University of BRISTOL

Spectral statistics of chaotic many-body systems

Rémy Dubertrand (Liège) and
Sebastian Müller (Bristol)

Université de Liège

Luchon, March 2015

EPSRC
Engineering and Physical Sciences Research Council

Single-particle systems

Single-particle systems

In the semiclassical limit chaotic systems display universal spectral statistics, in agreement with predictions from RMT.
(Bohigas, Giannoni, Schmit 84)

In the semiclassical limit chaotic systems display universal spectral statistics, in agreement with predictions from RMT.
(Bohigas, Giannoni, Schmit 84)

e.g. for two-point correlation function

$$R(\epsilon) = \langle d(E + \epsilon/2) d(E - \epsilon/2) \rangle$$

where $d(E) = \sum_j \delta(E - E_j)$, E measured in units of mean level spacing

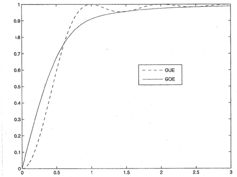
Single-particle systems

In the semiclassical limit chaotic systems display universal spectral statistics, in agreement with predictions from RMT.
(Bohigas, Giannoni, Schmit 84)

e.g. for two-point correlation function

$$R(\epsilon) = \langle d(E + \epsilon/2) d(E - \epsilon/2) \rangle$$

where $d(E) = \sum_j \delta(E - E_j)$, E measured in units of mean level spacing



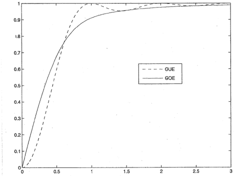
Single-particle systems

In the semiclassical limit chaotic systems display universal spectral statistics, in agreement with predictions from RMT.
(Bohigas, Giannoni, Schmit 84)

e.g. for two-point correlation function

$$R(\epsilon) = \langle d(E + \epsilon/2) d(E - \epsilon/2) \rangle$$

where $d(E) = \sum_j \delta(E - E_j)$, E measured in units of mean level spacing



explanation based on:

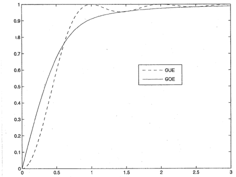
Single-particle systems

In the semiclassical limit chaotic systems display universal spectral statistics, in agreement with predictions from RMT.
(Bohigas, Giannoni, Schmit 84)

e.g. for two-point correlation function

$$R(\epsilon) = \langle d(E+\epsilon/2)d(E-\epsilon/2)\rangle$$

where $d(E) = \sum_j \delta(E - E_j)$, E measured in units of mean level spacing



explanation based on: Gutzwiller trace formula

$$d(E) \sim \bar{d}(E) + \frac{1}{\pi\hbar} \; \mathrm{Re} \sum_{\text{per. orbits } p} A_p \; e^{iS_p/\hbar}$$

Many-particle systems

Many-particle systems

e.g. Bose-Hubbard model

Many-particle systems

e.g. Bose-Hubbard model

$$\hat{H} = \underbrace{-\frac{J}{2}\sum_j (\hat{a}^\dagger_{j+1}\hat{a}_j + \hat{a}^\dagger_j \hat{a}_{j+1})}_{\text{jumps}} + \underbrace{\frac{U}{2}\sum_j (\hat{a}^\dagger_j)^2 \hat{a}_j^2}_{\text{interaction}}$$

Many-particle systems

e.g. Bose-Hubbard model

$$\hat{H} = \underbrace{-\frac{J}{2}\sum_j (\hat{a}^\dagger_{j+1}\hat{a}_j + \hat{a}^\dagger_j \hat{a}_{j+1})}_{\text{jumps}} + \underbrace{\frac{U}{2}\sum_j (\hat{a}^\dagger_j)^2 \hat{a}^2_j}_{\text{interaction}}$$

more general:

$$\hat{H} = \sum_{jk} h_{jk}\hat{a}^\dagger_j \hat{a}_k + \sum_{jklm} U_{jklm}\hat{a}^\dagger_j \hat{a}^\dagger_k \hat{a}_l \hat{a}_m$$

