Statistical physics of time-periodic systems

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mpipks

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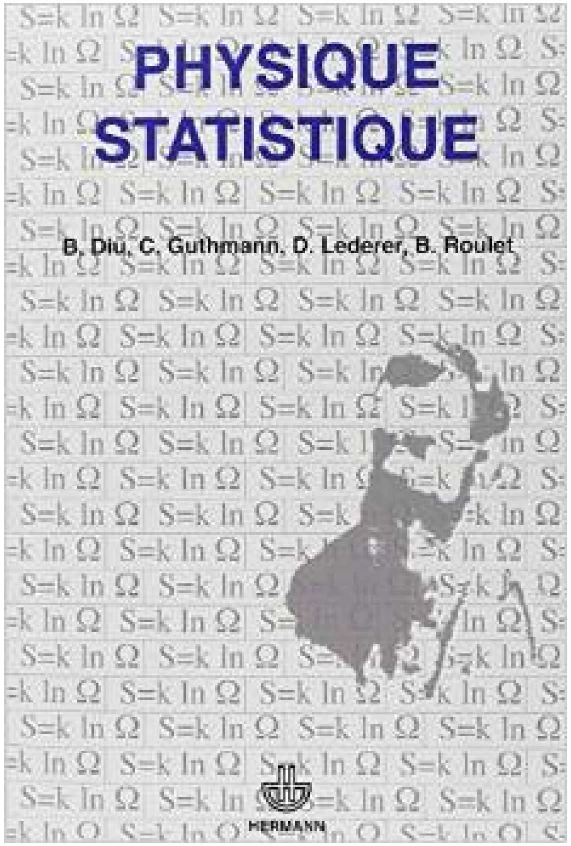


+ Time-Periodic Driving

1. Canonical ensemble

   Asymptotic probability to be in state i

   $$p_i = \frac{1}{Z} e^{-\beta E_i} \; ?$$

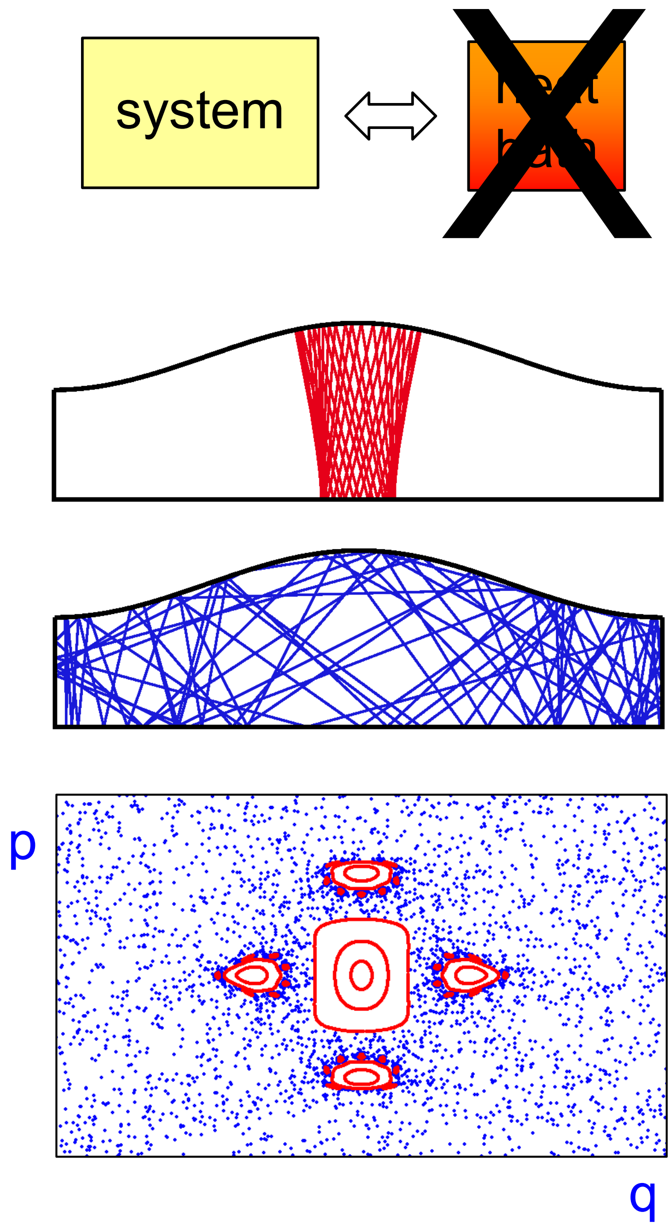
   Eigenstate properties essential

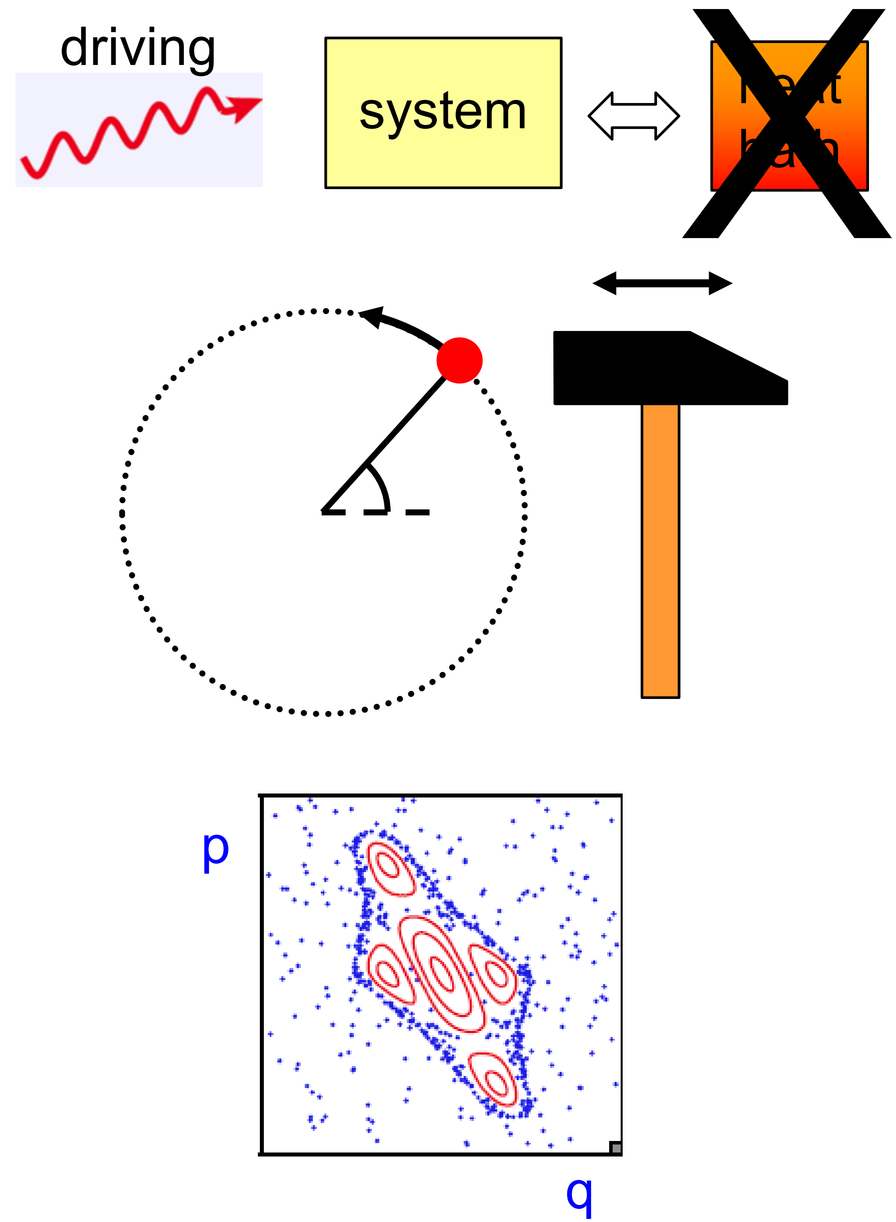
2. Ideal quantum gas

   Bose condensation ?

