Relaxation of isolated quantum systems beyond chaos

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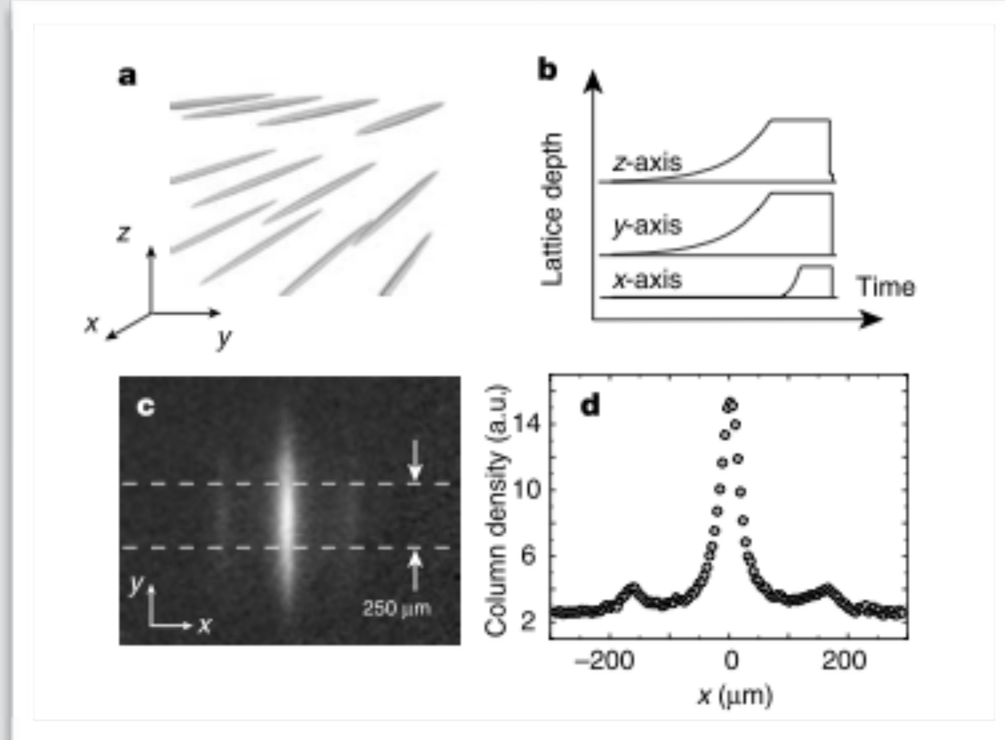
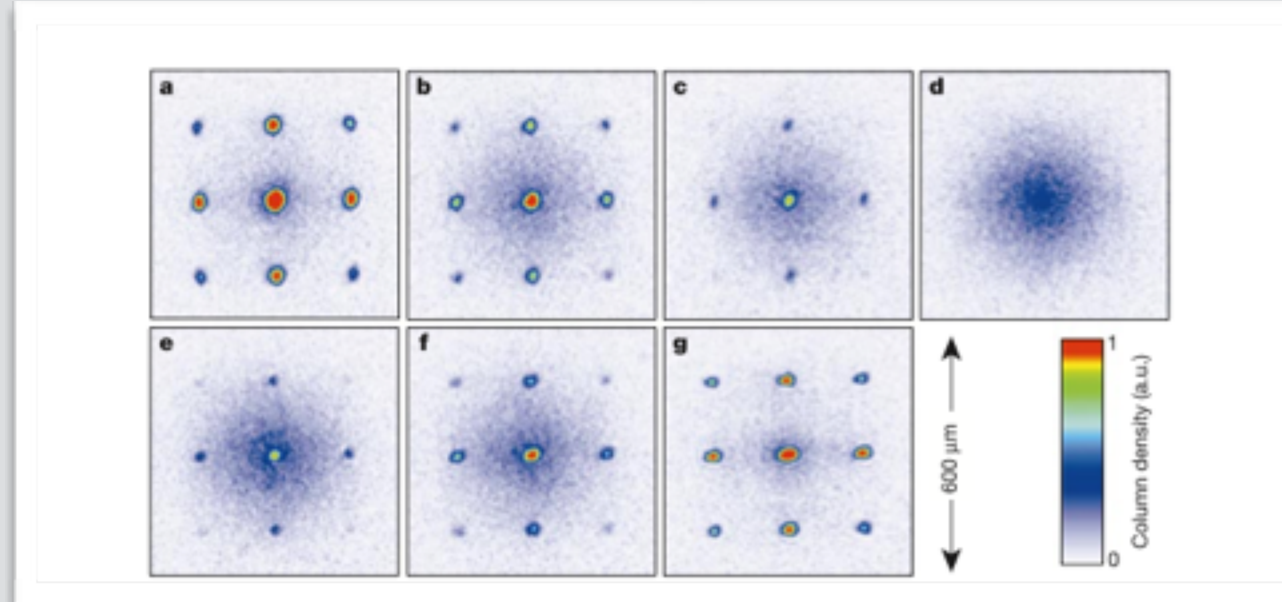
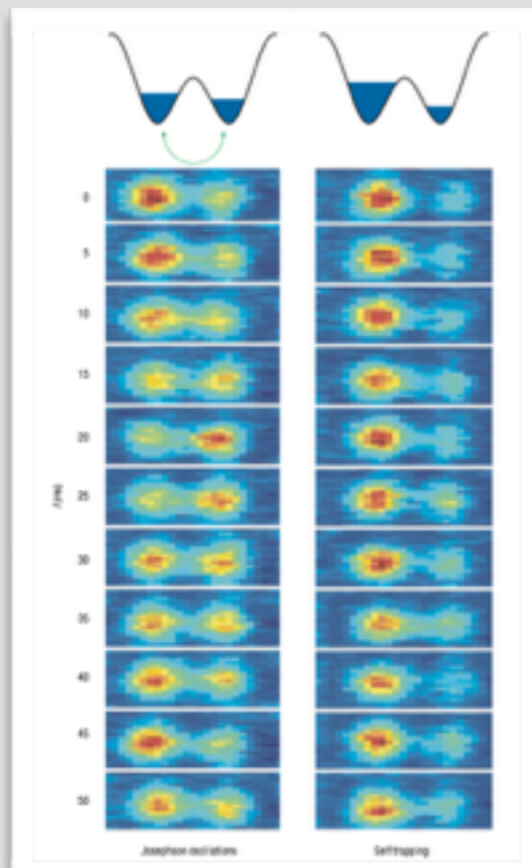
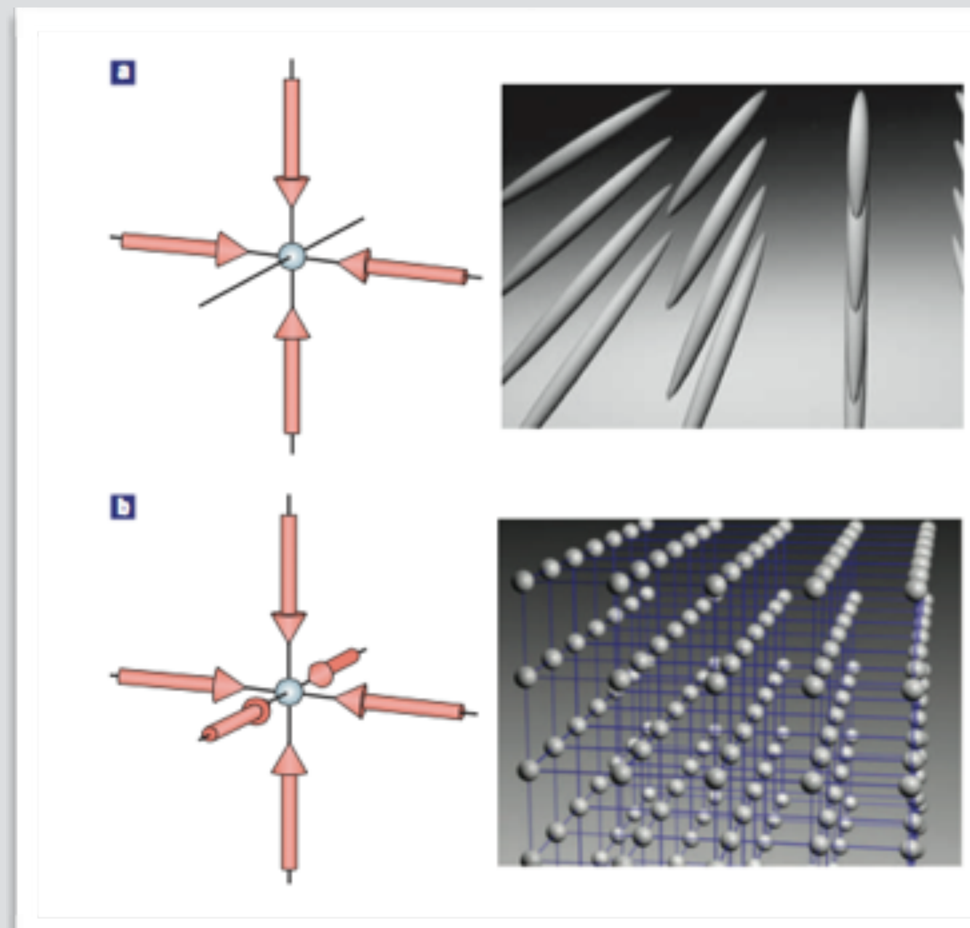
School for advanced sciences of Luchon
Quantum chaos: fundamentals and applications
Session Workshop I (W1), March 14 - 21, 2015


Buenos Aires
Mar del Plata


a
b
Lattice depth
z-axis
y-axis
x-axis
Time
z
x
y
c
250 μm
d
Column density (a.u.)
14
10
6
2
−200
0
200
x (μm)


a
b
c
d
e
f
g
600 μm
Column density (a.u.)
1
0


0
5
10
15
20
25
30
35
40
45
50
Josephson oscillations
Self-trapping


a
b

Isolated Quantum systems

non equilibrium dynamics — relaxation & universality

thermalization

PHYSICAL REVIEW E VOLUME 50, NUMBER 2 AUGUST 1994

Chaos and quantum thermalization

Mark Srednicki*
Department of Physics, University of California, Santa Barbara, Santa Barbara, California 93106
(Received 21 March 1994)

PHYSICAL REVIEW A VOLUME 43, NUMBER 4 15 FEBRUARY 1991

Brief Reports

Brief Reports are accounts of completed research which do not warrant regular articles or the priority handling given to Rapid Communications; however, the same standards of scientific quality apply. (Addenda are included in Brief Reports.) A Brief Report may be no longer than 3½ printed pages and must be accompanied by an abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.

Quantum statistical mechanics in a closed system

J. M. Deutsch
Department of Physics, University of California, Santa Cruz, California 95064
and The James Franck Institute, 5640 South Ellis Avenue, Chicago, Illinois 60637
(Received 4 December 1989)

A closed quantum-mechanical system with a large number of degrees of freedom does not necessarily give time averages in agreement with the microcanonical distribution. For systems where the different degrees of freedom are uncoupled, situations are discussed that show a violation of the usual statistical-mechanical rules. By adding a finite but very small perturbation in the form of a random matrix, it is shown that the results of quantum statistical mechanics are recovered. Expectation values in energy eigenstates for this perturbed system are also discussed, and deviations from the microcanonical result are shown to become exponentially small in the number of degrees of freedom.

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Quantum statistical mechanics in a closed system

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A closed qu
sarily give tim
different degr
al statistical-r
dom matrix, i
tion values in
the microcan
dom.

nature Vol 452 | 17 April 2008 | doi:10.1038/nature06838

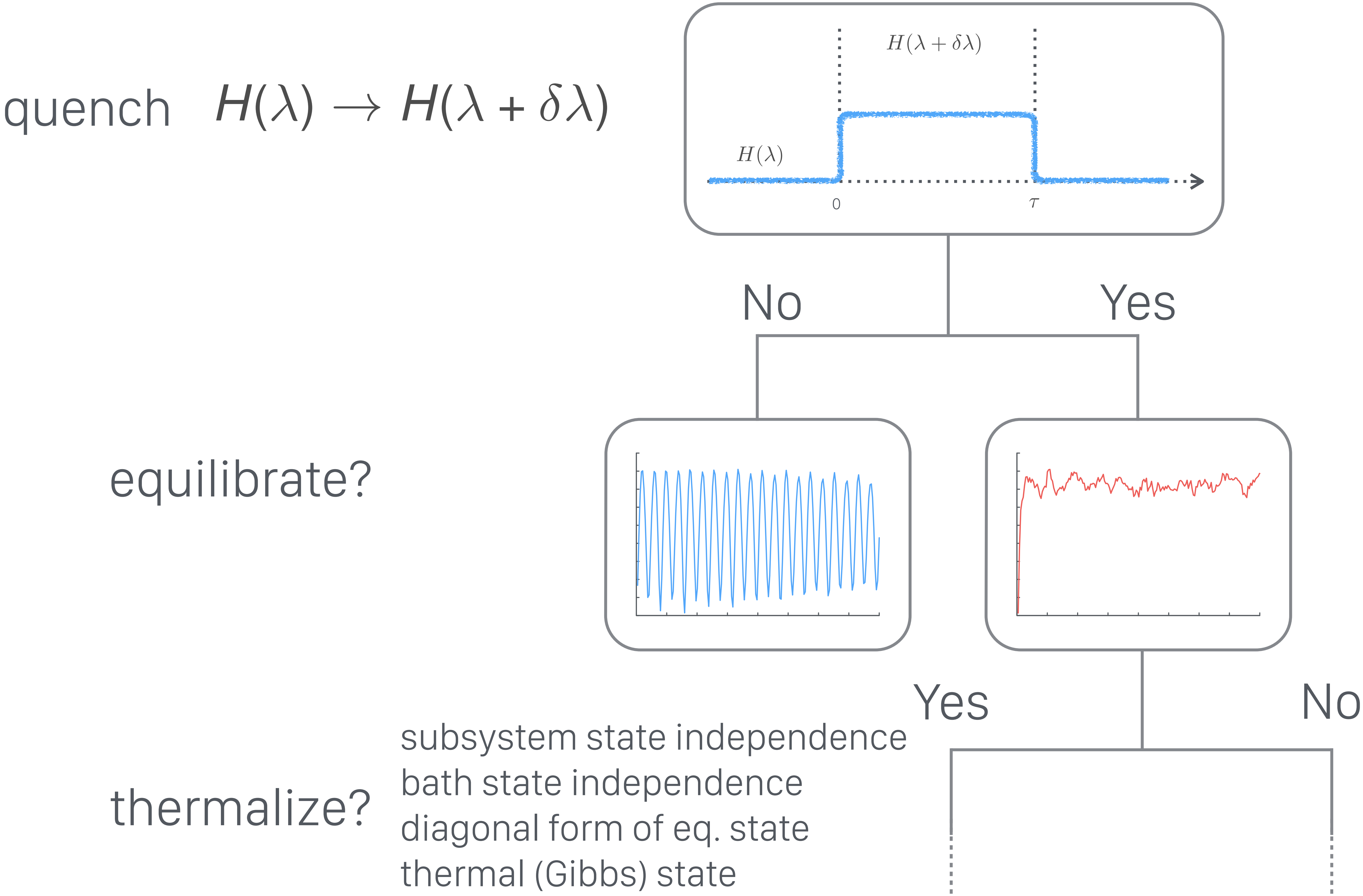
LETTERS

Thermalization and its mechanism for generic isolated quantum systems

Marcos Rigol[1,2], Vanja Dunjko[1,2] & Maxim Olshanii[2]

