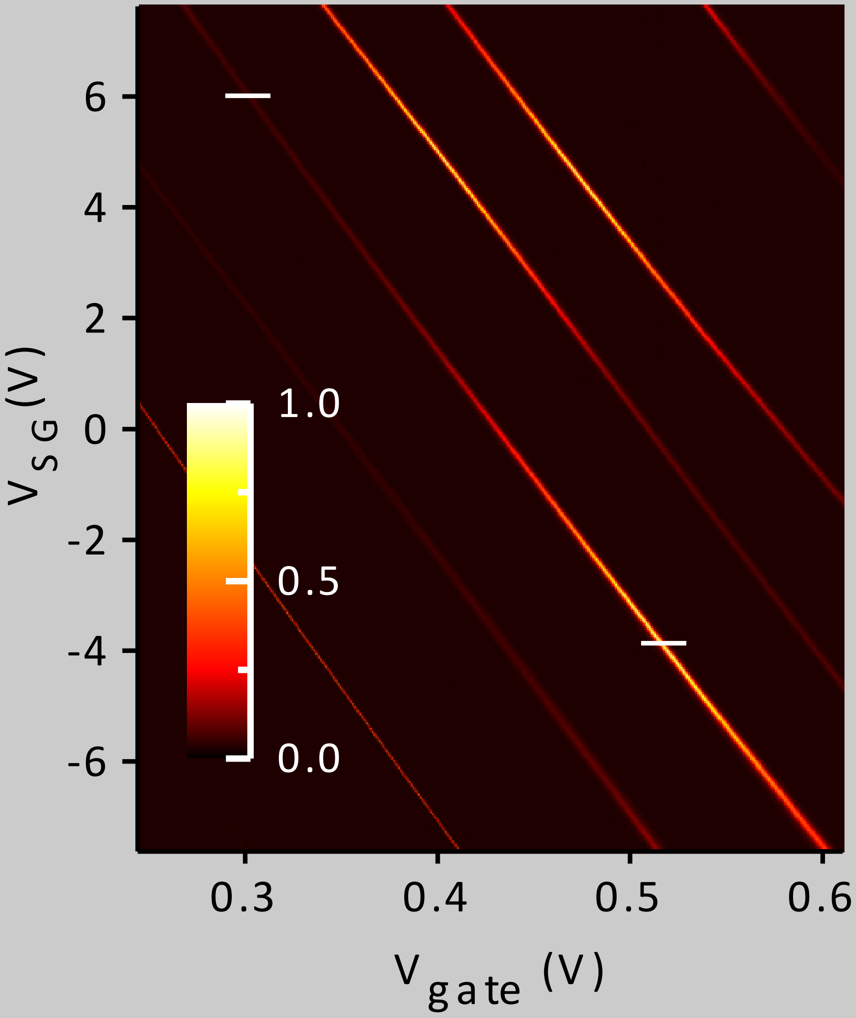
Short carbon nanotube (CNT) quantum dot (300 nm) connected to resistive leads via tunable tunnel barriers.

Sample: tuning the coupling asymmetry by the side gate

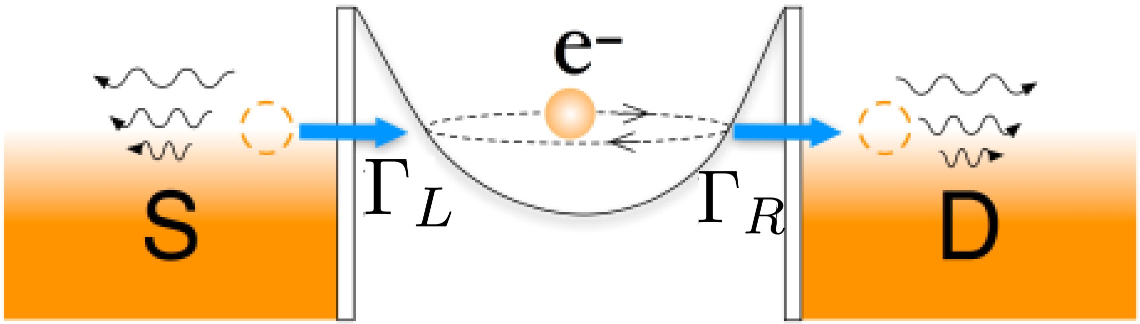


B=6 T (spinless case)

Conductance, in units of e^2/h



Resonant Tunneling



Ignore environment for now:
Tunneling through a double barrier → resonances (sharp)

$$T = \frac{4\Gamma_L\Gamma_R}{(\Delta\epsilon)^2 + (\Gamma_L + \Gamma_R)^2}$$

can tune
$\Delta\epsilon,\ \Gamma_L,\ \Gamma_R$

Symmetric coupling + on resonance → <u>perfect transmission</u>

Conductance is Transmission!
(Landauer viewpoint)

