Dynamics of a quantum particle in the presence of a time-dependent absorbing barrier

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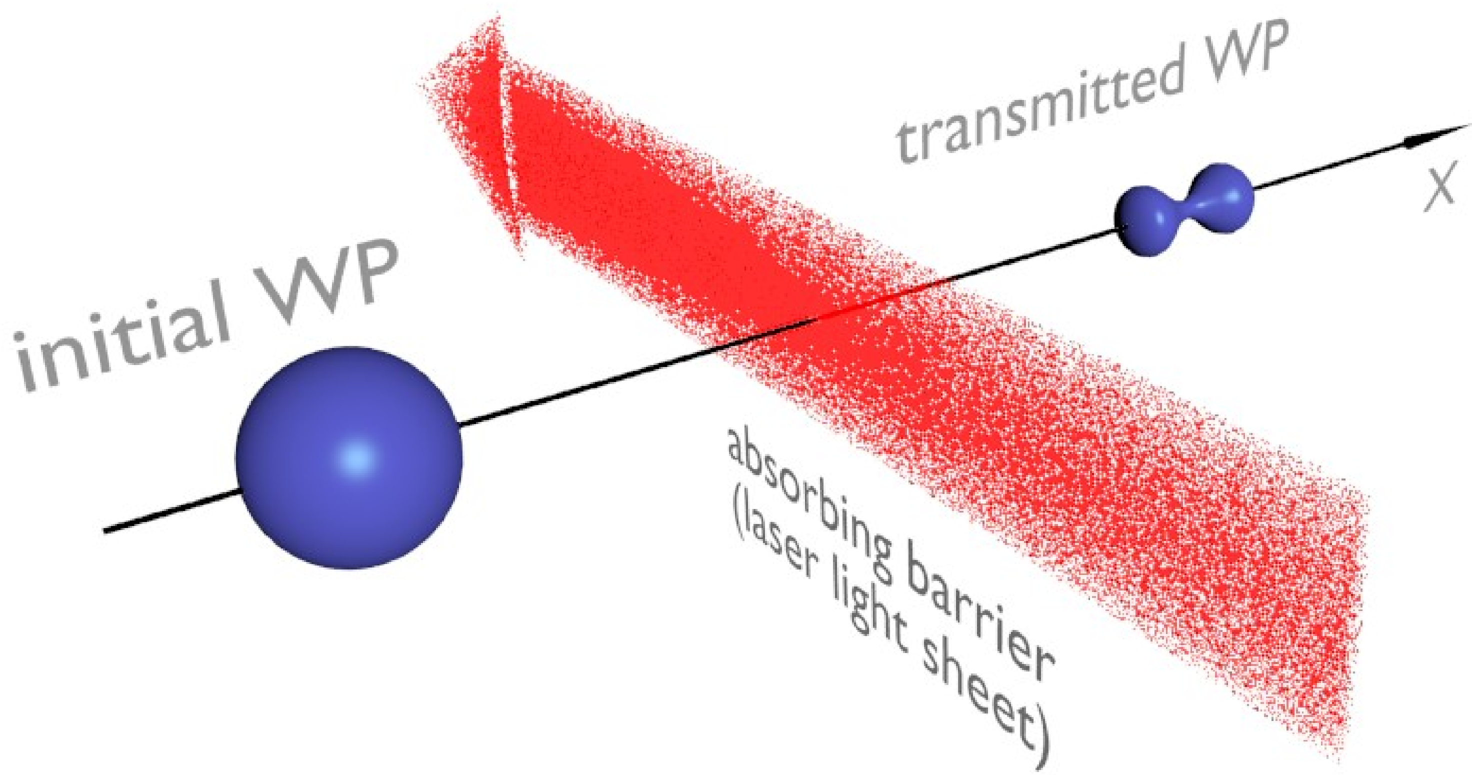


initial WP
transmitted WP
X
absorbing barrier
(laser light sheet)

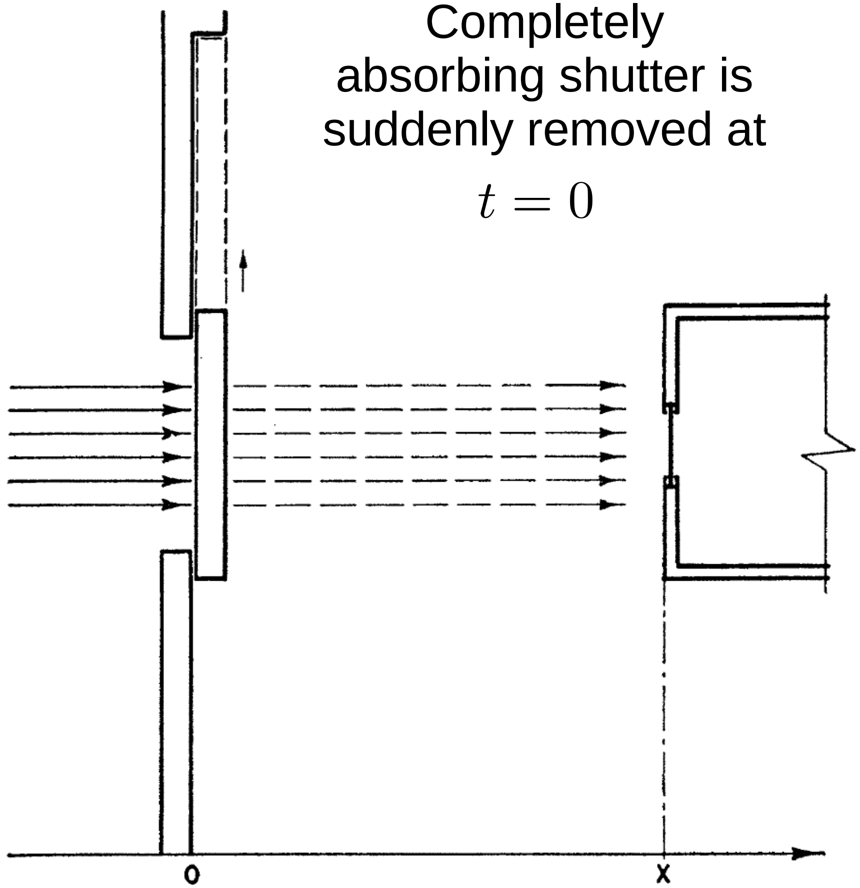
Outline

- Introduction *(Moshinsky problem, diffraction in time)*
- Exactly solvable model *(Kottler discontinuity)*
- Applications *(Diffraction at a time grating,*
  space-time diffraction,
  matter pulse carving)

Moshinsky problem

Completely absorbing shutter is suddenly removed at $t = 0$

Beam of "monoenergetic" particles



When will the beam hit the detector?