Many-particle systems

e.g. Bose-Hubbard model

$$\hat{H} = \underbrace{-\frac{J}{2}\sum_j (\hat{a}_{j+1}^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_{j+1})}_{\text{jumps}} + \underbrace{\frac{U}{2}\sum_j (\hat{a}_j^\dagger)^2 \hat{a}_j^2}_{\text{interaction}}$$

more general:

$$\hat{H} = \sum_{jk} h_{jk} \hat{a}_j^\dagger \hat{a}_k + \sum_{jklm} U_{jklm} \hat{a}_j^\dagger \hat{a}_k^\dagger \hat{a}_l \hat{a}_m$$

classical limit: $\hat{a}_j \to \psi_j$, $\hat{a}_j^\dagger \to \psi_j^*$

Many-particle systems

e.g. Bose-Hubbard model

$$\hat{H} = \underbrace{-\frac{J}{2}\sum_j (\hat{a}_{j+1}^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_{j+1})}_{\text{jumps}} + \underbrace{\frac{U}{2}\sum_j (\hat{a}_j^\dagger)^2 \hat{a}_j^2}_{\text{interaction}}$$

more general:

$$\hat{H} = \sum_{jk} h_{jk} \hat{a}_j^\dagger \hat{a}_k + \sum_{jklm} U_{jklm} \hat{a}_j^\dagger \hat{a}_k^\dagger \hat{a}_l \hat{a}_m$$

classical limit: $\hat{a}_j \to \psi_j$, $\hat{a}_j^\dagger \to \psi_j^*$

Hamilton equations give discrete nonlinear Schrödinger equation

$$i\hbar\dot{\psi}_j = -\frac{\partial H}{\partial \psi_j^*} = J(\psi_{j+1} + \psi_{j-1}) - U|\psi_j|^2\psi_j$$

Bose-Hubbard model

Bose-Hubbard model

numerical observations:

e.g. Kolovsky & Buchleitner 2004, Kolovsky 2007, Kollath et al. 2010, ...

Bose-Hubbard model

numerical observations:

e.g. Kolovsky & Buchleitner 2004, Kolovsky 2007, Kollath et al. 2010, ...

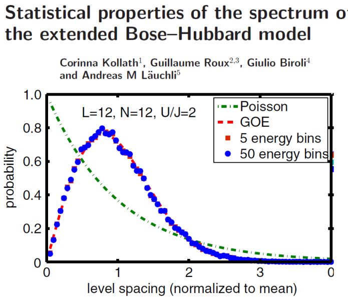
- discrete nonlinear Schrödinger equation is chaotic
  (for several sites, hopping and interaction term comparable,
  apart from stability islands)

Bose-Hubbard model

numerical observations:

e.g. Kolovsky & Buchleitner 2004, Kolovsky 2007, Kollath et al. 2010, ...

- discrete nonlinear Schrödinger equation is chaotic (for several sites, hopping and interaction term comparable, apart from stability islands)
- spectral statistics agrees with RMT under the same conditions

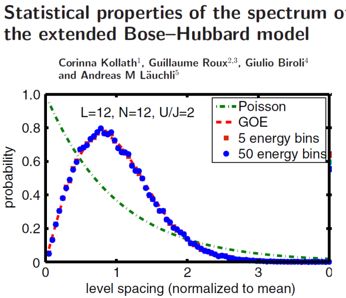


Bose-Hubbard model

numerical observations:

e.g. Kolovsky & Buchleitner 2004, Kolovsky 2007, Kollath et al. 2010, ...

- discrete nonlinear Schrödinger equation is chaotic
(for several sites, hopping and interaction term comparable, apart from stability islands)
- spectral statistics agrees with RMT under the same conditions



Why?

Semiclassical approach

Semiclassical approach

- path integral over all $\psi(t'), \psi(t')^*$ with $\psi(0) = \psi^{(i)}$, $\psi^*(t) = \psi^{(f)*}$

Semiclassical approach

- path integral over all $\psi(t'), \psi(t')^*$ with $\psi(0) = \psi^{(i)}$, $\psi^*(t) = \psi^{(f)*}$

$$\left\langle \psi^{(f)} \middle| e^{-\frac{i}{\hbar}\hat{H}t} \middle| \psi^{(i)} \right\rangle = \int D[\psi, \psi^*] e^{\frac{i}{\hbar}R[\psi,\psi^*]}$$

$$\frac{i}{\hbar}R[\psi, \psi^*] = \int_0^t dt' \left[\frac{\dot{\psi}^*(t') \cdot \psi(t') - \psi^*(t') \cdot \dot{\psi}(t')}{2} - H(\psi(t'), \psi^*(t')) \right] + \underbrace{\frac{\psi^{(f)*} \cdot \psi(t) + \psi^*(0) \cdot \psi^{(i)}}{2}}_{\text{due to coherent states approach}}$$

