Two scenarios for quantum multifractality breakdown

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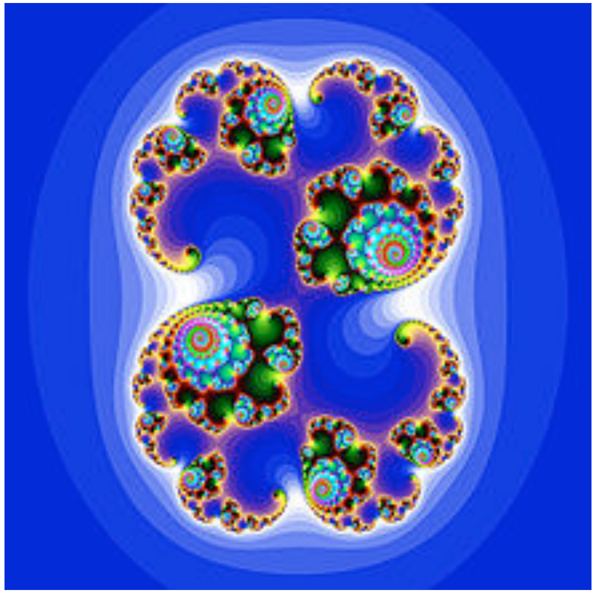
Fractals and multifractals

->Fractal behaviour : well-known in many areas

->Multifractal systems cannot be described by a single fractal dimension



->Observed in many fields of classical physics, from turbulence to stock market

->Much more recently predicted to occur in quantum mechanics

Different quantum states

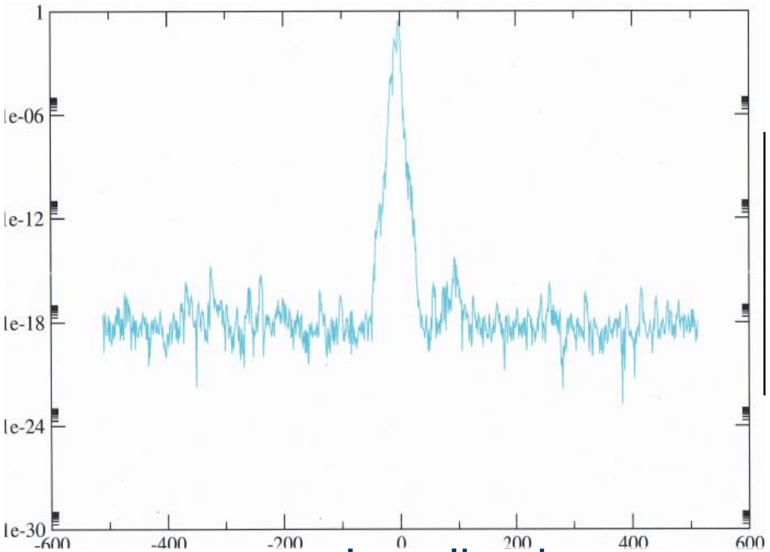
-> Ergodic states: wave functions
spread over the system
with random-like fluctuations

->Localized states: wave functions
exponentially localized

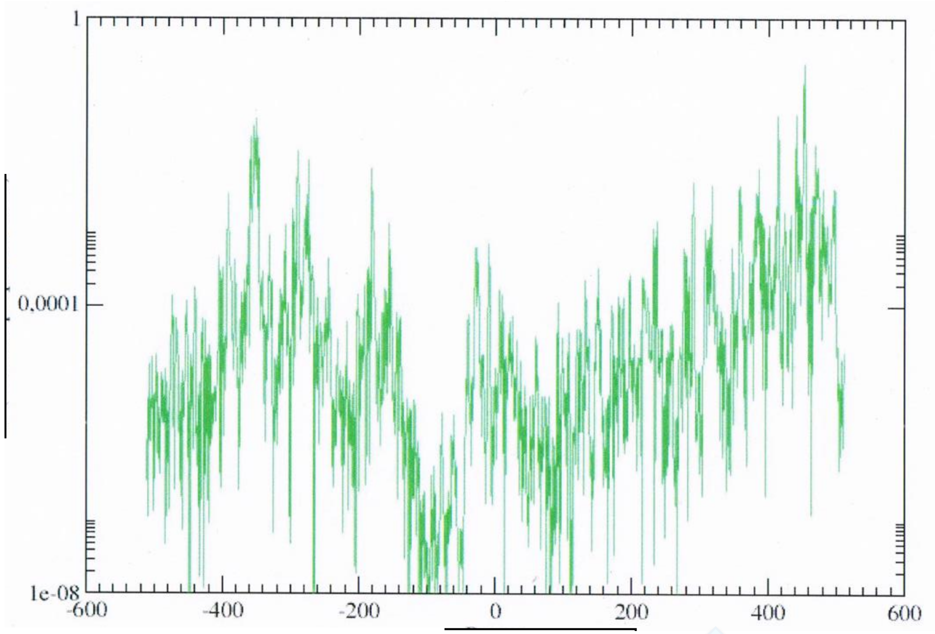
->Multifractal states: **large fluctuations**
all over the system

