Google matrix analysis of the multiproduct world trade network

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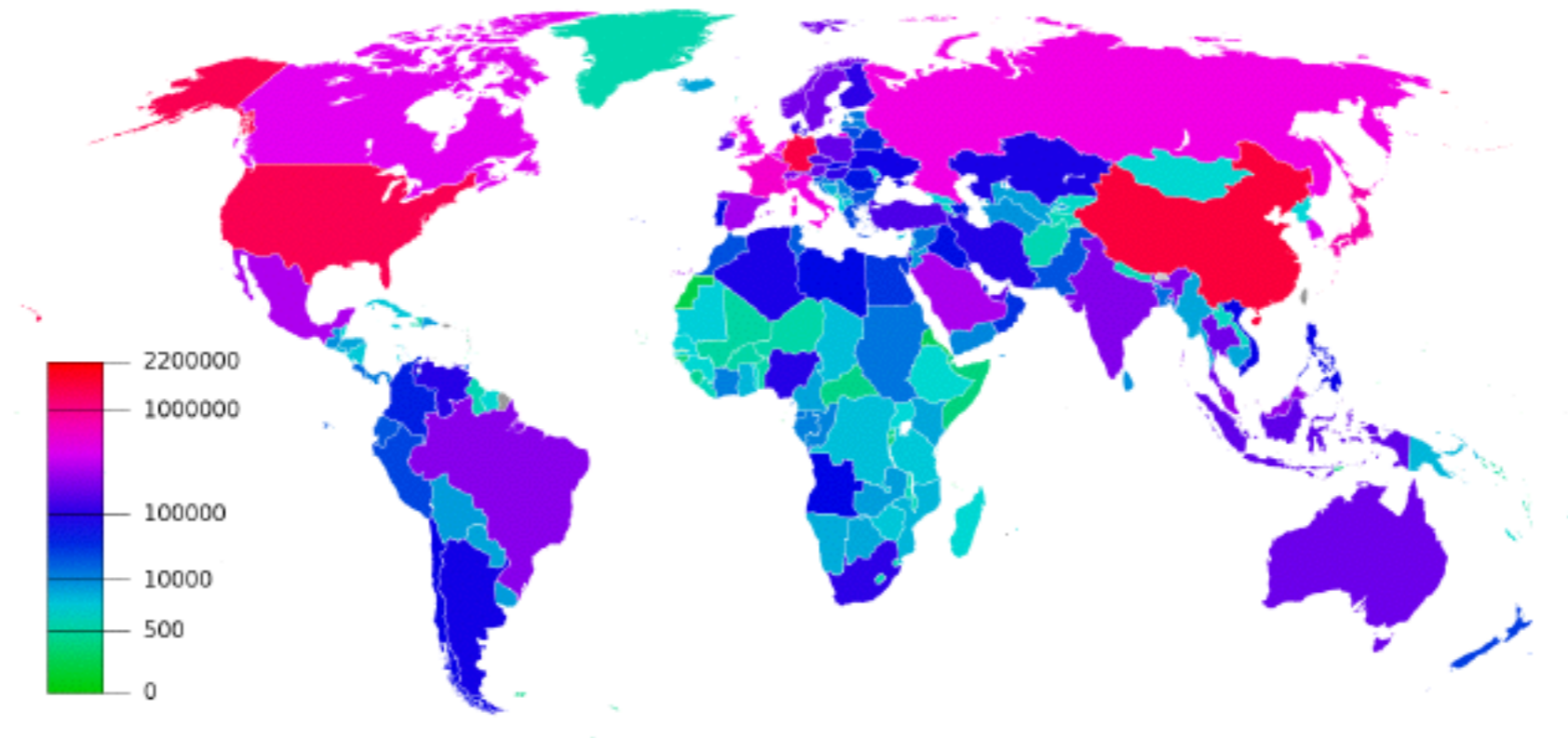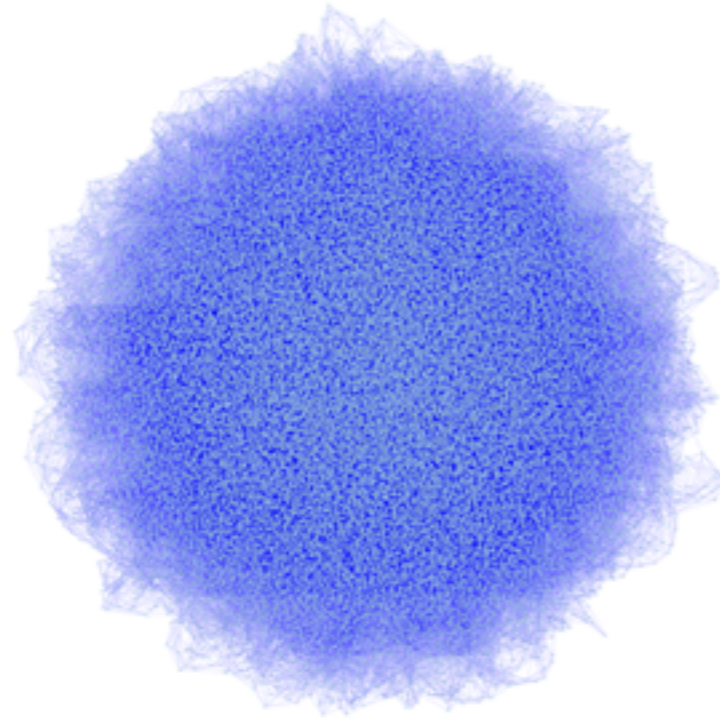Alexei Chepelianskii and Klaus Frahm

Quantum chaos: fundamentals and applications

March 15th, 2015

School for advanced sciences of Luchon

Motivations

Google approach to the World Trade Network
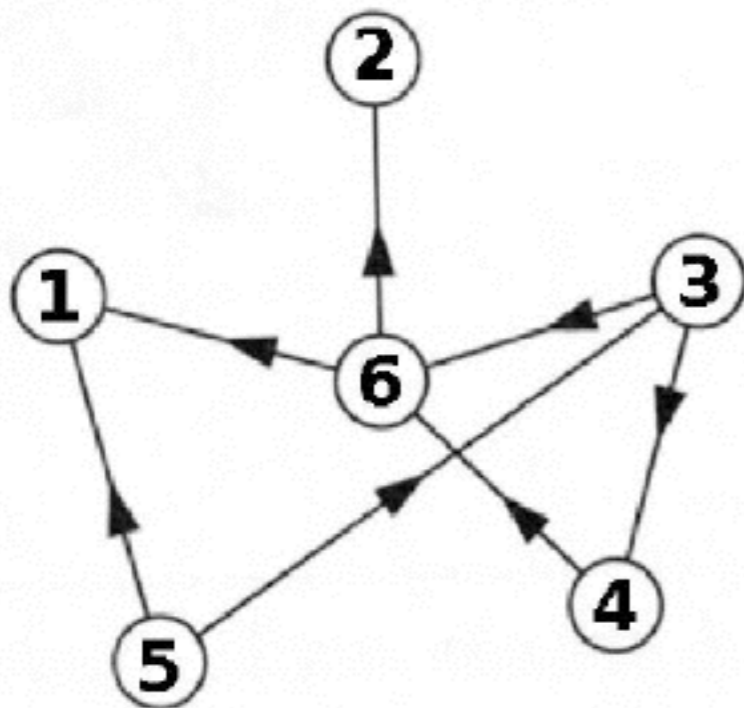
Google Matrix

thanks to Klaus introduction of G matrix

Brin and Page (1998)

centrality measure:

Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

directed network

adjacency matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$
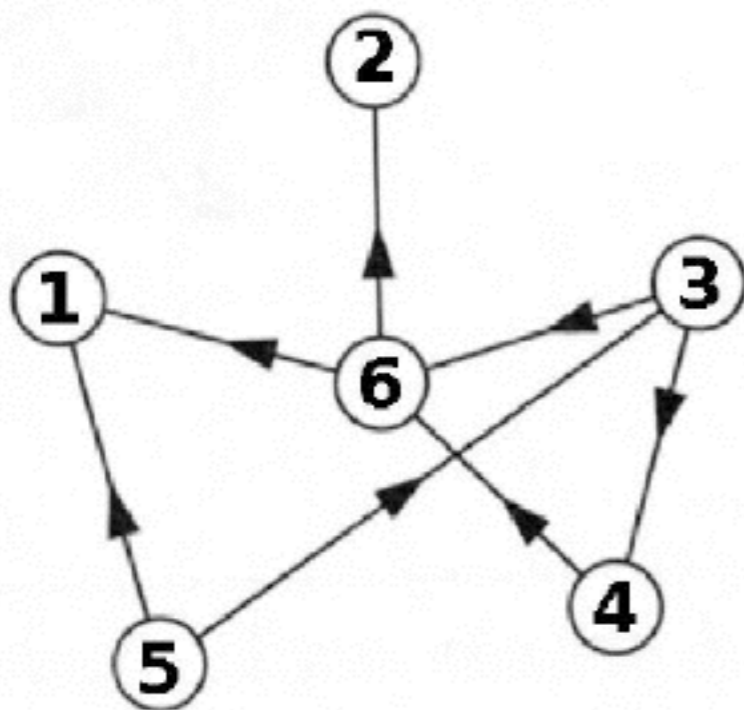
Google Matrix

thanks to Klaus introduction of G matrix

Brin and Page (1998)

centrality measure:

Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

directed network

weighted adjacency matrix and dangling nodes

$$S = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & 1 & 0 & 0 \end{pmatrix}$$

- $\sum_i S_{i,j} = 1$
- Perron-Frobenius (non-negative)
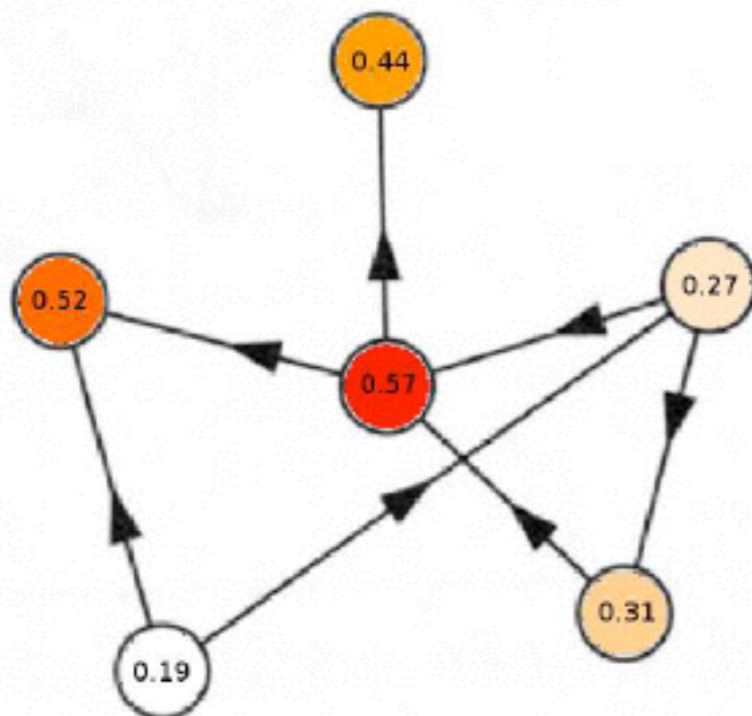- $\lambda_1 = 1$ (degeneracy)

Google Matrix

thanks to Klaus introduction of G matrix

centrality measure:

Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

Brin and Page (1998)

PageRank

$$\mathbf{G}P = P$$

$$\mathbf{G} = \alpha\mathbf{S} + (1-\alpha)\mathbf{E}/N \; (\alpha = 0.85)$$
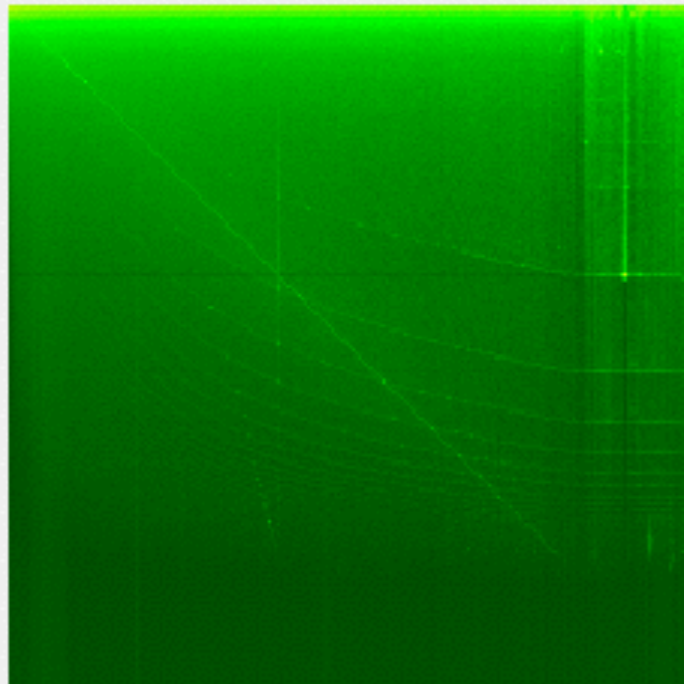
directed network

Google Matrix

$$G = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} & \frac{9}{20} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{9}{20} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{9}{20} & \frac{7}{8} & \frac{1}{40} & \frac{1}{40} \end{pmatrix}$$
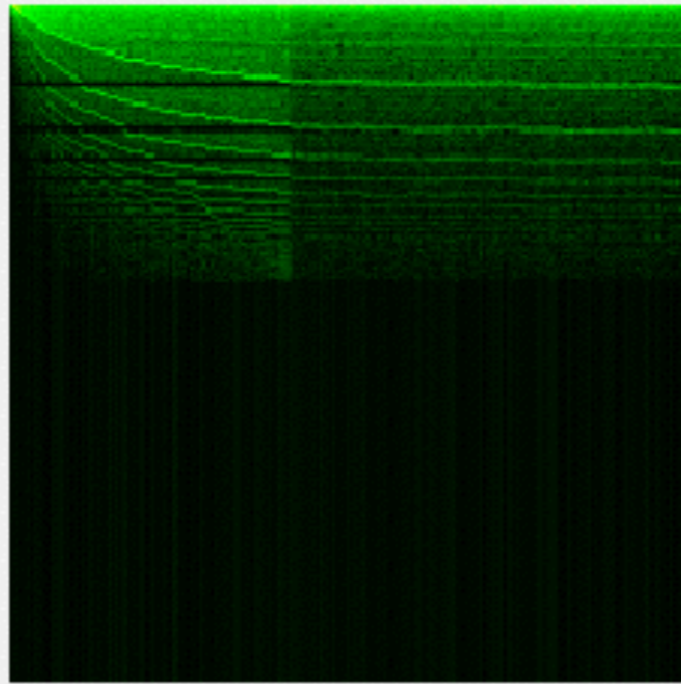
- $\alpha \to S, (1-\alpha) \to$ random node
- Perron-Frobenius (positive) $\lambda_1 = 1$
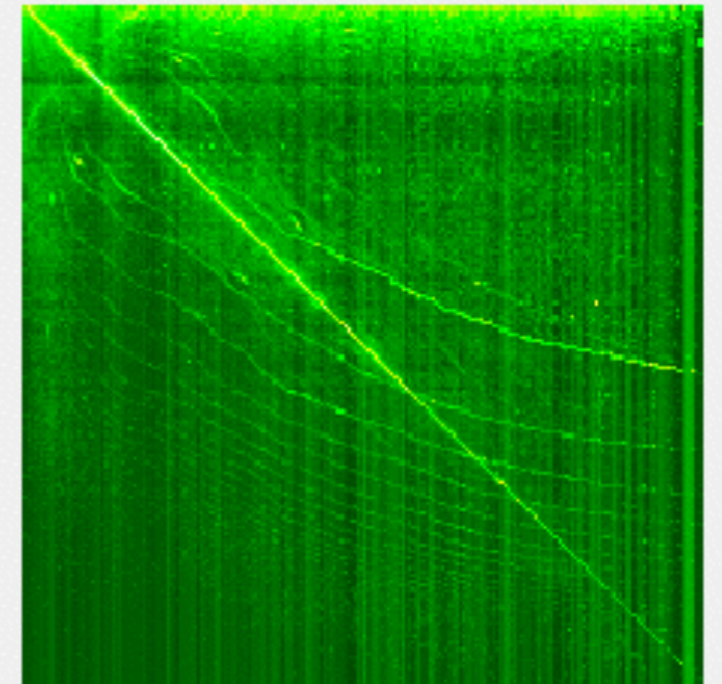- $\Delta \geq (1-\alpha)$ (global convergence)

Google Matrix

L.E, Chepelianskii and Shepelyansky, Jour. Phys. A 45, 275101 (2012).

