

Relaxation of isolated quantum systems beyond chaos

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**Quantum chaos: fundamentals and
applications**

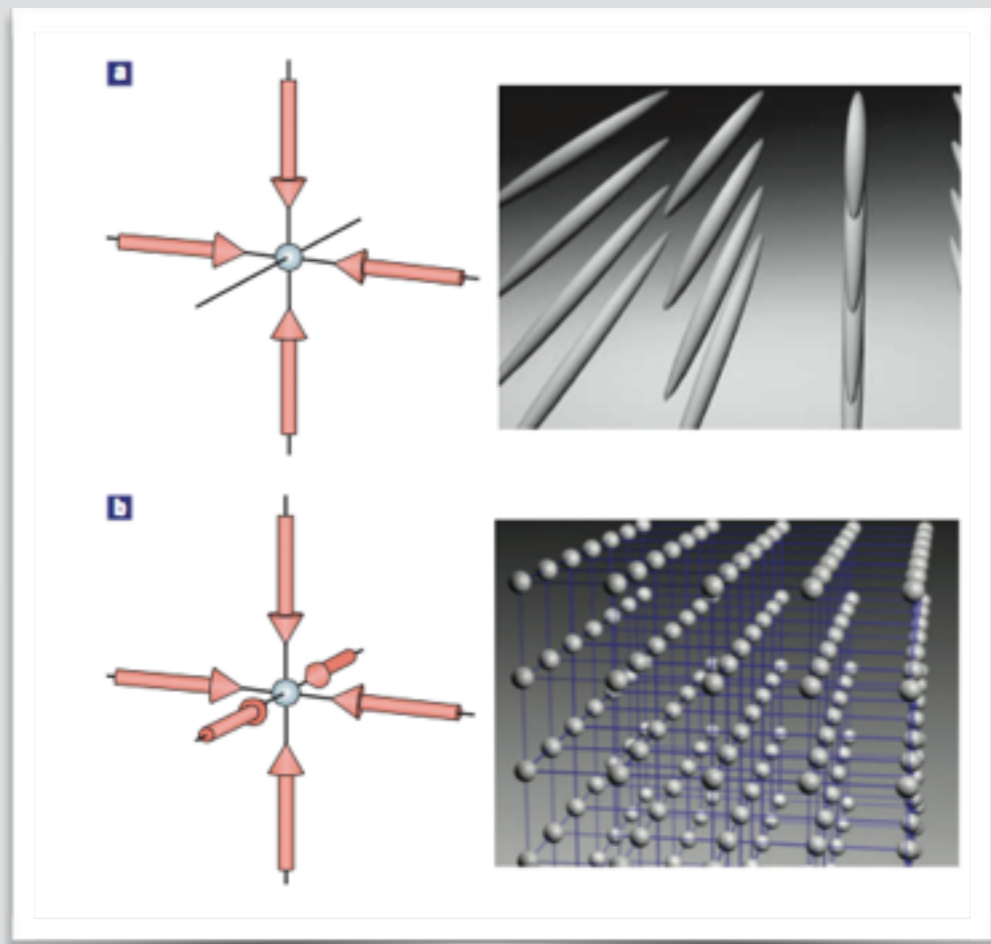
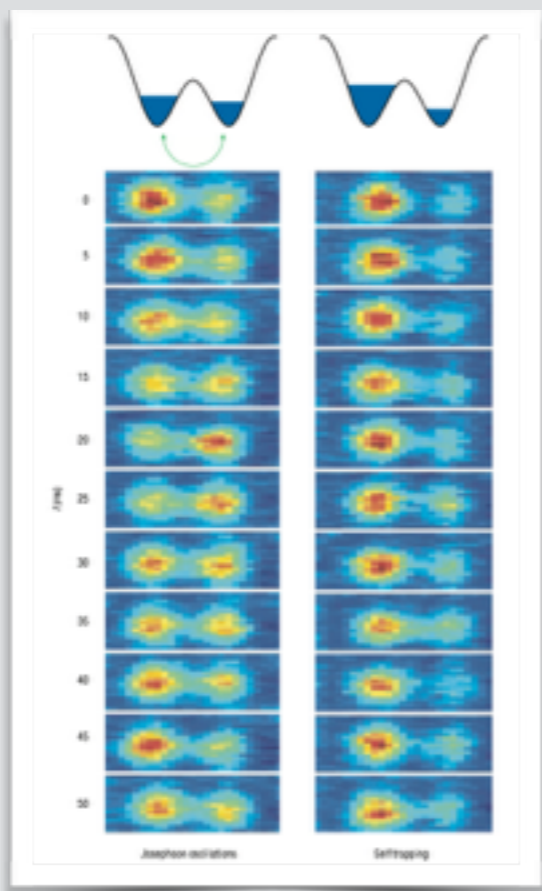
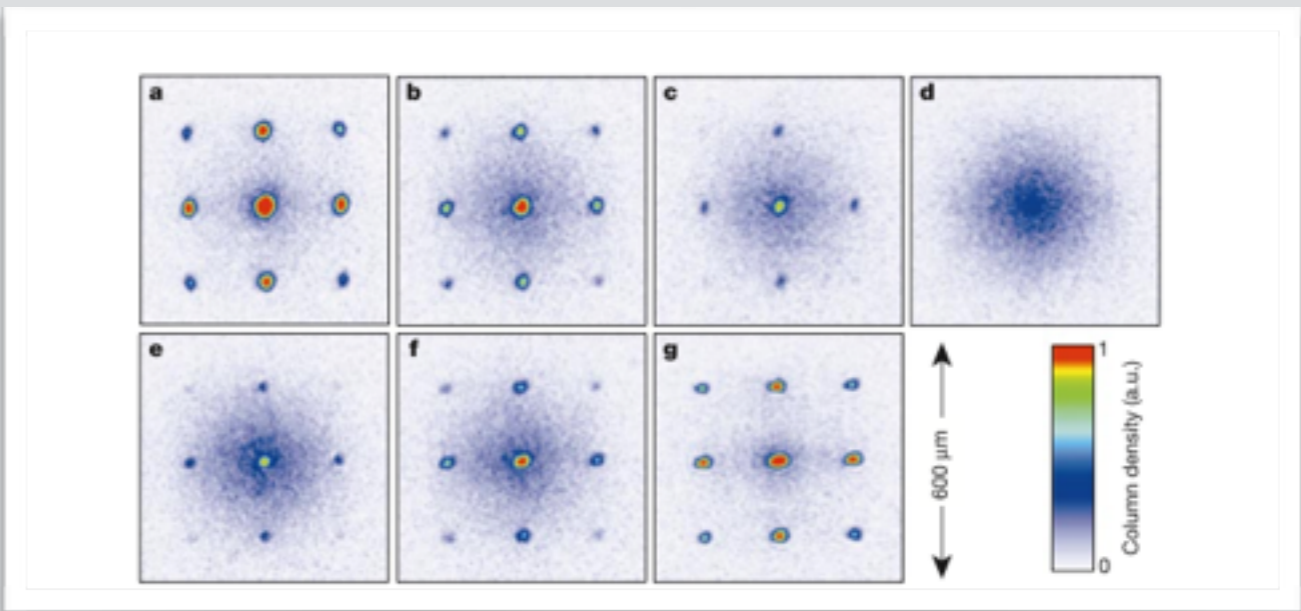
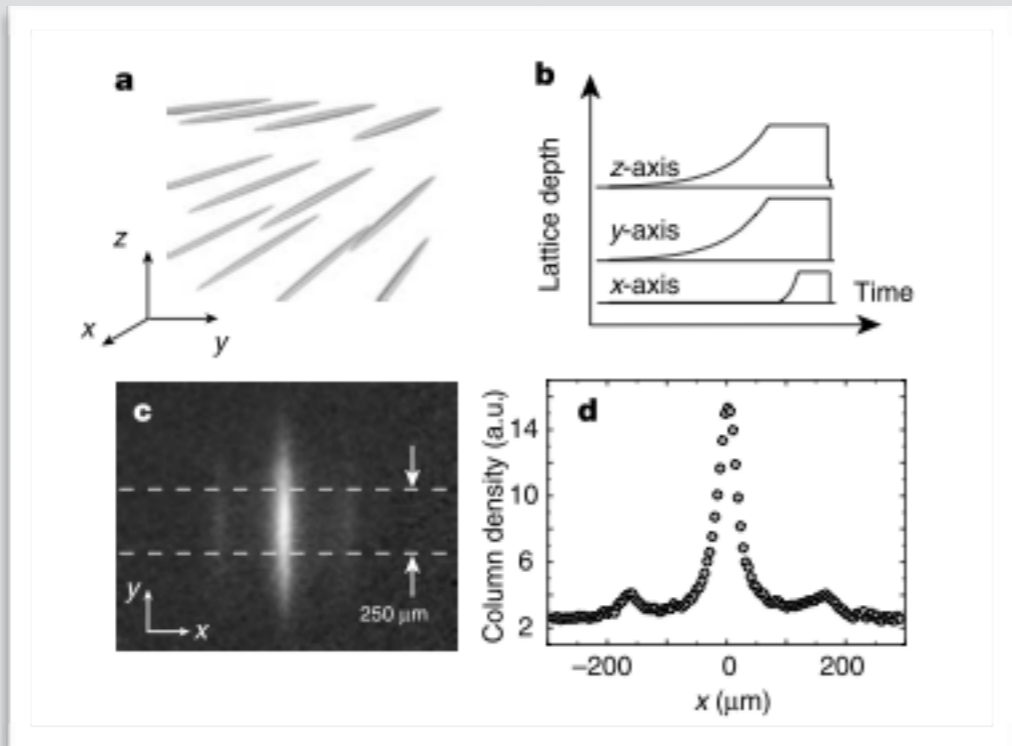
Session Workshop I (W1), March 14 - 21, 2015





Buenos Aires

Mar del Plata



Isolated Quantum systems

non equilibrium dynamics — relaxation & universality

thermalization

Chaos and quantum thermalization

Mark Srednicki*

Department of Physics, University of California, Santa Barbara, Santa Barbara, California 93106

(Received 21 March 1994)

Brief Reports

Brief Reports are accounts of completed research which do not warrant regular articles or the priority handling given to Rapid Communications; however, the same standards of scientific quality apply. (Addenda are included in Brief Reports.) A Brief Report may be no longer than 3½ printed pages and must be accompanied by an abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.

Quantum statistical mechanics in a closed system

J. M. Deutsch

*Department of Physics, University of California, Santa Cruz, California 95064**and The James Franck Institute, 5640 South Ellis Avenue, Chicago, Illinois 60637*

(Received 4 December 1989)

A closed quantum-mechanical system with a large number of degrees of freedom does not necessarily give time averages in agreement with the microcanonical distribution. For systems where the different degrees of freedom are uncoupled, situations are discussed that show a violation of the usual statistical-mechanical rules. By adding a finite but very small perturbation in the form of a random matrix, it is shown that the results of quantum statistical mechanics are recovered. Expectation values in energy eigenstates for this perturbed system are also discussed, and deviations from the microcanonical result are shown to become exponentially small in the number of degrees of freedom.

Chaos and quantum thermalization

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Quantum statistical mechanics in a closed system

J. M. Deutsch

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A closed quantum system will generically give time averages that agree with the predictions of equilibrium statistical mechanics. This is shown by diagonalizing the Hamiltonian matrix, and using the fact that the eigenvalues in the microcanonical ensemble are distributed as random numbers.

nature

Vol 452 | 17 April 2008 | doi:10.1038/nature06838

LETTERS

Thermalization and its mechanism for generic isolated quantum systemsMarcos Rigol^{1,2}, Vanja Dunjko^{1,2} & Maxim Olshanii²

Eigenstate thermalization hypothesis

$$A_{nn} = \langle n | \hat{A} | n \rangle$$

smooth
(approx. constant)

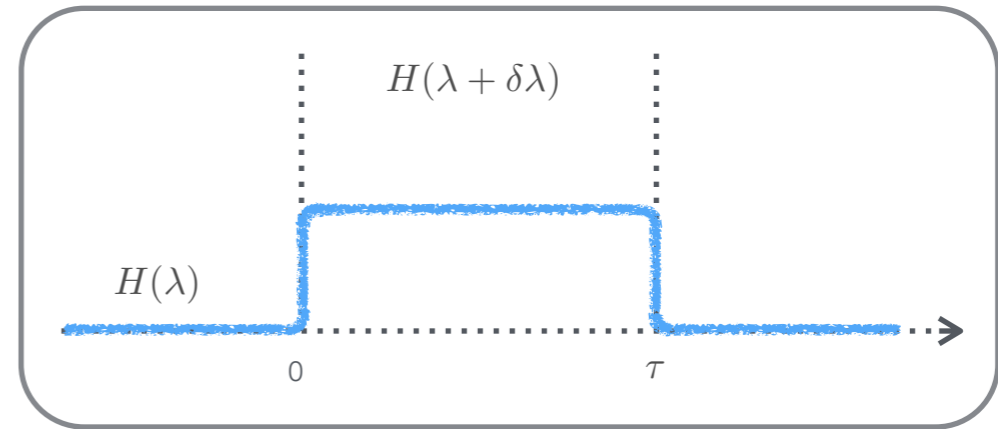
$$A_{nm} = \langle n | \hat{A} | m \rangle$$

very small

$$\langle A \rangle_t \approx \langle A \rangle_{\text{MC}}$$

Other mechanism: see C. Gogolin's thesis

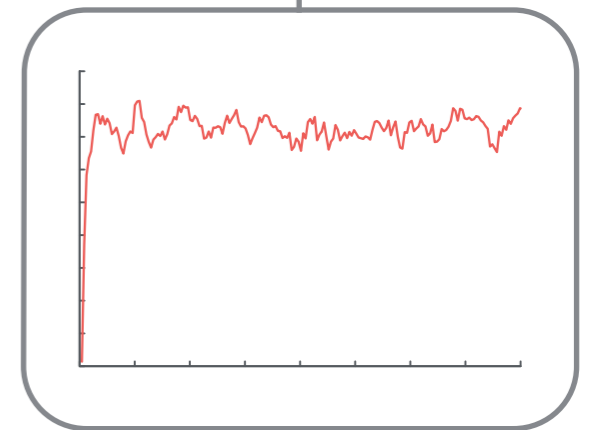
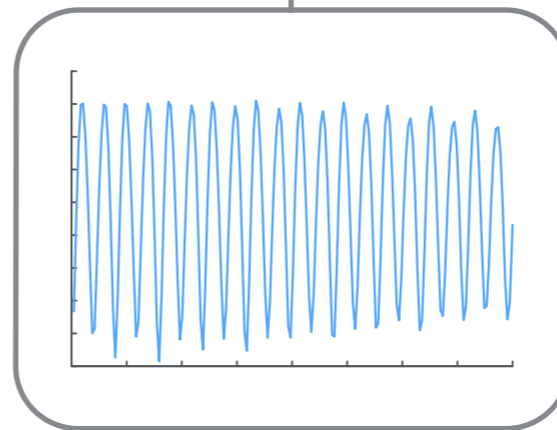
quench $H(\lambda) \rightarrow H(\lambda + \delta\lambda)$



No

Yes

equilibrate?



Yes

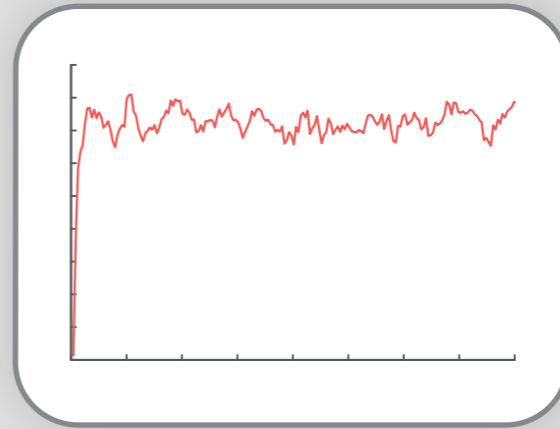
No

thermalize?

