Coherent backscattering in the Fock space of a disordered Bose-Hubbard system

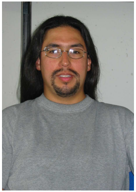
Peter Schlagheck

Université de Liège

20/3/2015

Coworkers

Thomas Engl
(Regensburg)



Juan Diego Urbina
(Regensburg)

Klaus Richter
(Regensburg)

Julien Dujardin
(Liège)

Arturo Argüelles
(now in Cali)

Outline

- Introduction to coherent backscattering (CBS)
- Semiclassical theory of Bose-Hubbard systems
- Numerical results for the backscattering probability
- Implication for quantum thermalization
- Proposal for an experimental verification
- Conclusion

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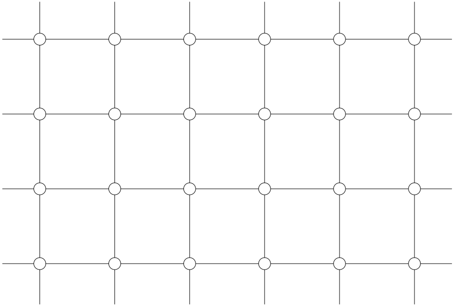
Proposal for a many-body CBS experiment



Consider an isolated 2D sheet of a 3D optical lattice . . .

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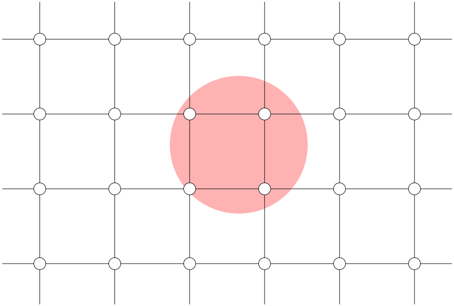

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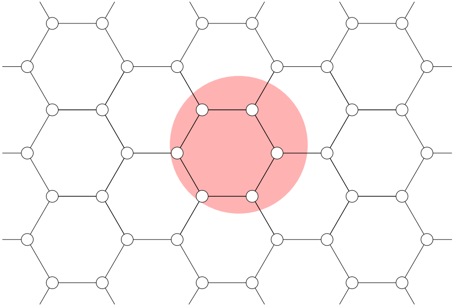

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L. Tarruell *et al.*, Nature 483, 302 (2012)

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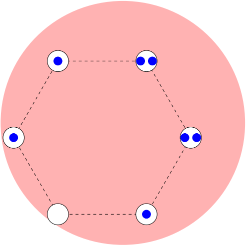

Experimental procedure:

1. Load the lattice with a well-defined number of (bosonic) atoms in the deep Mott-insulator regime

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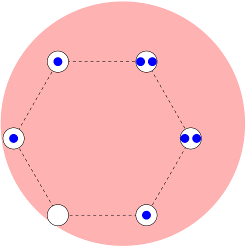

2. Add some disorder (by means of an optical speckle field) and/or randomly displace the focus of the red-detuned laser beam

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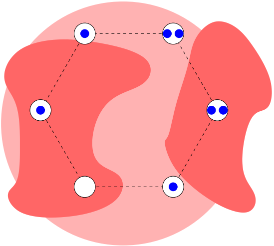

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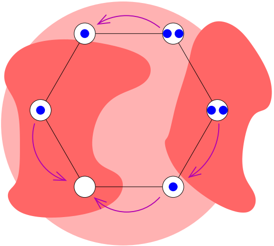

3. Switch on the inter-site hopping and let the atoms move . . .

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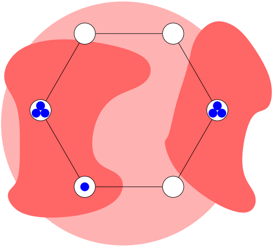

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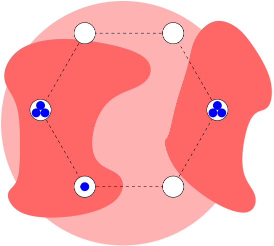