Moshinsky, *Phys. Rev.* **88**, 625 (1952)

Time-dependent Schrödinger equation

$$\left(i\partial_t + \frac{\hbar}{2m}\partial_x^2\right)\Psi(x,t) = 0 \qquad \text{for} \qquad x \in \mathbb{R}\,, \quad t > 0$$

with initial condition ("chopped" plane wave)

$$\Psi(x,0) = \Theta(-x)\exp(ipx/\hbar)$$

with $p = mv$ (average momentum)

Exact solution (Moshinsky function)

$$\Psi(x,t) = \exp\left[\frac{i}{\hbar}(px - Et)\right]\frac{1}{2}\mathrm{erfc}\left[\sqrt{\frac{m}{2i\hbar t}}(x - vt)\right]$$

with $E = p^2/(2m)$ (classical energy)

Moshinsky, *Phys. Rev.* **88**, 625 (1952)

Probability distribution

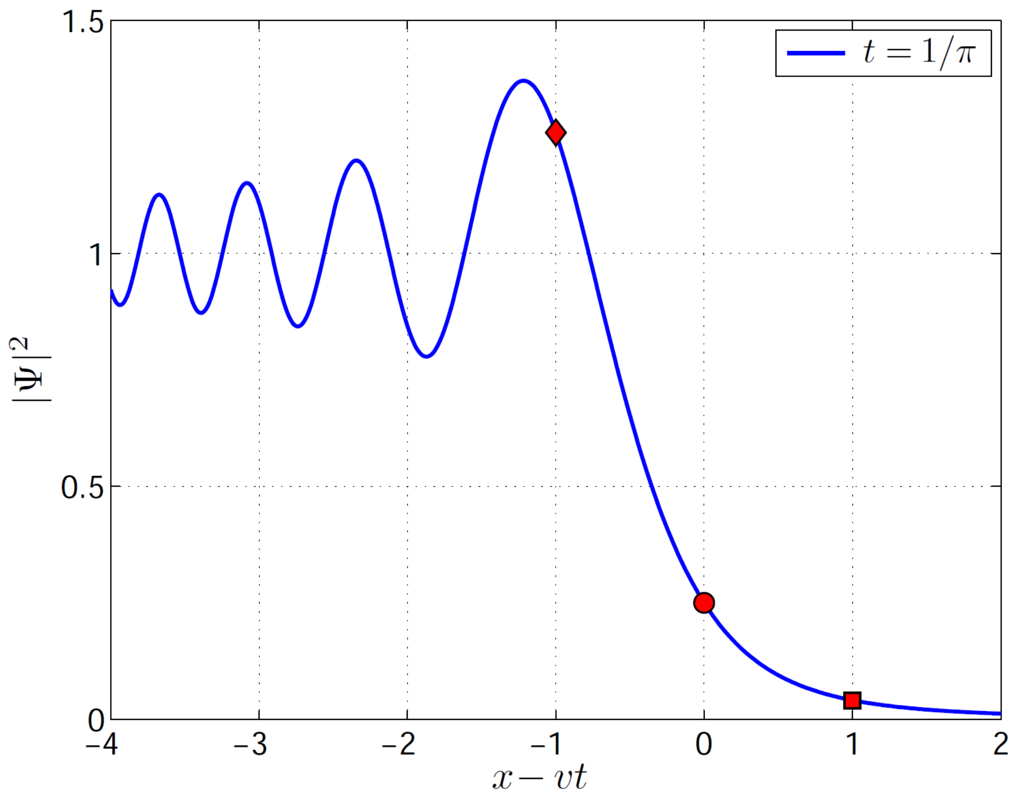
$$|\Psi|^2 = \frac{1}{2}\left\{\left[\frac{1}{2} + C(u)\right]^2 + \left[\frac{1}{2} + S(u)\right]^2\right\}$$

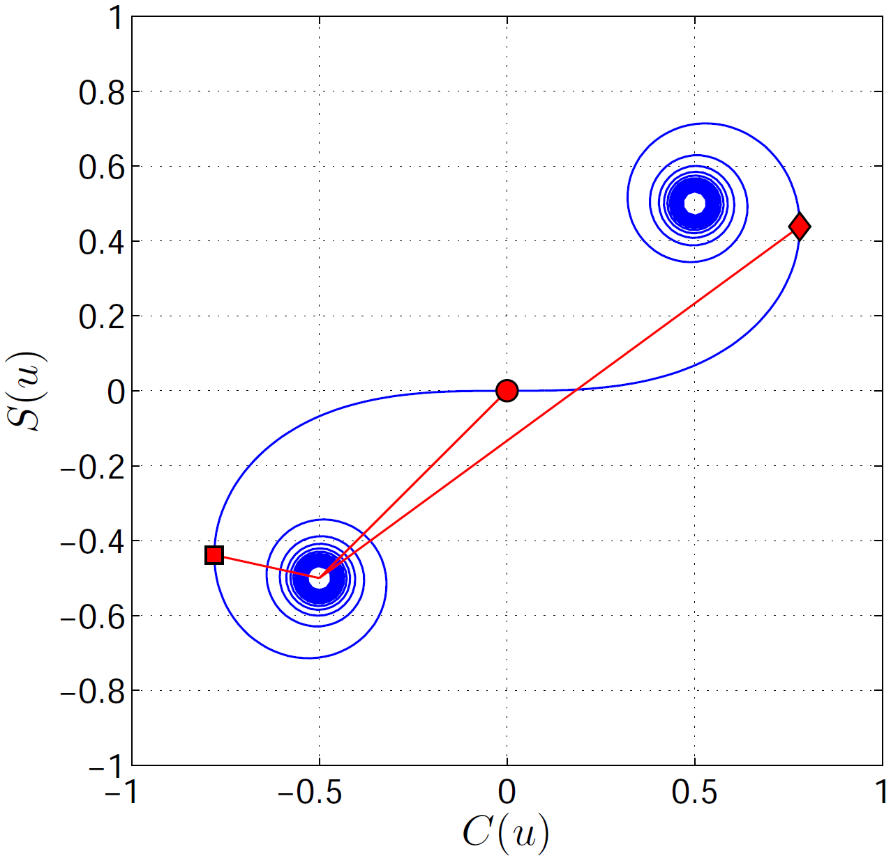
$$u = -\sqrt{\frac{m}{\pi\hbar t}}\,(x - vt)$$

Fresnel integrals and Cornu spiral

$$C(u) = \int_0^u \mathrm{d}z\, \cos\left(\frac{\pi}{2}z^2\right)$$

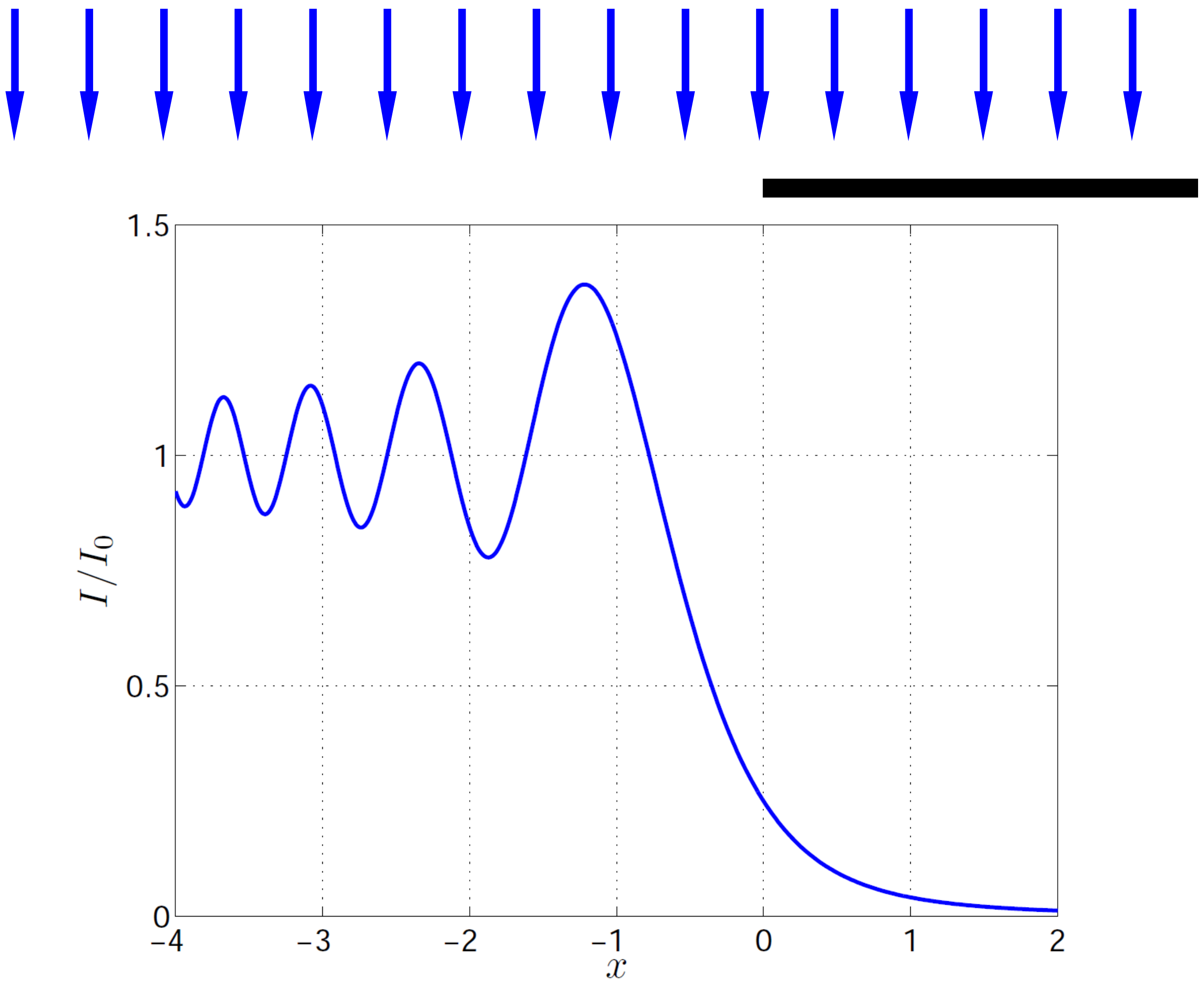
$$S(u) = \int_0^u \mathrm{d}z\, \sin\left(\frac{\pi}{2}z^2\right)$$





