Coherent backscattering in the Fock space of a disordered Bose-Hubbard system

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Outline

- Introduction to coherent backscattering (CBS)
- Semiclassical theory of Bose-Hubbard systems
- Numerical results for the backscattering probability
- Implication for quantum thermalization
- Proposal for an experimental verification
- Conclusion
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- Implication for quantum thermalization
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- Semiclassical theory of Bose-Hubbard systems
- Conclusion
Proposal for a many-body CBS experiment

Consider an isolated 2D sheet of a 3D optical lattice . . .
Proposal for a many-body CBS experiment

Consider an isolated 2D sheet of a 3D optical lattice within which you isolate a single plaquette (e.g. by means of a focused red-detuned laser beam)
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Proposal for a many-body CBS experiment

Experimental procedure:

1. Load the lattice with a well-defined number of (bosonic) atoms in the deep Mott-insulator regime
2. Add some disorder (by means of an optical speckle field) and/or randomly displace the focus of the red-detuned laser beam
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3. Switch on the inter-site hopping and let the atoms move . . .
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4. Quench back to the Mott regime after a given evolution time and detect the atomic population on each site

W. Bakr et al., Nature 462, 74 (2009)

S. Fölling et al., Nature 448, 1029 (2007) (Brillouin zone mapping)
Proposal for a many-body CBS experiment

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W. Bakr et al., Nature 462, 74 (2009)
S. Fölling et al., Nature 448, 1029 (2007)  (Brillouin zone mapping)
5. Repeat the experiment with the \textit{same initial state} but for a different disorder configuration.
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Proposal for a many-body CBS experiment

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Proposal for a many-body CBS experiment

Average population per site: $7/6 = 1.167$

... but we are now interested in the full statistical information of the experimental outcomes
Proposal for a many-body CBS experiment

General expectation from quantum statistical physics:

→ all quantum states that have about the same total energy as the initial state are equally likely to be detected

Coherent backscattering in Fock space  
Proposal for a many-body CBS experiment

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Statistics after 1000 measurements

Coherent backscattering in Fock space

Proposal for a many-body CBS experiment

General expectation from quantum statistical physics:

→ all quantum states that have about the same total energy as the initial state are equally likely to be detected

Statistics after 2000 measurements

Coherent backscattering in Fock space

Proposal for a many-body CBS experiment

General expectation from quantum statistical physics:

\[ \rightarrow \text{all quantum states that have about the same total energy as the initial state are equally likely to be detected} \]

Statistics after 5000 measurements

Coherent backscattering in Fock space

Proposal for a many-body CBS experiment

General expectation from quantum statistical physics:

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Statistics after 10000 measurements

Coherent backscattering in Fock space

Proposal for a many-body CBS experiment

General expectation from quantum statistical physics:

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Statistics after 20000 measurements

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Coherent backscattering in Fock space

Proposal for a many-body CBS experiment

General expectation from quantum statistical physics:

→ all quantum states that have about the same total energy as the initial state are equally likely to be detected

Statistics after 50000 measurements
Proposal for a many-body CBS experiment

General expectation from quantum statistical physics:

→ all quantum states that have about the same total energy as the initial state are equally likely to be detected

Statistics after 100000 measurements

Coherent backscattering in Fock space

Proposal for a many-body CBS experiment

General expectation from quantum statistical physics:

\[ \rightarrow \text{all quantum states that have about the same total energy as the initial state are equally likely to be detected} \]

This is not the case for the initial state which is twice as often detected as other states with comparable total energy.

Coherent backscattering in Fock space

Proposal for a many-body CBS experiment

General expectation from quantum statistical physics:

→ all quantum states that have about the same total energy as the initial state are equally likely to be detected

This is not the case for the initial state which is twice as often detected as other states with comparable total energy

→ signature of coherent backscattering in Fock space
Coherent backscattering in disordered systems

constructive wave interference between reflected classical paths and their time-reversed counterparts
Coherent backscattering in disordered systems

→ constructive wave interference between reflected classical paths and their time-reversed counterparts

Coherent backscattering of laser light in disordered media

M. P. Van Albada and A. Lagendijk, PRL 55, 2692 (1985)
P.-E. Wolf and G. Maret, PRL 55, 2696 (1985)
Coherent backscattering in disordered systems

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→ generalization to interacting many-body systems?