   Bose selection of multiple states (odd number)

Equilibrium vs. Non-Equilibrium Steady States





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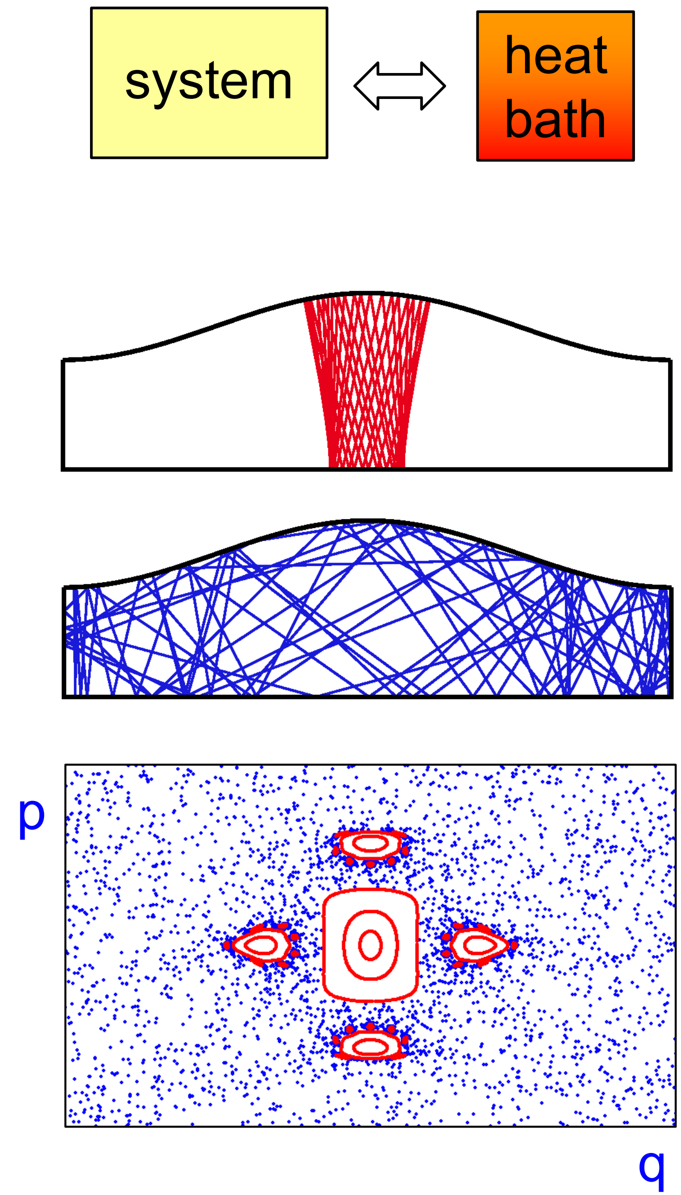
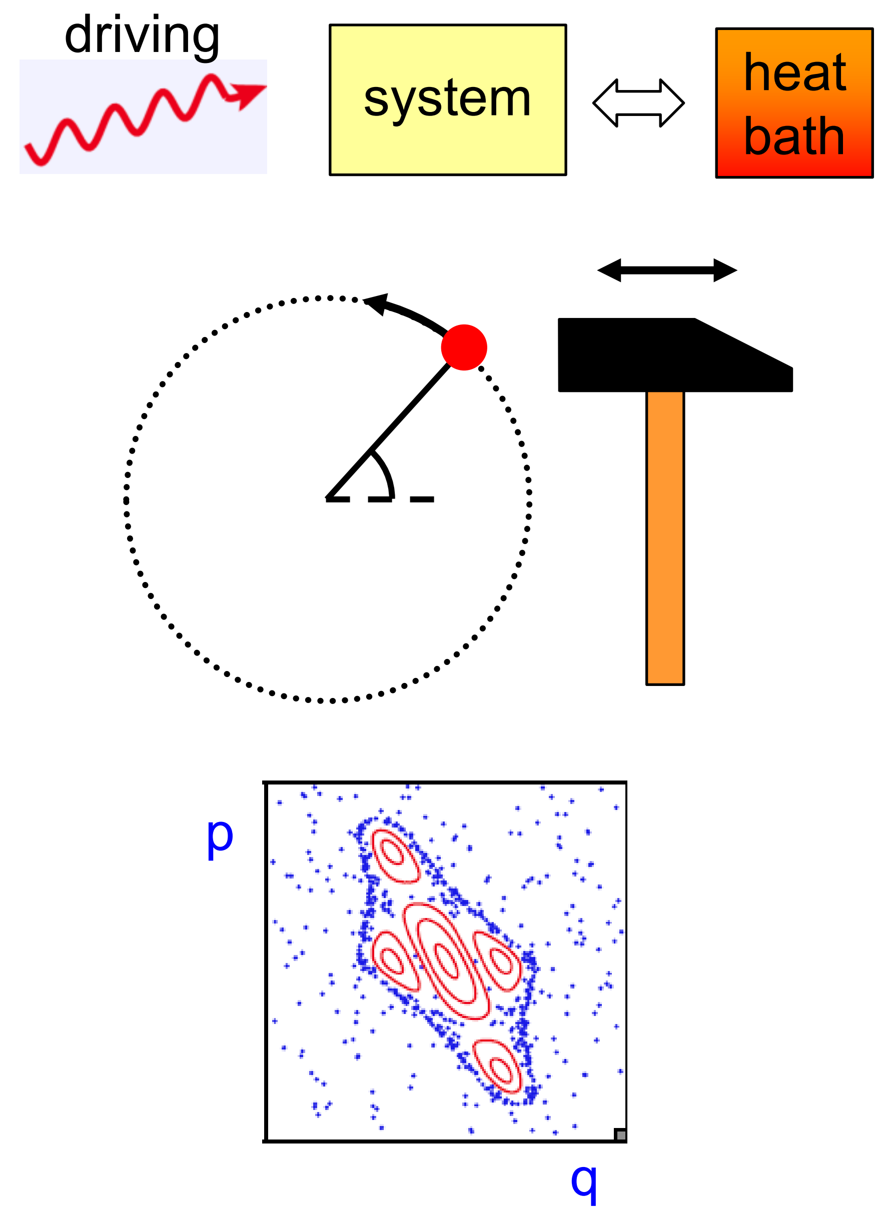
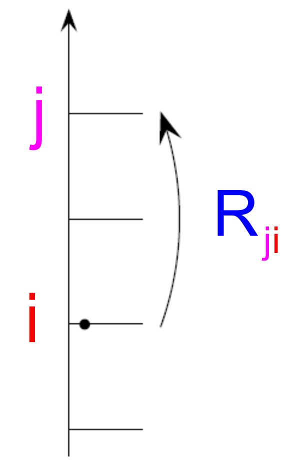




Weak coupling to heat bath

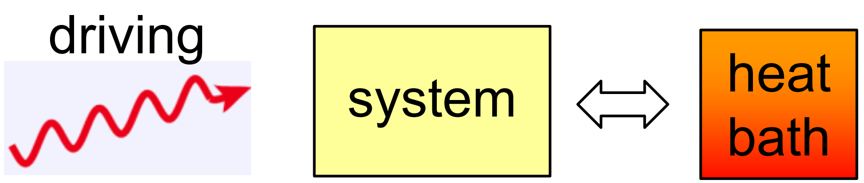
$$H = H_{system} + \gamma H_{coupl} + H_{bath} \qquad \gamma \ll 1$$