Moshinsky, *Phys. Rev.* **88**, 625 (1952)

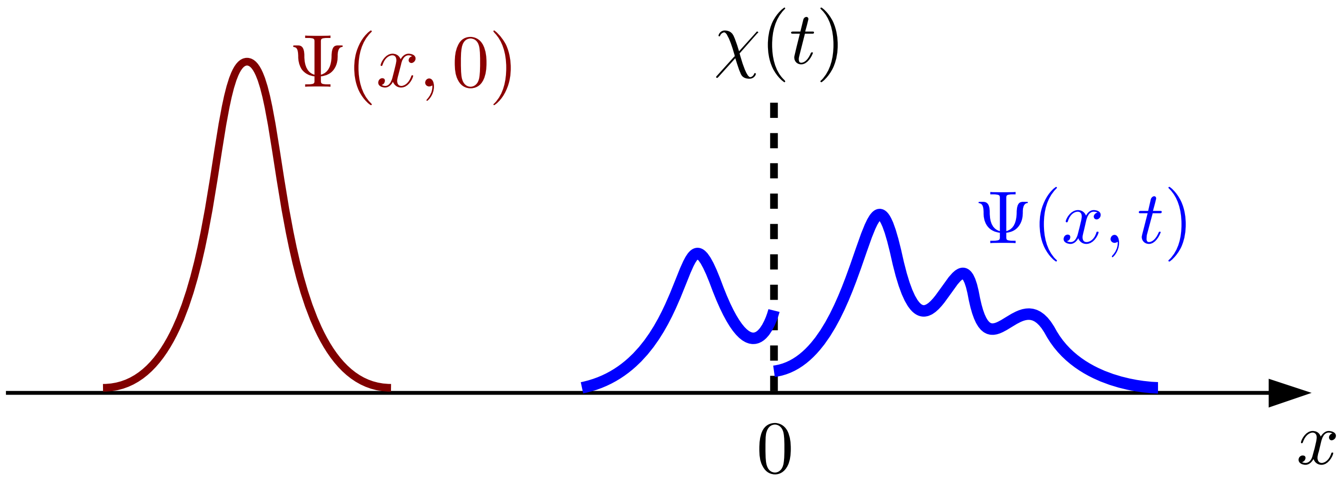
Monochromatic light of intensity I_0



Wave packet spreading in Moshinsky shutter problem $\Longleftrightarrow$ Fresnel diffraction of light at the edge of a semi-infinite screen

Moshinsky, *Phys. Rev.* **88**, 625 (1952)

Generalized Moshinsky problem

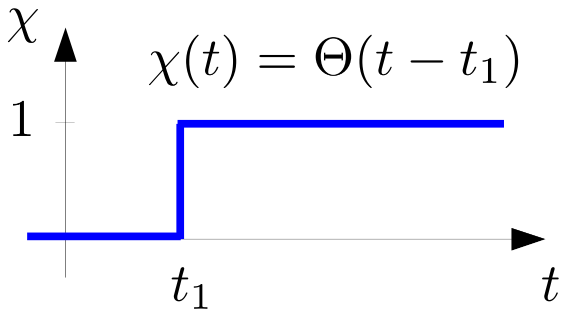


Transparency of an absorbing barrier depends on time in accordance with a real-valued aperture function $\chi(t)$ varying between 0 (complete absorption) and 1 (full transparency)

Previous results

“Time edge”

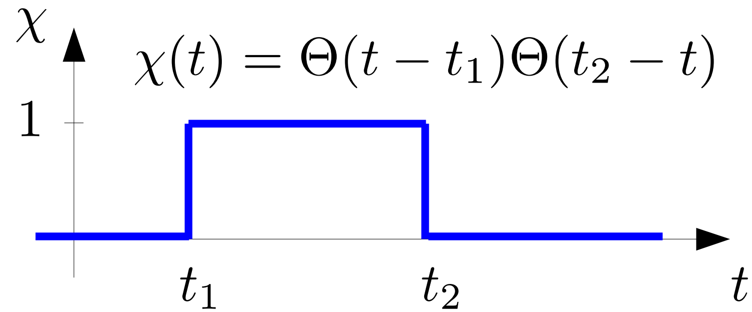
$$\chi(t) = \Theta(t - t_1)$$



Moshinsky, *Phys. Rev.* **88**, 625 (1952)

“Time slit”

$$\chi(t) = \Theta(t - t_1)\Theta(t_2 - t)$$



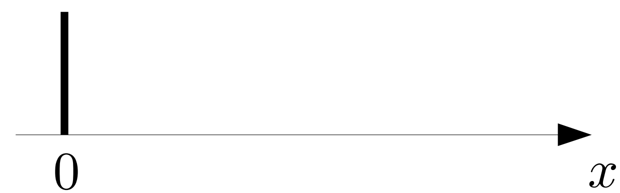
Moshinsky, *Am. J. Phys.* **44**, 1037 (1976)

Time-dependent “delta”-potential

$$V(x,t) = \delta(x)/\sqrt{at^2 + bt + c}$$

Scheitler & Kleber, *Z. Phys. D* **9**, 267 (1988)

Dodonov, Man'ko, Nikonov, *Phys. Lett. A* **162**, 359 (1992)



“Source” boundary approach

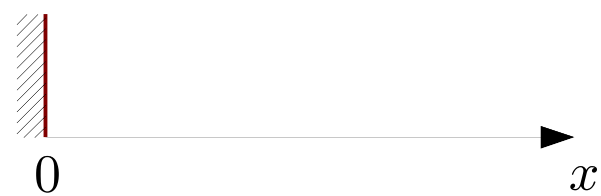
$$\Psi(0,t) = \chi(t)\Psi_{\text{in}}(0,t)$$

Brukner & Zeilinger, *Phys. Rev. A* **56**, 3804 (1997)

Hils et al., *Phys. Rev. A* **58**, 4784 (1998)

del Campo, Muga, Moshinsky, *J. Phys. B* **40**, 975 (2007)

Godoy, Olvera, del Campo, *Physica B* **396**, 108 (2007)

