Université Lille 1 Sciences et Technologies, Lille, France

Laboratoire de Physique des Lasers, Atomes et Molécules

Équipe Chaos Quantique

16 years of experiments on the atomic kicked rotor!

Chaos, disorder in dynamical ultracold atom systems

Jean-Claude Garreau

Workshop "Quantum chaos: fundamentals and applications"

Luchon-Superbagnères – 18 Mars 2015

Université Lille 1 Sciences et Technologies, Lille, France

Laboratoire de Physique des Lasers, Atomes et Molécules

Équipe Chaos Quantique

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LKB

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Nicolas Cherroret

LPT

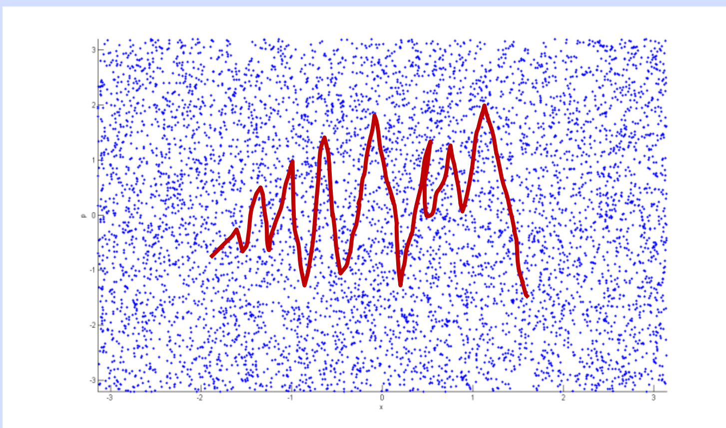
Gabriel Lemarié

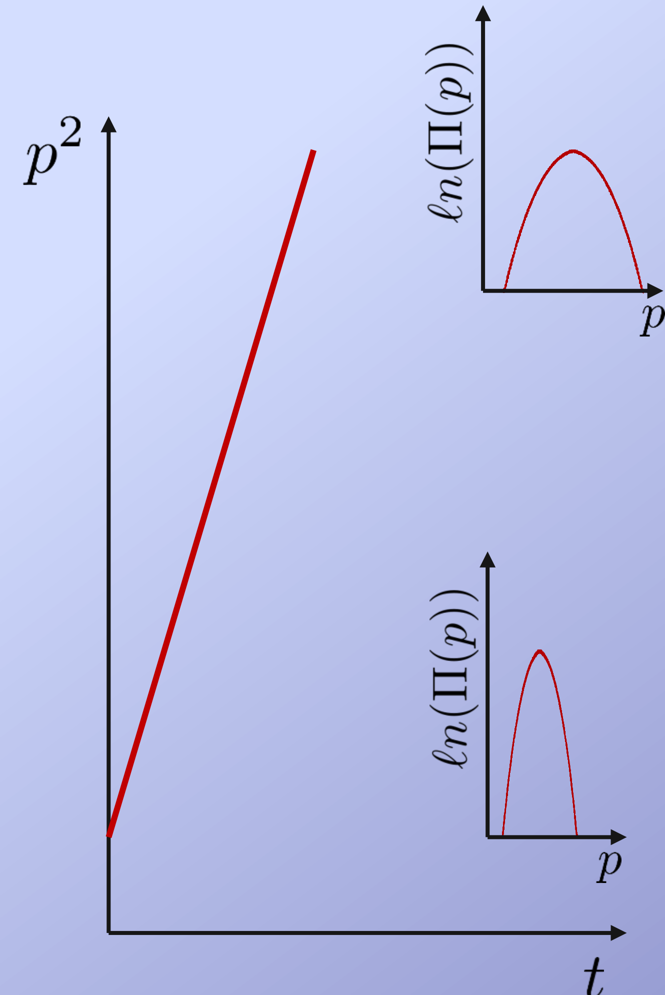
The kicked rotor: A paradigm of classical and quantum chaos

Kicked rotor: Chaotic diffusion in phase space

$$H = \frac{p^2}{2} + K \cos x \sum_n \delta(t-n)$$

$$K \geq 5$$

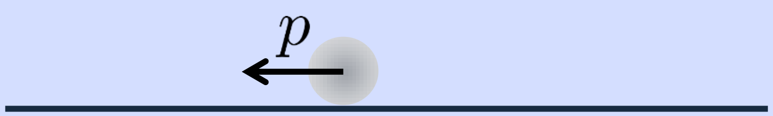




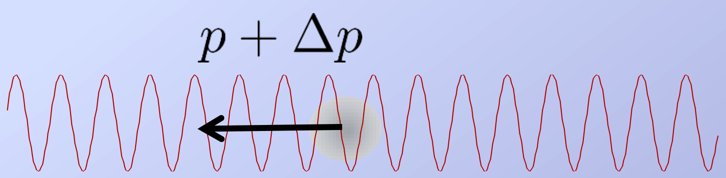
The atomic kicked rotor: An almost ideal "quantum simulator"