Baranger et. al. 2001

- stationary phase approximation leads to van Vleck propagator

$$\left\langle \psi^{(f)} \middle| e^{-\frac{i}{\hbar}\hat{H}t} \middle| \psi^{(i)} \right\rangle \approx \sum_\gamma \left| \frac{i}{\hbar} \frac{\partial^2 R_\gamma}{\partial \psi^{(f)*} \partial \psi^{(i)}} \right|^{1/2} \exp\left(\frac{i}{\hbar} R_\gamma - i\nu_\gamma \frac{\pi}{2} + \underbrace{\frac{i}{2\hbar} \int_0^t dt' \ \mathrm{tr} \frac{\partial^2 H}{\partial \psi \partial \psi^*} - \frac{|\psi^{(i)}|^2 + |\psi^{(f)}|^2}{2}}_{\text{due to coherent states approach}} \right)$$

sum over solutions of nonlinear Schrödinger equation

see also Engl, Dujardin, Argülles, Schlagheck, Richter, Urbina 2014

Semiclassical approach

- path integral over all $\psi(t'), \psi(t')^*$ with $\psi(0) = \psi^{(i)}$, $\psi^*(t) = \psi^{(f)^*}$

$$\left\langle \psi^{(f)} \middle| e^{-\frac{i}{\hbar}\hat{H}t} \middle| \psi^{(i)} \right\rangle = \int D[\psi, \psi^*] e^{\frac{i}{\hbar} R[\psi, \psi^*]}$$

$$\frac{i}{\hbar} R[\psi, \psi^*] = \int_0^t dt' \left[\frac{\dot{\psi}^*(t') \cdot \psi(t') - \psi^*(t') \cdot \dot{\psi}(t')}{2} - H(\psi(t'), \psi^*(t')) \right]$$

$$+ \underbrace{\frac{\psi^{(f)^*} \cdot \psi(t) + \psi^*(0) \cdot \psi^{(i)}}{2}}_{\text{due to coherent states approach}}$$

Baranger et. al. 2001

Semiclassical approach

- path integral over all $\psi(t'), \psi(t')^*$ with $\psi(0) = \psi^{(i)}$, $\psi^*(t) = \psi^{(f)^*}$

$$\left\langle \psi^{(f)} \middle| e^{-\frac{i}{\hbar}\hat{H}t} \middle| \psi^{(i)} \right\rangle = \int D[\psi, \psi^*] e^{\frac{i}{\hbar}R[\psi,\psi^*]}$$

$$\frac{i}{\hbar} R[\psi, \psi^*] = \int_0^t dt' \left[\frac{\dot{\psi}^*(t') \cdot \psi(t') - \psi^*(t') \cdot \dot{\psi}(t')}{2} - H(\psi(t'), \psi^*(t')) \right]$$

$$+ \underbrace{\frac{\psi^{(f)^*} \cdot \psi(t) + \psi^*(0) \cdot \psi^{(i)}}{2}}_{\text{due to coherent states approach}}$$

Baranger et. al. 2001

- stationary phase approximation leads to van Vleck propagator

Semiclassical approach

- path integral over all $\psi(t'), \psi(t')^*$ with $\psi(0) = \psi^{(i)}$, $\psi^*(t) = \psi^{(f)^*}$

$$\left\langle \psi^{(f)} \middle| e^{-\frac{i}{\hbar}\hat{H}t} \middle| \psi^{(i)} \right\rangle = \int D[\psi, \psi^*] e^{\frac{i}{\hbar} R[\psi,\psi^*]}$$

$$\frac{i}{\hbar} R[\psi, \psi^*] = \int_0^t dt' \left[\frac{\dot{\psi}^*(t') \cdot \psi(t') - \psi^*(t') \cdot \dot{\psi}(t')}{2} - H(\psi(t'), \psi^*(t')) \right]$$

$$+ \underbrace{\frac{\psi^{(f)^*} \cdot \psi(t) + \psi^*(0) \cdot \psi^{(i)}}{2}}_{\text{due to coherent states approach}}$$