- Reduced density operator
- Steady state: $\rho_\infty = \sum_{i=1}^{M} p_i |i\rangle\langle i|$
  (diagonal in eigenstates)
- Master equation: $\dot{p}_i = \sum_{j=1}^{M} (R_{ij} p_j - R_{ji} p_i) = 0$



- Floquet-Born-Markov Approach

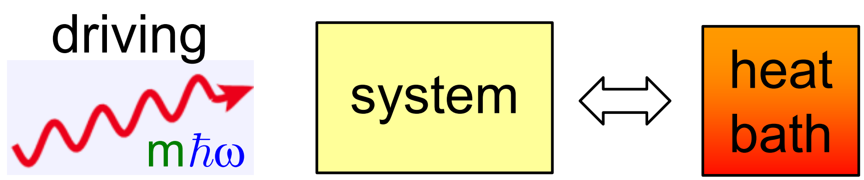
  Blümel et al.; PRA 1991
  Kohler, Dittrich, Hänggi; PRE 1997
  Breuer et al.; PRE 2000
  Kohn; J. Stat. Phys. 2001
  Hone, RK, Kohn; PRE 2009
  Langemeyer, Holthaus; PRE 2014



Rates

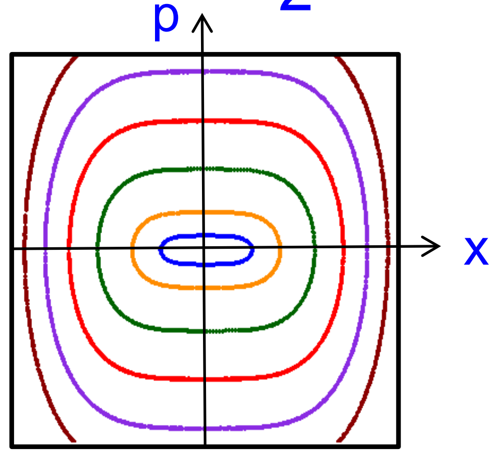
$$R_{ji} = \gamma^2 \cdot \left| \langle j | H_{coupl} | i \rangle \right|^2 \cdot g_\tau(E_j - E_i)$$

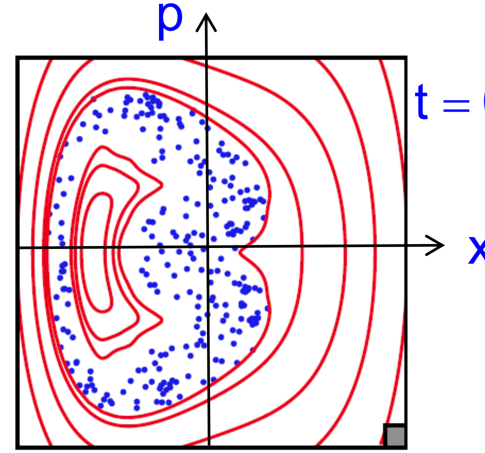
coupling strength — coupling matrix element — bath correlation function



$$R_{ji} = \sum_m R_{ji}^m \qquad R_{ji}^m = \gamma^2 \left| \frac{1}{T} \int_0^T dt\, e^{-im\omega t} \langle u_j(t) | H_{coupl} | u_i(t) \rangle \right|^2 g_\tau(\varepsilon_j - \varepsilon_i - m\hbar\omega)$$

Quartic Oscillator: $H(x,p,t) = \frac{p^2}{2} + x^4 \quad + A\,x\cos(\omega t)$ $\quad T = \frac{2\pi}{\omega}$

