Staircase-structure in tunneling splitting curve

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Quantum chaos: fundamentals and applications
Tuesday 17, March 2015 18h30 - 18h45

Outline

1. Tunneling splitting in 1-dimensional integrable systems
2. Tunneling splitting in non-integrable systems
3. Characterization of tunneling splitting in nearly integrable systems
4. Conclusion

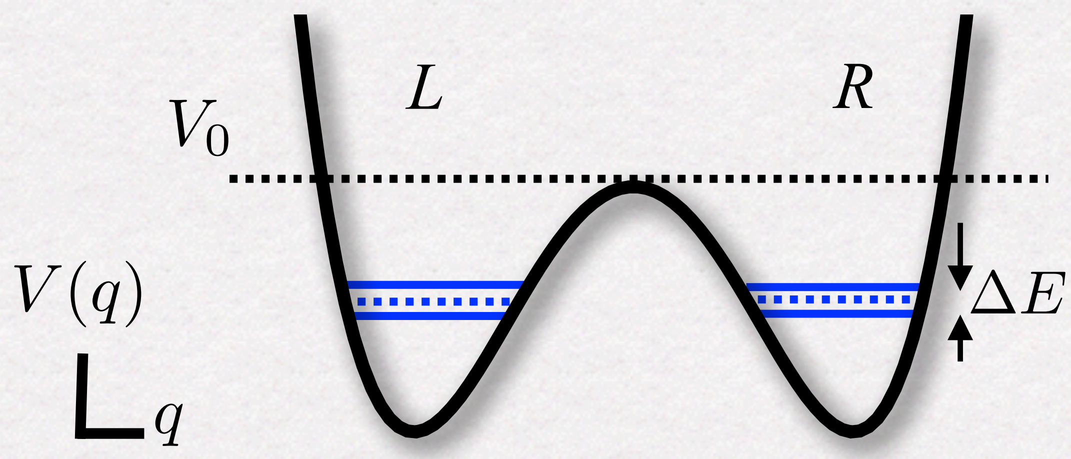
Aim

The aim of this talk is to propose a new analysis to characterize tunneling splitting in non-integrable systems. This is achieved by decomposition of the eigenfunction into a good integrable bases.

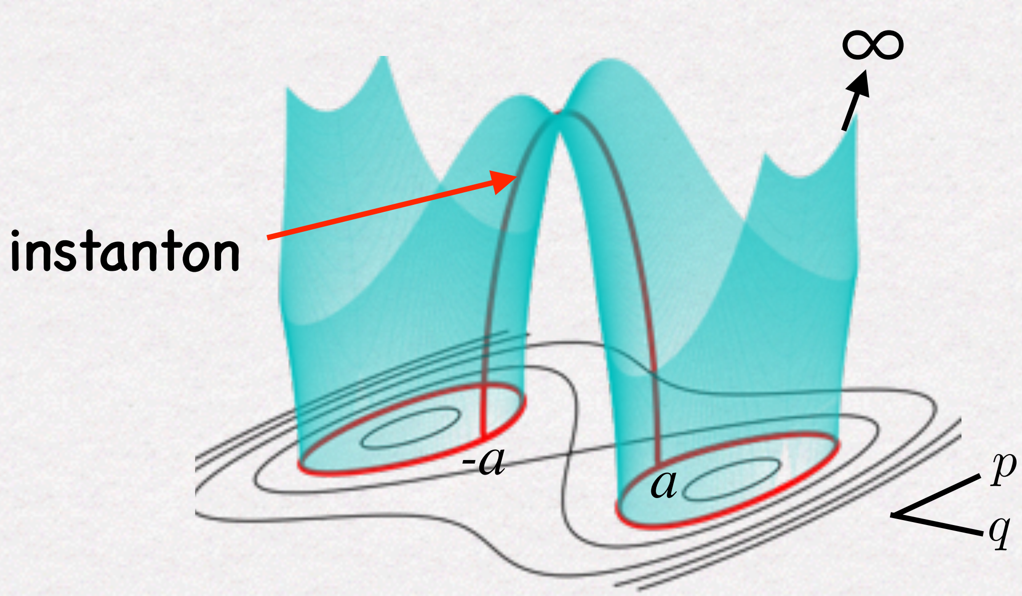
1. Tunneling splitting in 1-dimensional integrable systems

Tunneling in energy domain

e.g. 1-dim. double well potential



phase space



A period of the tunneling oscillation between symmetric wells

$$T = \frac{h}{\Delta E}$$

Semiclassical evaluation

$$\Delta E \underset{\hbar \to 0}{\sim} \alpha \hbar e^{-S/\hbar}$$

The exponent is given by instanton action

$$S = \int_{-a}^{a} \sqrt{2(V - E(q))}\, dq$$

1. Tunneling splitting in 1-dimensional integrable systems

Tunneling in energy domain

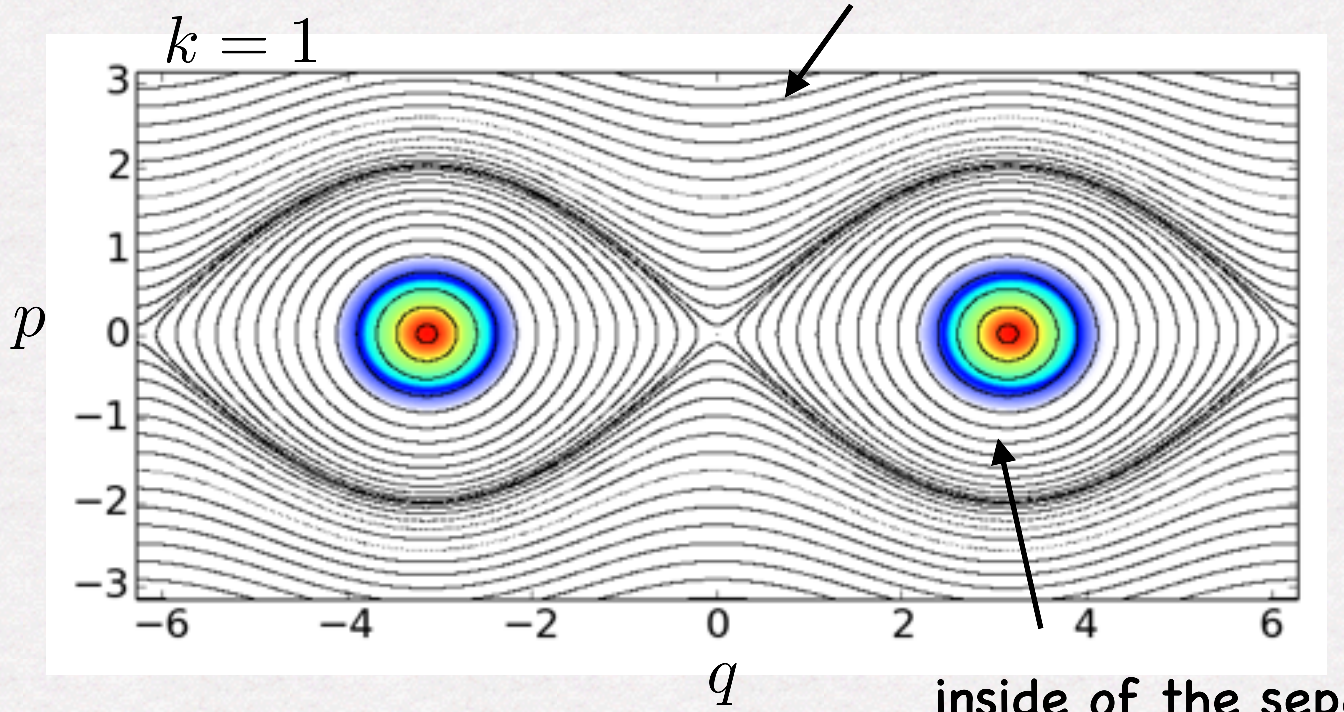
1-dim. Pendulum Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2} + k\cos\hat{q}, \qquad \hat{H}|J_n^{\pm}\rangle = E_n^{\pm}|J_n^{\pm}\rangle$$

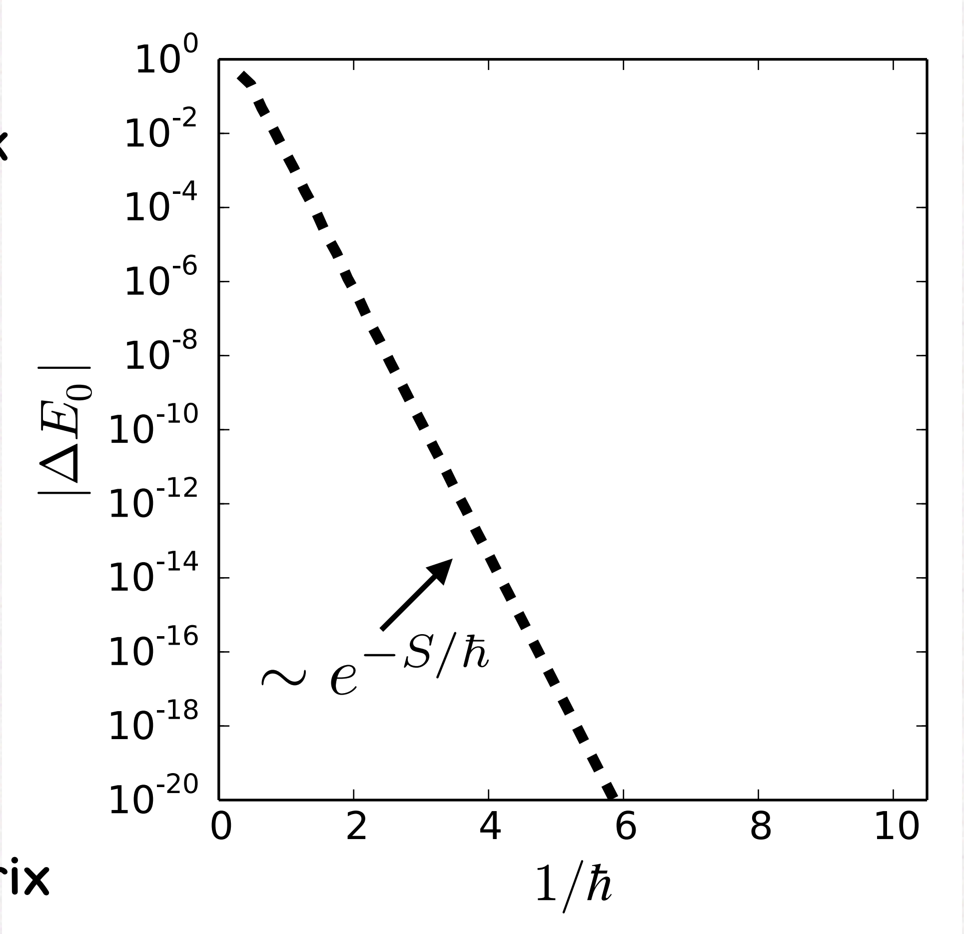
tunneling splitting

$$\Delta E_0 = E_0^{-} - E_0^{+}$$

outside of the separatrix



inside of the separatrix



2. Tunneling splitting in non-integrable systems

O. Bohigas, S. Tomsovic, D. Ullmo, *Phys Rep.* **223** (1993) 43
S. Tomsovic, D. Ullmo, *Phys. Rev. E* **50** (1994) 145
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O. Brodier, P. Schlagheck, D. Ullmo *Ann. Phys.* **300** (2002) 88
S. Löck, A. Bäcker, R. Ketzmerick, P. Schlagheck, PRL. **104** (2010) 114101
etc

Tunneling in energy domain

Symmetrized Standard map

$$T(p) = \frac{p^2}{2}, \quad V(q) = k\cos q$$

$$\hat{U} = e^{-\frac{i\tau}{2\hbar}V(\hat{q})} e^{-\frac{i\tau}{\hbar}T(\hat{p})} e^{-\frac{i\tau}{2\hbar}V(\hat{q})}, \qquad \hat{U}|\Psi_n^{\pm}\rangle = e^{-\frac{i}{\hbar}\tau E_n^{\pm}}|\Psi_n^{\pm}\rangle$$

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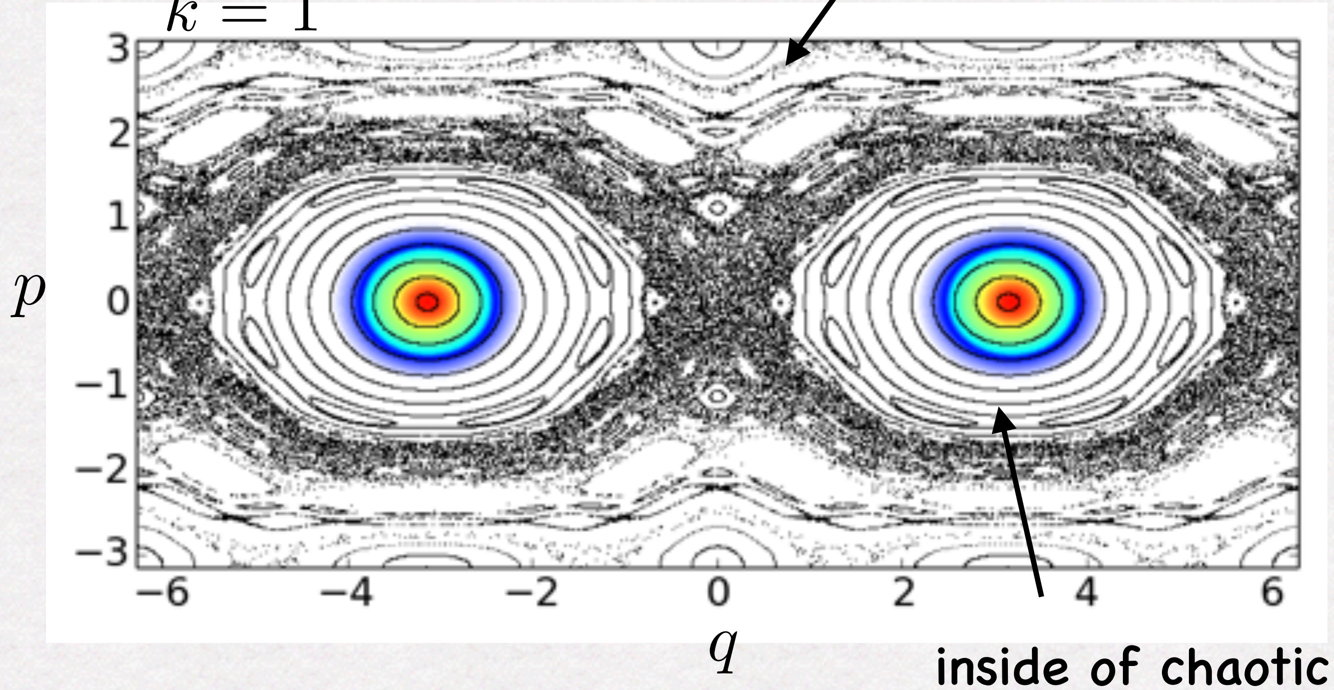
outside of chaotic sea