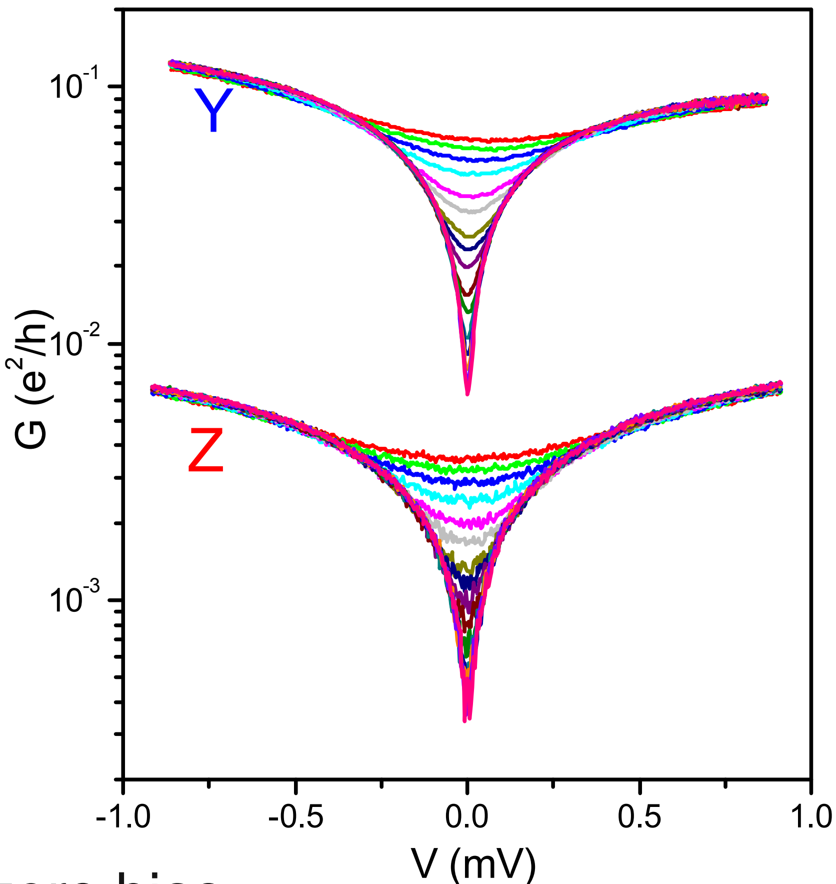
$$G = \frac{e^2}{h}T$$

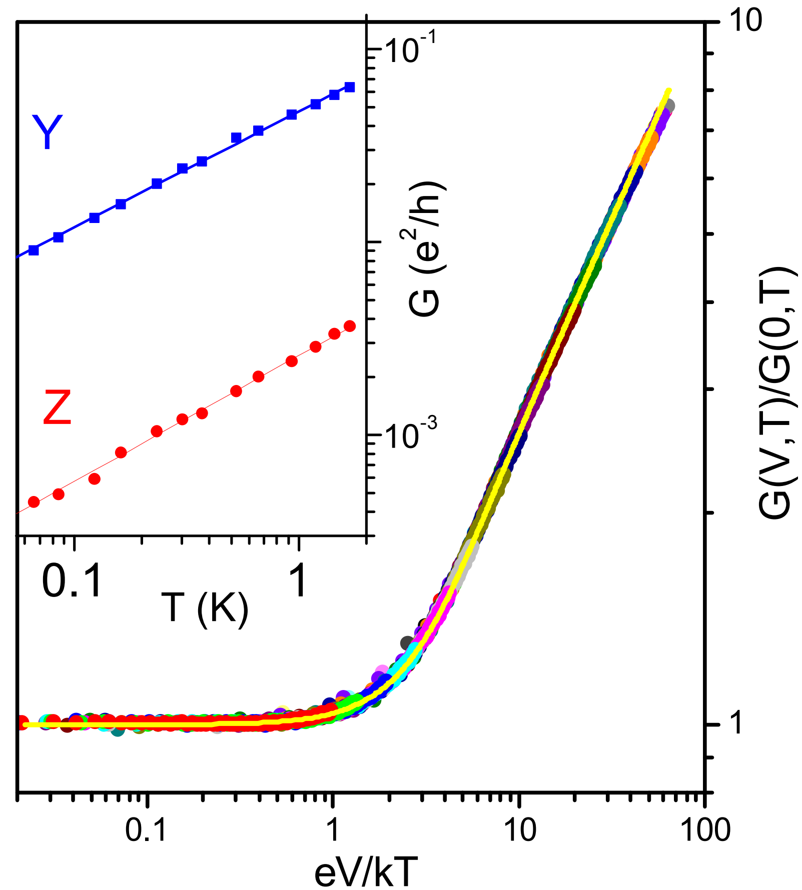
Now connect the environment-- what happens? is *T* suppressed?

B=6 T (<u>spinless case</u>)

Preliminary: Environmental Coulomb Blockade

Conductance far away from resonance → single barrier case

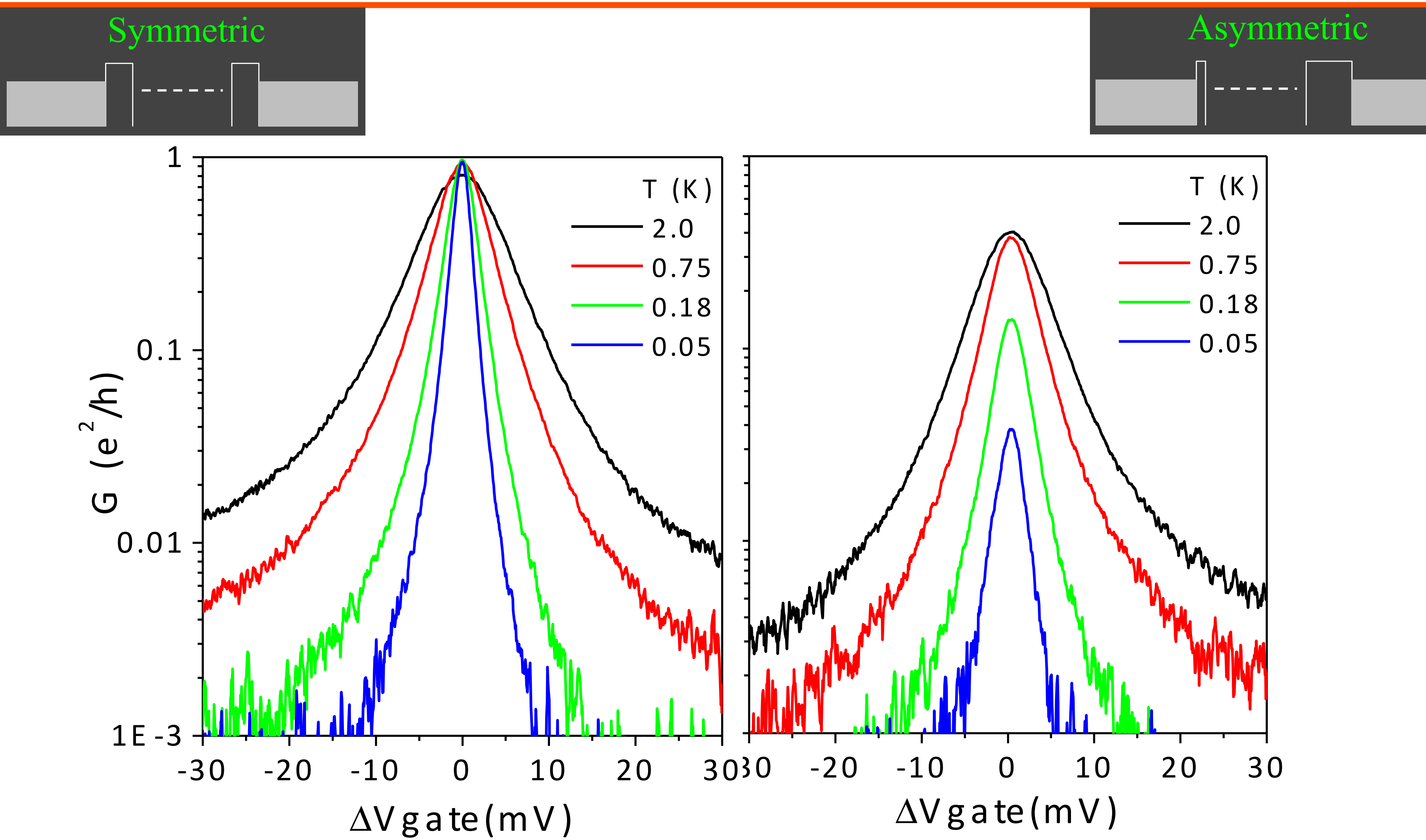




"zero bias anomaly"

$$G \sim \left(\max\{eV, k_B T\}\right)^{2r} \text{ with } r \equiv \frac{e^2}{h} R_{\text{leads}} \quad (\approx 0.3 \text{ here})$$

Conductance Resonance: Symmetric vs. Asymmetric Coupling

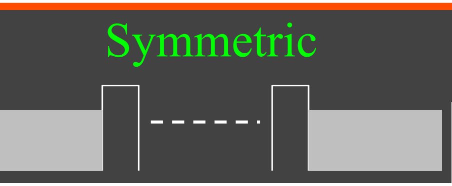


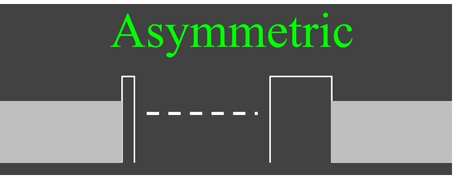
$$R_{\text{expt.}} = 0.75(h/e^2)$$

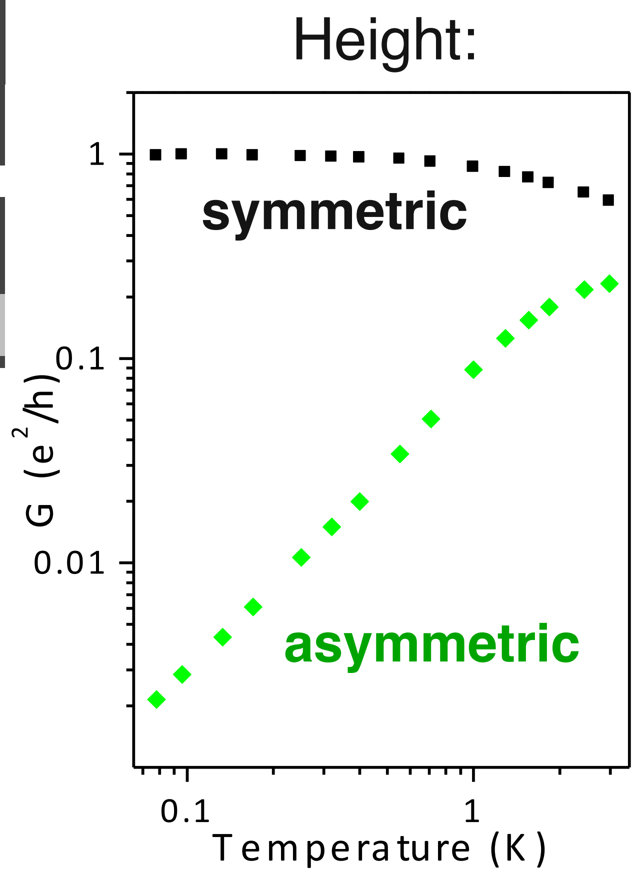
B=6 T (spinless case)