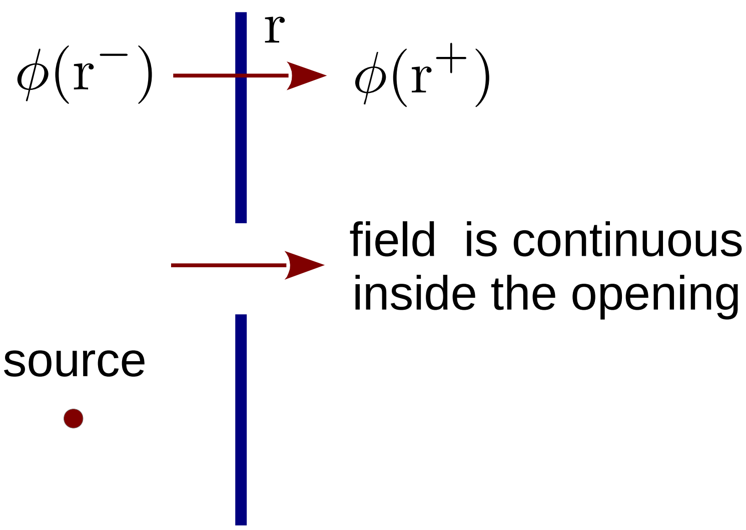


Absorbing boundary in stationary wave optics

Kottler discontinuity

Diffraction of stationary optical waves in three-dimensional space

$$(\nabla^2 + k^2)\,\phi(\mathrm{r}) = 0$$



The field and its normal derivative are postulated to change discontinuously across the absorbing screen:

$$\phi(\mathrm{r})\Big|_{\mathrm{r}^-}^{\mathrm{r}^+} = -\phi_0(\mathrm{r})$$

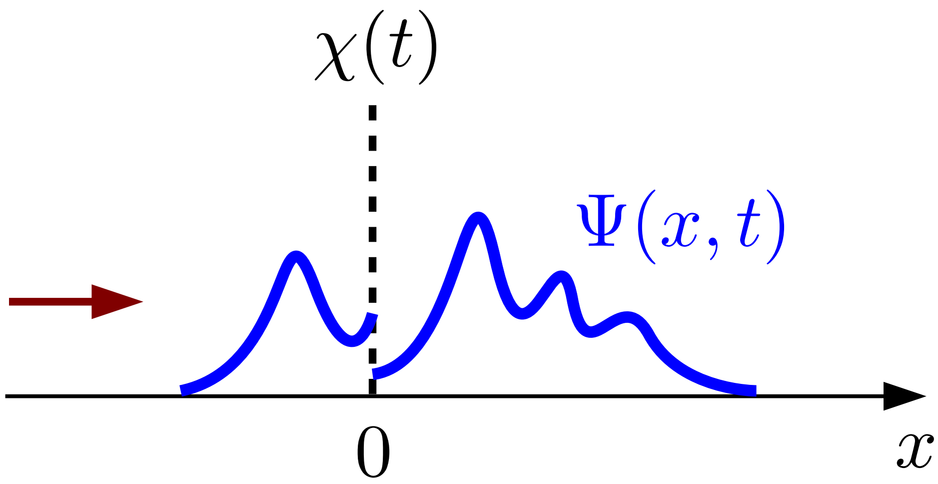
$$\partial_n\phi(\mathrm{r})\Big|_{\mathrm{r}^-}^{\mathrm{r}^+} = -\partial_n\phi_0(\mathrm{r})$$

where ϕ_0 = field in free space (in the absence of a screen)

The exact solution of Kottler problem is the wave filed predicted by Kirchhoff theory of diffraction!

Kottler, *Annln Phys.* **70**, 405 (1923); *Prog. Opt.* **4**, 281 (1965)

Kottler discontinuity in time-dependent quantum mechanics



$$\Psi(x,t)\Big|_{x=0^-}^{x=0^+} = -\left[1-\chi(t)\right]\Psi_0(x,t)\Big|_{x=0}$$

$$\partial_x\Psi(x,t)\Big|_{x=0^-}^{x=0^+} = -\left[1-\chi(t)\right]\partial_x\Psi_0(x,t)\Big|_{x=0}$$

where $\Psi_0(x,t)$ = free-particle wave function (in the absence of a barrier)

Goussev, Phys. Rev A **85**, 013626 (2012); Phys. Rev. A **87**, 053621 (2013)

The model

Propagator

$$\Psi(x,t) = \int_{-\infty}^{+\infty} \mathrm{d}x' \, K(x,x';t)\Psi(x',0)$$

Time-dependent Schrödinger equation

$$\left(i\partial_t + \frac{\hbar}{2m}\partial_x^2\right) K(x,x';t) = 0 \qquad \text{for} \qquad x, x' \neq 0$$

Initial condition

$$K(x,x';0^+) = \delta(x-x')$$

Boundary conditions

$$K(x,x';t)\Big|_{x=0^-}^{x=0^+} = \mathrm{sgn}(x')\left[1-\chi(t)\right] K_0(x-x',t)\Big|_{x=0}$$

$$\partial_x K(x,x';t)\Big|_{x=0^-}^{x=0^+} = \mathrm{sgn}(x')\left[1-\chi(t)\right] \partial_x K_0(x-x',t)\Big|_{x=0}$$

Free particle propagator

$$K_0(z,t) = \sqrt{\frac{m}{2\pi i\hbar t}} \exp\left(i\frac{m}{2\hbar t}z^2\right)$$

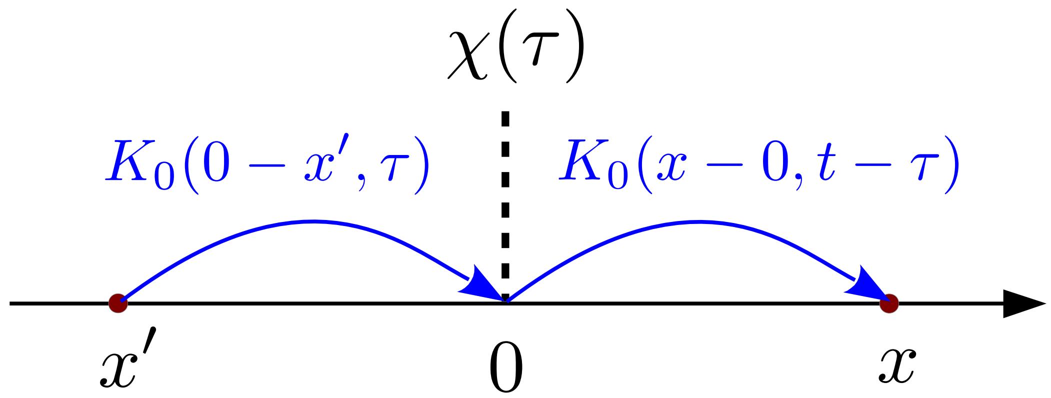
Exact solution

$$K(x,x';t) = \Xi(x,x')K_0(x-x',t) + K_1(x,x';t)$$

$$\Xi(x,x') = \begin{cases} 1\,, & x,x' \text{ lie on the same side of the barrier} \\ 0\,, & \text{otherwise} \end{cases}$$

$$K_1(x,x';t) = \int_0^t \mathrm{d}\tau\, u\, K_0(x,t-\tau)\chi(\tau)K_0(-x',\tau)$$

$$u(x,x';t,\tau) = -\frac{\mathrm{sgn}(x')}{2}\left(\frac{x}{t-\tau} - \frac{x'}{\tau}\right)$$



- Consistent with Moshinsky shutter propagator and Huygens-Fresnel principle
- Similarities (but also differences) with Brukner & Zeilinger, *Phys. Rev. A* **56**, 3804 (1997)