The "unfolded" kicked rotor

Free motion



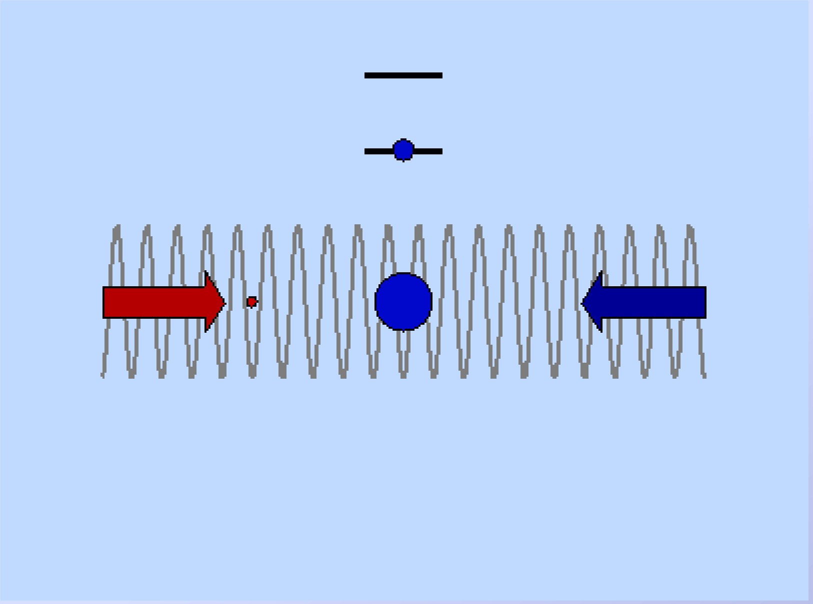
Kick



$$H = \frac{p^2}{2} + K \cos x \sum_n \delta(t - n)$$

$$[x, p] = \mathrm{i}\hbar$$

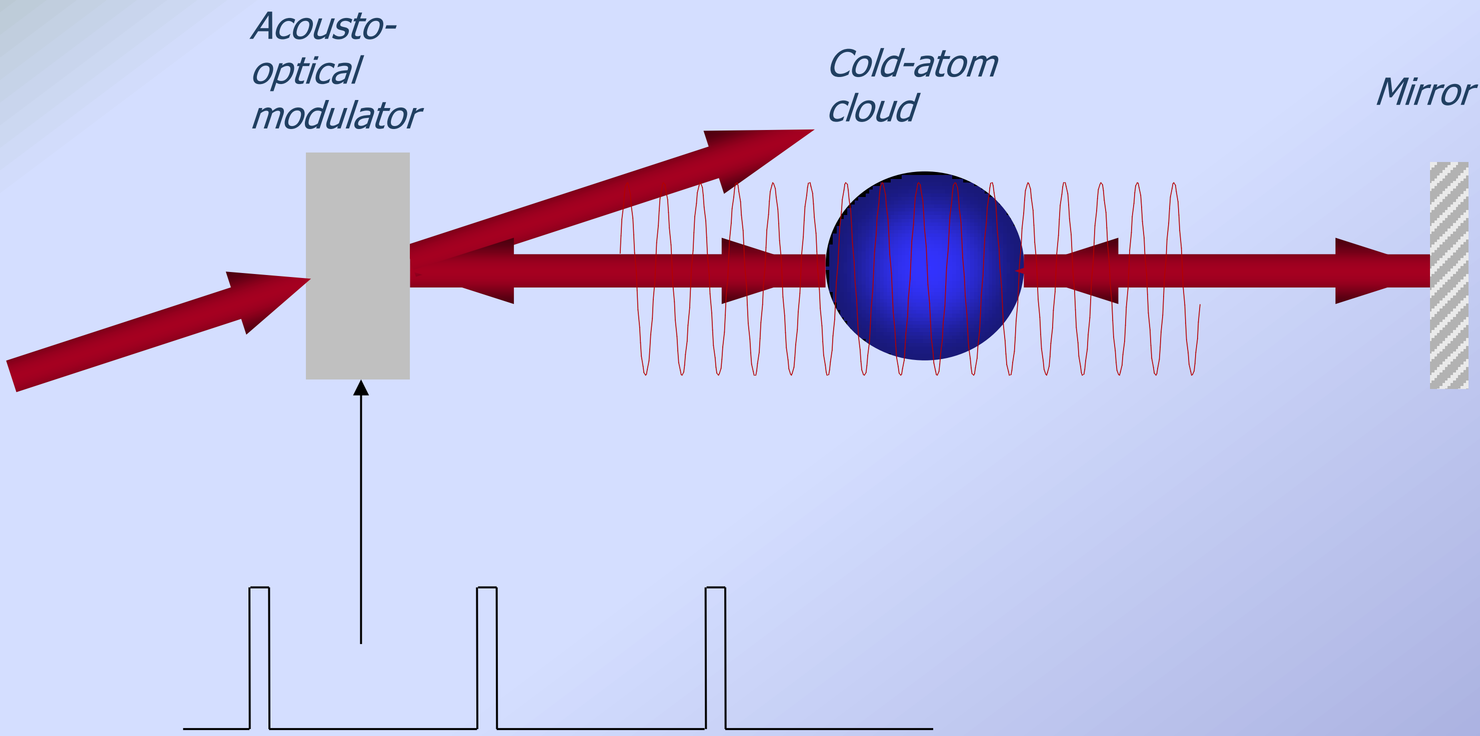
Doing it with cold atoms



$$p_{\text{after}} = p_{\text{before}} + 2\hbar k$$

"Optical potential" $V(x) \sim \frac{I}{\Delta} \to I_0 \cos(2kx)$

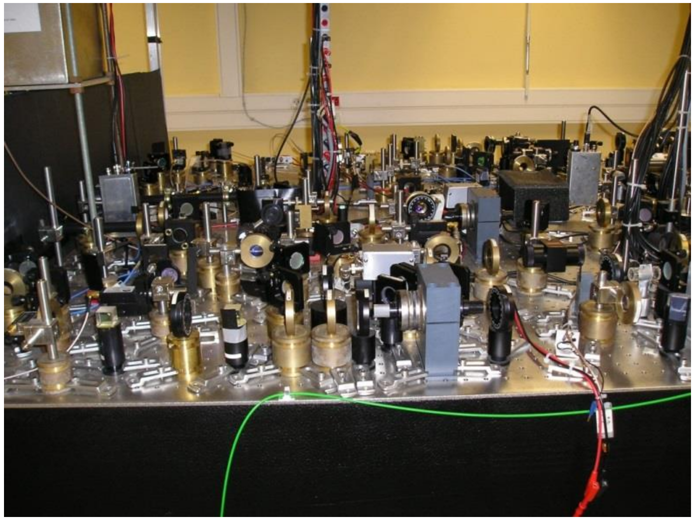
Doing it with cold atoms



$$H = \frac{p^2}{2} + K \cos x \sum_n \delta(t-n)$$

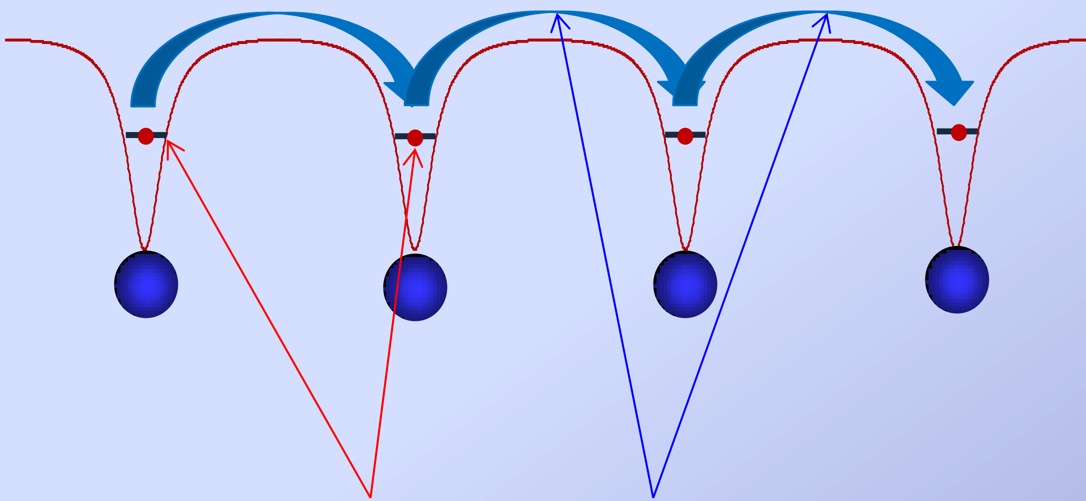
F. L. Moore *et al.*, *Atom optics realization of the quantum δ-kicked rotator*, Phys. Rev. Lett. **75**, 4598 (1995)

Doing it with cold atoms



Probing quantum disordered systems with ultracold atoms

Tight-binding

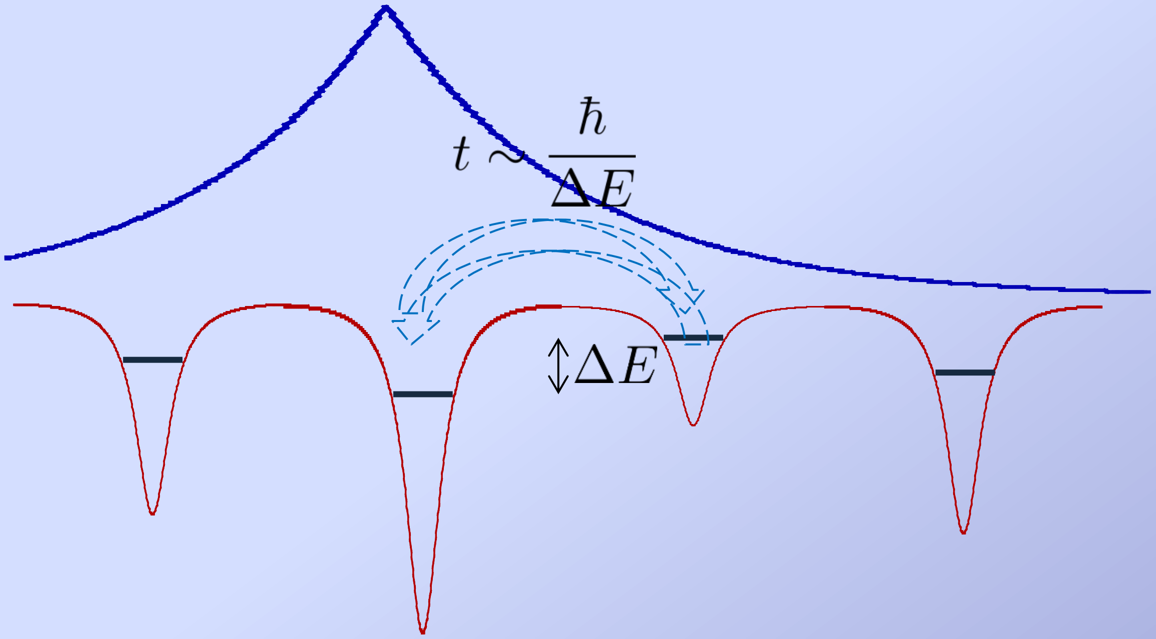


$$Hu_n = V_n u_n + T u_{n+1} + T u_{n-1}$$

Anderson

Random: $-\frac{W}{2} \leq V_n \leq -\frac{W}{2}$

P. W. Anderson, *Absence of Diffusion in Certain Random Lattices*, Phys. Rev. **109**, 1492--1505 (1958)



$$Hu_n = V_n u_n + T u_{n+1} + T u_{n-1}$$

Number of visited sites ~ $\dfrac{\textit{Absence time} \sim \hbar/\Delta E \sim \hbar/W}{\textit{Hopping time} \sim \hbar/T} \sim \dfrac{T}{W}$

$\frac{T}{W} \ll 1$ *Localization*

$\frac{T}{W} \gg 1$ *Diffusion (3D)*

3D: Quantum phase transition

The Anderson model

- 1D : Exponential localization of the eigenfunctions $$\psi \sim \exp\left(\frac{-|x-x_0|}{\xi}\right)$$
- Suppression of the diffusion → Insulator
- 3D → « Mobility edge » → Metal-insulator transition

Simulating condensed matter systems with ultracold atoms

Experiments in condensed-matter and ultracold atoms

Condensed matter

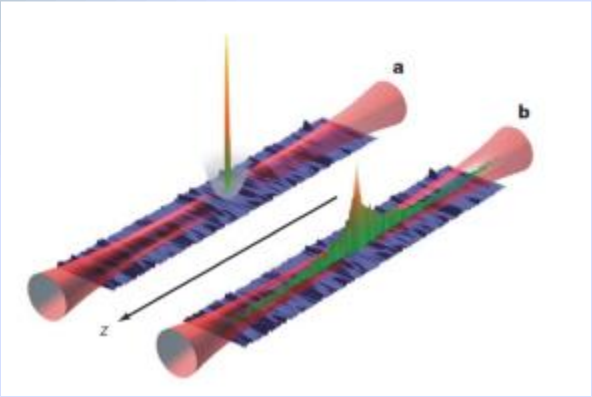
- Decoherence (ill-defined quantum phases)
- No access to the wave function
- Electron-electron Coulomb interactions

Ultracold atoms

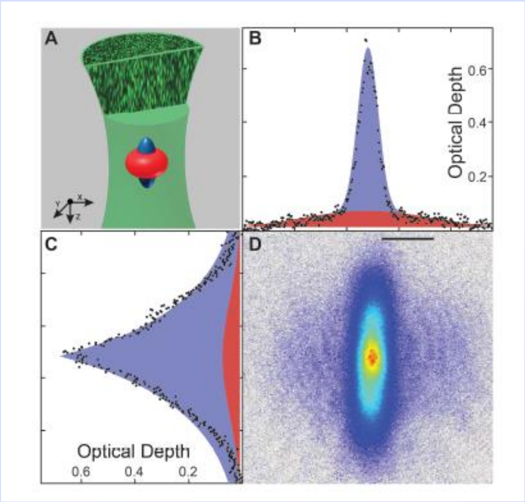
- Control of decoherence
- Access to probability distributions (and even the full wavefunction)
- Control of interactions (Feshbach resonances)

Doing with cold atoms

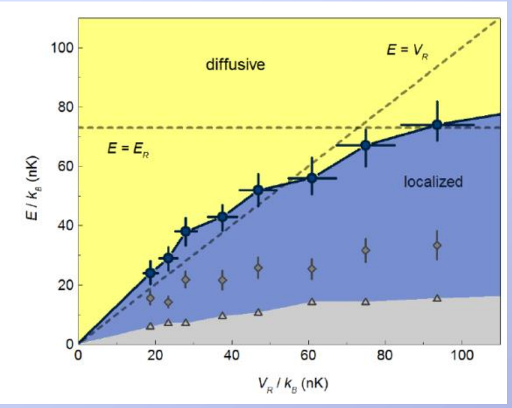
Palaiseau



Urbana-Champain



Florence



1D: J. Billy *et al.*, *Direct observation of Anderson localization of matter-waves in a controlled disorder*, Nature **453**, 891 (2008)