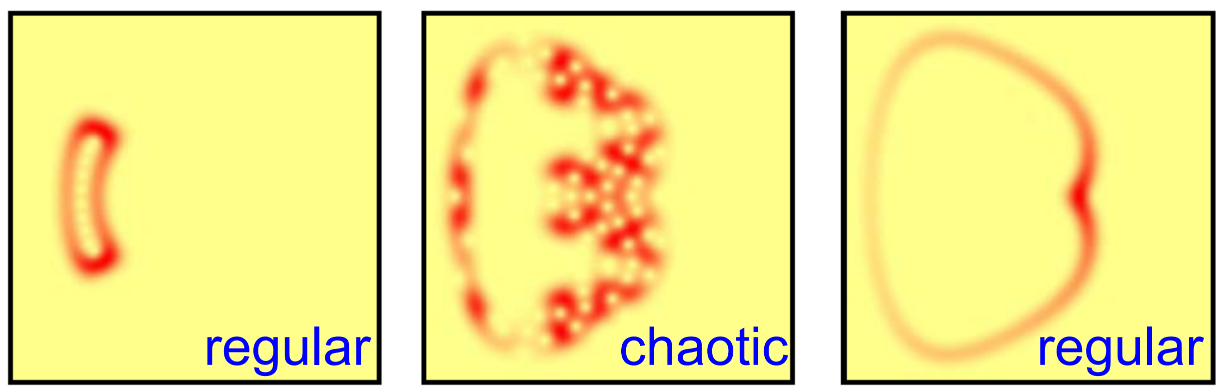




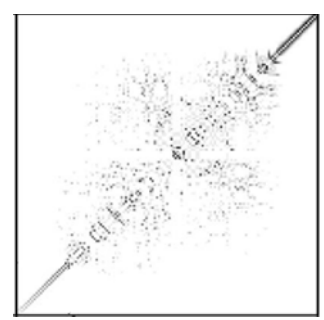
$t = 0, T, 2T, \ldots$

$A = 0.2 \quad \omega = 0.83$

Semiclassical
Eigenfunction
Hypothesis
Percival 1973
Berry 1977



Floquet-Born-Markov Approach $\Rightarrow$ R_{ji}

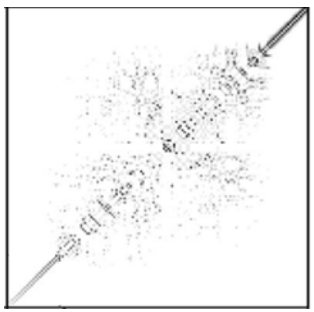


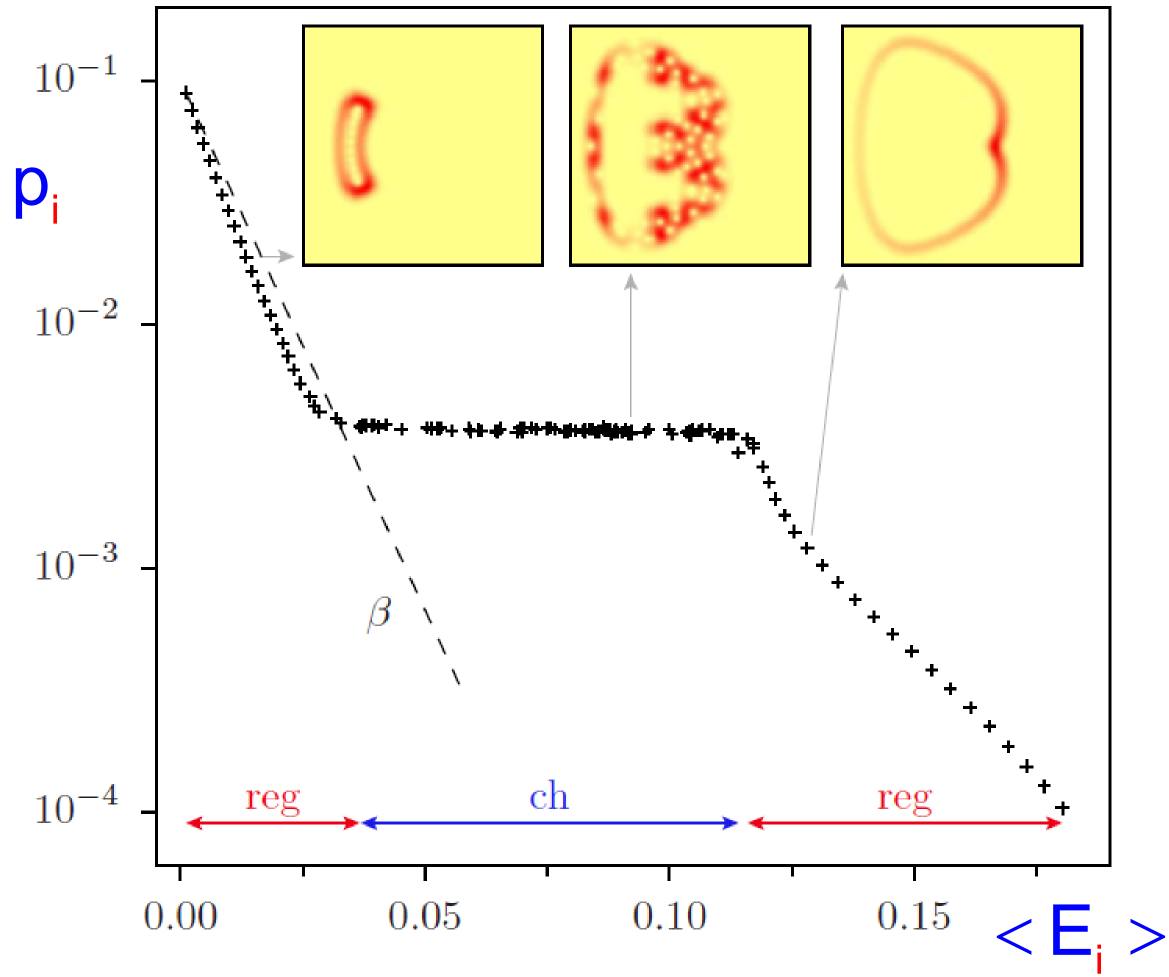
Occupation Probabilities: Quartic Oscillator