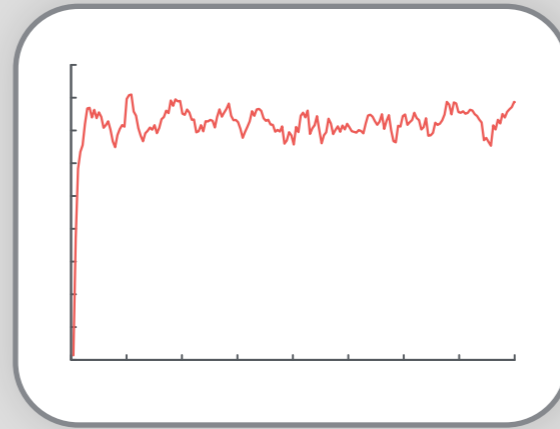
subsystem state independence
bath state independence
diagonal form of eq. state
thermal (Gibbs) state



equilibrate?



equilibrate?



how?

chaos?

quantum Entropy

von Neumann?

$$S_{\text{vN}}(\rho) = -\text{Tr}(\rho \ln \rho)$$

consistent with second law

von Neumann entropy

$$S_{\text{vN}}(\rho) = -\text{Tr}(\rho \ln \rho)$$

equilibrium

diagonal entropy

$$S_{\text{D}}(\rho) = -\sum_n \rho_{nn} \ln \rho_{nn}$$

non-equilibrium

external operations

Santos, Polkovnikov & Rigol, PRL **107**, 040601 (2011)

Polkovnikov, Ann. Phys. **326**, 486 (2011)

consistent with second law

diagonal entropy

$$S_D(\rho) = - \sum_n \rho_{nn} \ln \rho_{nn}$$

increases

conserved for
adiabatic process

uniquely related to P(E)

additive

Santos, Polkovnikov & Rigol, PRL **107**, 040601 (2011)

Polkovnikov, Ann. Phys. **326**, 486 (2011)

$$\text{Prob}[S_D(\rho_0) \leq S_D(\rho_\tau)] \sim 1$$

$$S_{\text{dec}} - \overline{S_D(\tau)} \leq 1 - \gamma$$

$$\gamma = 0.5772\dots$$

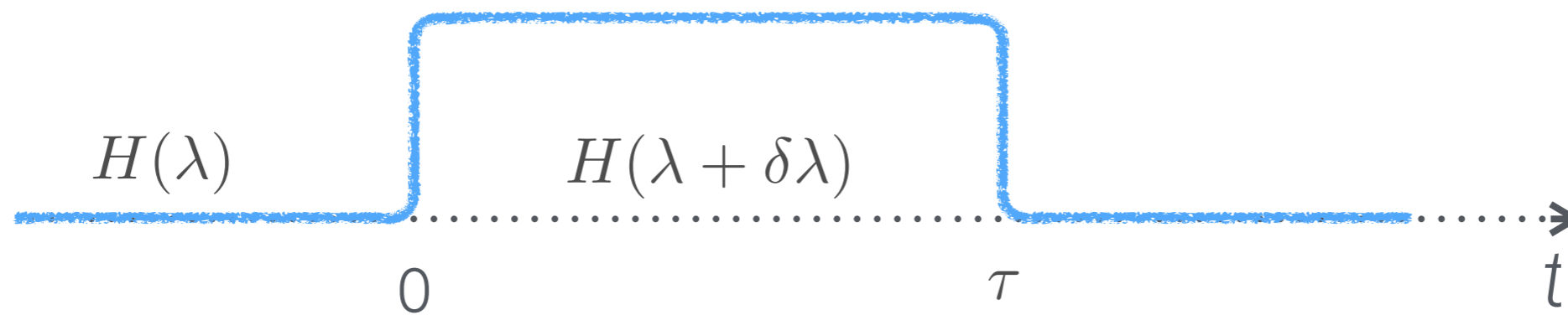
$$S_{\text{dec}} = S_D(\bar{\rho})$$

$$\text{Prob}[S_D(\rho_0) \leq S_D(\rho_\tau)] \sim 1$$

$$S_{\text{dec}} - \overline{S_D(\tau)} \leq 1 - \gamma$$

$$S_{\text{dec}} = S_D(\bar{\rho})$$

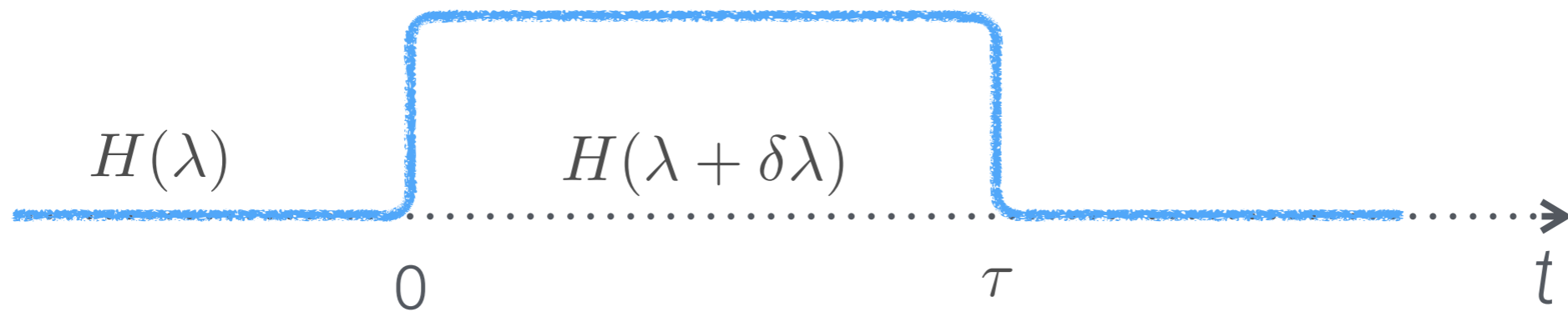
Quench dynamics



Cyclic process

$$\rho_0 = |n_0\rangle\langle n_0|$$

$$\rho(\tau) = e^{-H'\tau} \rho_0 e^{iH'\tau}$$



$$S_D = - \sum_n C_n(\tau) \ln C_n(\tau)$$

$$S_{\text{dec}} = S_D(\bar{\rho})$$

equilibrium

$$S_{\text{dec}} - \overline{S_{\text{D}}(\tau)}$$

$$\Delta S_{\text{D}}(\tau) / \overline{S_{\text{D}}(\tau)}$$

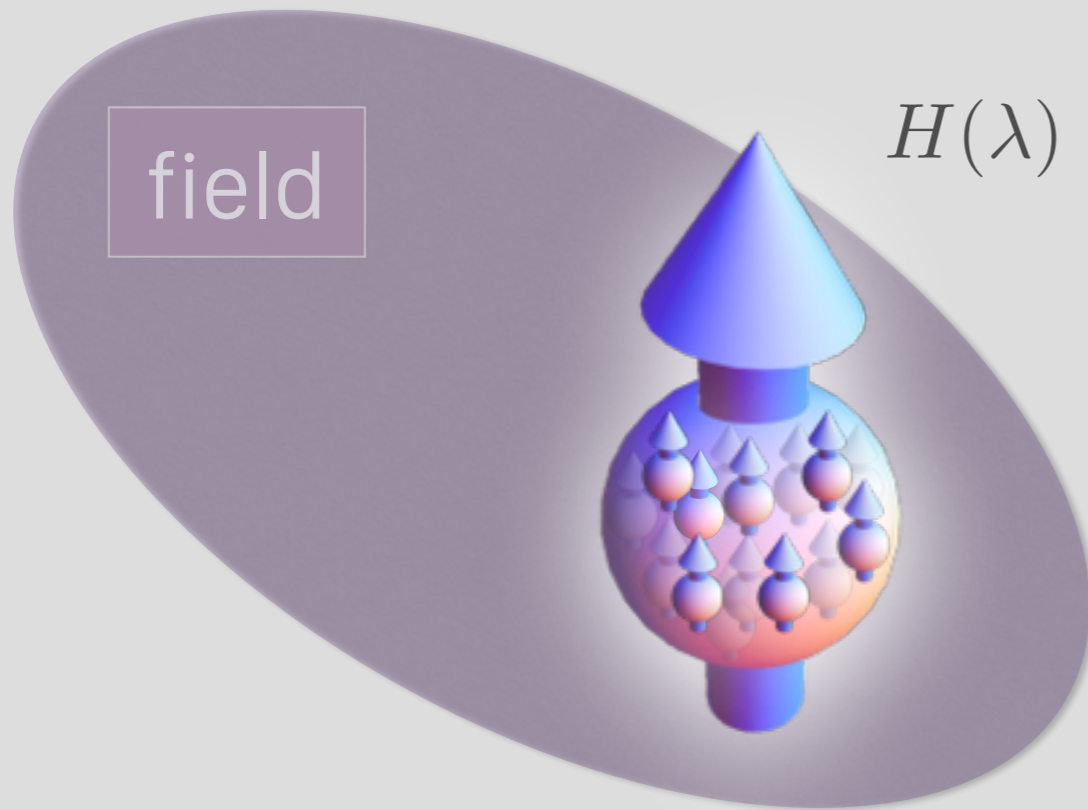
equilibrium

$$S_{\text{dec}} - \overline{S_{\text{D}}(\tau)} \rightarrow 1 - \gamma$$

$$\Delta S_{\text{D}}(\tau) / \overline{S_{\text{D}}(\tau)} \ll 1$$

two different models

Dicke model



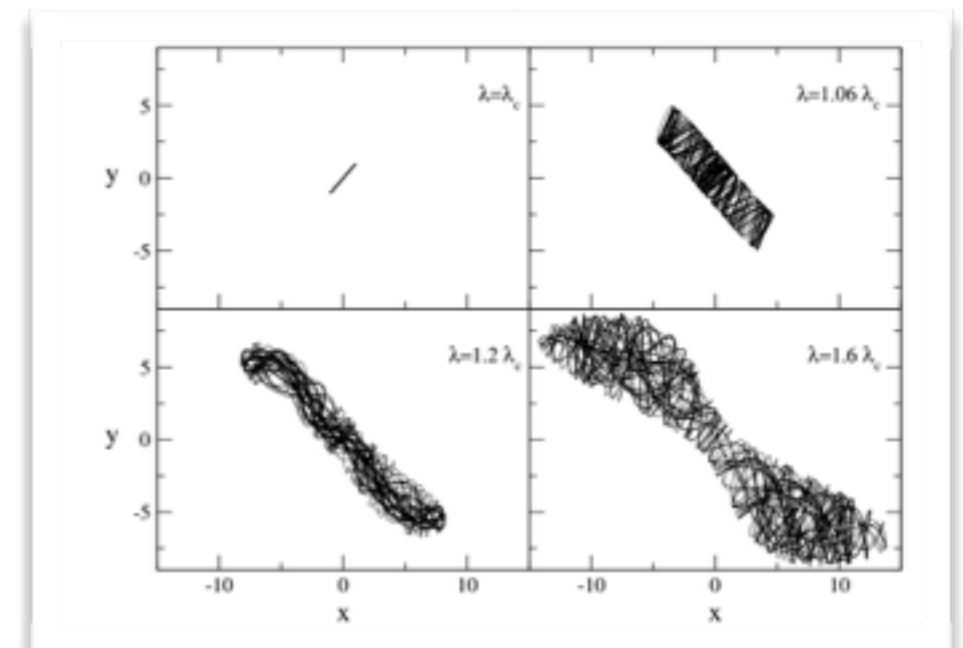
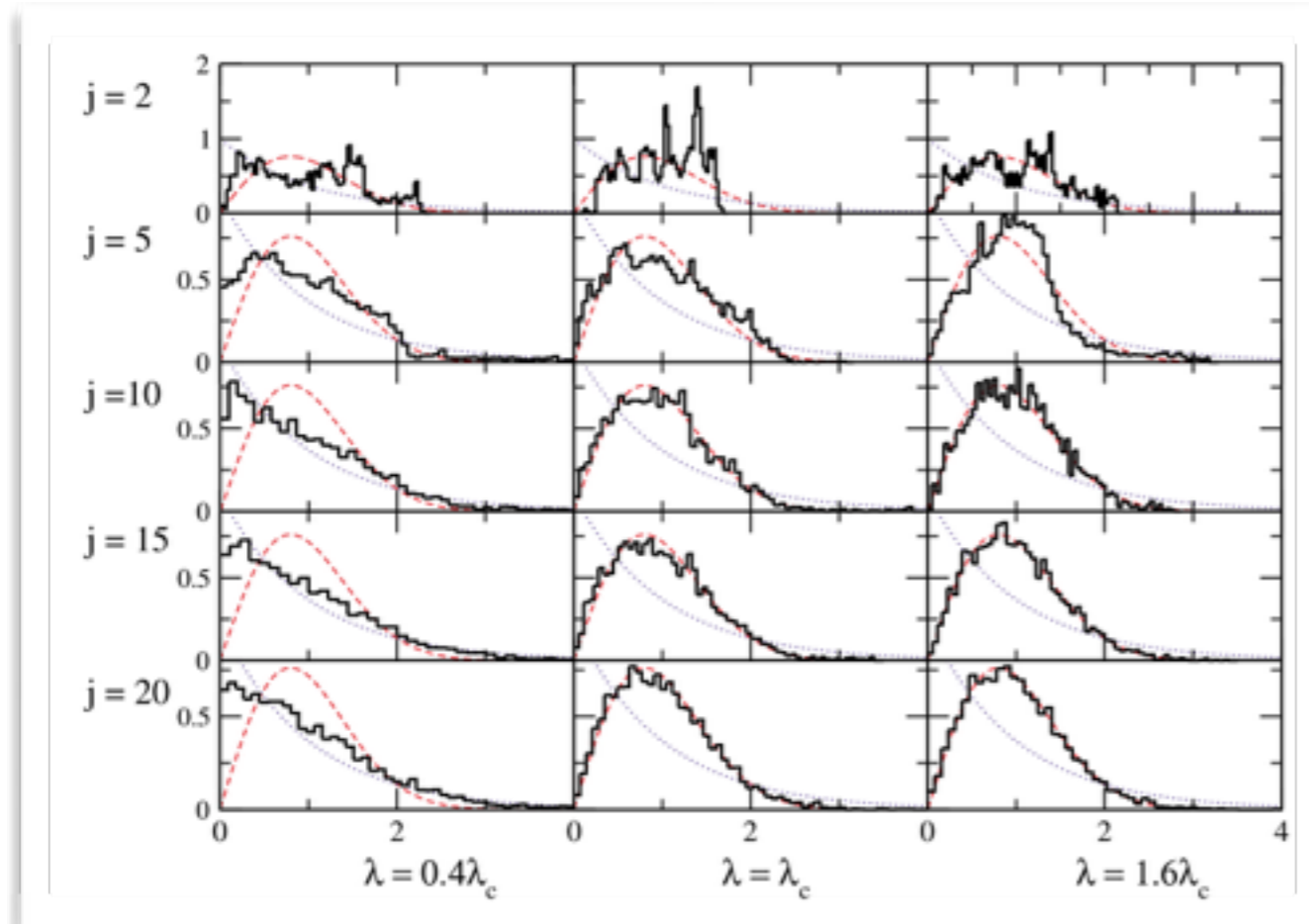
$$H(\lambda) = \omega_0 J_z + \omega a^\dagger a + \frac{\lambda}{\sqrt{2j}} (a^\dagger + a)(J_+ + J_-)$$

superradiant transition

$$\lambda_c = \frac{1}{2} \sqrt{\omega_0 \omega}$$

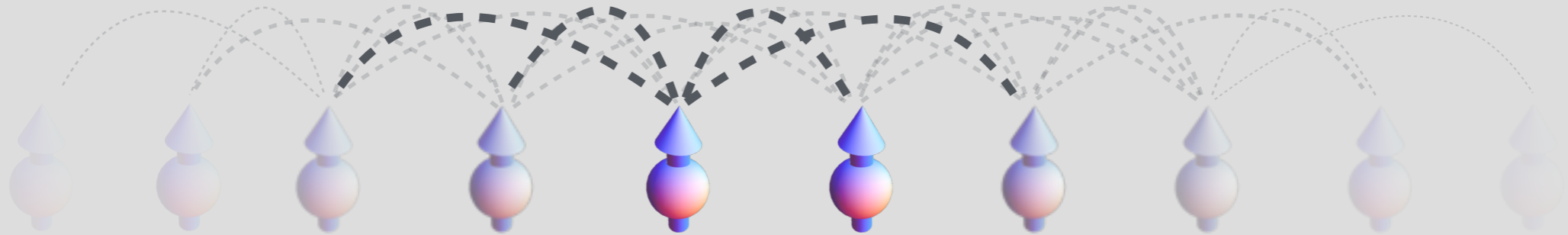
$$\omega_0 = \omega = \hbar = 1 \quad \lambda_c = 0.5$$

Dicke model



Emary & Brandes, PRL **90**, 044101 (2003)