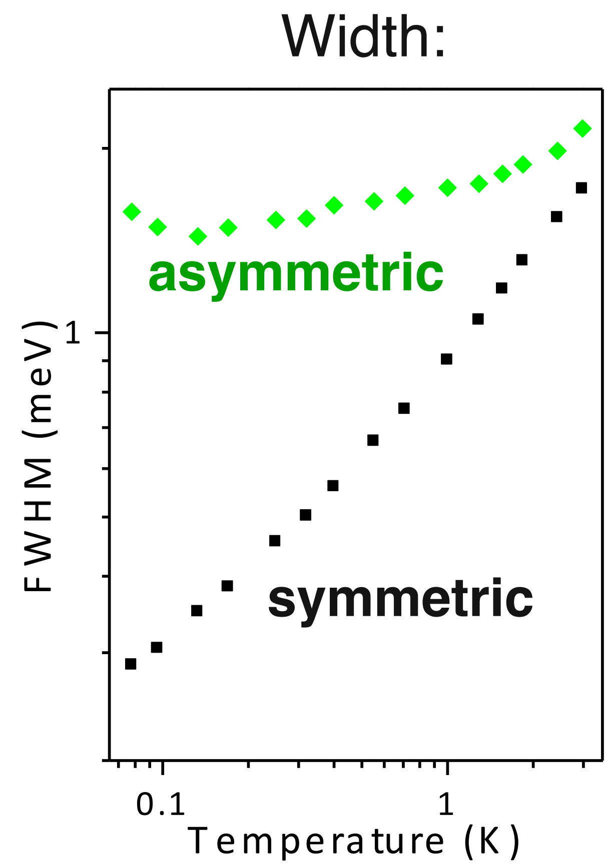
[Mebrahtu, et al., Nature 488, 61 (2012)]

Conductance Resonance: Height and Width Power Laws





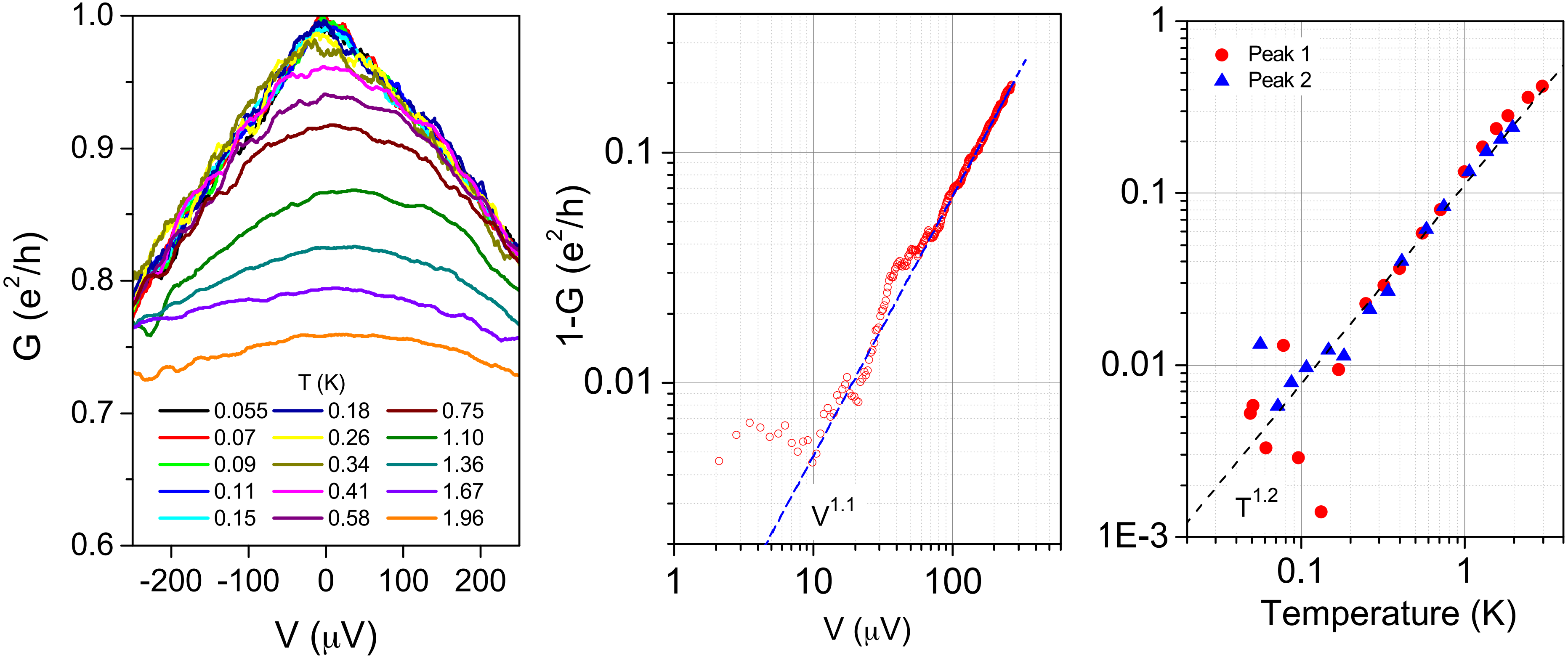




<u>Summary:</u>

- 1 special point— symmetric & on resonance— with perfect transmission, $G \longrightarrow 1$
- for all other parameters, conduction blocked, $G \longrightarrow 0$

G~1 : Unstable, Strong-Coupling Fixed Point



- unusual cusp in conductance!
- power law approach to full transparency
- V power and T power agree (quasi-linear)

[Henok Mebrahtu, et al., Nature Physics 2013]

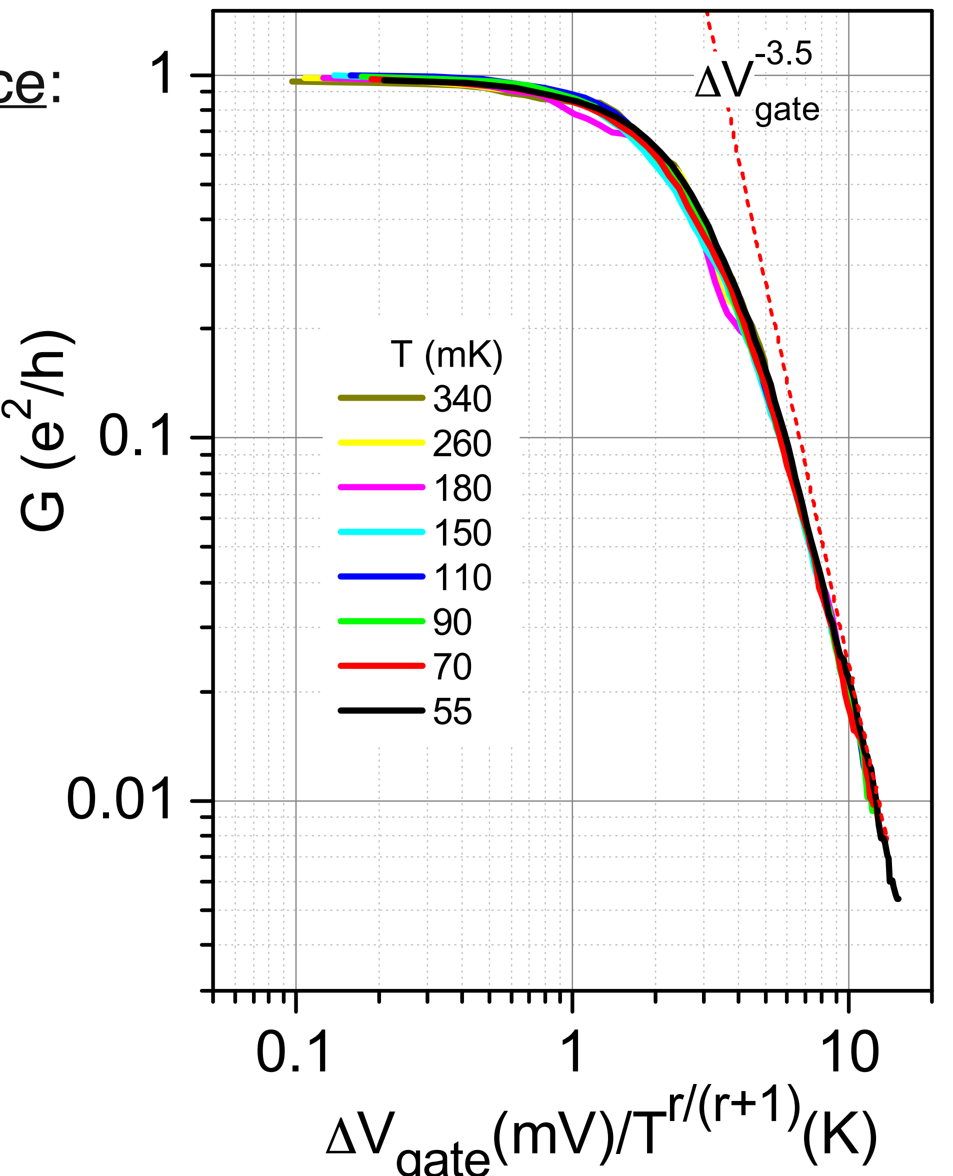
From G~1 to G=0: Flow Toward Weak-Coupling Fixed Point