4. Quench back to the Mott regime after a given evolution time and detect the atomic population on each site

W. Bakr *et al.*, Nature 462, 74 (2009)

J. Sherson *et al.*, Nature 467, 68 (2010)

S. Fölling *et al.*, Nature 448, 1029 (2007) (Brillouin zone mapping)

Proposal for a many-body CBS experiment

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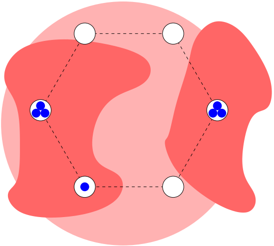

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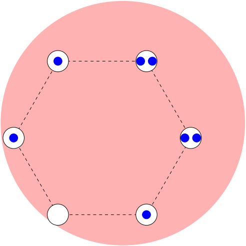

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5. Repeat the experiment with the same initial state but for a different disorder configuration

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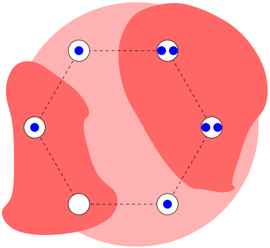

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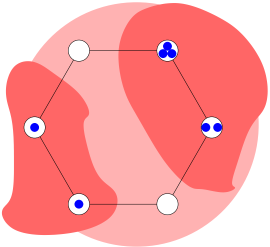

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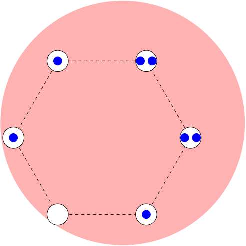

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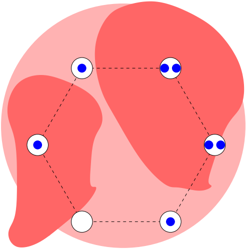

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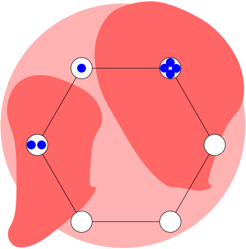

3	0	0	3	0	1
1	0	3	2	0	1
2	1	4	0	0	0

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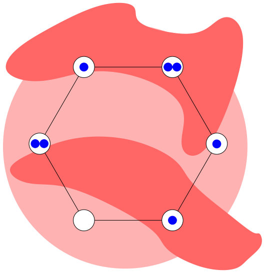

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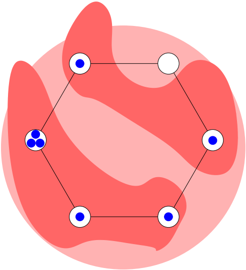

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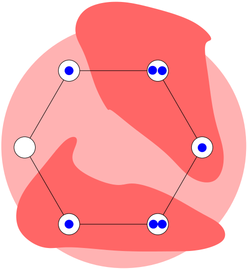

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Phys. Rev. Lett. 112, 140403 (2014)

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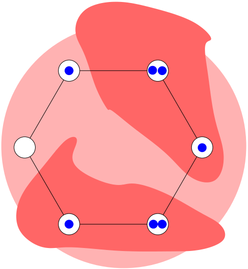

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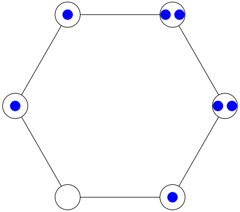
Average population per site: $7/6 = 1.167$

. . . but we are now interested in the full statistical information of the experimental outcomes

Proposal for a many-body CBS experiment

General expectation from quantum statistical physics:

$\longrightarrow$ all quantum states that have about the same total energy as the initial state are equally likely to be detected



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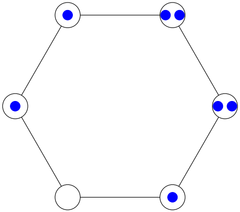

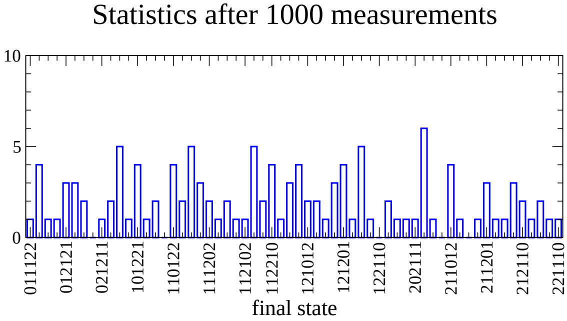

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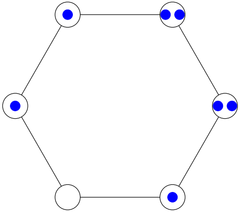