Exact solution: Alternative form

$$K(x,x';t) = \underbrace{\Xi(x,x')\big[1-\chi(t)\big]K_0(x-x',t)}_{\text{discontinuous}} + \underbrace{K_2(x,x';t)}_{\text{continuous}}$$

$$K_2(x,x';t) = \frac{1}{2}\left(\chi(0)+\chi(t)+\mathrm{sgn}(x')\int_0^t \mathrm{d}\tau\,\frac{\mathrm{d}\chi(\tau)}{\mathrm{d}\tau}\,\mathrm{erf}(\Phi)\right)K_0(x-x',t)$$

$$\Phi(x,x';t,\tau) = \sqrt{\frac{m}{2i\hbar t}}\left(x\sqrt{\frac{\tau}{t-\tau}}+x'\sqrt{\frac{t-\tau}{\tau}}\right)$$

Composition property (or absence of)

In general, the composition property is not fulfilled:

$$K(x,x';t) \neq \int_{-\infty}^{+\infty} \mathrm{d}\xi\, K(x,\xi;t-\tau)K(\xi,x';\tau)$$

However

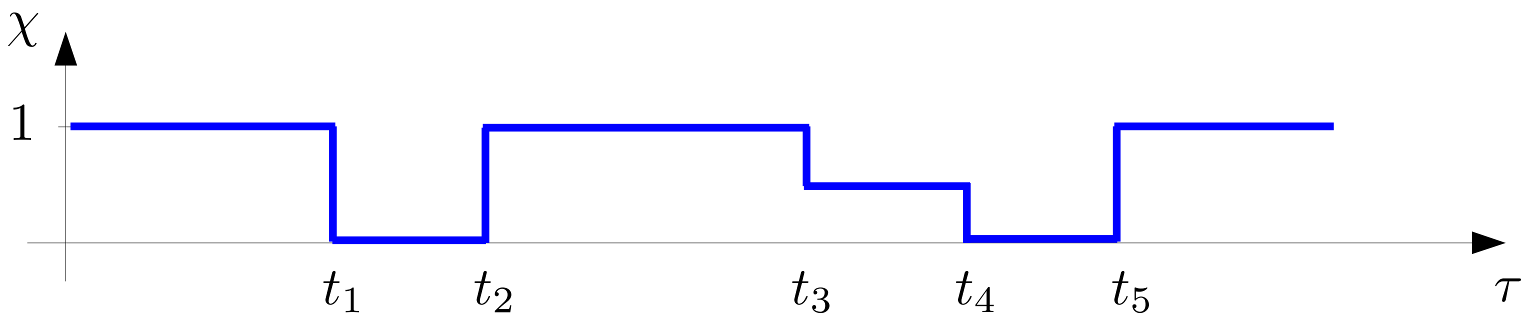
$$K(x,x';t) = \int_{-\infty}^{\infty} \mathrm{d}\xi\, K_0(x-\xi,t-\tau)K(\xi,x',\tau)$$

provided the absorbing barrier acts only up to some time t_f , i.e.,

$$\chi(\tau) = 1\,, \qquad 0 < t_\mathrm{f} < \tau < t$$

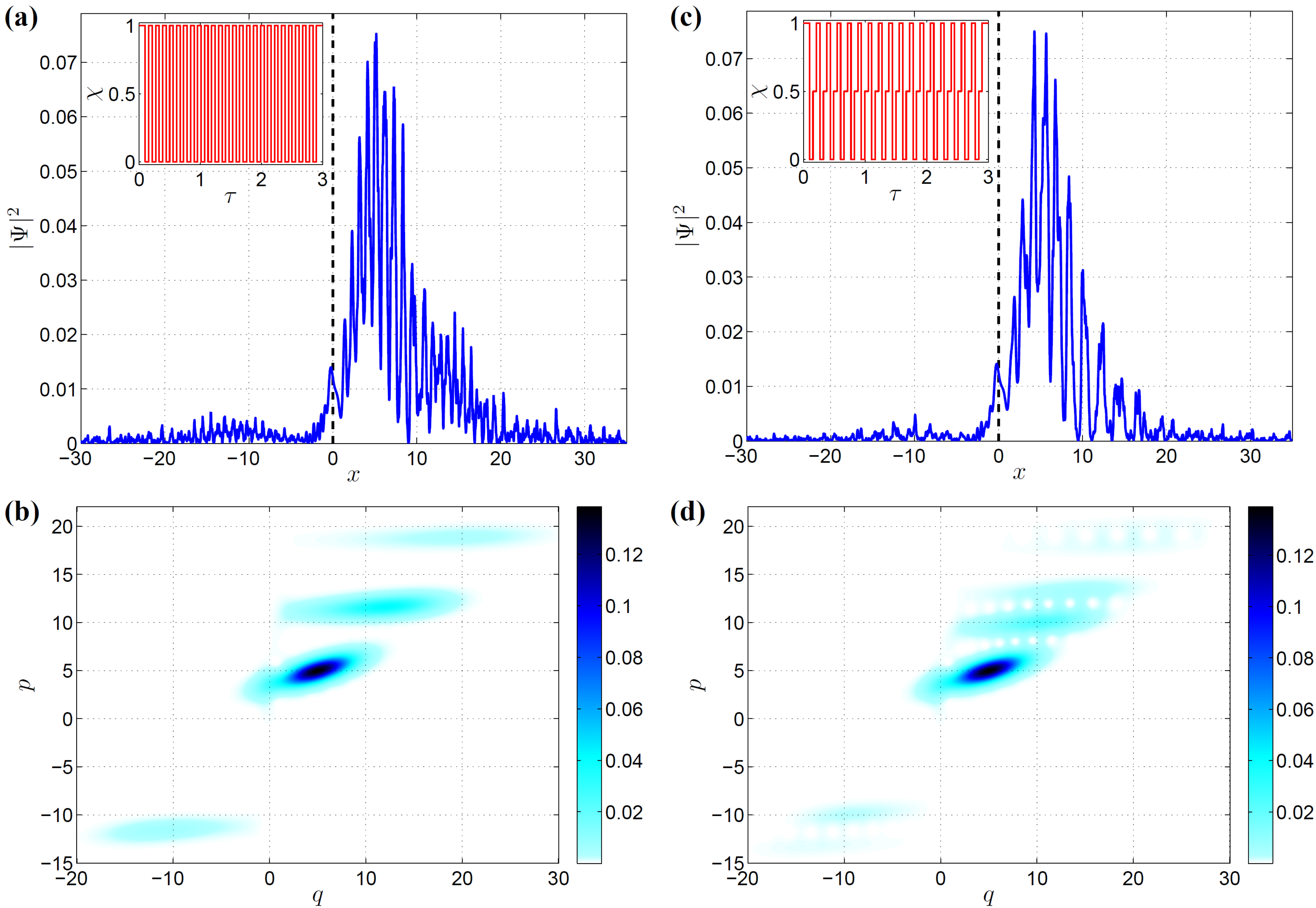
"Time grating"

$$\chi(\tau) = \chi_0 \Theta(t_1 - \tau) + \sum_{n=1}^{N-1} \chi_n \Theta(\tau - t_n)\Theta(t_{n+1} - \tau) + \chi_N \Theta(\tau - t_N)$$