Spin system

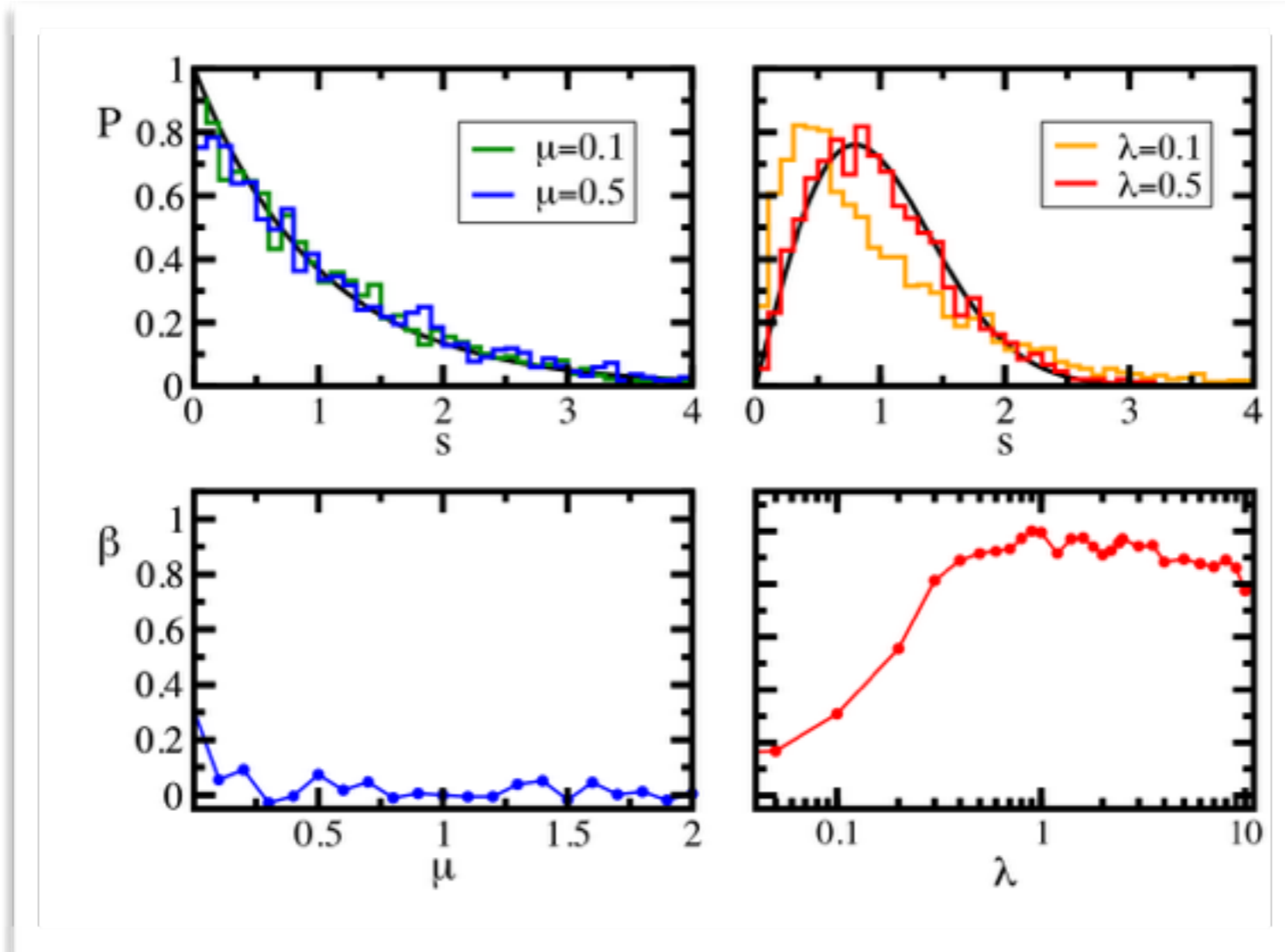


$$H(\lambda) = H_0 + \lambda V$$

$$H_0 = \sum_{i=1}^{L-1} J(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \mu S_i^z S_{i+1}^z)$$

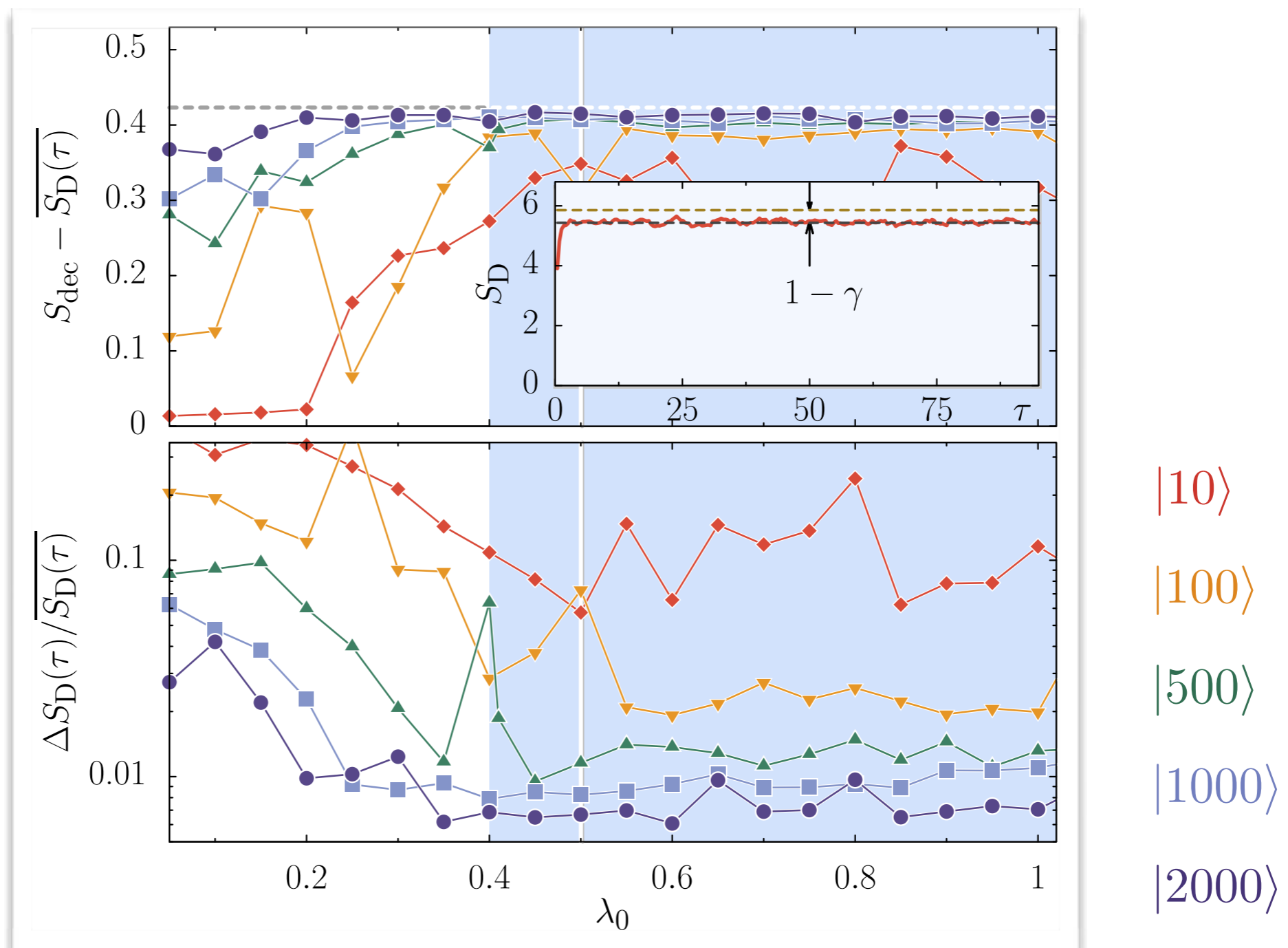
$$V = \sum_{i=0}^{L-2} J(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + \mu S_i^z S_{i+2}^z)$$

Spin system



Santos, Borgonovi & Izrailev, PRE **85**. 036209 (2012)

Dicke



$$j = 20, N = 250$$

$$\delta\lambda = 0.1$$

$$\lambda_c = 0.5$$

λ_0

$$S_{\text{dec}} - \overline{S_D}$$

$$\Delta S_D / \overline{S_D}$$

 $|n_0\rangle$

λ_0

structure
of initial state

$|n_0\rangle$

$$S_{\text{dec}} - \overline{S_D}$$

$$\Delta S_D / \overline{S_D}$$

IPR

$$\xi = \frac{1}{\sum_m |\langle n(\lambda) | m(\lambda + \delta\lambda) \rangle|^4}$$

basis

complexity of eigenstates

Localization

chaos

B.V. Chirikov, F.M. Izrailev and D.L. Shepelyansky Physica D 33 (1988) 77-88

Y. V. Fyodorov and A. D. Mirlin, Phys. Rev. B 52, R11580 (1995).

Ph. Jacquod and D. L. Shepelyansky, Phys. Rev. Lett. 75, 3501 (1995).