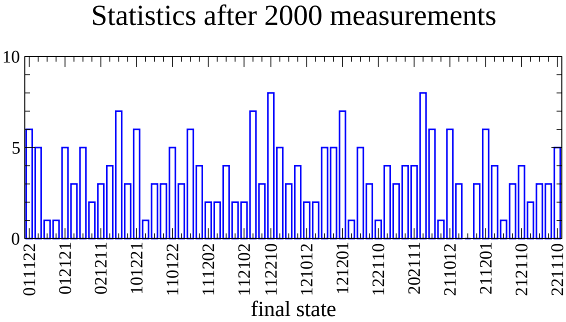

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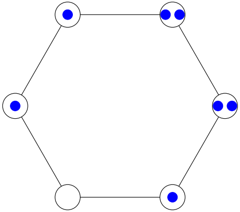

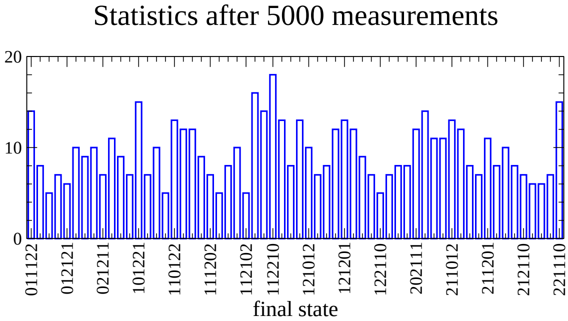

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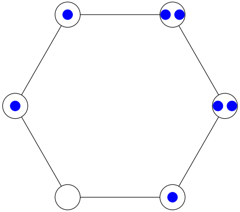

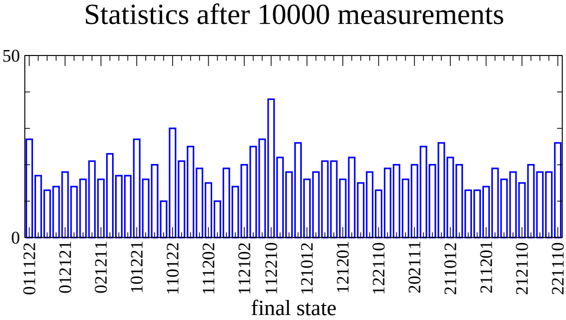

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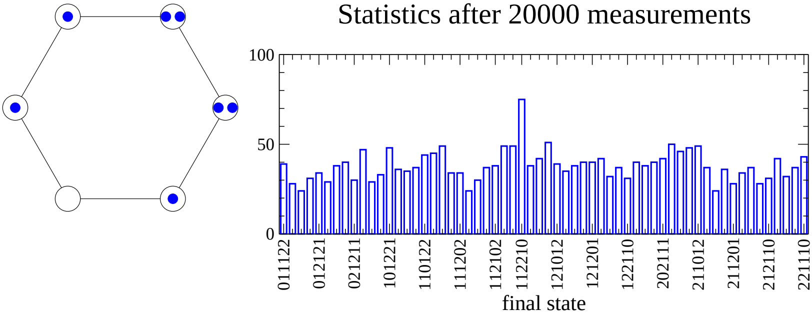

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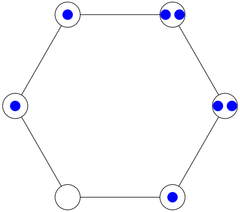

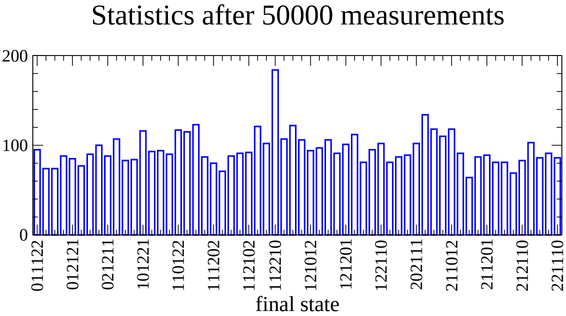

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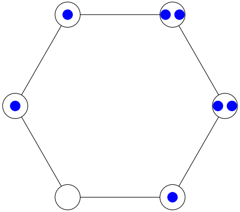