$k = 1$



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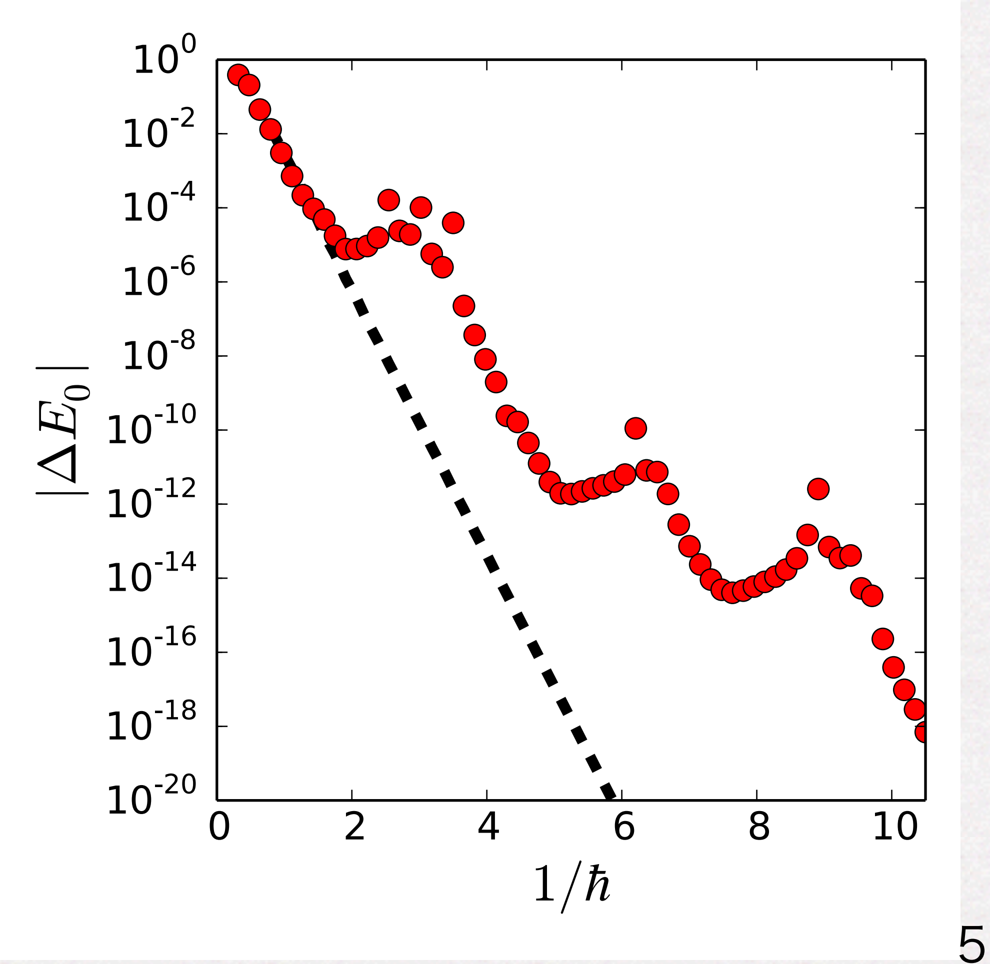

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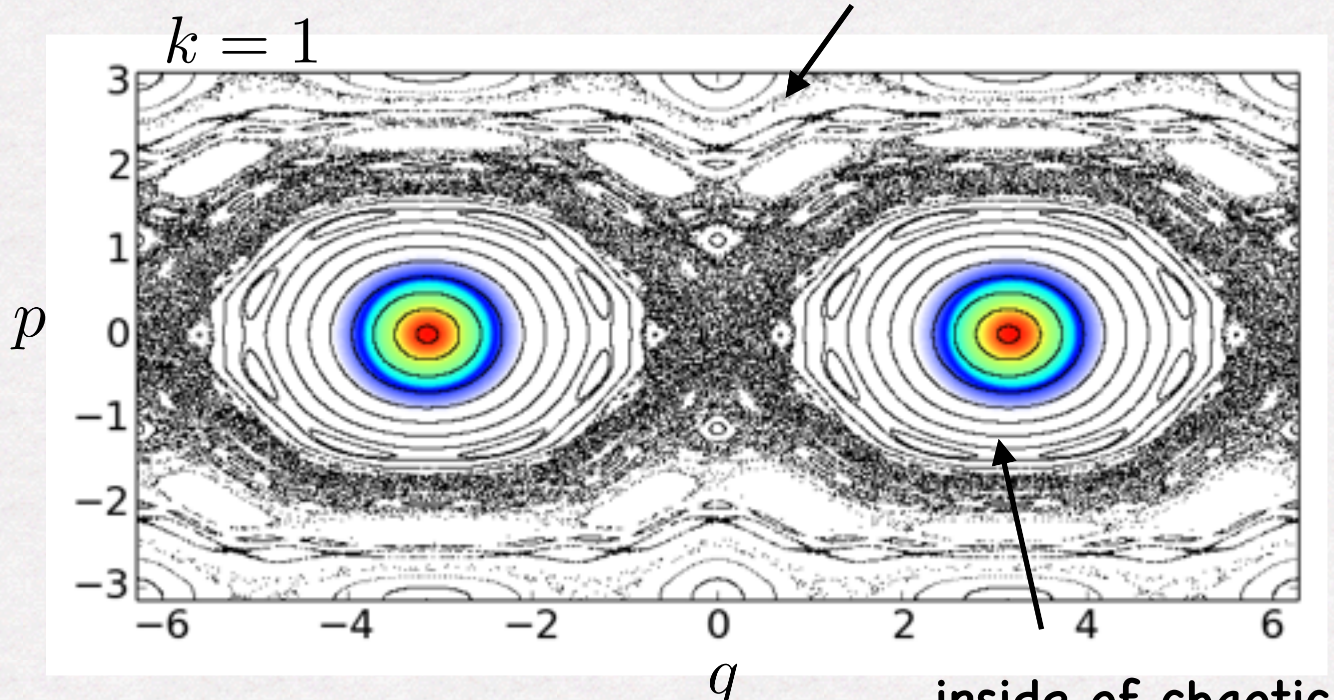
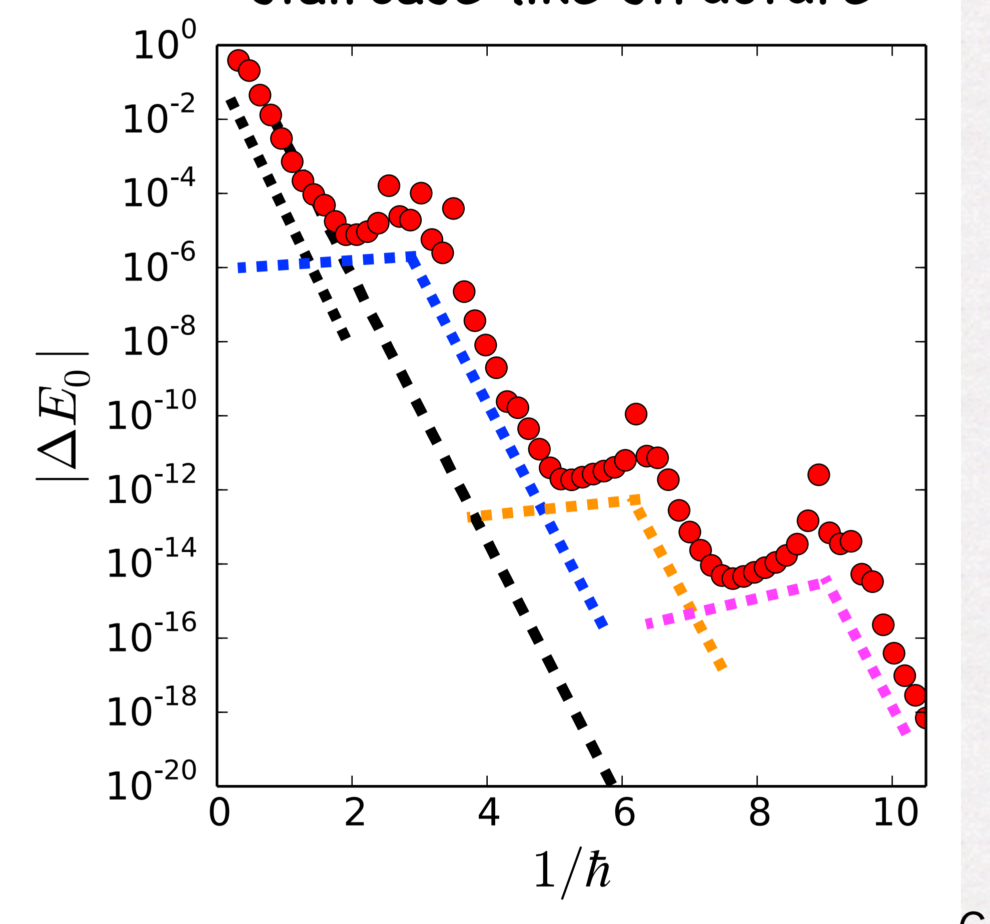


inside of chaotic sea

staircase-like structure



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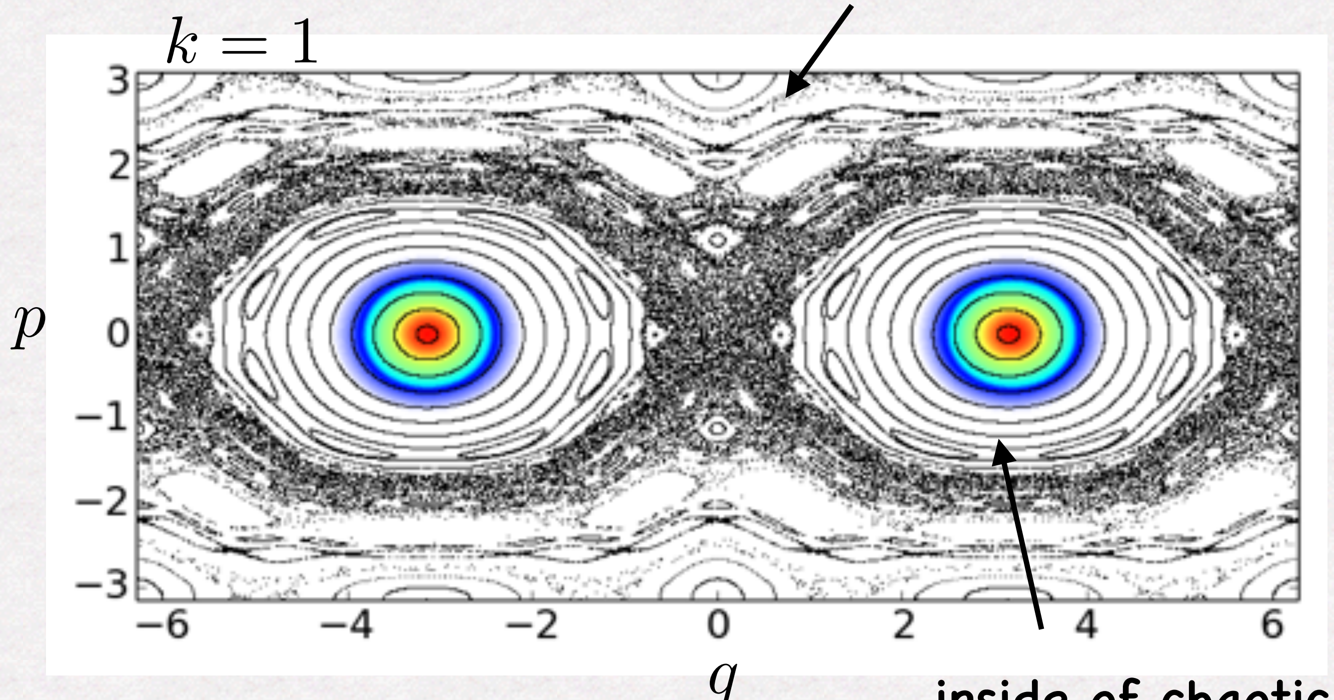