Google Matrix

FWL in Google matrices

Ulam method and fractal Weyl law for Perron-Frobenius operators

L. Ermann and D.L. Shepelyansky[a]

Laboratoire de Physique Théorique du CNRS (IRSAMC), Université de Toulouse, UPS, 118 route de Narbonne, 31062 Toulouse Cedex 4, France

Fractal Weyl law for Linux Kernel architecture

L. Ermann[1], A.D. Chepelianskii[2], and D.L. Shepelyansky[1,a]

[1] Laboratoire de Physique Théorique (IRSAMC), Université de Toulouse, UPS-CNRS, 31062 Toulouse, France
[2] LPS, Université Paris-Sud, CNRS, UMR8502, 91405 Orsay, France

2D rankings

wiki: K-K* plane
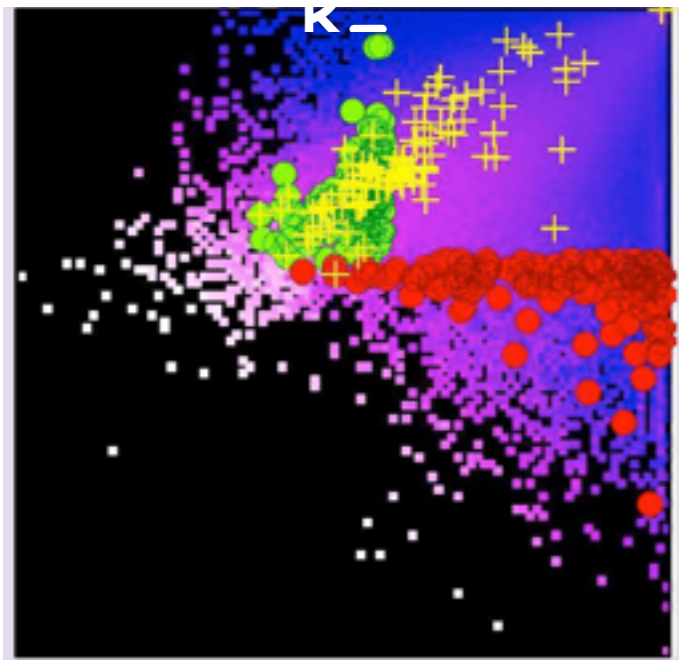
PageRank-CheiRank 2d ranking

PageRank: 1. Napoleon I of France, 2. George W. Bush, 3. Elizabeth II of the United Kingdom, 4. William Shakespeare, 5. Carl Linnaeus, 6. Adolf Hitler, 7. Aristotle, 8. Bill Clinton, 9. Franklin D. Roosevelt, 10. Ronald Reagan.

CheiRank: 1. Kasey S. Pipes, 2. Roger Calmel, 3. Yury G. Chernavsky, 4. Josh Billings (pitcher), 5. George Lyell, 6. Landon Donovan, 7. Marilyn C. Solvay, 8. Matt Kelley, 9. Johann Georg Hagen, 10. Chikage Oogi.

2DRank: 1. Michael Jackson, 2. Frank Lloyd Wright, 3. David Bowie, 4. Hillary Rodham Clinton, 5. Charles Darwin, 6. Stephen King, 7. Richard Nixon, 8. Isaac Asimov, 9. Frank Sinatra, 10. Elvis Presley.

Chepelianskii (2010) O. Zhirov and Shepelyansky,(2010), LE, Chepeliansskii, Shepelyansky JPA(2012)

World Trade Network

Import-Export trade database:

United Nation Commodities Trade Network
HTTP://COMTRADE.UN.ORG/DB/
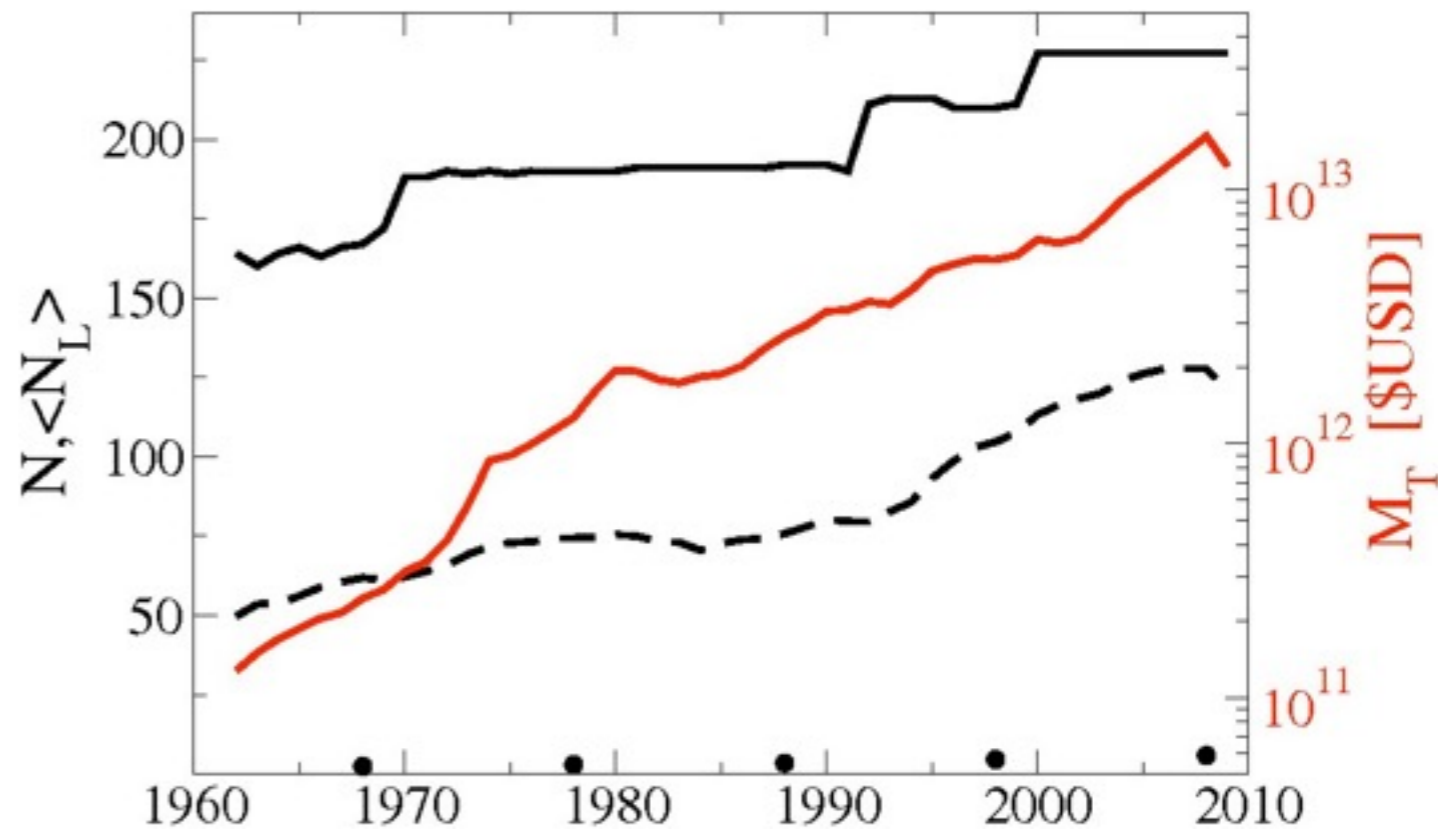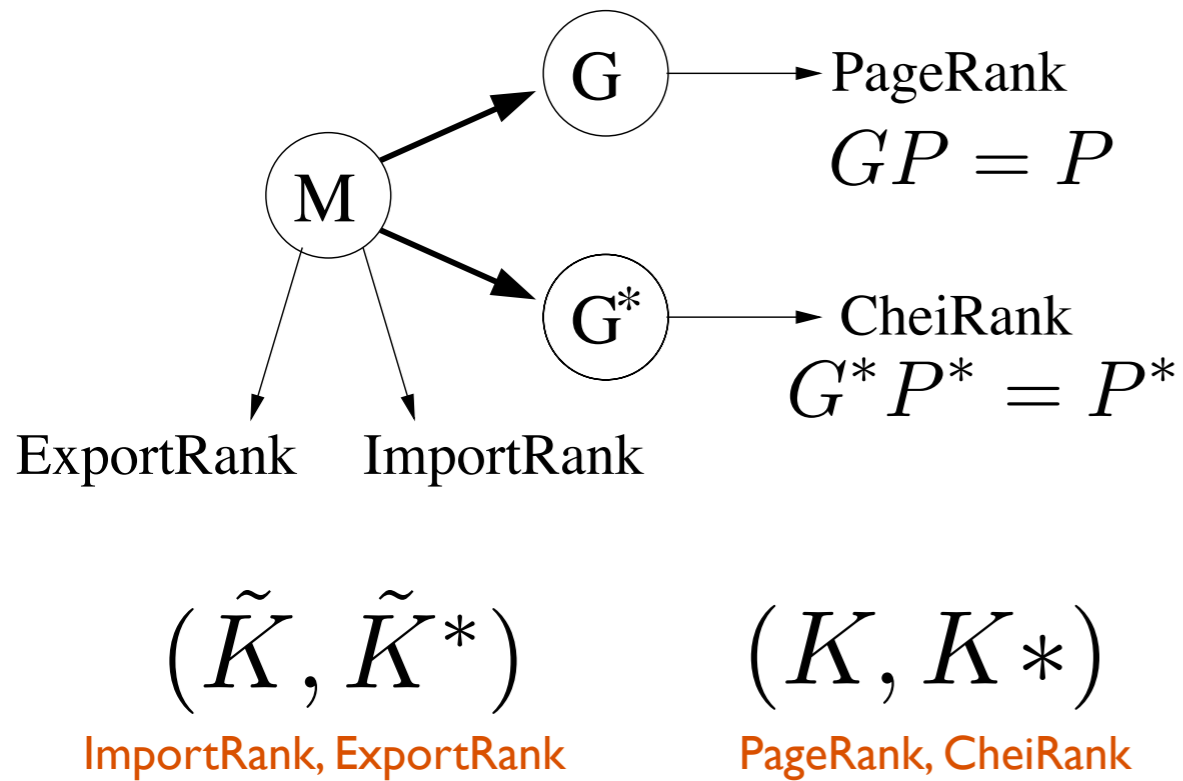
* Each year from 1962 to 2011 (2014)
* All UN countries: ~ 220 (Nc=227 in 2008)
* Product classification (SITC Rev. 1): Np=61
* Trade volume is given in USD (N=13847 x 50 years)

Money Matrices

$$M_{c,c'} = \$ \ (c' \rightarrow c) \qquad\qquad M_{c,c'}^{p} = \$ \ (c' \rightarrow c)$$

Google matrix of the WTN

$$GP = P$$ — PageRank

$$G^*P^* = P^*$$ — CheiRank

$(\tilde{K}, \tilde{K}^*)$ $(K, K*)$

ImportRank, ExportRank   PageRank, CheiRank

ExportRank   ImportRank

M → G → PageRank

M → G* → CheiRank

Democracy in countries but not in products

Google matrix of the WTN

1) WTN (all com. or 1 prod)
N=227

non-interacting products

2) WTN (multiprod)
N=13847

personalized vector $\longrightarrow$ prop to Vp of each c

2nd iteration $\longrightarrow$ reduced Pp for all c

$$v_i = \frac{V_c^p}{N_c \sum_{p'} V_c^{p'}} \;,\; v_i^* = \frac{V_c^{*p}}{N_c \sum_{p'} V_c^{*p'}} \;,$$

$$v'(i) = \frac{P_p}{N_c} \;,\; v'^*(i) = \frac{P_p^*}{N_c} \;.$$
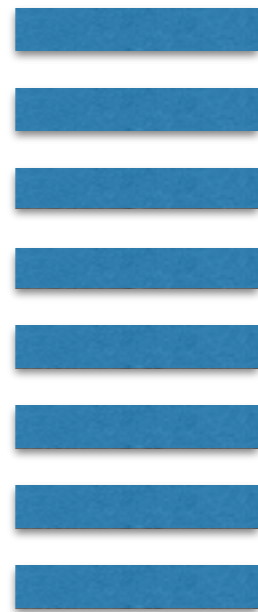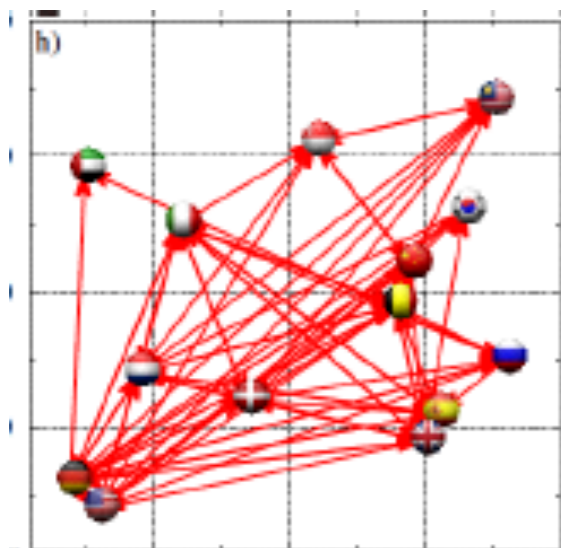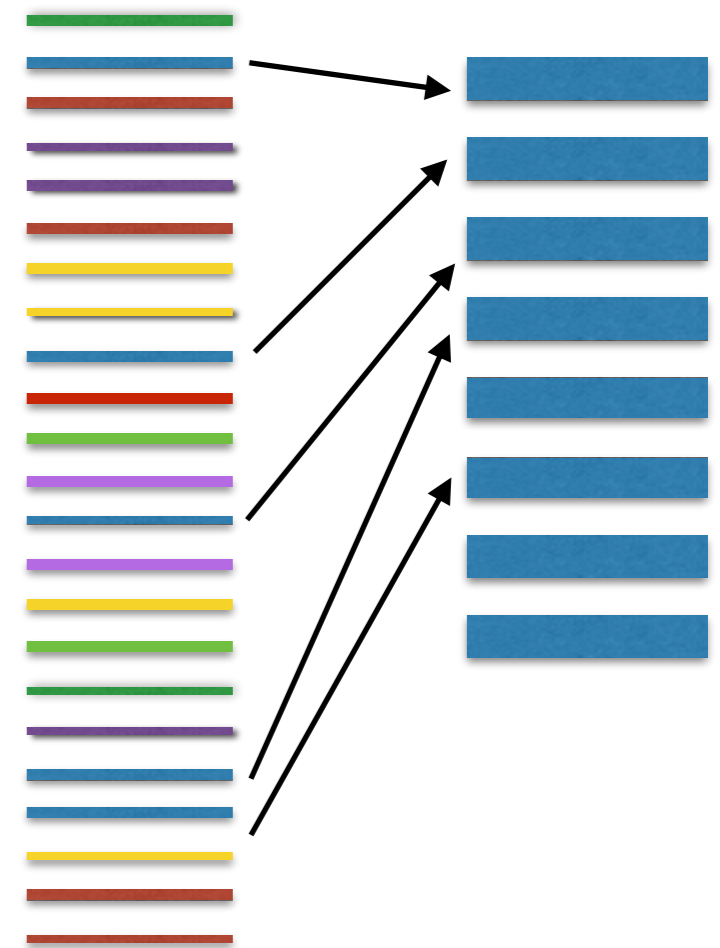
L. Ermann and D.L. Shepelyansky, APPA, Vol. 120, A-158 (2011),
http://www.quantware.ups-tlse.fr/QWLIB/tradecheirank

L. Ermann and D.L. Shepelyansky, EPJB (2015).

all commodities and given products (N=227)