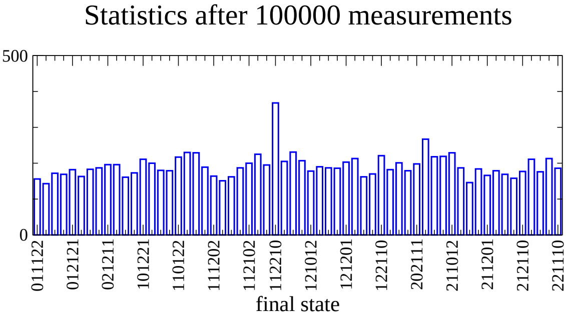

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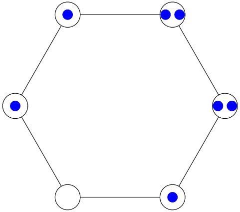

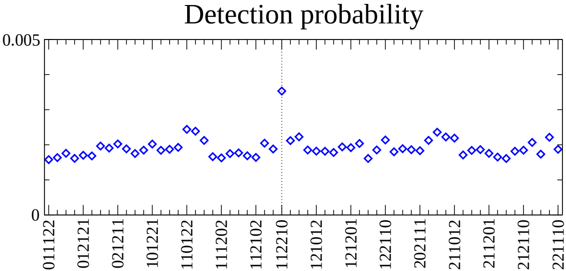

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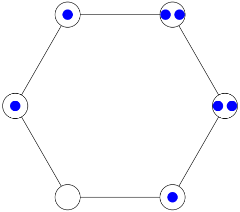

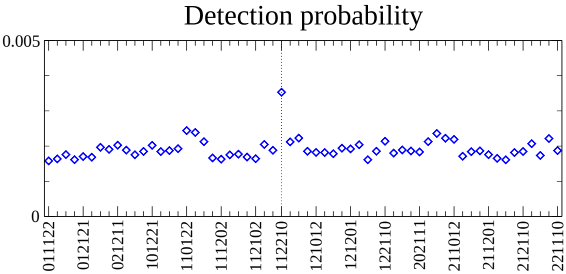

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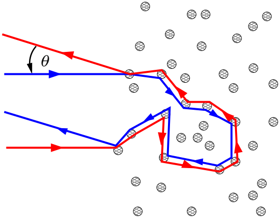
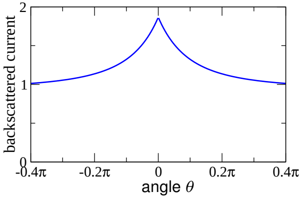
$\longrightarrow$ signature of coherent backscattering in Fock space

Coherent backscattering in disordered systems

→ constructive wave interference between reflected classical paths and their time-reversed counterparts





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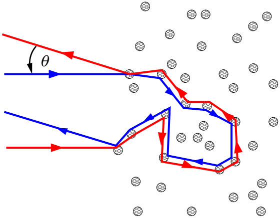

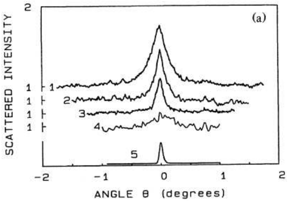

- Coherent backscattering of laser light in disordered media

  M. P. Van Albada and A. Lagendijk, PRL 55, 2692 (1985)

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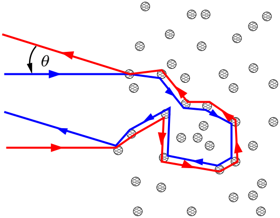

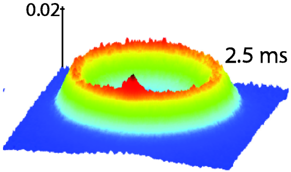

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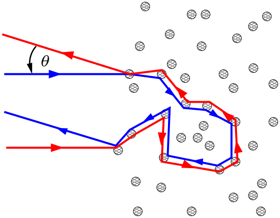

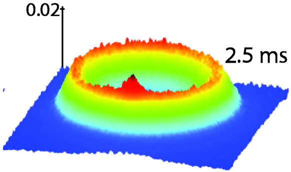

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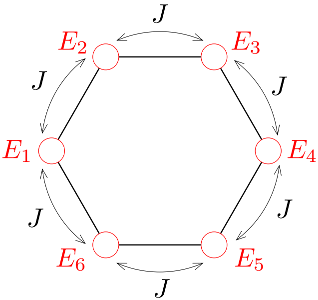
→ generalization to interacting many-body systems?