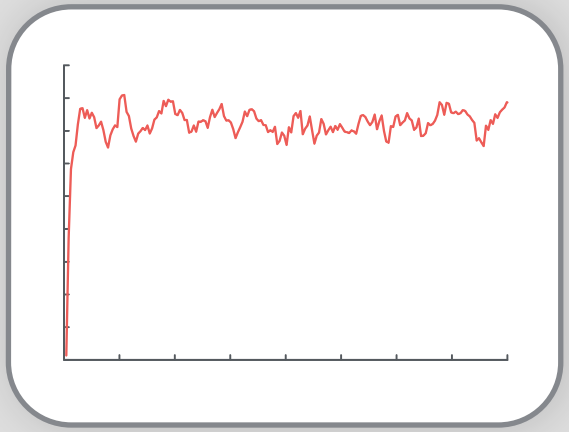
Eigenstate thermalization hypothesis		
$A_{nn} = \langle n \vert \hat{A} \vert n \rangle$	smooth (approx. constant)	$\langle A \rangle_t \approx \langle A \rangle_{\mathrm{MC}}$
$A_{nm} = \langle n \vert \hat{A} \vert m \rangle$	very small	

Other mechanism: see C. Gogolin's thesis

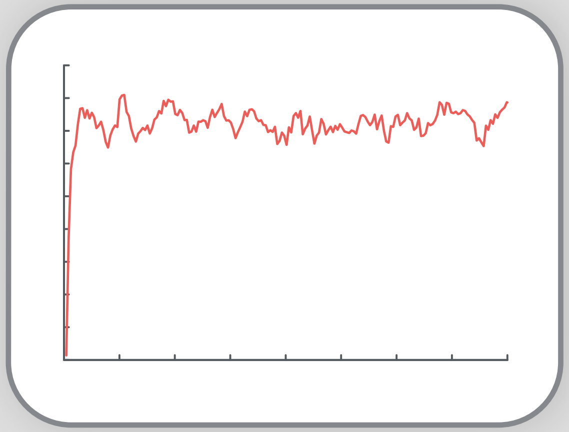


quench $H(\lambda) \rightarrow H(\lambda + \delta\lambda)$
$H(\lambda + \delta\lambda)$
$H(\lambda)$
0
τ
No
Yes
equilibrate?
Yes
No
thermalize?
subsystem state independence
bath state independence
diagonal form of eq. state
thermal (Gibbs) state

equilibrate?



equilibrate?



how?

chaos?

quantum Entropy

von Neumann?

$$S_{\mathrm{vN}}(\rho) = -\mathrm{Tr}(\rho \ln \rho)$$

	consistent with second law
von Neumann entropy $S_{\mathrm{vN}}(\rho) = -\mathrm{Tr}(\rho \ln \rho)$	equilibrium
diagonal entropy $S_{\mathrm{D}}(\rho) = -\sum_n \rho_{nn} \ln \rho_{nn}$	non-equilibrium external operations

Santos, Polkovnikov & Rigol, PRL **107**, 040601 (2011)

Polkovnikov, Ann. Phys. **326**, 486 (2011)

consistent with second law

diagonal entropy

$$S_{\mathrm{D}}(\rho) = -\sum_n \rho_{nn} \ln \rho_{nn}$$

increases

conserved for
adiabatic process

uniquely related to P(E)

additive

Santos, Polkovnikov & Rigol, PRL **107**, 040601 (2011)
Polkovnikov, Ann. Phys. **326**, 486 (2011)

$$\mathrm{Prob}[S_{\mathrm{D}}(\rho_0)] \leq S_{\mathrm{D}}(\rho_\tau)] \sim 1$$

$$S_{\mathrm{dec}} - \overline{S_{\mathrm{D}}(\tau)} \leq 1 - \gamma$$

$\gamma = 0.5772\ldots$

$$S_{\mathrm{dec}} = S_{\mathrm{D}}(\overline{\rho})$$

Ikeda, Sakumichi, Polkovnikov, & Ueda. Ann. Phys **354** (2015) 338-352

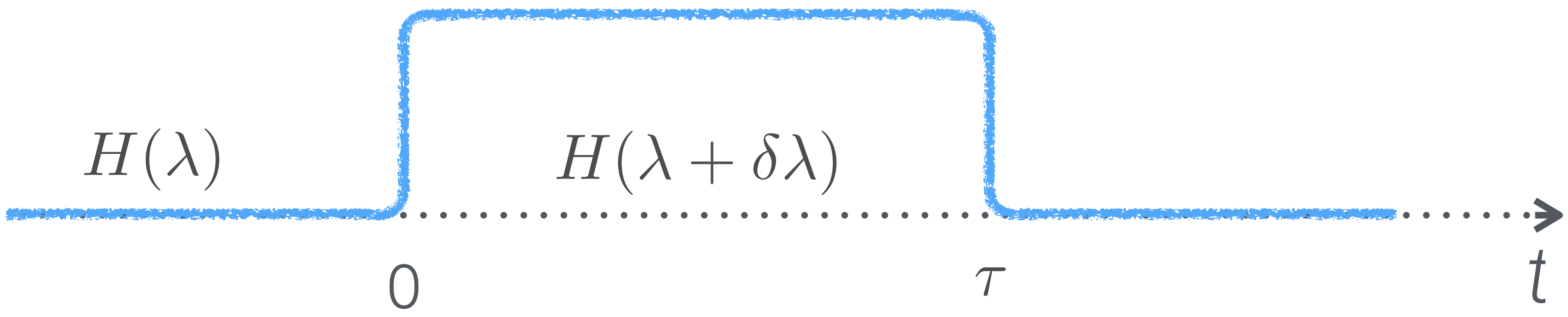
$$\mathrm{Prob}[S_{\mathrm{D}}(\rho_0)] \leq S_{\mathrm{D}}(\rho_\tau)] \sim 1$$

$$S_{\mathrm{dec}} - \overline{S_{\mathrm{D}}(\tau)} \leq 1 - \gamma$$

$$S_{\mathrm{dec}} = S_{\mathrm{D}}(\overline{\rho})$$

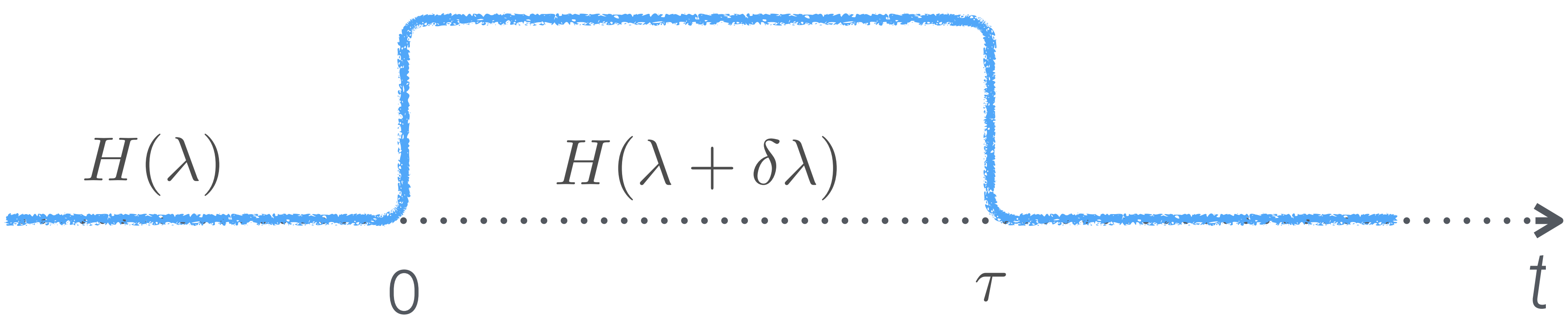
Ikeda, Sakumichi, Polkovnikov, & Ueda. Ann. Phys **354** (2015) 338–352

Quench dynamics



Cyclic process

$$\rho_0 = |n_0\rangle\langle n_0| \qquad \rho(\tau) = e^{-H'\tau}\rho_0 e^{iH'\tau}$$



$$S_{\mathrm{D}} = -\sum_n C_n(\tau) \ln C_n(\tau)$$

$$S_{\mathrm{dec}} = S_{\mathrm{D}}(\overline{\rho})$$

equilibrium

$$S_{\mathrm{dec}} - \overline{S_{\mathrm{D}}(\tau)}$$

$$\Delta S_{\mathrm{D}}(\tau)/\overline{S_{\mathrm{D}}(\tau)}$$

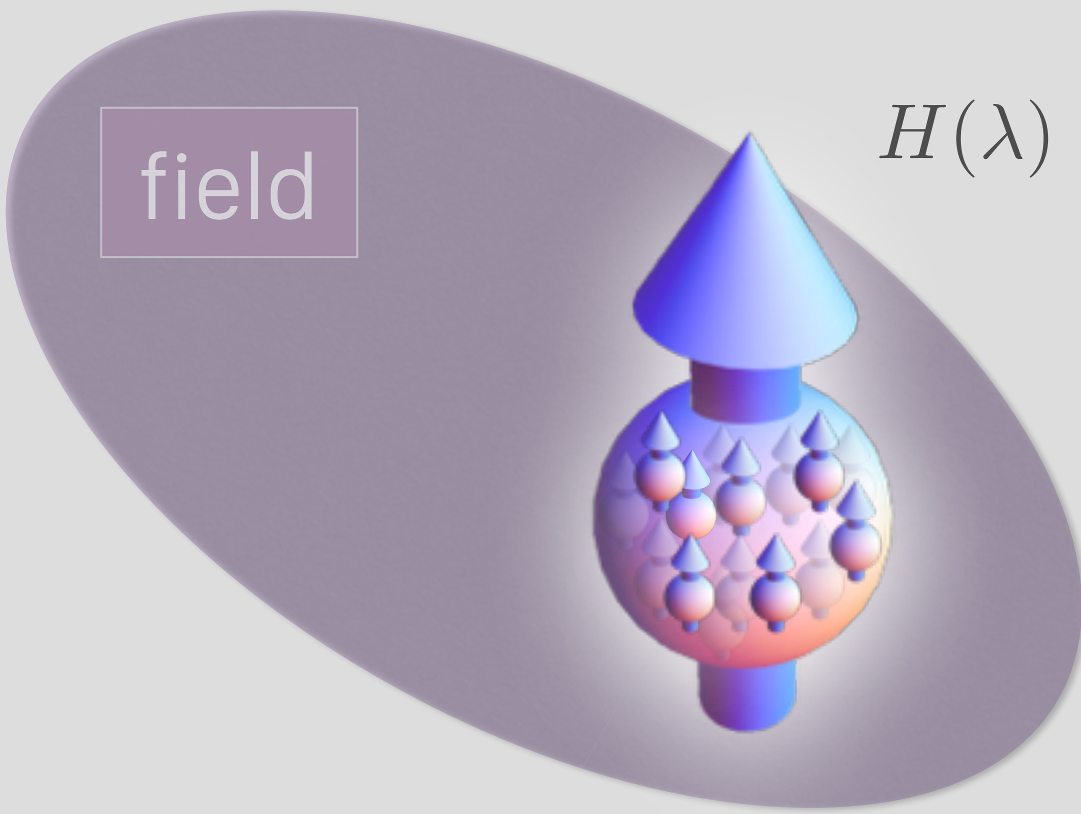
equilibrium

$$S_{\rm dec} - \overline{S_{\rm D}(\tau)} \to 1 - \gamma$$

$$\Delta S_{\rm D}(\tau)/\overline{S_{\rm D}(\tau)} \ll 1$$

two different models

Dicke model



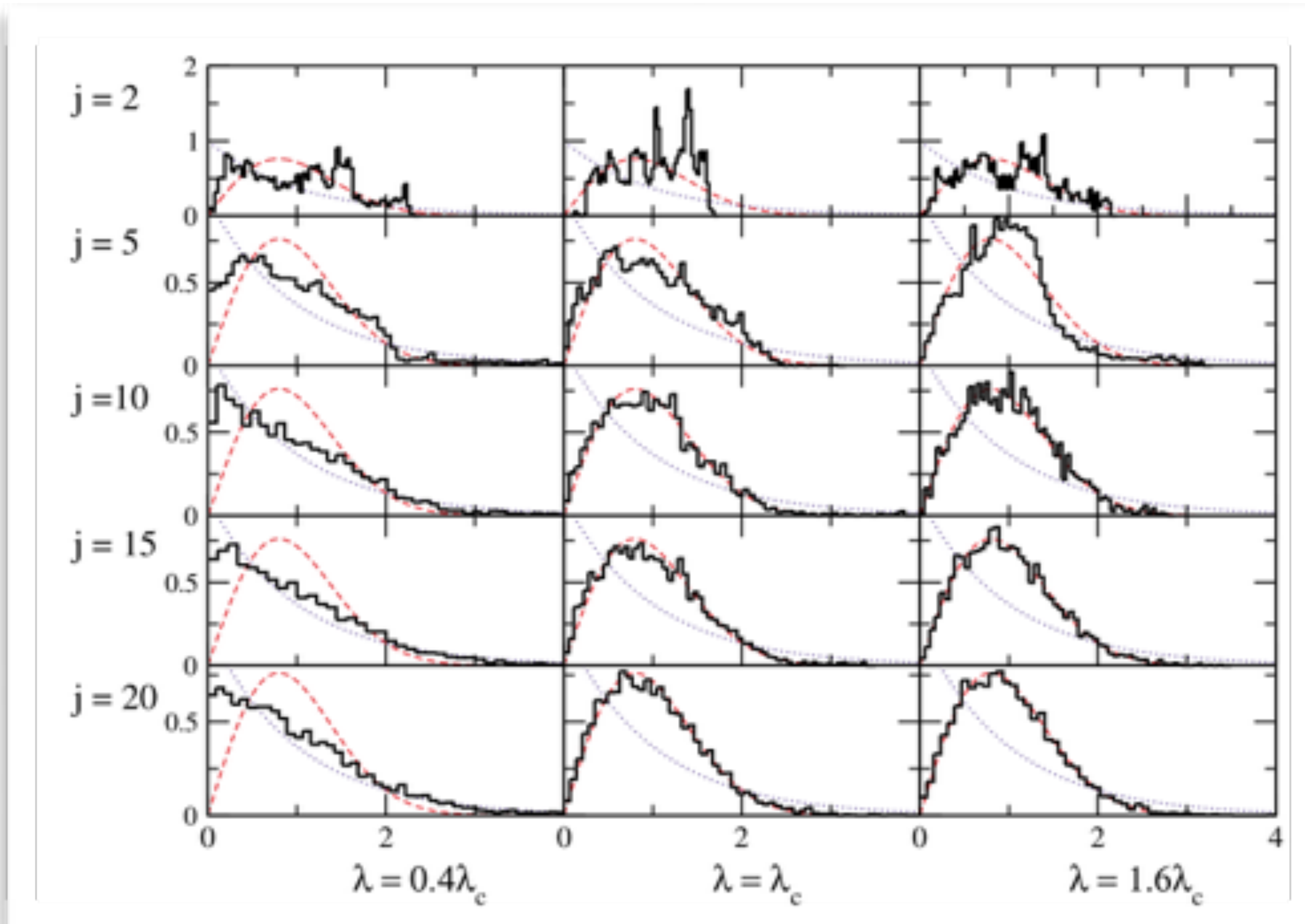
$$H(\lambda) = \omega_0 J_z + \omega a^\dagger a + \frac{\lambda}{\sqrt{2j}}(a^\dagger + a)(J_+ + J_-)$$