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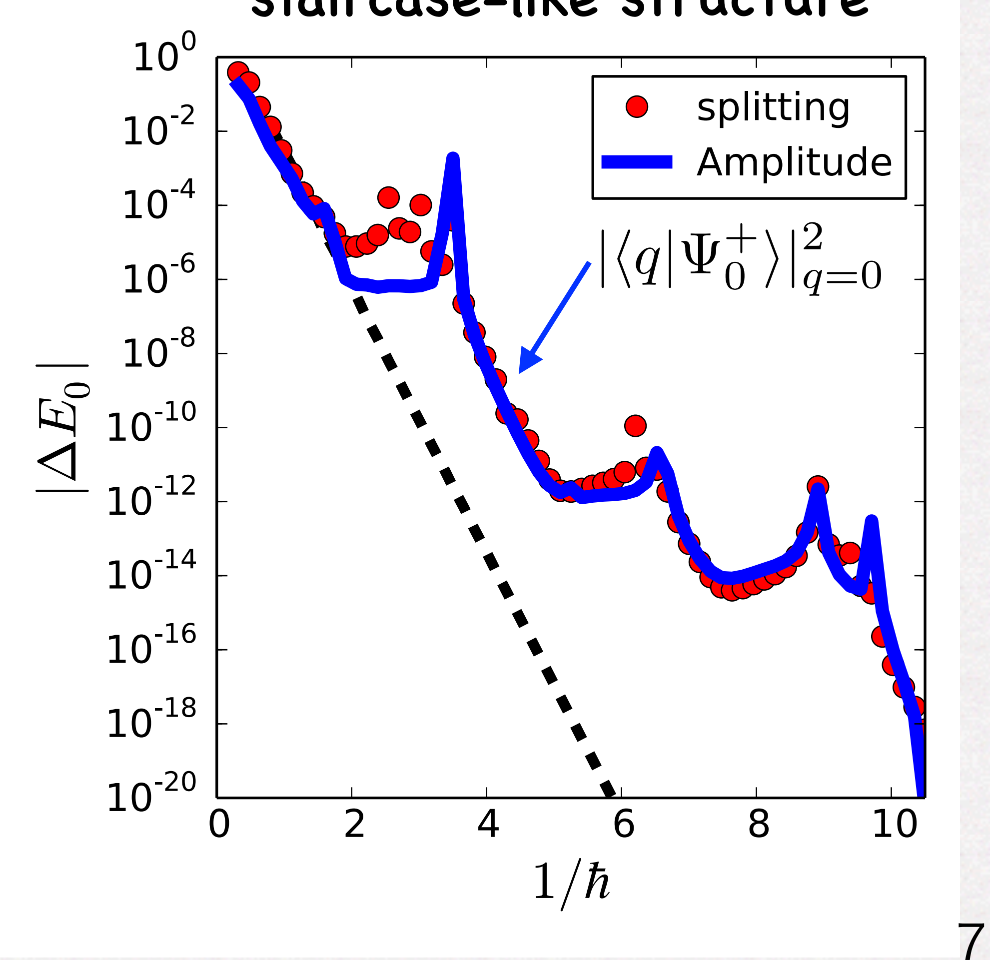

3. Characterization of tunneling splitting in nearly integrable systems

A. Shudo, Y. Hanada, T. Okushima, K.S. Ikeda, Europhys. Lett. **108** (2014) 50004
Y. Hanada, A. Shudo, K.S. Ikeda, submitted to PRE

Contribution mode decomposition

$$\langle q|\Psi_n\rangle = \sum_{\ell} \mathrm{Con}_{n,\ell}^{(M)}(q)$$

Contribution spectrum

$$\mathrm{Con}_{n,\ell}^{(M)}(q) \equiv \langle q|J_\ell^{(M)}\rangle\langle J_\ell^{(M)}|\Psi_n\rangle$$

Integrable approximation of quantum map using Baker-Campbell-Hausdorff expansion

R. Scharf, *J. Phys. A* **21** (1988) 2007

$$\hat{U} = e^{-\frac{i\tau}{2\hbar}V(\hat{q})}e^{-\frac{i\tau}{\hbar}T(\hat{p})}e^{-\frac{i\tau}{2\hbar}V(\hat{q})}$$

$$\simeq e^{-\frac{i\tau}{\hbar}H_{\mathrm{eff}}^{(M)}} \equiv \hat{U}_{\mathrm{eff}}^{(M)}$$

Effective integrable Hamiltonian

$$\hat{H}_{\mathrm{eff}}^{(M)}(\hat{q},\hat{p}) = \sum_{j\in odd\ int.}^{M} \left(-\frac{i\tau}{\hbar}\right)^{j-1} \hat{H}_j(\hat{q},\hat{p})$$

$$\hat{H}_{\mathrm{eff}}^{(M)}|J_\ell^{(M)}\rangle = E_\ell^{(M)}|J_\ell^{(M)}\rangle$$

$$\hat{H}_1 = T + V$$

$$\hat{H}_3 = \frac{1}{12}([T,[T,V]] - 2[V,[V,T]])$$

$$\vdots$$

[· , ·] : commutator

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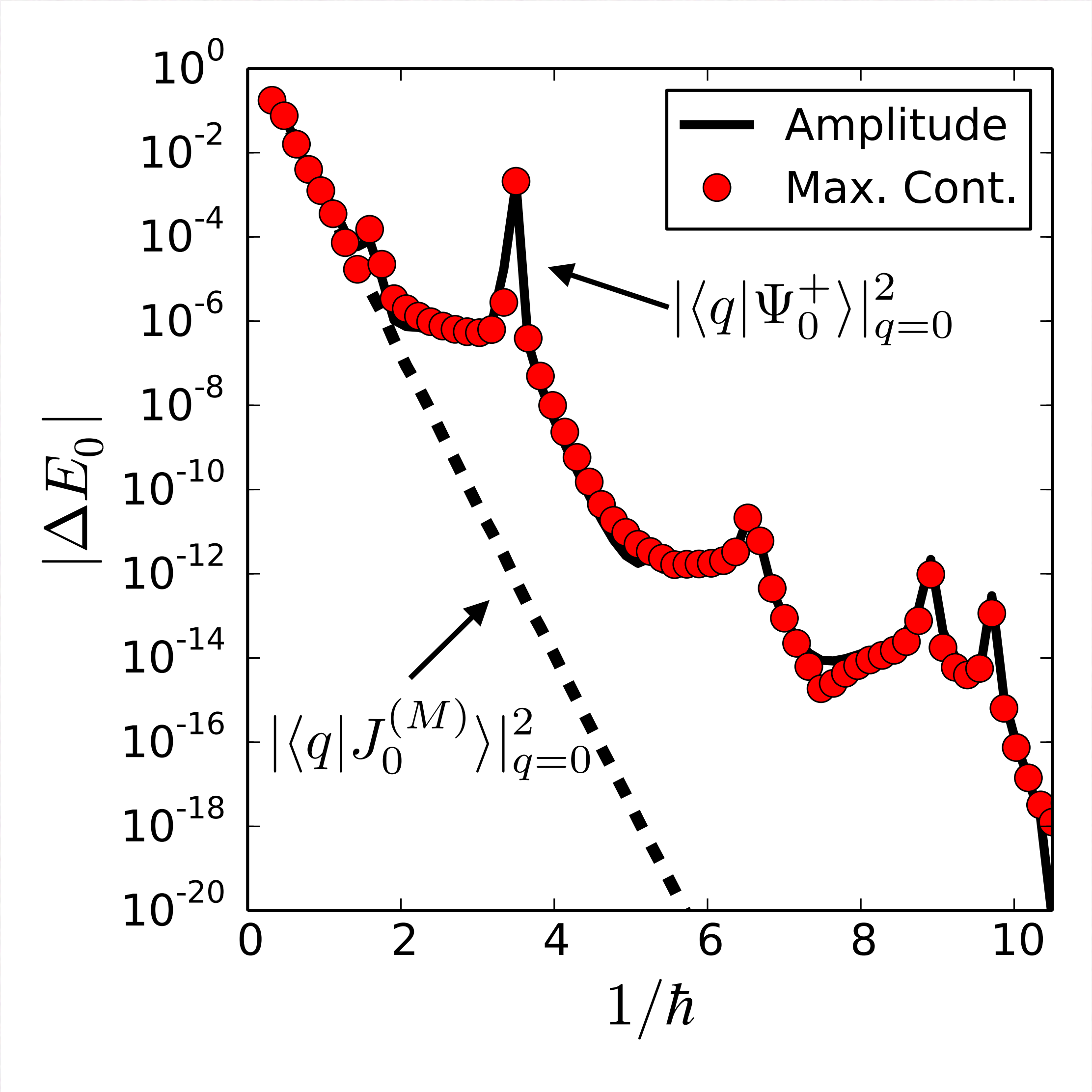

Maxmal mode of contribution spectrum

$$\max(\mathrm{Con}_{n,\ell}^{(M)}(q))$$

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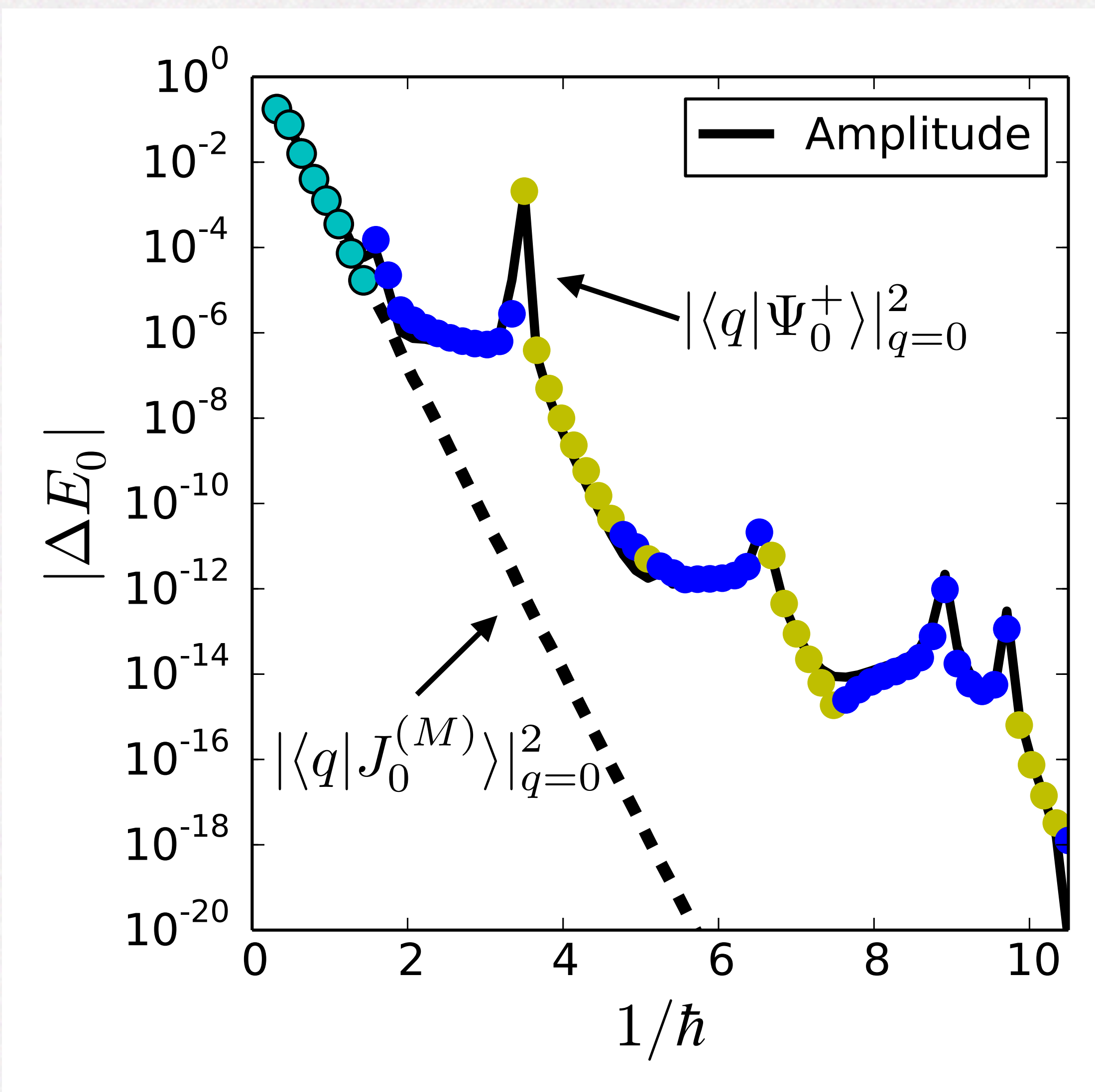

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whose support is located on

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of effective integrable Hamiltonian $H^{(M)}_{\mathrm{eff}}$

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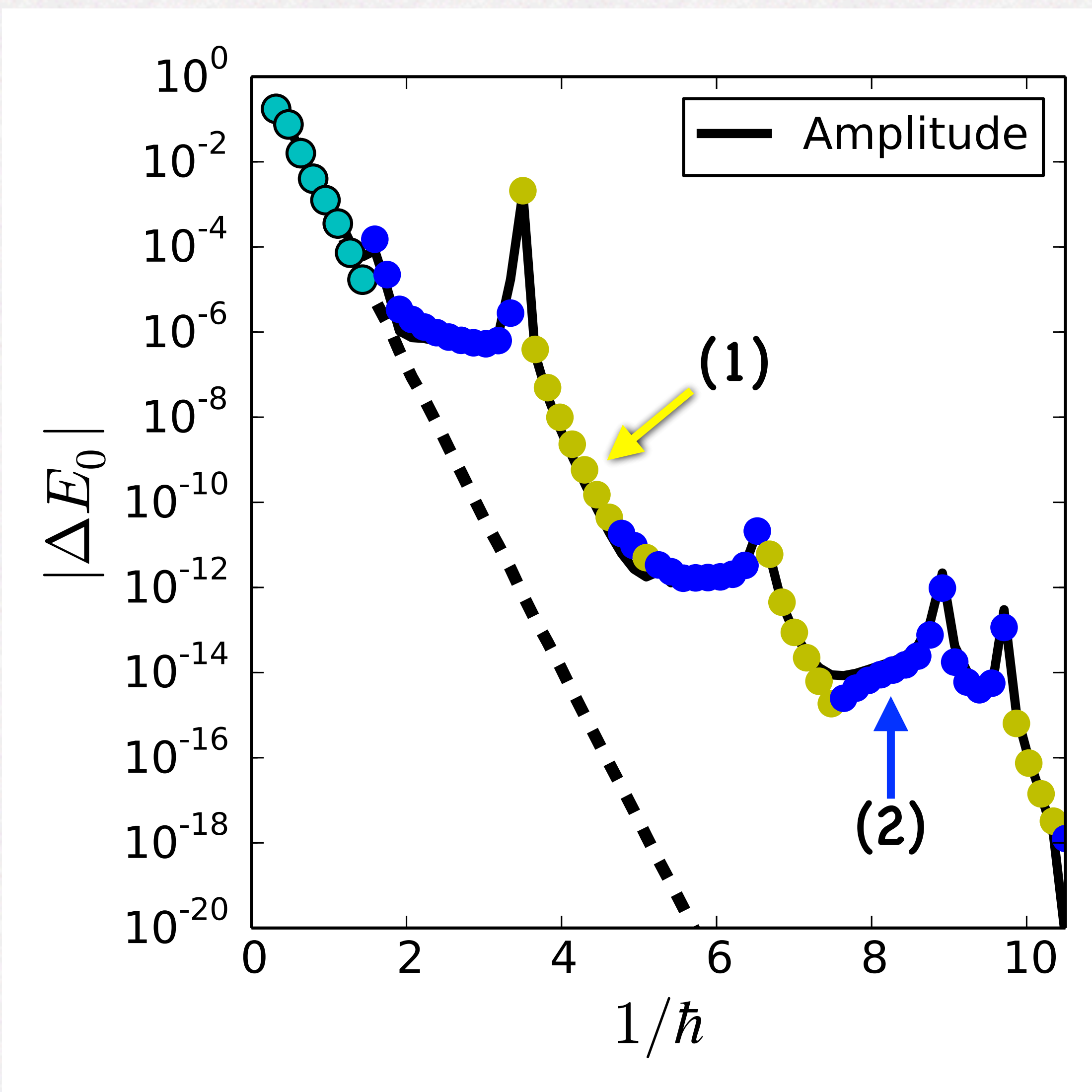