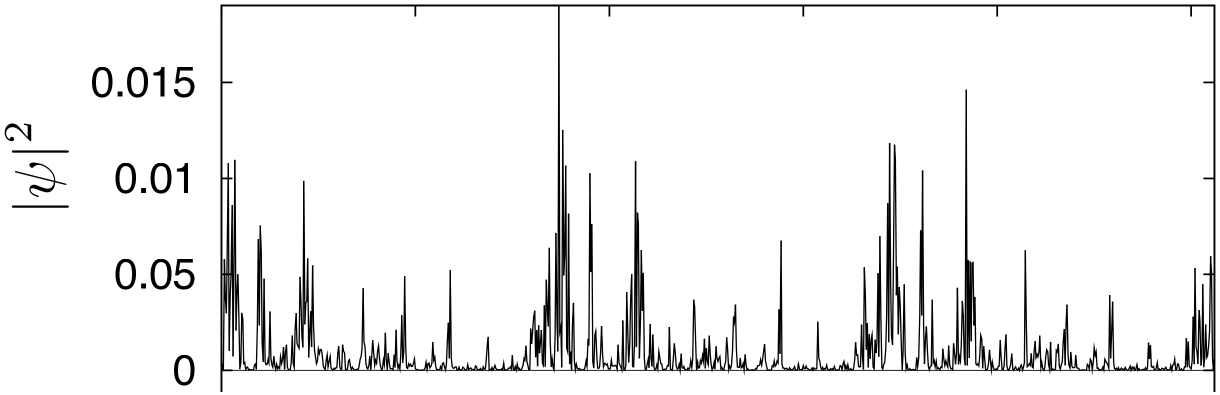
These different states give rise to
specific spectral statistics

Localized vs multifractal states



Localized



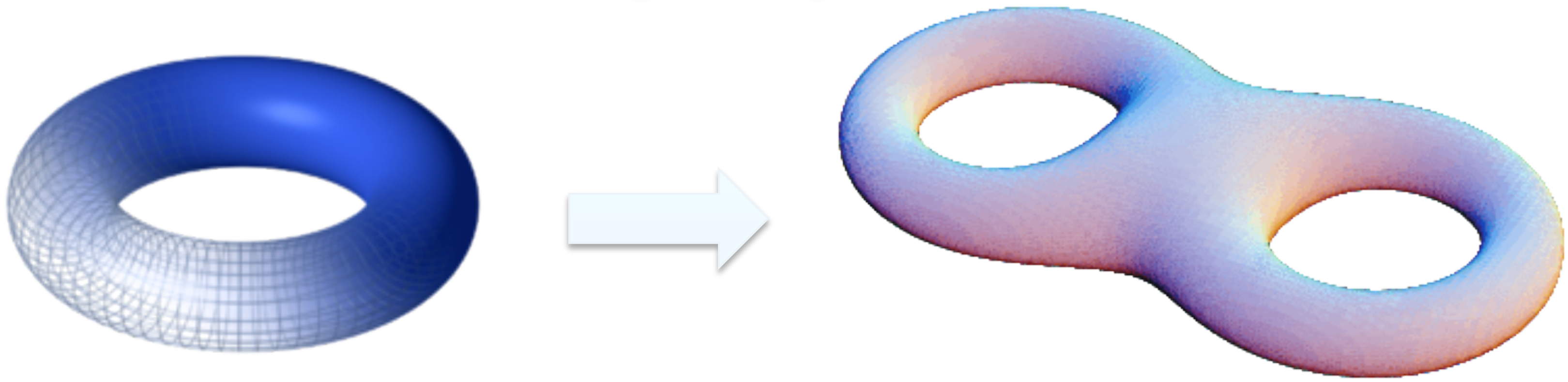


Multifractal

Systems with quantum multifractality

->3D Anderson model at metal-insulator transition:
disordered system form solid-state physics

->Pseudo integrable systems, dynamical systems in between integrable and chaotic systems:
classical motion takes place not on tori as for integrability, but on **surfaces of higher genus**:



How to observe multifractality?