RK, Wustmann
PRE 2010

$$\sum_{j=1}^{M}\left(R_{ij}p_j - R_{ji}p_i\right) = 0$$

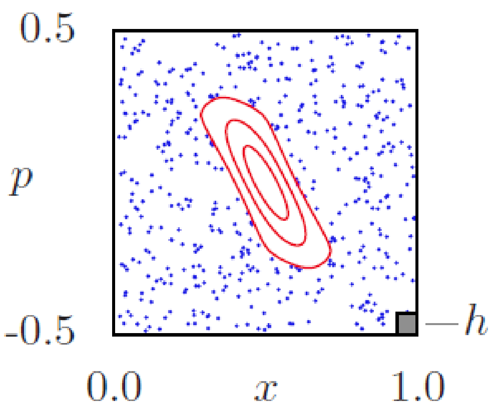
R_{ji}



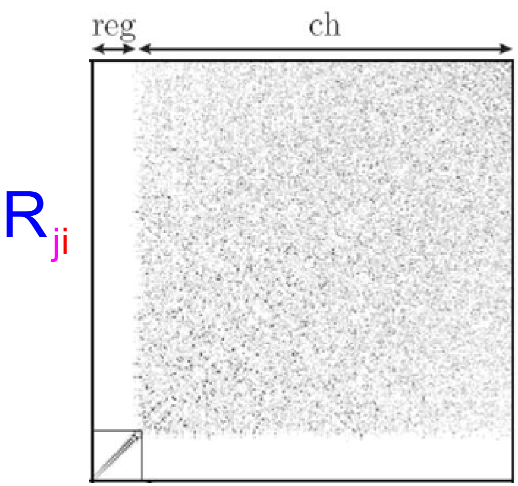


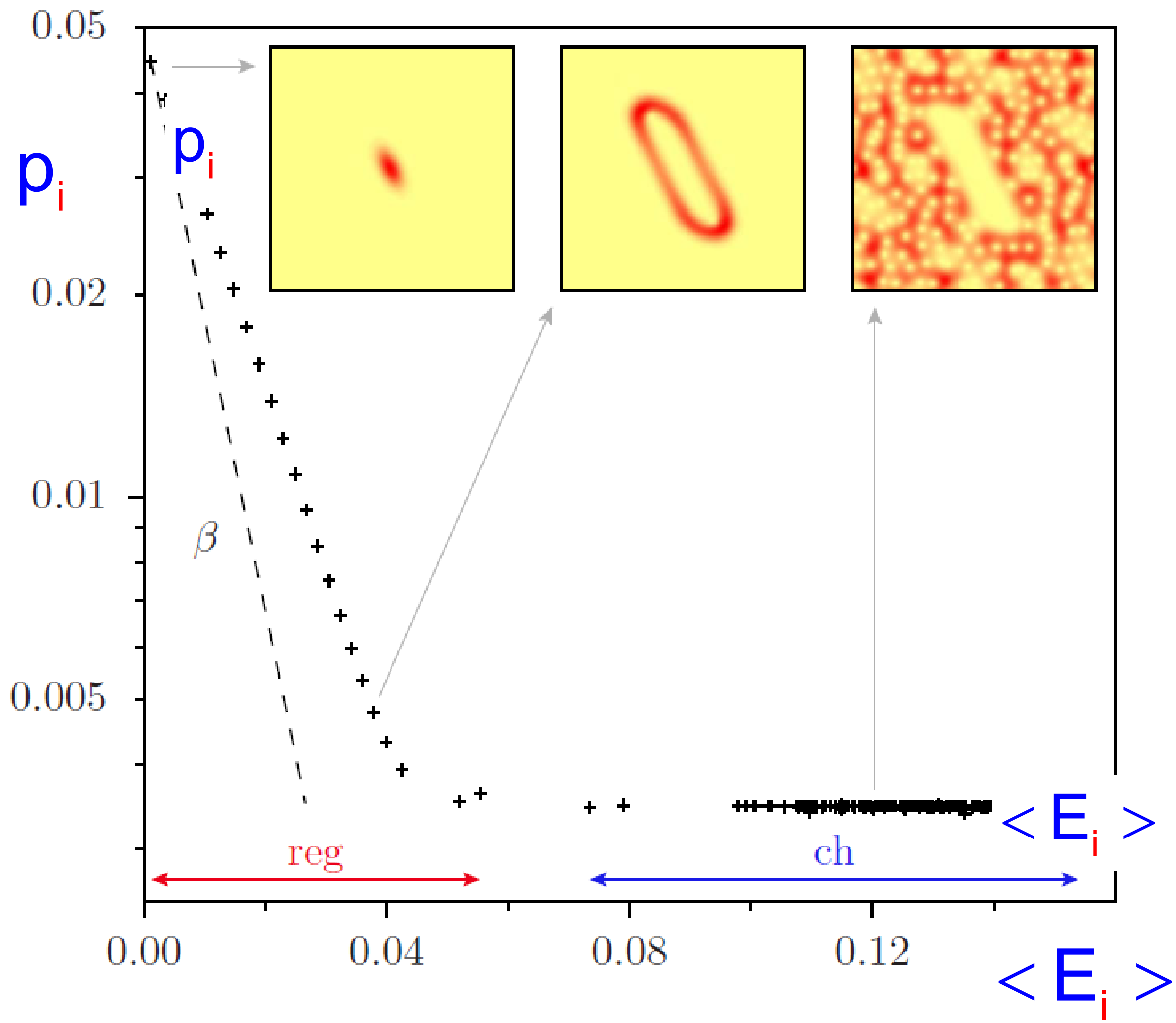
Occupation Probabilities: Kicked Rotor

RK, Wustmann
PRE 2010



K = 2.9





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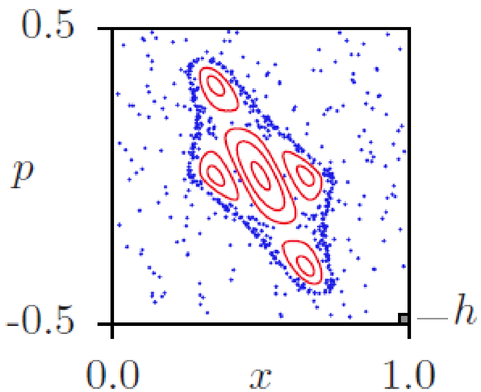
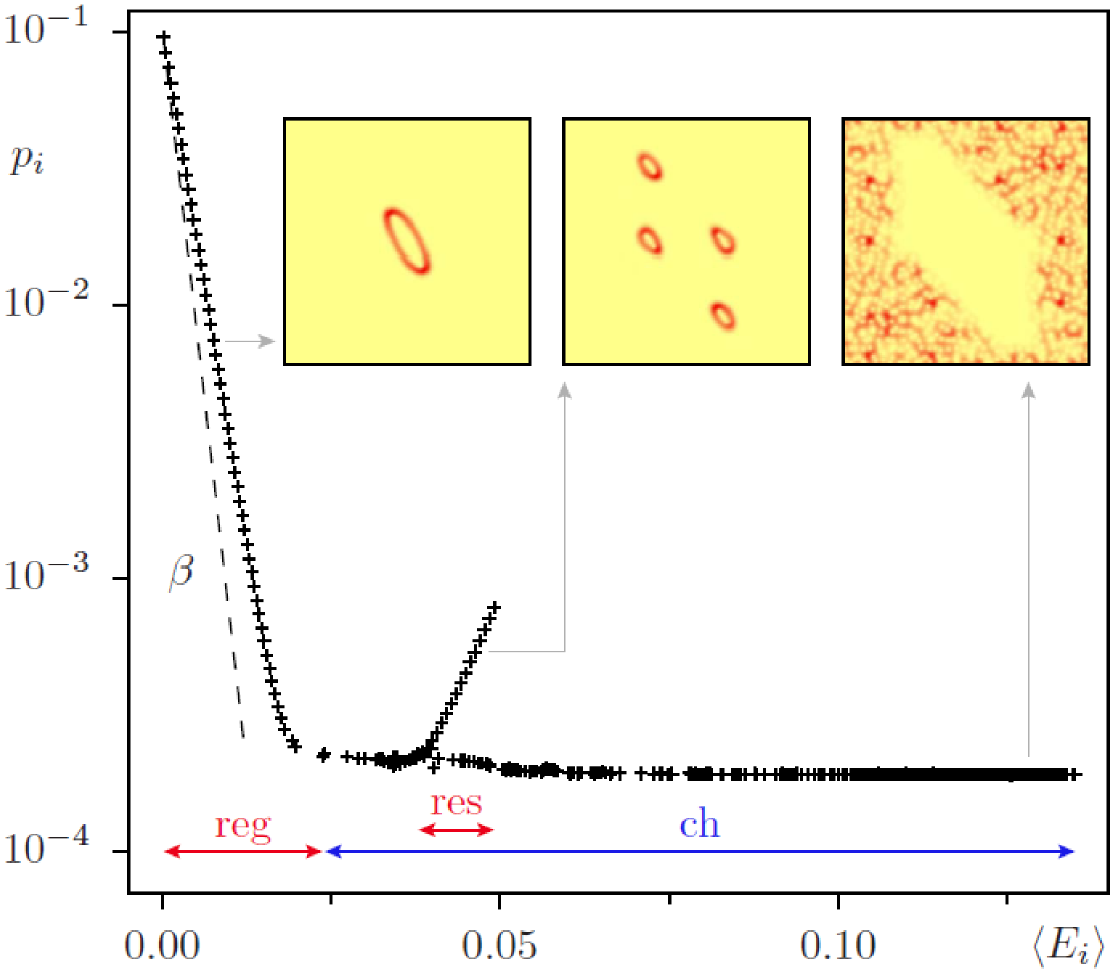


K = 2.35

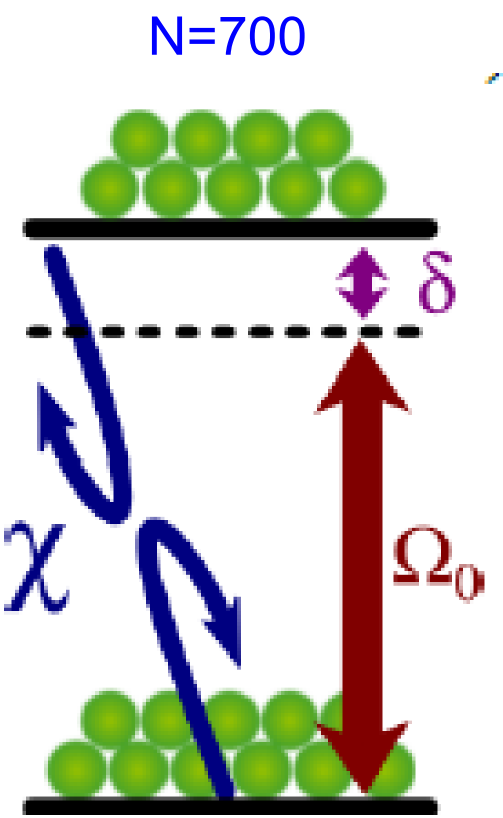


⇒ Eigenstate properties essential

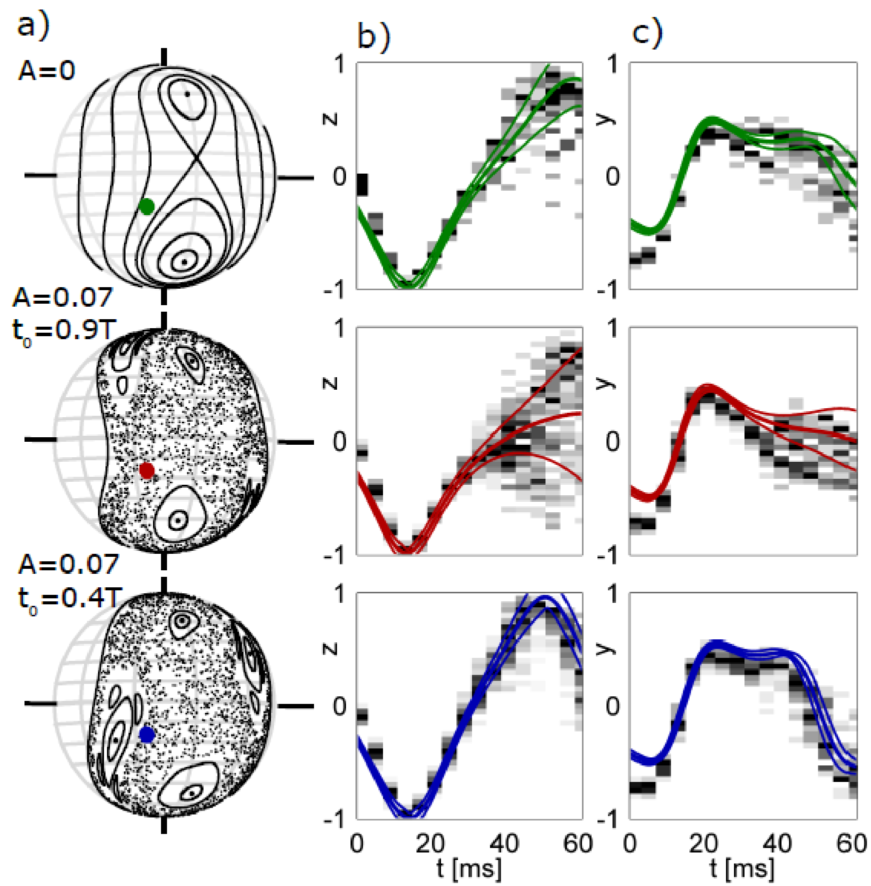
… on to many-body systems …

Many-body + periodic driving (no heat bath)

Experiment: Tomkovic, Müssel, Oberthaler   Theory: Schlagheck, Löck, R.K.



