# The elastic enhancement factor in a transient region between regular and chaotic dynamics

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INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY

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# Plan of the talk:

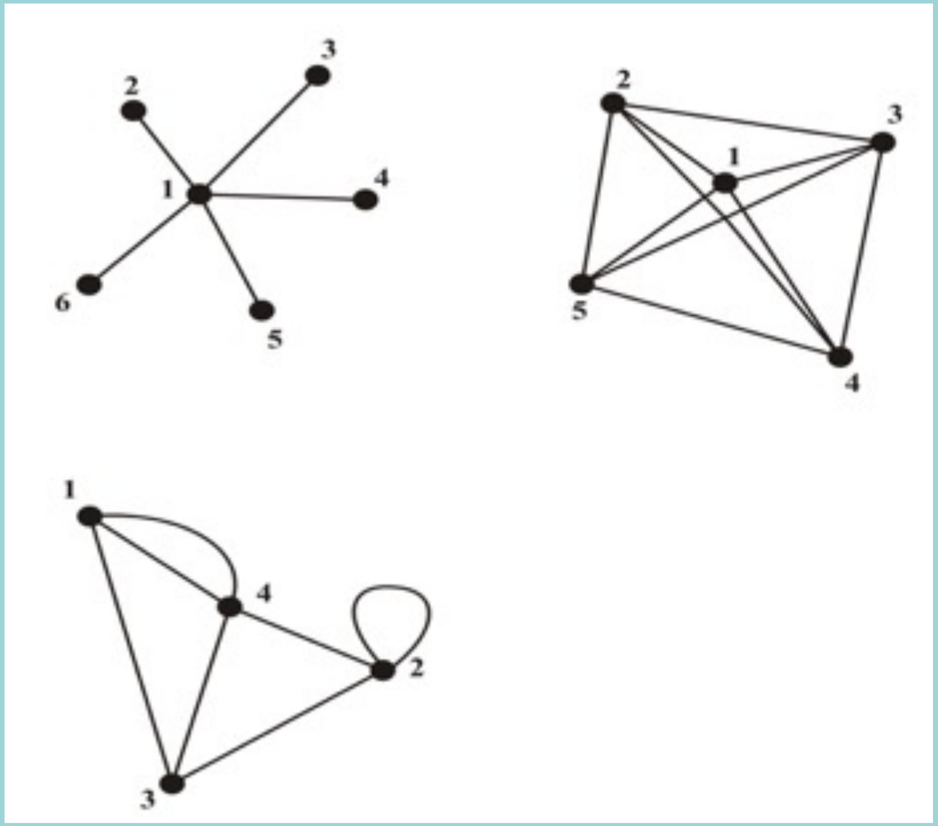
1. Microwave networks of coaxial cables and cavities as experimental realizations of quantum graphs and billiards