->Multifractal states are difficult to observe experimentally

-> Multifractality has been seen with acoustic waves (S. Faez, A. Strybulevych, J. H. Page, A. Lagendijk and B. A. van Tiggelen, Phys. Rev. Lett. 103,155703 (2009)) but in a quantum context, only indirect evidences up to now

->Important to assess how multifractality resists perturbation

Anderson model

particles on a lattice of sites

$$\mathcal{H} = \sum_i \mu_i |i\rangle\langle i| + \sum_{\langle i,j\rangle} |i\rangle\langle j|$$

where the random on-site energies μ_i are uniformly distributed in $[-W/2, W/2]$ and $\langle i, j\rangle$ denote nearest neighbors.

->classically: diffusion

-> in 1D or 2D: quantum particles localized

-> in 3D metal-insulator transition at $W_c \approx 16.53$

-> At the transition point, multifractal states

Quantum map

->One-dimensional quantum map

Hamiltonian $H(p,q,t)= p^2/2 - \gamma \{ q\} \Sigma_n \delta(t-n)$,

periodically kicked by a discontinuous linear potential

p is momentum and q the space coordinate;{q} is fractional part of q, γ is a real parameter, and the sum runs over all integers.

Classical system integrated over one period:

$$p_{n+1} = p_n + \gamma \mod 1 \ , \quad q_{n+1} = q_n + 2p_{n+1} \mod 1$$

For γ irrational, ergodic dynamics

For γ rational=a/b with a,b integers

iterates cover b circles=pseudo-integrable system

Quantum map

Quantum dynamics: $$\psi^{n+1} = U\psi^n$$

With U NxN matrix such that

$$U_{kl} = \frac{e^{-2\pi i k^2/N}}{N} \frac{1 - e^{2i\pi\gamma N}}{1 - e^{2i\pi(k-l+\gamma N)/N}}$$

->For γ irrational, Random Matrix Theory

->For γ rational=a/b with a,b integers, eigenstates are multifractal, the more so if b is small

The box-counting method

->A system of linear size L is divided
into L/ℓ boxes of size ℓ

->A measure for each box k is $\mu_k = \sum_i |\psi_i|^2$
where the indices run over sites inside box k

-> Moments are defined by $P_q = \sum_k \mu_k^q$

-> Multifractality=power-law behaviour of moments

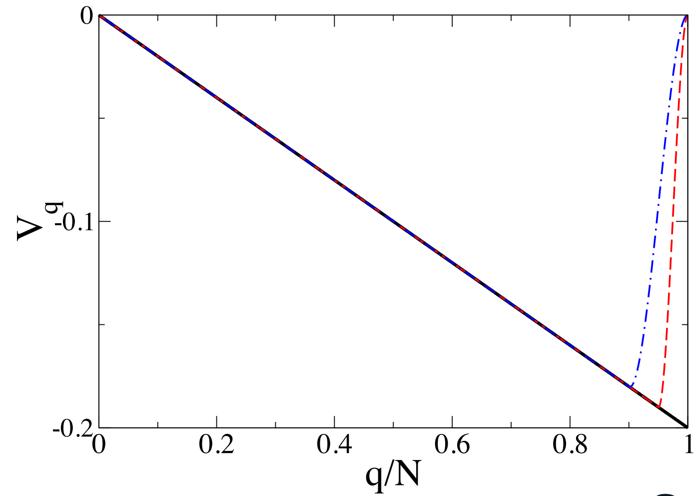
$$P_q \sim (\ell/L)^{D_q(q-1)}$$

Smoothing the singular potential

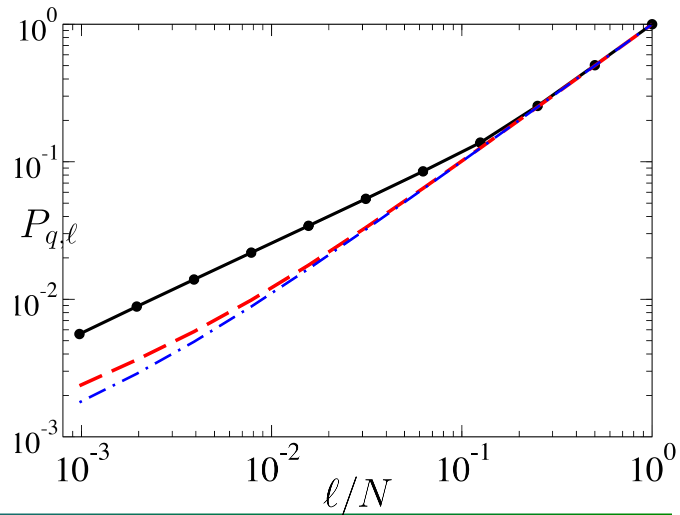
-> First perturbation:
smoothing the singularity of the quantum map