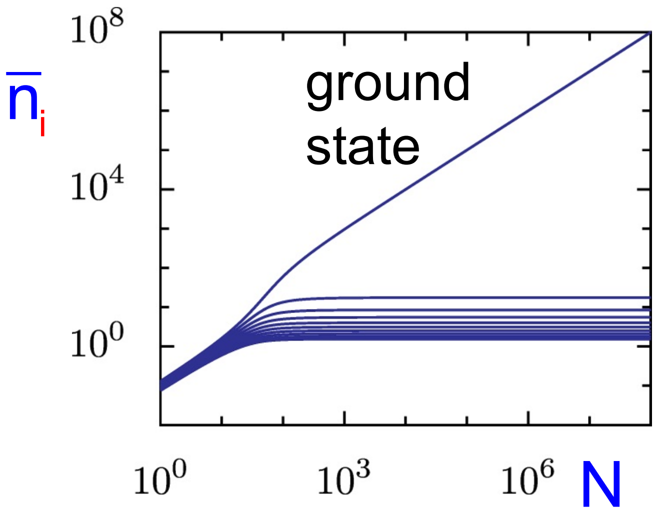
- Poincare-Birkhoff scenario
- Spreading in regular vs. chaotic region



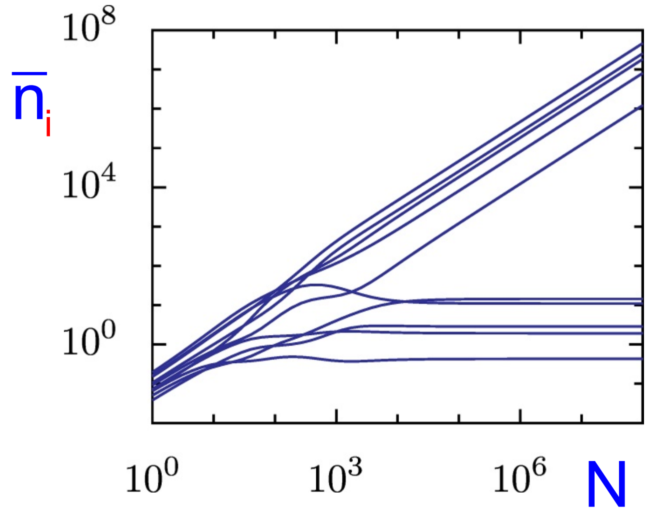
Vorberg, Wustmann, R.K., Eckardt, PRL 2013

Generalization of Bose-Einstein Condensation in Non-Equilibrium Steady States

Bose-Einstein Condensation



Bose Selection



Which states?
How many?

odd

André Eckardt

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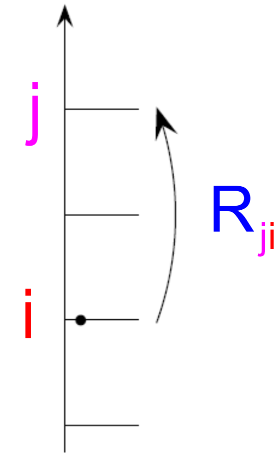
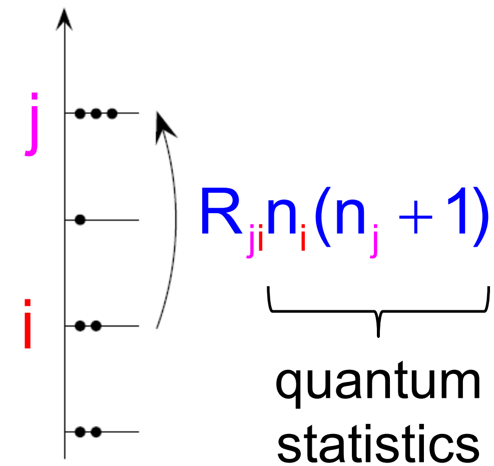


Ideal Bose gas:

$$\vec{n} = (n_1, \ldots, n_i, \ldots, n_j, \ldots, n_M)$$

$$\dot{p}_{\vec n} = \sum_{i,j=1}^{M} \left(R_{ij} p_{\vec n_{ji}} - R_{ji} p_{\vec n} \right) n_i (n_j + 1) = 0$$



solution: exists, unique

Occupation of single particle states

(mean-field approximation)

$$\dot{\bar{n}}_i = \sum_{j=1}^{M} \left[R_{ij}\bar{n}_j(\bar{n}_i + 1) - R_{ji}\bar{n}_i(\bar{n}_j + 1) \right] = 0$$

Asymptotic theory for $n \to \infty$:

Odd number M_S of states selected

Zero determinant of antisymmetric matrix $R_{ji} - R_{ij}$ when dimension odd.

Which states selected?

- existence and uniqueness (H. Schomerus)
- algorithm
- M_S=1 (Bose condensation)