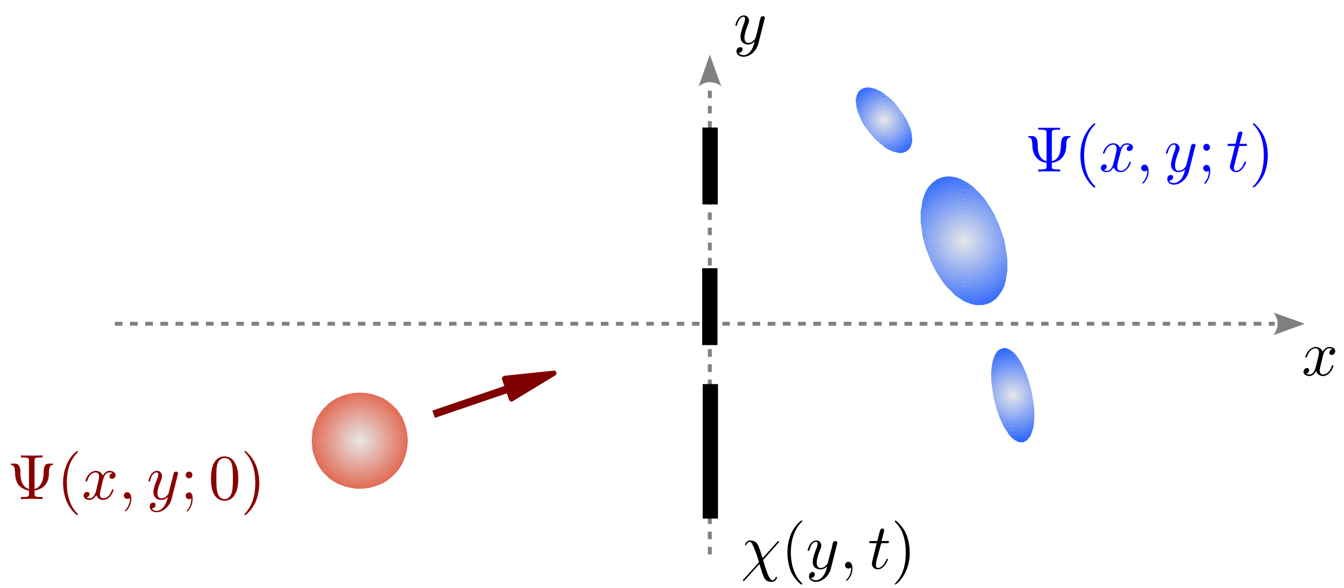


The full propagator admits a closed form expression!

Diffraction of Gaussian wave packets



Extension to two and three spatial dimensions

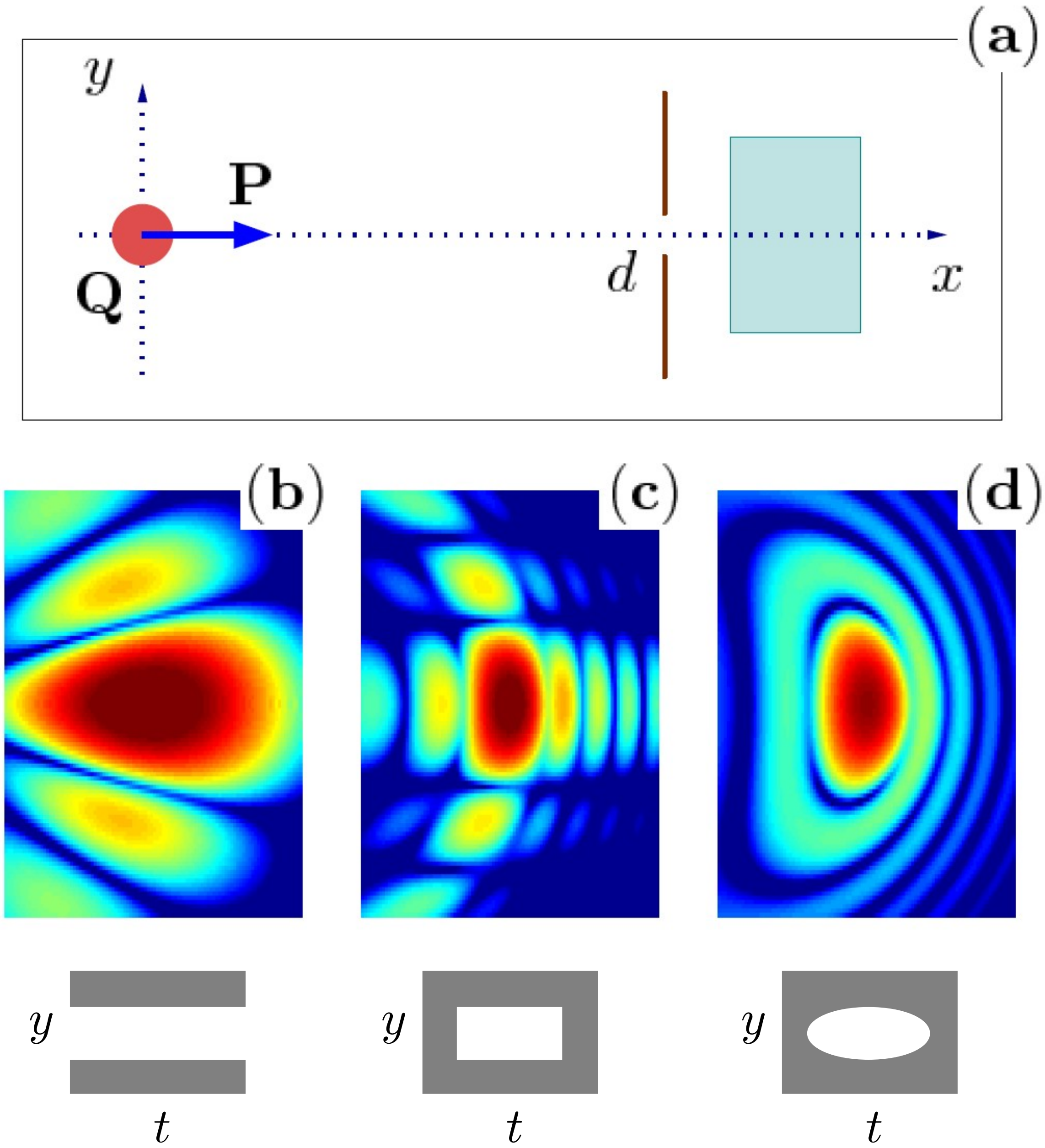


In the transmission region:

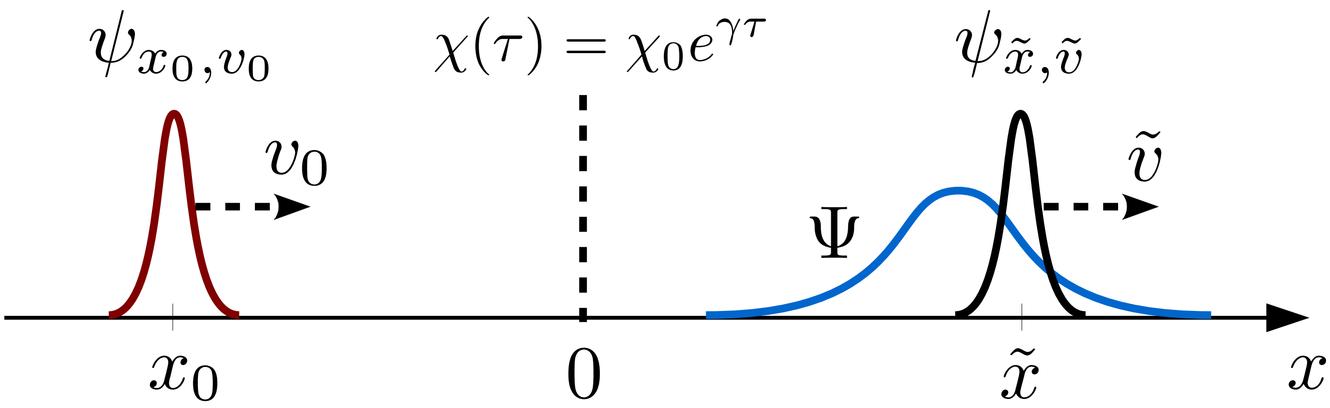
$$K_1(x,y;x',y';t)$$

$$= \int_0^t \mathrm{d}\tau \int_{-\infty}^{+\infty} \mathrm{d}\eta\, u\, K_0(x,y-\eta;t-\tau)\chi(\eta,\tau)K_0(-x',\eta-y';\tau)$$