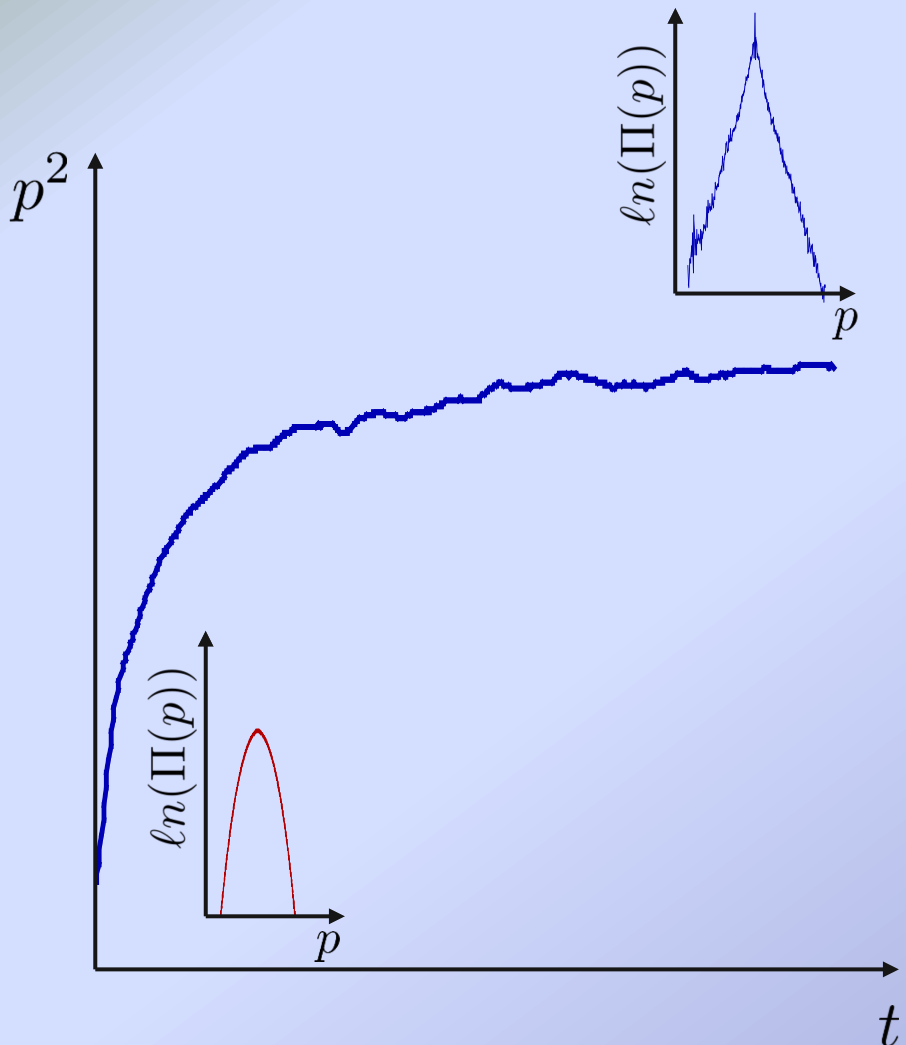
3D : F. Jendrzejewski *et al.*, *Three-dimensional localization of ultracold atoms in an optical disordered potential*, Nature Physics **8**, 398 (2012)

3D : S. S. Kondov *et al.*, *Three-Dimensional Anderson Localization of Ultracold Fermionic Matter*, Science **334**, 66 (2011)

3D : G. Semeghini, *et al.*, *Measurement of the mobility edge for 3D Anderson localization*, arXiv:1404.3528 (2014)

Simulating the Anderson model with the atomic kicked rotor

Dynamical localization



Exponential "localization"

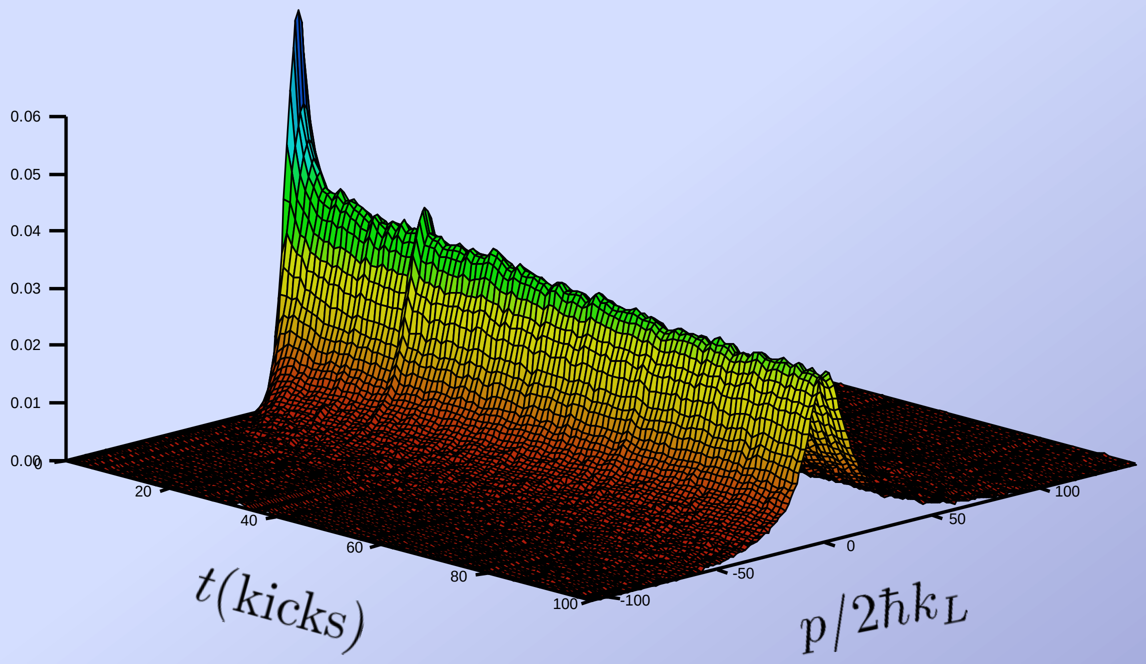
in momentum space → "dynamical" localization

Can be mathematically mapped into a 1D "Anderson model" which discribes disorder in quantum system. Predicts exponential localization in real space

G. Casati *et al., Stochastic behavior of a quantum pendulum under periodic perturbation,* Lect. Notes Phys. **93**, 334 (1979)

S. Fishman *et al., Chaos, Quantum Recurrences, and Anderson Localization*, Phys. Rev. Lett. **49**, 509—512 (1982)

Dynamical localization, experiment with the atomic kicked rotor

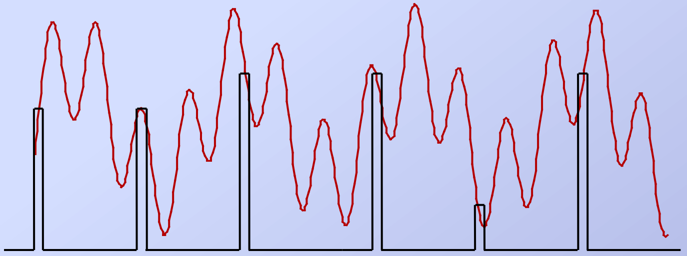


The Anderson transition

In 3D "the Anderson model predicts a quantum metal-insulator transition

How to do it with the atomic kicked rotor ?

$$H = \frac{p^2}{2} + K \cos x \left(1 + \epsilon \cos(\omega_2 t) \cos(\omega_3 t)\right) \sum_n \delta(t-n)$$

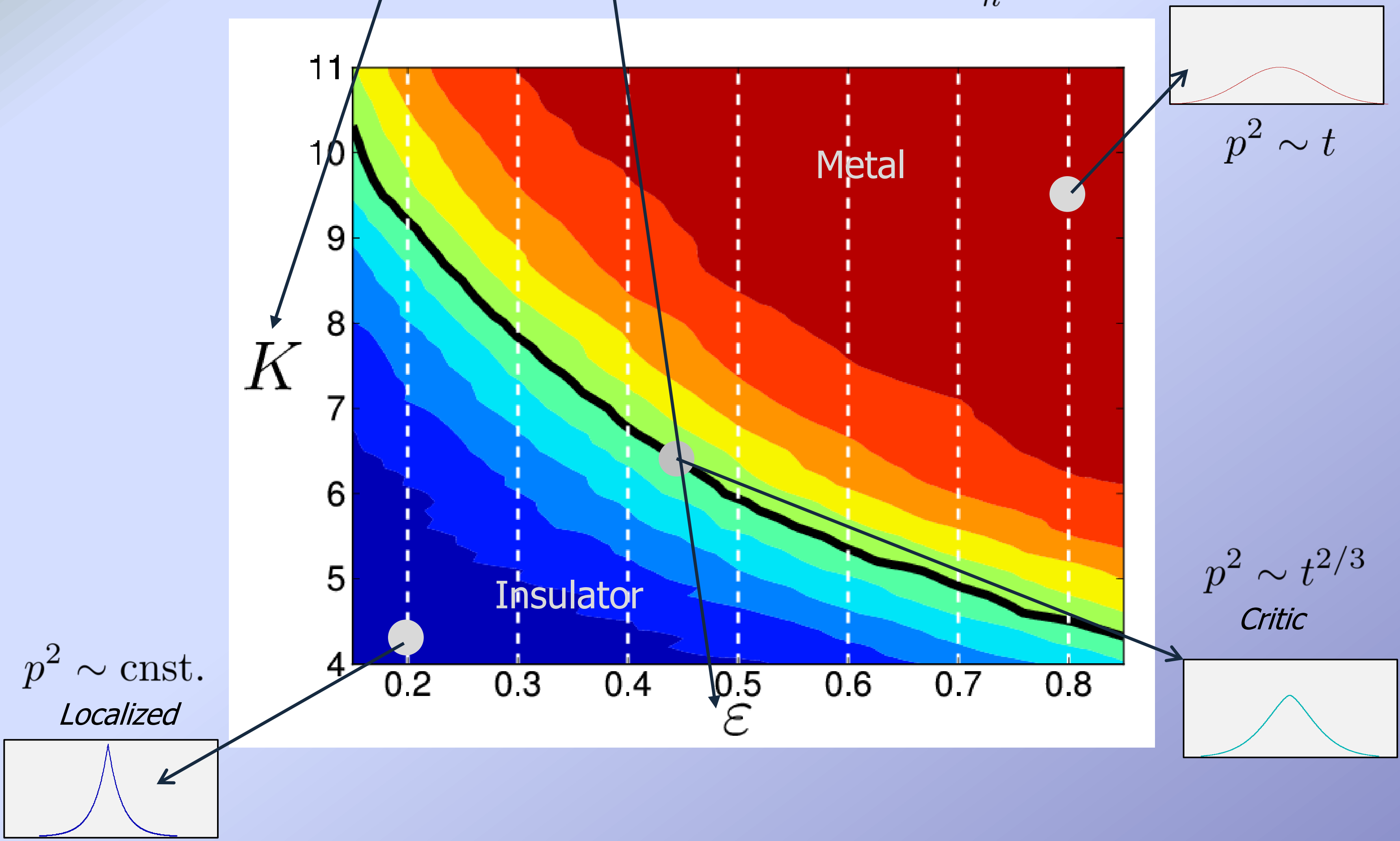


Maps onto a **3D** *Anderson model !!!*

G. Casati *et al.*, *Anderson Transition in a One-Dimensional System with Three Incommensurate Frequencies*, Phys. Rev. Lett. **62**, 345--348 (1989)