M   G   G*

all commodities
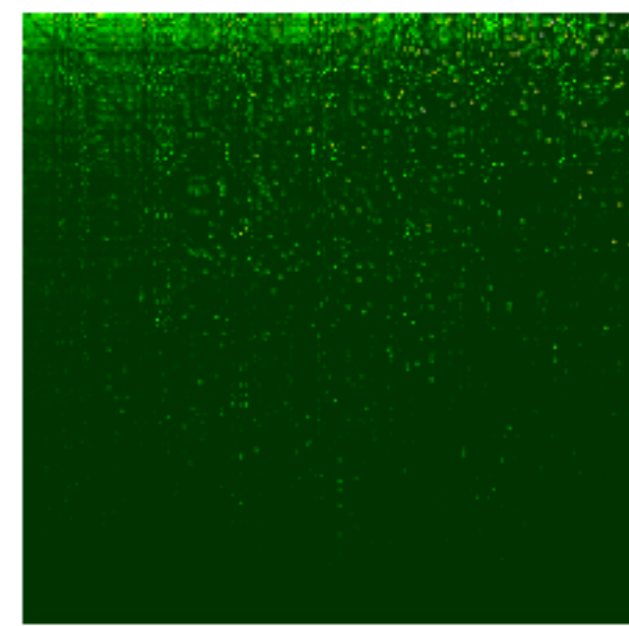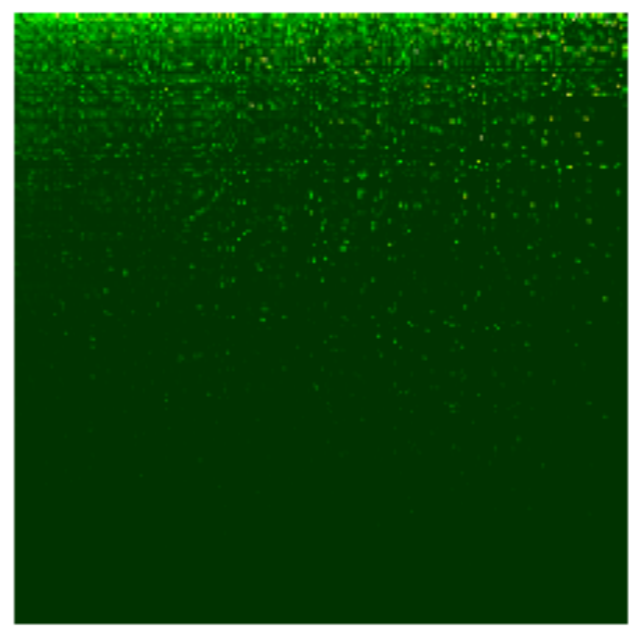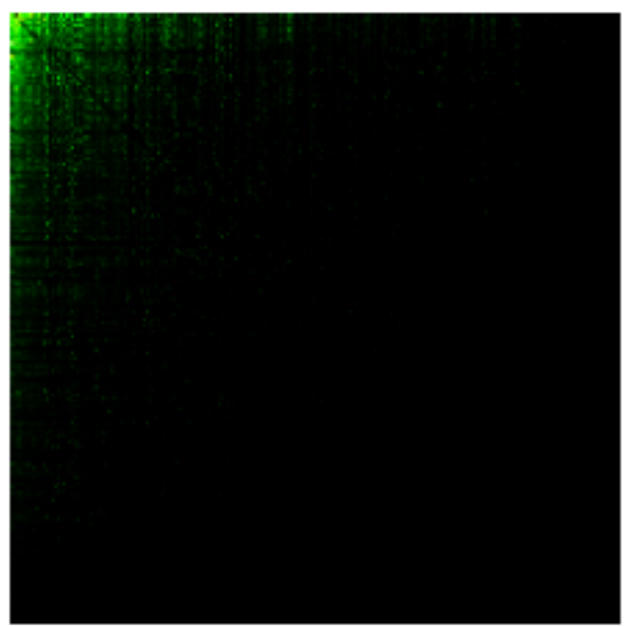
PageRank, CheiRank, ImportRank, ExportRank $\alpha = 0.5$

Zipf law P~1/K

$P, P^*, \tilde{P}, \tilde{P}^*$

$K, K^*, \tilde{K}, \tilde{K}^*$

Spectra $\alpha = 1$

λ_{all}

λ_{food}

$\lambda_{cereals}$

less symmetric

λ_{barley}

2d rank evolution

1962
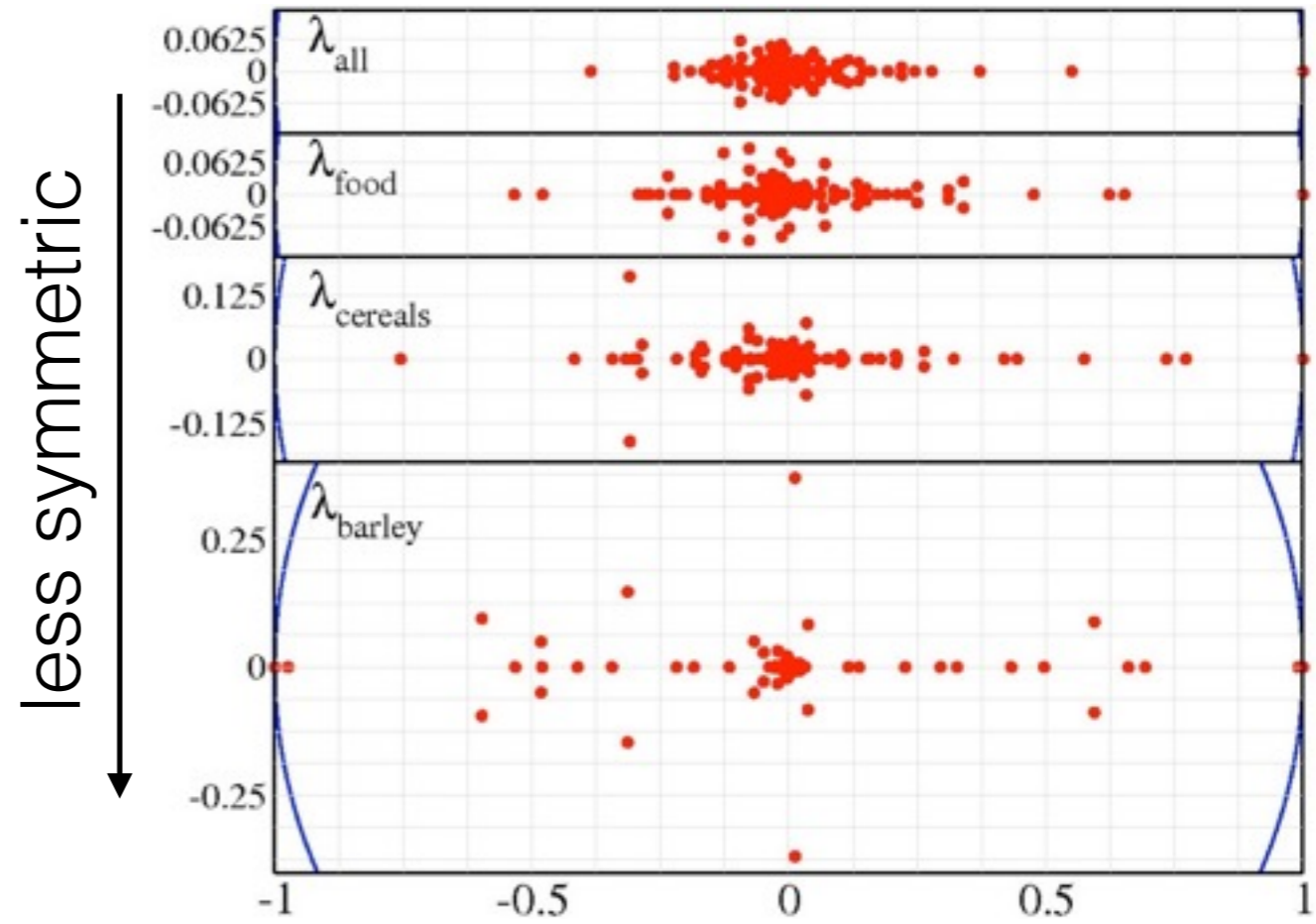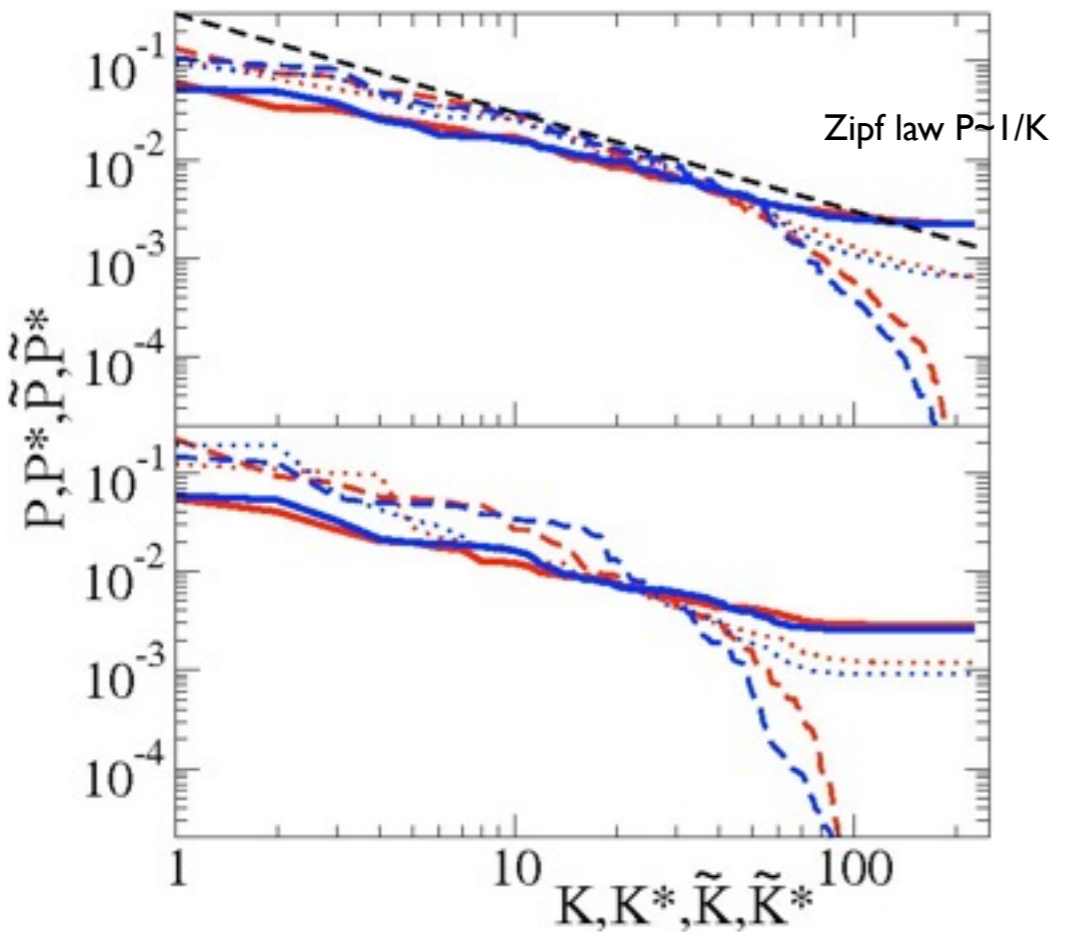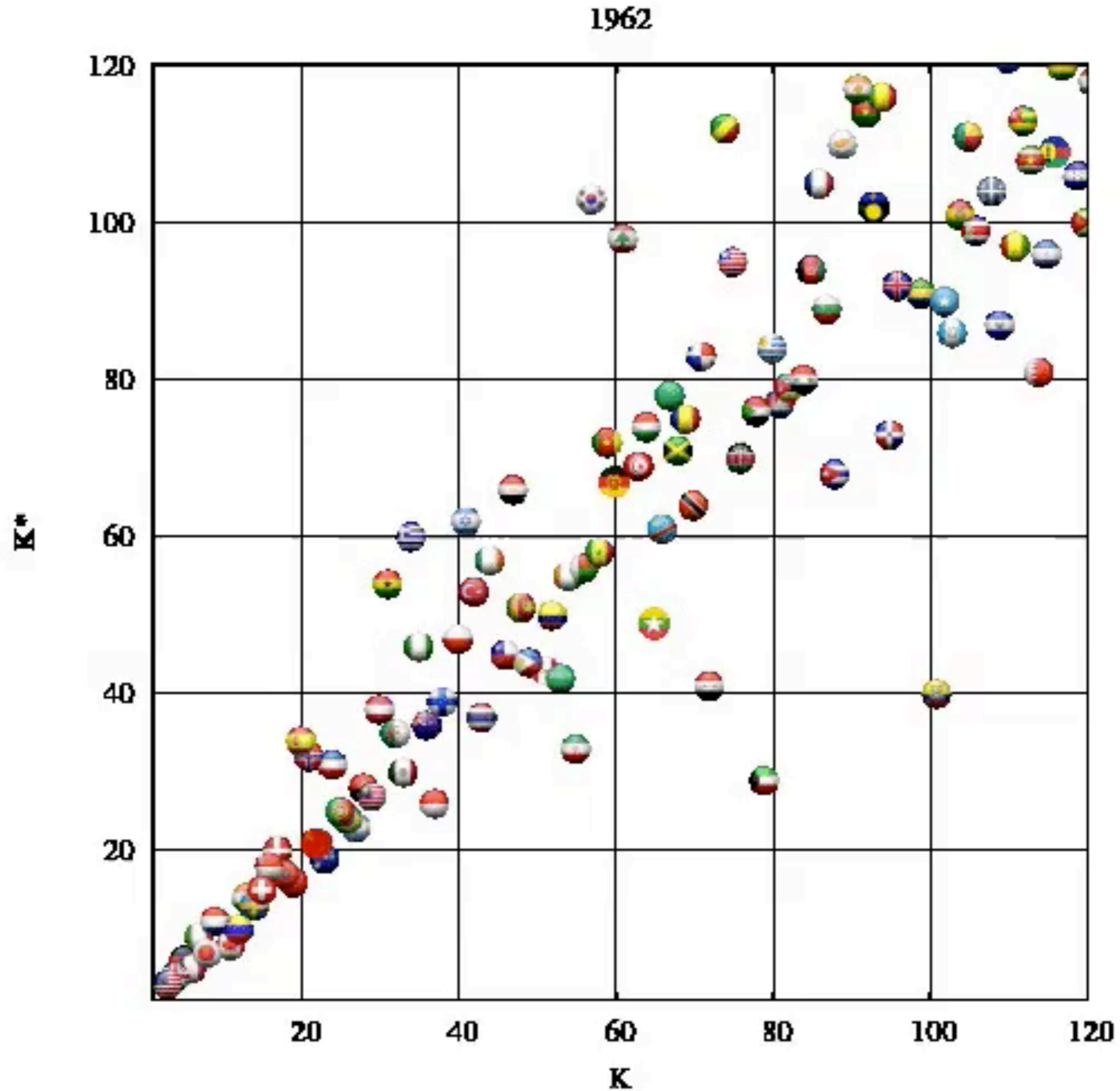
2d rank evolution