Diffraction in space and time



Exponentially opening/closing barrier



Initial wave packet: $\psi_{x_0,v_0}(x) = Ce^{-(x-x_0)^2/(2\sigma^2)+imv_0(x-x_0)/\hbar}$

Evolved wave packet: $\Psi(x,t) = \int dx' K(x,x',t)\psi_{x_0,v_0}(x')$

Husimi representation: $H(\tilde{x},\tilde{v}) = \left|\langle\psi_{\tilde{x},\tilde{v}}|\Psi\rangle\right|^2$

Steepest descent evaluation for

$$1 \ll \frac{|x_0|}{\sigma} \lesssim \frac{v_0 t}{2\sigma} \ll \frac{mv_0\sigma}{2\hbar} \quad \text{and} \quad |\gamma| \ll \frac{2|x_0|v_0}{\sigma^2}$$

$\Longrightarrow$ transmitted wave packet is spatially shifted by $\boxed{\Delta x = -\frac{\gamma\sigma^2}{v_0}}$

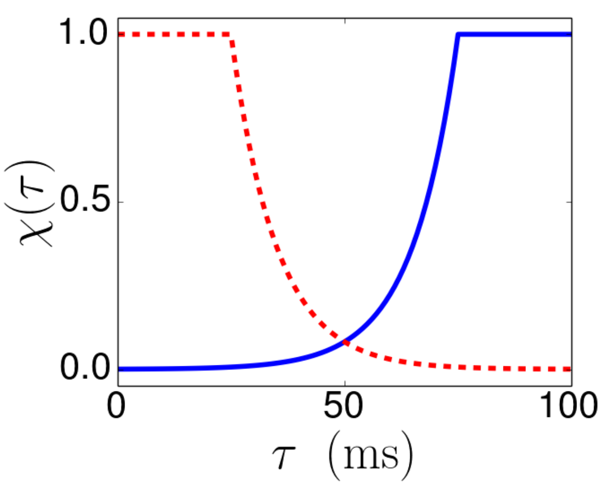
Goussev, *arXiv:1503.00031*

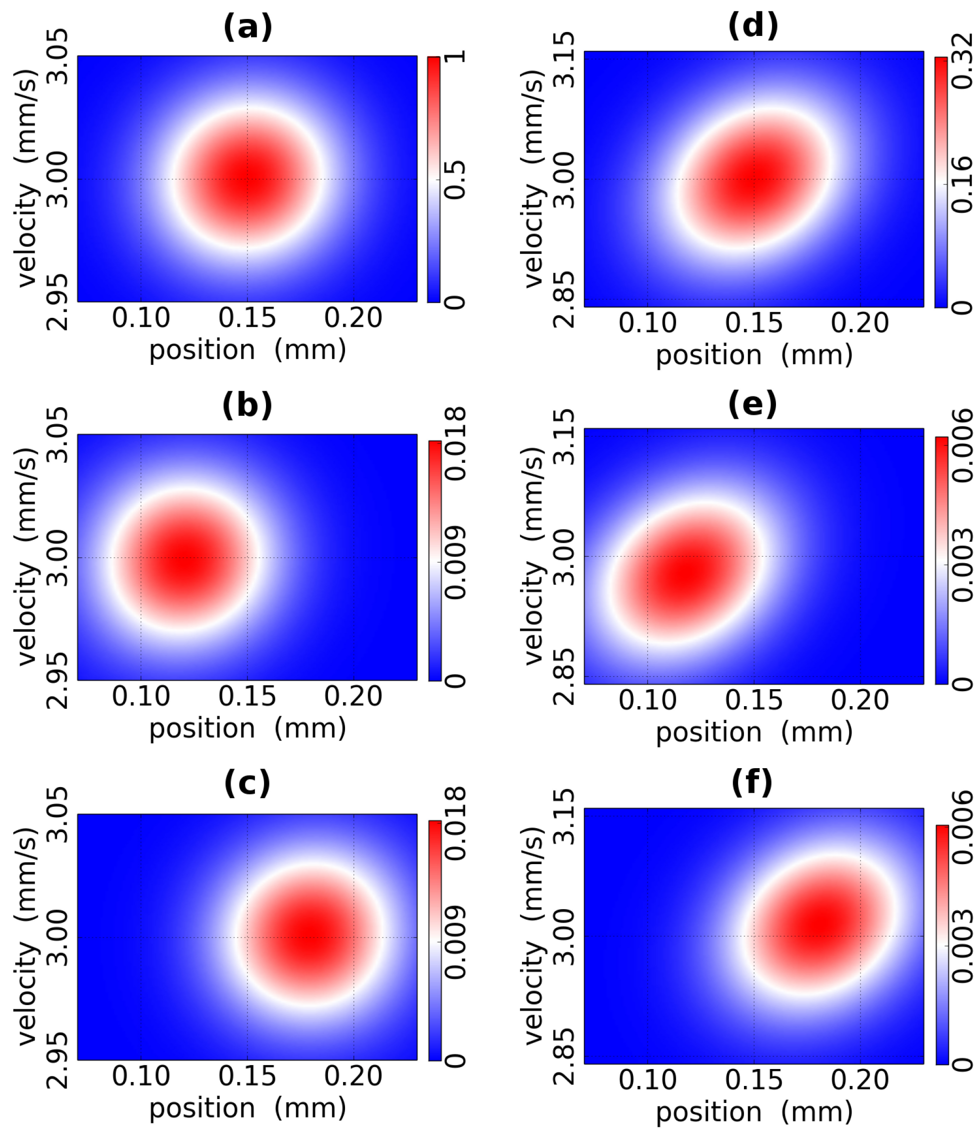
Exponentially changing aperture: Shifting

Parameters

$$m = 86.909\,\mathrm{u} \quad ({}^{87}\mathrm{Rb})$$
$$\sigma = 30\,\mu\mathrm{m}$$
$$v_0 = 3\,\mathrm{mm/s}$$
$$\Delta v = 0.1\,\mathrm{mm/s}$$
$$t = 100\,\mathrm{ms}$$