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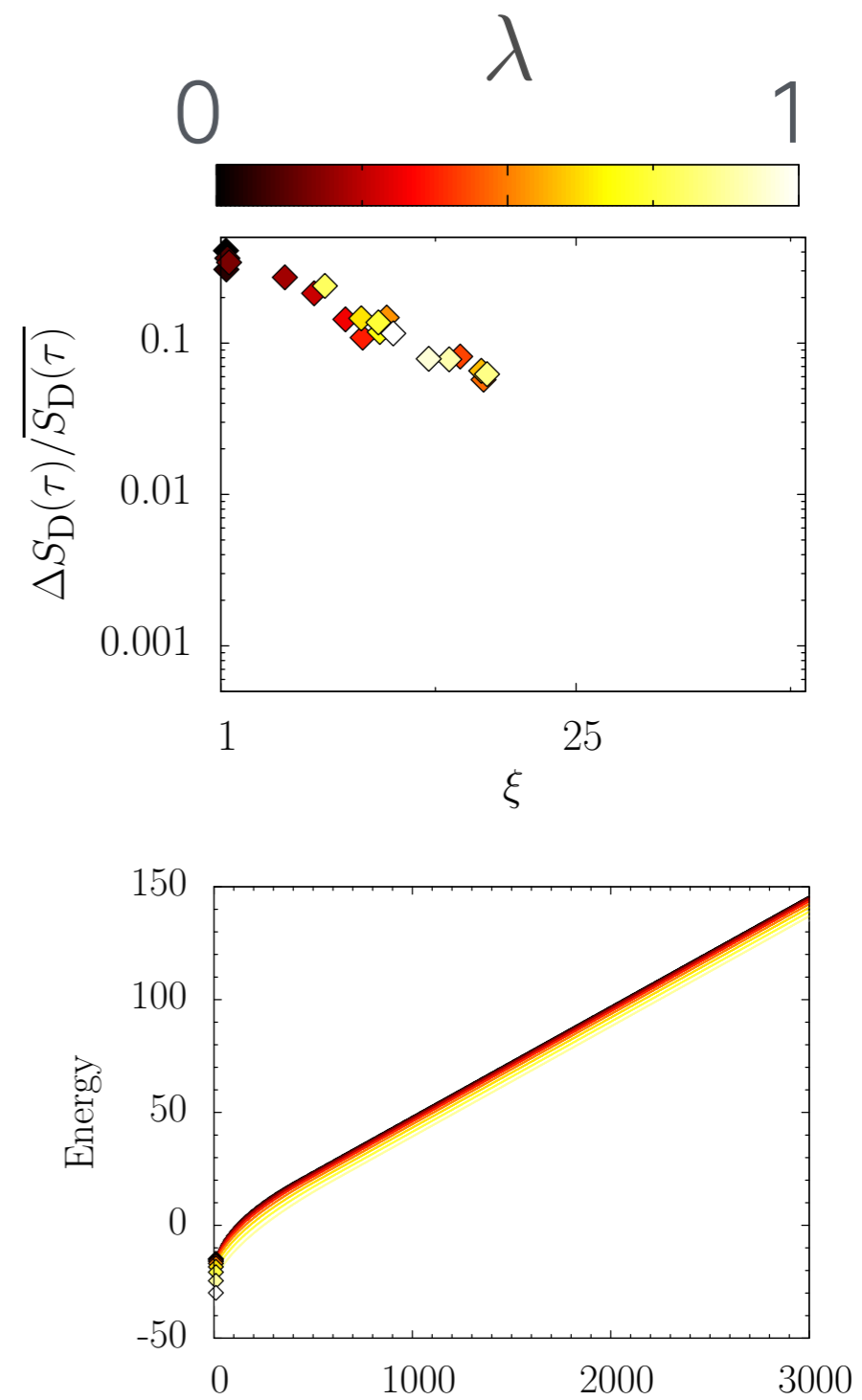
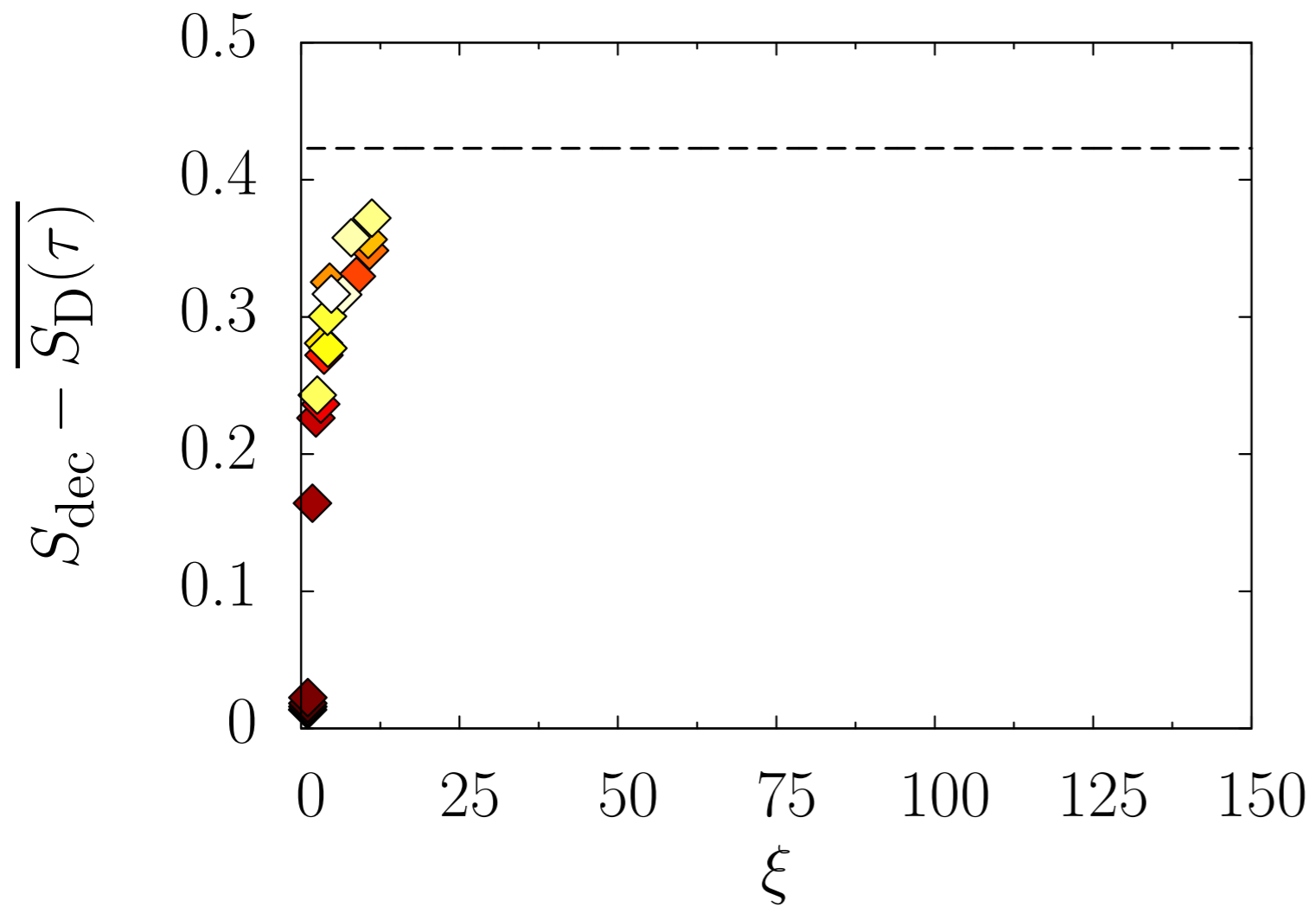
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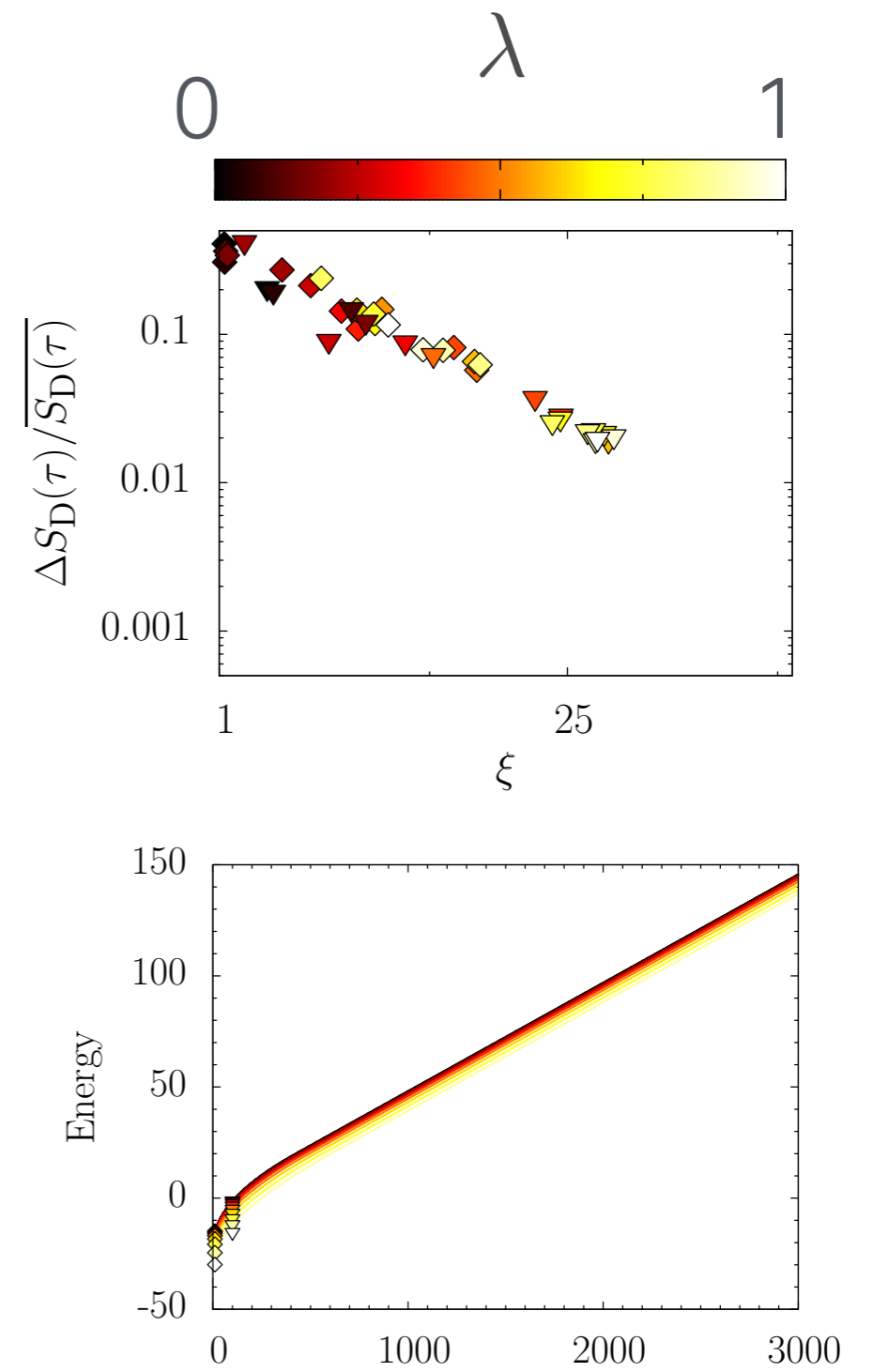
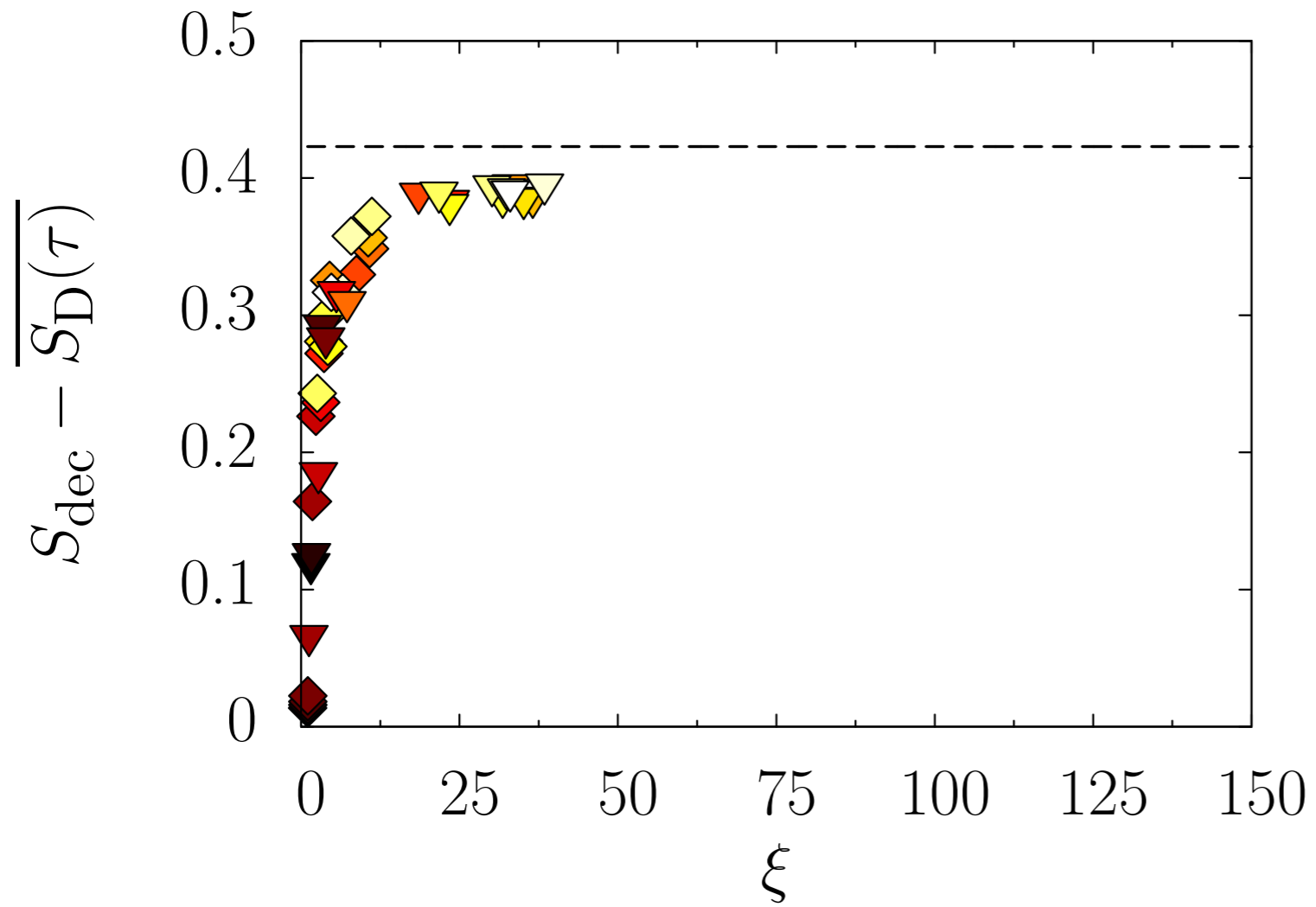
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L. F. Santos, F. Borgonovi, and F. M. Izrailev, PRE 85, 036209 (2012)

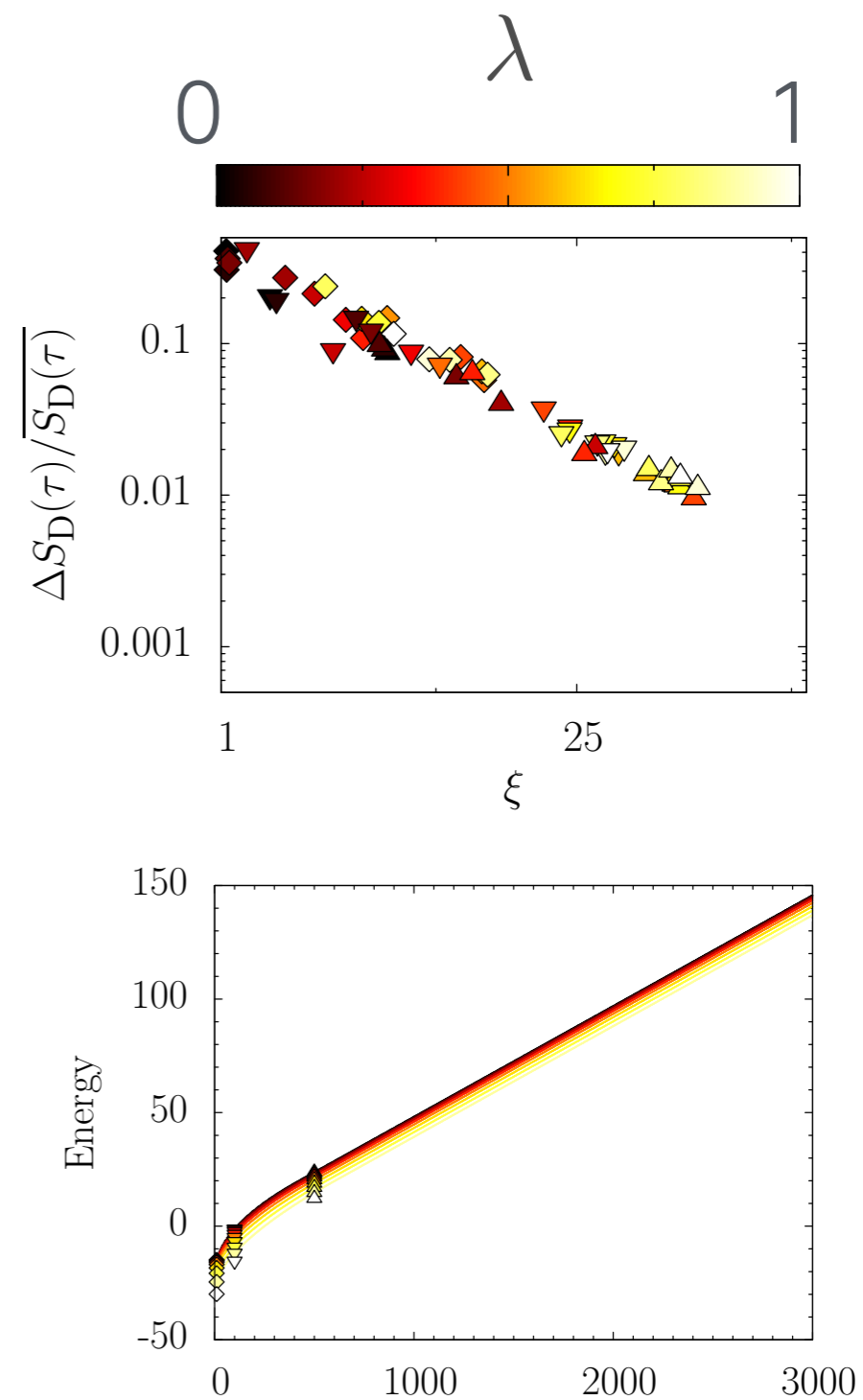
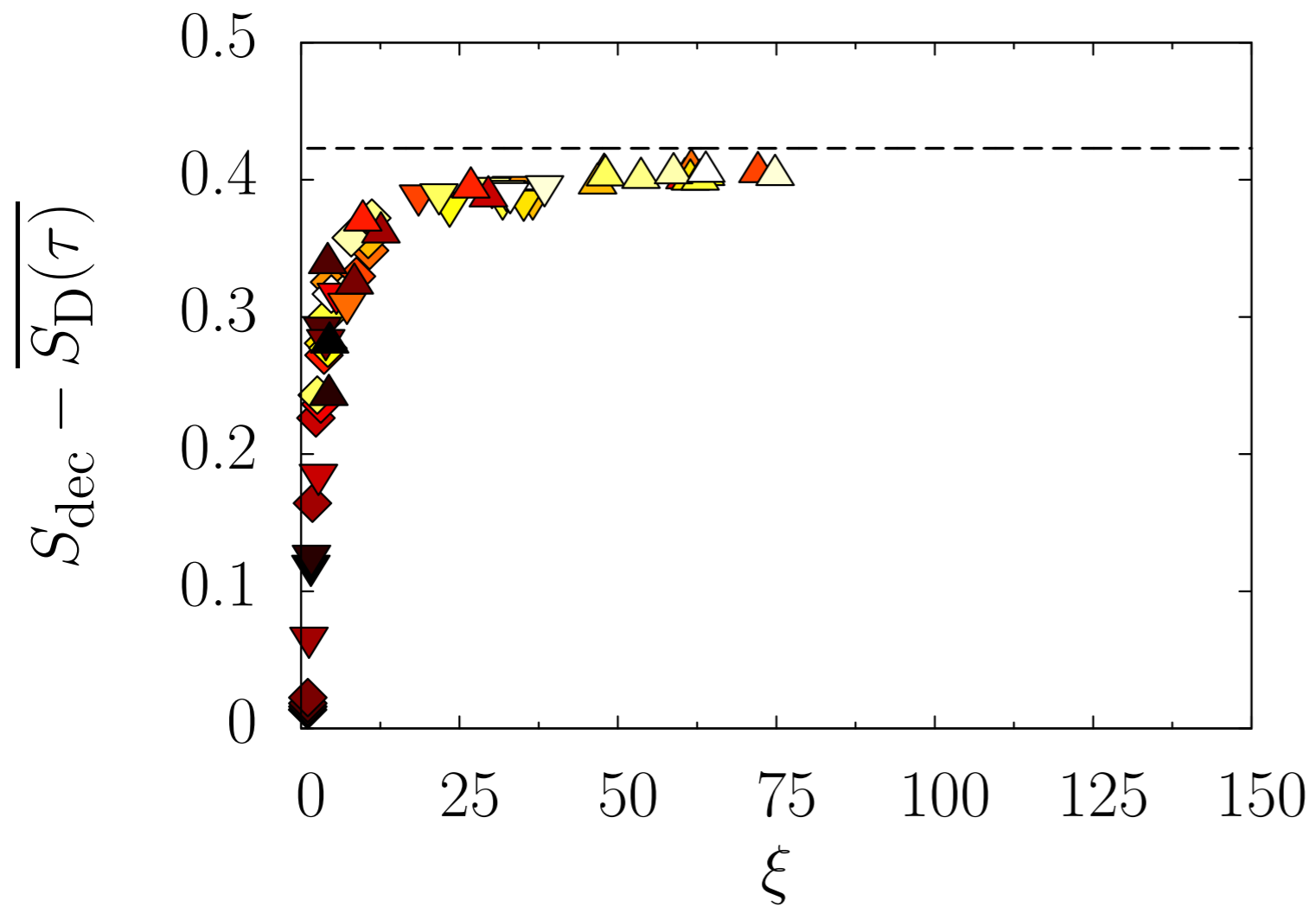
Dicke model