Shape of conductance resonance:

use V_{gate} to tune resonant level through the chemical potential

Remember simple double barrier result:

$$T = \frac{4\Gamma_L\Gamma_R}{(\Delta\epsilon)^2 + (\Gamma_L + \Gamma_R)^2}$$



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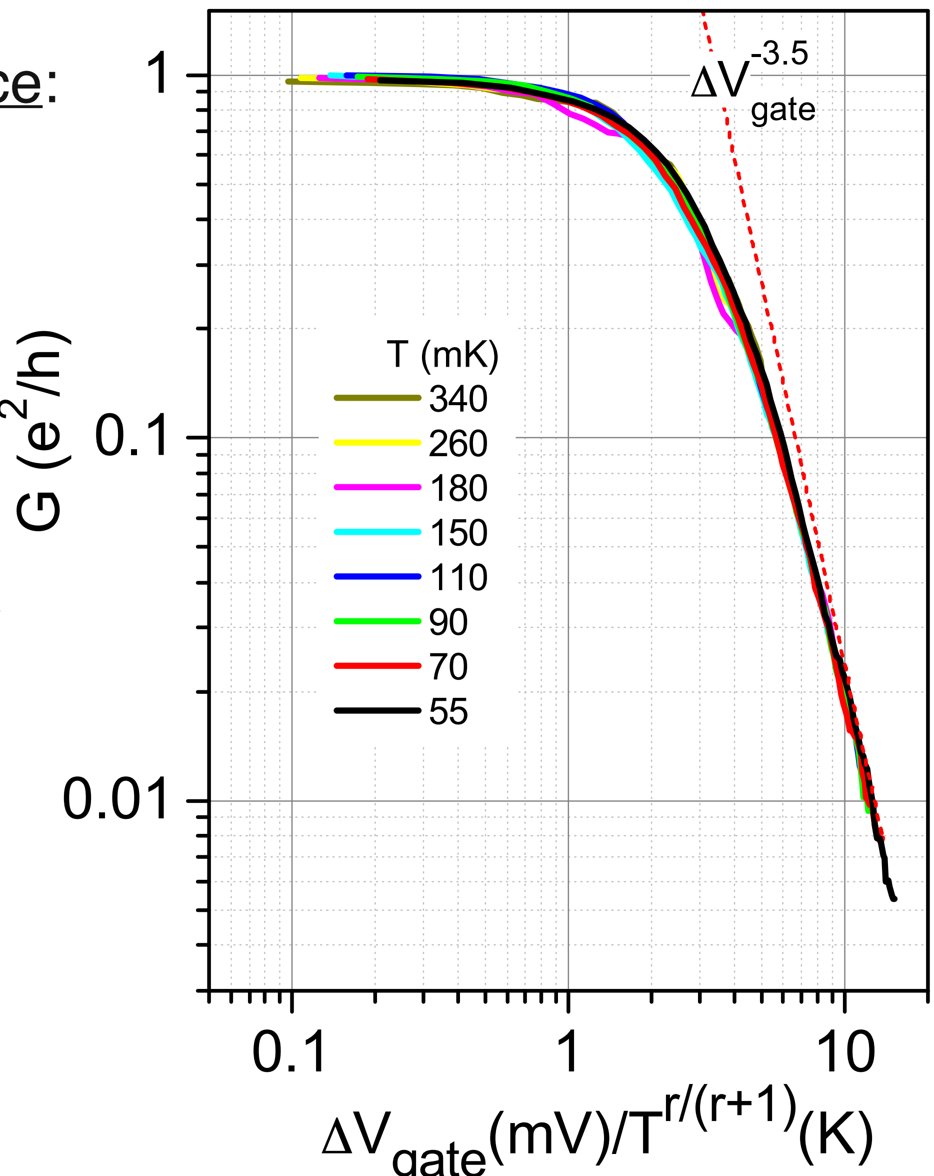

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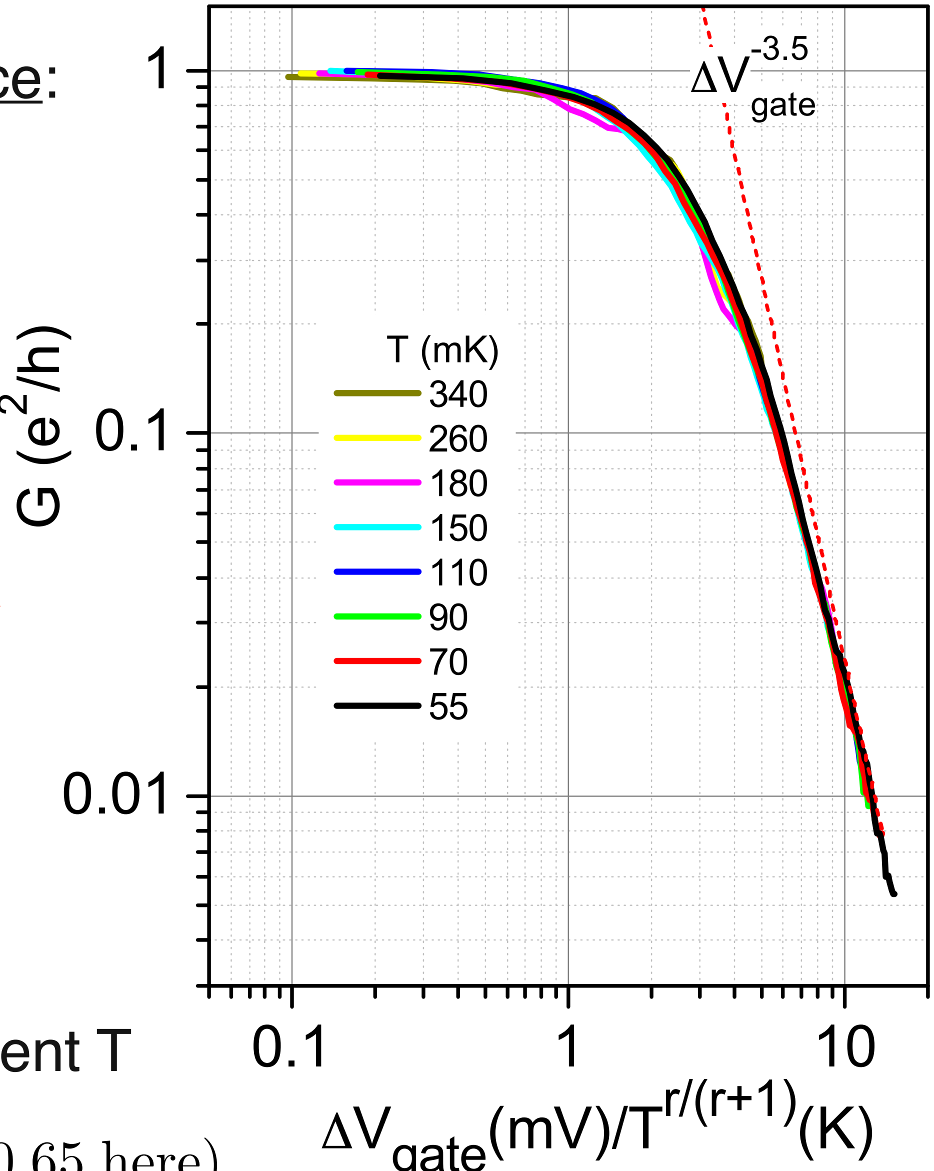