Velocity square vs. K+K*

$$\Delta v^2 = [K(t) - K(t-1)]^2 + [K^*(t) - K^*(t-1)]^2$$

average per $K + K^*$

average in $[K + K^* - 10, K + K^* + 10]$
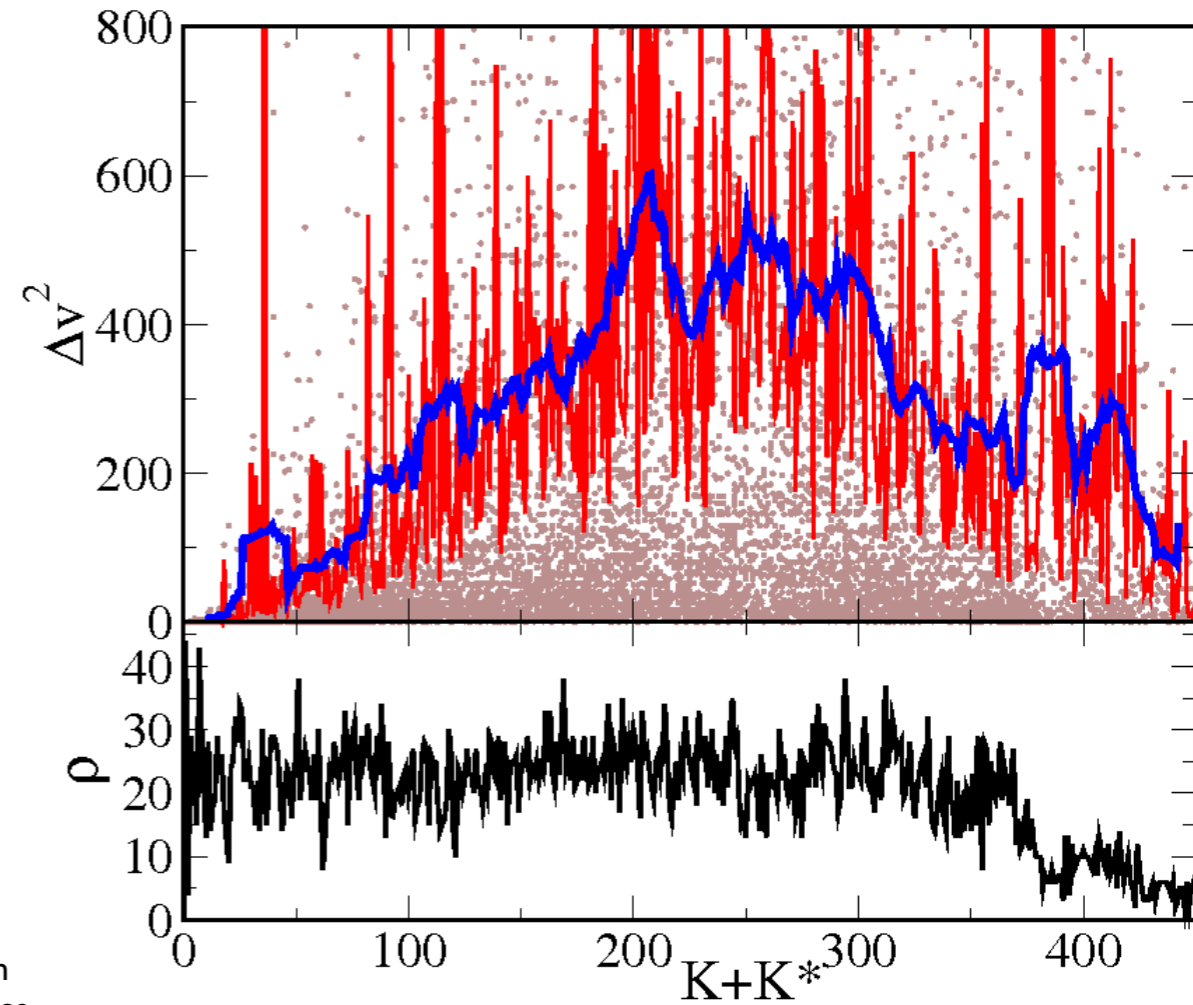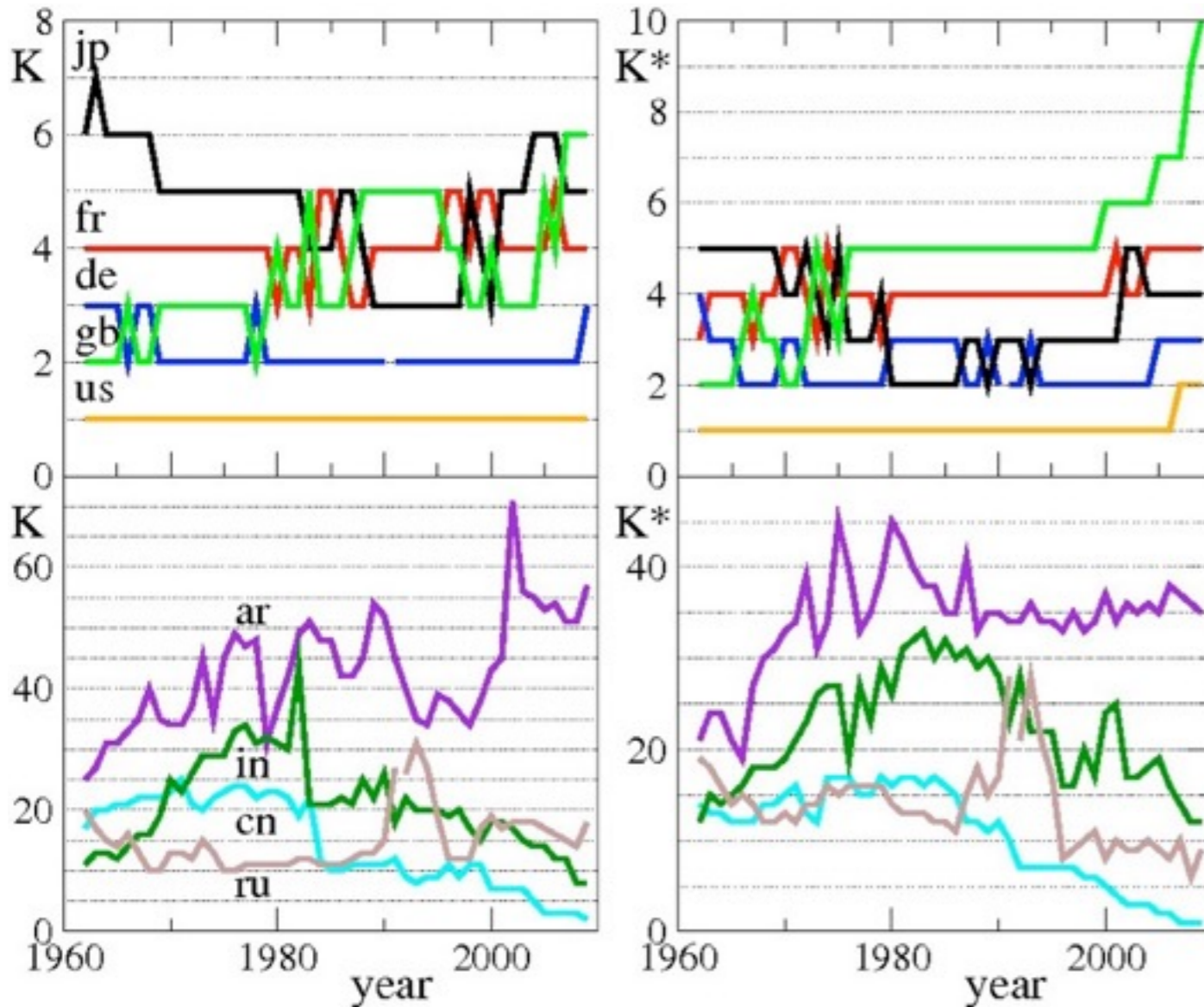
Japan
France
Fed. Rep. of Germany and Germany
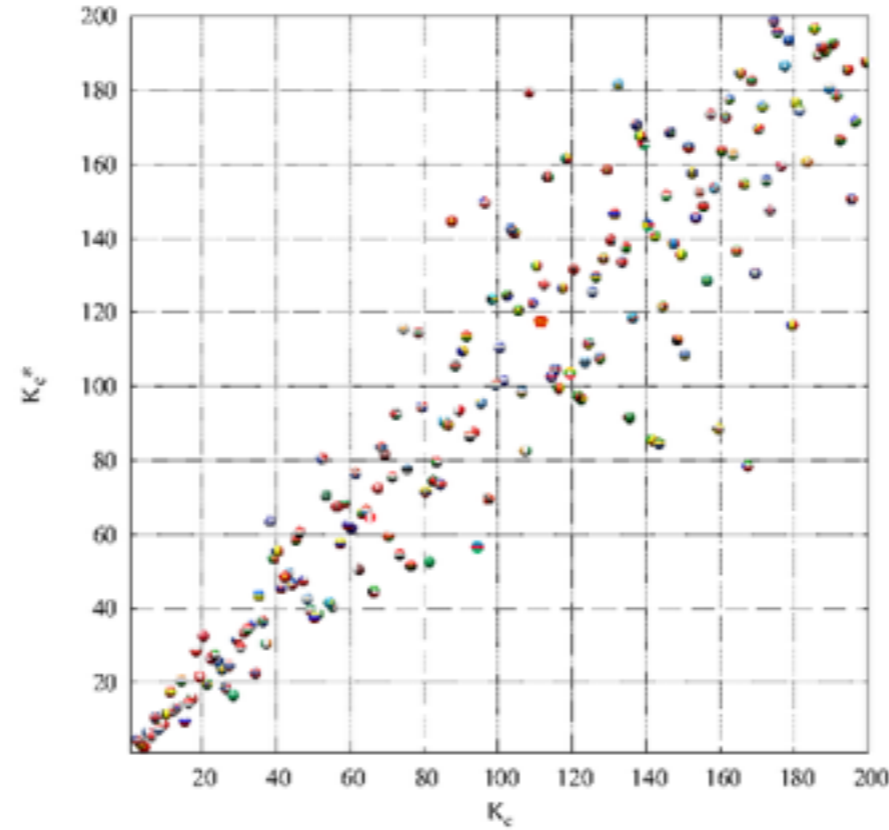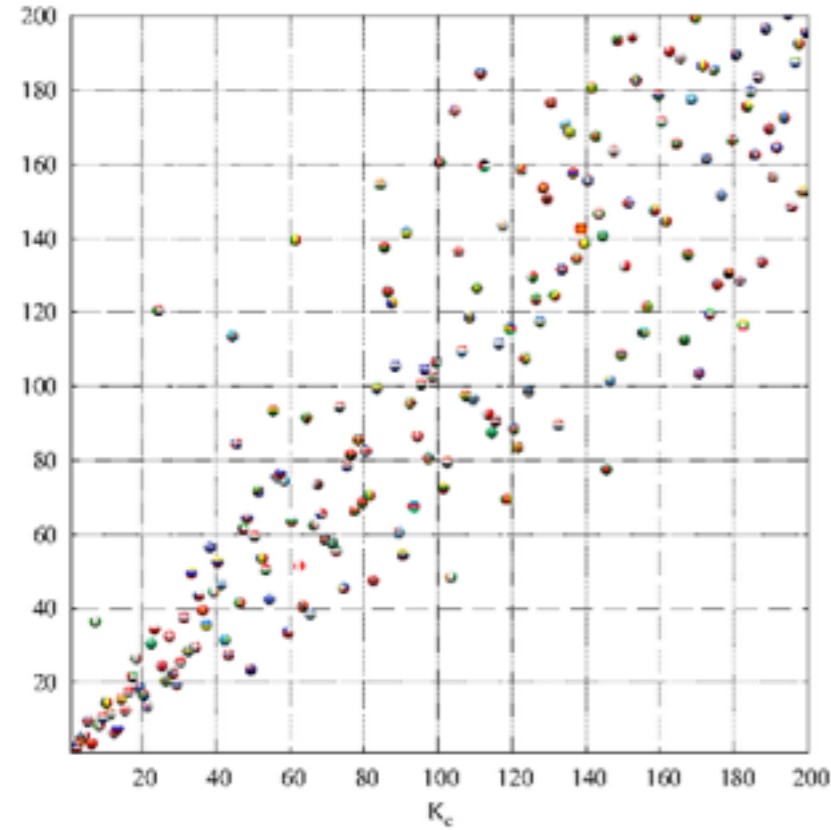Great Britain (sublimation?)
USA

Argentina
 India
China (deposition)
USSR and Russian Fed.

2d ranking of countries (multiproducts)
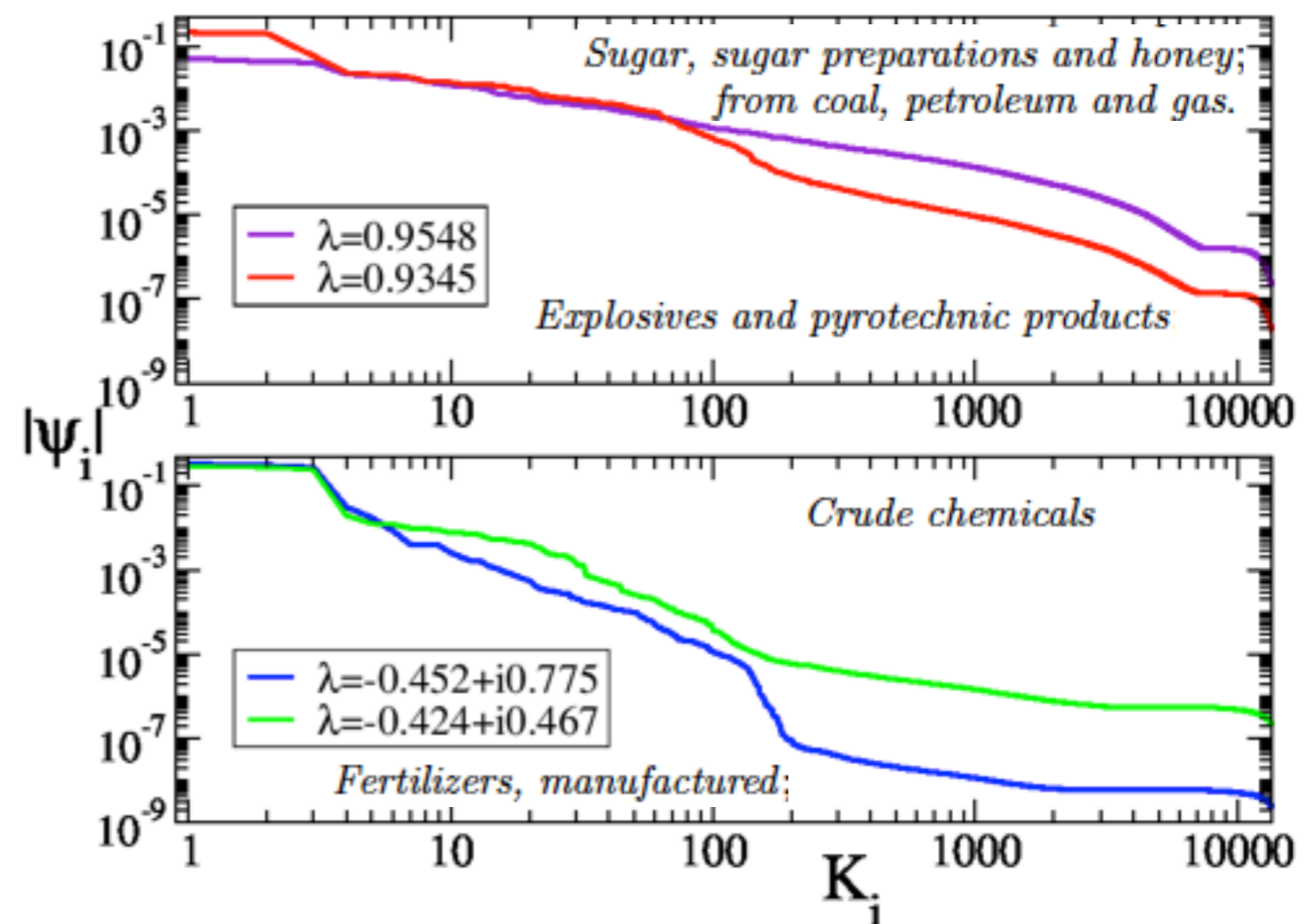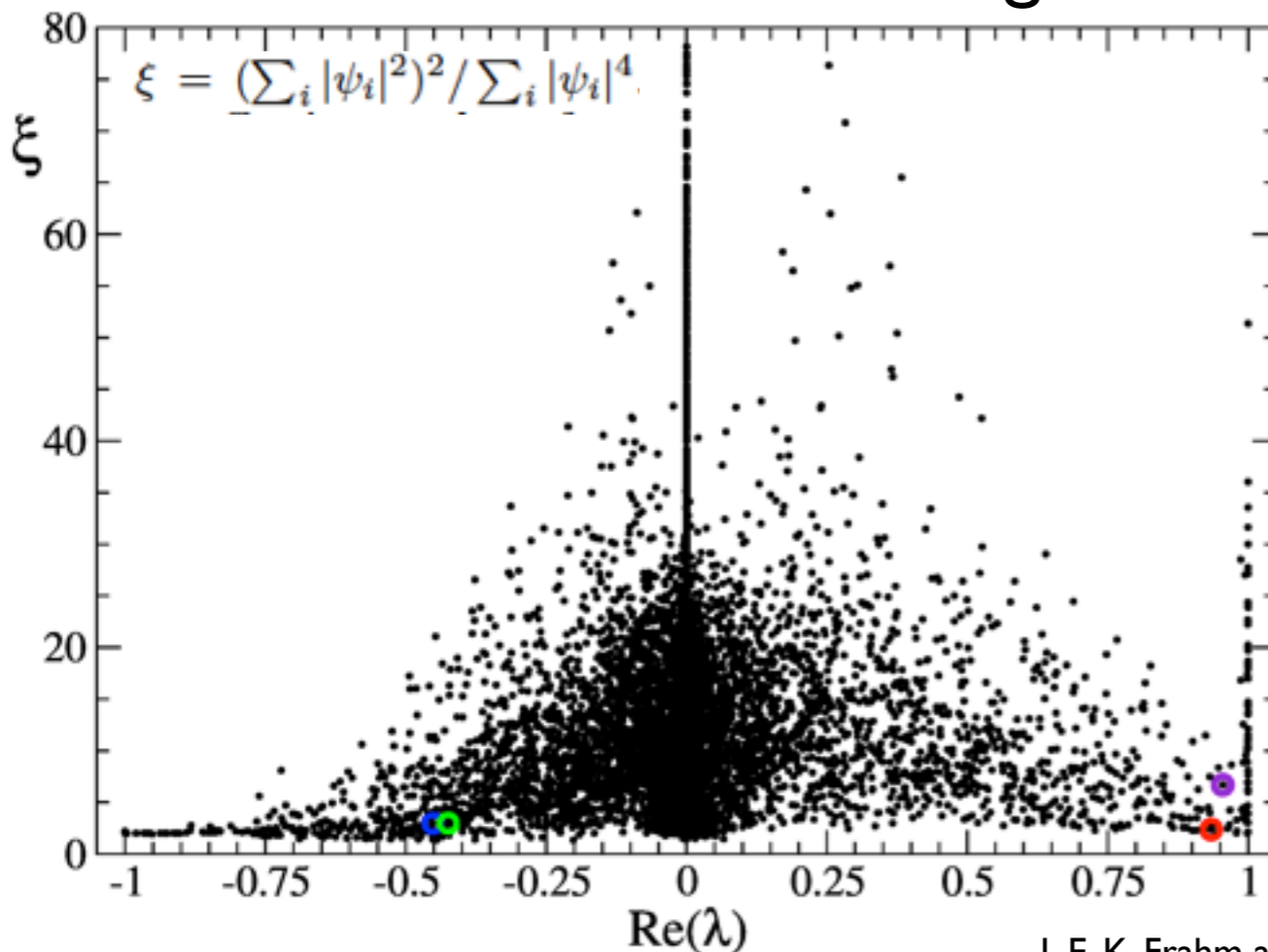