The Anderson transition

$$H = \frac{p^2}{2} + K \cos x \,(1 + \varepsilon \cos(\omega_2 t) \cos(\omega_3 t)) \sum_n \delta(t - n)$$



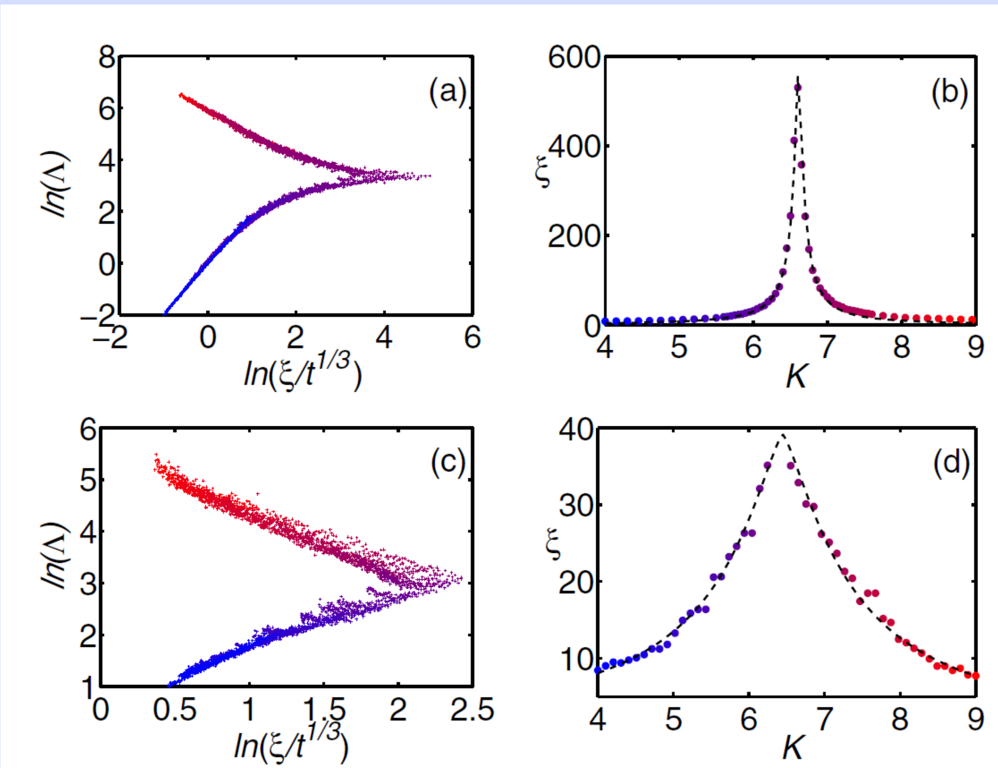
J. Chabé *et al.*, *Experimental Observation of the Anderson Metal-Insulator Transition with Atomic Matter Waves*, Phys. Rev. Lett. **101**, 255702 (2008)

Critical exponent

Selected for a Viewpoint in *Physics*
PRL **101,** 255702 (2008) PHYSICAL REVIEW LETTERS week ending 19 DECEMBER 2008

Experimental Observation of the Anderson Metal-Insulator Transition with Atomic Matter Waves

Julien Chabé,[1] Gabriel Lemarié,[2] Benoît Grémaud,[2] Dominique Delande,[2] Pascal Szriftgiser,[1] and Jean Claude Garreau[1]
[1]*Laboratoire de Physique des Lasers, Atomes et Molécules, Université des Sciences et Technologies de Lille, CNRS; CERLA; F-59655 Villeneuve d'Ascq Cedex, France**
[2]*Laboratoire Kastler-Brossel, Université Pierre et Marie Curie-Paris 6, ENS, CNRS; 4 Place Jussieu, F-75005 Paris, France*



KR, experimental

$$\nu = 1.4 \pm 0.3$$
$$= 1.63 \pm 0.05$$

KR, numerical

$$\nu = 1.59 \pm 0.01$$

Anderson, numerical

$$\nu = 1.571 \pm 0.008$$

K. Slevin and T. Ohtsuki, *Critical exponent for the Anderson transition in the three-dimensional orthogonal universality class*, New J. Phys 16, 015012 (2014)
G. Lemarié *et al.*, *Universality of the Anderson transition with the quasiperiodic kicked rotor*, EPL **87**, 37007 (2009)

Critical wave function

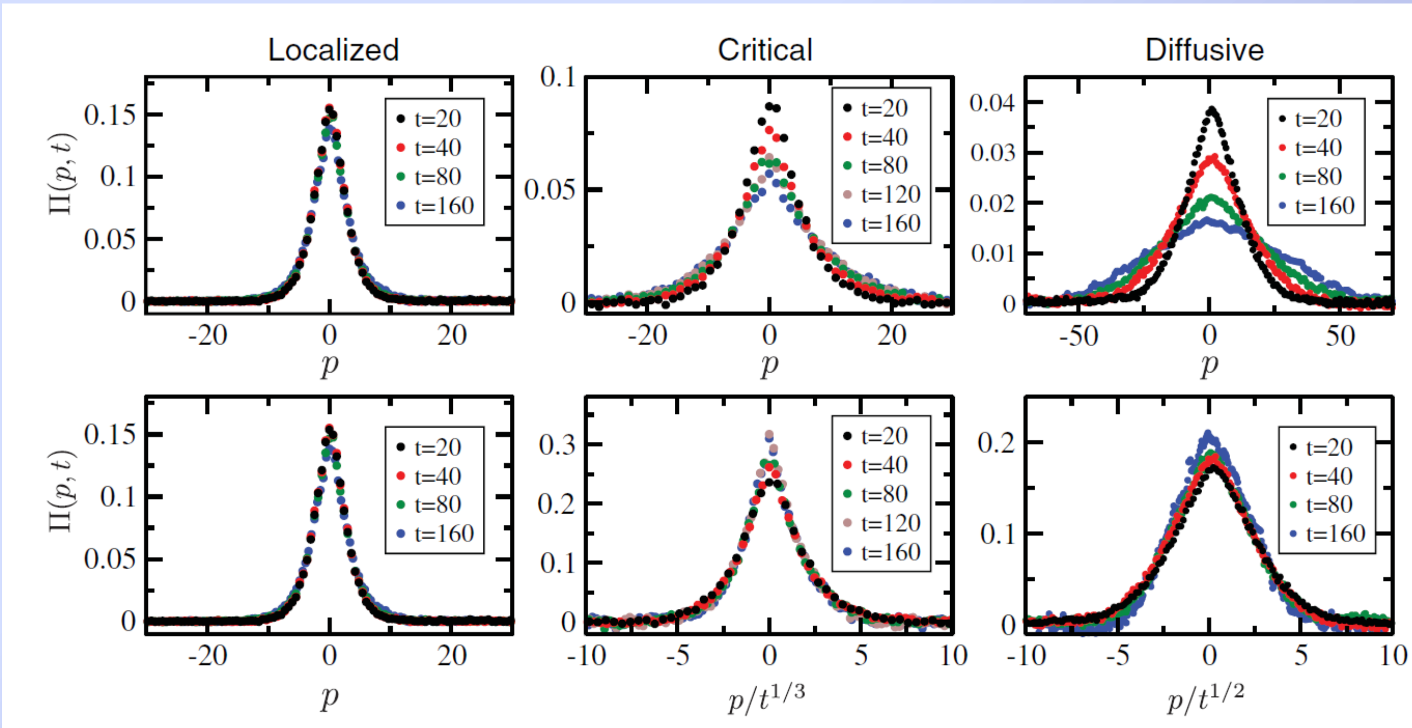
PRL **105,** 090601 (2010) PHYSICAL REVIEW LETTERS week ending 27 AUGUST 2010

Critical State of the Anderson Transition: Between a Metal and an Insulator

Gabriel Lemarié,[1,*] Hans Lignier,[2,†] Dominique Delande,[1] Pascal Szriftgiser,[2] and Jean Claude Garreau[2]

[1]*Laboratoire Kastler Brossel, UPMC-Paris 6, ENS, CNRS; 4 Place Jussieu, F-75005 Paris, France*

[2]*Laboratoire de Physique des Lasers, Atomes et Molécules, Université Lille 1 Sciences et Technologies, UMR CNRS 8523; F-59655 Villeneuve d'Asca Cedex, France*[‡]