Baranger et. al. 2001

- stationary phase approximation leads to van Vleck propagator

$$\left\langle \psi^{(f)} \middle| e^{-\frac{i}{\hbar}\hat{H}t} \middle| \psi^{(i)} \right\rangle \approx \sum_\gamma \left| \frac{i}{\hbar} \frac{\partial^2 R_\gamma}{\partial \psi^{(f)^*} \partial \psi^{(i)}} \right|^{1/2}$$

$$\exp\left(\frac{i}{\hbar} R_\gamma - i\nu_\gamma \frac{\pi}{2} + \underbrace{\frac{i}{2\hbar} \int_0^t dt' \ \mathrm{tr} \frac{\partial^2 H}{\partial \psi \partial \psi^*} - \frac{|\psi^{(i)}|^2 + |\psi^{(f)}|^2}{2}}_{\text{due to coherent states approach}} \right)$$

Semiclassical approach

- path integral over all $\psi(t'), \psi(t')^*$ with $\psi(0) = \psi^{(i)}$, $\psi^*(t) = \psi^{(f)*}$

$$\left\langle \psi^{(f)} \middle| e^{-\frac{i}{\hbar}\hat{H}t} \middle| \psi^{(i)} \right\rangle = \int D[\psi, \psi^*] e^{\frac{i}{\hbar}R[\psi,\psi^*]}$$

$$\frac{i}{\hbar}R[\psi,\psi^*] = \int_0^t dt' \left[\frac{\dot{\psi}^*(t') \cdot \psi(t') - \psi^*(t') \cdot \dot{\psi}(t')}{2} - H(\psi(t'), \psi^*(t')) \right]$$

$$+ \underbrace{\frac{\psi^{(f)*} \cdot \psi(t) + \psi^*(0) \cdot \psi^{(i)}}{2}}_{\text{due to coherent states approach}}$$

Baranger et. al. 2001

- stationary phase approximation leads to van Vleck propagator

$$\left\langle \psi^{(f)} \middle| e^{-\frac{i}{\hbar}\hat{H}t} \middle| \psi^{(i)} \right\rangle \approx \sum_\gamma \left| \frac{i}{\hbar} \frac{\partial^2 R_\gamma}{\partial \psi^{(f)*} \partial \psi^{(i)}} \right|^{1/2}$$

$$\exp\left(\frac{i}{\hbar} R_\gamma - i\nu_\gamma \frac{\pi}{2} + \underbrace{\frac{i}{2\hbar} \int_0^t dt' \ \mathrm{tr} \frac{\partial^2 H}{\partial \psi \partial \psi^*} - \frac{|\psi^{(i)}|^2 + |\psi^{(f)}|^2}{2}}_{\text{due to coherent states approach}} \right)$$

sum over solutions of nonlinear Schrödinger equation

see also Engl, Dujardin, Argülles, Schlagheck, Richter, Urbina 2014

Limit

Limit

- $\hbar \to 0$

Limit

- $\hbar \to 0$
- or particle number $N \to \infty$

Limit

- $\hbar \to 0$
- or particle number $N \to \infty$

  recall Bose-Hubbard model: $H = -\frac{J}{2}\sum_j(\hat{a}^\dagger_{j+1}\hat{a}_j + \hat{a}^\dagger_j\hat{a}_{j+1}) + \frac{U}{2}\sum_j(\hat{a}^\dagger_j)^2\hat{a}^2_j$

Limit

- $\hbar \to 0$
- or particle number $N \to \infty$

  recall Bose-Hubbard model: $H = -\frac{J}{2}\sum_j(\hat{a}_{j+1}^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_{j+1}) + \frac{U}{2}\sum_j(\hat{a}_j^\dagger)^2 \hat{a}_j^2$

  - we have $\sum_j \hat{a}_j^\dagger \hat{a}_j = \hat{N}$ but want to keep $\sum_j |\psi_j|^2$ fixed

Limit

- $\hbar \to 0$
- or particle number $N \to \infty$

  recall Bose-Hubbard model: $H = -\frac{J}{2}\sum_j(\hat{a}^\dagger_{j+1}\hat{a}_j + \hat{a}^\dagger_j\hat{a}_{j+1}) + \frac{U}{2}\sum_j(\hat{a}^\dagger_j)^2\hat{a}^2_j$

  - we have $\sum_j \hat{a}^\dagger_j \hat{a}_j = \hat{N}$ but want to keep $\sum_j |\psi_j|^2$ fixed
    $\Rightarrow$ better scale $\hat{a}_j \to \sqrt{N}\psi_j$, $\hat{a}^\dagger_j \to \sqrt{N}\psi^*_j$