Scaling collapse of data at different T

$$r \equiv \frac{e^2}{h} R_{\text{leads}} \quad (\approx 0.65 \text{ here})$$

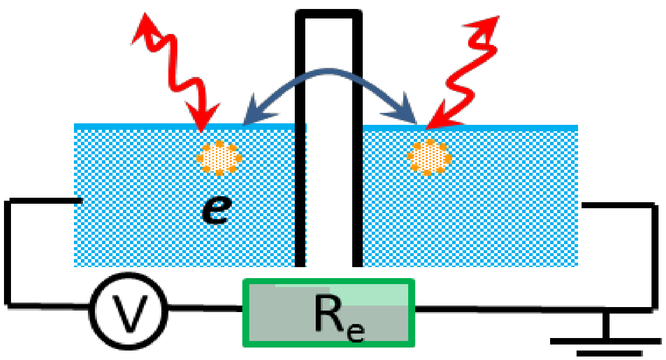
[Henok Mebrahtu, et al., Nature Physics 2013]

Tunneling with Dissipation ↔ Luttinger Liquid physics

Theoretical approach: [D. Liu, H. Zheng, S. Florens (Grenoble), HUB]

- model in which tunneling event excites the environment
- exploit formal correspondence to interacting 1D electrons
- analyze resulting 1D quantum field theory
- power laws come from scaling dimension of irrelevant and relevant operators near the strong- and weak- coupling fixed points

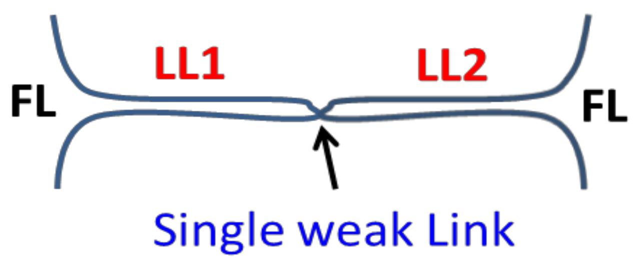
Single barrier (environmental Coulomb blockade):



mapping

$$g = \frac{1}{1+r}$$

[Safi & Saleur, PRL 04]

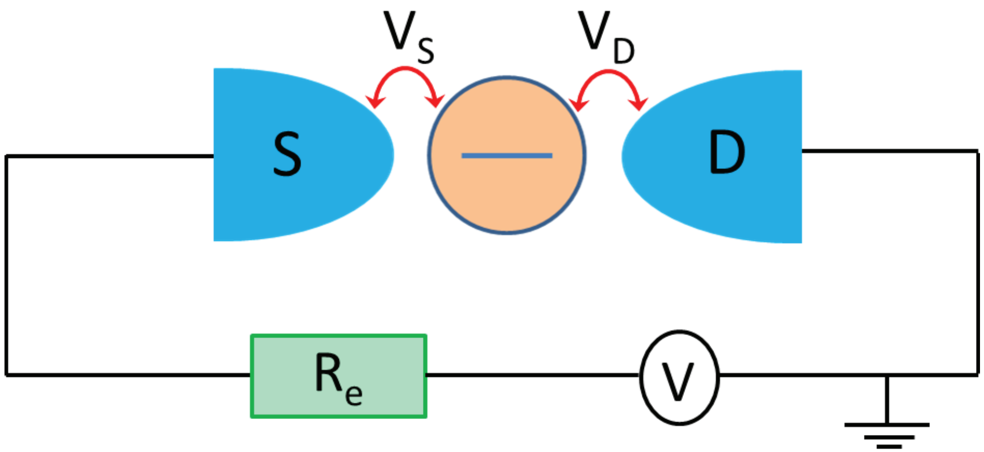


$$G \sim T^{2r}, \quad \text{with} \;\; r \equiv R_e/(h^2/e)$$

$$G \sim T^{2(1/g-1)} \quad ✓$$

Model of Resonant Tunneling with Environment

After: Ingold & Nazarov review 1992, K. LeHur et al., C.-H. Chung et al., S. Florens et al.



$$H = H_{\text{Dot}} + H_{\text{Leads}} + \underline{H_{\text{T}}} + H^{\text{T}}_{\text{Env}}$$

$$H_{\text{Dot}} = \epsilon_d d^\dagger d$$

$$H_{\text{Leads}} = \sum_k \epsilon_k c^\dagger_{kS} c_{kS} + \sum_k \epsilon_k c^\dagger_{kD} c_{kD}$$

Need quantum description of electrical properties of the junctions S and D, i.e. a **quantum capacitor**:

introduce conjugate charge and phase fluctuations on the junctions:

$$\varphi_{S,D}(t) = \frac{e}{\hbar} \int_{-\infty}^{t} dt' \; \delta V(t')$$

$$[\varphi, Q] = ie$$

$$H = \frac{Q^2}{2C} + \frac{\varphi^2}{2L}\left(\frac{\hbar}{e}\right)^2$$

Model of Tunneling Junctions

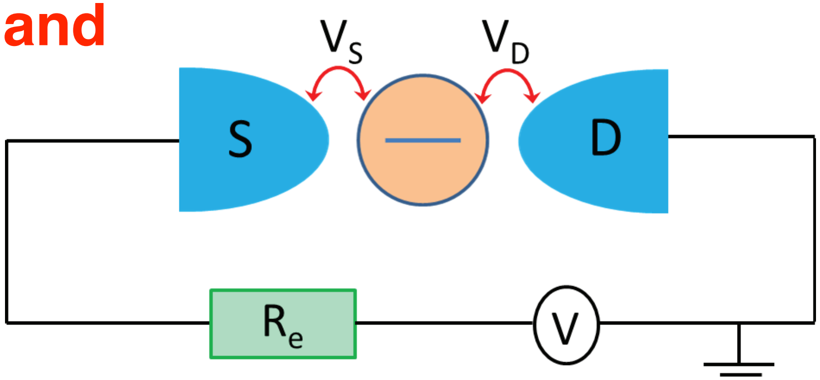
operator $e^{i\varphi}$ increments charge on capacitor by 1:

$$e^{i\varphi} Q e^{-i\varphi} = Q - e$$ [remember action of $e^{i\hat{p}}$ on position]

$$H_{\mathrm{T}} = V_S \sum_k (c^\dagger_{kS} e^{-i\varphi_S} d + \text{h.c.}) + V_D \sum_k (c^\dagger_{kD} e^{-i\varphi_D} d + \text{h.c.})$$

electron destroyed in dot

quasi-particle appears in metallic lead **and** charge on junction shifts by 1e



convenient to use sum and difference variables:

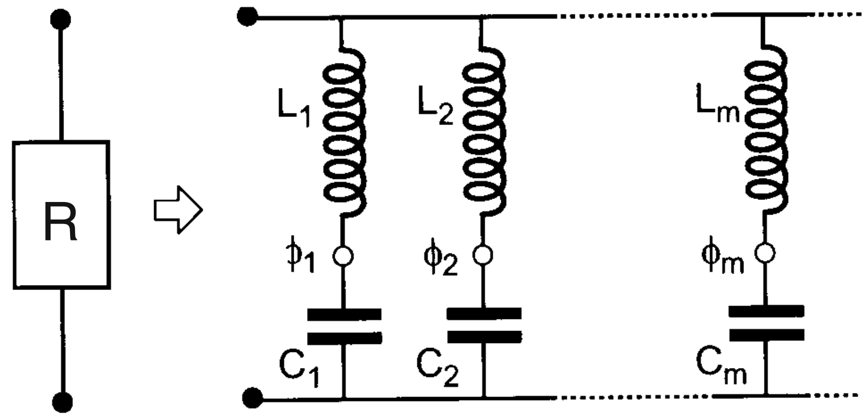
$$\varphi_c \equiv (\varphi_S + \varphi_D)/2$$ conjugate to total charge on dot

$$\varphi \equiv \varphi_S - \varphi_D$$ $e^{i\varphi}$ moves charge around circuit ✷

Model of the Environment

Couple phase to bath of LC oscillators:

choose oscillators such that the impedance is ohmic (resistance R)



[Devoret, LesHouches 95]

$$H^{\mathrm{T}}_{\mathrm{Env}} = \frac{q^2}{2C} + \sum_m \left[\frac{q_m^2}{2C_m} + \left(\frac{\hbar}{e}\right)^2 \frac{1}{2L_m} (\varphi - \varphi_m)^2 \right]$$

Integrate out bath degrees of freedom to get correlation of phi:

$$\left\langle e^{i\varphi(t)} e^{-i\varphi(0)} \right\rangle \propto \frac{1}{t^{2r}}$$

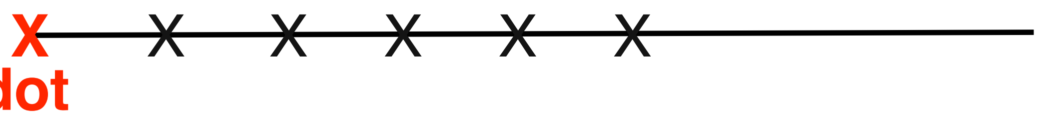
$$r \equiv \frac{R_{\mathrm{environ.}}}{R_{\mathrm{Quant.}}}$$

decay of quantum fluctuations of charge moving around the circuit are controlled by the resistance

Bosonic Description of Electrons

in 1D, bosonize the fermionic leads

[**Wait! why 1D??** a local quantum system couples to only 1 continuous degree of freedom:



mathematically: can always tri-diagonalize H starting from given state]

Leads: free fermions
→ chiral fermions → bosonize

$$c_{S,D}(x) = \frac{1}{\sqrt{2\pi a}} F_{S,D} \exp[i\phi_{S,D}(x)]$$

$\phi_S(x)$ and $\phi_D(x)$ are standard chiral bosonic fields:

density fluctuations of the electrons in the leads

Mapping to Luttinger Liquid

Goal: combine these lead fields with the environmental phase

$$\phi_{S,D} = (\phi_c^0 \pm \phi_f^0)/\sqrt{2}$$

$$H_{\mathrm{T}} = \sum_{S,D} \frac{V_{S,D} F_{S,D}}{\sqrt{2\pi a}} e^{-i\frac{1}{\sqrt{2}}[\phi_c^0(x=0) \pm \phi_f^0(x=0)]} e^{-i(\varphi_c \pm \varphi/2)} d + \mathrm{h.c.}$$

Combine so that fields in leads remain free; boundary op. changes:

$$\phi_f \equiv \sqrt{g_f}\left(\phi_f^0 + \frac{1}{\sqrt{2}}\varphi\right)$$

$$g_f \equiv \frac{1}{1+r} \leq 1 \quad \text{with } r \equiv \frac{R_e}{R_Q}$$

$$H = H_{\mathrm{Dot}} + \frac{v_F}{4\pi} \int_{-\infty}^{\infty} dx \left[(\partial_x \phi_c)^2 + (\partial_x \phi_f)^2 \right]$$

$$+ \sum_{S,D} \left[\frac{V_{S,D} F_{S,D}}{\sqrt{2\pi a}} e^{\mp i \frac{1}{\sqrt{2 g_f}} \phi_f(x=0)} e^{-i\frac{1}{\sqrt{2}}\phi_c(x=0)} d + \mathrm{h.c.} \right]$$

Emulate Luttinger liquid physics using E&M environment

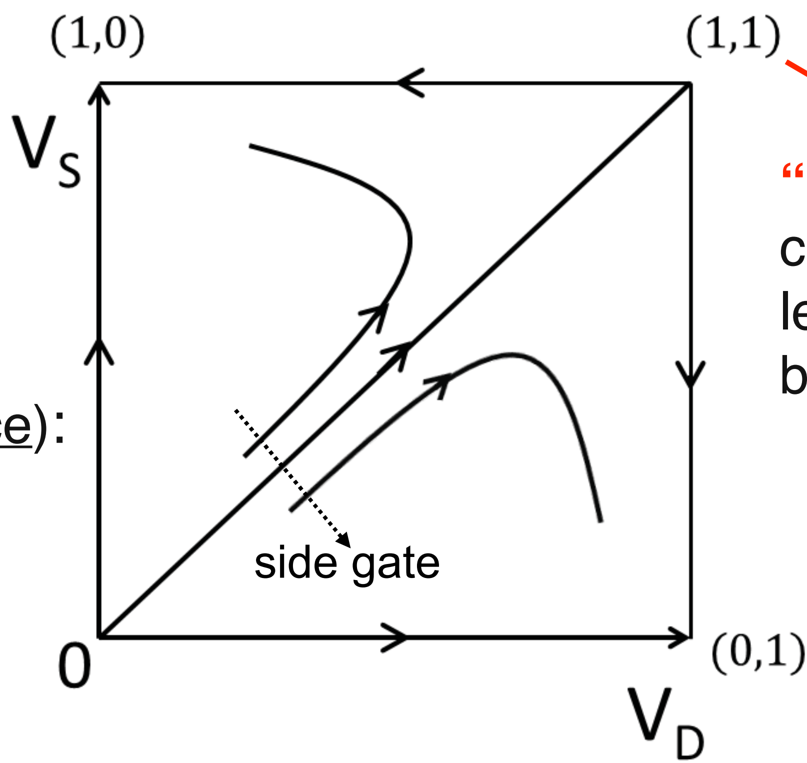
Analysis of the Effective 1D Interacting Model

To solve: draw on enormous literature on LL physics

[resonant level: Kane&Fisher 92, Eggert&Affleck 92, Furusaki 98, Nazarov&Glazman 03, Komnik&Gogolin 03, Meden, et al. 05, Goldstein&Berkovits 10, ...]

- integrate out all quadratic degrees of freedom
- perform perturbative RG-- "Coulomb gas RG"
- at strong-coupling fixed point, analyze likely dominant operators

resulting RG flow diagram (on resonance):



"healed chain" fixed point: competition between the two leads prevents the level from being absorbed into either one

G~1

Operators Controlling the Flow

Most important effect on transmission:

$2k_F$ scattering (back scattering) from the barriers

in bosonized form, this corresponds to the operator $\cos\left(2\sqrt{\pi}\phi_{x=0}\right)$

This operator is <u>relevant</u>-- causes flow to weak coupling

→ for low conductance, $G \sim T^{2r}$

near unitary conductance, $1 - G \sim T^{\frac{r}{r+1}}$

If the coefficient of this leading operator is exactly 0

(ie. on-resonance and with symmetric barriers),

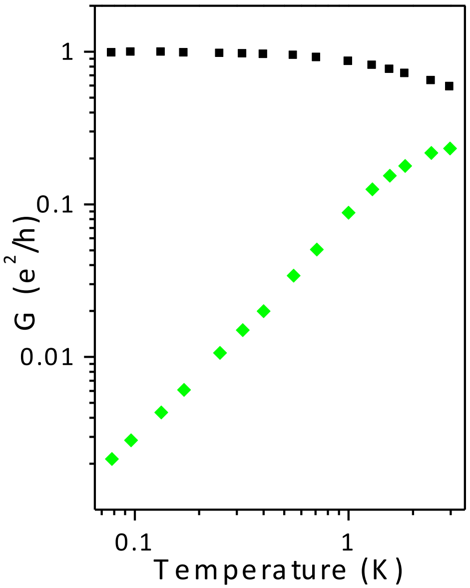
then the next-to-leading operator is $\cos\left(2\sqrt{\pi}\phi_{x=0}\right)\partial_x\phi_{x=0}$

corresponds to $2k_F$ scattering off the Friedel oscillation in the density

This operator is <u>irrelevant</u>-- controls flow into the QCP along symmetric line

→ near unitary conductance, $1 - G \sim T^{\frac{2}{r+1}}$

Comparing Exponents to Experiment



G_{peak} vs T (asym.)