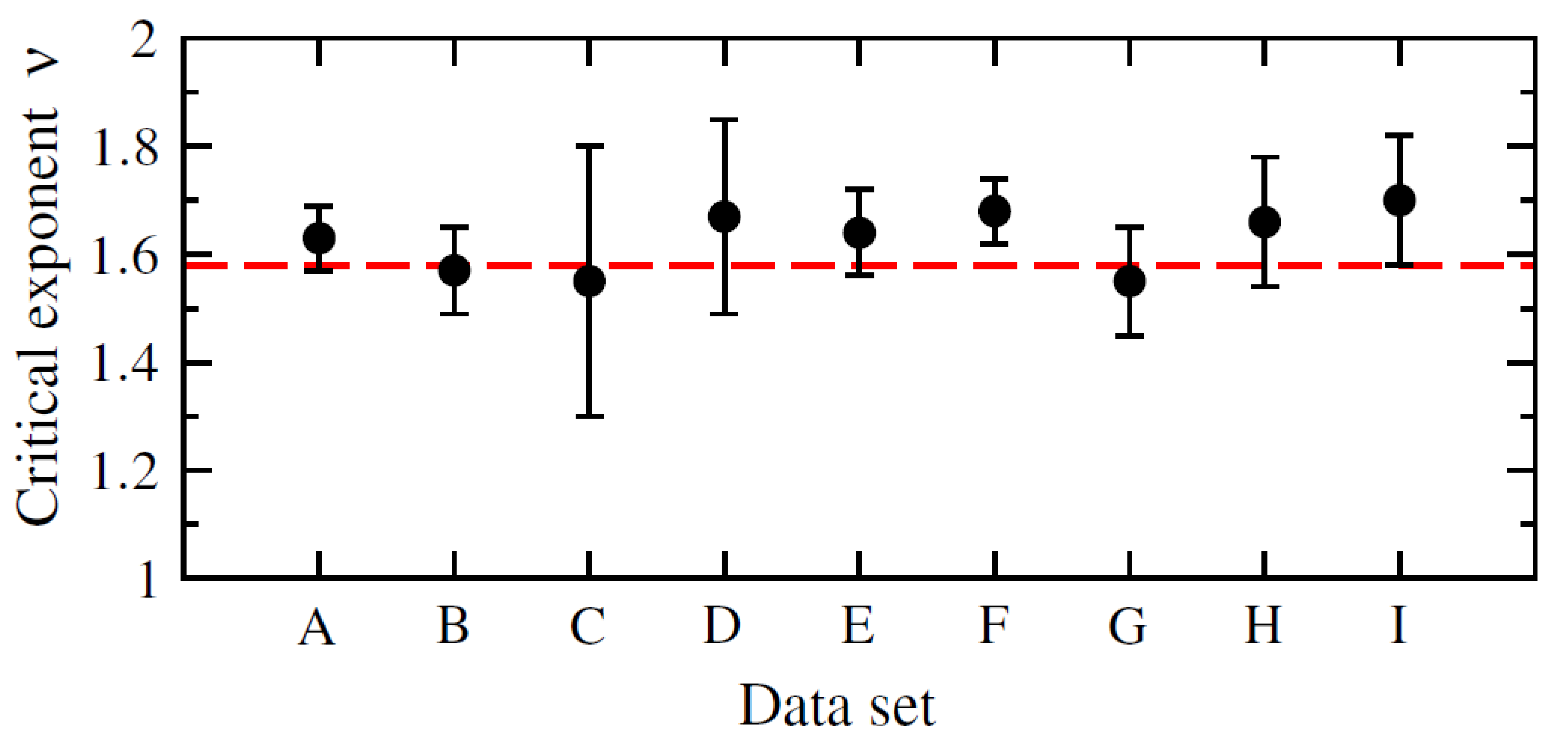


Universality

PRL **108,** 095701 (2012) PHYSICAL REVIEW LETTERS week ending 2 MARCH 2012

Experimental Test of Universality of the Anderson Transition

Matthias Lopez,[1] Jean-François Clément,[1] Pascal Szriftgiser,[1] Jean Claude Garreau,[1] and Dominique Delande[2]
[1]*Laboratoire de Physique des Lasers, Atomes et Molécules, Université Lille 1 Sciences et Technologies, CNRS; F-59655 Villeneuve d'Ascq Cedex, France*[*]
[2]*Laboratoire Kastler-Brossel, UPMC-Paris 6, ENS, CNRS; 4 Place Jussieu, F-75005 Paris, France*



Phase diagram

Phase diagram of the anisotropic Anderson transition with the atomic kicked rotor: theory and experiment

Matthias Lopez[1], **Jean-François Clément**[1], **Gabriel Lemarié**[2,3], **Dominique Delande**[3], **Pascal Szriftgiser**[1] **and Jean Claude Garreau**[1,4]

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New Journal of Physics **15** (2013) (

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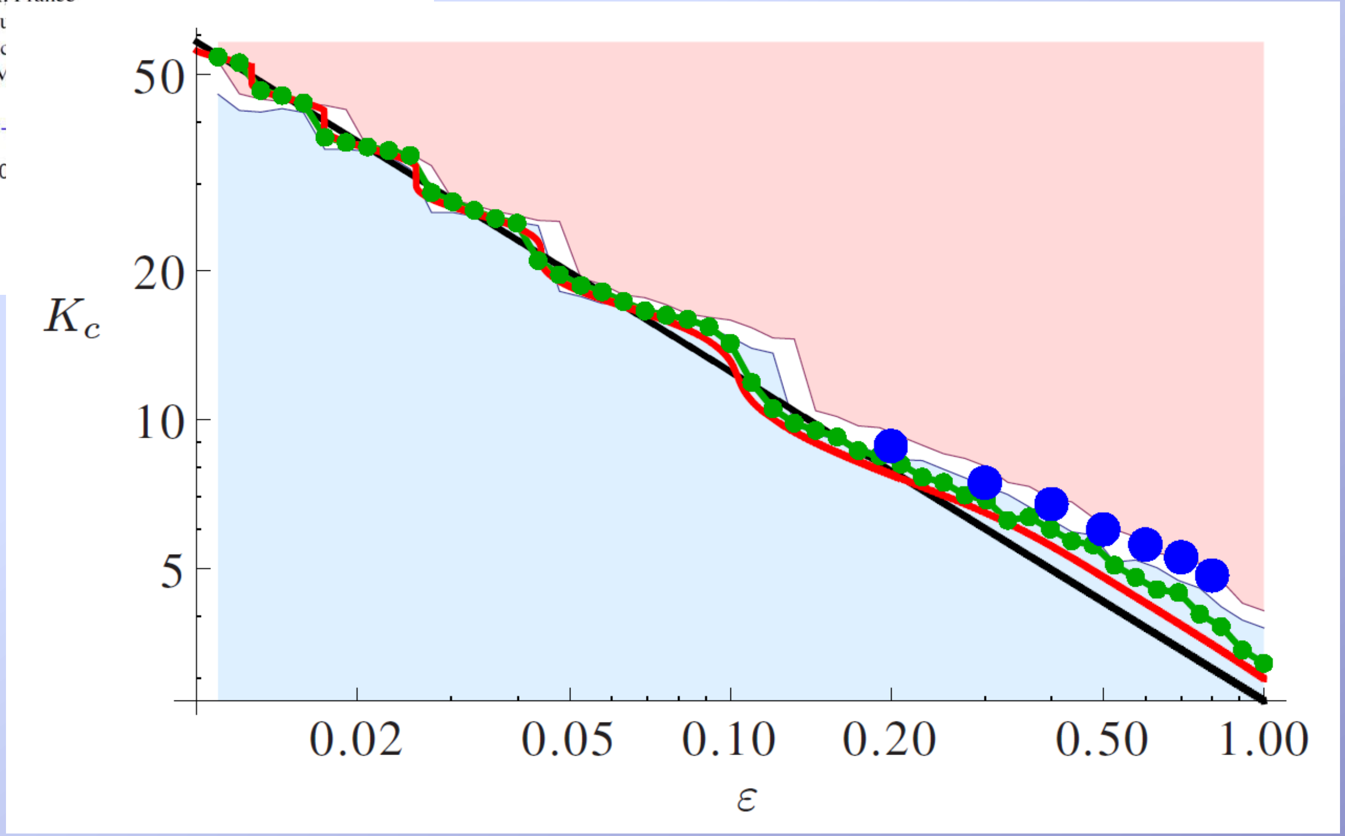
doi:10.1088/1367-2630/15/6/065013

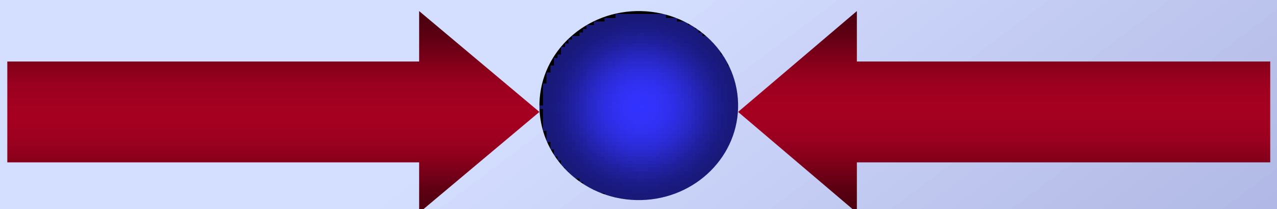


M. Lopez et al., *Phase diagram of the anisotropic Anderson transition with the atomic kicked rotor: theory and experiment*, New J. Phys **15**, 065013 (2013).

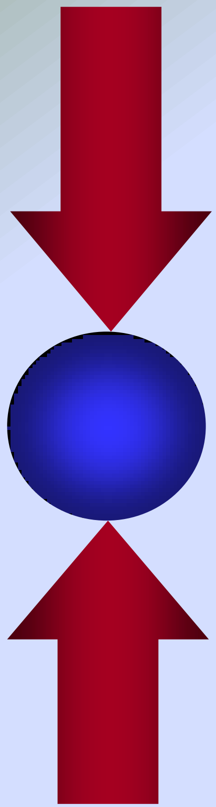
D =2 is the "lower critical dimension" for Anderson physics

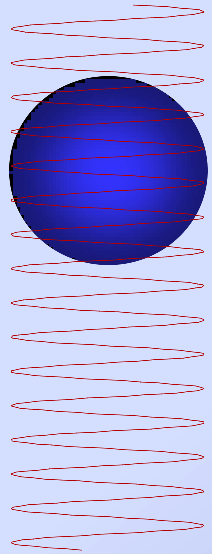
All states are localized but with exponentially large localization length

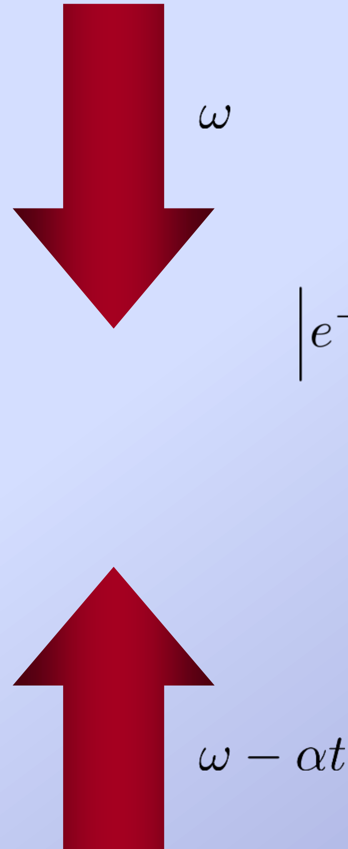
$$H = \frac{p^2}{2} + K \cos x \,(1 + \varepsilon \cos(\omega_2 t)) \sum_n \delta(t - n)$$