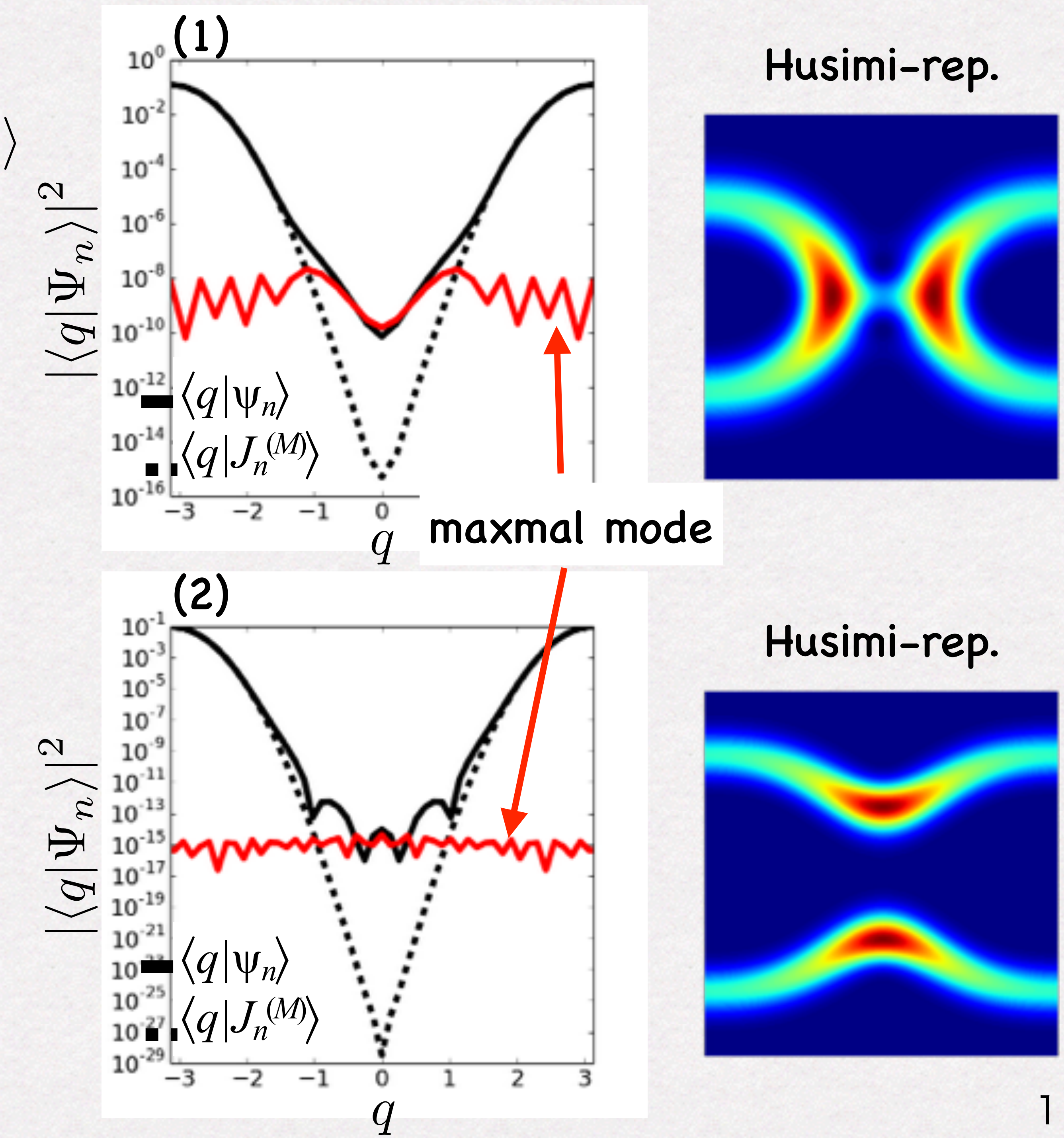

Conclusion

We have explored the origin of the staircase structure in the splitting curve. The maximal mode of the contribution spectrum has the capability of reproducing the exact amplitude.

The maximal mode analysis tells us that the staircase structure consists of the two regions:

- Coupling with outside of separatrix → plateau (slowly decaying)
- Coupling with inside of separatrix → steeply decaying

The successive switching of the position of the maximal mode generates the staircase structure.

A. Shudo, Y. Hanada, T. Okushima, K.S. Ikeda, Europhys. Lett. **108** (2014) 50004
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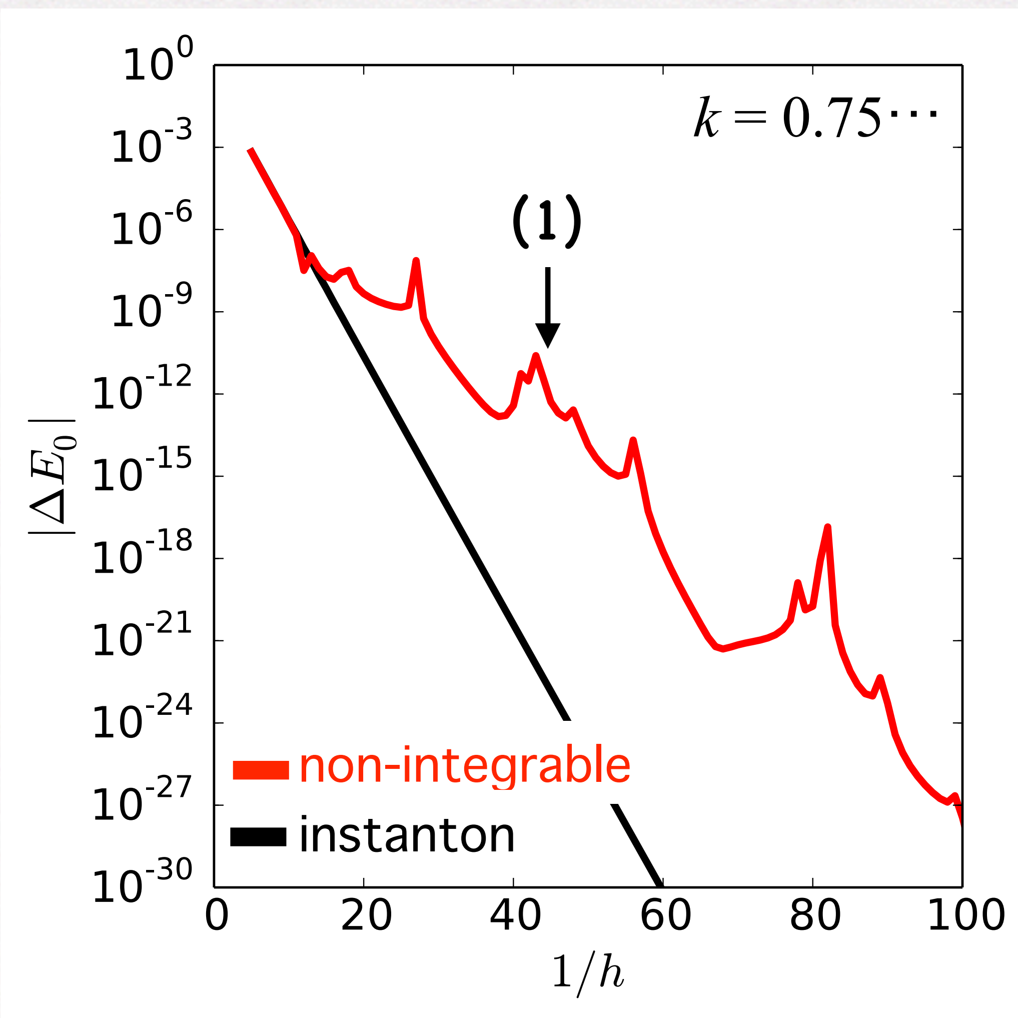
Supplemental Slide1:
Absorption of the energy level resonance

Absorbing operator

$$\hat{P} = \mathbb{1} - \frac{\Gamma}{2}\sum_{\ell \in L} |J_\ell\rangle\langle J_\ell|$$



spikes with the energy level resonance

3rd state

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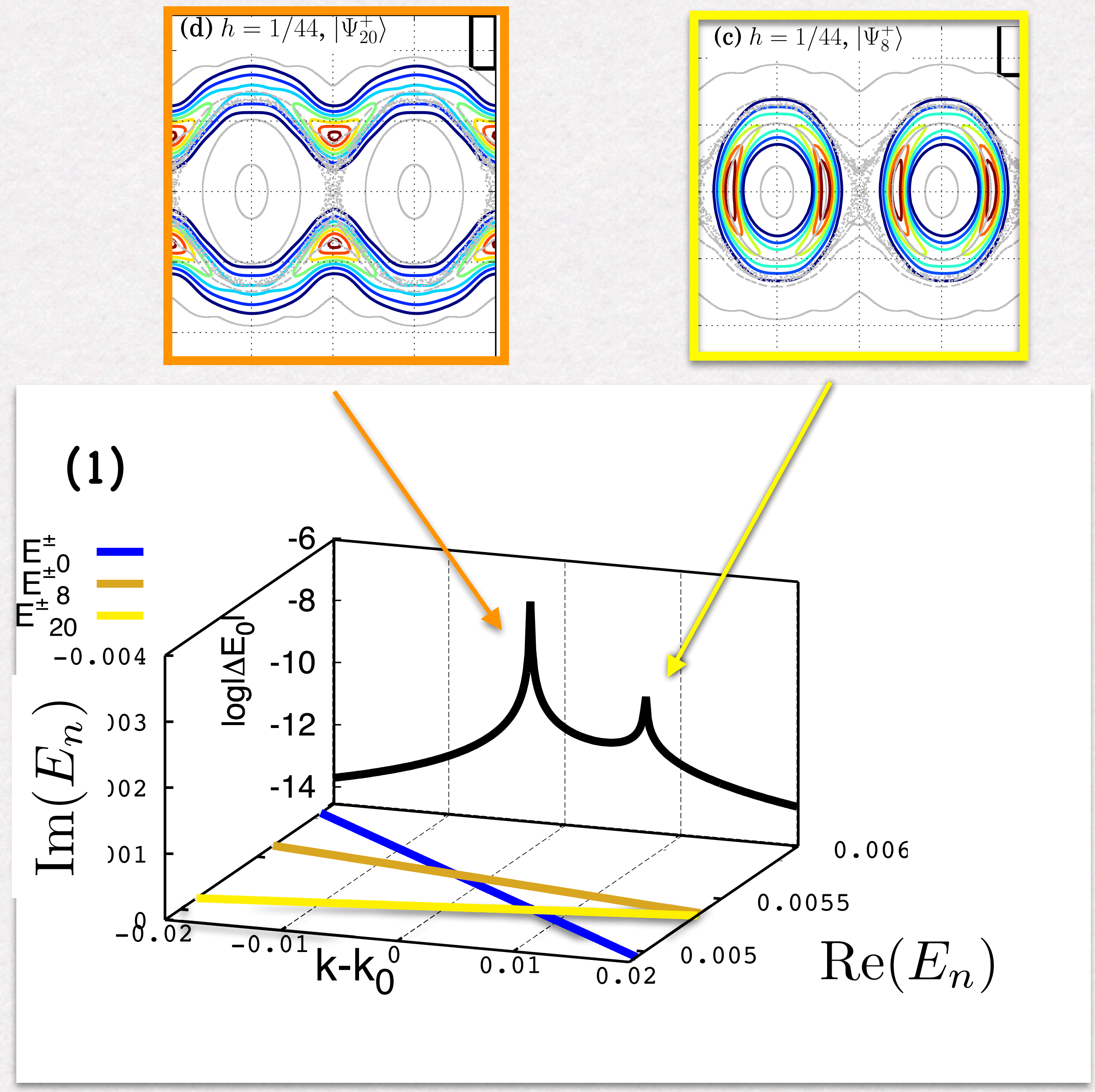