Limit

- $\hbar \to 0$
- or particle number $N \to \infty$

  recall Bose-Hubbard model: $H = -\frac{J}{2}\sum_j(\hat{a}_{j+1}^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_{j+1}) + \frac{U}{2}\sum_j(\hat{a}_j^\dagger)^2\hat{a}_j^2$

  - we have $\sum_j \hat{a}_j^\dagger \hat{a}_j = \hat{N}$ but want to keep $\sum_j |\psi_j|^2$ fixed
    $\Rightarrow$ better scale $\hat{a}_j \to \sqrt{N}\psi_j$, $\hat{a}_j^\dagger \to \sqrt{N}\psi_j^*$
  - for agreement with $\hbar \to 0$ need $U \sim \frac{u}{N}$

Trace formula

accessed from van Vleck propagator as for single-particle systems

Trace formula

accessed from van Vleck propagator as for single-particle systems

but: particle number conservation, $\sum_j |\psi_j|^2 = 1$

Trace formula

accessed from van Vleck propagator as for single-particle systems

but: particle number conservation, $\sum_j |\psi_j|^2 = 1$

- canonical transformation with $P = \sum_j |\psi_j|^2$ as a new generalised momentum

Trace formula

accessed from van Vleck propagator as for single-particle systems

but: particle number conservation, $\sum_j |\psi_j|^2 = 1$

- canonical transformation with $P = \sum_j |\psi_j|^2$ as a new generalised momentum
- H independent of Q, and P can be replaced by 1

Trace formula

accessed from van Vleck propagator as for single-particle systems

but: particle number conservation, $\sum_j |\psi_j|^2 = 1$

- canonical transformation with $P = \sum_j |\psi_j|^2$ as a new generalised momentum
- H independent of Q, and P can be replaced by 1
- system with less dimensions and no conservation law

Trace formula

accessed from van Vleck propagator as for single-particle systems

but: particle number conservation, $\sum_j |\psi_j|^2 = 1$

- canonical transformation with $P = \sum_j |\psi_j|^2$ as a new generalised momentum
- H independent of Q, and P can be replaced by 1
- system with less dimensions and no conservation law

Trace formula

$$d(E) = \bar{d}(E) + \frac{1}{\pi\hbar}\mathrm{Re} \sum_{\text{per. solutions p}} A_p \, e^{iS_p/\hbar}$$

Trace formula

accessed from van Vleck propagator as for single-particle systems

but: particle number conservation, $\sum_j |\psi_j|^2 = 1$

- canonical transformation with $P = \sum_j |\psi_j|^2$ as a new generalised momentum
- H independent of Q, and P can be replaced by 1
- system with less dimensions and no conservation law

Trace formula

$$
\begin{aligned}
d(E) &= \bar{d}(E) + \frac{1}{\pi\hbar}\mathrm{Re}\sum_{\text{per. solutions p}} A_p\, e^{iS_p/\hbar} \\
A_p &= \frac{T_p^{\mathrm{prim}} e^{-i\mu_p\frac{\pi}{2}}}{\sqrt{|\det(M_p - 1)|}}
\end{aligned}
$$

Trace formula

accessed from van Vleck propagator as for single-particle systems

but: particle number conservation, $\sum_j |\psi_j|^2 = 1$

- canonical transformation with $P = \sum_j |\psi_j|^2$ as a new generalised momentum
- H independent of Q, and P can be replaced by 1
- system with less dimensions and no conservation law

Trace formula

$$d(E) = \bar{d}(E) + \frac{1}{\pi\hbar}\mathrm{Re}\sum_{\text{per. solutions p}} A_p\, e^{iS_p/\hbar}$$

$$A_p = \frac{T_p^{\mathrm{prim}} e^{-i\mu_p \frac{\pi}{2}}}{\sqrt{|\det(M_p - 1)|}}$$

M_p = stability matrix relating initial and final deviations in reduced phase space

Spectral statistics

Spectral statistics

Two point correlation function: $R(\epsilon) = \left\langle d\left(E + \frac{\epsilon}{2}\right) d\left(E - \frac{\epsilon}{2}\right)\right\rangle$

Spectral statistics

Two point correlation function: $R(\epsilon) = \left\langle d\left(E + \frac{\epsilon}{2}\right) d\left(E - \frac{\epsilon}{2}\right)\right\rangle$

insert trace formula:

Spectral statistics

Two point correlation function: $R(\epsilon) = \left\langle d\left(E + \frac{\epsilon}{2}\right) d\left(E - \frac{\epsilon}{2}\right)\right\rangle$

insert trace formula:

$$R(\epsilon) \quad \sim \quad 1 + \mathrm{Re} \sum_{p,p'} \left\langle A_p A_{p'}^* e^{i(S_p(E+\epsilon/2) - S_{p'}(E-\epsilon/2))/\hbar} \right\rangle$$

Spectral statistics

Two point correlation function: $R(\epsilon) = \left\langle d\left(E + \frac{\epsilon}{2}\right) d\left(E - \frac{\epsilon}{2}\right)\right\rangle$

insert trace formula:

$$R(\epsilon) \quad \sim \quad 1 + \mathrm{Re} \sum_{p,p'} \left\langle A_p A_{p'}^* \, e^{i(S_p(E+\epsilon/2) - S_{p'}(E-\epsilon/2))/\hbar} \right\rangle$$

$\Rightarrow$ need pairs of orbits with small action difference

Spectral statistics

Two point correlation function: $R(\epsilon) = \left\langle d\left(E + \frac{\epsilon}{2}\right) d\left(E - \frac{\epsilon}{2}\right)\right\rangle$

insert trace formula:

$$R(\epsilon) \quad \sim \quad 1 + \mathrm{Re} \sum_{p,p'} \left\langle A_p A_{p'}^* e^{i(S_p(E+\epsilon/2) - S_{p'}(E-\epsilon/2))/\hbar} \right\rangle$$

$\Rightarrow$ need pairs of orbits with small action difference

- identical and time-reversed orbits (diagonal approximation)

  Berry 1985; Hannay & Ozorio de Almeida 1985

Spectral statistics

Two point correlation function: $R(\epsilon) = \left\langle d\left(E + \frac{\epsilon}{2}\right) d\left(E - \frac{\epsilon}{2}\right)\right\rangle$

insert trace formula:

$$R(\epsilon) \quad \sim \quad 1 + \mathrm{Re} \sum_{p,p'} \left\langle A_p A_{p'}^* e^{i(S_p(E+\epsilon/2) - S_{p'}(E-\epsilon/2))/\hbar} \right\rangle$$

$\Rightarrow$ need pairs of orbits with small action difference

- identical and time-reversed orbits (diagonal approximation)

  Berry 1985; Hannay & Ozorio de Almeida 1985

- pairs of orbits differing in encounters

Spectral statistics

Two point correlation function: $R(\epsilon) = \left\langle d\left(E + \frac{\epsilon}{2}\right) d\left(E - \frac{\epsilon}{2}\right)\right\rangle$

insert trace formula:

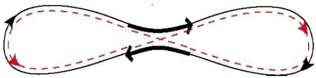
$$R(\epsilon) \quad \sim \quad 1 + \mathrm{Re} \sum_{p,p'} \left\langle A_p A_{p'}^* e^{i(S_p(E+\epsilon/2) - S_{p'}(E-\epsilon/2))/\hbar} \right\rangle$$

$\Rightarrow$ need pairs of orbits with small action difference

- identical and time-reversed orbits (diagonal approximation)

  Berry 1985; Hannay & Ozorio de Almeida 1985

- pairs of orbits differing in encounters



Spectral statistics

Two point correlation function: $R(\epsilon) = \left\langle d\left(E + \frac{\epsilon}{2}\right) d\left(E - \frac{\epsilon}{2}\right)\right\rangle$

insert trace formula:

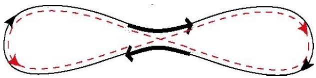
$$R(\epsilon) \quad \sim \quad 1 + \mathrm{Re} \sum_{p,p'} \left\langle A_p A_{p'}^* e^{i(S_p(E+\epsilon/2) - S_{p'}(E-\epsilon/2))/\hbar} \right\rangle$$

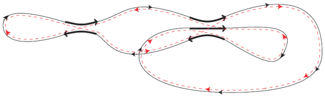
$\Rightarrow$ need pairs of orbits with small action difference

- identical and time-reversed orbits (diagonal approximation)

  Berry 1985; Hannay & Ozorio de Almeida 1985

- pairs of orbits differing in encounters





Spectral statistics

Two point correlation function: $R(\epsilon) = \left\langle d\left(E + \frac{\epsilon}{2}\right) d\left(E - \frac{\epsilon}{2}\right)\right\rangle$

insert trace formula:

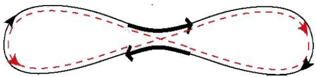
$$R(\epsilon) \quad \sim \quad 1 + \mathrm{Re} \sum_{p,p'} \left\langle A_p A_{p'}^* \, e^{i(S_p(E+\epsilon/2) - S_{p'}(E-\epsilon/2))/\hbar} \right\rangle$$

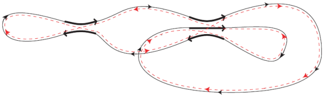
$\Rightarrow$ need pairs of orbits with small action difference

- identical and time-reversed orbits (diagonal approximation)

  Berry 1985; Hannay & Ozorio de Almeida 1985

- pairs of orbits differing in encounters





Sieber & Richter 2001; S.M., Heusler, Braun, Haake Altland 2004 & 2005

Spectral statistics

Spectral statistics

- these explain non-oscillatory terms in

Spectral statistics

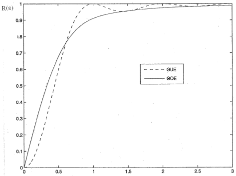
- these explain non-oscillatory terms in

$$R(\epsilon) = \begin{cases} \mathrm{Re}\left(1 - \frac{1}{2(\pi\epsilon)^2} + \frac{1}{2(\pi\epsilon)^2} e^{2\pi i\epsilon} \right) & \text{no time rev. inv. (GUE)} \\ \mathrm{Re}\left(\sum_n c_n \left(\frac{1}{\epsilon}\right)^n + \sum_n d_n \left(\frac{1}{\epsilon}\right)^n e^{2\pi i\epsilon} \right) & \text{with time rev. inv. (GOE)} \end{cases}$$

Spectral statistics

- these explain non-oscillatory terms in

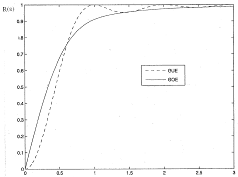
$$R(\epsilon) = \begin{cases} \mathrm{Re}\left(1 - \frac{1}{2(\pi\epsilon)^2} + \frac{1}{2(\pi\epsilon)^2} e^{2\pi i \epsilon} \right) & \text{no time rev. inv. (GUE)} \\ \mathrm{Re}\left(\sum_n c_n \left(\frac{1}{\epsilon}\right)^n + \sum_n d_n \left(\frac{1}{\epsilon}\right)^n e^{2\pi i \epsilon} \right) & \text{with time rev. inv. (GOE)} \end{cases}$$



Spectral statistics

- these explain non-oscillatory terms in

$$R(\epsilon) = \begin{cases} \mathrm{Re}\left(1 - \frac{1}{2(\pi\epsilon)^2} + \frac{1}{2(\pi\epsilon)^2} e^{2\pi i\epsilon} \right) & \text{no time rev. inv. (GUE)} \\ \mathrm{Re}\left(\sum_n c_n \left(\frac{1}{\epsilon}\right)^n + \sum_n d_n \left(\frac{1}{\epsilon}\right)^n e^{2\pi i\epsilon} \right) & \text{with time rev. inv. (GOE)} \end{cases}$$

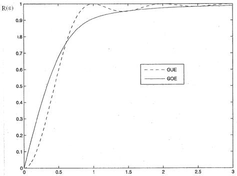


- for oscillatory terms:

Spectral statistics

- these explain non-oscillatory terms in

$$R(\epsilon) = \begin{cases} \mathrm{Re}\left(1 - \frac{1}{2(\pi\epsilon)^2} + \frac{1}{2(\pi\epsilon)^2} e^{2\pi i\epsilon} \right) & \text{no time rev. inv. (GUE)} \\ \mathrm{Re}\left(\sum_n c_n \left(\frac{1}{\epsilon}\right)^n + \sum_n d_n \left(\frac{1}{\epsilon}\right)^n e^{2\pi i\epsilon} \right) & \text{with time rev. inv. (GOE)} \end{cases}$$

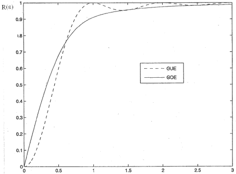


- for oscillatory terms: improved semiclassical approximation,

Spectral statistics

- these explain non-oscillatory terms in

$$R(\epsilon) = \begin{cases} \mathrm{Re}\left(1 - \frac{1}{2(\pi\epsilon)^2} + \frac{1}{2(\pi\epsilon)^2} e^{2\pi i\epsilon} \right) & \text{no time rev. inv. (GUE)} \\ \mathrm{Re}\left(\sum_n c_n \left(\frac{1}{\epsilon}\right)^n + \sum_n d_n \left(\frac{1}{\epsilon}\right)^n e^{2\pi i\epsilon} \right) & \text{with time rev. inv. (GOE)} \end{cases}$$