Experiment limited to a few ms







$$\left| e^{-\mathrm{i}(kz-\omega t)} + e^{\mathrm{i}(kz+\omega t-\alpha t^2)} \right|^2$$

$$\sim 1 + \cos(2kz - \alpha t^2)$$

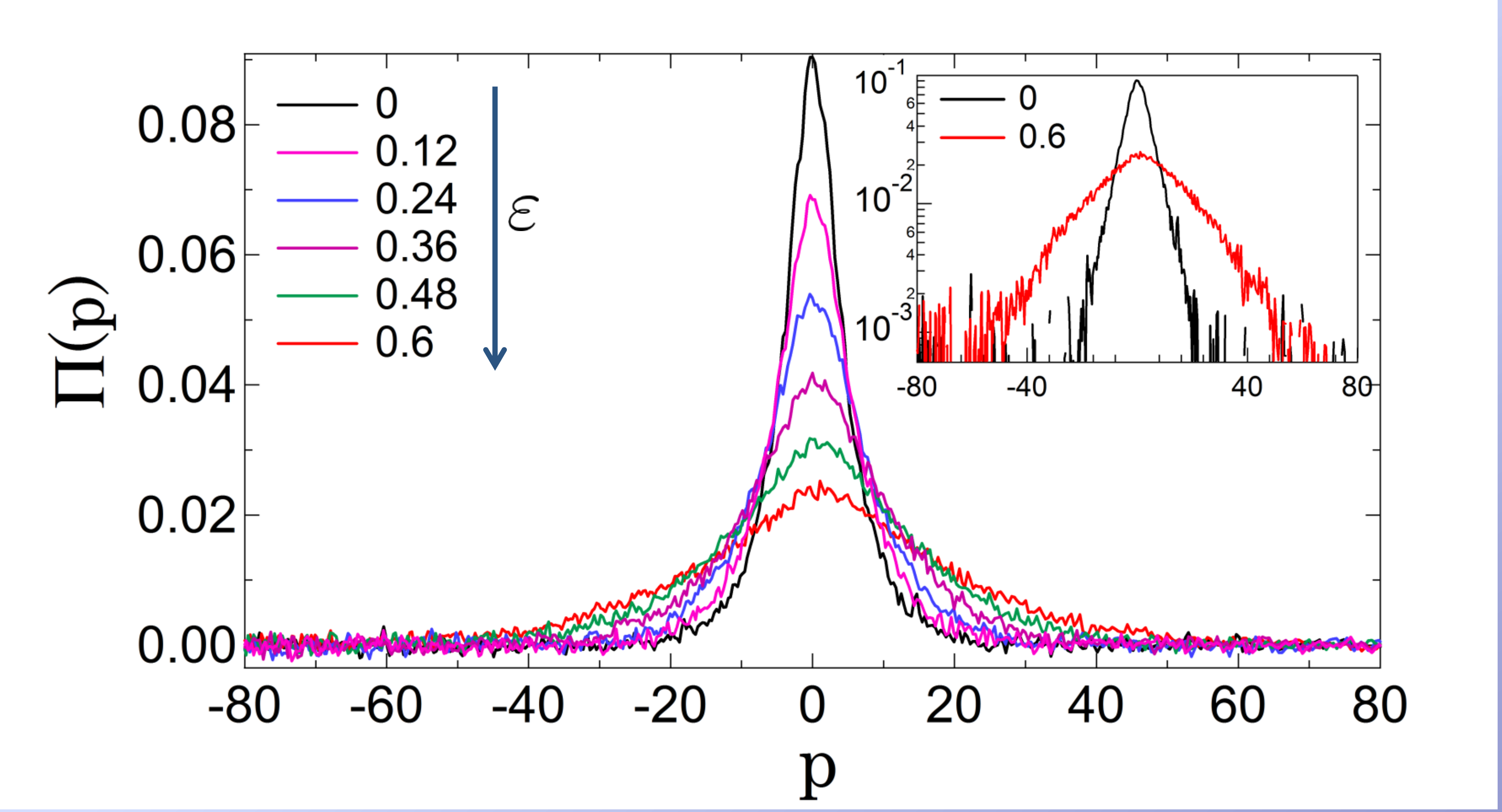
$$z_n = \frac{n\pi + \alpha t^2}{2k}$$

$$g' = \alpha / k$$

Not *a kicked rotor*
(kicked accelerator)

Recent results: 2D Anderson localization (unpublished)

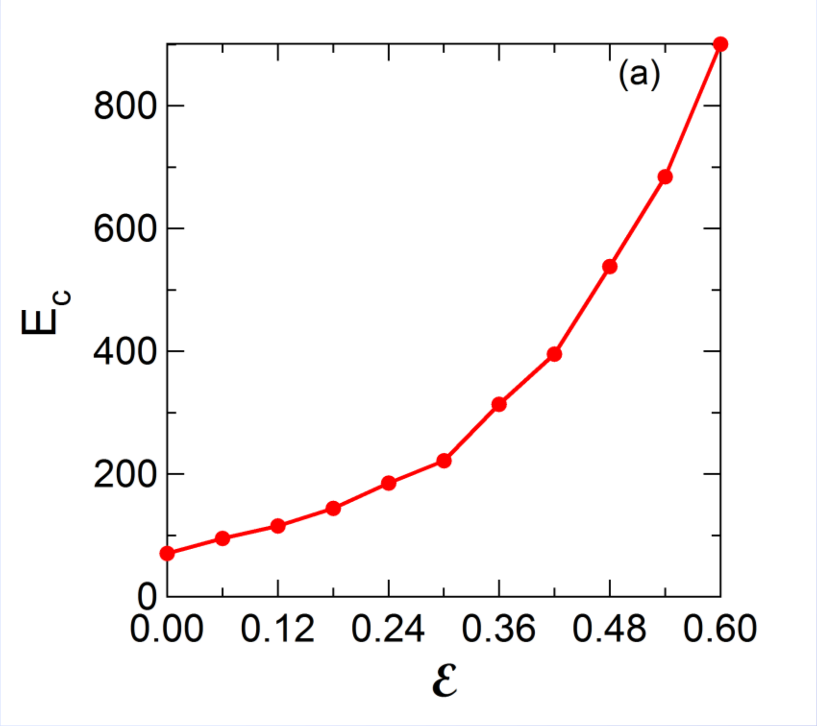
$$H = \frac{p^2}{2} + K\cos x\,(1+\varepsilon\cos(\omega_2 t))\sum_n \delta(t-n)$$

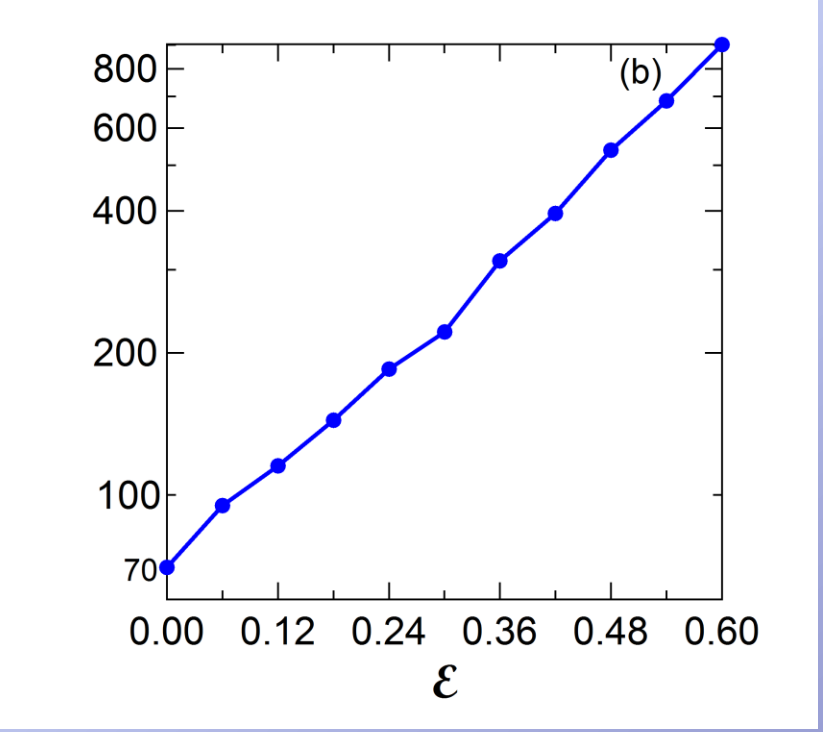


I. Manai *et al.*, *Experimental observation of two-dimensional Anderson localization with the atomic kicked rotor*, to be published

Recent results: 2D Anderson localization (unpublished)