Coherent backscattering in Fock space
Many-body CBS in disordered Bose-Hubbard rings

\[ \hat{H} = \sum_{l=1}^{L} \left[ E_l \hat{a}^\dagger_l \hat{a}_l - J \left( \hat{a}_l^\dagger \hat{a}_{l-1} + \hat{a}_{l-1}^\dagger \hat{a}_l \right) + \frac{U}{2} \hat{a}_l^\dagger \hat{a}_l \hat{a}_l^\dagger \hat{a}_l \right] \]
Many-body CBS in disordered Bose-Hubbard rings

\[ \hat{H} = \sum_{l=1}^{L} \left[ E_l \hat{a}_l \hat{a}_l \hat{a}_l + J \left( \hat{a}_l \hat{a}_{l-1} + \hat{a}_{l-1} \hat{a}_l \right) + \frac{U}{2} \hat{a}_l \hat{a}_l \hat{a}_l \hat{a}_l \right] \]

Classical description: discrete Gross-Pitaevskii equation

\[ i\hbar \frac{\partial}{\partial t} \psi_l(t) = E_l \psi_l(t) - J \left[ \psi_{l+1}(t) + \psi_{l-1}(t) \right] + U \left( |\psi_l(t)|^2 - 1 \right) \psi_l(t) \]
Semiclassical van Vleck-Gutzwiller theory:

Represent the quantum transition amplitude

\[ \langle n^f | \hat{U} | n^i \rangle \equiv \langle n^f | \exp[-i\frac{\hbar}{\hbar} t \hat{H}] | n^i \rangle = \sum_\gamma A_\gamma e^{iR_\gamma / \hbar} \]

in terms of classical (Gross-Pitaevskii) trajectories \( \gamma \) going from \( \psi_l(0) = \sqrt{n_i^l + 0.5} e^{i\theta_i^l} \) to \( \psi_l(t) = \sqrt{n_f^l + 0.5} e^{i\theta_f^l} \) for all \( l = 1, \ldots, L \) with some arbitrary phases \( 0 \leq \theta_i^l / f < 2\pi \)

\( |n^{i(f)}\rangle \equiv |n_1^{i(f)} \ldots n_L^{i(f)}\rangle \): initial (final) Fock state on the ring

\( R_\gamma \) = classical action of the trajectory \( \gamma \)

\( A_\gamma \) = stability amplitude (related to Lyapunov exponent) of \( \gamma \)

Coherent backscattering in Fock space

Semiclassical van Vleck-Gutzwiller theory:

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in terms of classical (Gross-Pitaevskii) trajectories $\gamma$ going from $\psi_l(0) = \sqrt{n_i^l + 0.5 e^{i\theta^l_i}}$ to $\psi_l(t) = \sqrt{n_{f}^l + 0.5 e^{i\theta^f_l}}$

for all $l = 1, \ldots, L$ with some arbitrary phases $0 \leq \theta_i^{i/f} < 2\pi$

Average detection probability of the Fock state $| n^f \rangle$:

$$|\langle n^f | \hat{U} | n^i \rangle|^2 = \sum_{\gamma, \gamma'} A_{\gamma} A_{\gamma'} e^{i(R_{\gamma} - R_{\gamma'})/\hbar}$$

$$= 0 \text{ if } R_{\gamma} \neq R_{\gamma'}$$

Coherent backscattering in Fock space

Semiclassical van Vleck-Gutzwiller theory:

\[ \langle n^f | \hat{U} | n^i \rangle \equiv \langle n^f | \exp[-i \frac{\hat{t}}{\hbar} \hat{H}] | n^i \rangle = \sum_{\gamma} A_{\gamma} e^{i R_{\gamma} / \hbar} \]

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for all \( l = 1, \ldots, L \) with some arbitrary phases \( 0 \leq \theta_i^l / \theta_f^l < 2\pi \)