$$\chi(\tau) = \min\left\{e^{\gamma(\tau - t_1)},\, 1\right\}$$





$\gamma = 0$

$\gamma = 100\,\mathrm{s}^{-1}$

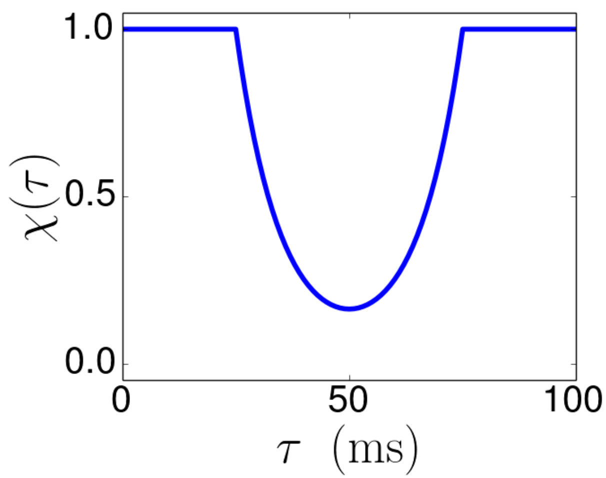
$\gamma = -100\,\mathrm{s}^{-1}$

Pure state

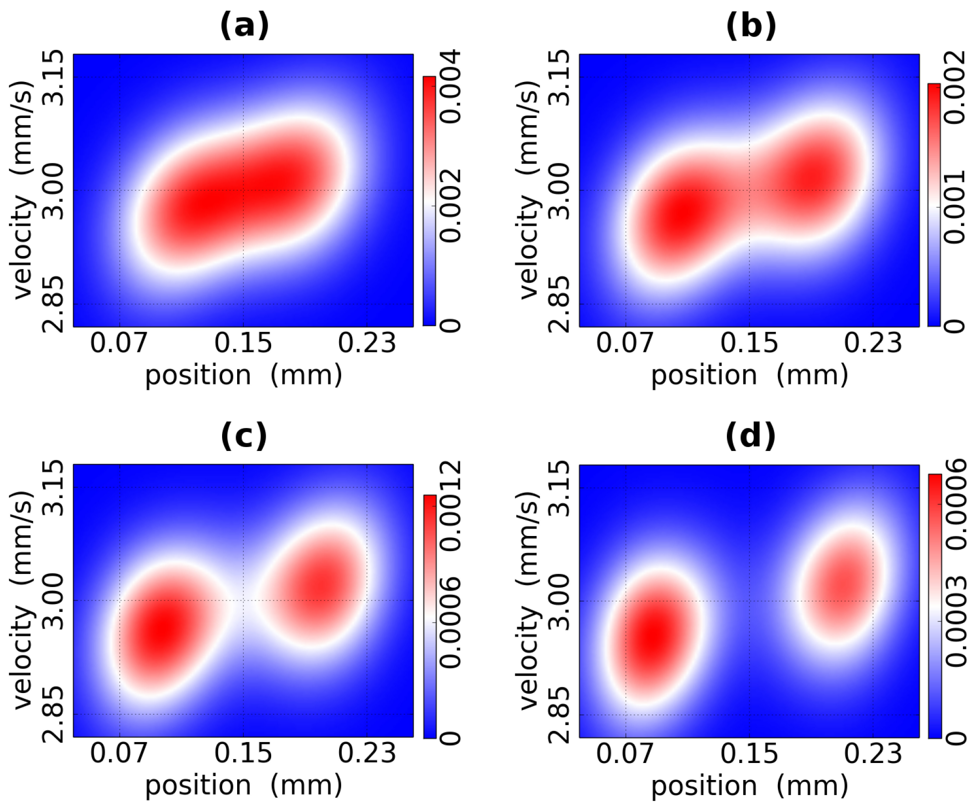
Mixed state

Exponentially changing aperture: Splitting

$$\chi(\tau) = \min\left\{ \frac{\cosh[\gamma(\tau - t_0)]}{\cosh(\gamma t_0/2)}\,,\ 1 \right\}$$



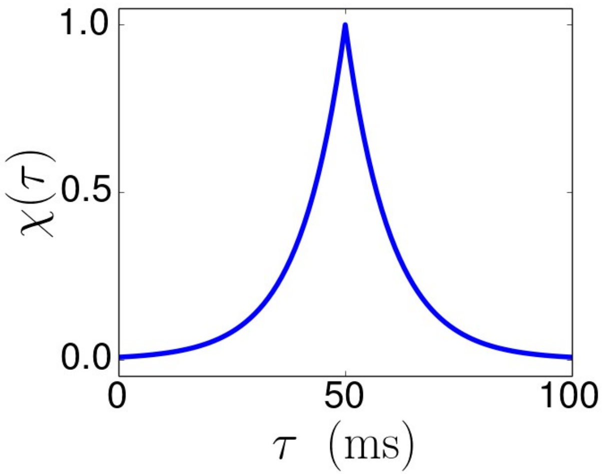
$\gamma = 125\,\mathrm{s}^{-1}$ $\gamma = 150\,\mathrm{s}^{-1}$

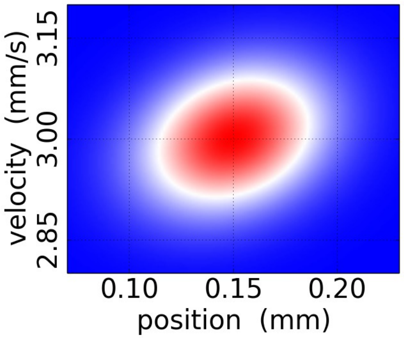


$\gamma = 175\,\mathrm{s}^{-1}$ $\gamma = 225\,\mathrm{s}^{-1}$

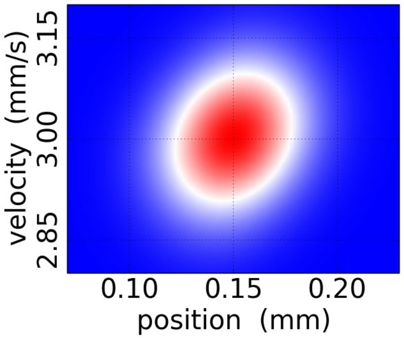
Exponentially changing aperture: Squeezing

$$\chi(\tau) = e^{-\gamma|\tau - t_0|}$$

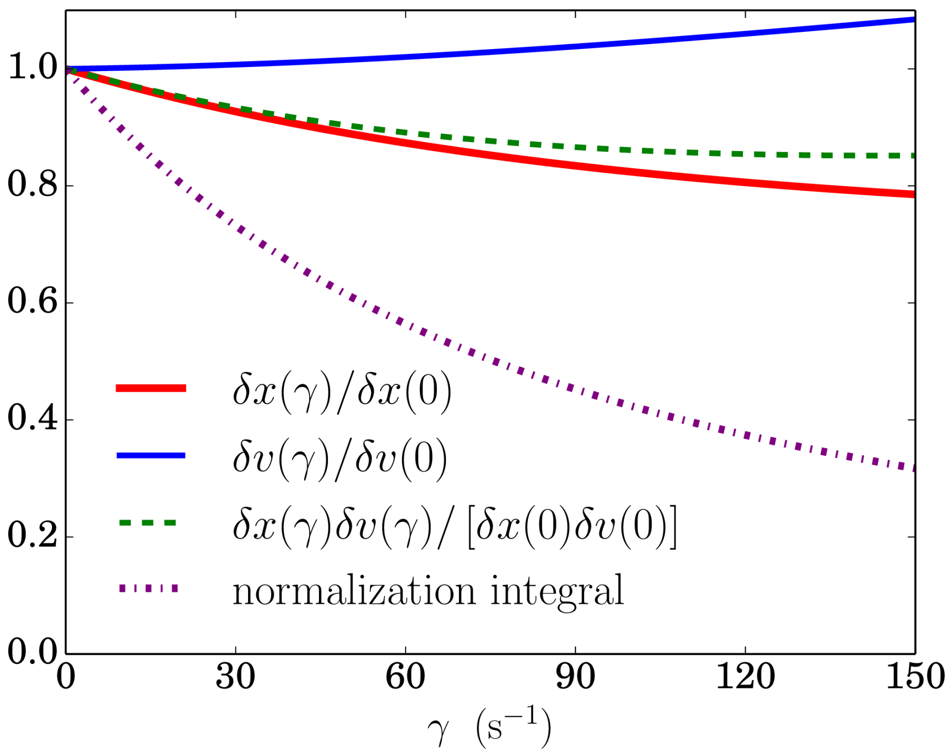




$$\delta x(m\delta v)|_{\gamma=0} \simeq 3.12\,\hbar$$



$$\delta x(m\delta v)|_{\gamma=150\,\mathrm{s}^{-1}} \simeq 2.66\,\hbar$$



Outlook

- ➡ Comparison with a realistic barrier
- ➡ Optimal matter pulse carving
- ➡ Extension to the case of two interacting particles
- ➡ Experiments welcome!

References

"Matter pulse carving: Manipulating quantum wave packets via time-dependent absorption", arXiv:1503.00031

"Diffraction in time: An exactly solvable model", *Phys. Rev. A* **87**, 053621 (2013)

"Huygens-Fresnel-Kirchhoff construction for quantum propagators with application to diffraction in space and time", *Phys. Rev. A* **85**, 013626 (2012)