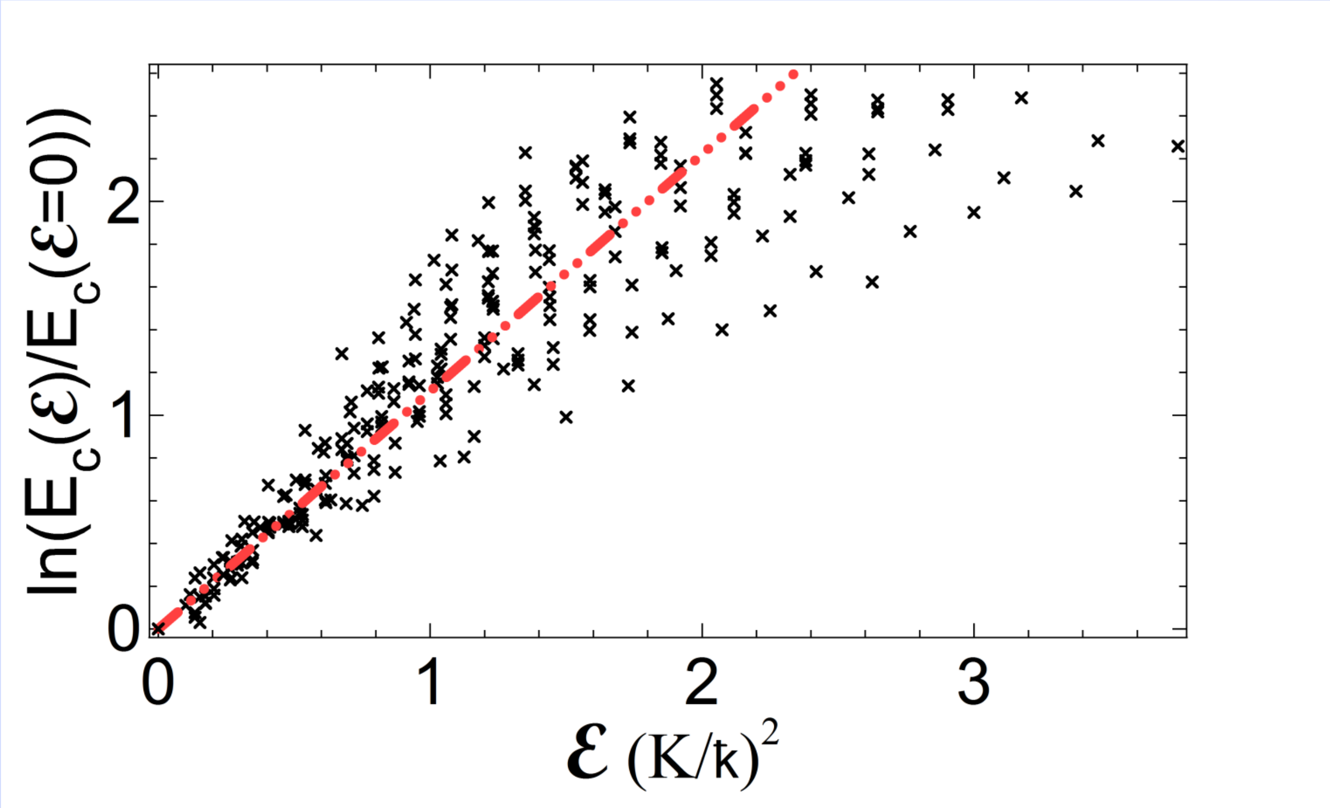
$$H = \frac{p^2}{2} + K \cos x \left(1 + \varepsilon \cos(\omega_2 t)\right) \sum_n \delta(t-n)$$





I. Manai *et al.*, *Experimental observation of two-dimensional Anderson localization with the atomic kicked rotor*, to be published

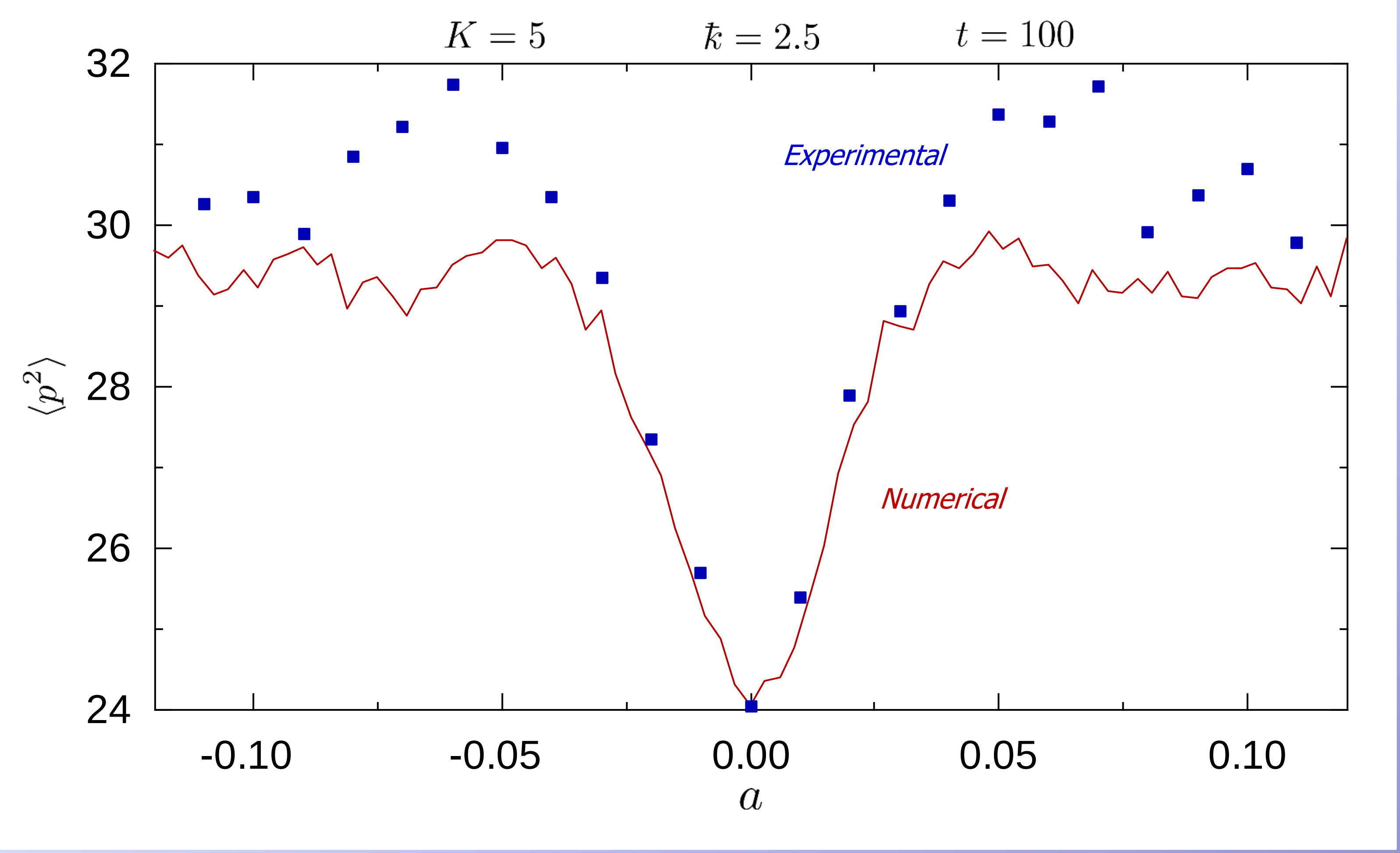
Comparison with the self-consistent prediction



I. Manai *et al.*, *Experimental observation of two-dimensional Anderson localization with the atomic kicked rotor*, to be published

Breaking the orthogonal symmetry (brand-new preliminary results !)

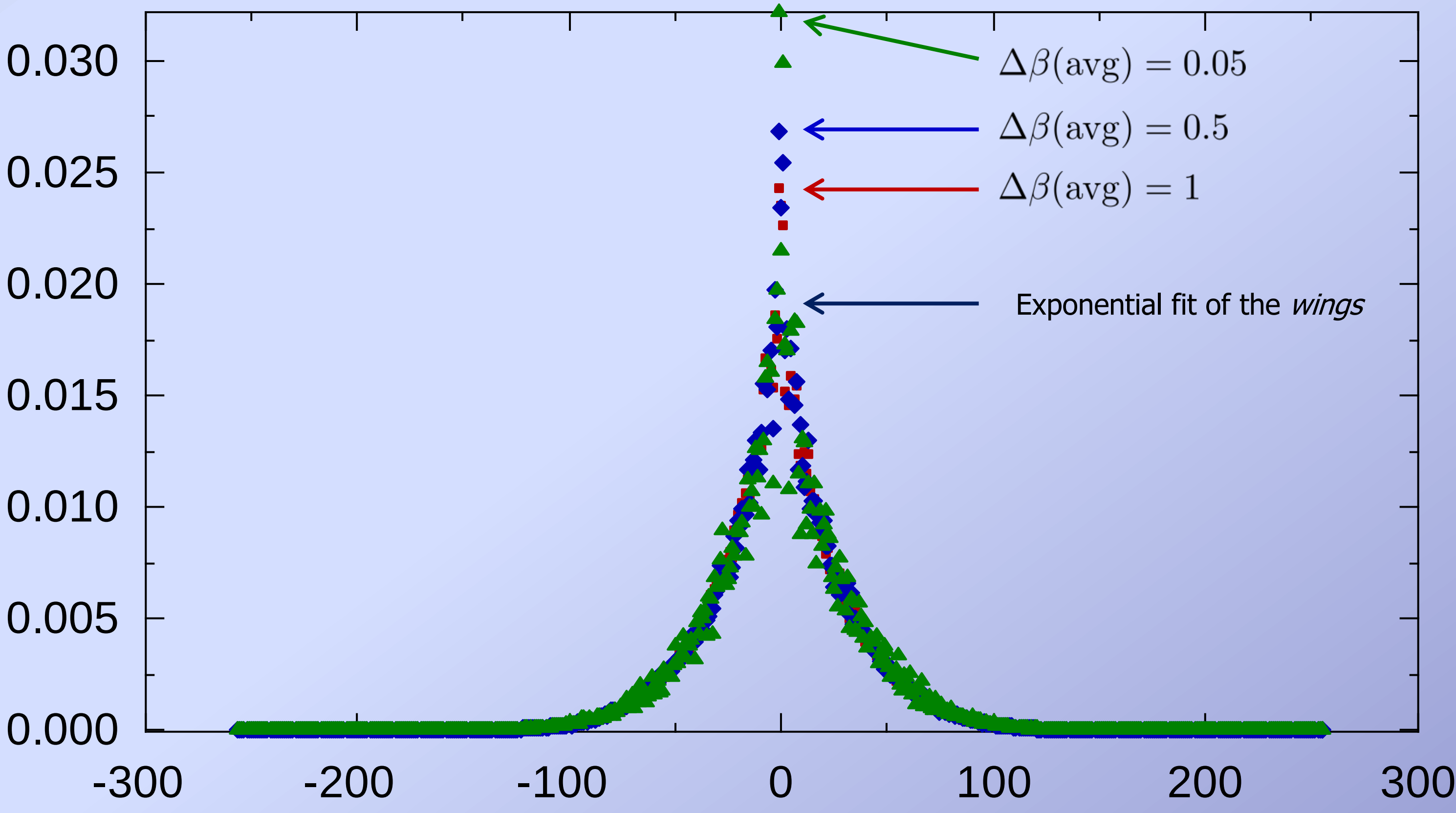
$$H = \frac{p^2}{2} + K \sum_n \left[\delta(t-2n)\cos x + \delta(t-2n-1)\cos(x+a)\right]$$



C. Tian et al., *Weak Dynamical Localization in Periodically Kicked Cold Atomic Gases*, Phys. Rev. Lett. **93**, 124101 (2004)