Many-body CBS in disordered Bose-Hubbard rings

$$\hat{H} = \sum_{l=1}^{L} \left[E_l \hat{a}_l^\dagger \hat{a}_l - J \left(\hat{a}_l^\dagger \hat{a}_{l-1} + \hat{a}_{l-1}^\dagger \hat{a}_l \right) + \frac{U}{2} \hat{a}_l^\dagger \hat{a}_l^\dagger \hat{a}_l \hat{a}_l \right]$$



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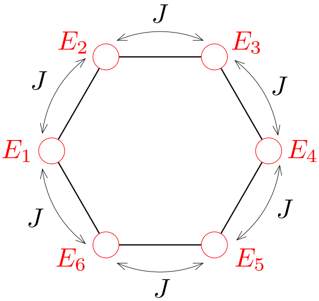

Classical description: discrete Gross-Pitaevskii equation

$$i\hbar \frac{\partial}{\partial t} \psi_l(t) = E_l \psi_l(t) - J \left[\psi_{l+1}(t) + \psi_{l-1}(t) \right] + U \left(|\psi_l(t)|^2 - 1 \right) \psi_l(t)$$

Many-body CBS in disordered Bose-Hubbard rings

Semiclassical van Vleck-Gutzwiller theory:

$\longrightarrow$ Represent the quantum transition amplitude

$$\langle \mathbf{n}^{\mathrm{f}} | \hat{U} | \mathbf{n}^{\mathrm{i}} \rangle \equiv \langle \mathbf{n}^{\mathrm{f}} | \exp[-\tfrac{i}{\hbar} t \hat{H}] | \mathbf{n}^{\mathrm{i}} \rangle = \sum_\gamma A_\gamma e^{i R_\gamma / \hbar}$$

in terms of classical (Gross-Pitaevskii) trajectories γ going from $\psi_l(0) = \sqrt{n_l^{\mathrm{i}} + 0.5}\, e^{i\theta_l^{\mathrm{i}}}$ to $\psi_l(t) = \sqrt{n_l^{\mathrm{f}} + 0.5}\, e^{i\theta_l^{\mathrm{f}}}$ for all $l = 1, \ldots, L$ with some arbitrary phases $0 \leq \theta_l^{\mathrm{i/f}} < 2\pi$

$|\mathbf{n}^{\mathrm{i(f)}}\rangle \equiv |n_1^{\mathrm{i(f)}} \ldots n_L^{\mathrm{i(f)}}\rangle$: initial (final) Fock state on the ring

$R_\gamma =$ classical action of the trajectory γ

$A_\gamma =$ stability amplitude (related to Lyapunov exponent) of γ

Many-body CBS in disordered Bose-Hubbard rings

Semiclassical van Vleck-Gutzwiller theory:

$\longrightarrow$ Represent the quantum transition amplitude

$$\langle \mathbf{n}^{\mathrm{f}} | \hat{U} | \mathbf{n}^{\mathrm{i}} \rangle \equiv \langle \mathbf{n}^{\mathrm{f}} | \exp[-\tfrac{i}{\hbar} t \hat{H}] | \mathbf{n}^{\mathrm{i}} \rangle = \textstyle\sum_\gamma A_\gamma e^{i R_\gamma / \hbar}$$

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Average detection probability of the Fock state $|\mathbf{n}^{\mathrm{f}}\rangle$:

$$\overline{|\langle \mathbf{n}^{\mathrm{f}} | \hat{U} | \mathbf{n}^{\mathrm{i}} \rangle|^2} = \textstyle\sum_{\gamma,\gamma'} \underbrace{\overline{A_\gamma A_{\gamma'} e^{i(R_\gamma - R_{\gamma'})/\hbar}}}_{= 0 \text{ if } R_\gamma \neq R_{\gamma'}}$$