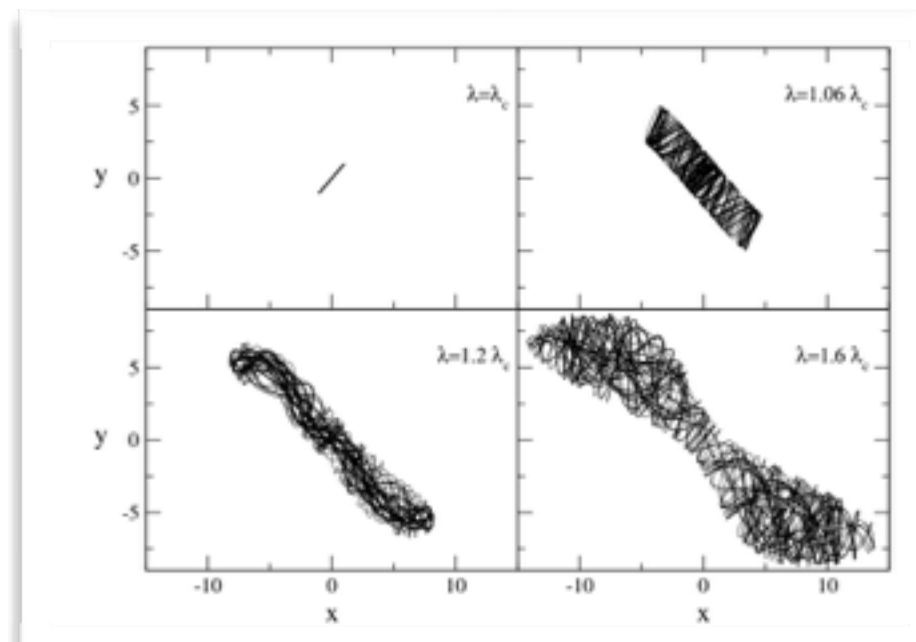
superradiant transition

$$\lambda_c = \frac{1}{2}\sqrt{\omega_0 \omega}$$

$\omega_0 = \omega = \hbar = 1 \qquad \lambda_c = 0.5$

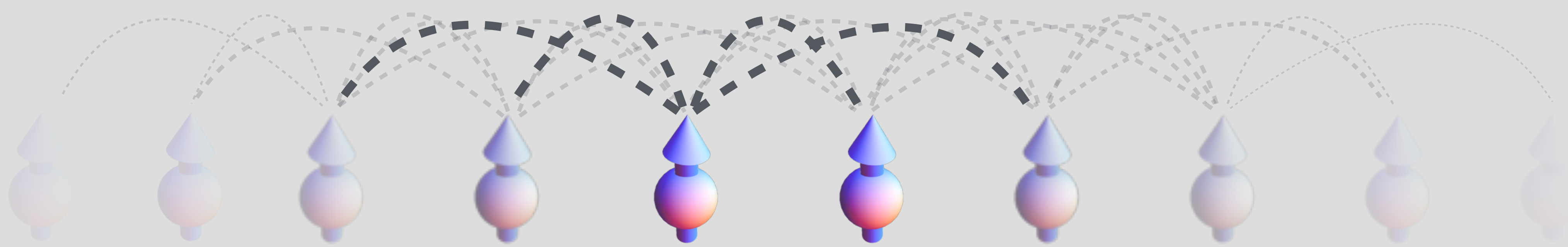
Dicke model





Emary & Brandes, PRL **90**, 044101 (2003)

Spin system

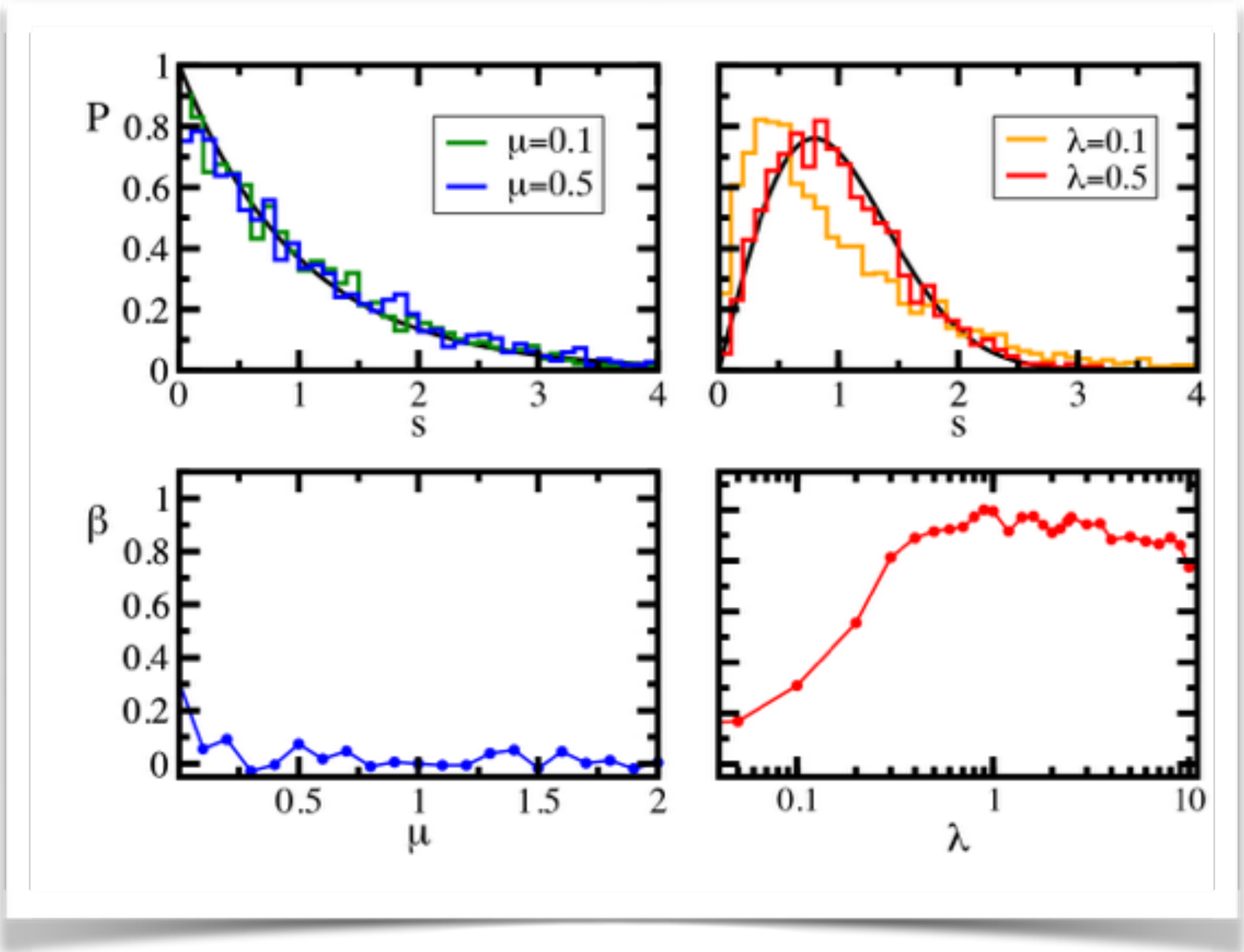


$$H(\lambda) = H_0 + \lambda V$$

$$H_0 = \sum_{i=1}^{L-1} J(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \mu S_i^z S_{i+1}^z)$$

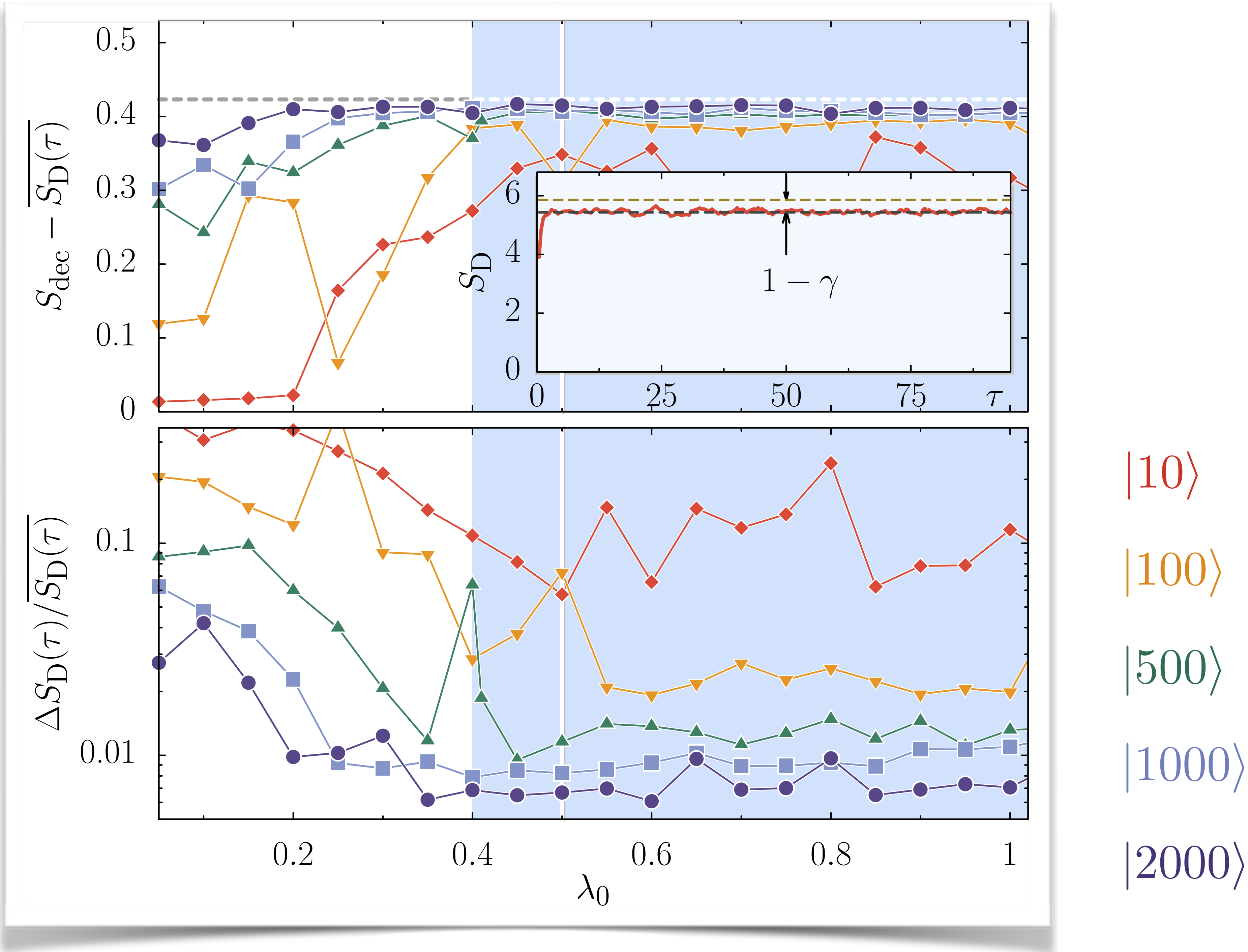
$$V = \sum_{i=0}^{L-2} J(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + \mu S_i^z S_{i+2}^z)$$