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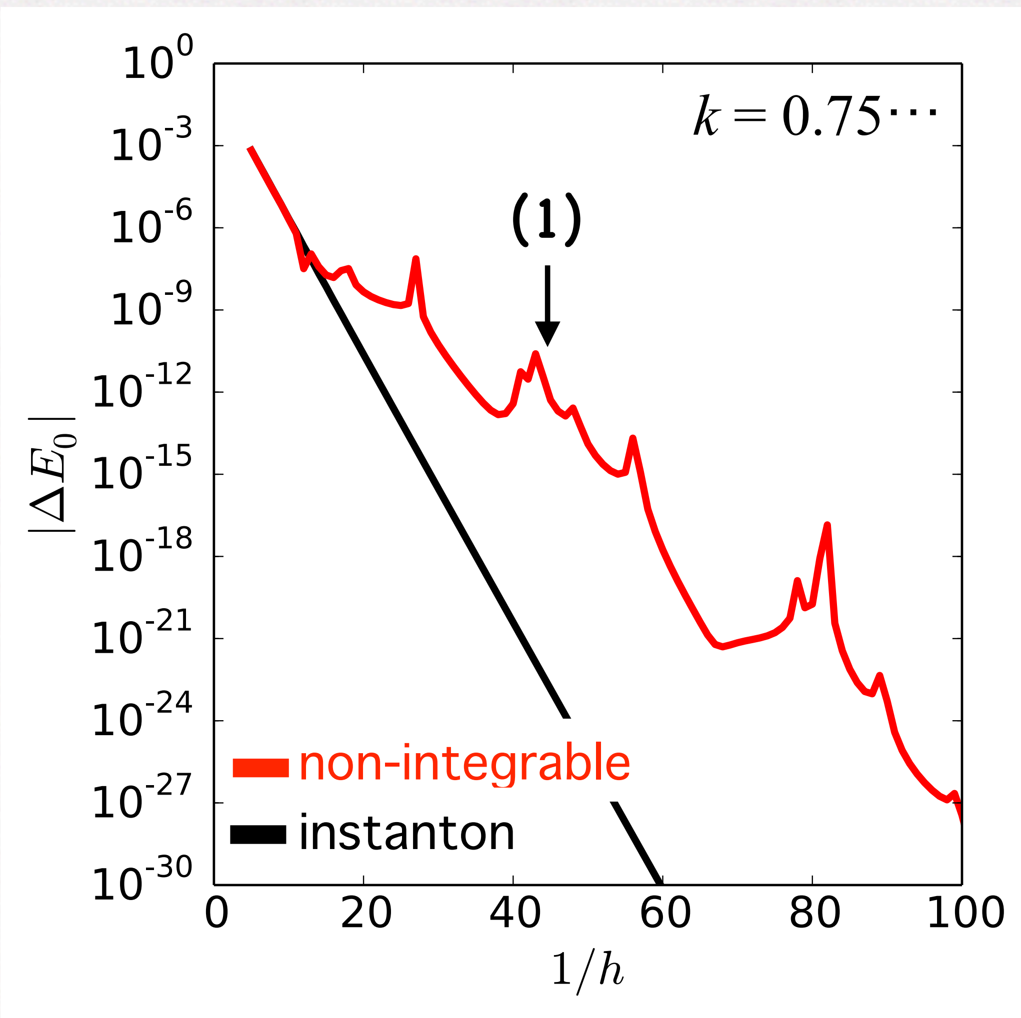

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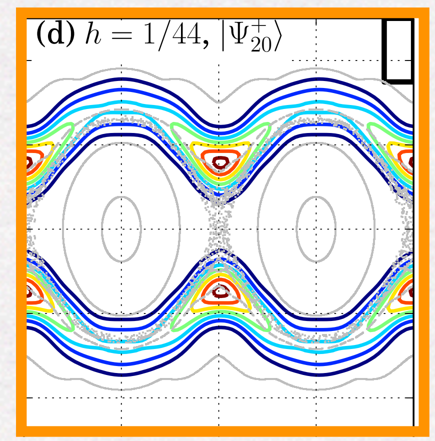

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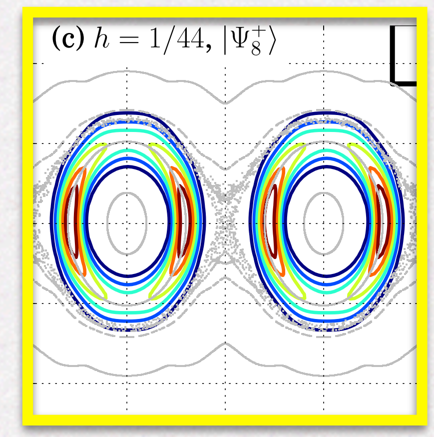

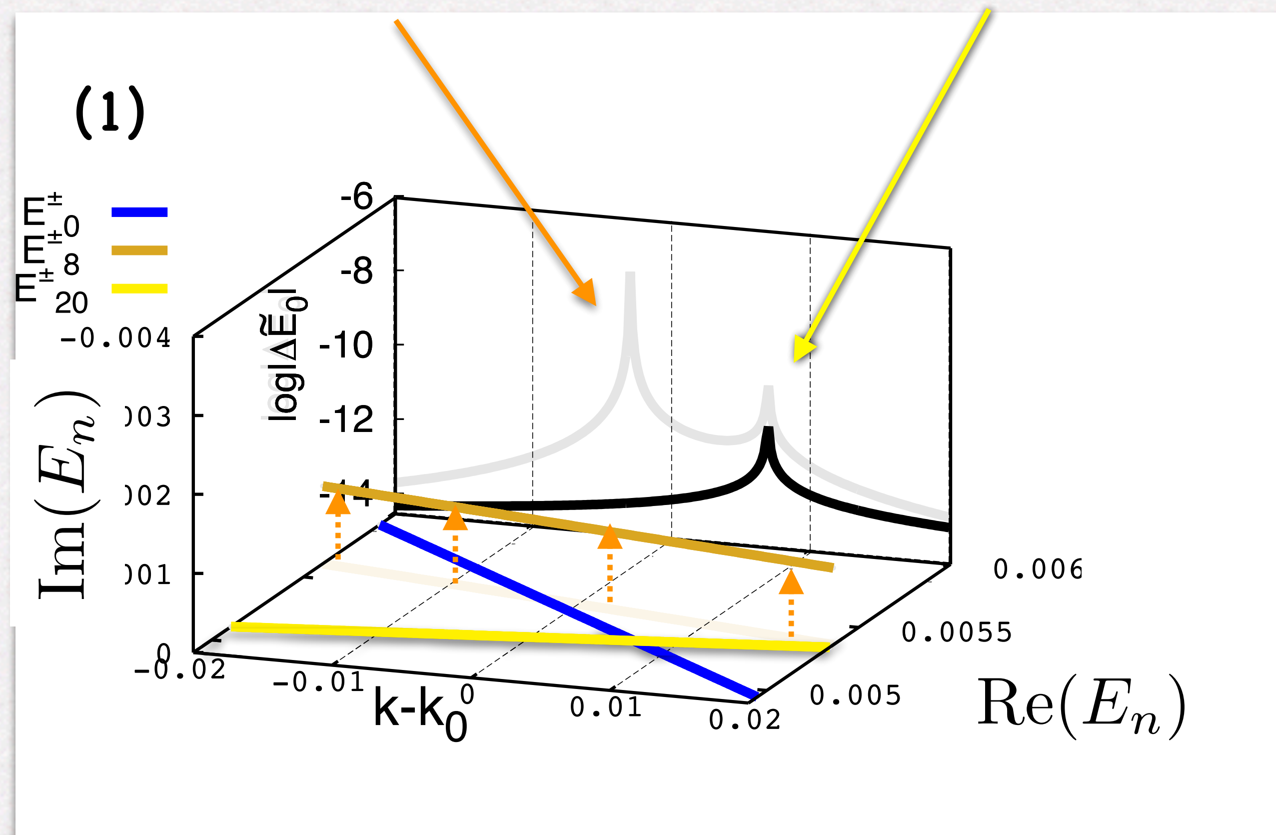

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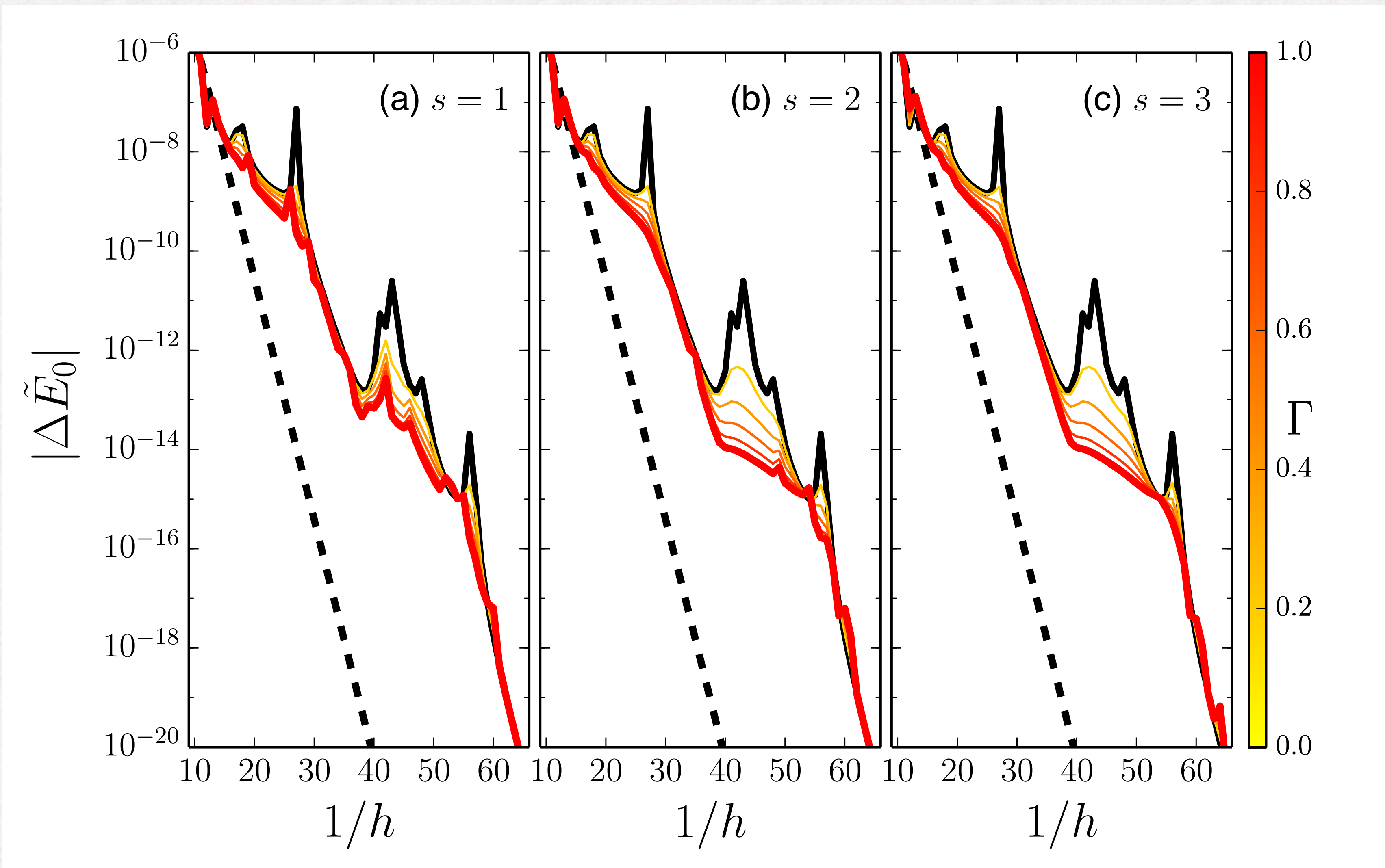


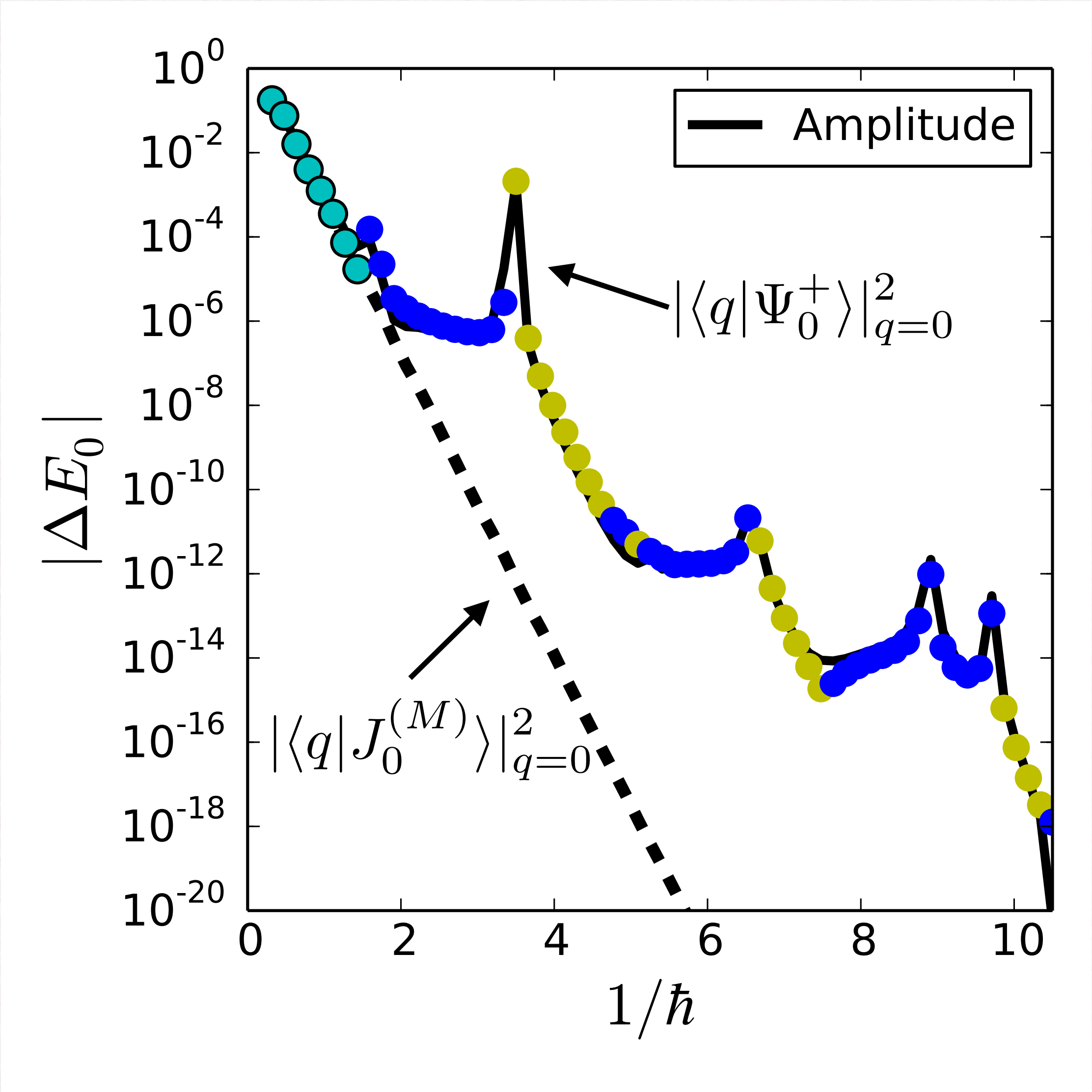
Supplemental Slide2: Origin of the staircase-structure in the splitting curve