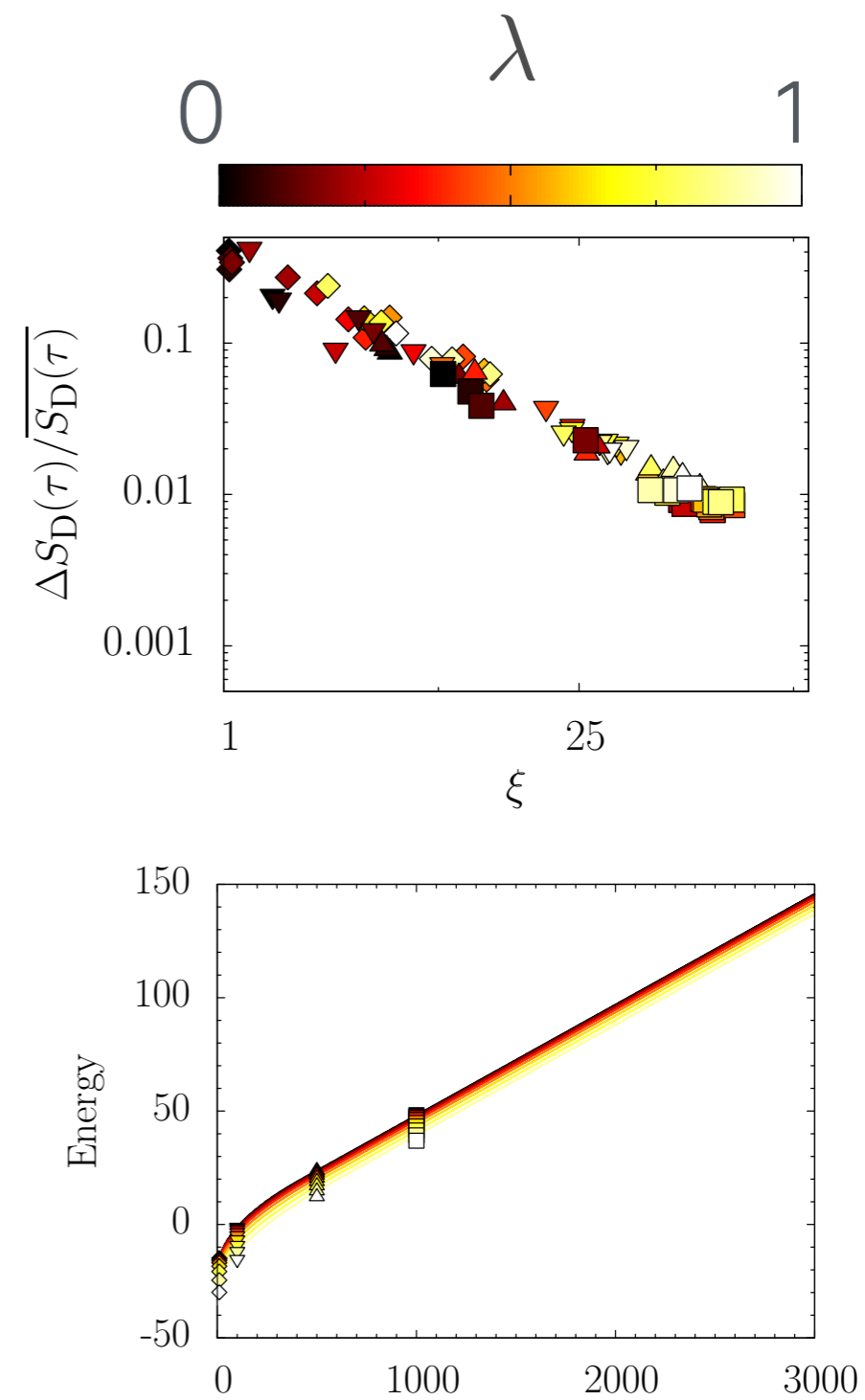
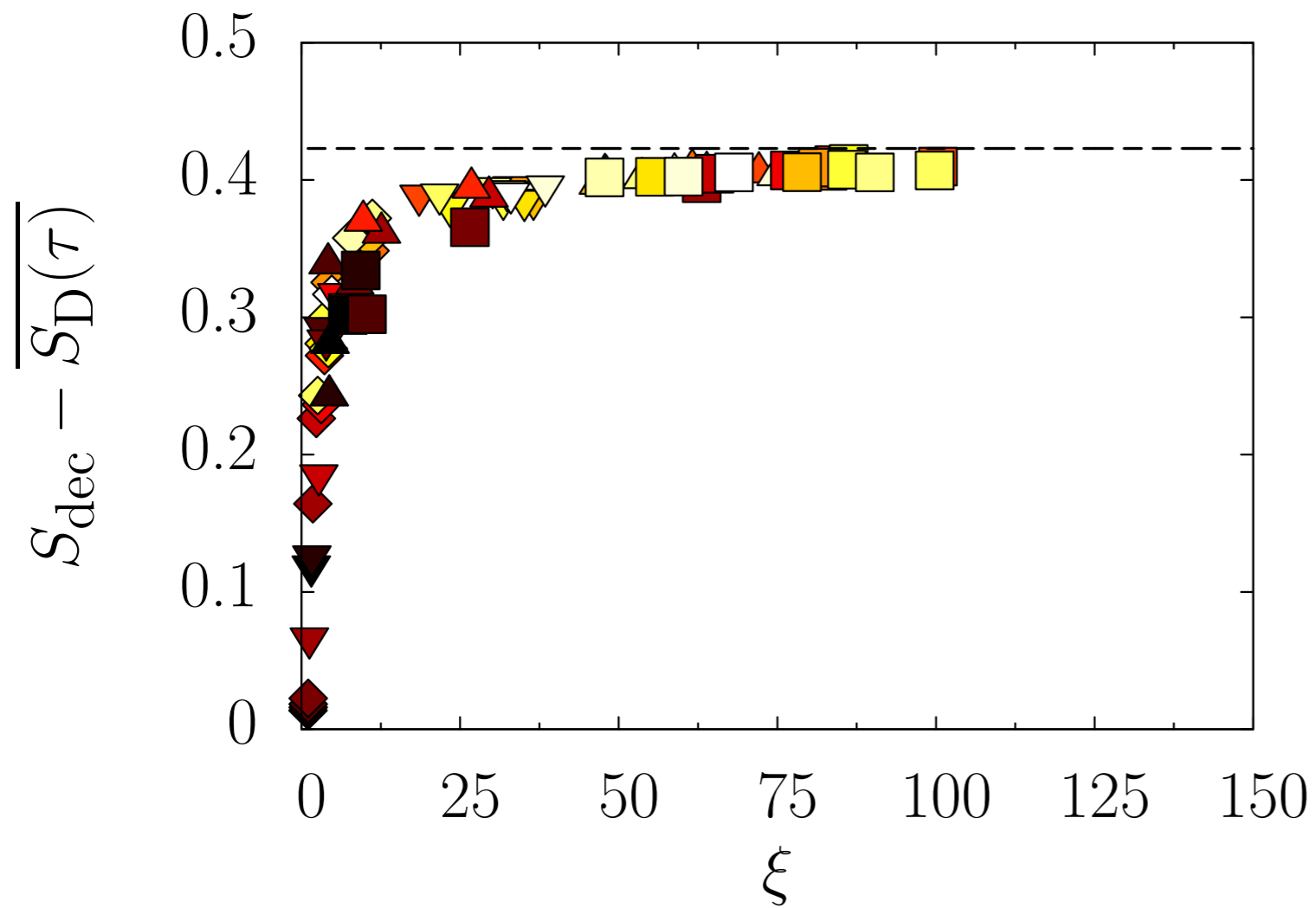
Dicke model



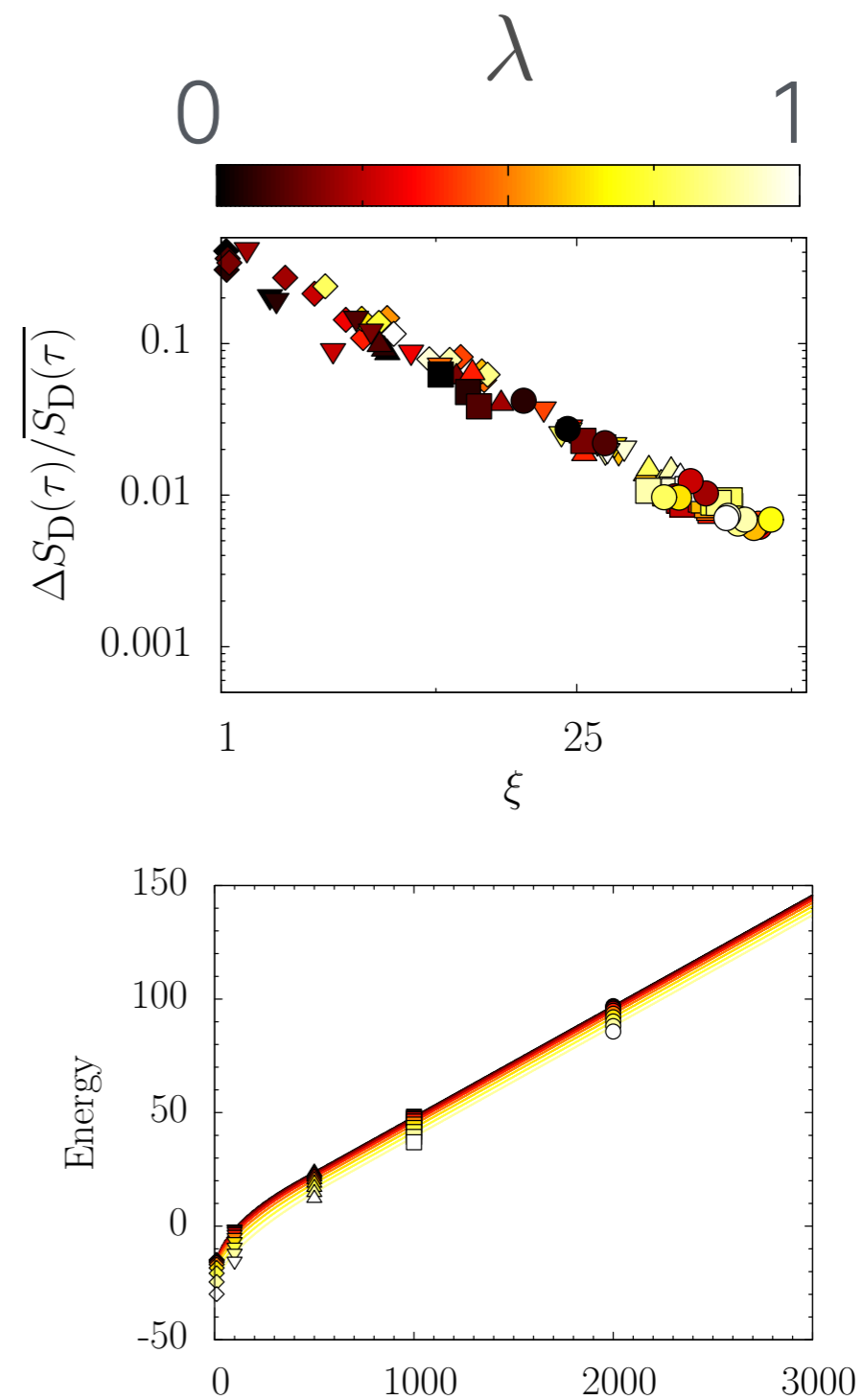
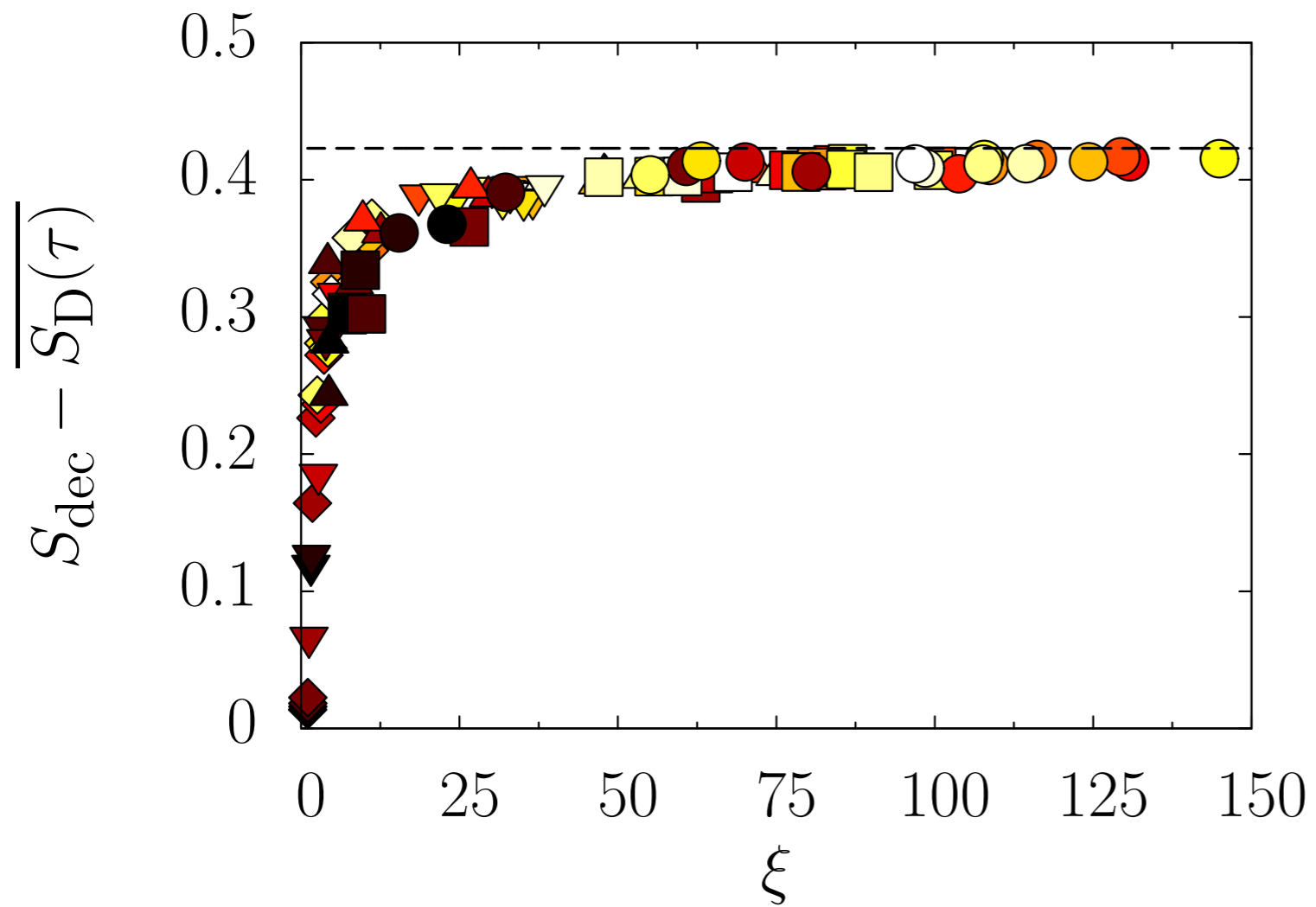
Dicke model