Spin system



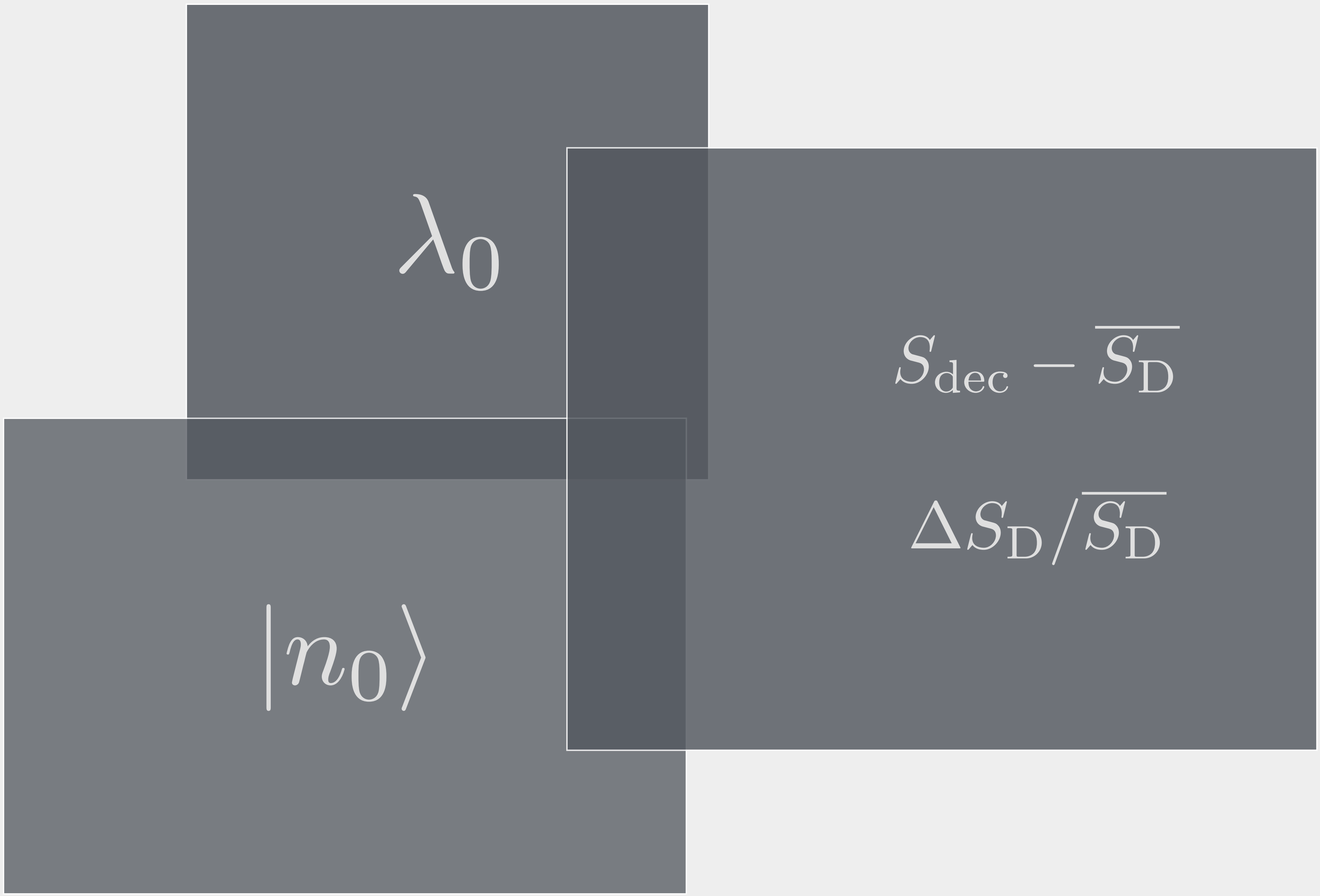
Santos, Borgonovi & Izrailev, PRE **85**. 036209 (2012)

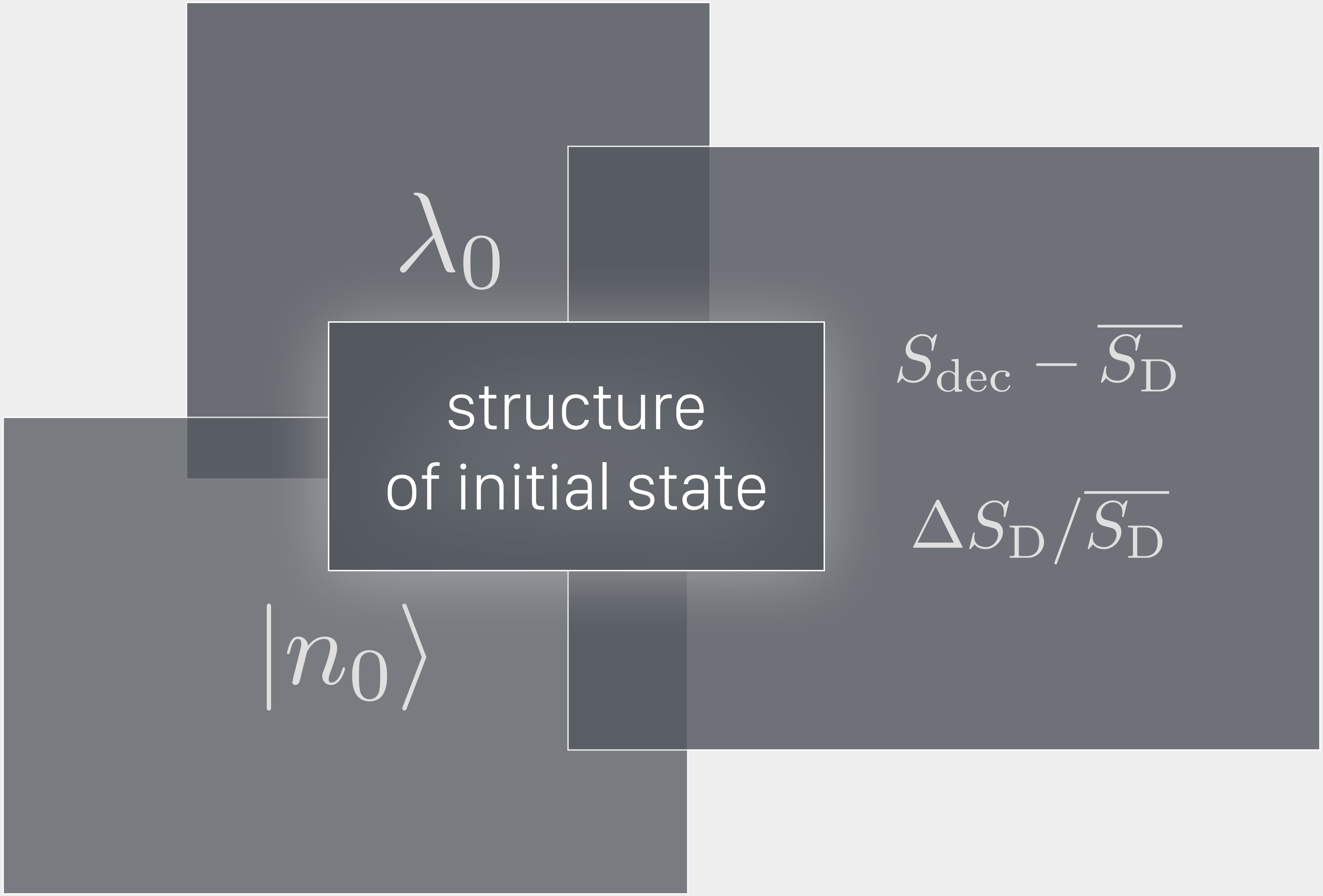
Dicke



$j = 20, N = 250$ $\delta\lambda = 0.1$

$\lambda_c = 0.5$

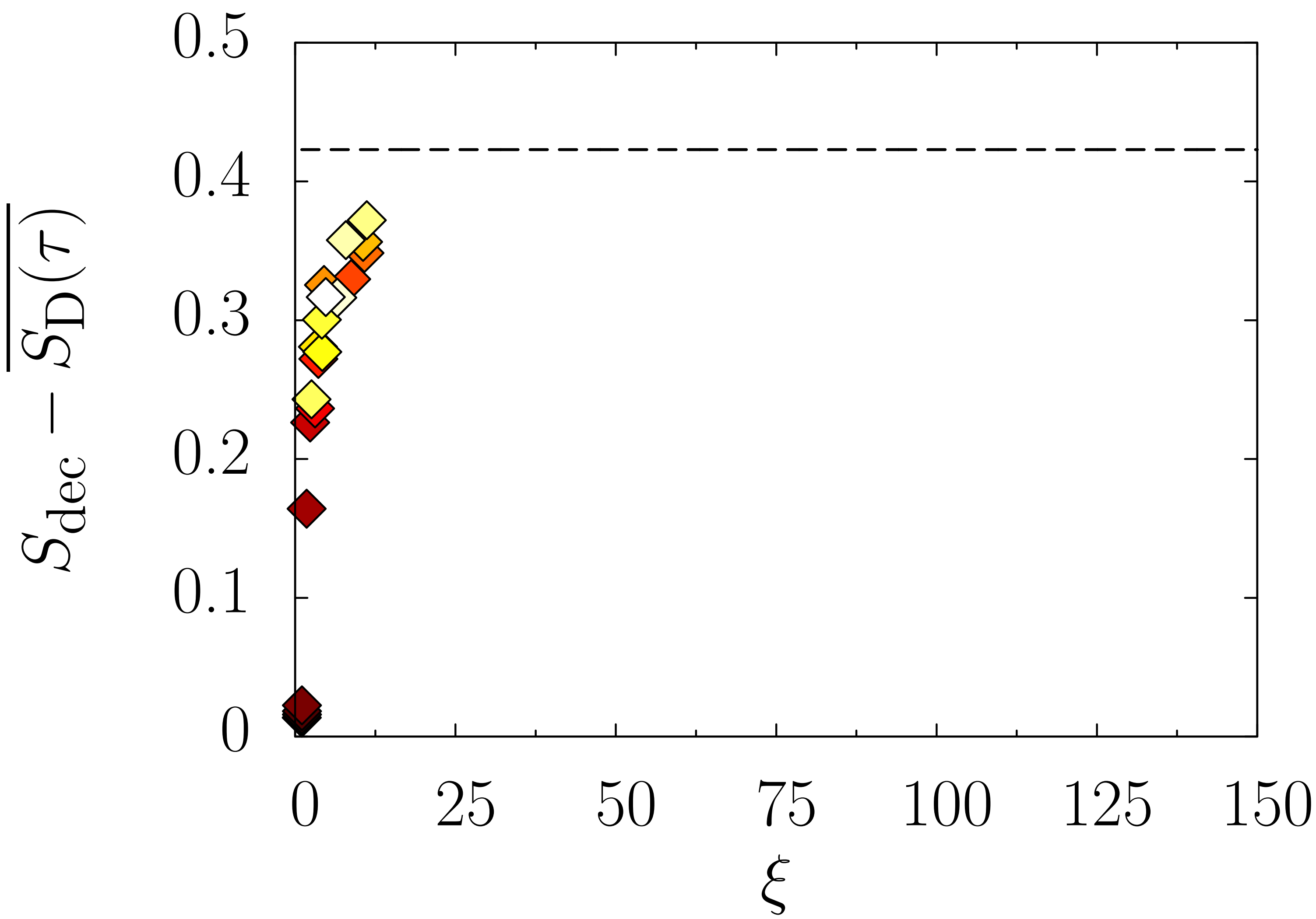

λ_0
$S_{\mathrm{dec}} - \overline{S_{\mathrm{D}}}$
$\Delta S_{\mathrm{D}} / \overline{S_{\mathrm{D}}}$
$|n_0\rangle$

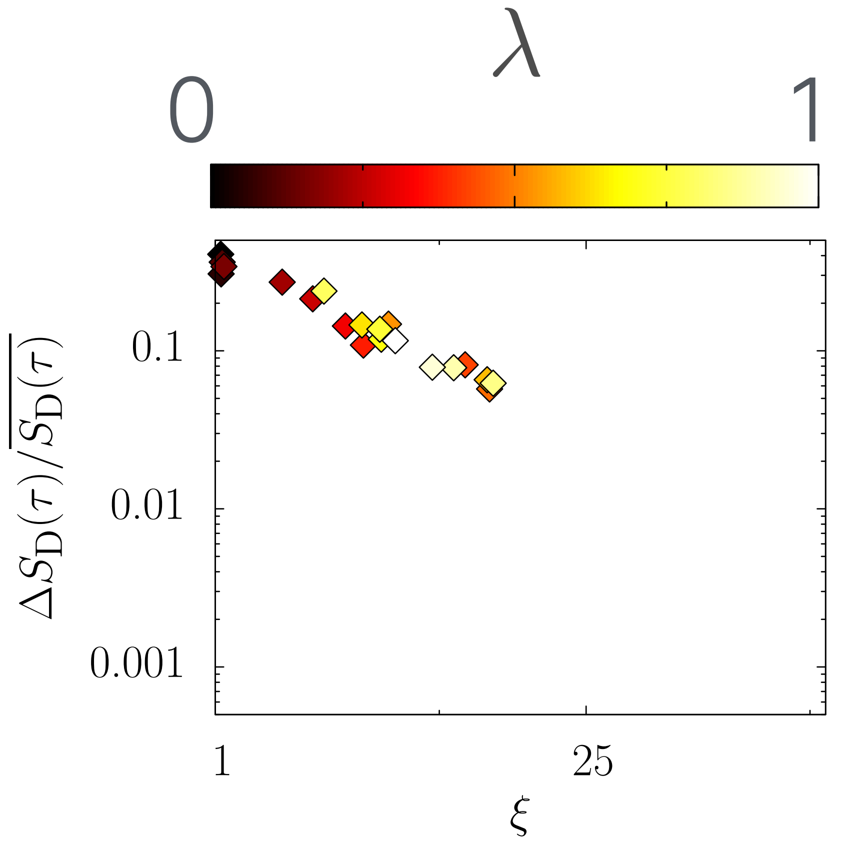


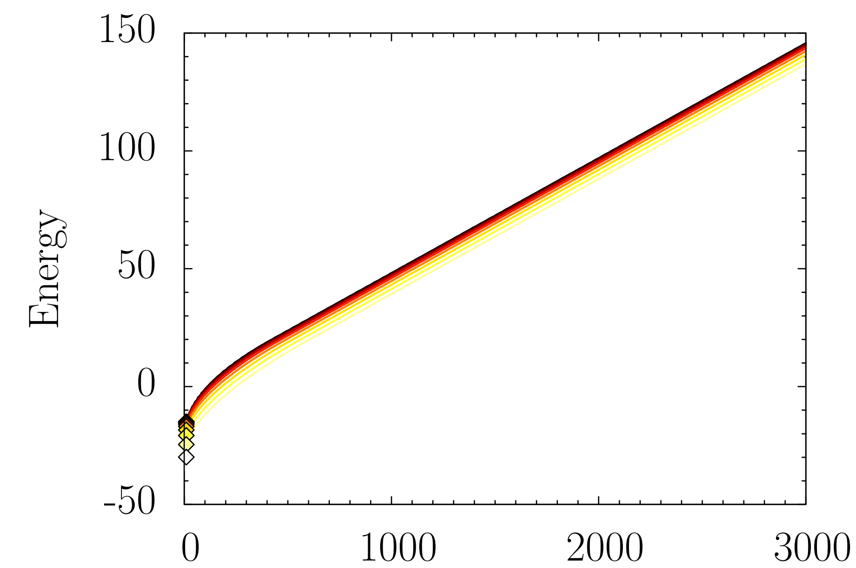
λ_0
structure
of initial state
$S_{\mathrm{dec}} - \overline{S_{\mathrm{D}}}$
$\Delta S_{\mathrm{D}}/\overline{S_{\mathrm{D}}}$
$|n_0\rangle$