PageRank–CheiRank
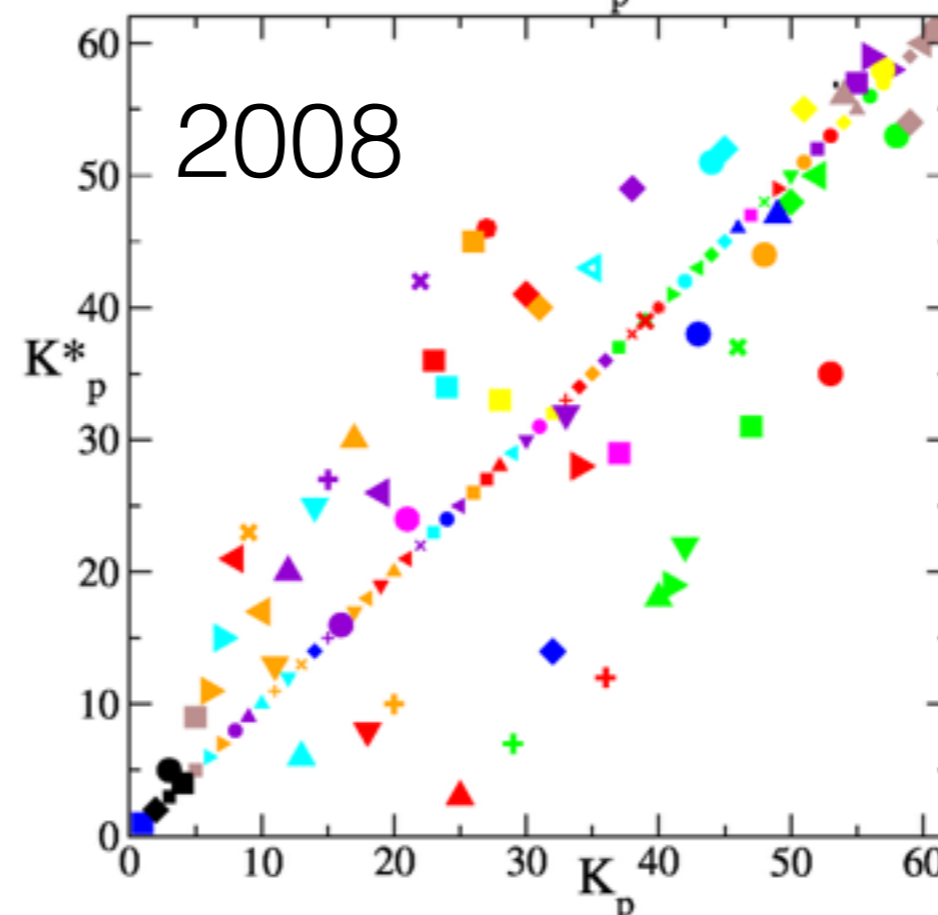
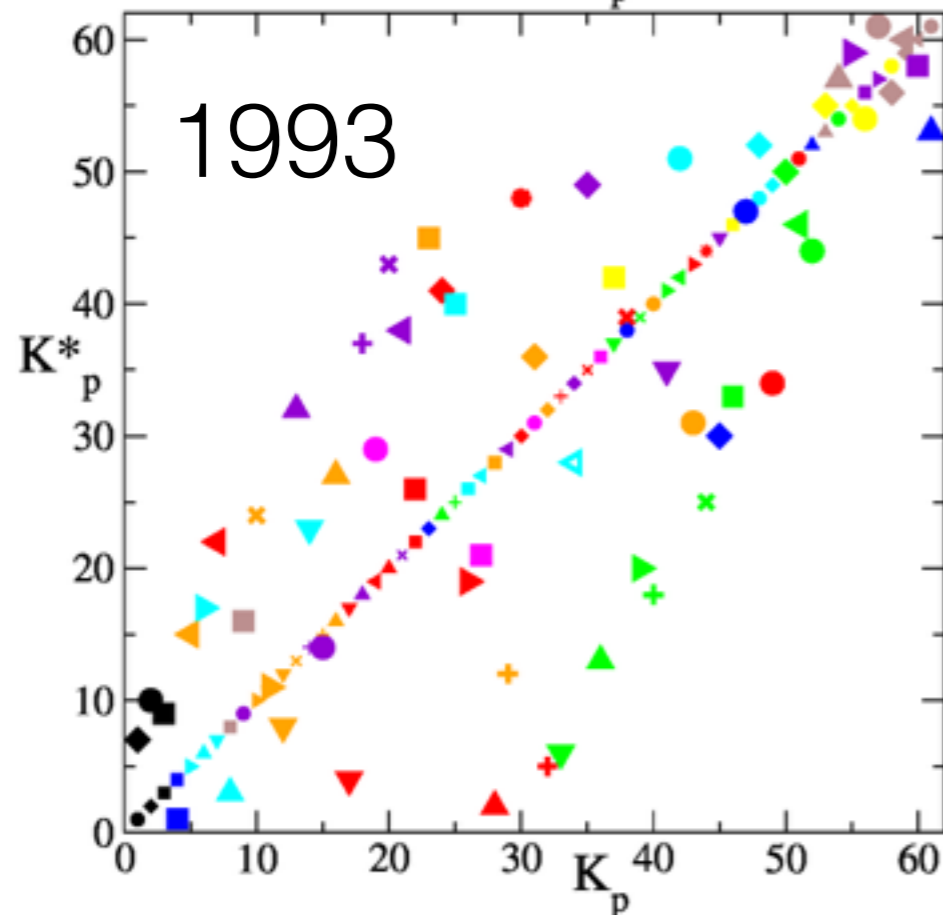ImportRank–ExportRank

PageRank–CheiRank: all commodities
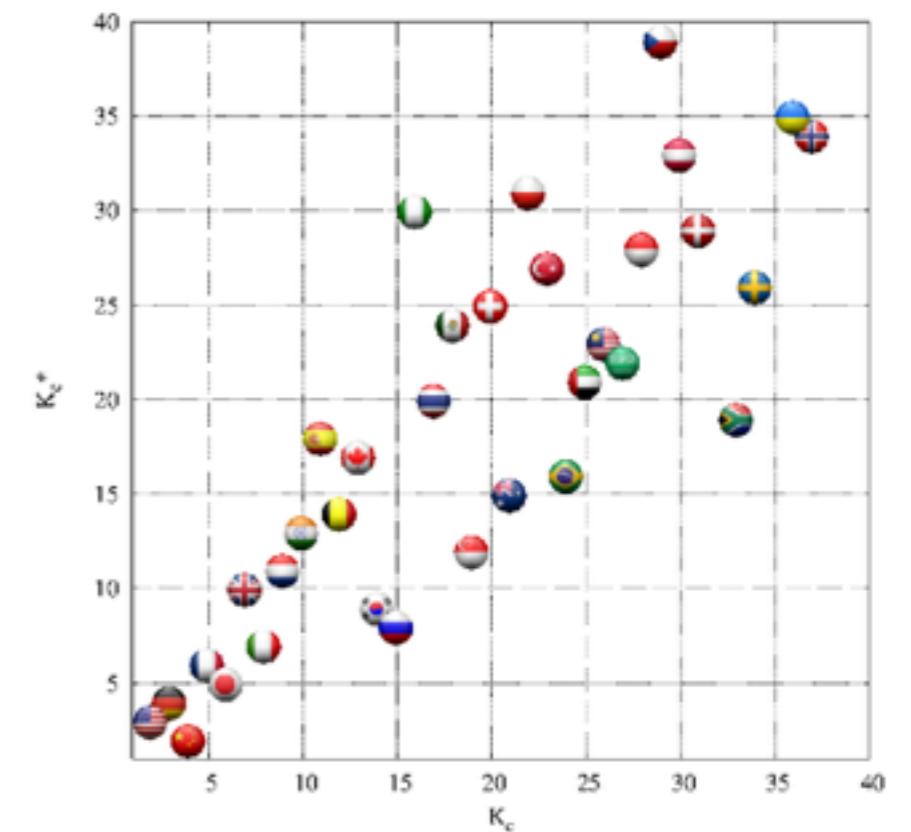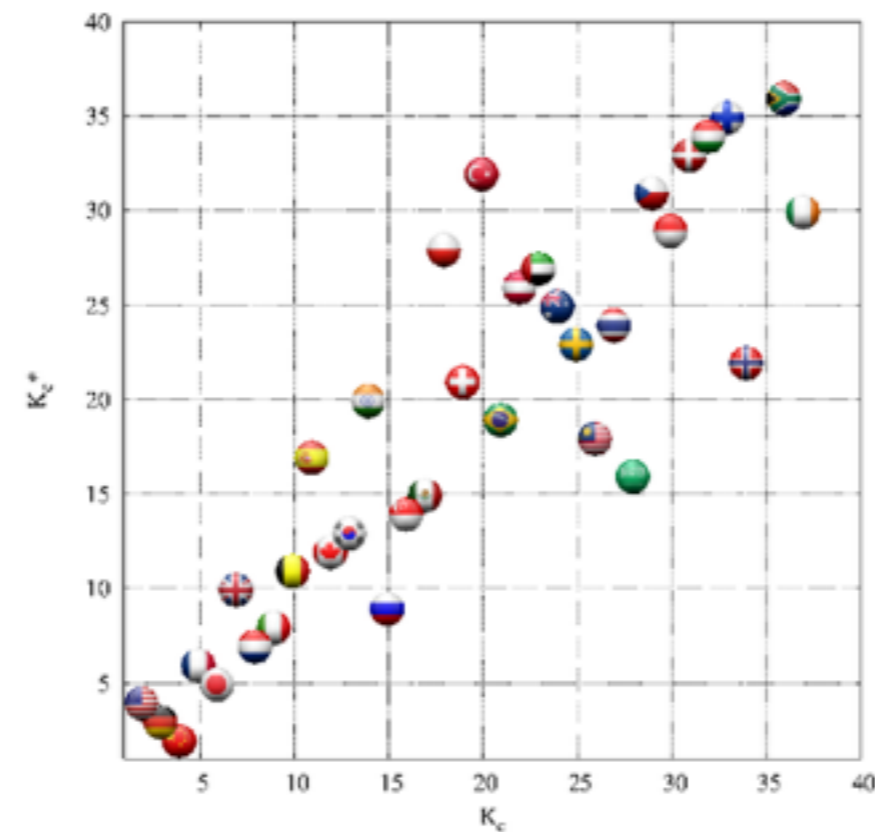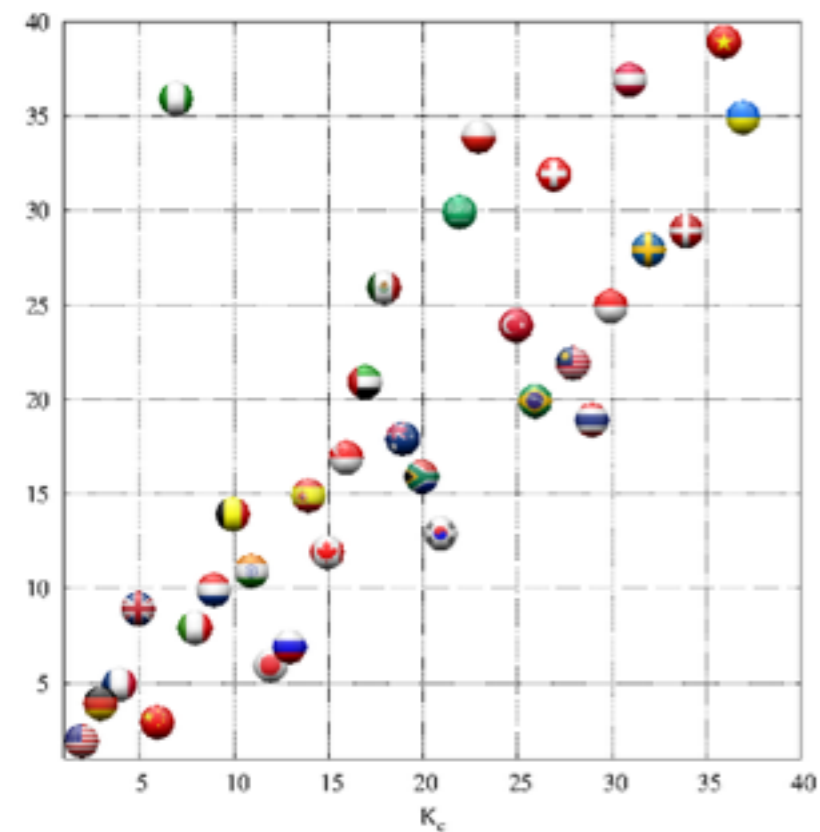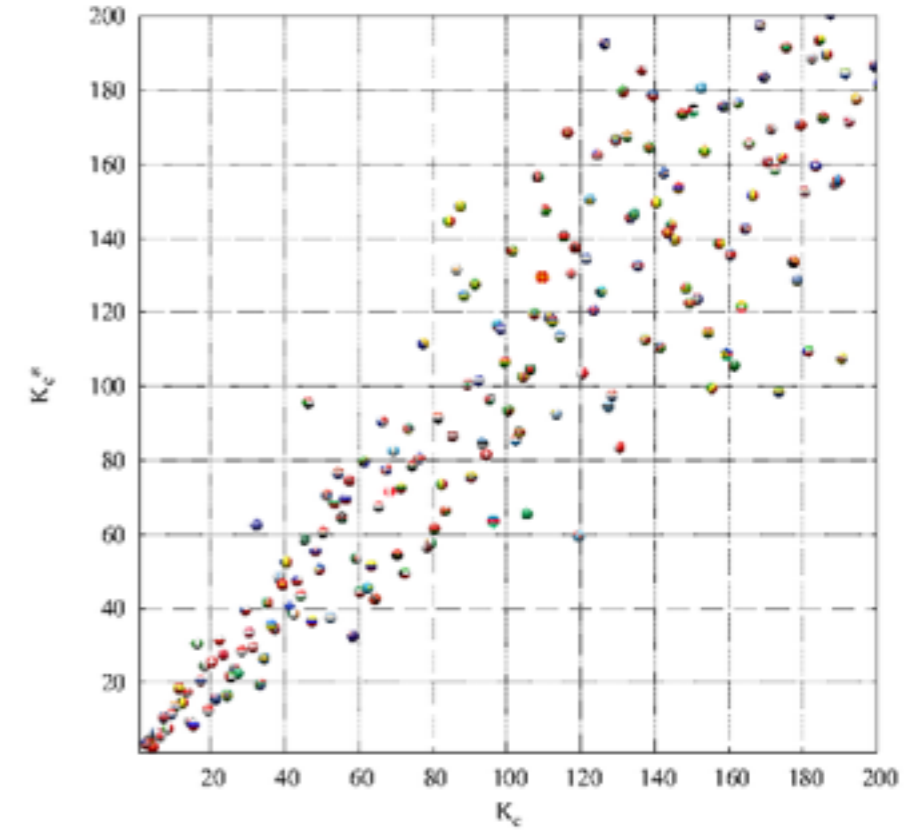
multi-prod WTN spectrum

K_i	$|\psi_i|$	country prod: 57	$|\psi_i|$	country prod:06	$|\psi_i|$	country prod:56	$|\psi_i|$	country prod:52
1	0.052	USA	0.216	Mali	0.332	Brazil	0.288	Japan
2	0.044	Tajikistan	0.201	Guinea	0.304	Bolivia	0.279	Rep. of Korea
3	0.042	Kyrgyzstan	0.059	USA	0.274	Paraguay	0.245	China
4	0.022	France	0.023	Germany	0.031	Argentina	0.020	Australia
5	0.021	Mexico	0.021	Mexico	0.017	Uruguay	0.013	USA
6	0.018	Italy	0.021	Canada	0.009	Chile	0.012	U Arab Em
7	0.018	Canada	0.018	UK	0.004	Portugal	0.010	Canada
8	0.015	Germany	0.015	Israel	0.004	Angola	0.010	Singapore
9	0.013	U Arab Em	0.015	C d'Ivoire	0.004	Spain	0.009	Germany
10	0.012	Qatar	0.014	Japan	0.003	France	0.008	New Zealand

eigenstate communities

$\xi = (\sum_i |\psi_i|^2)^2 / \sum_i |\psi_i|^4$

Sugar, sugar preparations and honey; from coal, petroleum and gas.