->We replace the potential jump by a smooth interpolation of width ε

-> Moments have different scaling laws depending on scale



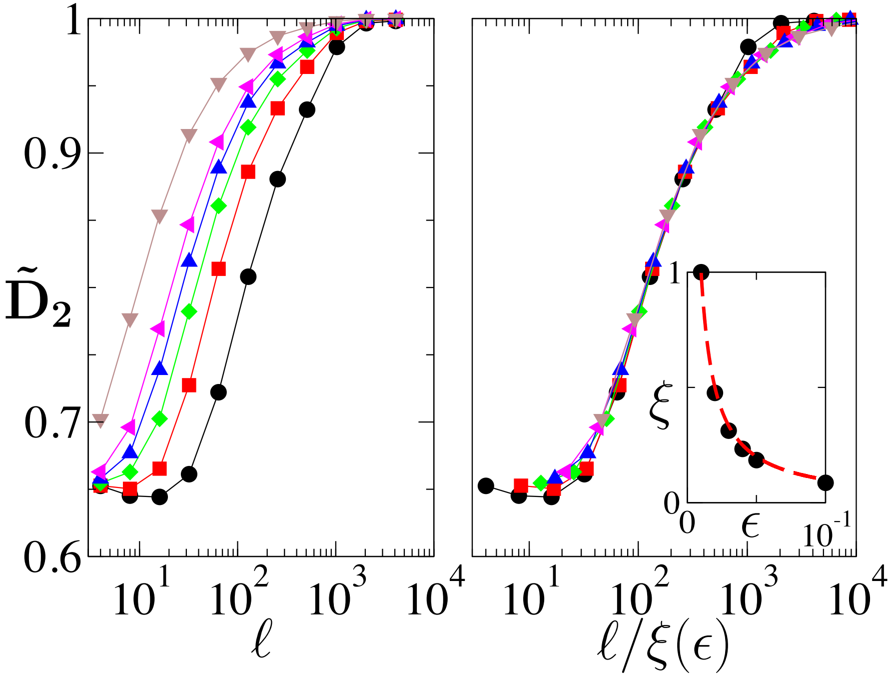
ε



Scale-dependent multifractality

->We define scale-dependent multifractal exponent $\tilde{D}_q(\ell)$ with ℓ denoting the scale

->Numerical result: finite-size scaling theory collapses different perturbations strength onto one curve



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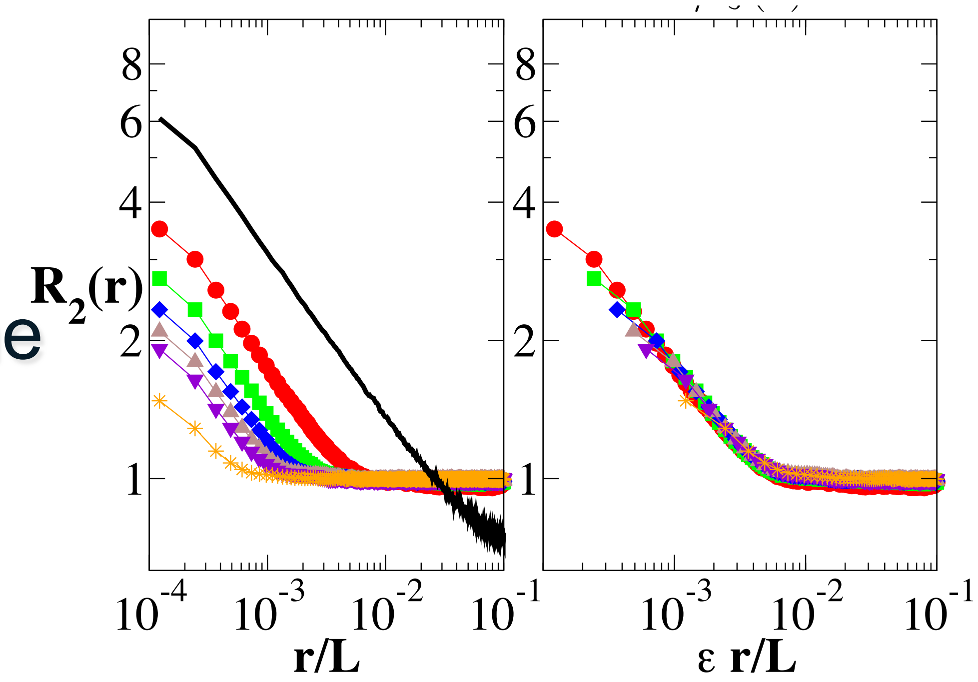
$$\tilde{D}_q(\ell) = G_q\left(\frac{\ell}{\xi(\epsilon)}\right), \qquad \text{with } \xi(\epsilon) \propto \frac{1}{\epsilon}$$