IPR	$\xi = \frac{1}{\sum_m |\langle n(\lambda) | m(\lambda + \delta\lambda)\rangle|^4}$
basis complexity of eigenstates Localization chaos	B.V. Chirikov,F.M Izrailev and D.L. Shepelyansky Physica D 33 (1988) 77-88 Y. V. Fyodorov and A. D. Mirlin, Phys. Rev. B 52, R11580 (1995). Ph. Jacquod and D. L. Shepelyansky, Phys. Rev. Lett. 75, 3501 (1995). B. Georgeot and D. L. Shepelyansky, Phys. Rev. Lett. 79, 4365 (1997). R. Berkovits and Y. Avishai, Phys. Rev. Lett. 80, 568 (1998). V. V. Flambaum, A. A. Gribakina, G. F. Gribakin, and M. G. Kozlov, Phys. Rev. A 50, 267 (19 ... ... L. F. Santos F. Borgonovi, and F. M. Izrailev, PRE 85, 036209 (2012)

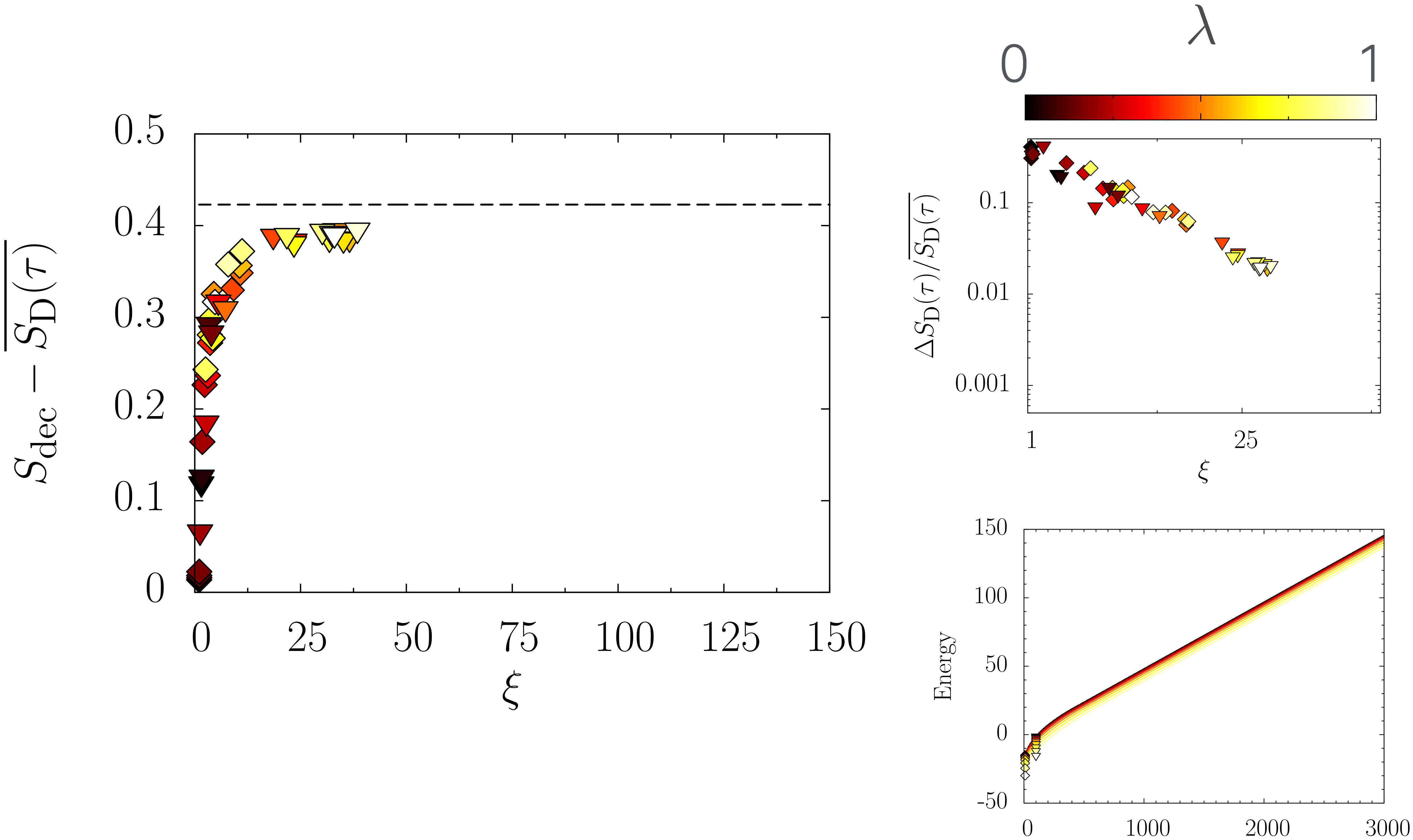
Dicke model


0.5
0.4
0.3
0.2
0.1
0
0
25
50
75
100
125
150
ξ
$S_{\mathrm{dec}} - \overline{S_{\mathrm{D}}(\tau)}$


λ
0
1
0.1
0.01
0.001
1
25
ξ
$\Delta S_{\mathrm{D}}(\tau)/\overline{S_{\mathrm{D}}(\tau)}$

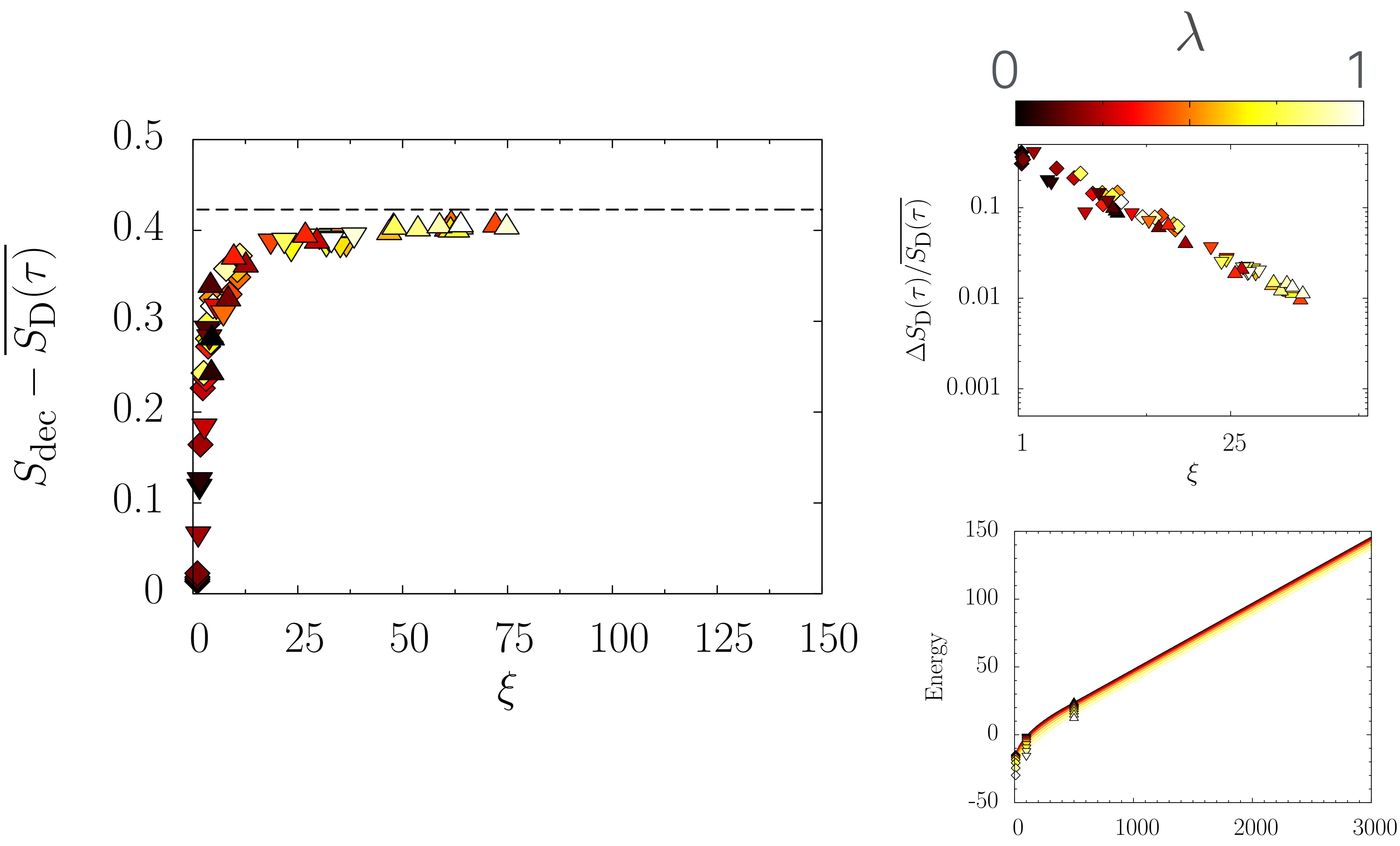

150
100
50
0
-50
0
1000
2000
3000
Energy

Dicke model



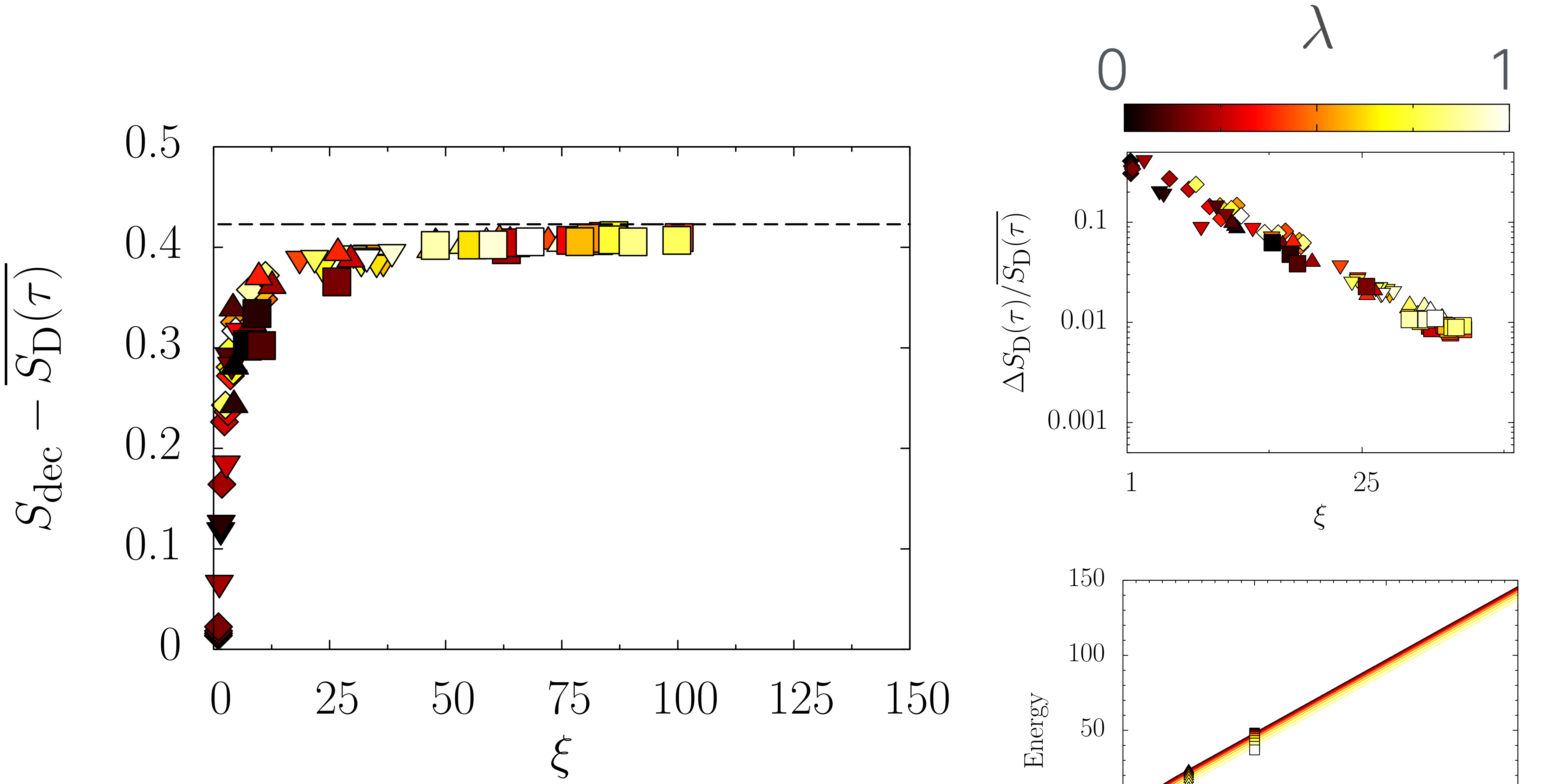
λ
0
1
$S_{\mathrm{dec}} - \overline{S_{\mathrm{D}}(\tau)}$
0.5
0.4
0.3
0.2
0.1
0
0
25
50
75
100
125
150
ξ
$\Delta S_{\mathrm{D}}(\tau)/\overline{S_{\mathrm{D}}(\tau)}$
0.1
0.01
0.001
1
25
ξ
Energy
150
100
50
0
-50
0
1000
2000
3000