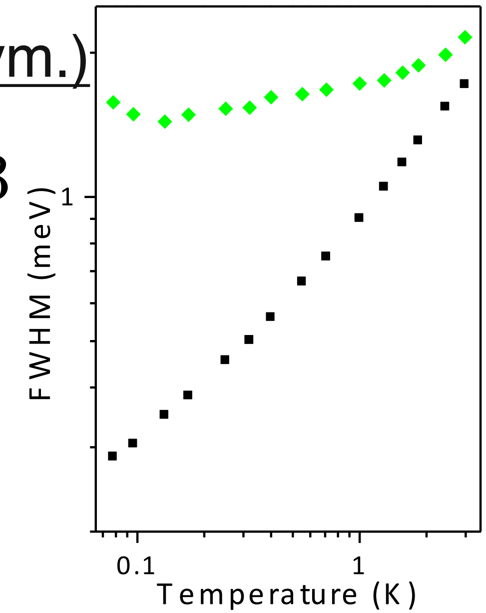
$$2r = 1.5$$

used to extract
r=0.75

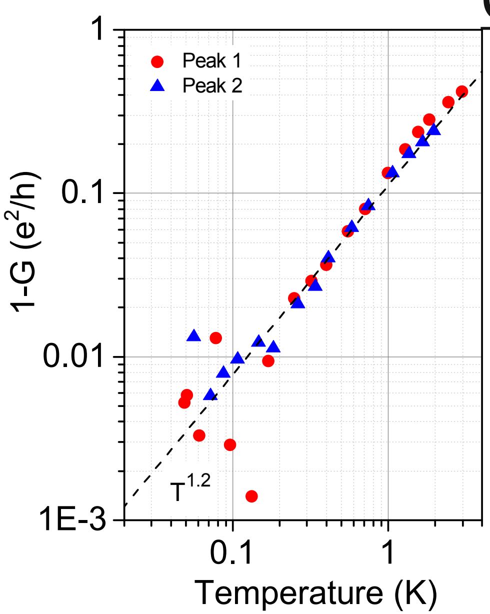
G_{width} vs T (sym.)

$$\frac{r}{r+1} = 0.43$$

0.45



G_{peak} vs T or V (sym.)



$$\frac{2}{r+1} = 1.14$$

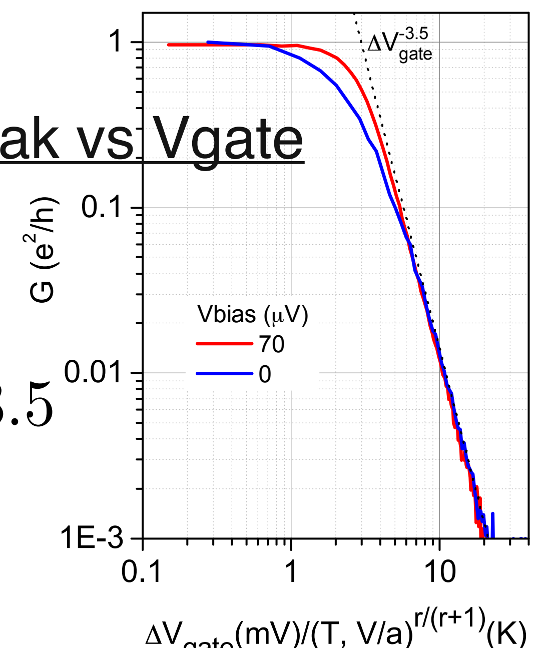
1.1 or 1.2

✔✔✔

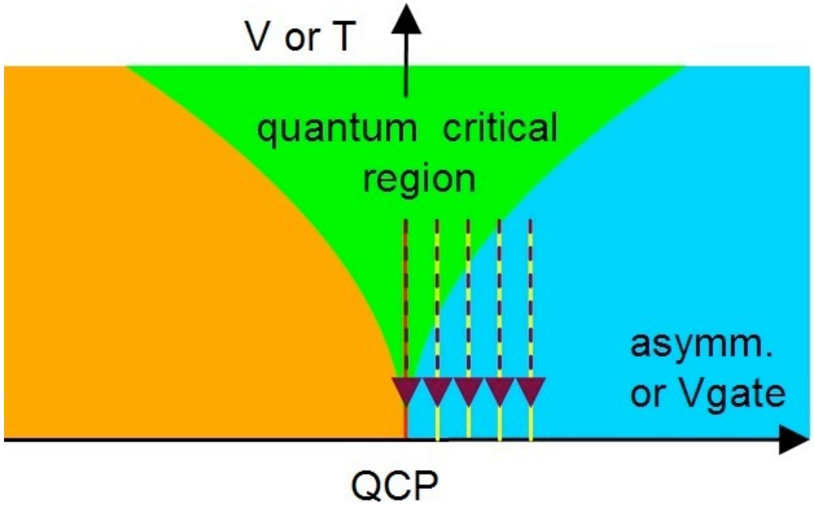
Tail of peak vs Vgate

$$2(r+1) = 3.5$$

3.4



Strong Coupling Critical State: Majorana Fermion



What is special about the model Hamiltonian at the critical point?

$$H_T \sim \sum_{S,D} \left[\frac{V_{S,D} F_{S,D}}{\sqrt{2\pi a}} e^{\mp i \frac{1}{\sqrt{2 g_f}} \phi_f(x=0)} e^{-i \frac{1}{\sqrt{2}} \phi_c(x=0)} d + \text{h.c.} \right]$$

reorganize, drop inessential factors

$$\sim e^{-i \frac{1}{\sqrt{2 g_f}} \phi_f(x=0)} \left[V_S\, d + V_D\, d^\dagger \right] + e^{+i \frac{1}{\sqrt{2 g_f}} \phi_f(x=0)} \left[V_D\, d + V_S\, d^\dagger \right]$$

If $V_S = V_D$, only the combination $d + d^\dagger$ appears!

(current around the loop involves both destruction and creation of the dot electron)

Majorana Hamiltonian for the QCP

First, unitary transform eliminates ϕ_c field from tunneling term:

$U = \exp\left[i(d^\dagger d - 1/2)\phi_c(0)/\sqrt{2}\right]$, generating a density-density term

$$H = H_{\rm Dot} + H_{\rm Leads} - \pi\hbar v_F (d^\dagger d - 1/2)\partial_x \phi_c(x=0)$$
$$+ \left[\frac{V_S F_S}{\sqrt{2\pi a}} e^{-i\frac{1}{\sqrt{2g_f}}\phi_f(x=0)} d + \frac{V_D F_D}{\sqrt{2\pi a}} e^{+i\frac{1}{\sqrt{2g_f}}\phi_f(x=0)} d + {\rm h.c.} \right]$$

Second, refermionize for g=1/2 (ie. r=1): $\psi_f^\dagger \equiv \frac{F_S}{\sqrt{2\pi a}} e^{-i\phi_f(x=0)}$

[as in Komnik & Gogolin PRL 03; like Emery & Kivelson for 2 channel Kondo]

competition between leads is like competition between channels
in 2CK-- leads to Majorana representation in same way

Majorana Hamiltonian for the QCP

Write the state in the quantum dot as two Majorana's: $d^\dagger = a - ib$

$$H_{\text{Majorana}} = H_{\text{Leads}} + (V_S - V_D)\big[\psi_f^\dagger(0) - \psi_f(0)\big]a$$
$$+ i(V_S + V_D)\big[\psi_f^\dagger(0) + \psi_f(0)\big]b$$
$$+ 2i\epsilon_d ab - 2i\pi\hbar v_F \psi_c^\dagger(0)\psi_c(0)ab$$