Scale-dependent multifractality

-> 2-point correlation function R_2 is related to the multifractal exponent D_2 for r/L ->0
(L is system size)



-> Again, finite-size scaling defines a perturbation-dependent scale below which multifractality survives unchanged
->This scale varies as inverse of the smoothing length

First scenario: multifractality unchanged at small scale

-> First scenario for multifractality breakdown

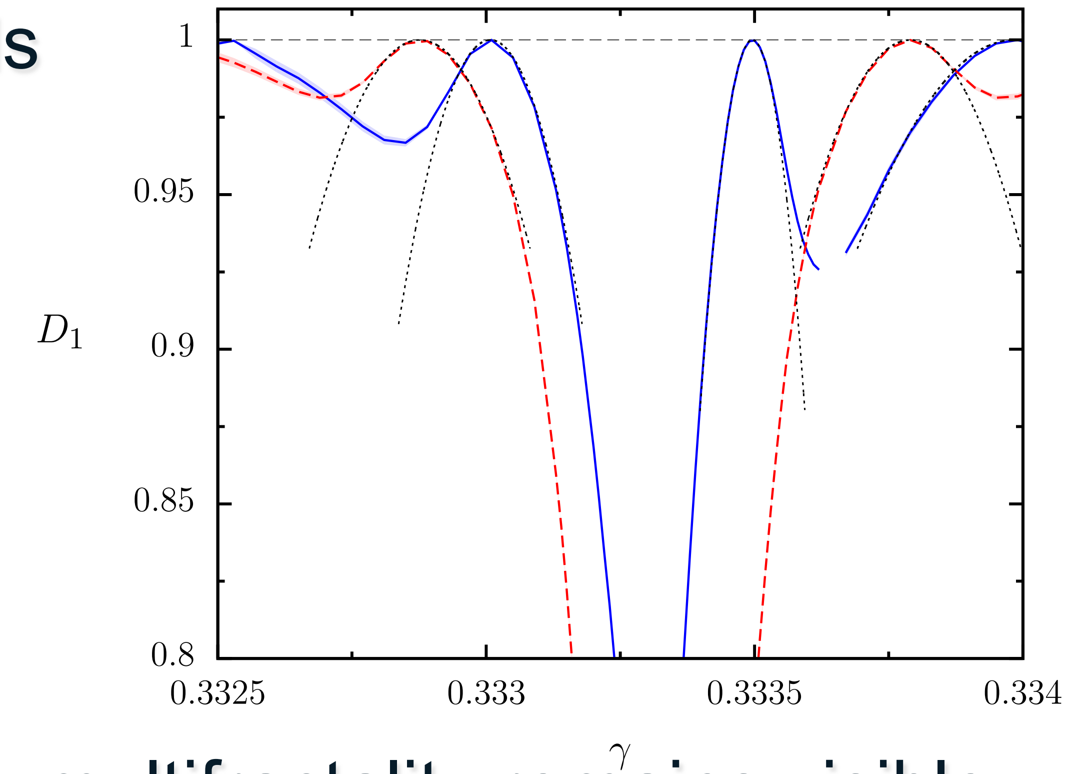
-> Same scenario known to hold for Anderson model away from critical point :
E. Cuevas, V. E. Kravtsov, Phys. Rev. B 76, 235 119 (2007): “multifractal metal”, “multifractal insulator”→multifractality survives below a certain scale

-> in this scenario, experimental imperfections can be compensated by looking at smaller scales.

Changing the slope of the potential for the quantum map

->Multifractality depends on the slope γ of the potential

->For infinite size, no multifractality for all irrational values of γ



-> For finite size system, multifractality remains visible outside rational points: how does it disappear?