Many-body CBS in disordered Bose-Hubbard rings

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$\longrightarrow$ Represent the quantum transition amplitude

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$\overline{|\langle \mathbf{n}^{\mathrm{f}} | \hat{U} | \mathbf{n}^{\mathrm{i}} \rangle|^2} = \sum_\gamma |A_\gamma|^2$ if $\mathbf{n}^{\mathrm{f}} \neq \mathbf{n}^{\mathrm{i}}$

$\overline{|\langle \mathbf{n}^{\mathrm{i}} | \hat{U} | \mathbf{n}^{\mathrm{i}} \rangle|^2} = 2 \sum_\gamma |A_\gamma|^2$ due to CBS

in the presence of chaos (ergodicity)

Many-body CBS in disordered Bose-Hubbard rings

Semiclassical van Vleck-Gutzwiller theory:

$\longrightarrow$ Represent the quantum transition amplitude

$$\langle \mathbf{n}^{\mathrm{f}} | \hat{U} | \mathbf{n}^{\mathrm{i}} \rangle \equiv \langle \mathbf{n}^{\mathrm{f}} | \exp[-\tfrac{i}{\hbar} t \hat{H}] | \mathbf{n}^{\mathrm{i}} \rangle = \sum_\gamma A_\gamma e^{i R_\gamma / \hbar}$$

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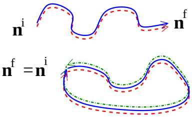
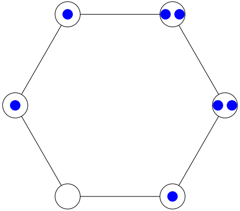
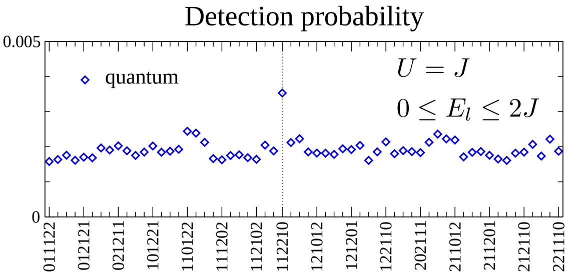


Comparison with numerical data

$$\hat{H} = \sum_{l=1}^{L} \left[E_l \hat{a}_l^\dagger \hat{a}_l - J \left(\hat{a}_l^\dagger \hat{a}_{l-1} + \hat{a}_{l-1}^\dagger \hat{a}_l \right) + \frac{U}{2} \hat{a}_l^\dagger \hat{a}_l^\dagger \hat{a}_l \hat{a}_l \right]$$





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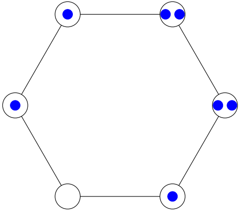

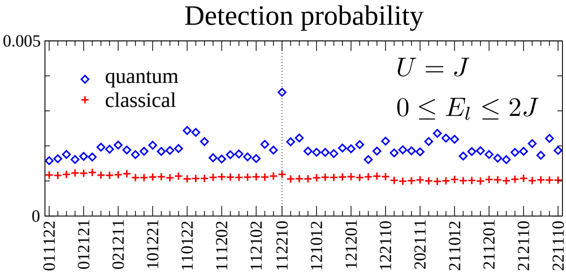

$$\overline{\langle \mathbf{n}^{\mathrm{f}} | \hat{U} | \mathbf{n}^{\mathrm{i}} \rangle |^2}^{\text{classical}} = \sum_{\gamma} |A_\gamma|^2$$

$$= \int_0^{2\pi} \frac{d\theta_2^{\mathrm{i}}}{2\pi} \cdots \int_0^{2\pi} \frac{d\theta_L^{\mathrm{i}}}{2\pi} \prod_{l=2}^{L} \delta \left(n_l^{\mathrm{f}} + 0.5 - |\psi_l(t; n_1^{\mathrm{i}}, 0, n_2^{\mathrm{i}}, \theta_2^{\mathrm{i}} \ldots n_L^{\mathrm{i}}, \theta_L^{\mathrm{i}})|^2 \right)$$