2. The elastic enhancement factor $W_{S,\beta}$

3. Results for chaotic and transient systems

4. Conclusions

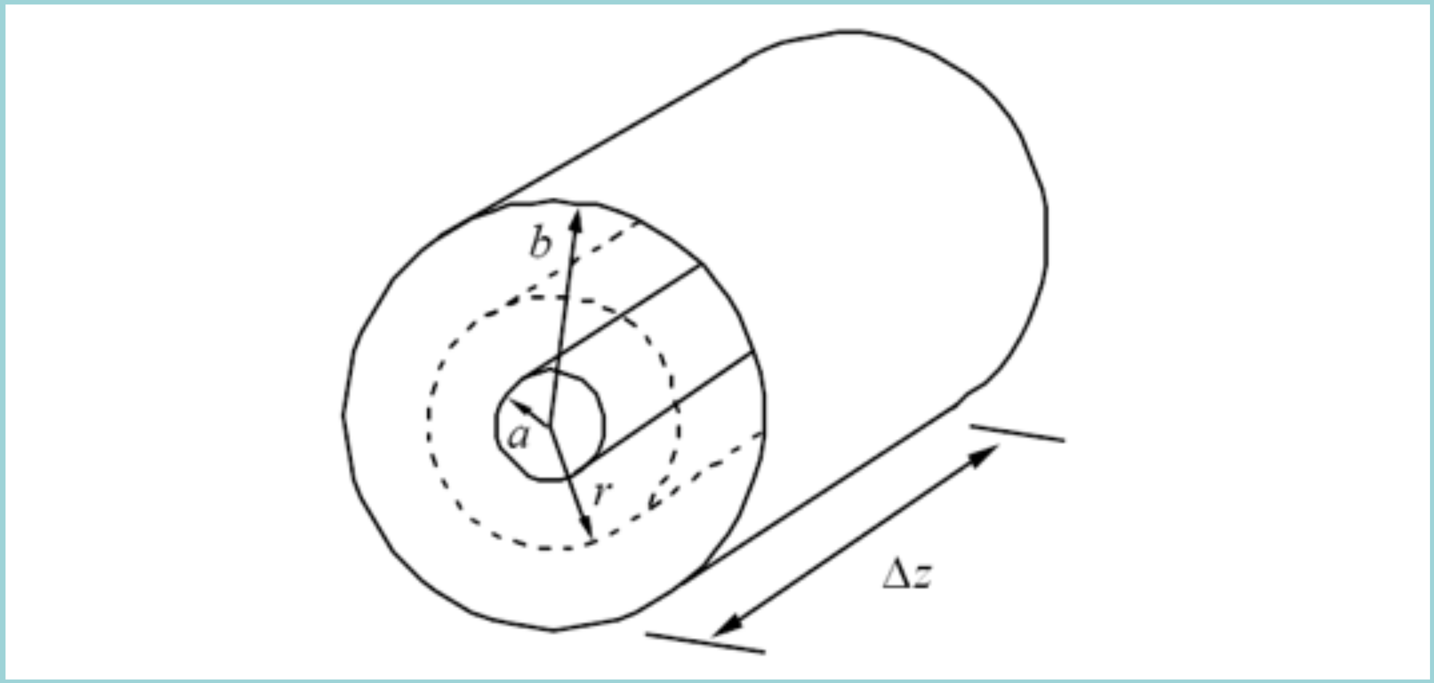
# Quantum graphs




Quantum graphs were introduced by Linus Pauling in 1936.

They are excellent examples of quantum chaotic systems.

An example of a microwave graph with time reversal symmetry




A coaxial cable segment
($a$ = 0.05 cm, $b$ = 0.15 cm)

# Propagation inside a coaxial cable

In order to find propagation of a wave inside the coaxial cable joining the i-th and the j-th vertex of the microwave graph we begin with the continuity equation for the charge and the current on the considered cable (bond)

$$\frac{de_{ij}(x,t)}{dt} = -\frac{dJ_{ij}(x,t)}{dx} \tag{1}$$

where $e_{ij}(x,t)$ and $J_{ij}(x,t)$ are the charge and the current per unit length on the surface of the inner conductor of a coaxial cable.