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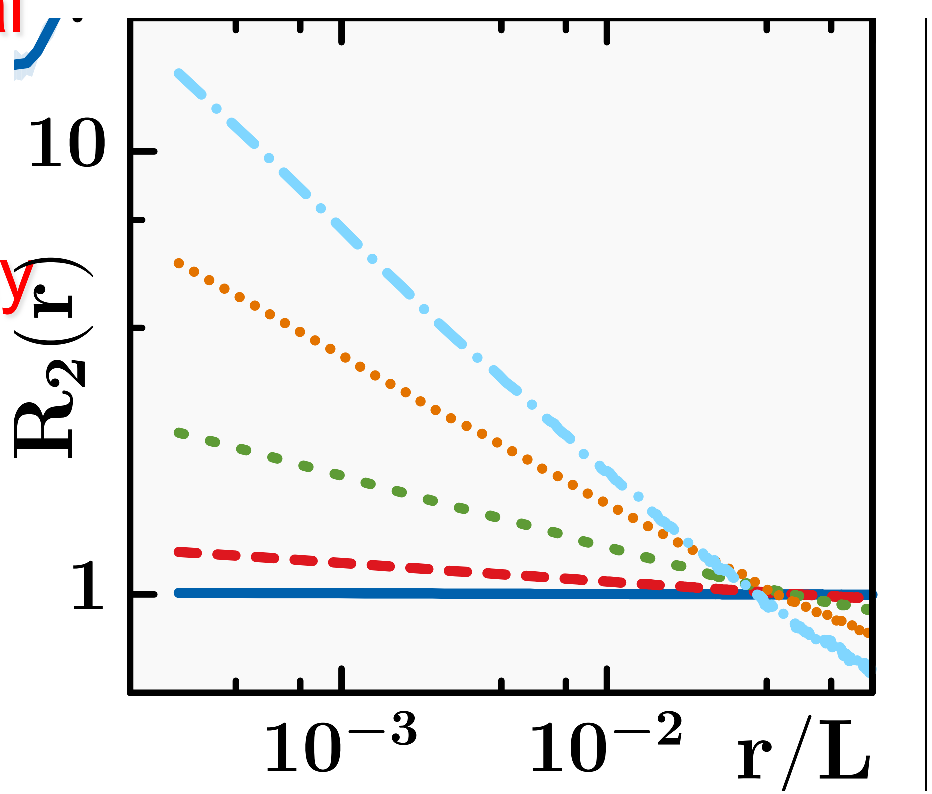
The second scenario

->Analytical and numerical results show that multifractality is now destroyed in the same way at all scales.

->Different scenario from previous one



->In this scenario, experimental imperfections cannot be compensated by looking at smaller scales