λ=0.9548
λ=0.9345

Explosives and pyrotechnic products

Crude chemicals

λ=-0.452+i0.775
λ=-0.424+i0.467

Fertilizers, manufactured;

2d ranking of products

L. ERMANN G matrix analysis of multiprod WTN

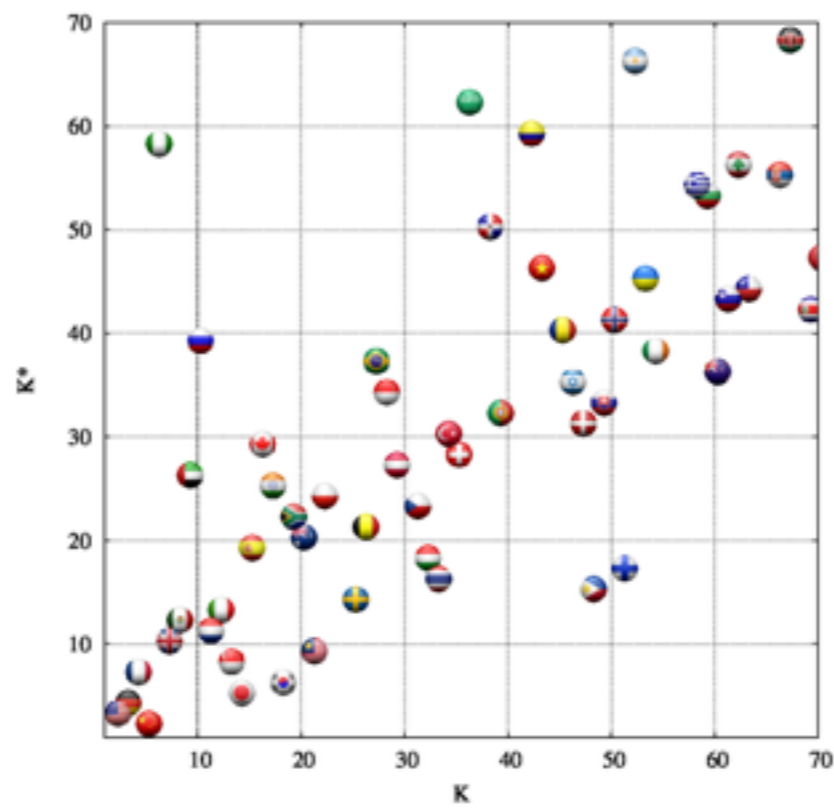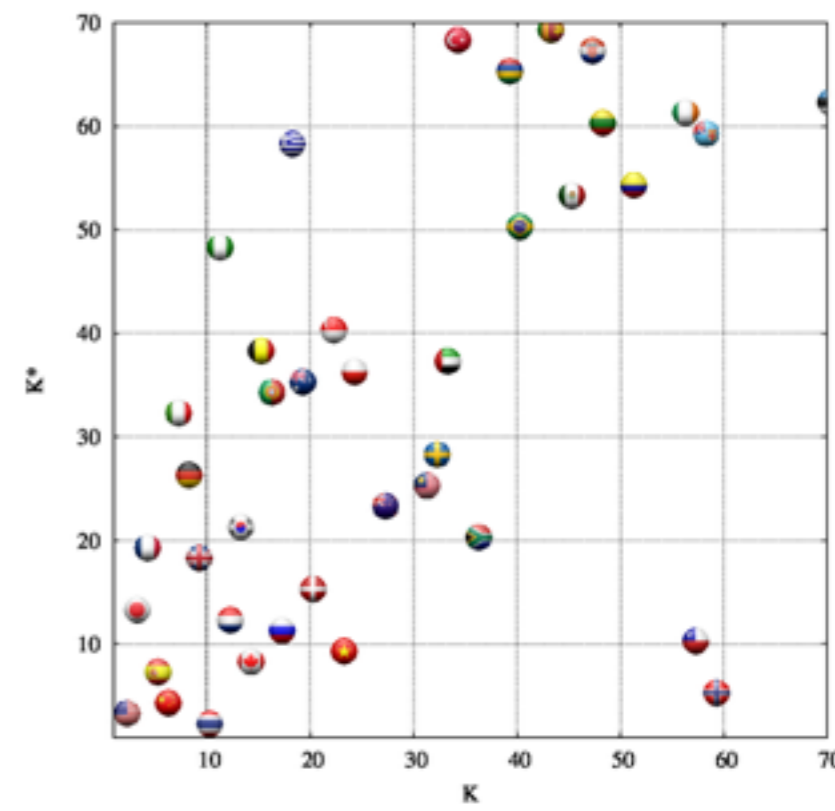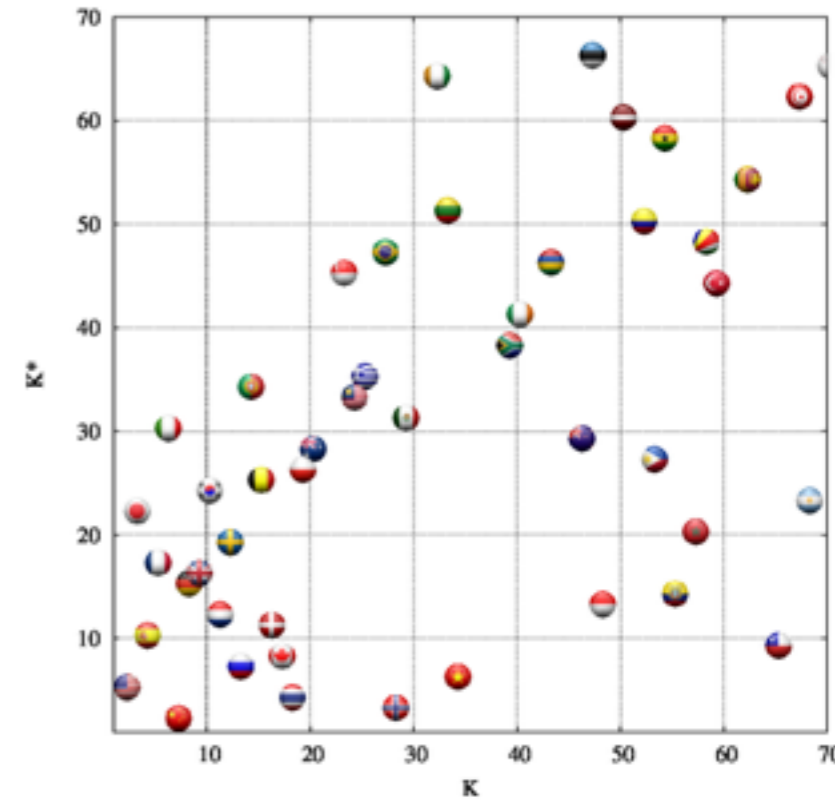# 2d reduced ranks

33 – Petroleum and petroleum products

72 – Electrical machinery, apparatus, and appliances

03 – Fish and fish preparations

P-C

I-E

PageRank CheiRank correlator

$$\kappa = N \sum_i P(i)P^*(i) - 1$$

Legend:
- ▲ Wikipedia
- ✖ Brain Model
- ● British Universities
- ■ Kernel Linux
- ⬢ Yeast Transcription
- ✚ Esch. Coli Transcr.
- ◆ Business Proc. Man.

I-E

P-C

$$\kappa_{pp'} = N_c \sum_{c=1}^{N_c} \left[\frac{P(p + (c-1)N_p)P^*(p' + (c-1)N_p)}{\sum_{c'} P(p + (c'-1)N_p) \sum_{c''} P^*(p' + (c''-1)N_p)} \right] - 1$$

$$\kappa_p = \kappa_{pp'}\delta_{p,p'}$$

Sensitivity to price variation

$$D = dP/d\delta = \Delta P/\delta$$

$$D^* = dP^*/d\check{\delta} = \Delta P^*/\delta.$$

$$\delta' = 0.01, 0.03, 0.05$$

$$\bar{D_l} = D/P \text{ and } \bar{D_l^*} = D^*/P^*$$

$$K_2 \leq 50,$$
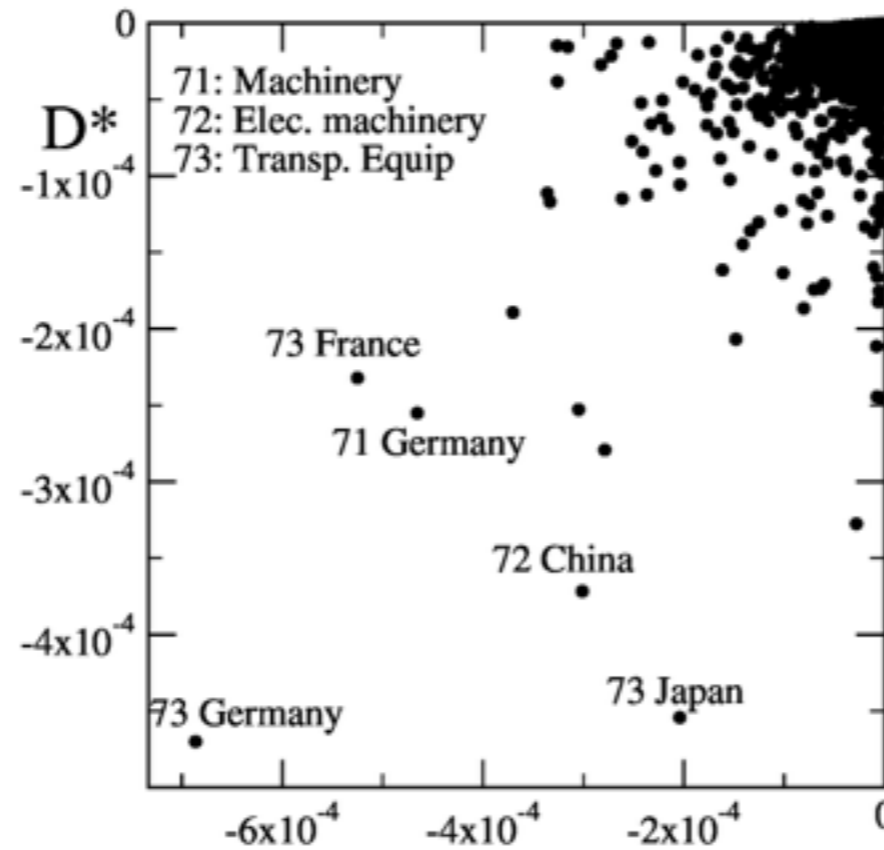
Country balance

$$B_c = \sum_p (P_{cp}^* - P_{cp}) / \sum_p (P_{cp}^* + P_{cp}) = (P_c^* - P_c)/(P_c^* + P_c).$$