Contribution mode decomposition

$$\langle q|\Psi_n\rangle = \sum_\ell \mathrm{Con}_{n,\ell}^{(M)}(q)$$

Contribution spectrum

$$\mathrm{Con}_{n,\ell}^{(M)}(q) \equiv \langle q|J_\ell^{(M)}\rangle\langle J_\ell^{(M)}|\Psi_n\rangle$$



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action representation of $|\Psi_n\rangle$
(projection onto $|J_\ell^{(M)}\rangle$)

Amplitude of $|J_\ell^{(M)}\rangle$ at $q=0$

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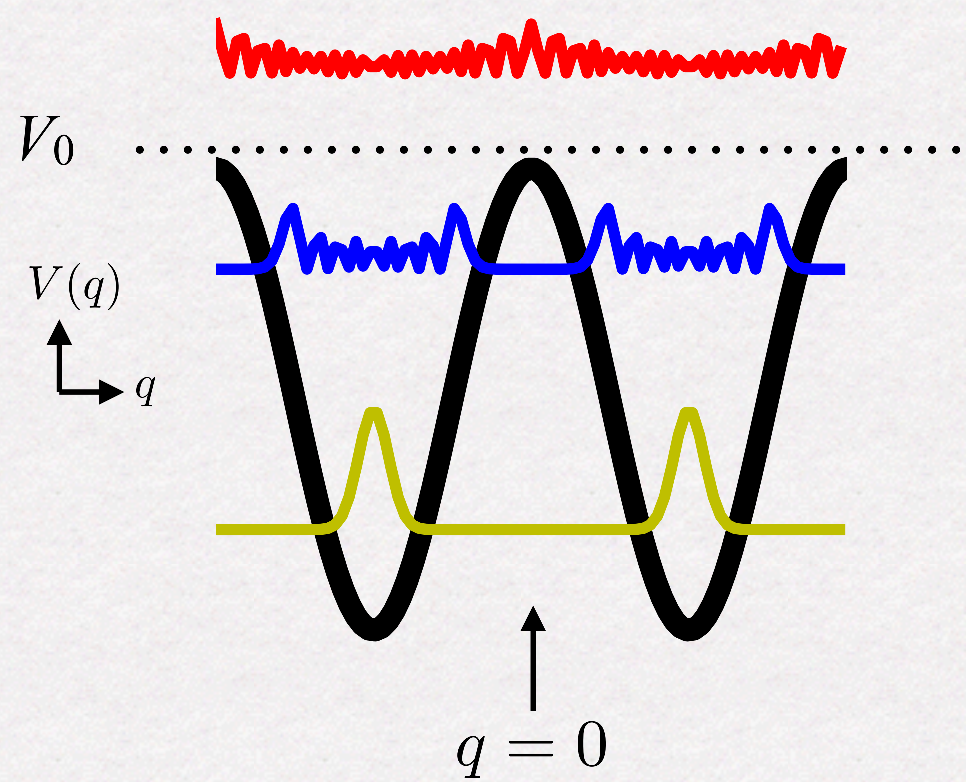
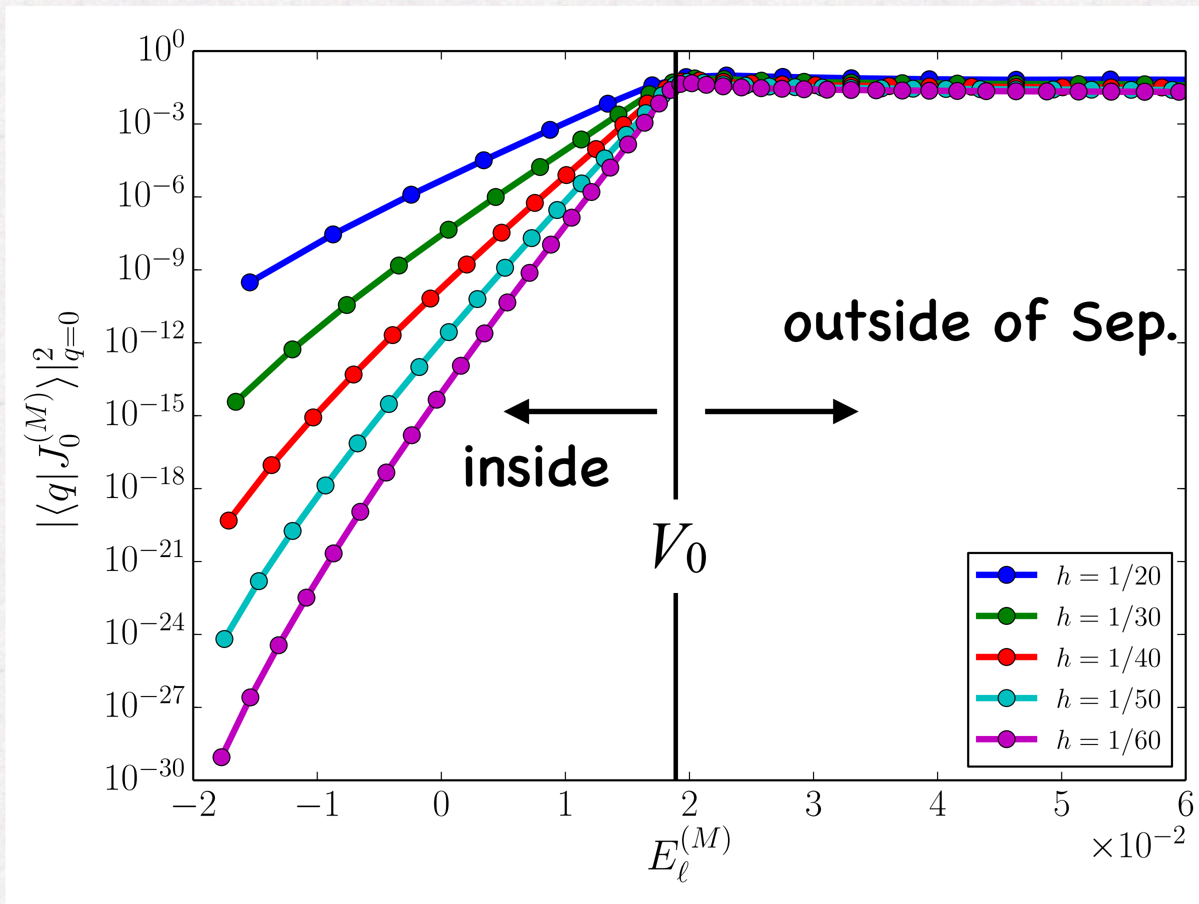
effective (BCH) integrable Hamiltonian

$$\hat{H}_{\mathrm{eff}}^{(M)}|J_\ell^{(M)}\rangle = E_\ell^{(M)}|J_\ell^{(M)}\rangle$$





Amplitude of $\langle \mathrm{q}|J_l^{(M)}\rangle$ behaves in a trivially way

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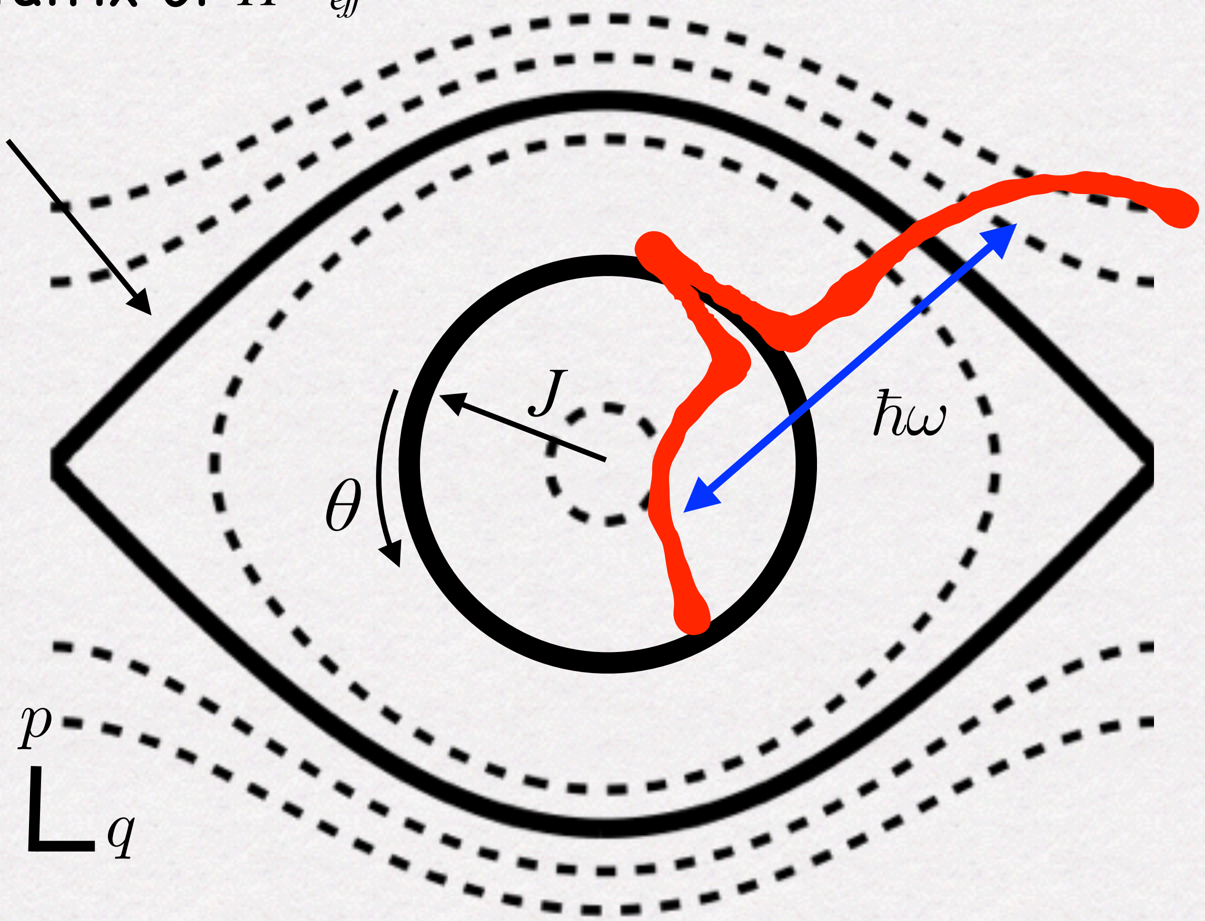
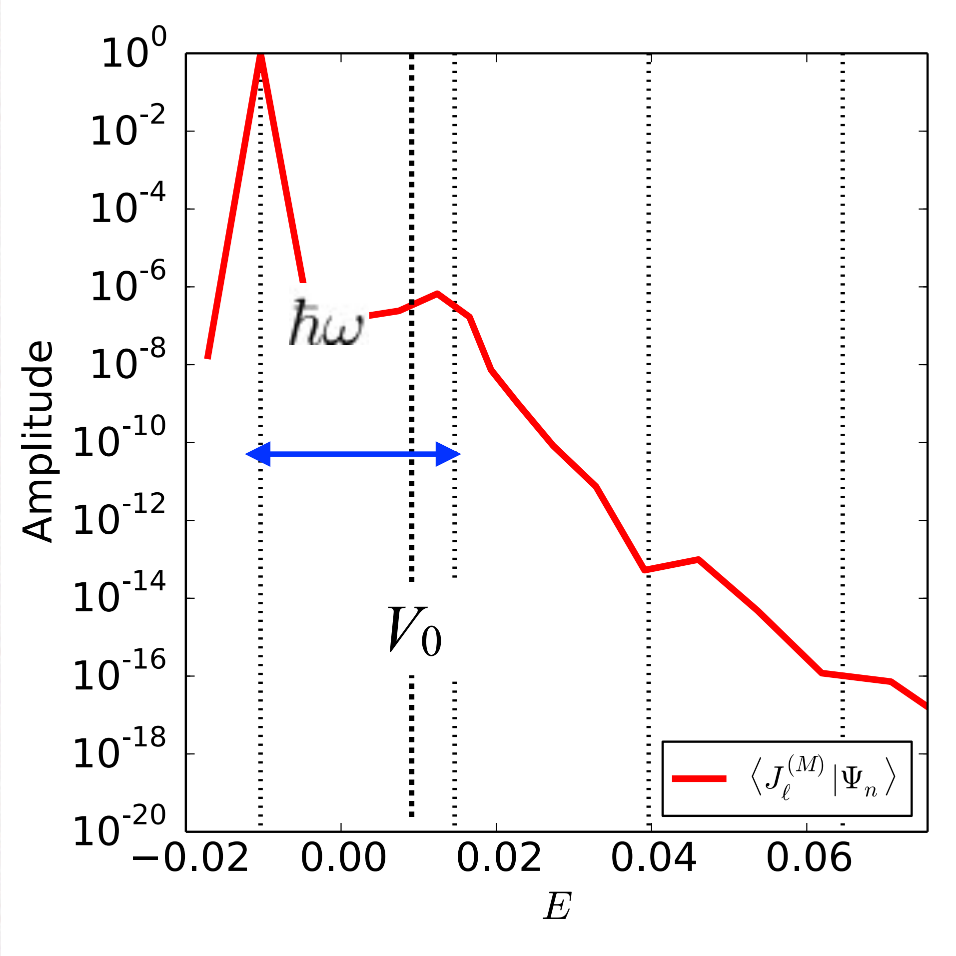
ω is frequency of external field (kicking).