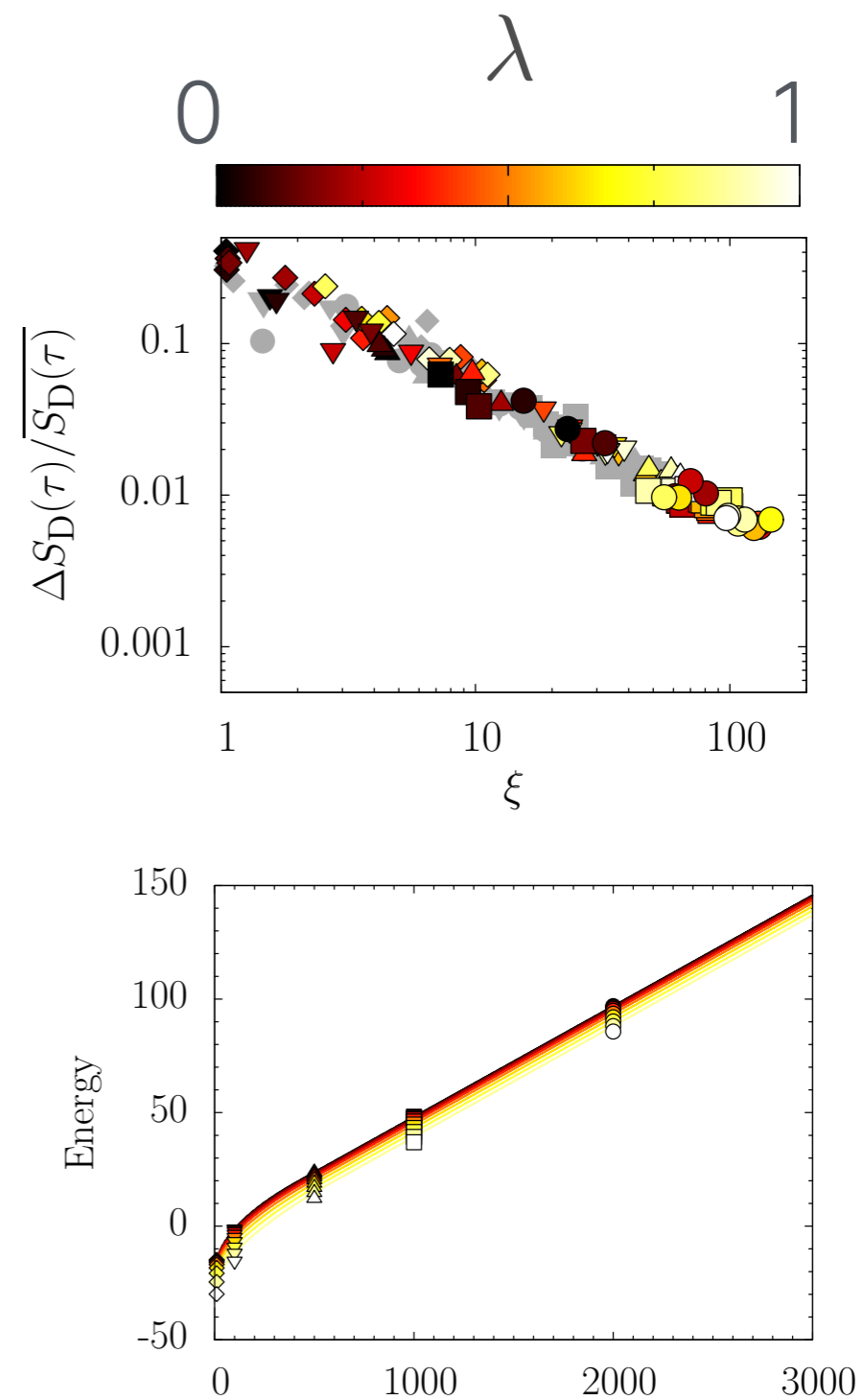
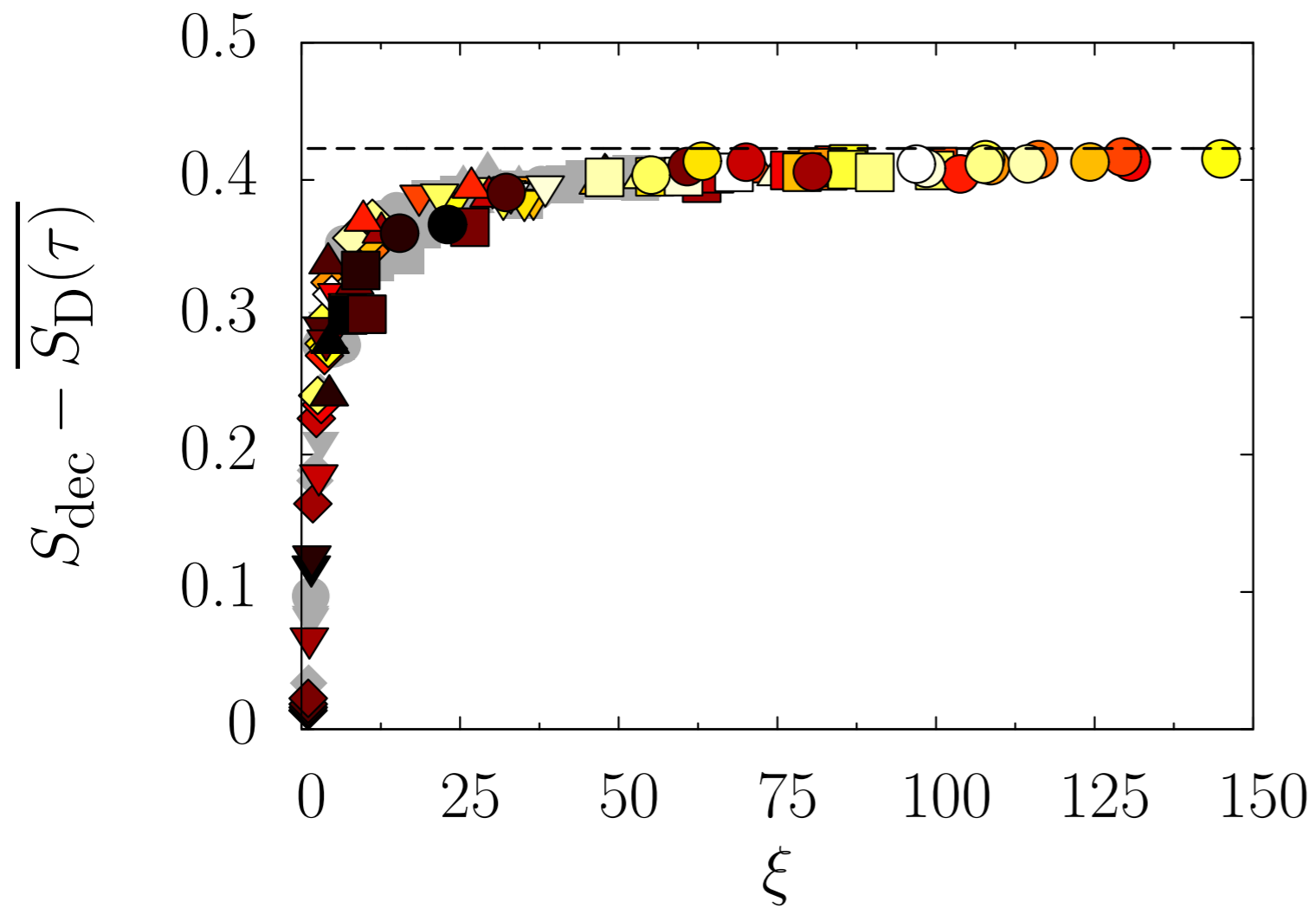
Dicke model



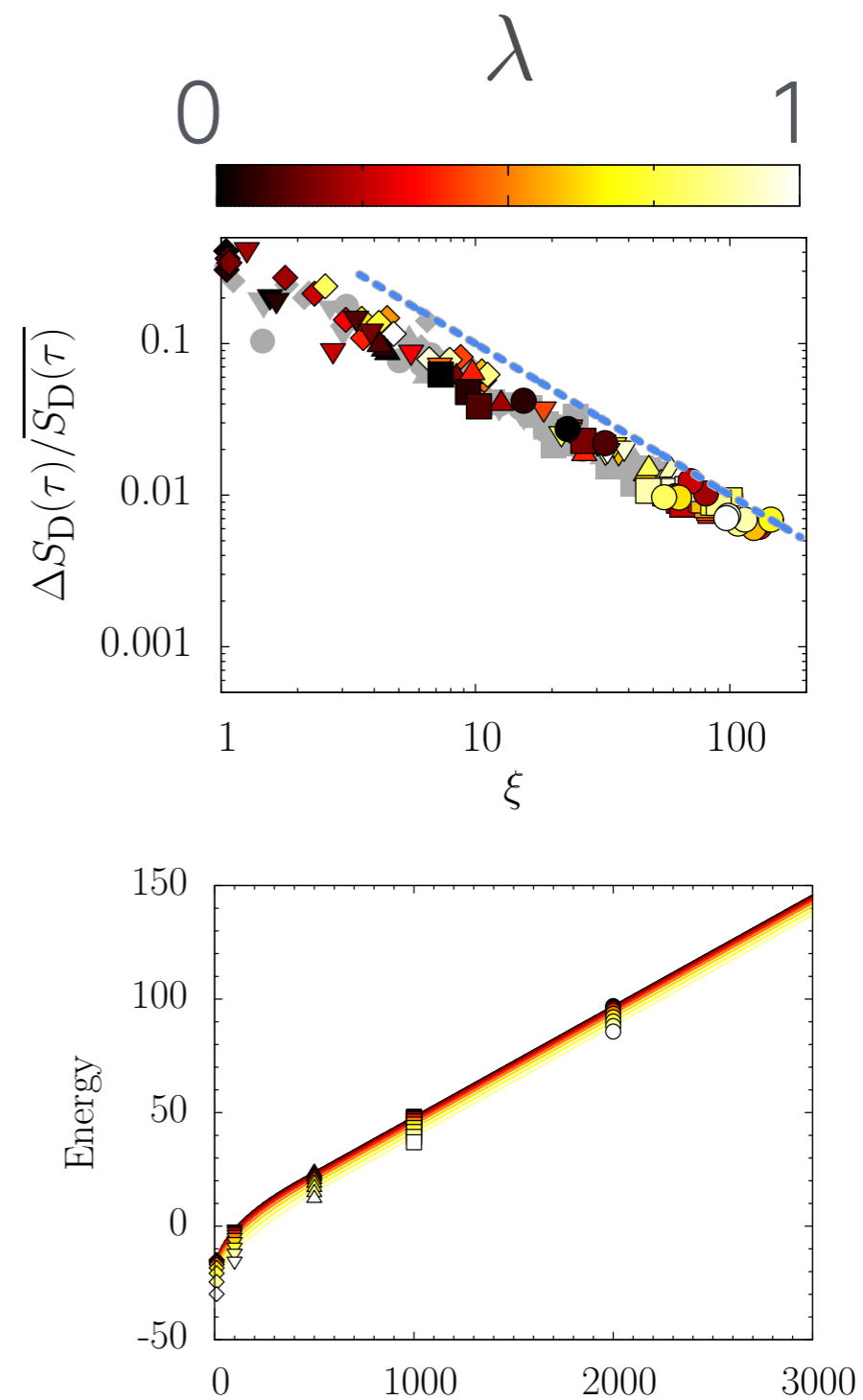
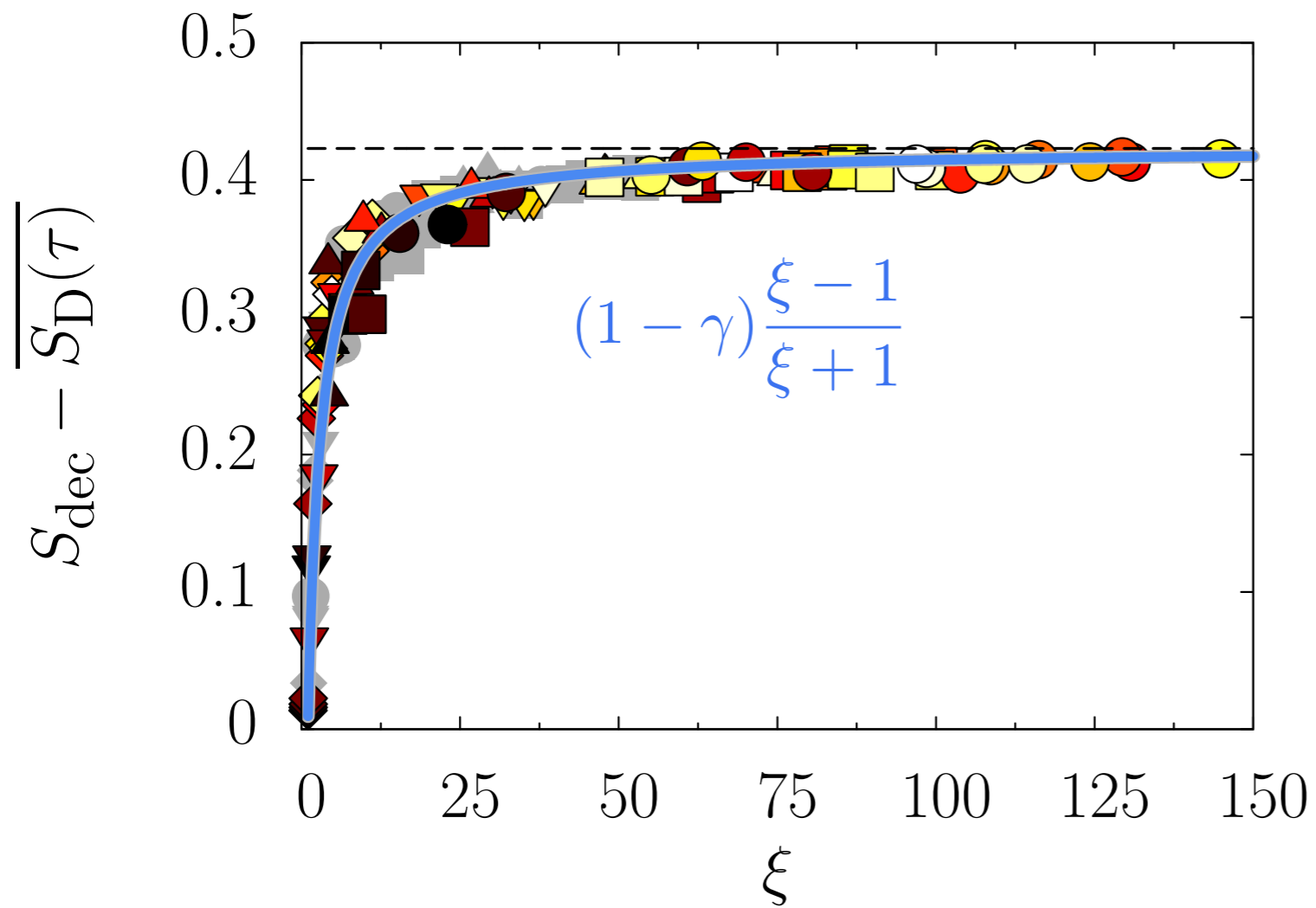
Dicke model