P-C

0.44

0.0

-0.52

I-E

0.79

0

-0.95

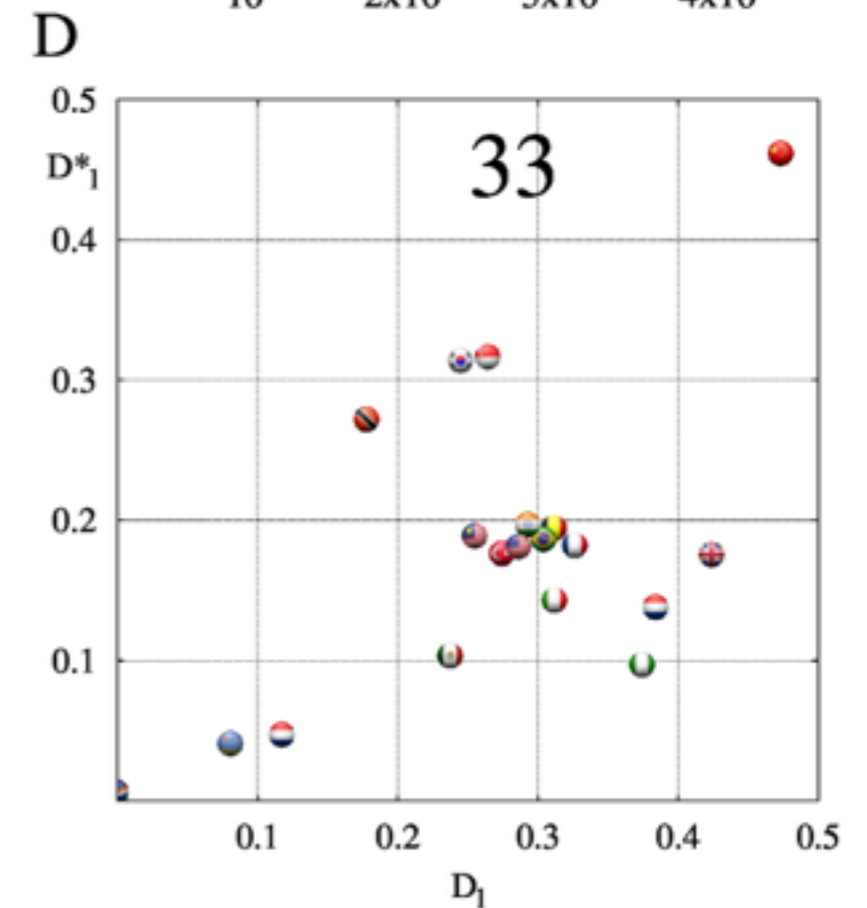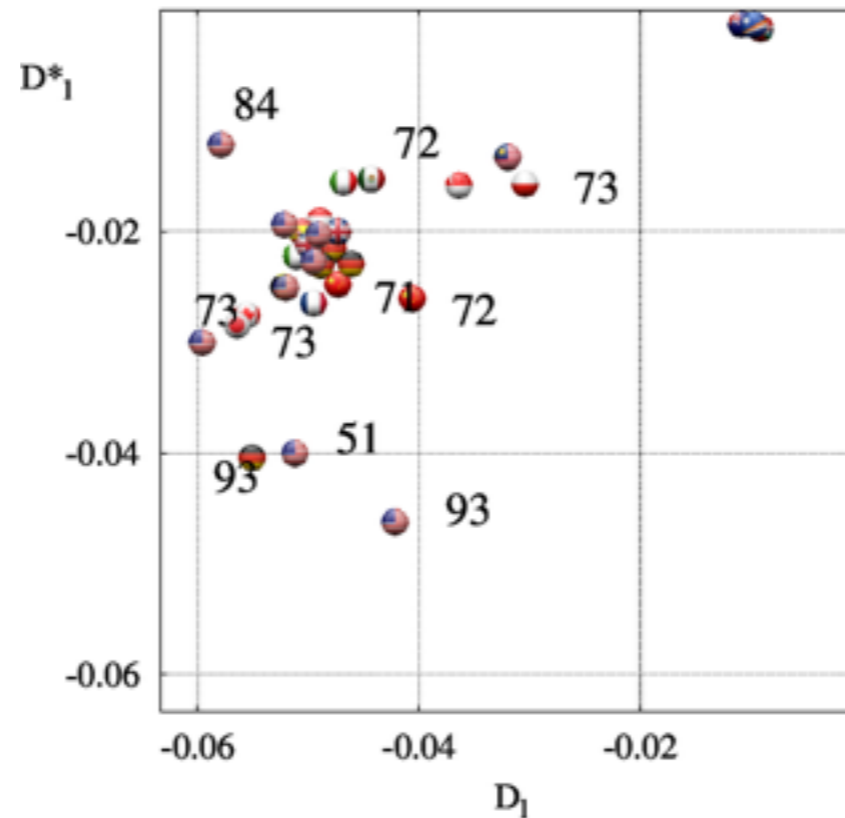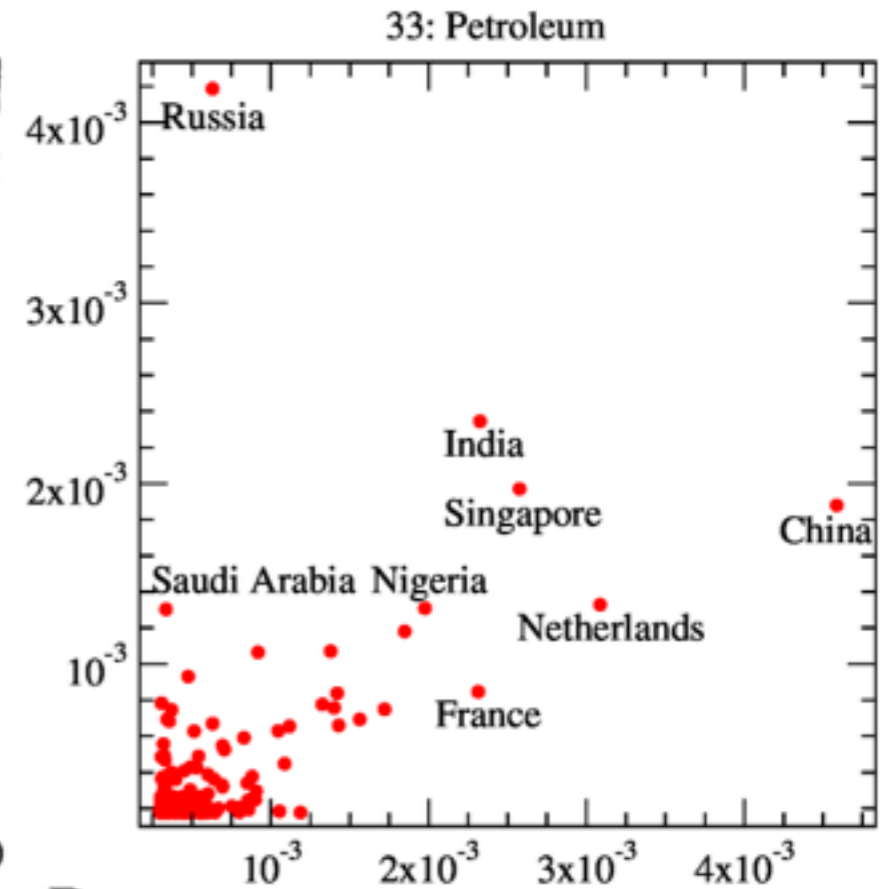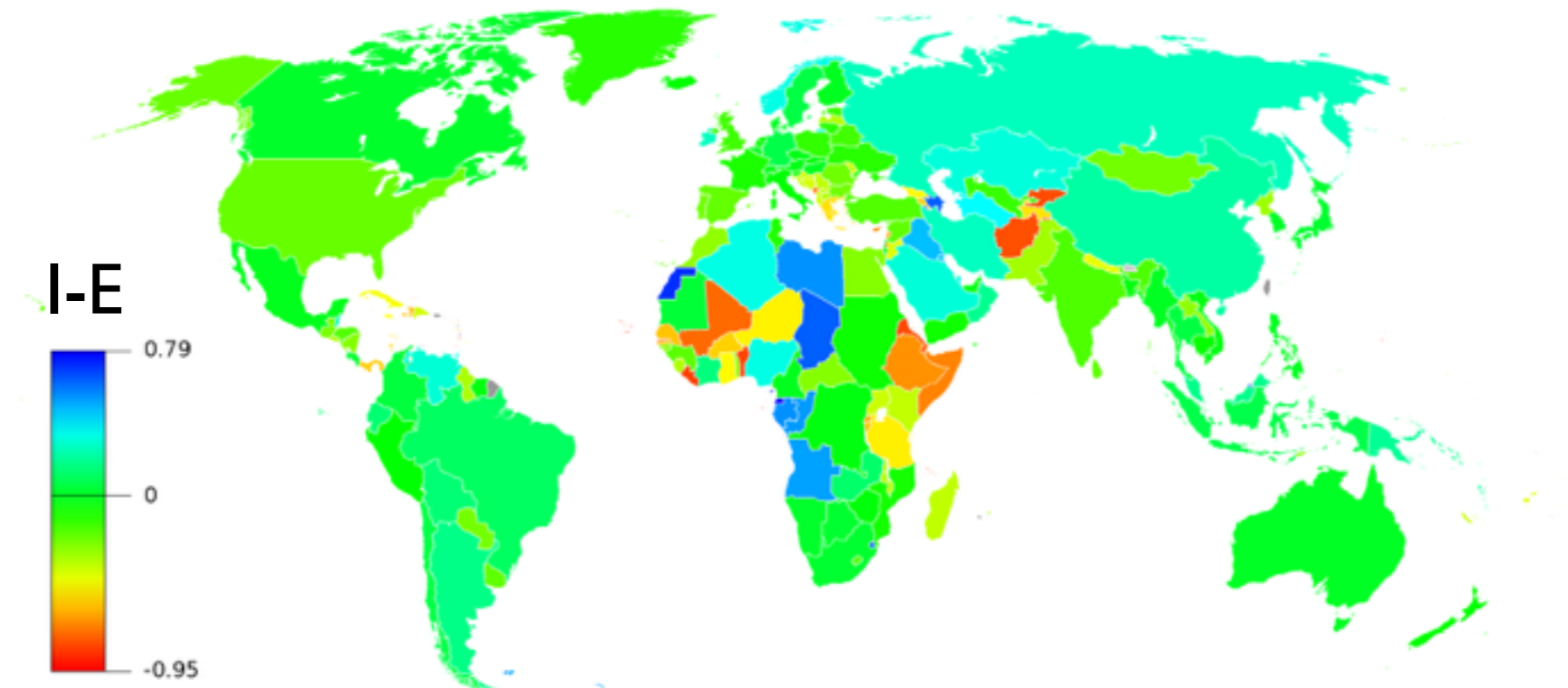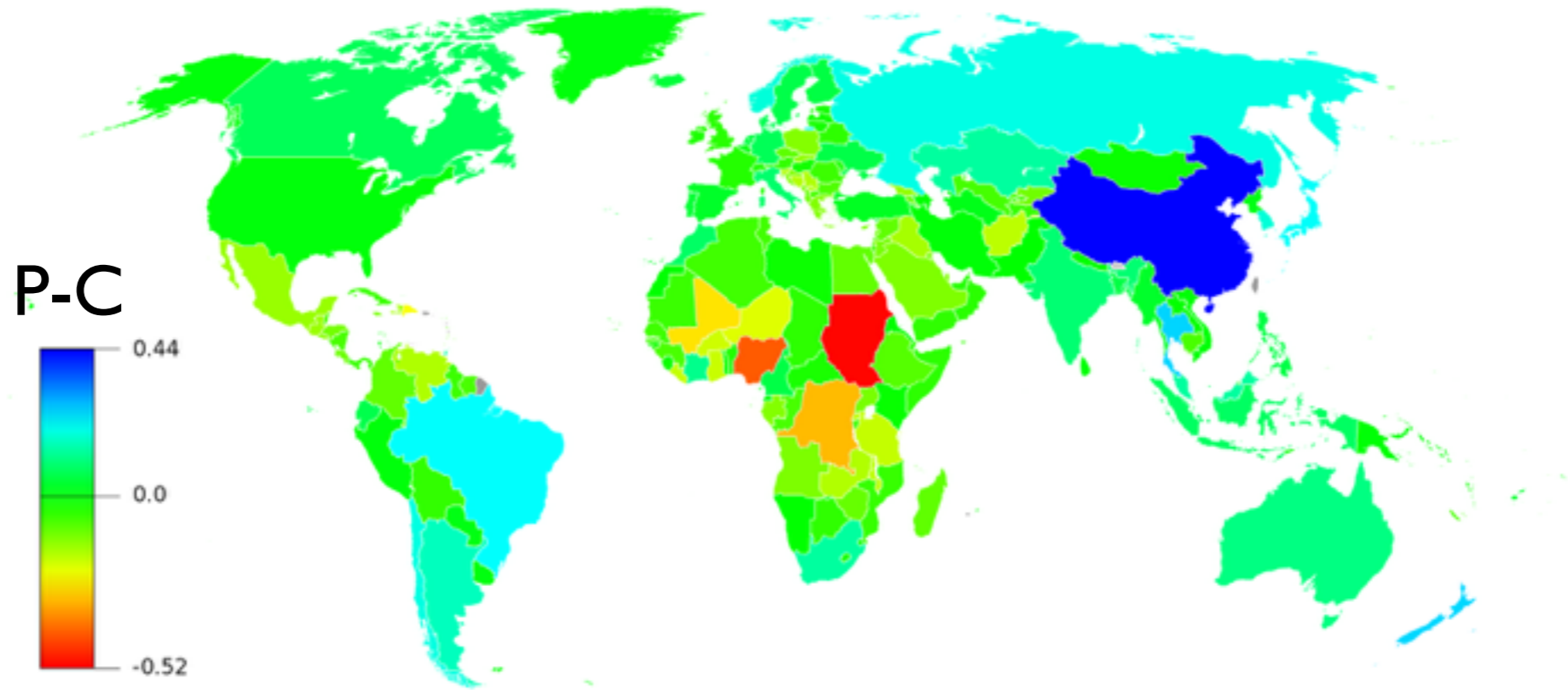# Sensitivity to price variation II
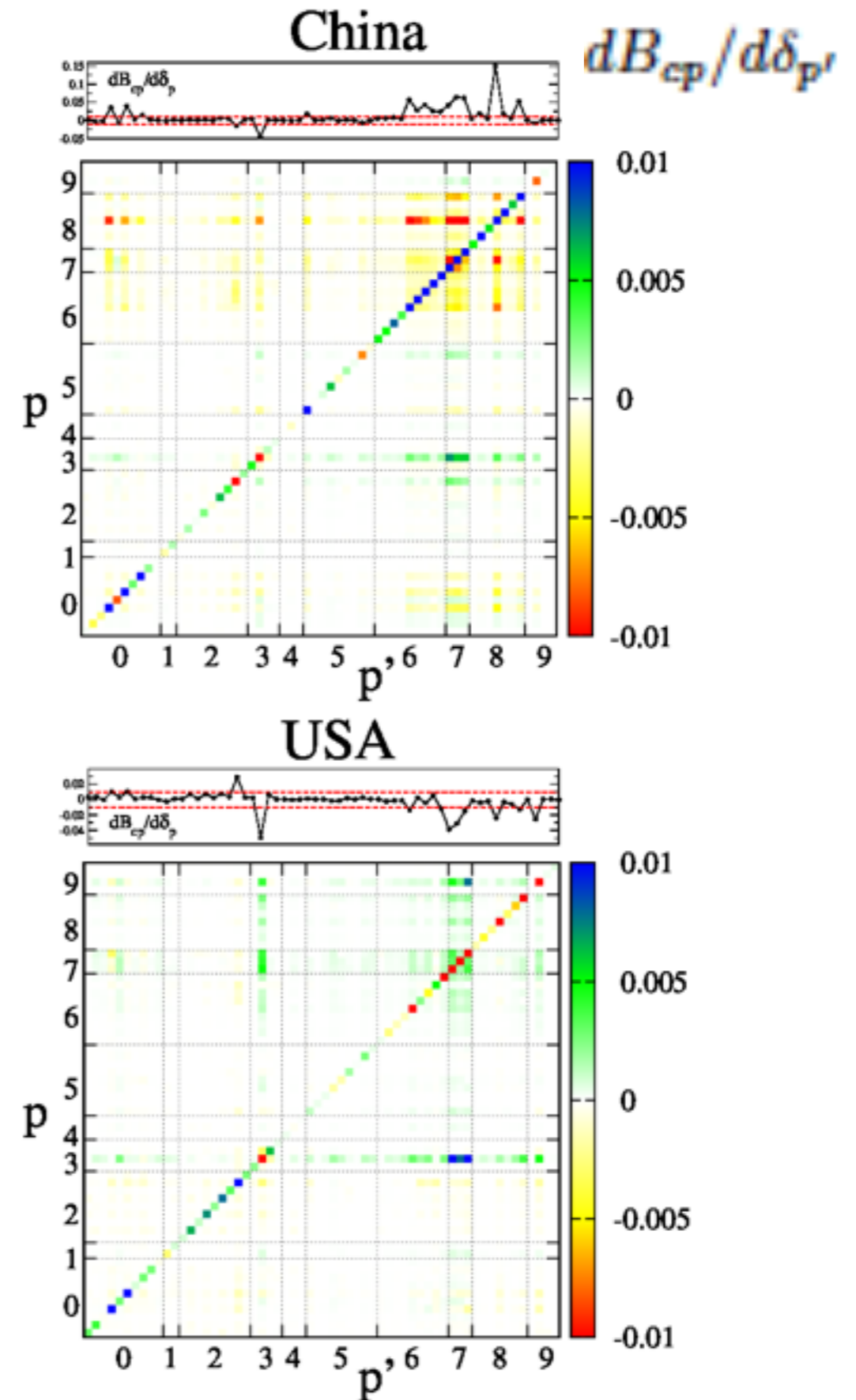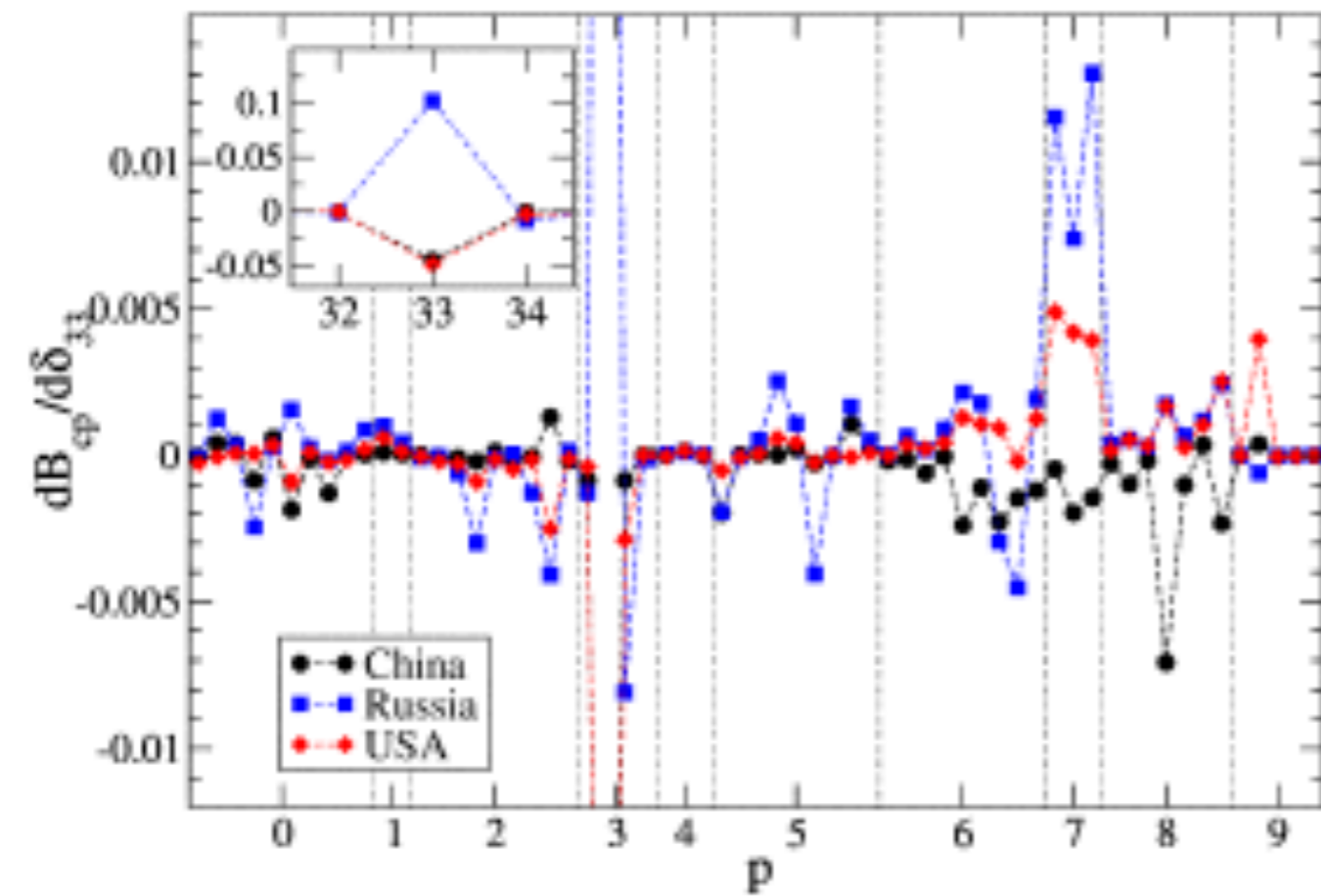
Conclusions

- Google matrix of the WTN (democratic in countries, global network properties):
  1) one product of all comm. (Nc)
  2) multiprod (Nc x Np)
  2d-ranking, spectrum, communities in eigenstates, correlation between P-C, comparison with I-E, new tool for trade analysis
- Asymmetry in products
- Time evolution analysis
- Sensitivity to price variation (weak coupling between products) would lead to prediction of crisis and time evolution