Dicke model



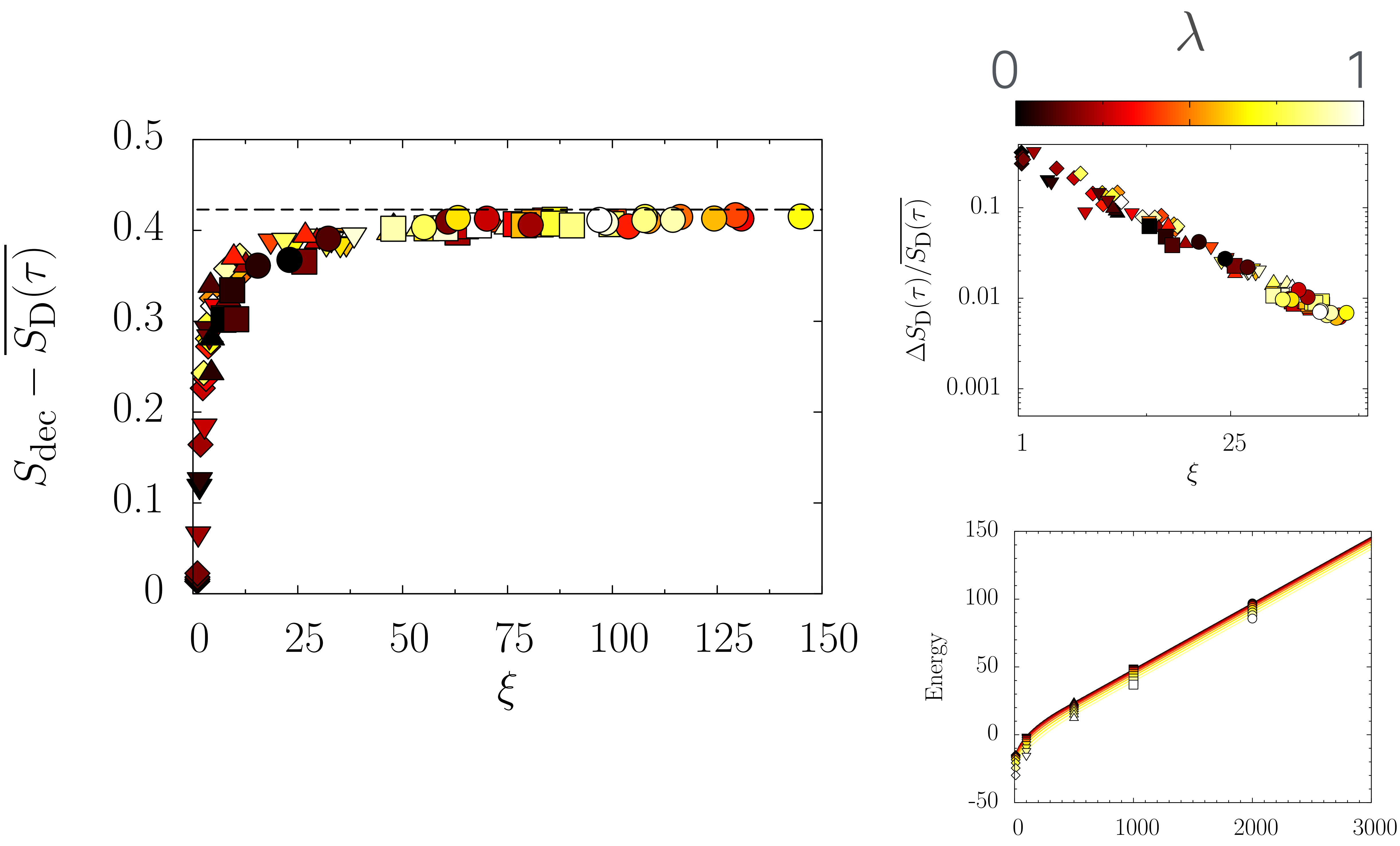
λ
0
1
0.5
0.4
0.3
0.2
0.1
0
$S_{\rm dec} - \overline{S_{\rm D}(\tau)}$
0
25
50
75
100
125
150
ξ
0.1
0.01
0.001
$\Delta S_{\rm D}(\tau)/\overline{S_{\rm D}(\tau)}$
1
25
ξ
150
100
50
0
-50
Energy
0
1000
2000
3000

Dicke model

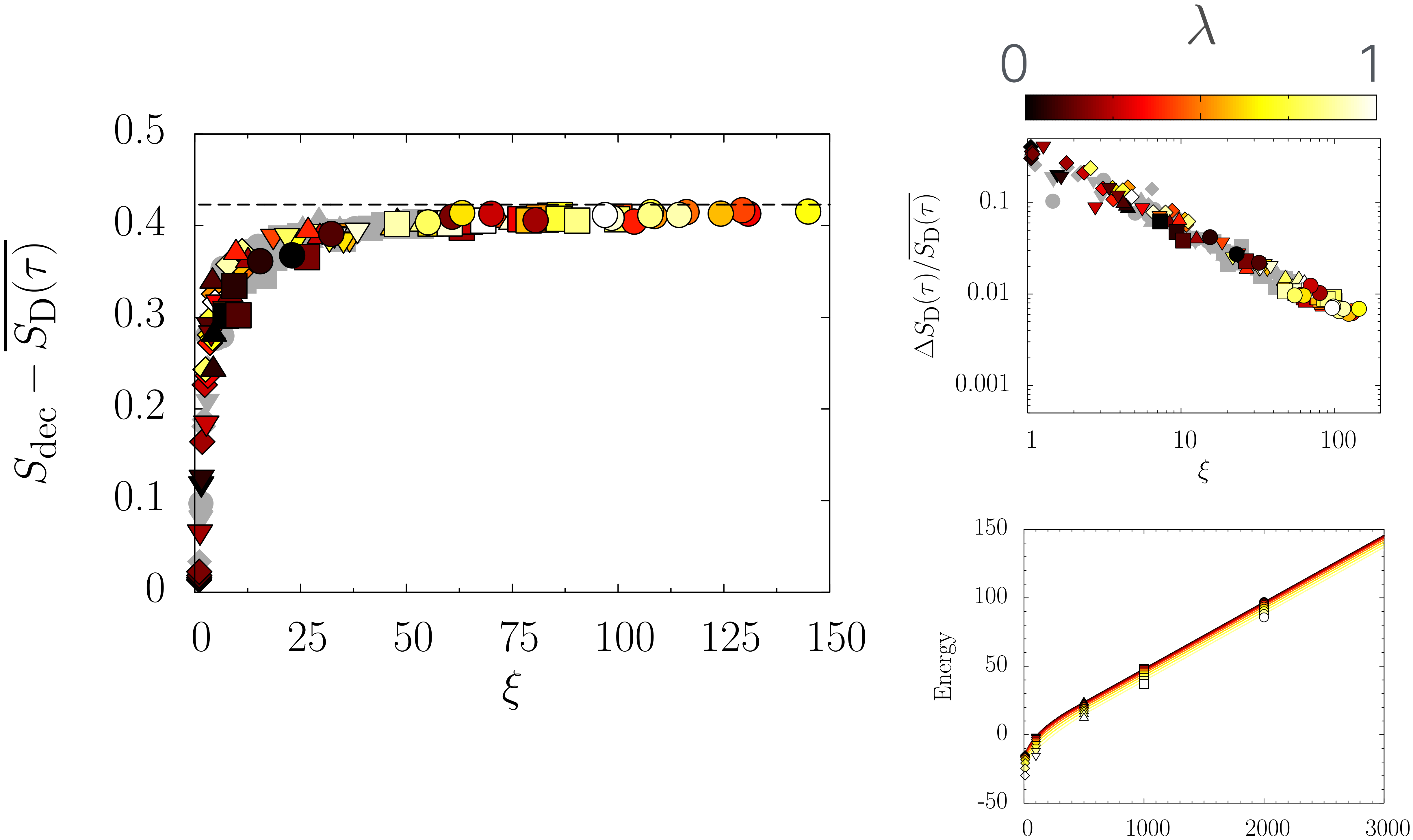


$S_{\rm dec} - \overline{S_{\rm D}(\tau)}$
0.5
0.4
0.3
0.2
0.1
0
0
25
50
75
100
125
150
ξ
λ
0
1
$\Delta S_{\rm D}(\tau)/\overline{S_{\rm D}(\tau)}$
0.1
0.01
0.001
1
25
ξ
Energy
150
100
50
0
-50
0
1000
2000
3000

Dicke model

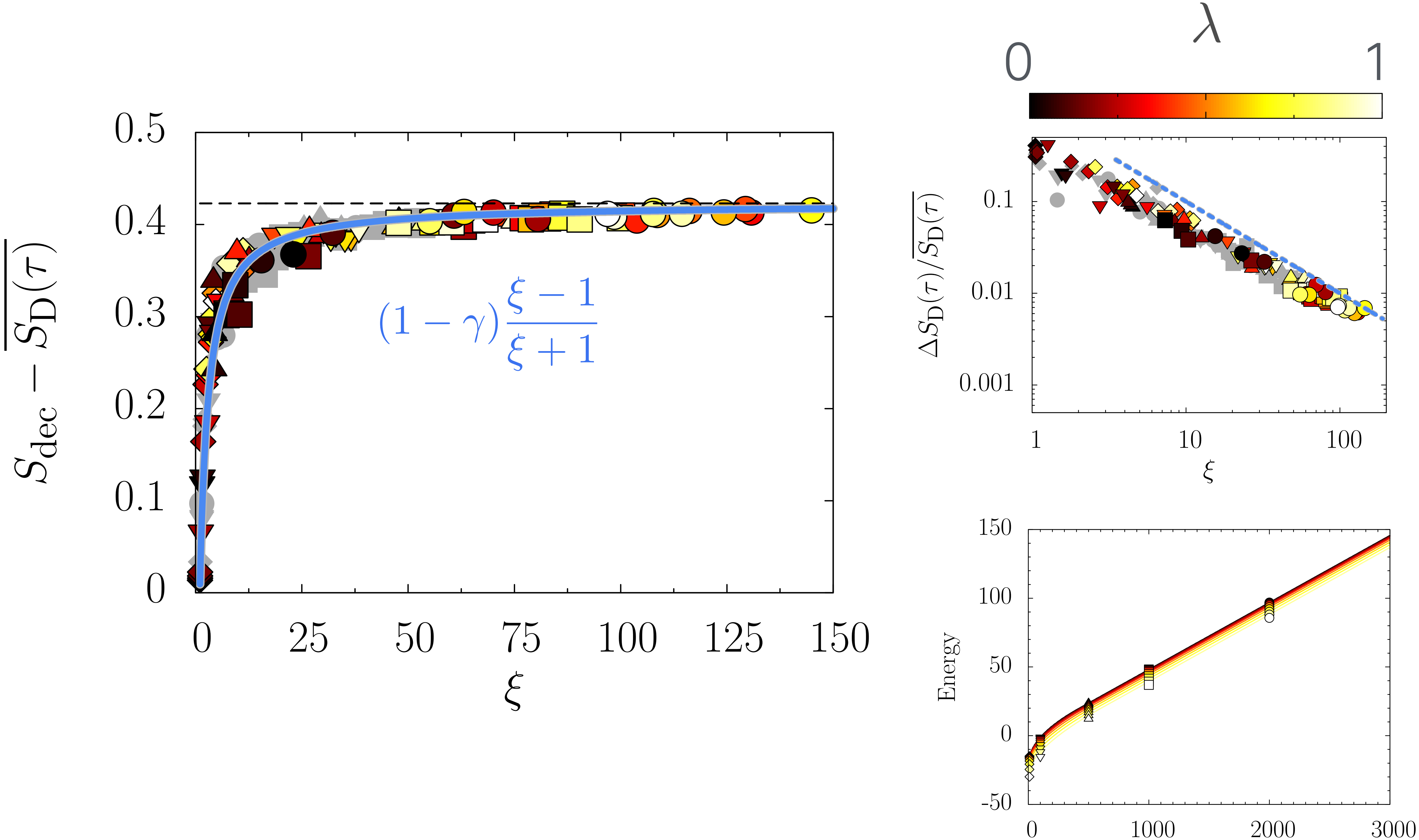

λ
0
1
0.5
0.4
0.3
0.2
0.1
0
0
25
50
75
100
125
150
ξ
$S_{\mathrm{dec}} - \overline{S_{\mathrm{D}}(\tau)}$
0.1
0.01
0.001
1
25
ξ
$\Delta S_{\mathrm{D}}(\tau)/\overline{S_{\mathrm{D}}(\tau)}$
150
100
50
0
-50
0
1000
2000
3000
Energy

Dicke model



λ
0
1
0.5
0.4
0.3
0.2
0.1
0
0
25
50
75
100
125
150
ξ
$S_{\rm dec} - \overline{S_{\rm D}(\tau)}$
$\Delta S_{\rm D}(\tau)/\overline{S_{\rm D}(\tau)}$
0.1
0.01
0.001
1
10
100
ξ
Energy
150
100
50
0
-50
0
1000
2000
3000