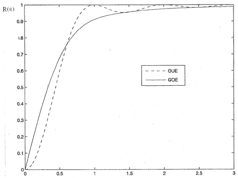


- for oscillatory terms: improved semiclassical approximation, via spectral determinant $\det(E - H)$

Spectral statistics

- these explain non-oscillatory terms in

$$R(\epsilon) = \begin{cases} \mathrm{Re}\left(1 - \frac{1}{2(\pi\epsilon)^2} + \frac{1}{2(\pi\epsilon)^2} e^{2\pi i\epsilon} \right) & \text{no time rev. inv. (GUE)} \\ \mathrm{Re}\left(\sum_n c_n \left(\frac{1}{\epsilon}\right)^n + \sum_n d_n \left(\frac{1}{\epsilon}\right)^n e^{2\pi i\epsilon} \right) & \text{with time rev. inv. (GOE)} \end{cases}$$



- for oscillatory terms: improved semiclassical approximation, via spectral determinant $\det(E - H)$

Berry & Keating 1990; Heusler et al 2007; Keating & S.M. 2007; S.M., Heusler, Altland, Braun, Haake 2009

Discrete symmetries

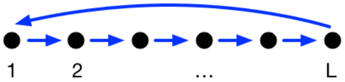
Discrete symmetries

using Robbins 89; Keating, Robbins 97; Joyner, S.M., Sieber 12

Discrete symmetries

using Robbins 89; Keating, Robbins 97; Joyner, S.M., Sieber 12

discrete translation symmetry

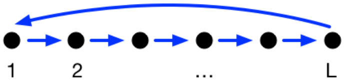


reflection symmetry

Discrete symmetries

using Robbins 89; Keating, Robbins 97; Joyner, S.M., Sieber 12

discrete translation symmetry



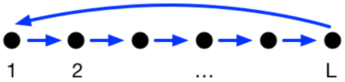
reflection symmetry

- consider statistics in subspectra associated to symmetries

Discrete symmetries

using Robbins 89; Keating, Robbins 97; Joyner, S.M., Sieber 12

discrete translation symmetry



reflection symmetry

- consider statistics in subspectra associated to symmetries
- here all subspectra have GOE statistics

Conditions

Conditions

- for full agreement with RMT:

Conditions

- for full agreement with RMT:
  - ergodicity

Conditions

- for full agreement with RMT:
    - ergodicity
    - hyperbolicity

Conditions

- for full agreement with RMT:
  - ergodicity
  - hyperbolicity
  - for sums over orbits: need spectral gap

Conditions

- for full agreement with RMT:
  - ergodicity
  - hyperbolicity
  - for sums over orbits: need spectral gap

    i.e. Frobenius Perron operator P_t for phase space densities $\rho(\boldsymbol{x})$

    $$\rho_t(\boldsymbol{x}) = (P_t\, \rho_0)(\boldsymbol{x})$$

    has eigenvalues $e^{-\nu_j t}$ with $e^{-\nu_1 t} = 1$ (ergodic mode) and other eigenvalues bounded away from unit circle

Conditions

- for full agreement with RMT:
  - ergodicity
  - hyperbolicity
  - for sums over orbits: need spectral gap

    i.e. Frobenius Perron operator P_t for phase space densities $\rho(\boldsymbol{x})$

    $$\rho_t(\boldsymbol{x}) = (P_t\,\rho_0)(\boldsymbol{x})$$

    has eigenvalues $e^{-\nu_j t}$ with $e^{-\nu_1 t} = 1$ (ergodic mode) and other eigenvalues bounded away from unit circle
- in practice: small stability islands

Conclusions

Conclusions

- properties of many-body quantum systems approximated as sum over solutions of nonlinear Schödinger equation

Conclusions

- properties of many-body quantum systems approximated as sum over solutions of nonlinear Schödinger equation

  can study interference between solutions

Conclusions

- properties of many-body quantum systems approximated as sum over solutions of nonlinear Schödinger equation

  can study interference between solutions

- chaotic many-body systems e.g. Bose Hubbard model have spectral statistics in line with RMT (under certain conditions)