Dicke model

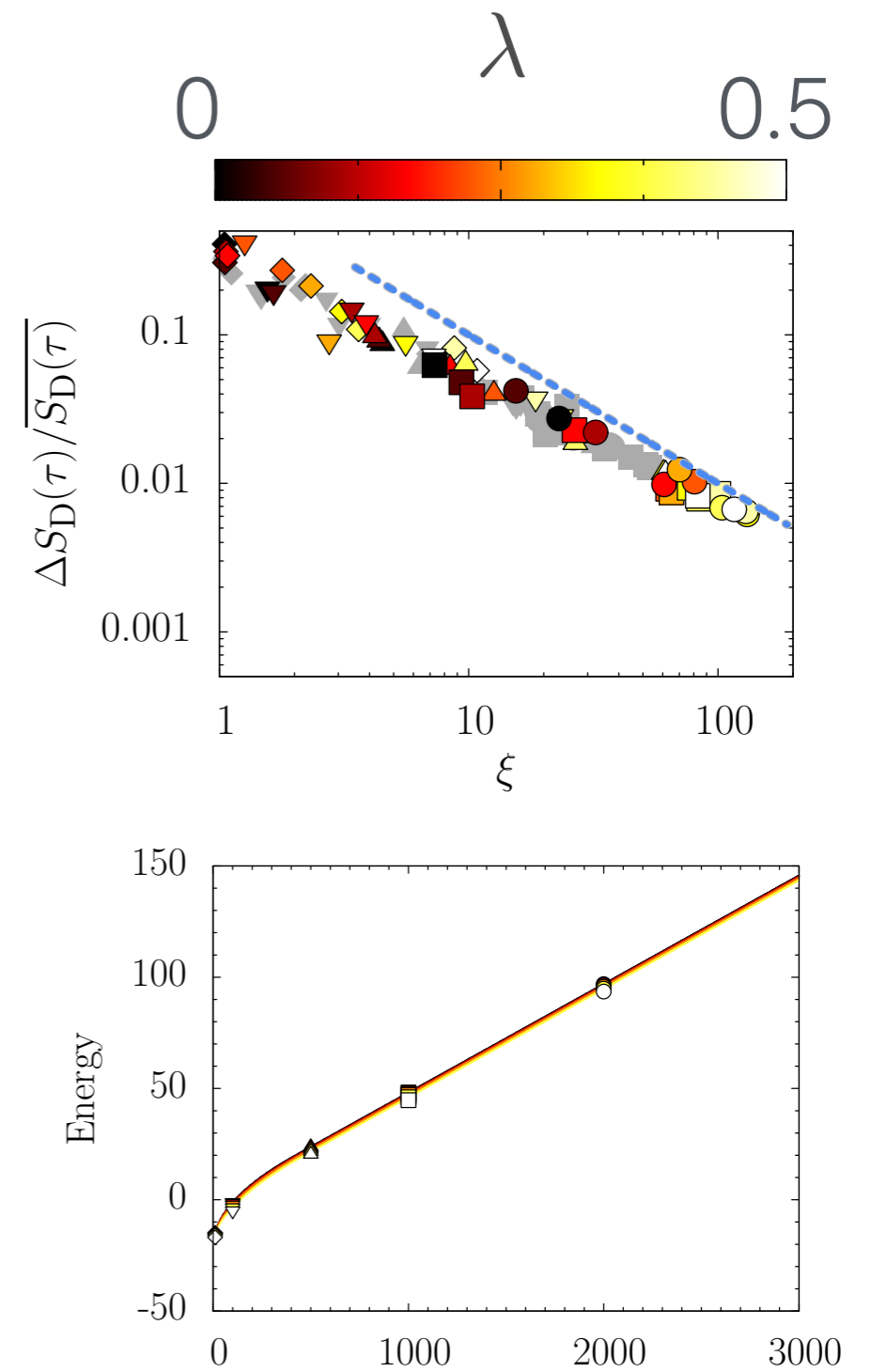
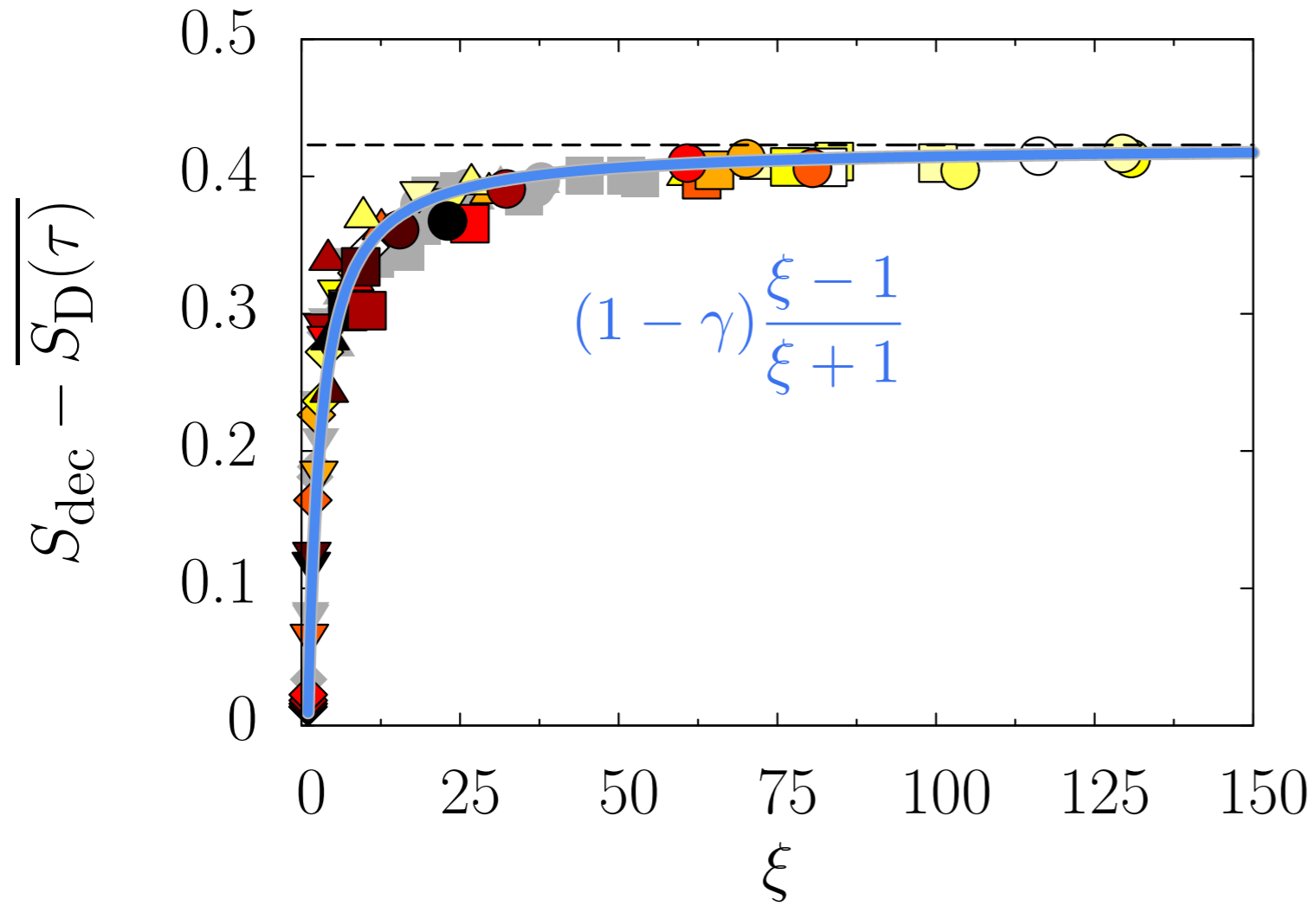


Dicke model

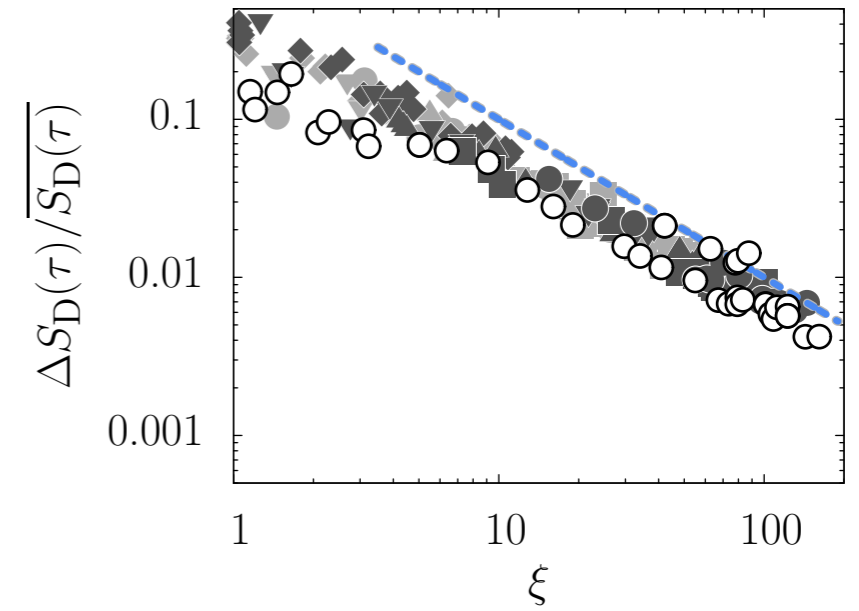
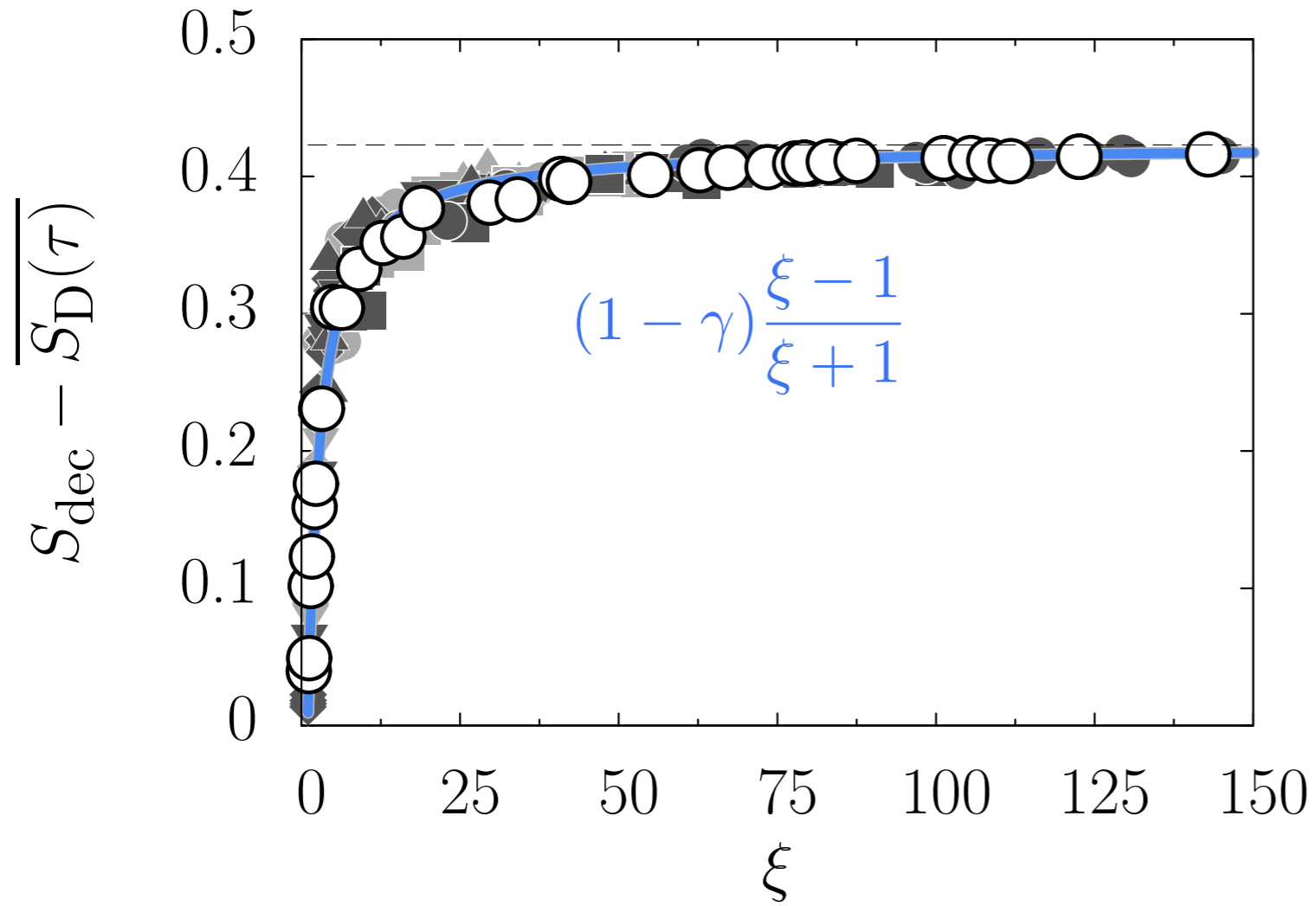