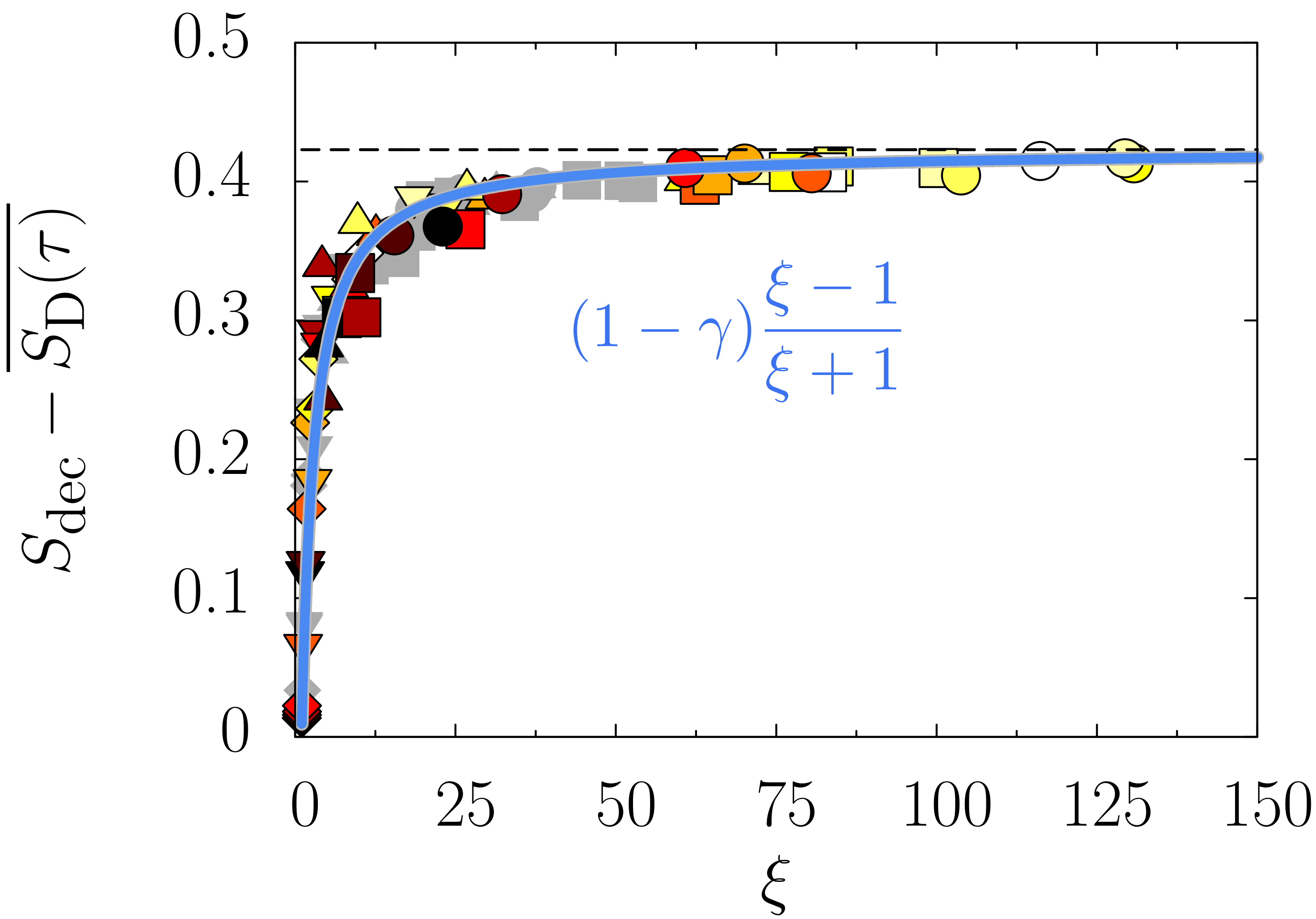
Dicke model

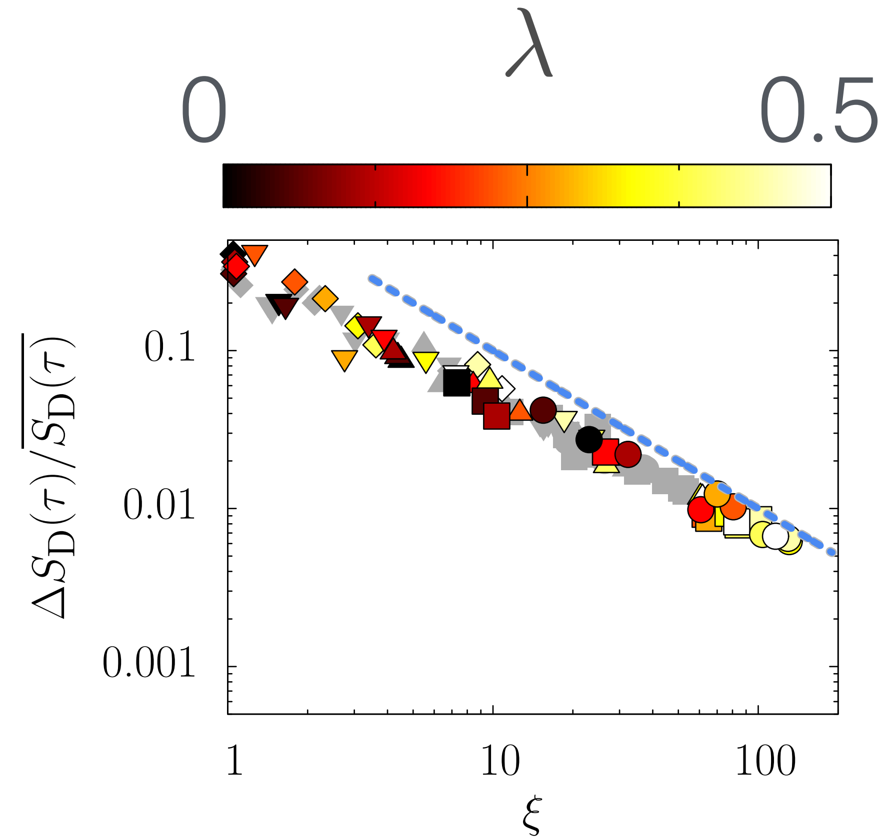


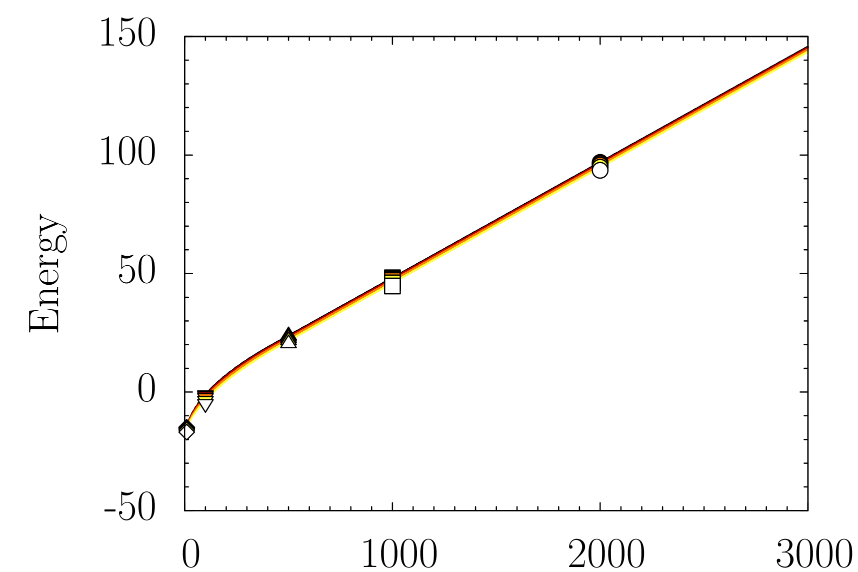
λ
0
1
$S_{\rm dec} - \overline{S_{\rm D}(\tau)}$
ξ
$(1-\gamma)\frac{\xi-1}{\xi+1}$
0
0.1
0.2
0.3
0.4
0.5
25
50
75
100
125
150
$\Delta S_{\rm D}(\tau)/\overline{S_{\rm D}(\tau)}$
0.001
0.01
10
Energy
-50
1000
2000
3000

Dicke model

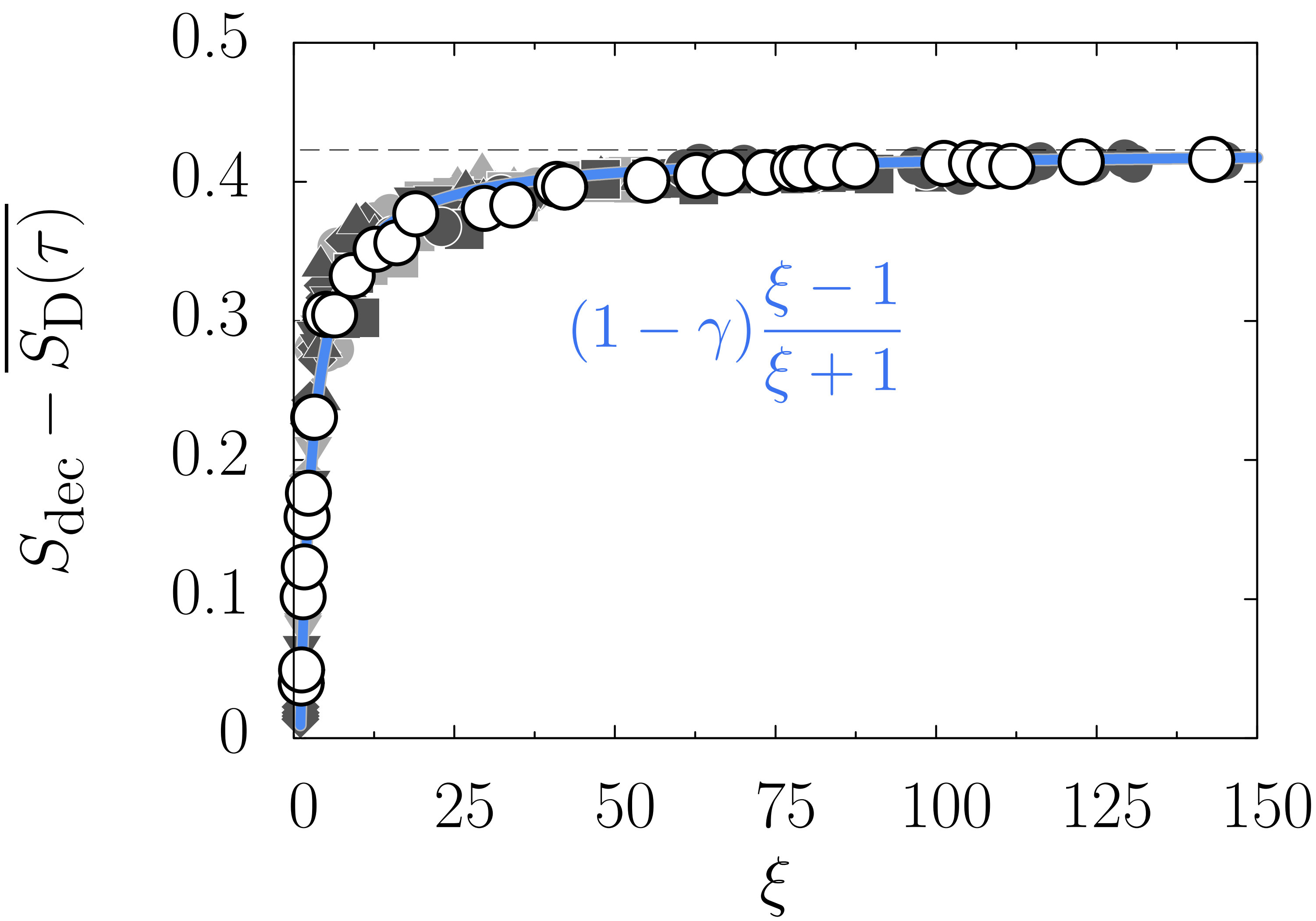
$\lambda < \lambda_c$

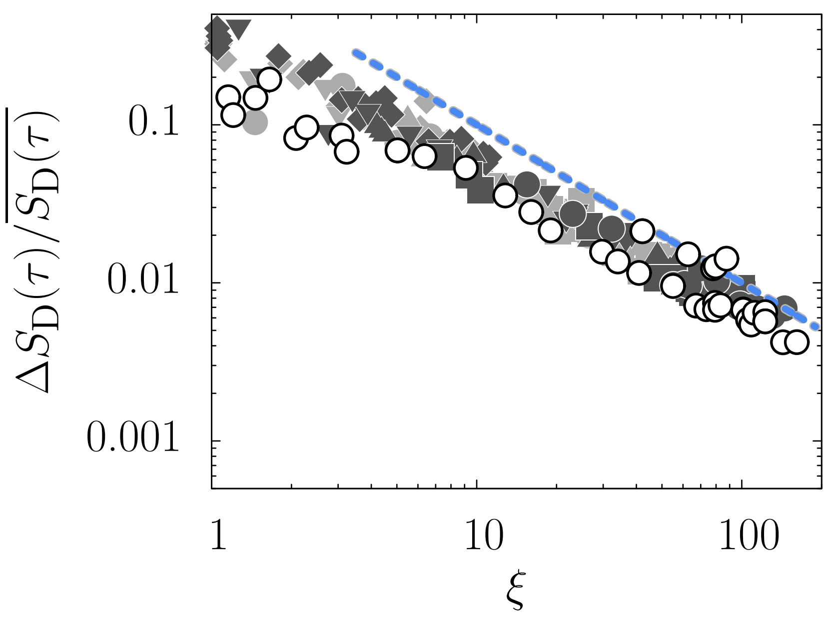






Dicke model & spin system





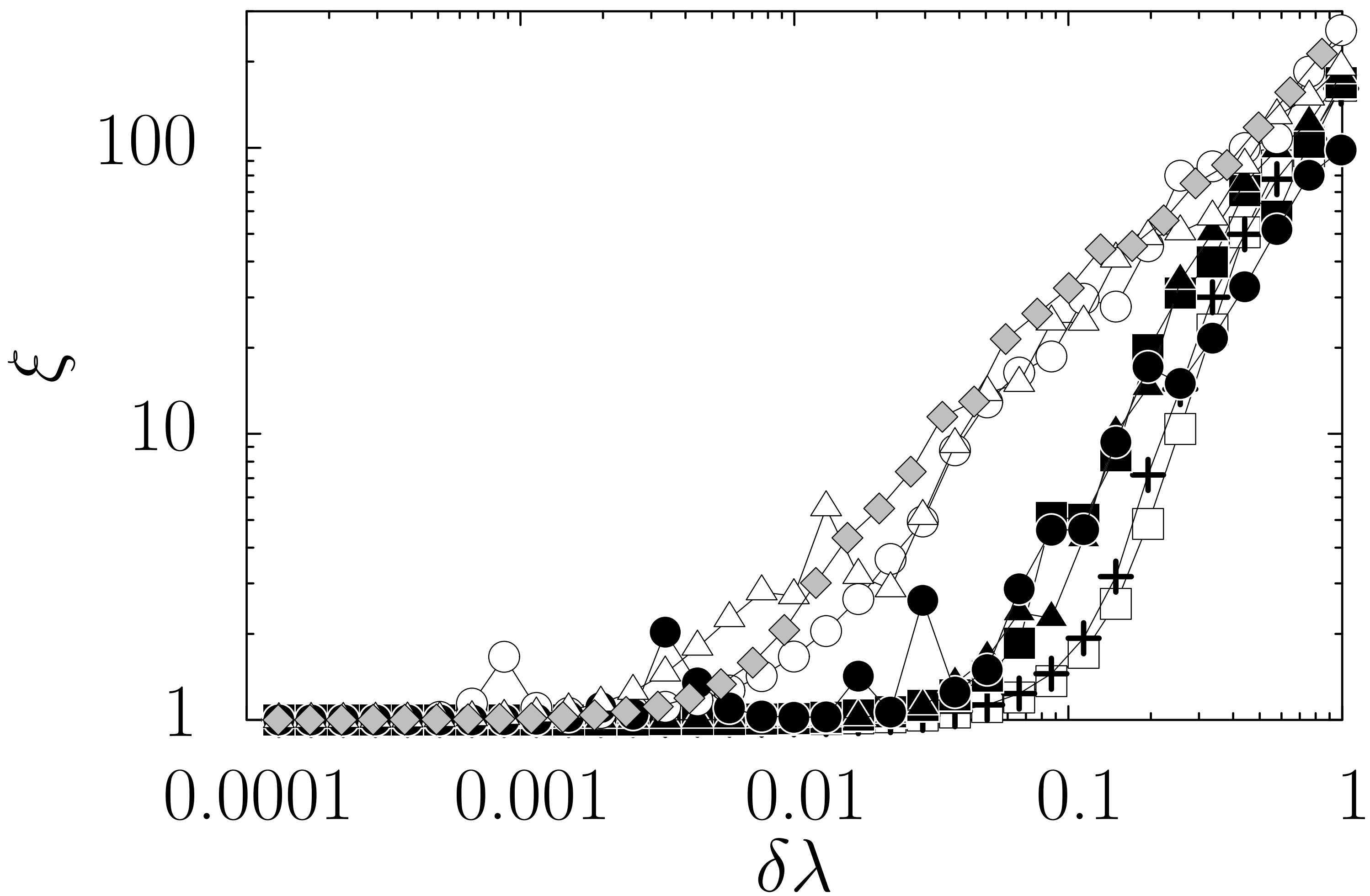
equilibration and chaos isolated systems	two different models universal behavior