Thank you

2d global ranks
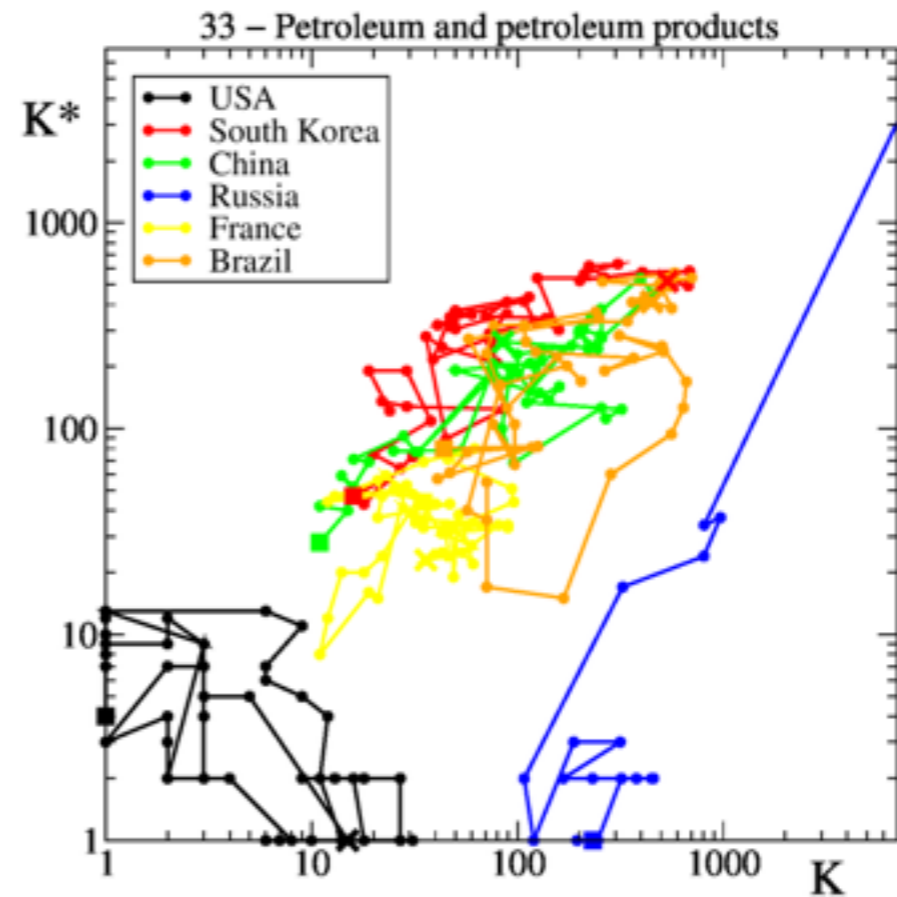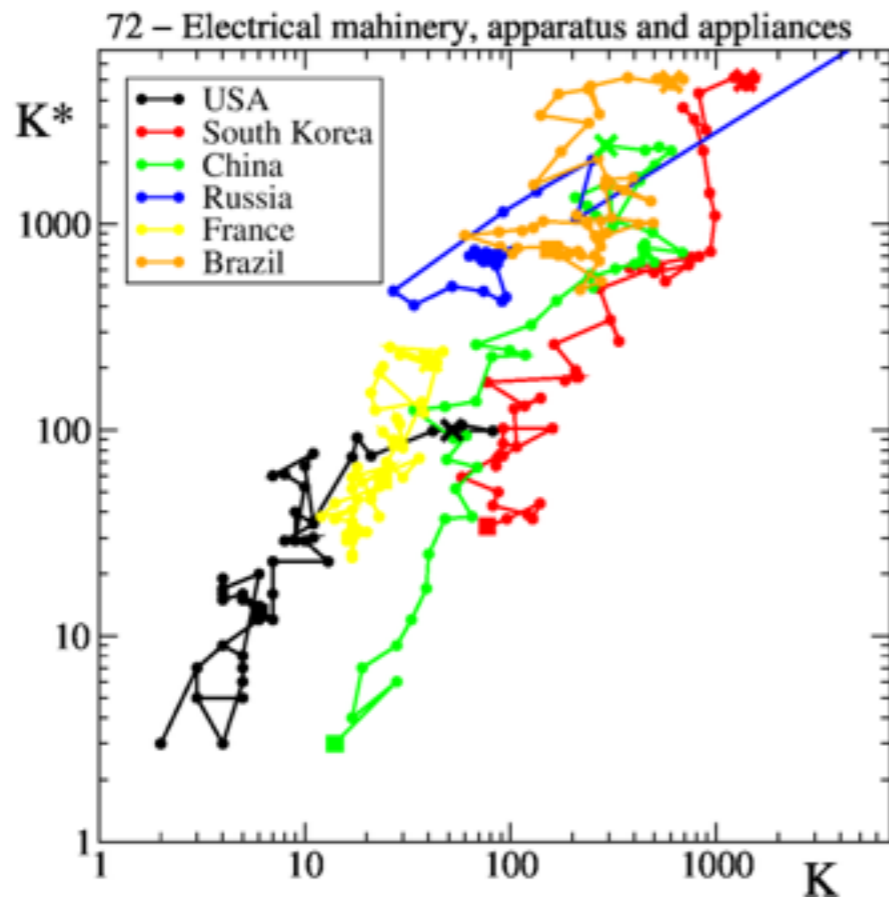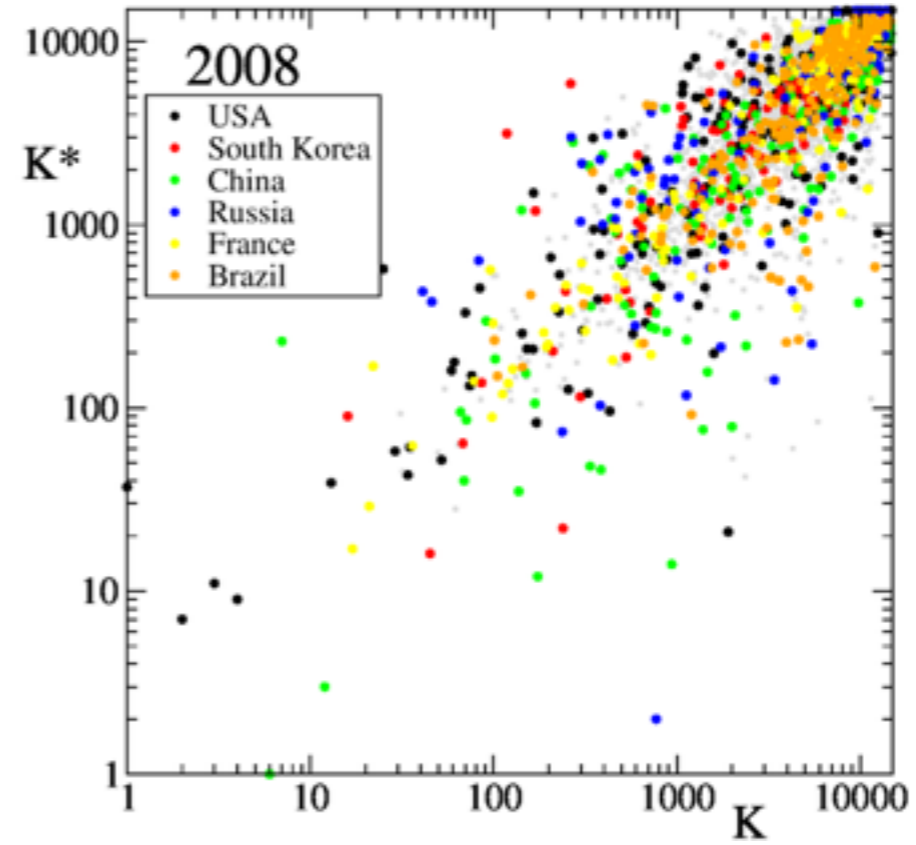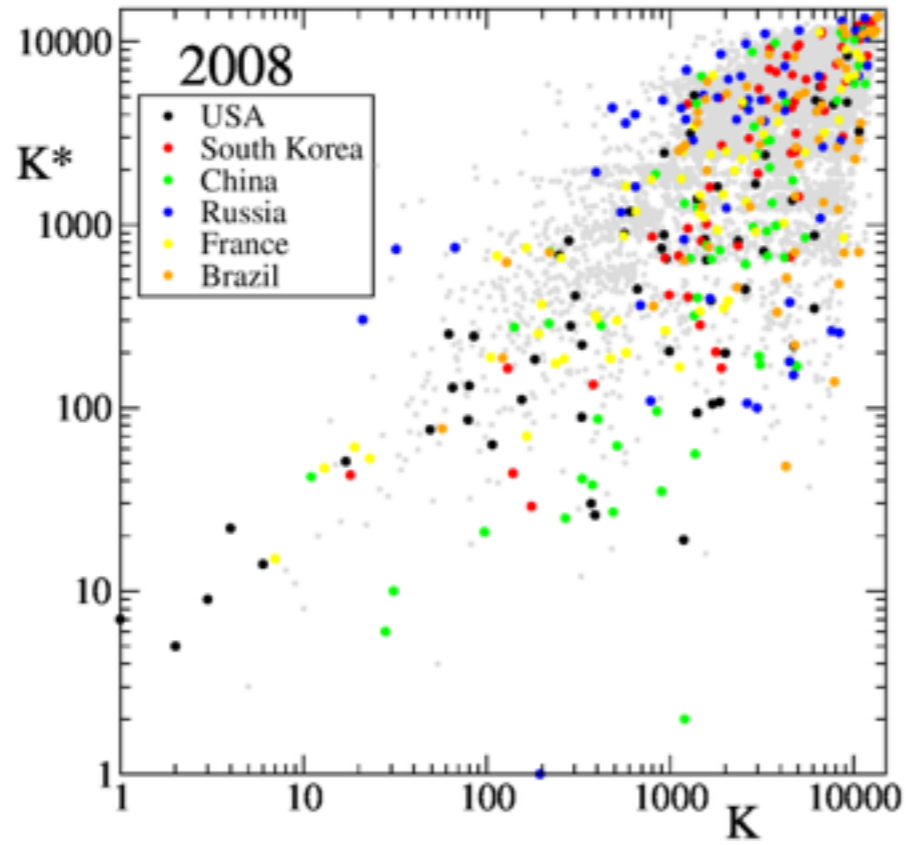
2d global ranks
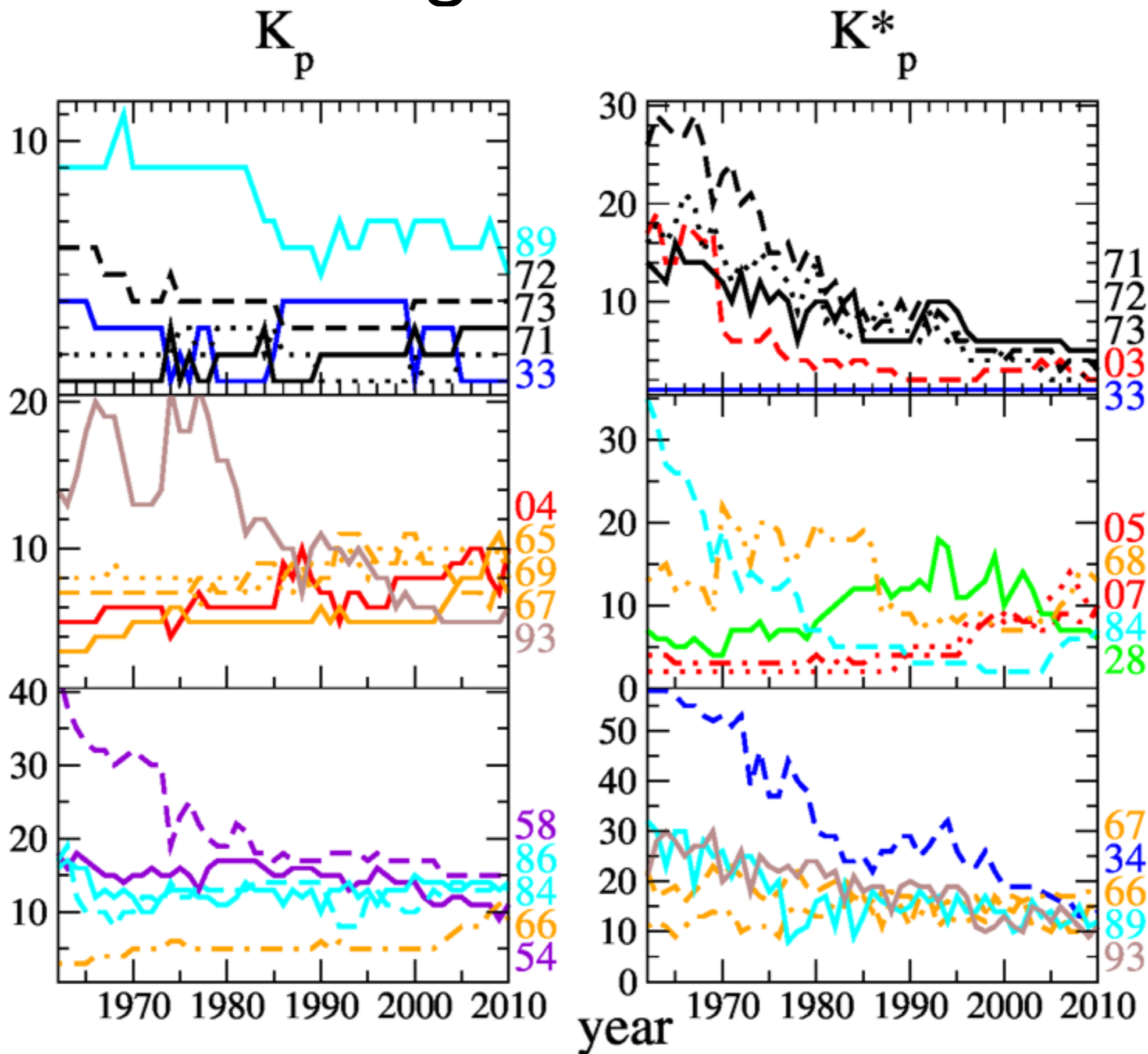
2d global ranks

product names

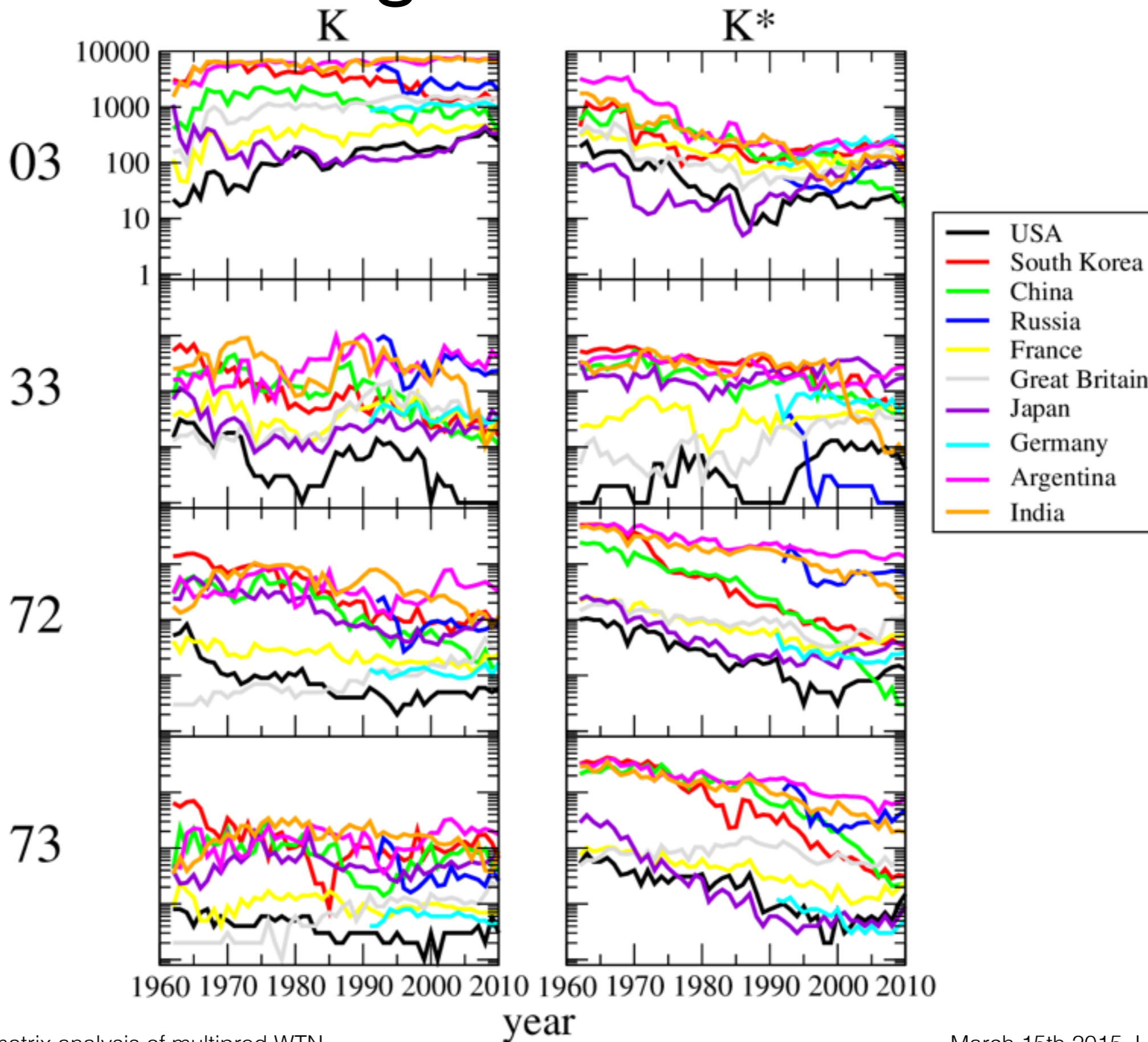| code	name	code	name
00	Live animals	54	Medicinal and pharmaceutical products
01	Meat and meat preparations	55	Perfume materials, toilet & cleansing preptions
02	Dairy products and eggs	56	Fertilizers, manufactured
03	Fish and fish preparations	57	Explosives and pyrotechnic products
04	Cereals and cereal preparations	58	Plastic materials, etc.
05	Fruit and vegetables	59	Chemical materials and products, nes
06	Sugar, sugar preparations and honey	61	Leather, lthr. Manufs., nes & dressed fur skins
07	Coffee, tea, cocoa, spices & manufacs. Thereof	62	Rubber manufactures, nes
08	Feed. Stuff for animals excl. Unmilled cereals	63	Wood and cork manufactures excluding furniture
09	Miscellaneous food preparations	64	Paper, paperboard and manufactures thereof
11	Beverages	65	Textile yarn, fabrics, made up articles, etc.
12	Tobacco and tobacco manufactures	66	Non metallic mineral manufactures, nes
21	Hides, skins and fur skins, undressed	67	Iron and steel
22	Oil seeds, oil nuts and oil kernels	68	Non ferrous metals
23	Crude rubber including synthetic and reclaimed	69	Manufactures of metal, nes
24	Wood, lumber and cork	71	Machinery, other than electric
25	Pulp and paper	72	Electrical machinery, apparatus and appliances
26	Textile fibres, not manufactured, and waste	73	Transport equipment
27	Crude fertilizers and crude minerals, nes	81	Sanitary, plumbing, heating and lighting fixt.
28	Metalliferous ores and metal scrap	82	Furniture
29	Crude animal and vegetable materials, nes	83	Travel goods, handbags and similar articles
32	Coal, coke and briquettes	84	Clothing
33	Petroleum and petroleum products	85	Footwear
34	Gas, natural and manufactured	86	Scientif & control instrum, photogr gds, clocks
35	Electric energy	89	Miscellaneous manufactured articles, nes
41	Animal oils and fats	91	Postal packages not class. According to kind
42	Fixed vegetable oils and fats	93	Special transact. Not class. According to kind
43	Animal and vegetable oils and fats, processed	94	Animals, nes, incl. Zoo animals, dogs and cats
51	Chemical elements and compounds	95	Firearms of war and ammunition therefor
52	Crude chemicals from coal, petroleum and gas	96	Coin, other than gold coin, not legal tender
53	Dyeing, tanning and colouring materials		