Dicke model

$$\lambda < \lambda_c$$



Dicke model & spin system



equilibration and chaos
isolated systems

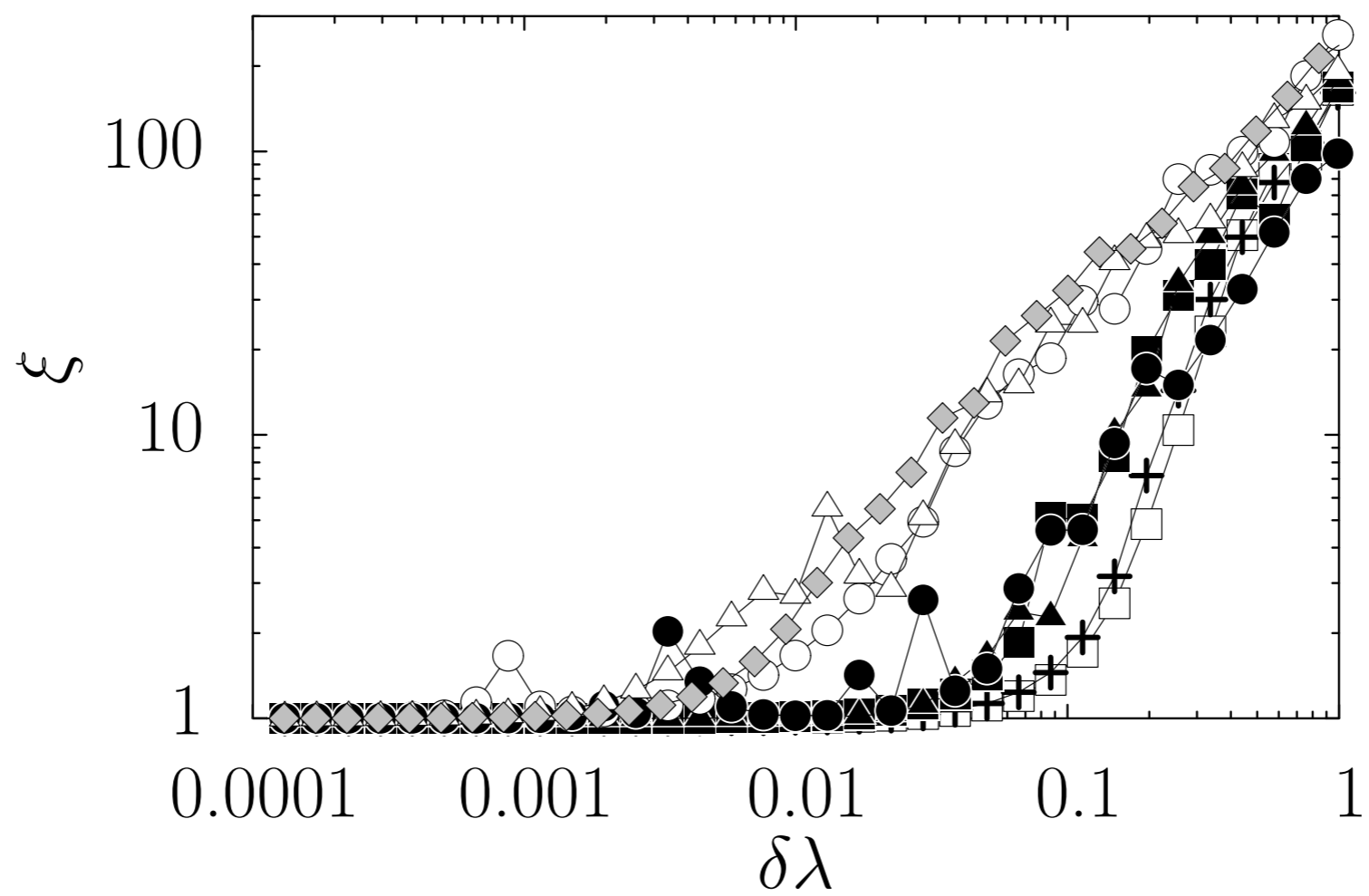
two different models
universal behavior

complexity is key

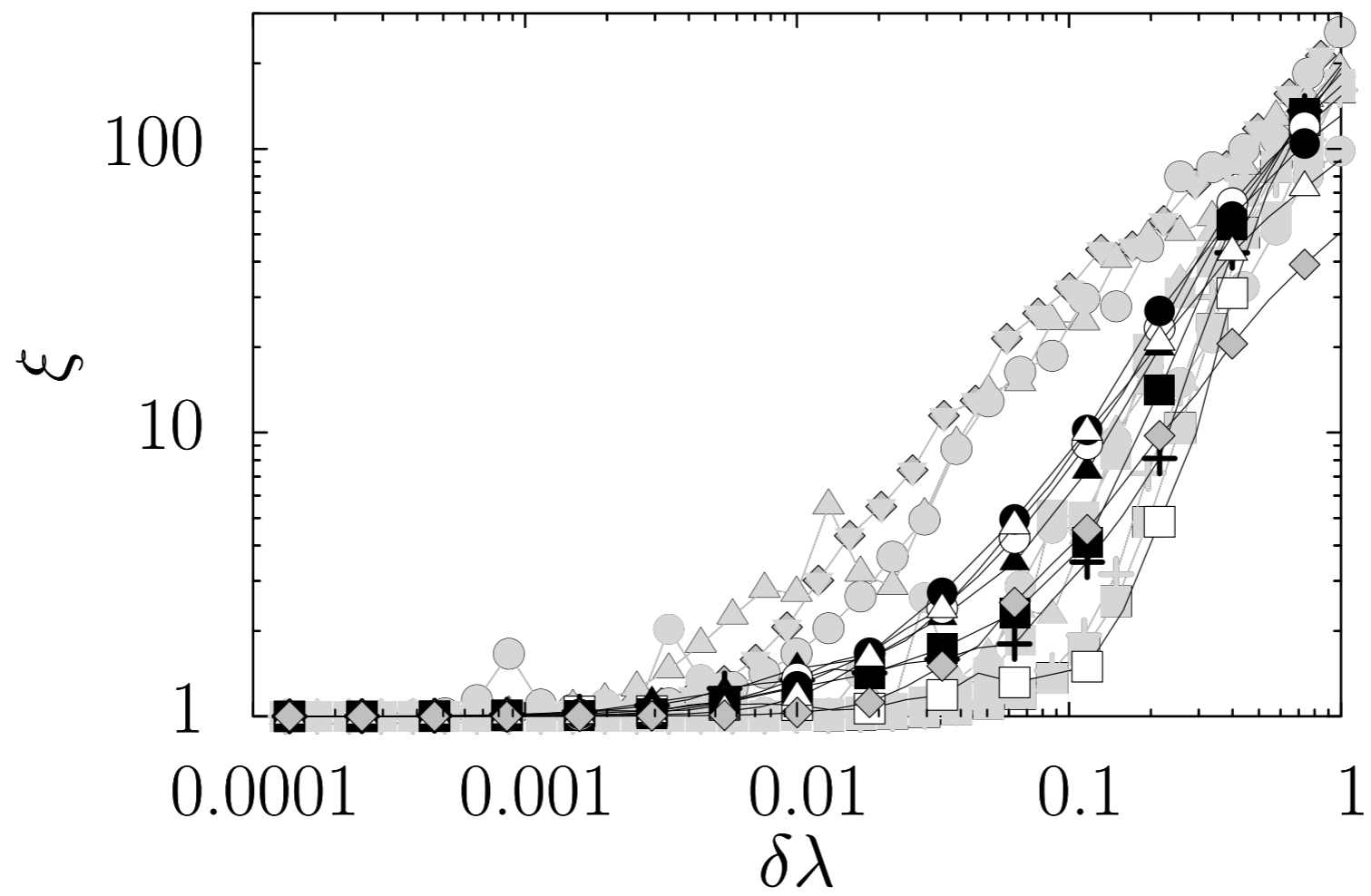
to be done

$$S_{\text{dec}} - \overline{S_D(\tau)} = (1 - \gamma) \frac{\xi - 1}{\xi + 1}$$

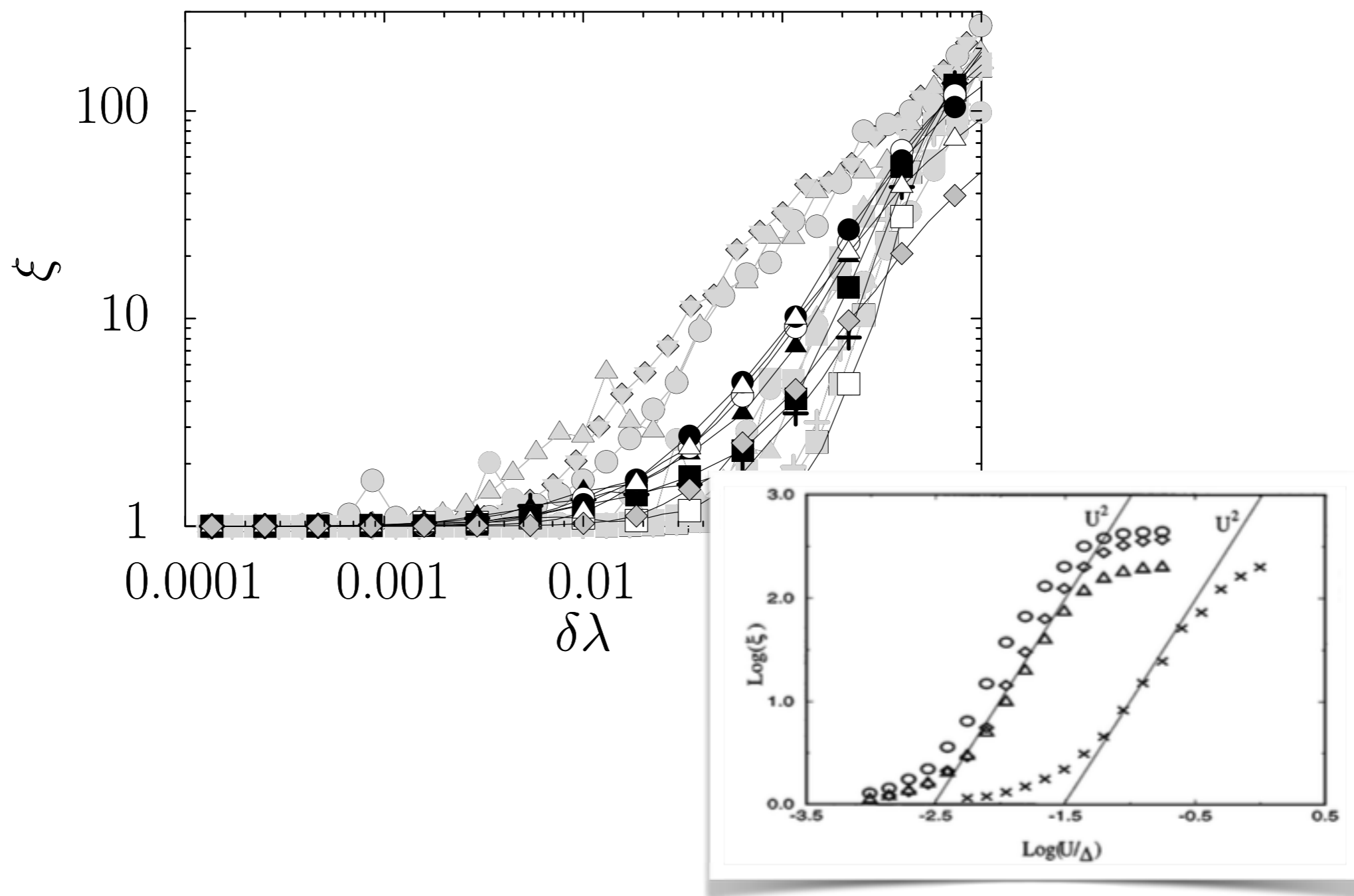
to be done



to be done



to be done



Relaxation of isolated quantum systems beyond chaos

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²Consejo Nacional de Investigaciones Científicas y Tecnológicas (CONICET), Argentina

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In classical statistical mechanics there is a clear correlation between relaxation to equilibrium and chaos. In contrast, for isolated quantum systems this relation is—to say the least—fuzzy. In this work we try to unveil the intricate relation between the relaxation process and the transition from integrability to chaos. We study the approach to equilibrium in two different many-body quantum systems that can be parametrically tuned from regular to chaotic. We show that a universal relation between relaxation and delocalization of the initial state in the perturbed basis can be established regardless of the chaotic nature of system.

PRE 91, 010902 (R) (2015)

Diego Wisniacki
Augusto Roncaglia
IGM &

GARCÍA-MATA, RONCAGLIA, AND WISNIACKI

d-entropy at time τ is

$$S_D(\tau) = - \sum_n C_n(\tau) \ln C_n(\tau), \quad (2)$$

where $C_n(\tau) = \langle n | \rho(\tau) | n \rangle = |\langle n | e^{-iH\tau} | n_0 \rangle|^2$, and $|n\rangle$ is an eigenstate of H . The d-entropy satisfies, for typical operational times, that is, after the approach to equilibrium and after an equilibration time scale it stabilizes to a constant value. Since the d-entropy is a nonlinear function of the density matrix, its time average is not equal to the time-averaged state S_{dec} . If the initial state is $\rho_{in} = \overline{\rho}(\tau)$, where $\overline{f}(\tau) = \lim_{T \rightarrow \infty} T^{-1} \int_0^T f(\tau) d\tau$, is $S_{dec} = - \sum_n \mu_n \ln \mu_n$ with $\mu_n = \langle n | \rho_{dec} | n \rangle$. It is conjectured [14] that the relaxation to equilibrium is accompanied by the following subextensive correction to the d-entropy

$$S_{dec} - \overline{S_D(\tau)} \sim \gamma \ln N$$

where $\gamma = 0.3772 \dots$ is Euler's constant. This relation holds if the initial state is pure. As a consequence, the fluctuations of the d-entropy will also be reflected in the fluctuations of the d-entropy at equilibrium, which should decrease in a maximum of $\gamma \ln N$ in the limit of large N . In this paper, we study the relation between the transition from integrability to chaos and the relaxation process, we study the approach to equilibrium in two different many-body quantum systems that can be parametrically tuned from regular to chaotic.

We start with the paradigmatic Dicke model [15], which is especially known for its quantum phase transition [16]. It has been observed recently with a superfluid gas in an optical cavity [16]. The single mode DM describes the (dipole) interaction between an ensemble of N two-level atoms with level splitting ω_0 and a single mode of a bosonic field of frequency ω :

$$H(\lambda) = \omega_0 J_+ + \omega_0 J_- + \frac{\lambda}{\sqrt{2}} (a^\dagger + a) \quad (4)$$

where J_\pm are the collective angular momentum operators for a paraxial geometry [17], a (a^\dagger) are the bosonic annihilation (creation) operators of the field. In the thermodynamic limit ($N \rightarrow \infty$), there is a superfluid phase transition [16] at $\lambda_c = \sqrt{2}\omega_0/2$. For finite N there is also a transition at λ_c [17]. In the integrability regime, where level spacing statistics is Poissonian, the approach to equilibrium is slow. Interestingly, the chaotic behavior could also be observed in a semiclassical model [18]. We consider $\omega_0 = 1$ so that $\lambda_c = 0.5$ and $\hbar = 1$. The Dicke Hamiltonian is invariant under parity transformations so we will constrain our analysis to the even subspace.

We consider the behavior of the d-entropy for different quenches, where an initial Hamiltonian $H = H(\lambda_0)$ is perturbed by $H' = H(\lambda_0 + \delta\lambda)$, where $\delta\lambda$ is the quench amplitude. We did straightforward diagonalization in the Fock basis (taking parity into account). The phonon basis was truncated at a value $n_{max} \sim 250$. The typical behavior of the d-entropy as a function of τ (for $\lambda_0 > \lambda_c$) can be seen in the inset of Fig. 1, for the DM ($\lambda_0 = 0.65$, $\delta\lambda = 0.1$). After a short period of time the d-entropy settles approximately to a constant value. The dashed line corresponds to S_{dec} and the difference



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The second law is the cornerstone upon which the strength of thermodynamics law [1]. It states that the entropy of an isolated system always increases. It delivers an initial equilibrium state to which the system is assumed that during the evolution for sufficiently long times an equilibrium state is achieved. In statistical mechanics relaxation can be formalized by the concept of weak mixing [2].

In quantum mechanics the situation is more subtle. First, there is no straightforward translation of the concept of classical chaos to the quantum realm. The definition of phase space trajectories and mixing [3]. Although these two notions in quantum systems are devoid of meaning, there are certainly other ways to define quantum chaos, such as the level statistics [4] and properties of eigenstates. The main reason is that the straightforward translation of the second law to quantum physics, the von Neumann entropy $S_N = -\text{Tr}(\rho \ln \rho)$, is preserved for any process in closed systems. Thus it does not comply with the second law for systems out of equilibrium. For this reason alternative definitions of entropy have been proposed. One good candidate is the diagonal entropy (d-entropy) [6,7], defined as

$$S_D = - \sum_n \rho_{nn} \ln \rho_{nn} \quad (1)$$

where ρ_{nn} are the diagonal elements of ρ in the energy eigenbasis. It is the Shannon entropy of the probability distribution corresponding to the energy measurement. If the density matrix is a convex combination of energy eigenstates, i.e., for stationary states, the d-entropy coincides with the S_N . On the other hand, S_D increases for systems out of equilibrium, and satisfies most of the requirements of a thermodynamic entropy [6,7].

The goal of this communication is to elucidate the approach to equilibrium of isolated quantum systems whose dynamics is governed by a Hamiltonian that can be tuned from integrable to chaotic. Equilibration [8,9] is a less restrictive property but which is (generally) deemed necessary for thermalization, i.e.,

the study of how isolated quantum systems can relax to states of equilibrium. In this work we try to unveil the intricate relation between the relaxation process and the transition from integrability to chaos. We study the approach to equilibrium in two different many-body quantum systems that can be parametrically tuned from regular to chaotic. We show that a universal relation between relaxation and delocalization of the initial state in the perturbed basis can be established regardless of the chaotic nature of system.

Hamiltonian is quenched at some initial time, then the system is left to evolve unitarily and finally a reversion of the original quench is applied. We consider that the quench is implemented by a sudden change of the (chaos) tuning parameter. Relaxation is then achieved by the approach to equilibrium. In terms of fluctuations, the paradigmatic Ising model [11] and a spin chain with nearest-neighbor and next-nearest-neighbor couplings. At equilibrium, the d-entropy is constant in time, thus in this case it will only depend on the time of the final quench τ . Given that the Hamiltonian for $t \gg \tau$ is H , the

We consider the following process. Initially the system is at state ρ_0 and the dynamics is given by a Hamiltonian H . At time $t = 0$ a quench is applied and the system evolves unitarily with the new (time-independent) Hamiltonian H' . Finally at time $t = \tau$ another quench changes the Hamiltonian to H'' . For simplicity, we consider a cyclic operation: $H'' = H$. The state of the system at time τ is $\rho(\tau) = e^{-iH'\tau} \rho_0 e^{iH'\tau}$, where we picked $\rho_0 = |n_0\rangle \langle n_0|$, with $|n_0\rangle$ an eigenstate of H . For time-independent Hamiltonians the d-entropy is constant in time, thus in this case it will only depend on the time of the final quench τ . Given that the Hamiltonian for $t \gg \tau$ is H , the

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Thank you



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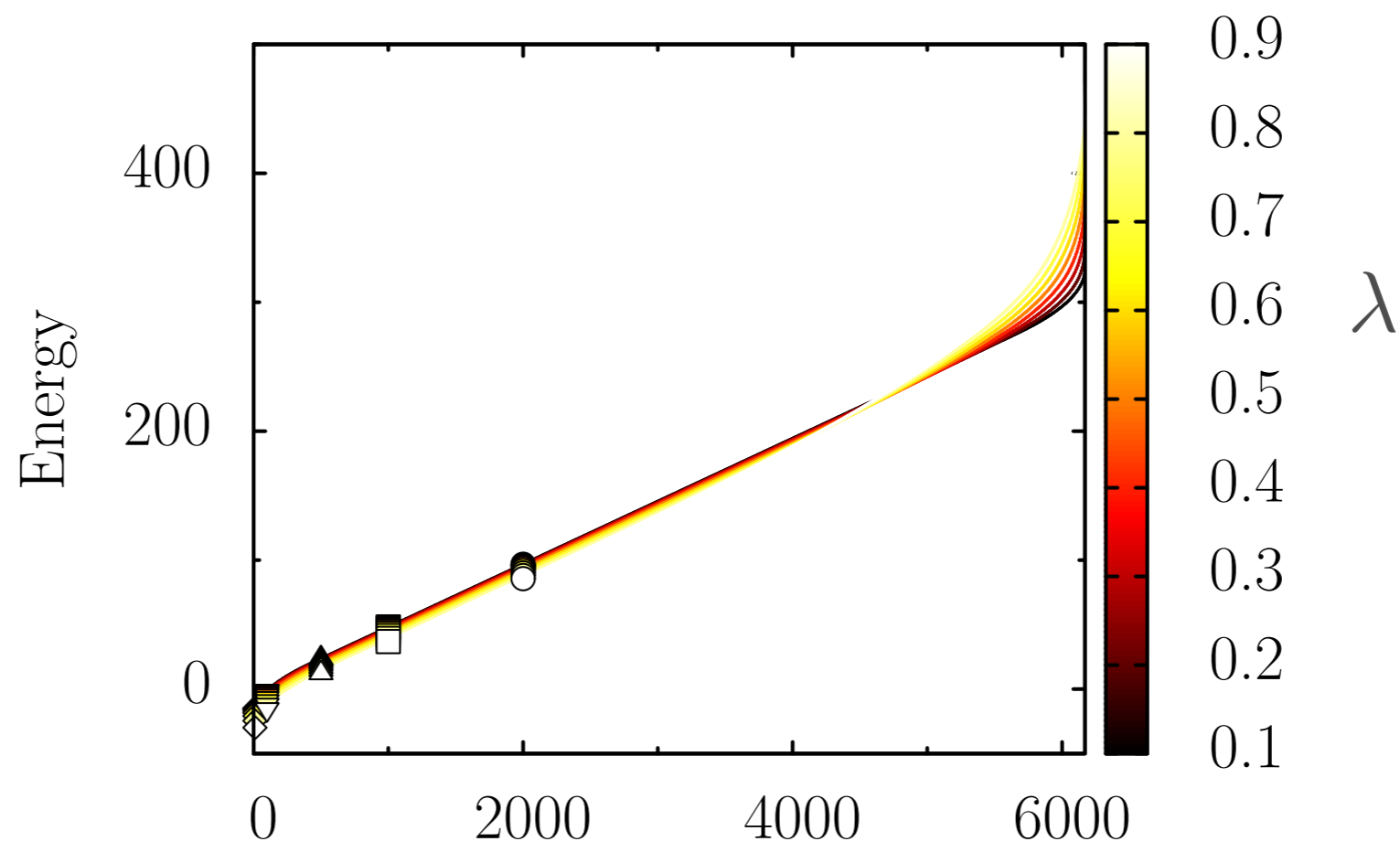
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Thank you

questions





$$j = 20, N = 250$$