separatrix of $H^{(M)}_{eff}$





Eigenstate $|\Psi_n\rangle$ expands beyond the separatrix

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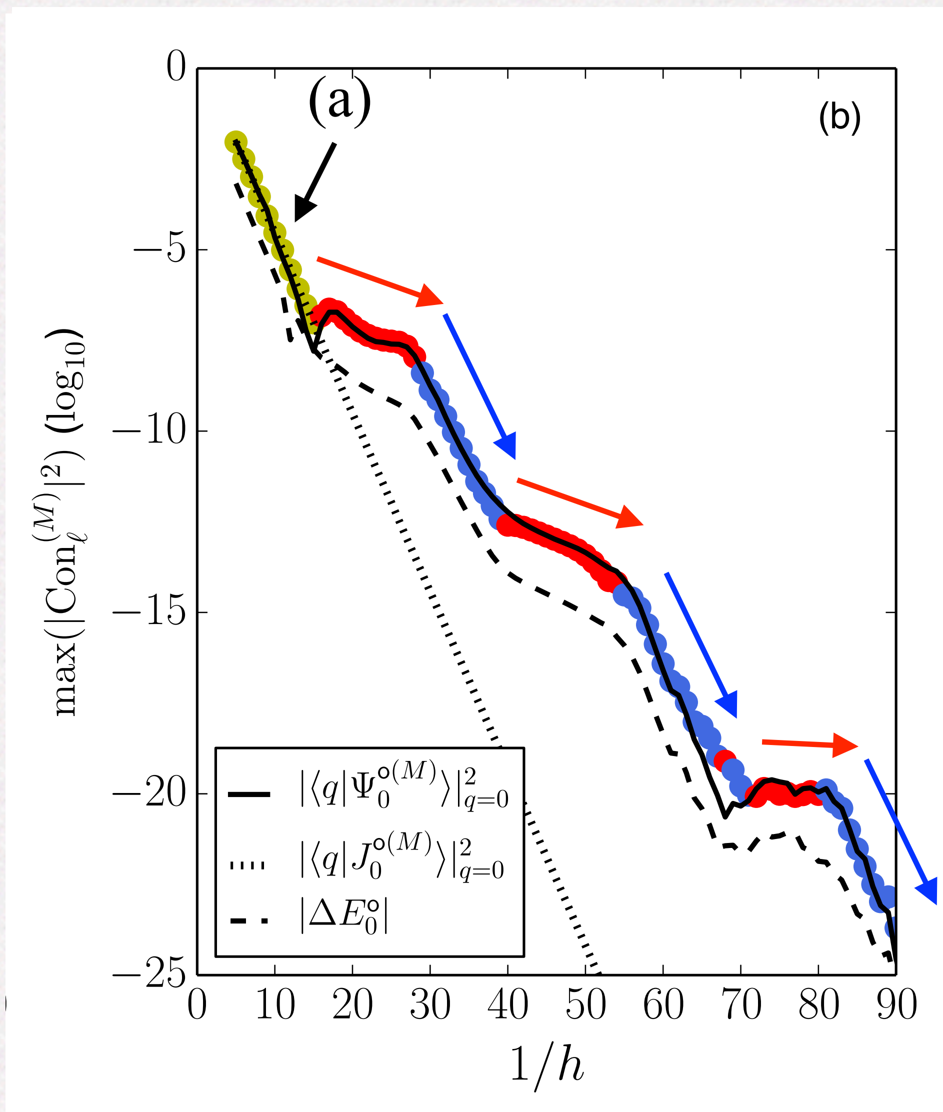

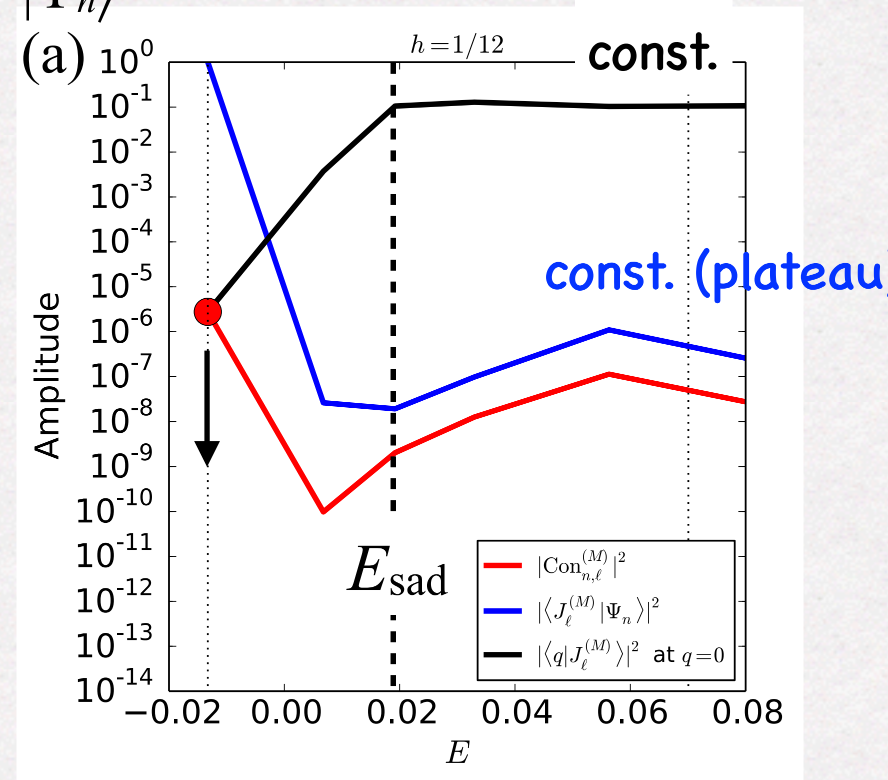

instanton mode exponentially decays
with increasing the value of 1/hbar

modes located on the outside of separatrix
keep almost constant values

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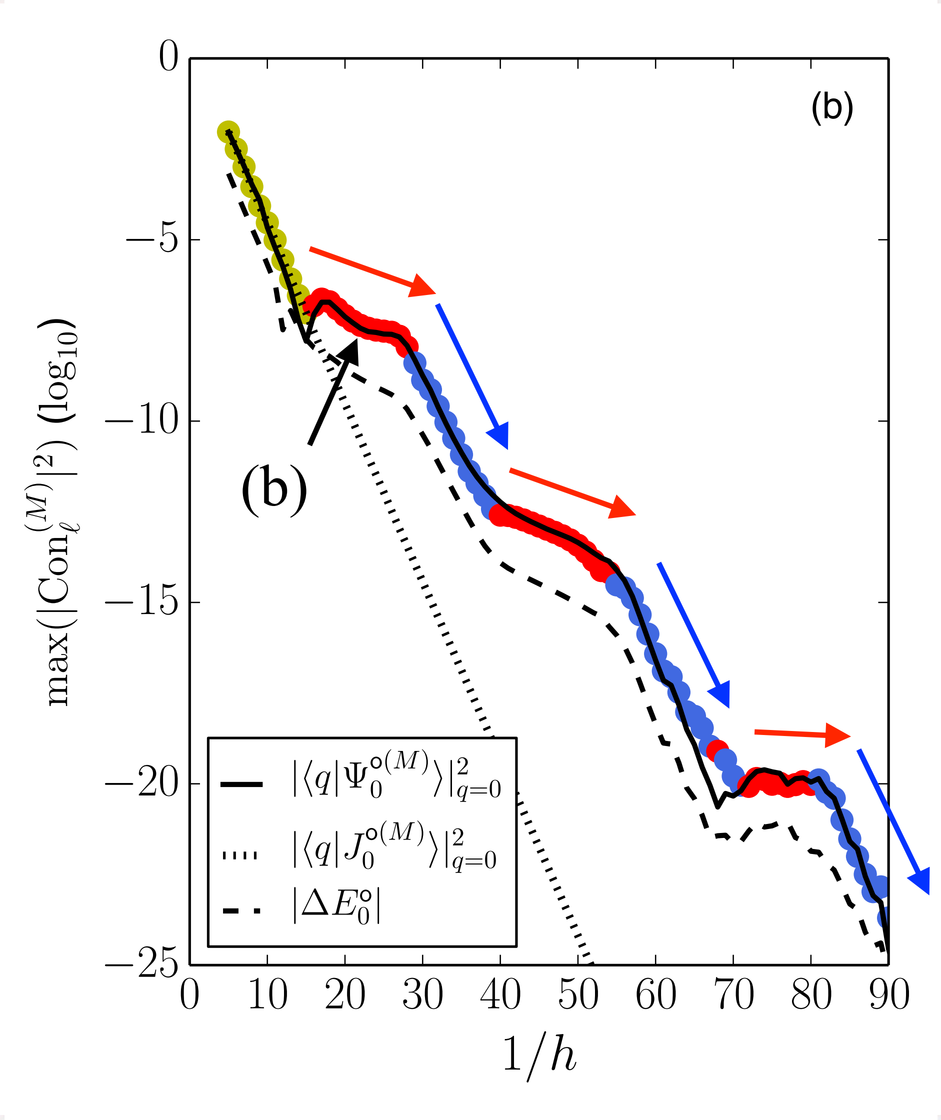

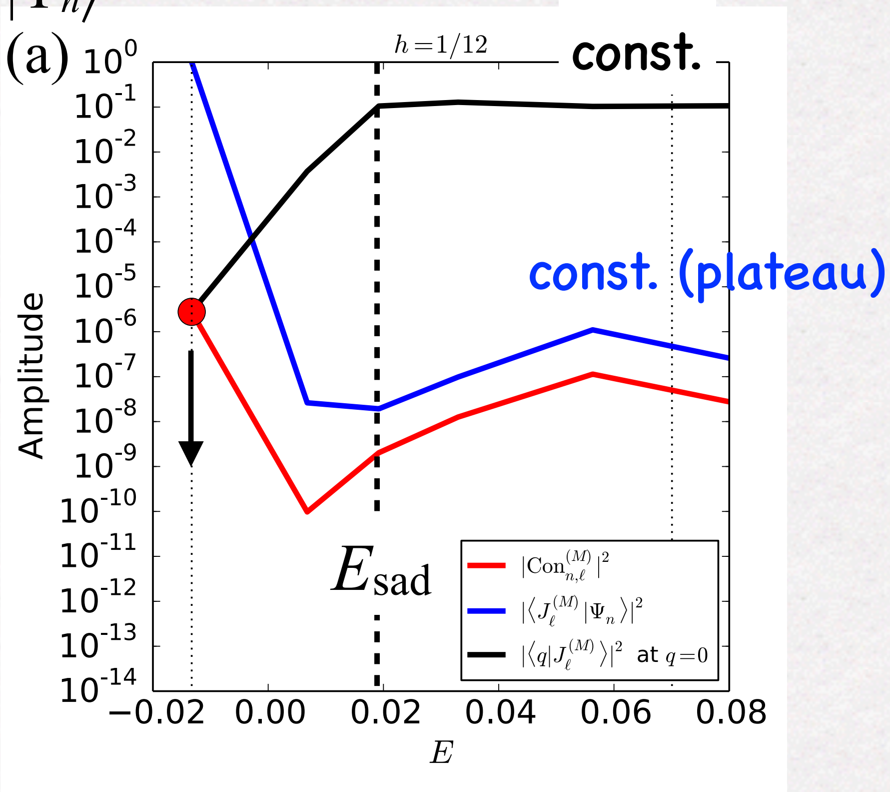

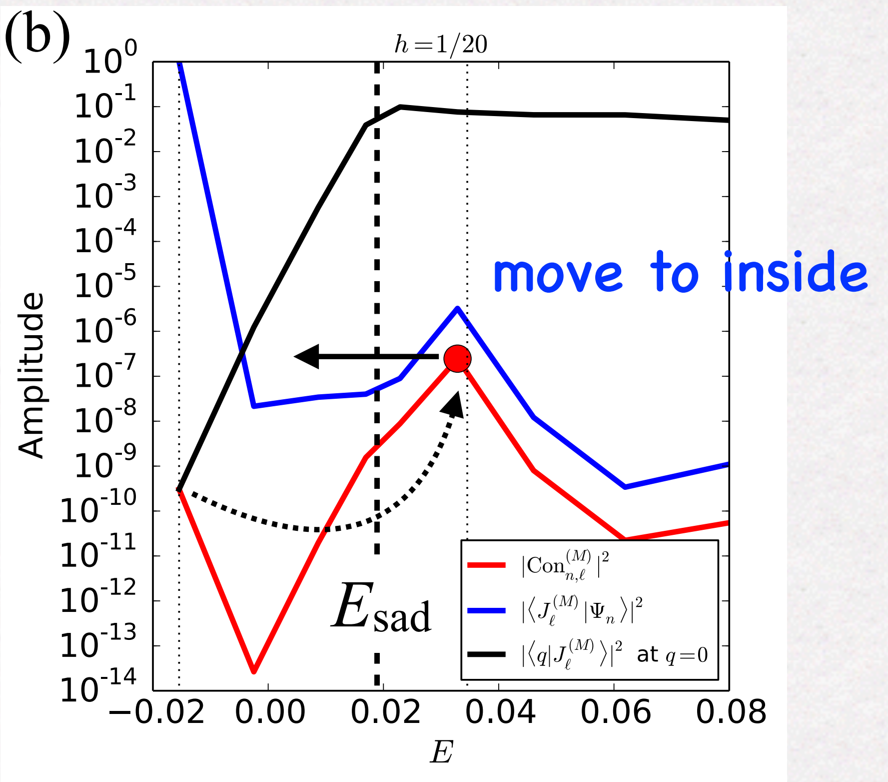

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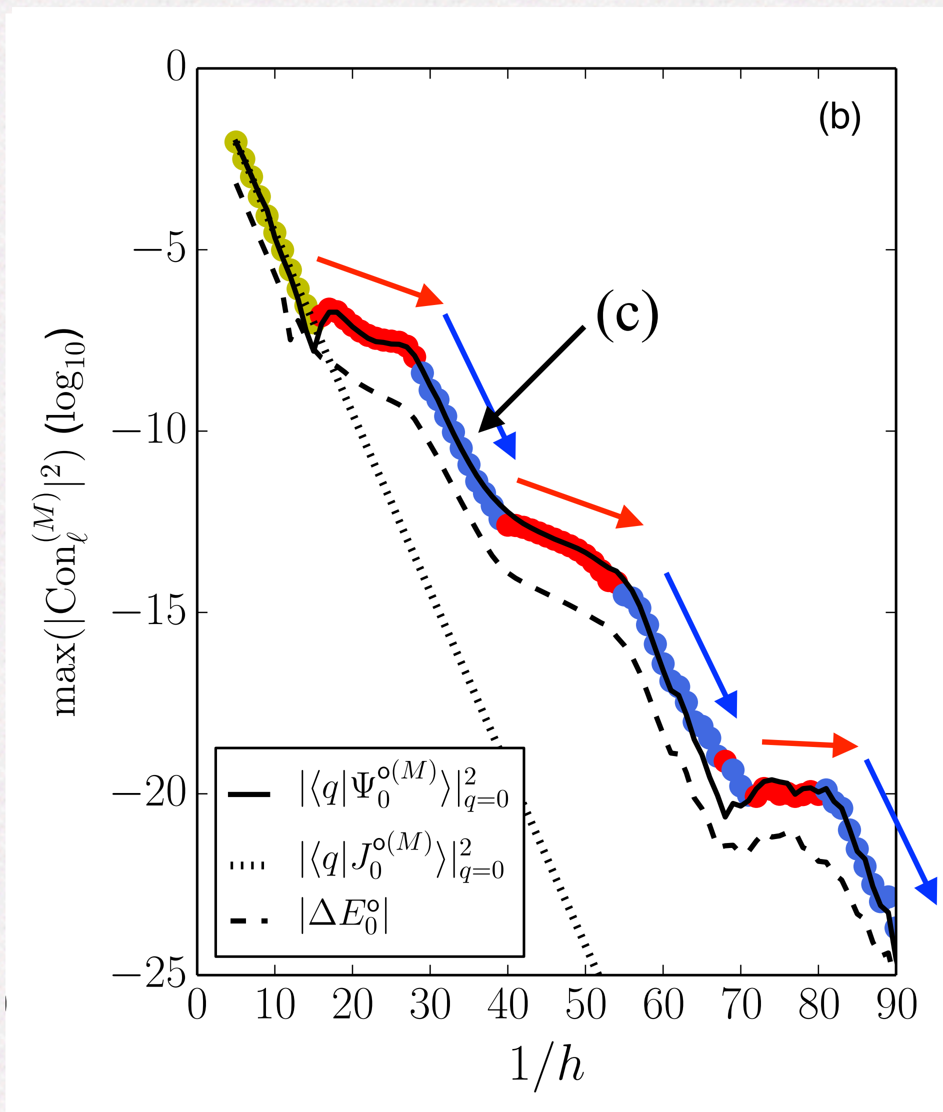