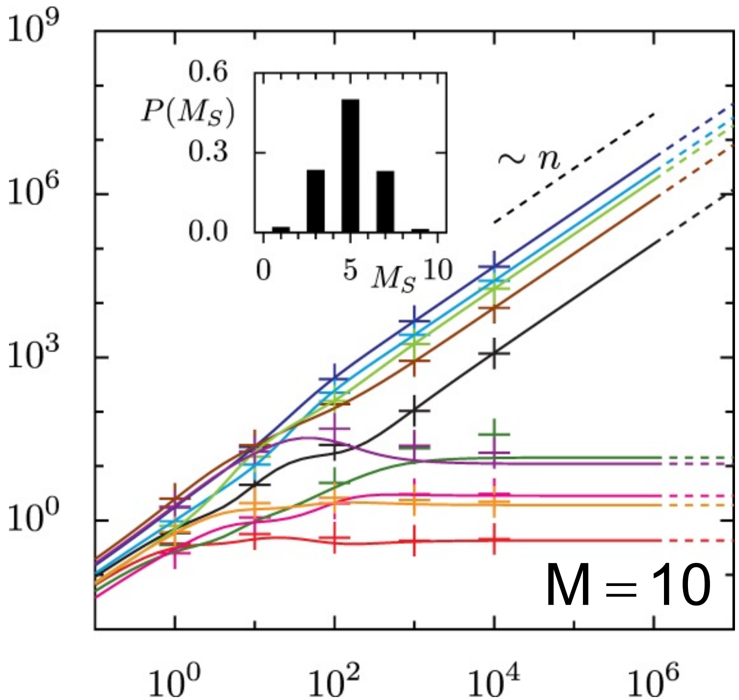
  iff

  $\exists$ „ground state“ k with $R_{ki} > R_{ik}$ for all i

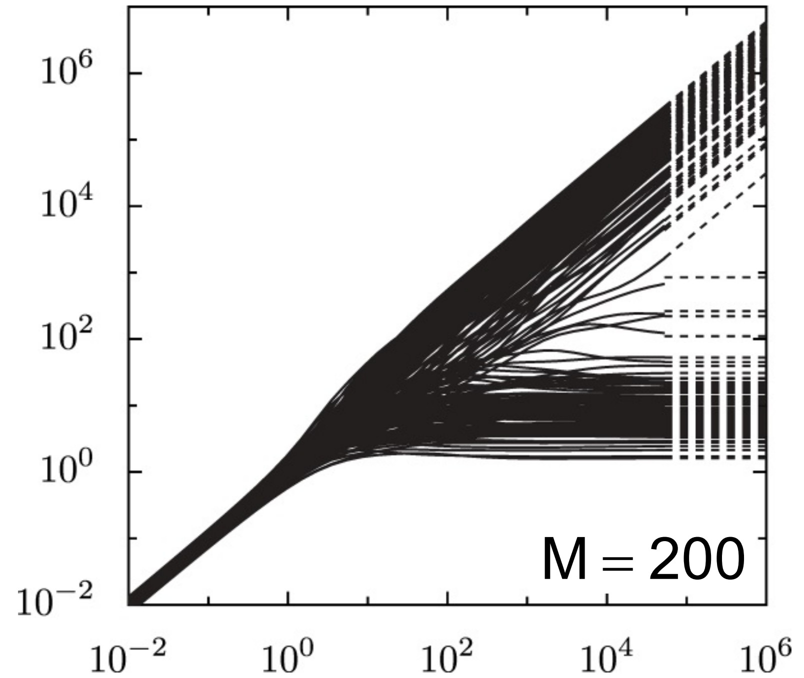
otherwise $M_S \geq 3$

Random rate matrix: R_{ji} from exponential distribution

mean occupation $\bar{n}_i$



$n = N/M$



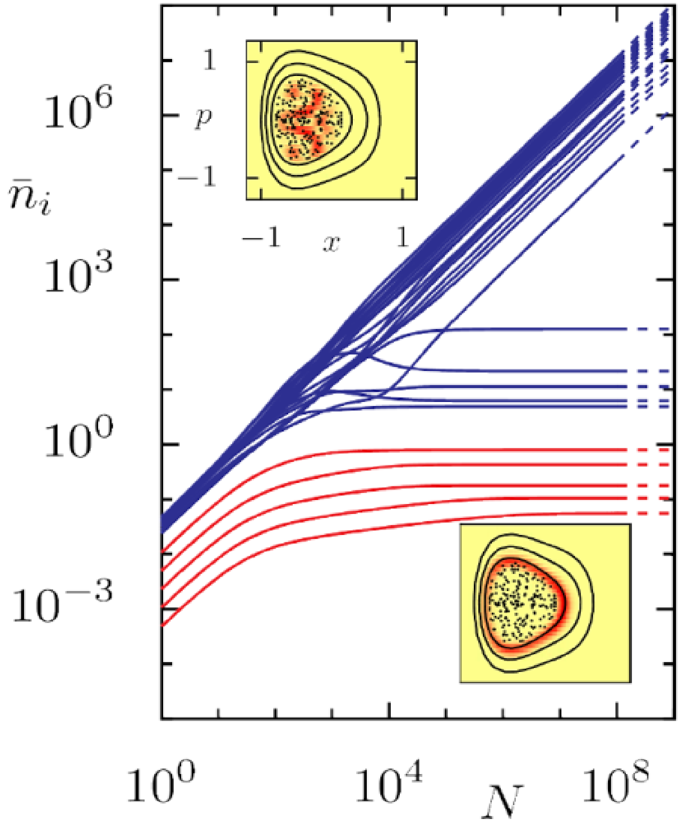
n

\+ quantum jump Monte-Carlo simulation

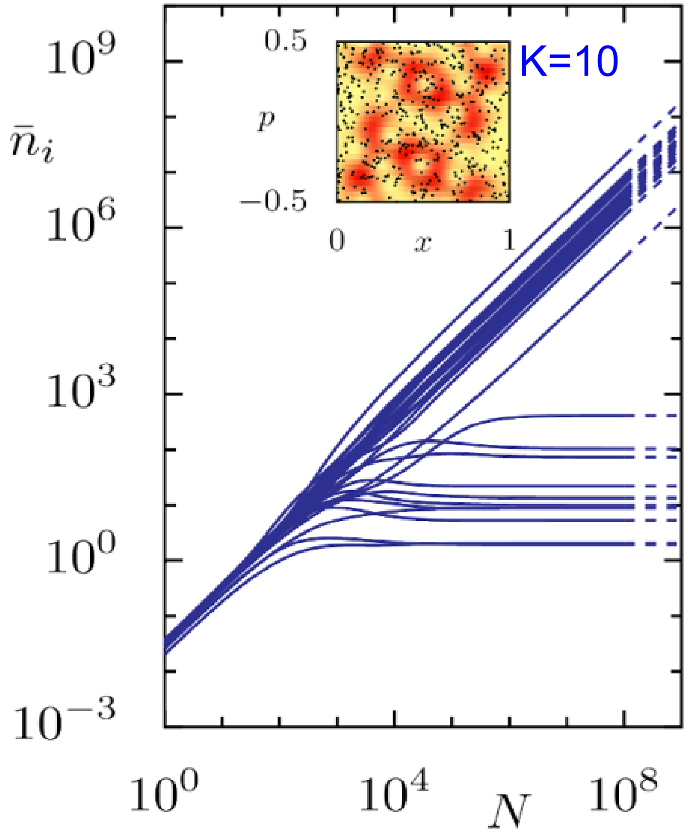
— mean field approximation

Examples for Bose selection

Periodically driven quartic oscillator

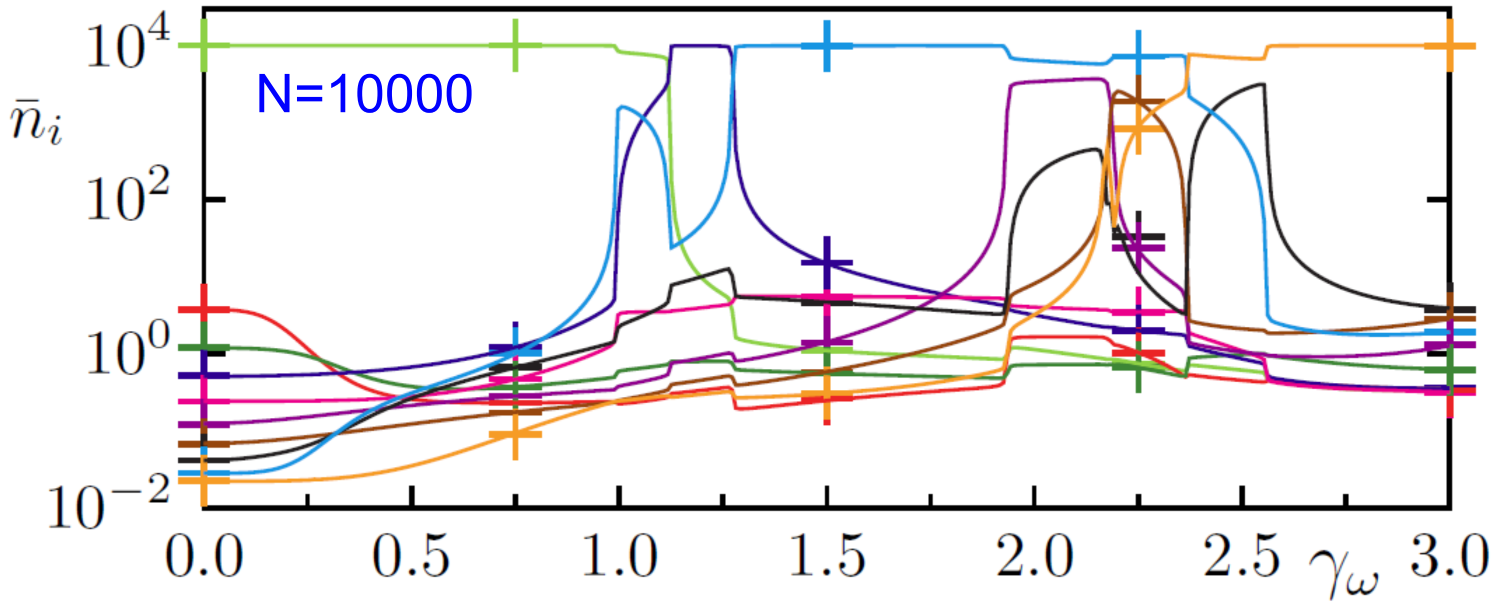


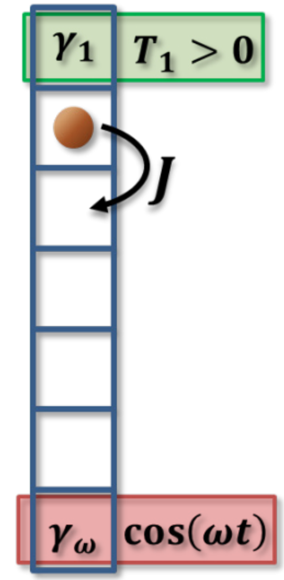
Kicked rotor



Examples for Bose selection