For the potential difference we can write down

$$U_{ij}(x,t) = V_2^{ij}(x,t) - V_1^{ij}(x,t) = \frac{e_{ij}(x,t)}{C} \tag{2}$$

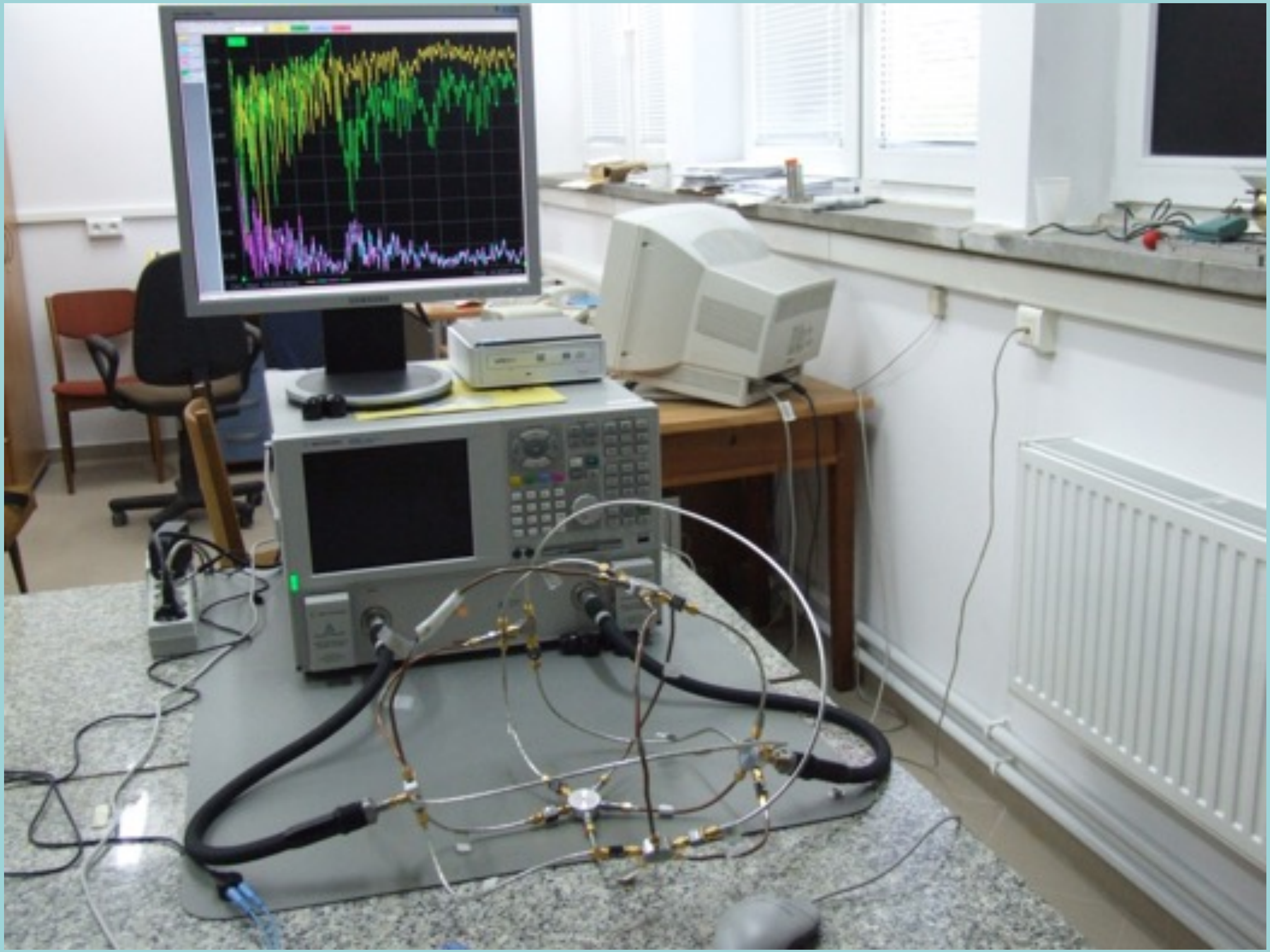
where $V_1^{ij}(x,t)$ and $V_2^{ij}(x,t)$ are the potentials of the inner and the outer conductors of a coaxial cable and $C$ is the capacitance per unit length of a cable.

Making use of the equations (1) and (2) for an ideal lossless coaxial cable and assuming that along the cable propagates a monochromatic wave, one can derive the telegraph equation on the microwave network