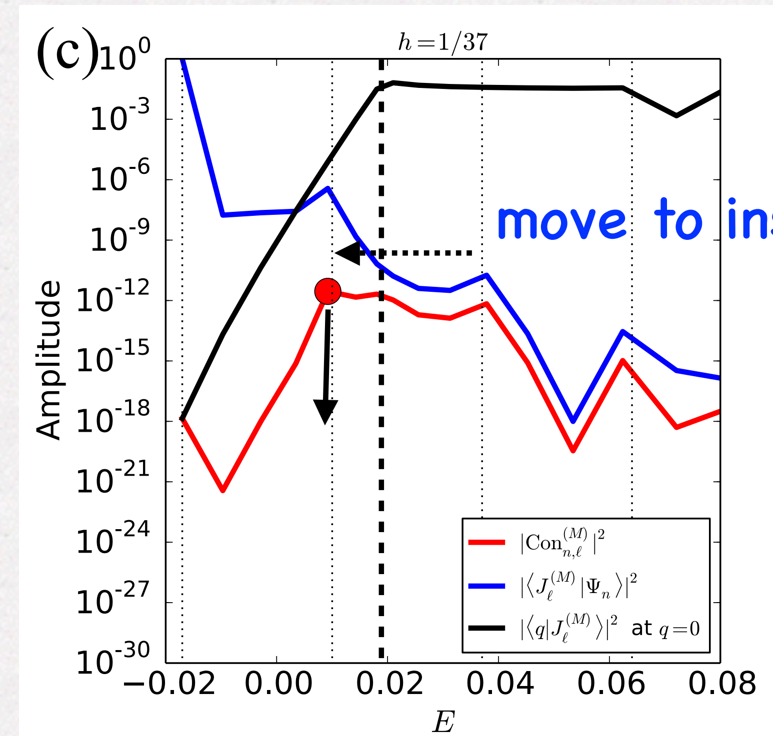

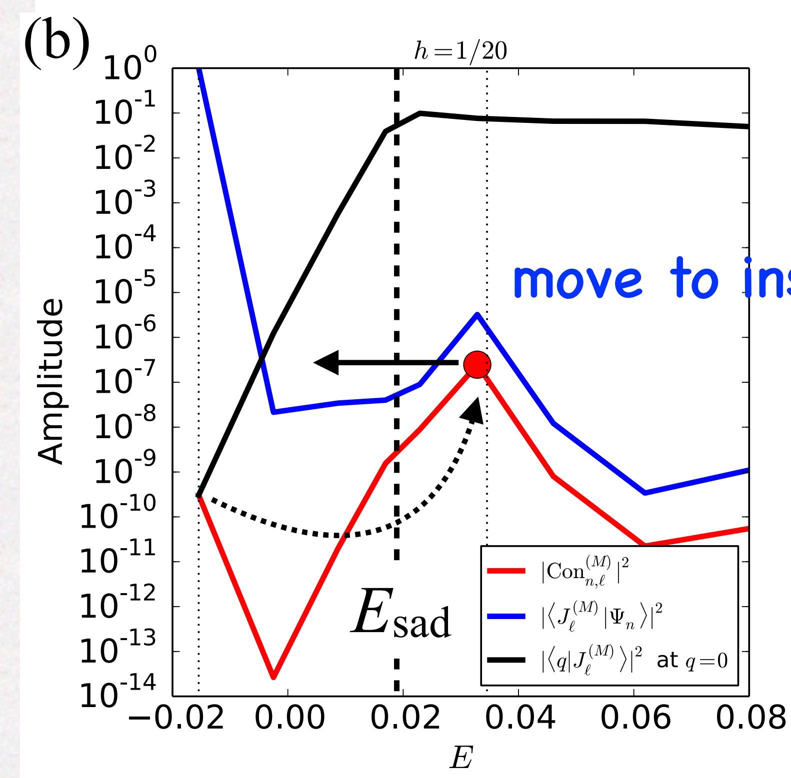

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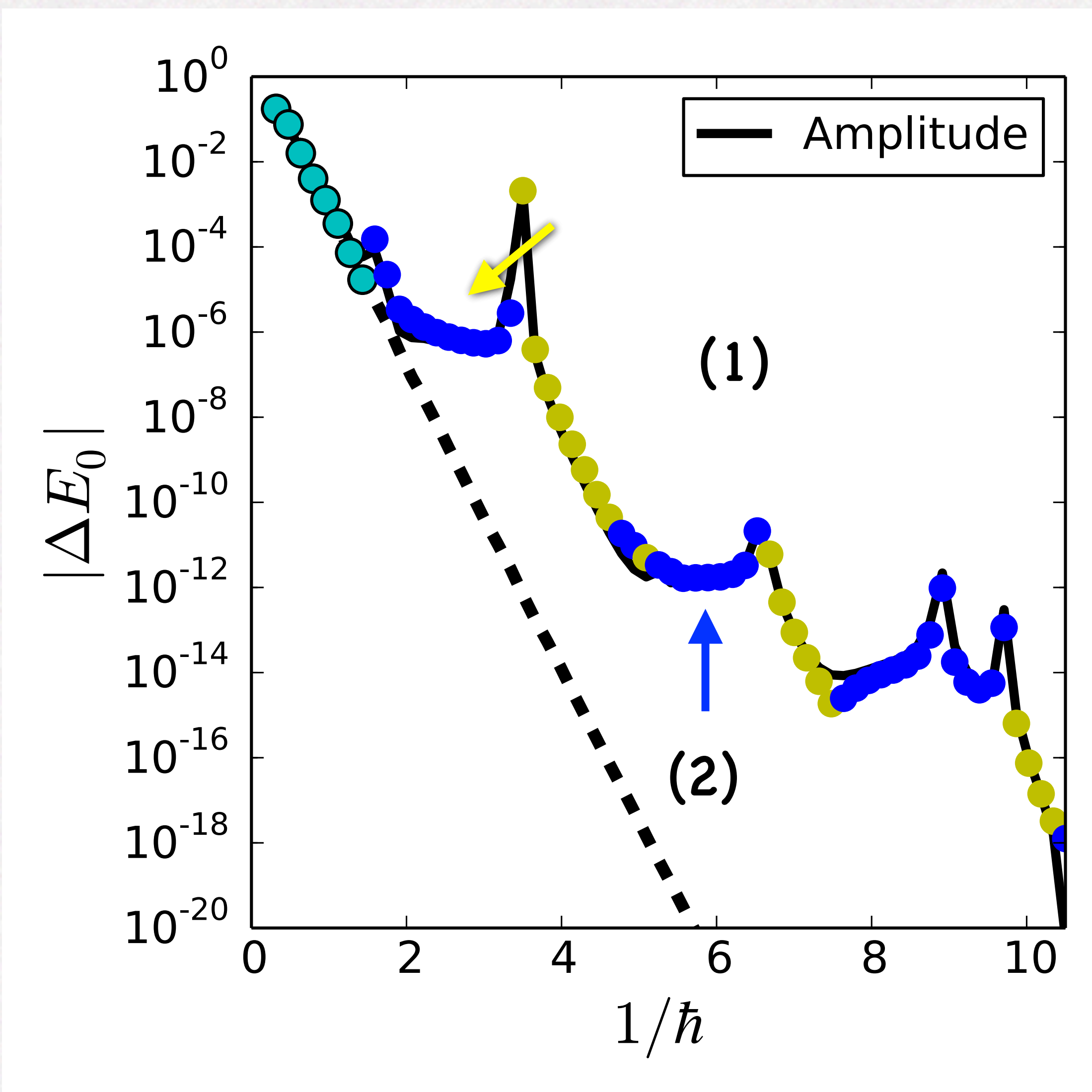

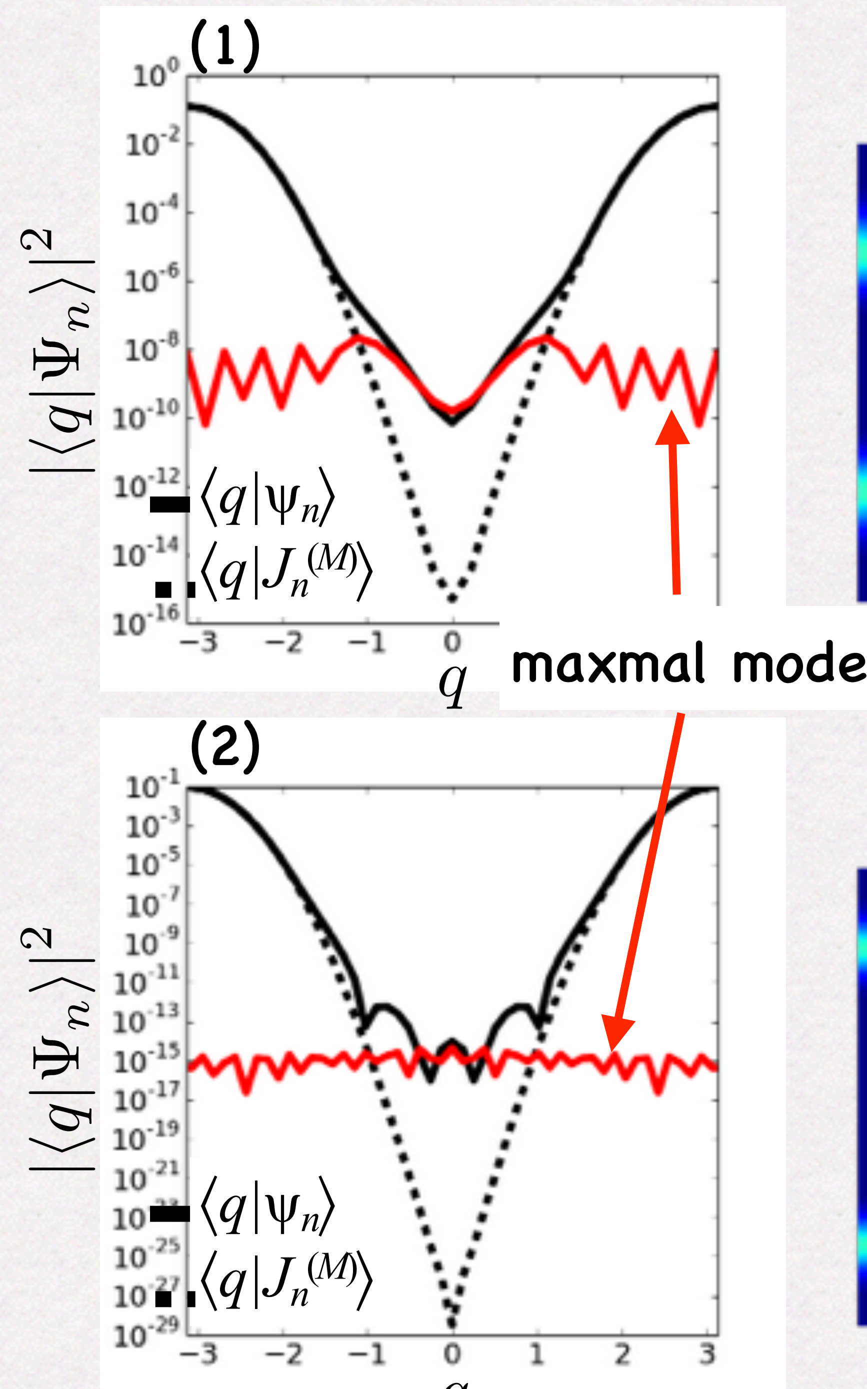

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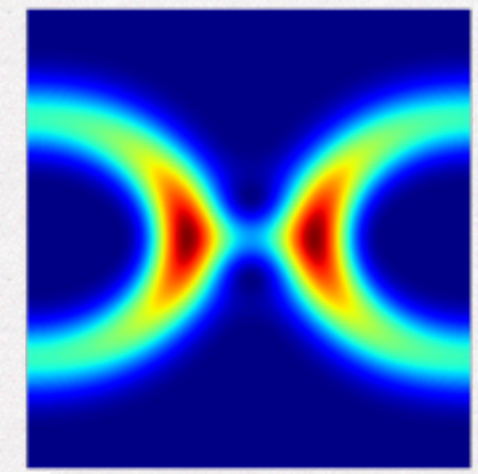

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