Tight-Binding Chain (10 sites): periodically driven

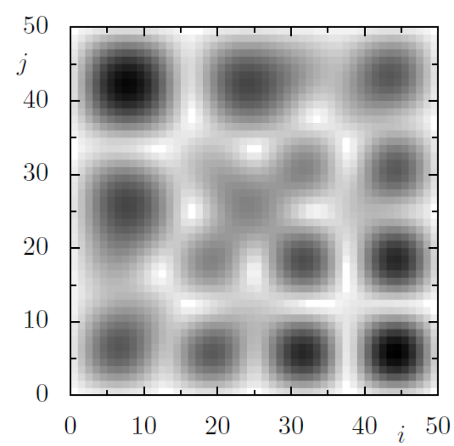




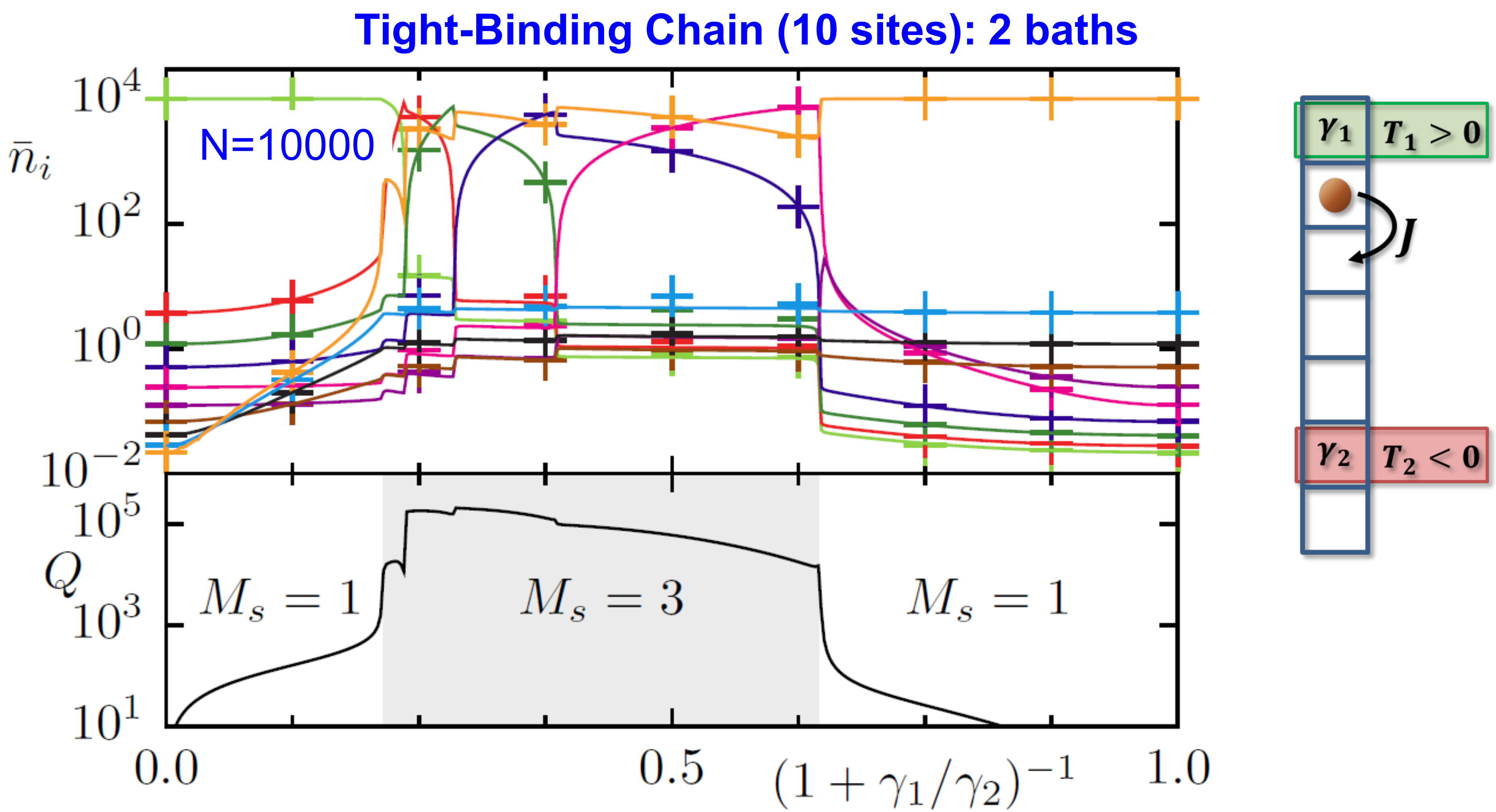
driving strength

always $M_S \leq 3$!

R_{ji}



Examples for Bose selection

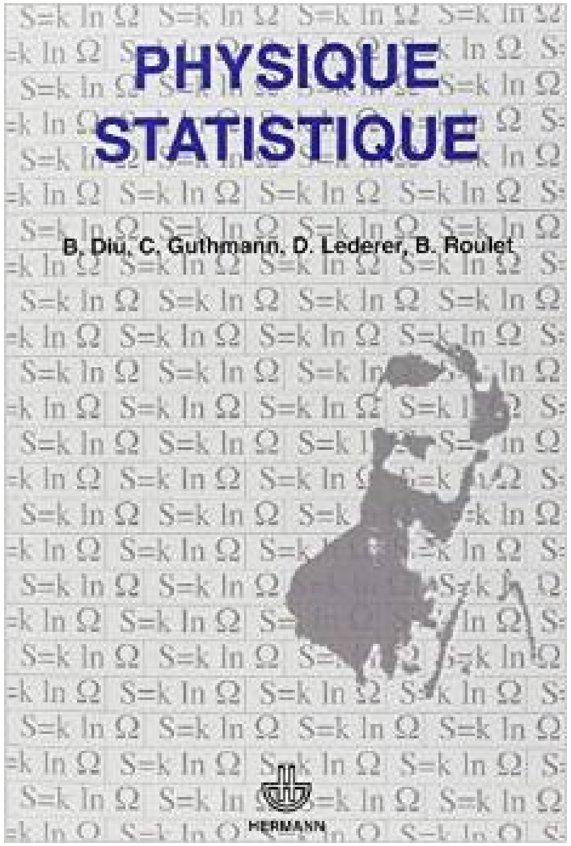


↑ only bath 1

relative bath coupling

↑ only bath 2

⇒ Quantum switch for heat



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Open directions: ⇒ anomalous long-range order in 1D (A. Schnell)
⇒ particle reservoir gives even number (D. Vorberg)

- interaction
- experiment (cold atom, quantum dot)
- connection to lasers (J. Wiersig)