- one Majorana hybridizes with the leads
- the other is uncoupled (when symmetric and on resonance)
  [much as in describing the two-channel Kondo problem]

an independent Majorana zero mode $S = \frac{1}{2}\log 2$

In experiment, r is not quite 1 :

interacting leads with $\tilde{g} \propto (1-r)$ $S = \frac{1}{2}\log(1+r)$ [Wong & Affleck]

If neglect contact interaction: Majorana resonant level
→ non-interacting model, so dependence on T and V is quadratic

Majorana Hamiltonian for the QCP

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$$H_{\text{Majorana}} = H_{\text{Leads}} + \cancel{(V_S - V_D)\left[\psi_f^\dagger(0) - \psi_f(0)\right]a}$$
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Majorana Hamiltonian: Linear Conductance from Interactions

If neglect contact interaction: Majorana resonant level
→ non-interacting model, so dependence on T and V is quadratic

But in system studied, the interaction is **large**, ~E_F
Nevertheless, try perturbation theory:

$$-2i\pi\hbar v_F \psi_c^\dagger(0)\psi_c(0)ab$$

physical current in transformed basis: $\hat{I} = V\left[\psi_f(0) - \psi_f^\dagger(0)\right]b$

ψ_f unaffected by interaction
→ correction to G=1 comes from self-energy of b Majorana!

$$G_b(t) \equiv \langle b^+(0)b(t)\rangle \propto 1/t$$

$$G_{\psi_c(x=0)}(t) \propto 1/t$$

$$G_a(t) = \langle a^+(0)a(t)\rangle \propto 1$$

→ $\Sigma_b(t) \propto v_F^2 G_a(t) G_{\psi_c}^2(t) \propto 1/t^2$

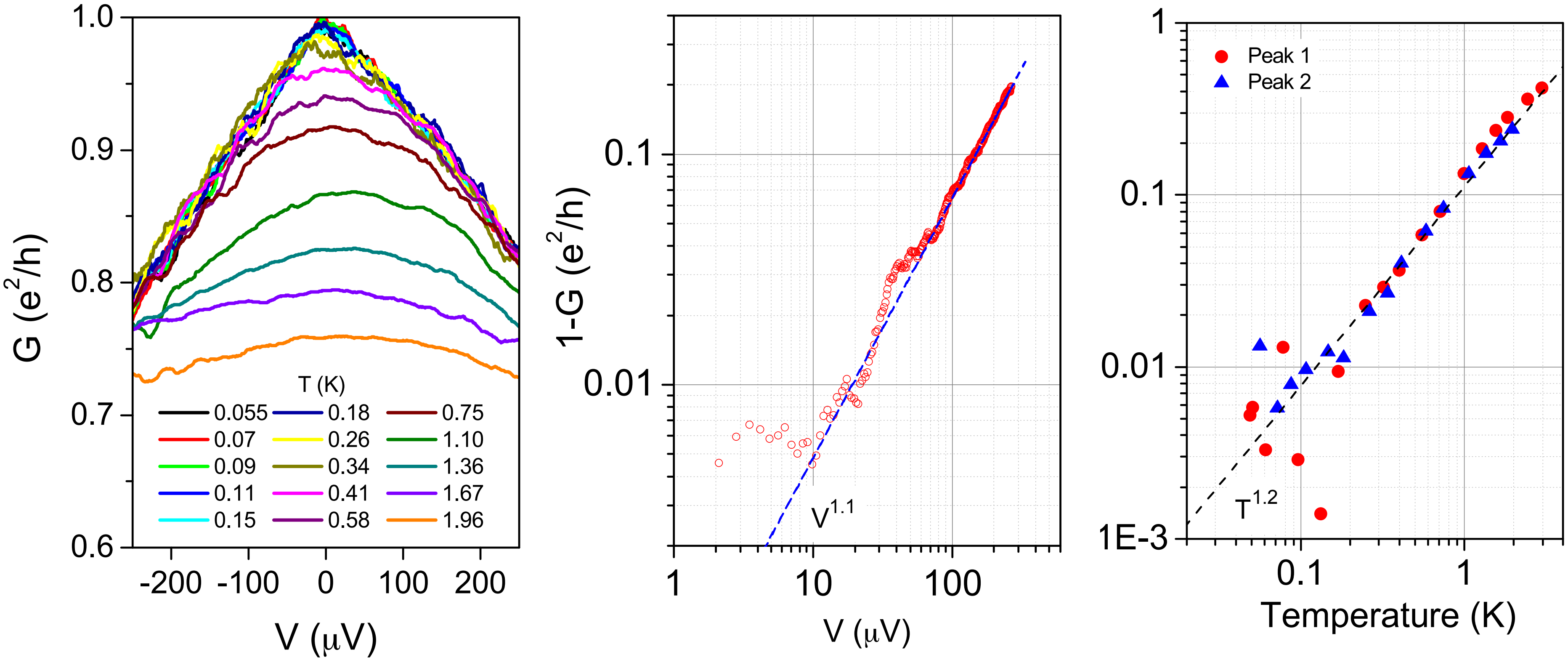
→ $1 - G \propto T^1$ by Fourier transform

linear dependence on T or V is signature of Majorana fermion

If r not 1, ~near linear dependence...

[H. Zheng, S. Florens, HB, PRB 2014]

G~1 : Unstable, Strong-Coupling Fixed Point



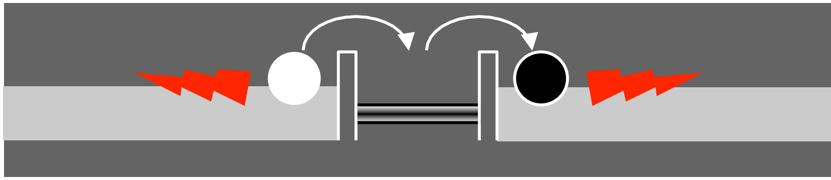
- unusual cusp in conductance!
- power law approach to full transparency
- V power and T power agree (quasi-linear)

non-quadratic dependence is signature of Majorana zero mode

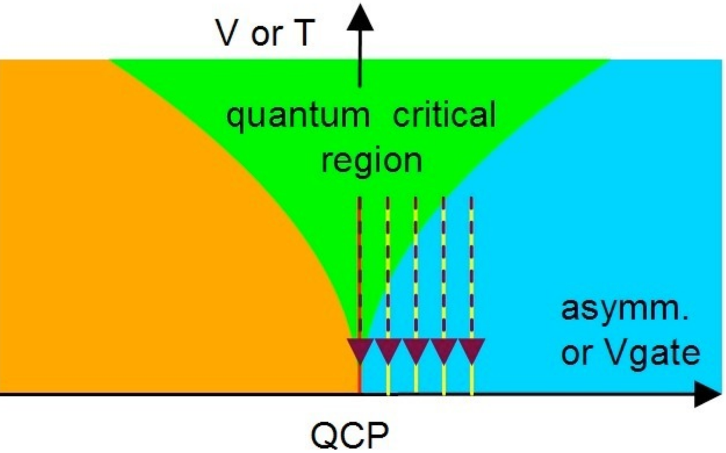
[Henok Mebrahtu, et al., Nature Physics 2013]

CONCLUSIONS

Majorana Quantum Critical Behavior for Resonant Level + Dissipative Environment



- **Resonant tunneling in a dissipative environment**
  - dissipation + symmetry of coupling → competition → QPT
- **1. Experiment:** Beautiful data accessing both strong- and weak- coupling fixed points
- **2. Theory:** mapping to interacting 1D model-- emulation of LL uncoupled Majorana state → ~linear dependence of 1-G



D. Liu, H. Zheng, S. Florens, HUB
H. Mebrahtu, I. Borzenets, Y. Bomze, A. Smirnov, G. Finkelstein

[Mebrahtu, et al., Nature 488, 61 (2012), Nature Physics (2013)]

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