Average detection probability of the Fock state \( |n^f\rangle \): \[
|\langle n^f | \hat{U} | n^i \rangle|^2 = \sum_{\gamma} |A_{\gamma}|^2 \text{ if } n^f \neq n^i \]

\[
|\langle n^i | \hat{U} | n^i \rangle|^2 = 2 \sum_{\gamma} |A_{\gamma}|^2 \text{ due to CBS}
\]

in the presence of chaos (ergodicity)
Semiclassical van Vleck-Gutzwiller theory:

\[ \langle n^f | \hat{U} | n^i \rangle \equiv \langle n^f | \exp[-i \frac{t}{\hbar} \hat{H}] | n^i \rangle = \sum_{\gamma} A_{\gamma} e^{i R_{\gamma} / \hbar} \]

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Average detection probability of the Fock state \( |n^f\rangle \):

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\]
\[
\hat{H} = \sum_{l=1}^{L} \left[ E_l \hat{a}_l^{\dagger} \hat{a}_l - J \left( \hat{a}^{\dagger}_l \hat{a}_{l-1} + \hat{a}^{\dagger}_{l-1} \hat{a}_l \right) + \frac{U}{2} \hat{a}^{\dagger}_l \hat{a}^{\dagger}_l \hat{a}_l \hat{a}_l \right]
\]

Detection probability

Coherent backscattering in Fock space

\[U = J\]

\[0 \leq E_l \leq 2J\]
Comparison with numerical data

\[ \hat{H} = \sum_{l=1}^{L} \left[ E_l \hat{a}_l^{\dagger} \hat{a}_l - J \left( \hat{a}_l^{\dagger} \hat{a}_{l-1} + \hat{a}_{l-1}^{\dagger} \hat{a}_l \right) + \frac{U}{2} \hat{a}_l^{\dagger} \hat{a}_l^{\dagger} \hat{a}_l \hat{a}_l \right] \]

Detection probability

\[ \langle n^f | \hat{U} | n^i \rangle \rvert^2 \text{classical} = \sum_{\gamma} |A_{\gamma}|^2 \]

\[ = \int_0^{2\pi} \frac{d\theta_2^i}{2\pi} \cdots \int_0^{2\pi} \frac{d\theta_L^i}{2\pi} \prod_{l=2}^{L} \delta \left( n_l^f + 0.5 - |\psi_l(t; n_1^i, 0, n_2^i, \theta_2^i \ldots n_L^i, \theta_L^i)|^2 \right) \]

Coherent backscattering in Fock space

Comparison with numerical data

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\[ \hat{H} = \sum_{l=1}^{L} \left[ E_l \hat{a}_l \hat{a}_l^\dagger - J \left( \hat{a}_l \hat{a}_{l-1} + \hat{a}_{l-1} \hat{a}_l \right) + \frac{U}{2} \hat{a}_l \hat{a}_l^\dagger \hat{a}_l \hat{a}_l^\dagger \right] \]

Detection probability

Ultimate experimental verification of CBS:

\[ \rightarrow \text{break time-reversal invariance by a synthetic gauge field} \]


J. Struck et al., Science 333, 996 (2011)
Comparison with numerical data

\[ \hat{H} = \sum_{l=1}^{L} \left[ E_l \hat{a}_l^\dagger \hat{a}_l - J \left( \hat{a}_l^\dagger \hat{a}_{l-1} e^{i\phi} + \hat{a}_{l-1}^\dagger \hat{a}_l e^{-i\phi} \right) + \frac{U}{2} \hat{a}_l^\dagger \hat{a}_{l-1}^\dagger \hat{a}_{l-1} \hat{a}_l \right] \]

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Some conclusions

Coherent backscattering in the Fock space of a Bose-Hubbard system

- privileges, on average, the initial Fock state as compared to other states with comparable energy,
- significantly affects quantum ergodicity in finite systems, even if the classical dynamics is fully ergodic,
- can be experimentally detected with ultracold atoms,
- relies on time-reversal invariance and can therefore be switched off with a synthetic gauge field,

T. Engl, J. Dujardin, A. Argüelles, P.S., K. Richter, and J. D. Urbina,