complexity is key

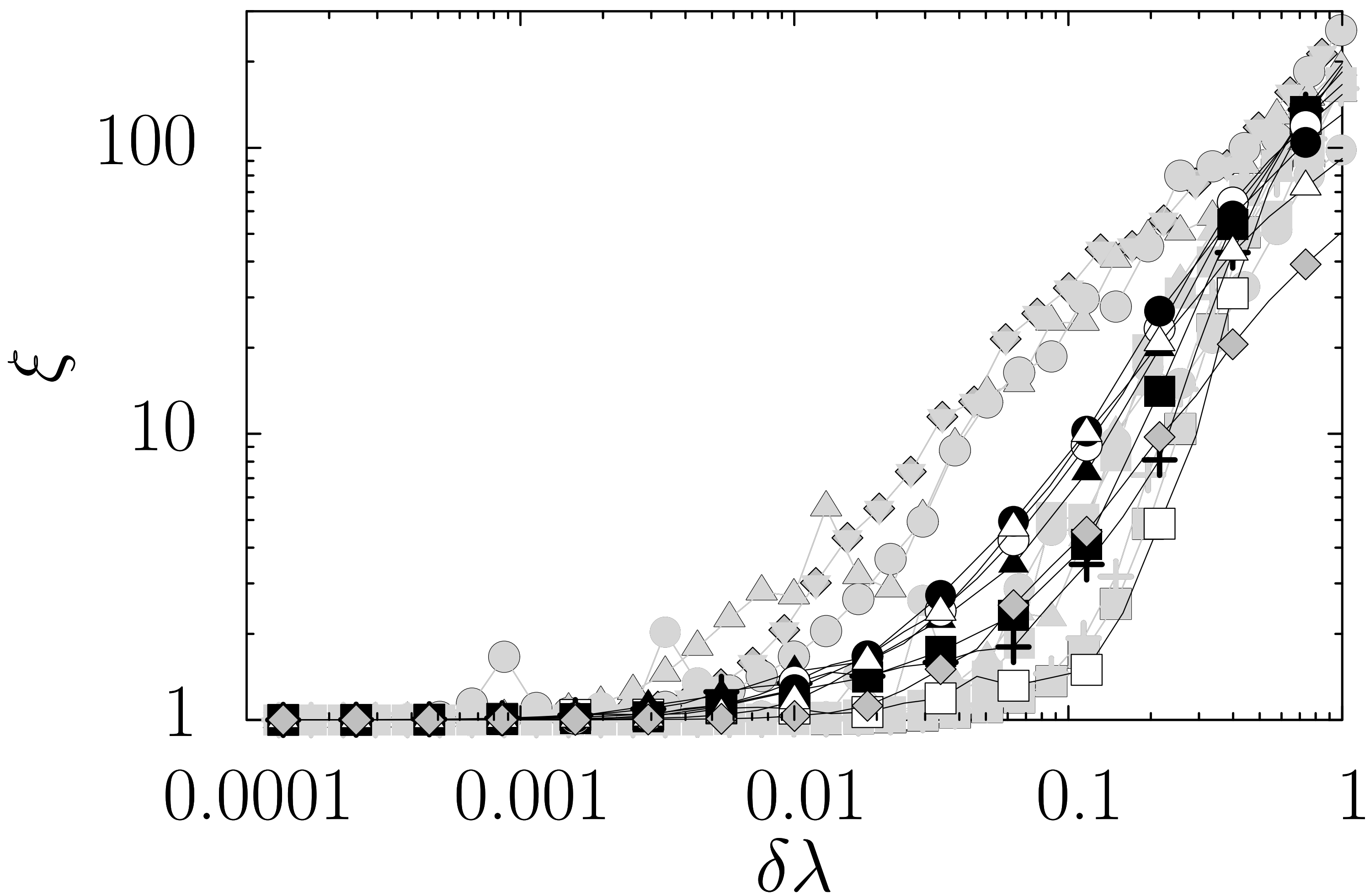
to be done

$$S_{\mathrm{dec}} - \overline{S_{\mathrm{D}}(\tau)} = (1-\gamma)\frac{\xi - 1}{\xi + 1}$$

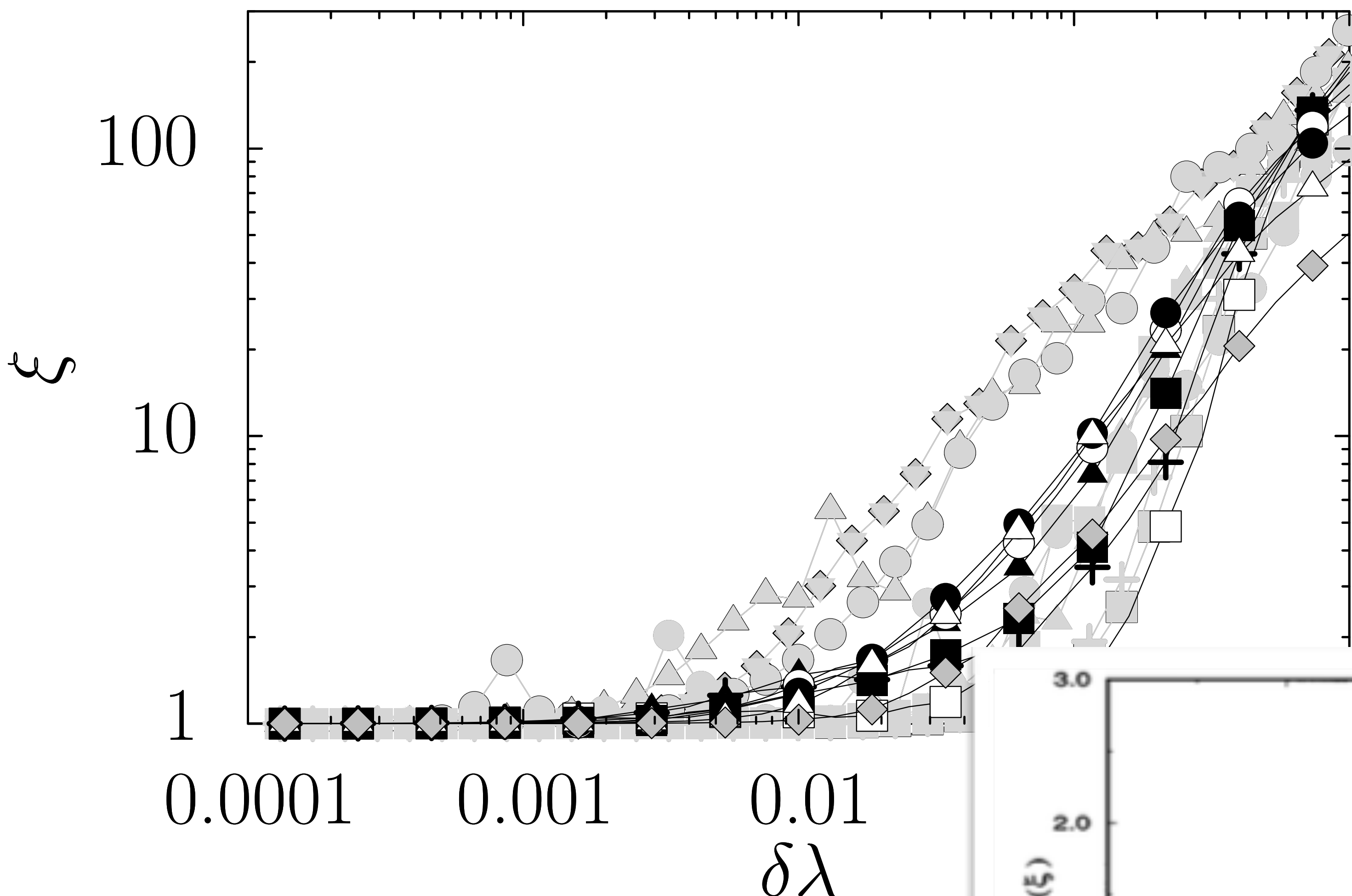
to be done

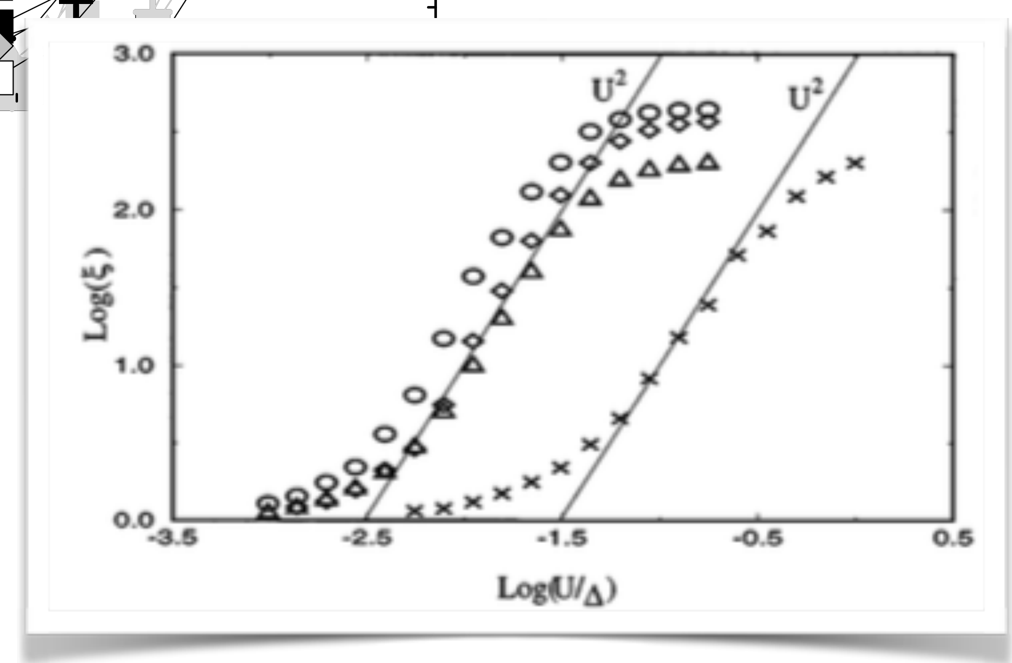


to be done



to be done





B. Georgeot and D. L. Shepelyansky, Phys. Rev. Lett. 79, 4365 (1997).

PRE **91**, 010902 (R) (2015)

IGM & Diego Wisniacki
Augusto Roncaglia

CONICET

UBACYT

AGENCIA

Thank you

CONICET

UBACYT

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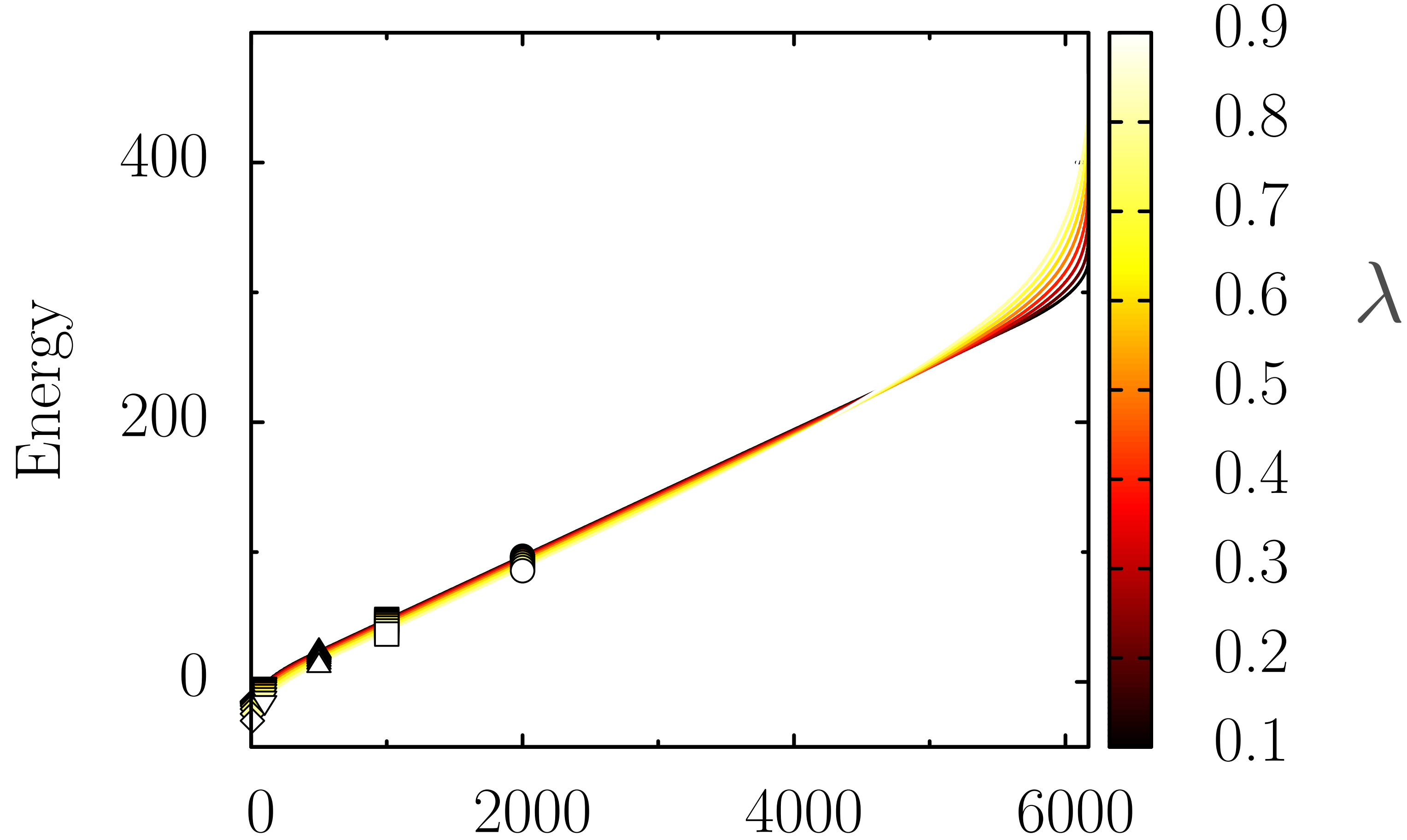
Thank you

questions



$j = 20, N = 250$