Changing basis

->Multifractality depends on the basis of observation

-> In experiments, basis of measurement cannot be chosen at will

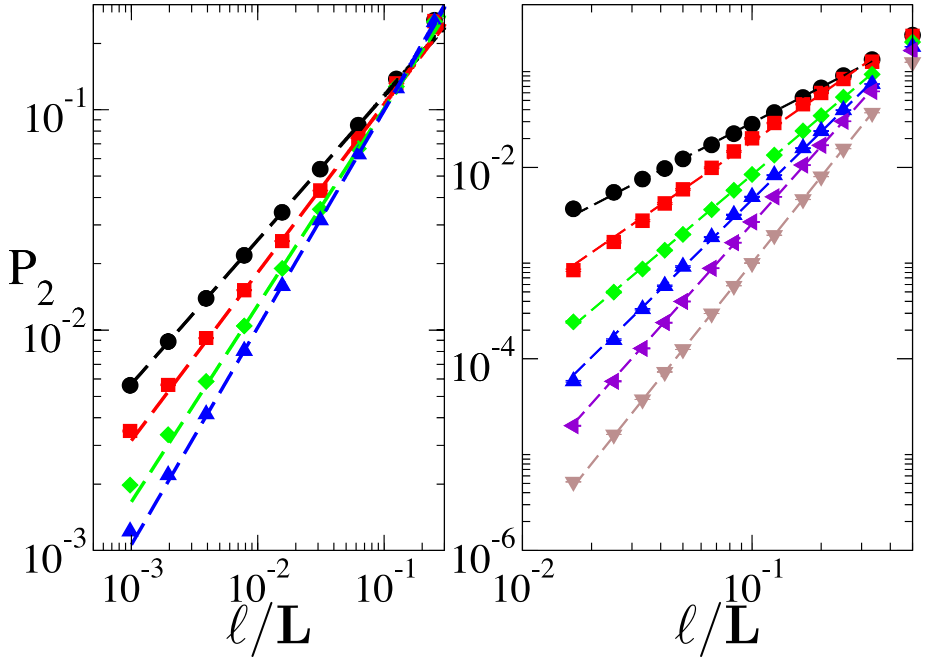


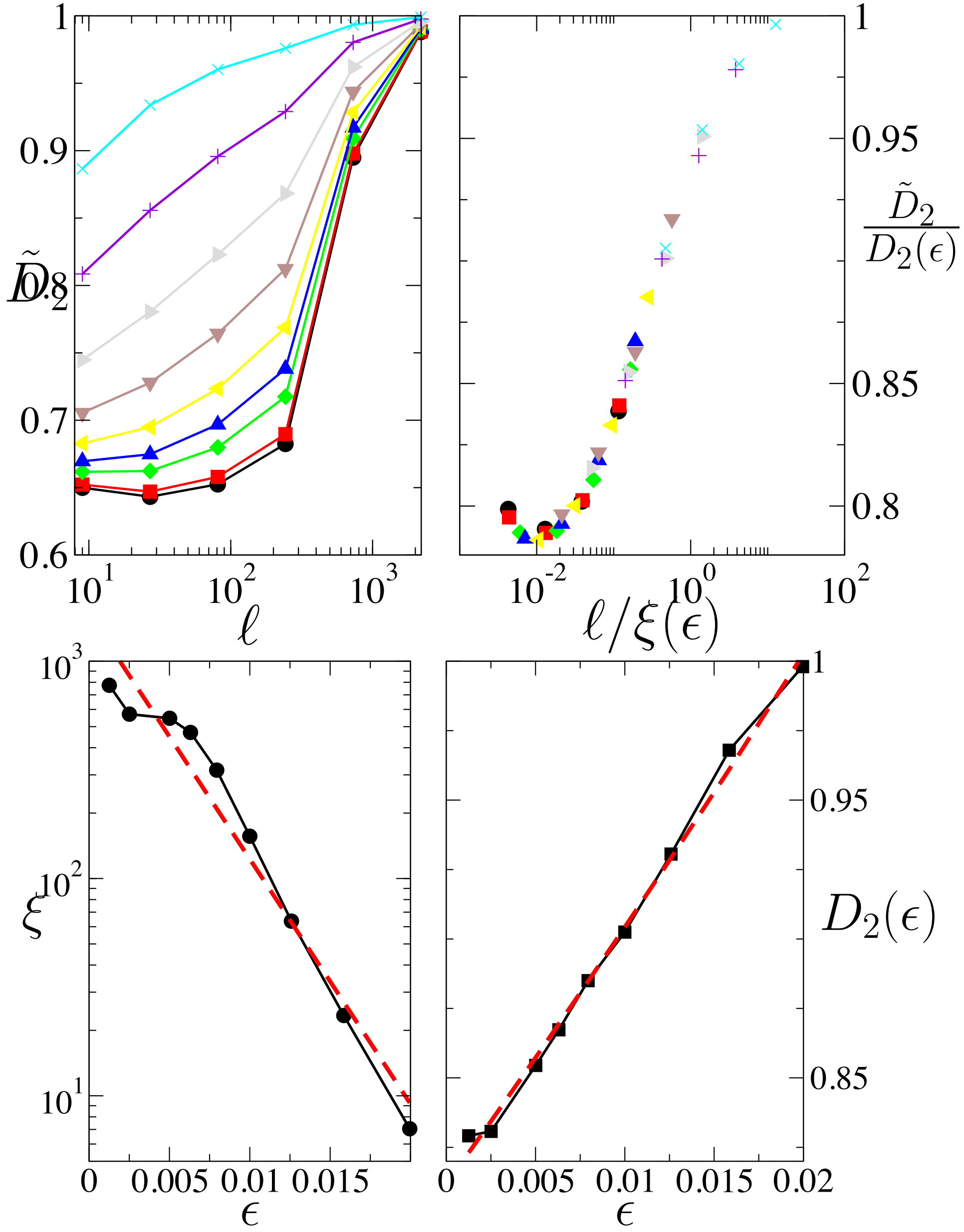
Figure: moments for quantum map and Anderson (size=120^3), for different perturbations

-> Our results show that changing basis destroys multifractality at all scales (second scenario) for both Anderson model and quantum maps

Two-parameter scaling

-> In some systems, changing basis leads to a variant of the second scenario with presence of a characteristic length

->Two-parameter scaling collapses the curves into one curve, with uniform destruction of multifractality below the characteristic length



Changing basis: system size scaling, quantum map

->Multifractality destruction depends on perturbation strength ε and system size N

->Quantum map: perturbation is $\tilde{U} = \exp(\mathrm{i}\epsilon H)$ with H Random GOE Matrix