Comparison with numerical data

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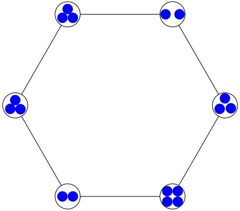

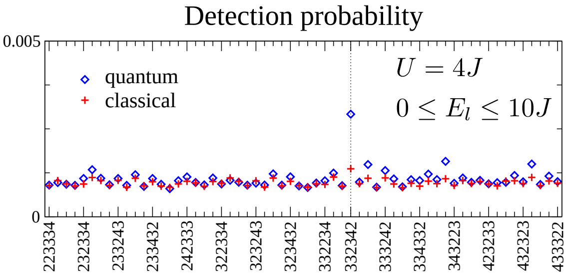

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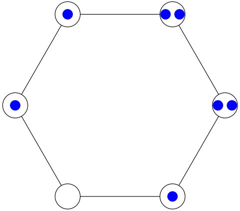
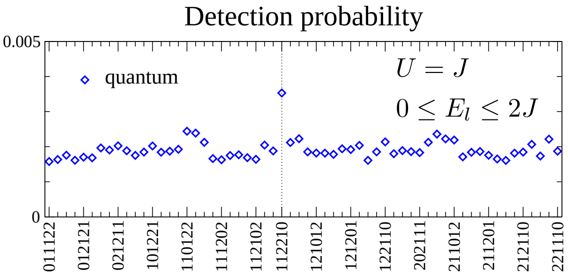




Ultimate experimental verification of CBS:

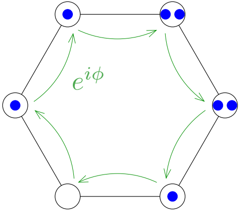
$\longrightarrow$ break time-reversal invariance by a synthetic gauge field

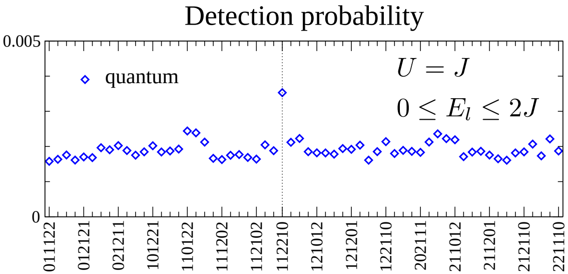
Y.-J. Lin *et al.*, Nature 462, 628 (2009)

J. Struck *et al.*, Science 333, 996 (2011)

Comparison with numerical data

$$\hat{H} = \sum_{l=1}^{L} \left[E_l \hat{a}_l^\dagger \hat{a}_l - J \left(\hat{a}_l^\dagger \hat{a}_{l-1} e^{i\phi} + \hat{a}_{l-1}^\dagger \hat{a}_l e^{-i\phi} \right) + \frac{U}{2} \hat{a}_l^\dagger \hat{a}_l^\dagger \hat{a}_l \hat{a}_l \right]$$





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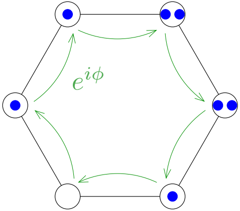

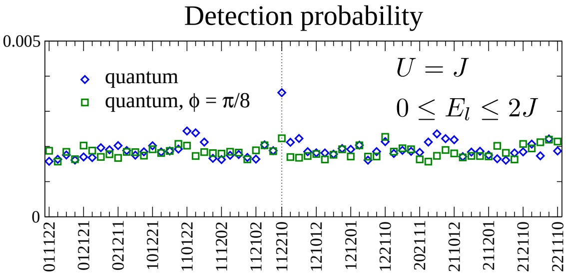

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Some conclusions

Coherent backscattering in the Fock space of a Bose-Hubbard system

- privileges, on average, the initial Fock state as compared to other states with comparable energy,
- significantly affects quantum ergodicity in finite systems, even if the classical dynamics is fully ergodic,
- can be experimentally detected with ultracold atoms,
- relies on time-reversal invariance and can therefore be switched off with a synthetic gauge field,

T. Engl, J. Dujardin, A. Argüelles, P.S., K. Richter, and J. D. Urbina,
Phys. Rev. Lett. 112, 140403 (2014)