C. Hainaut *et al.*, to be published

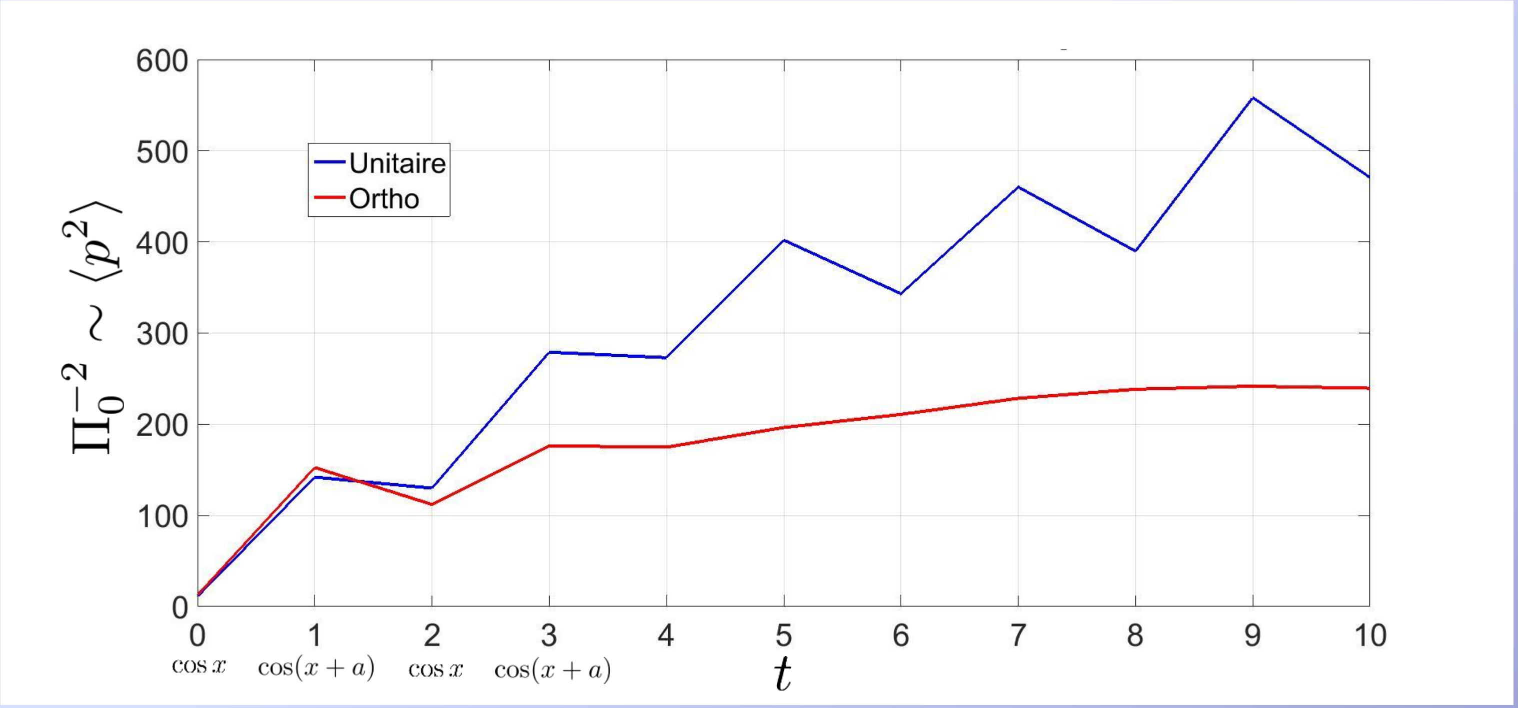
Coherent back-scattering (brand-new preliminary results !)



C. Hainaut *et al.*, to be published

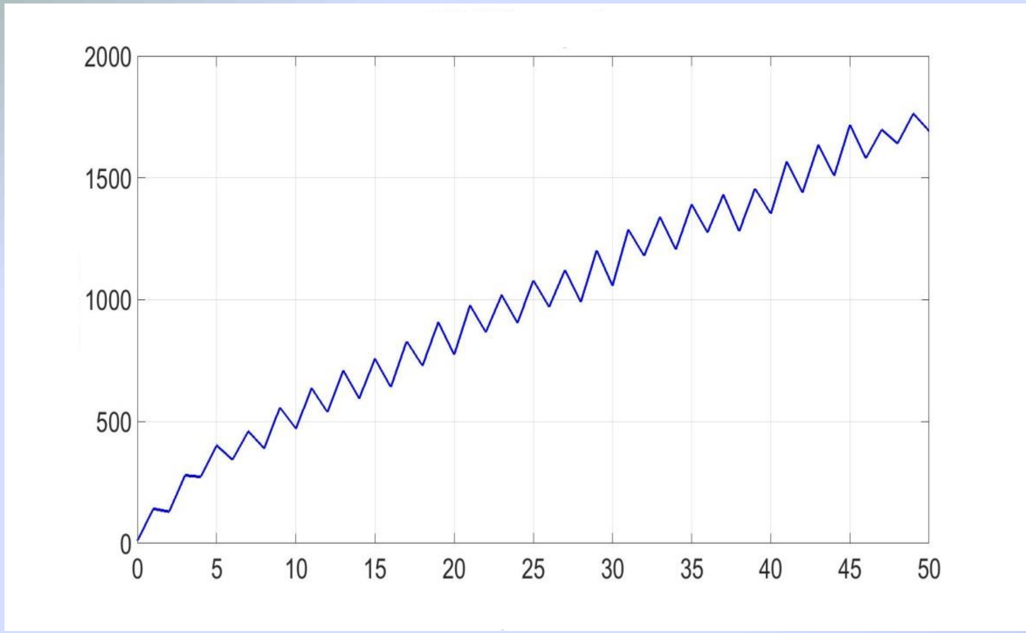
Coherent back-scattering (brand-new preliminary results !)

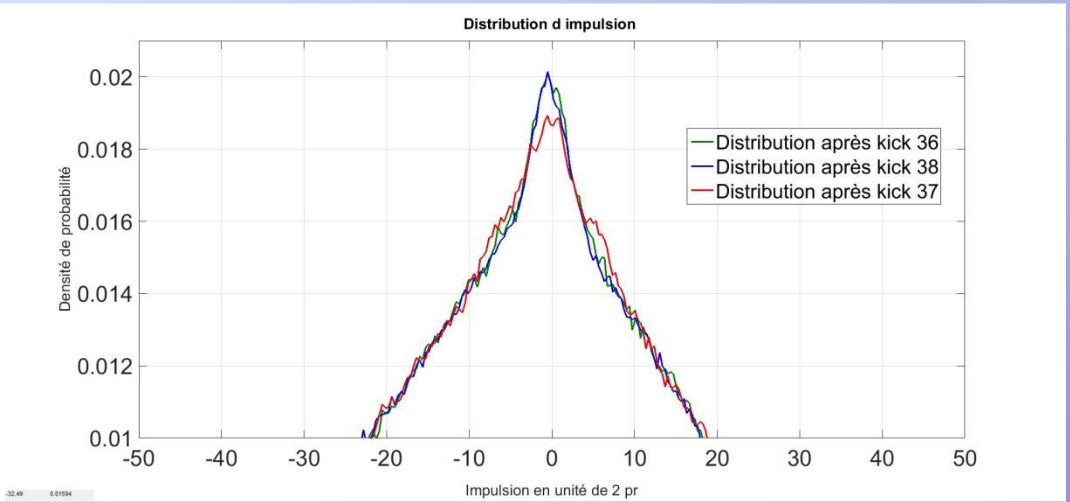
In the unitary case the CBS peak is there only after two kicks!



C. Hainaut *et al.*, to be published

Coherent back-scattering (brand-new preliminary results !)





C. Hainaut *et al.*, to be published

Next: Nonlinear interacting systems!

The funniest is still to come…

Use a Bose-Einstein condensate

Individual atoms $\rightarrow$ Coherent matter wave

$$\mathrm{i}\frac{\partial\psi}{\partial t} = -\frac{\Delta\psi}{2} + V\psi \quad \rightarrow \quad \mathrm{i}\frac{\partial\psi}{\partial t} = -\frac{\Delta\psi}{2} + V\psi + g\left|\psi\right|^2\psi$$

$$Hu_n = V_n u_n + T_n\left(u_{n-1} + u_{n+1}\right) + g\left|u_n\right|^2 u_n$$

Nonlinear quantum disorder!

(the ultimate dream – or nightmare – of a quantum-chaotician?)

CEMPI CENTRE EUROPÉEN POUR LES MATHÉMATIQUES, LA PHYSIQUE ET LEURS INTERACTIONS

The K-BEC project:

Potassium BEC-nonlinear QKR, started 2014, first results expected 2016

PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR

A. S. Pikovsky and D. L. Shepelyansky, *Destruction of Anderson Localization by a Weak Nonlinearity*, Phys. Rev. Lett. **100**, 094101 (2008)

B. Vermersch and J. C. Garreau, *Spectral description of the dynamics of ultracold interacting bosons in disordered lattices*, New J. Phys **15**, 045030 (2013)

N. Cherroret *et al.*, *How Nonlinear Interactions Challenge the Three-Dimensional Anderson Transition*, Phys. Rev. Lett. **112**, 170603 (2014)

L. Ermann and D. L. Shepelyansky, *Destruction of Anderson localization by nonlinearity in kicked rotator at different effective dimensions*, J. Phys. A: Math. Theor. **47**, 335101 (2014)

Conclusion

- Ultracold atom physics is very powerful tool for the study of quantum complexity

- KR as "quantum simulator"

  - *4D Anderson transition*

  - *Anderson transition and interactions*

    N. Cherroret *et al.*, Phys. Rev. Lett. **112**, 170603 (2014)

  - *3D Unitary class (critical exponent)*

    M. Thaha *et al.*, Phys. Rev. E **48**, 1764--1781 (1993)

    C. Tian *et al.*, Phys. Rev. Lett. **93**, 124101 (2004)

  - *Harper model and the Hofstadter butterfly*

    R. Lima and D. Shepelyansky, Phys. Rev. Lett. **67**, 1377-1380 (1991)

    J. Wang and J. Gong, Phys. Rev. A **77**, 031405(R) (2008)

  - *Spin-orbit coupled KR: Topological and quantum Hall physics*

    J. P. Dahlhaus *et al.,* Phys. Rev. B **84**, 115133 (2011)

    Y. Chen and C. Tian, Phys. Rev. Lett. **113**, 216802 (2014)