->Analytical theory predicts scaling in $\epsilon\sqrt{N}$ confirmed by numerics

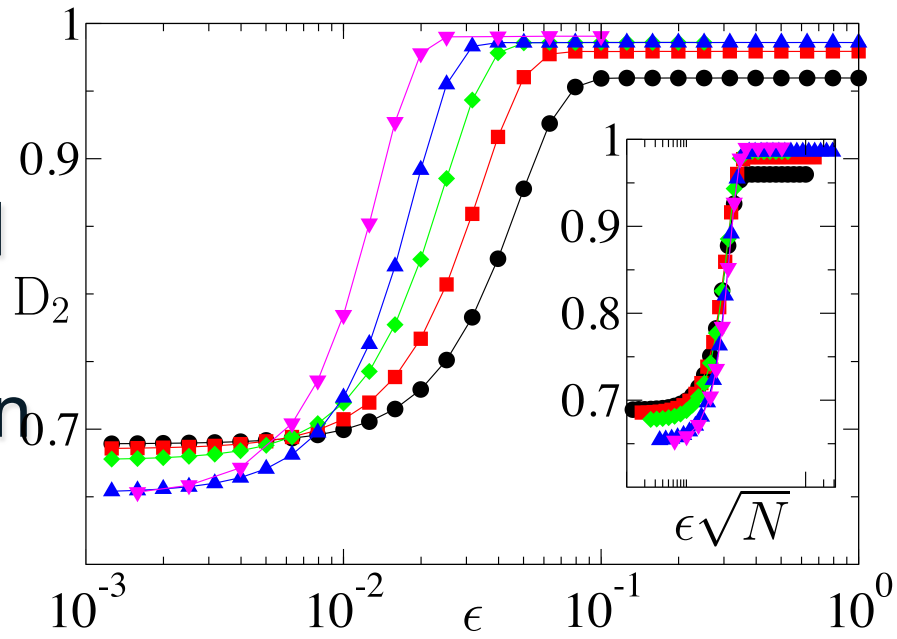
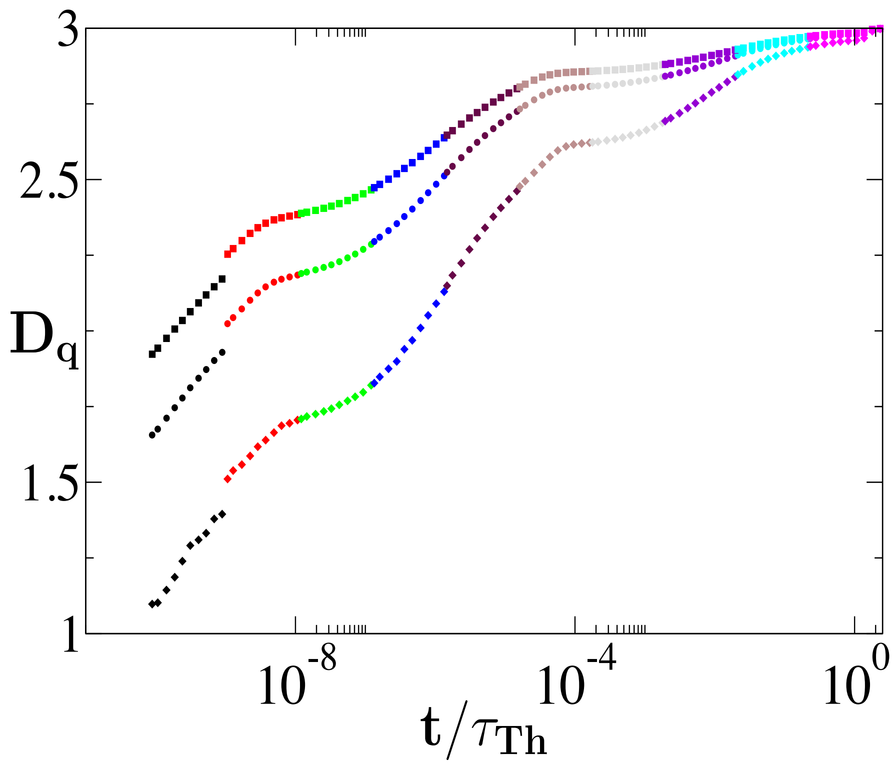


Figure: D_2 for quantum map, for different perturbations

Changing basis: Anderson model

->In Anderson model, due to enormous system size ($N=L^3$ with L=120) the basis changes was modeled by a diffusive system (quasiperiodic kicked rotor)



->Different theory for system size scaling than for map

->Relevant parameter is now the Thouless time L^2/D

->Confirmed by numerical simulations over many orders of magnitude

Conclusion

->we have studied how multifractality of wave functions is destroyed when a perturbation is applied.

->We have studied different perturbations of two representative models

-> We find that multifractality can be destroyed in two ways.

->In the first scenario, multifractality survives below a perturbation-dependent scale.

-> In the second scenario, it is destroyed at all scales

-> Both scenarios have implications for experimental observation of quantum multifractality.

-> In addition, our results imply that Anderson model remains critical when basis is changed at W_c