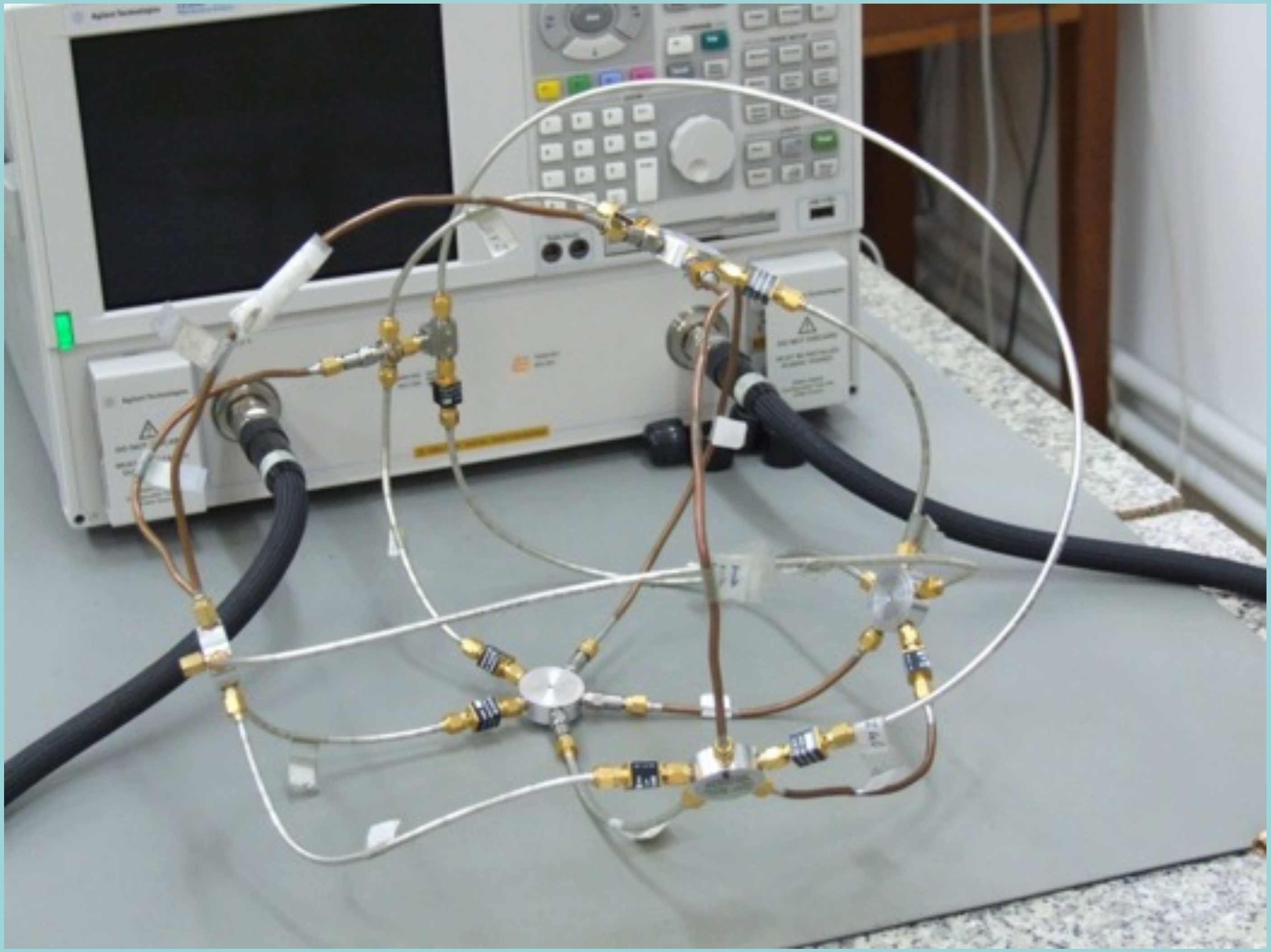
$$\frac{d^2}{dx^2} U_{ij}(x) + \frac{\omega^2 \varepsilon}{c^2} U_{ij}(x) = 0 \qquad (3)$$

Assuming the following correspondence $\Psi_{ij}(x) \Leftrightarrow U_{ij}(x)$ and $k^2 \Leftrightarrow \frac{\omega^2 \varepsilon}{c^2}$, the equation (3) is formally equivalent to the one-dimensional Schrödinger equation (with $\hbar = 2m = 1$) on the graph

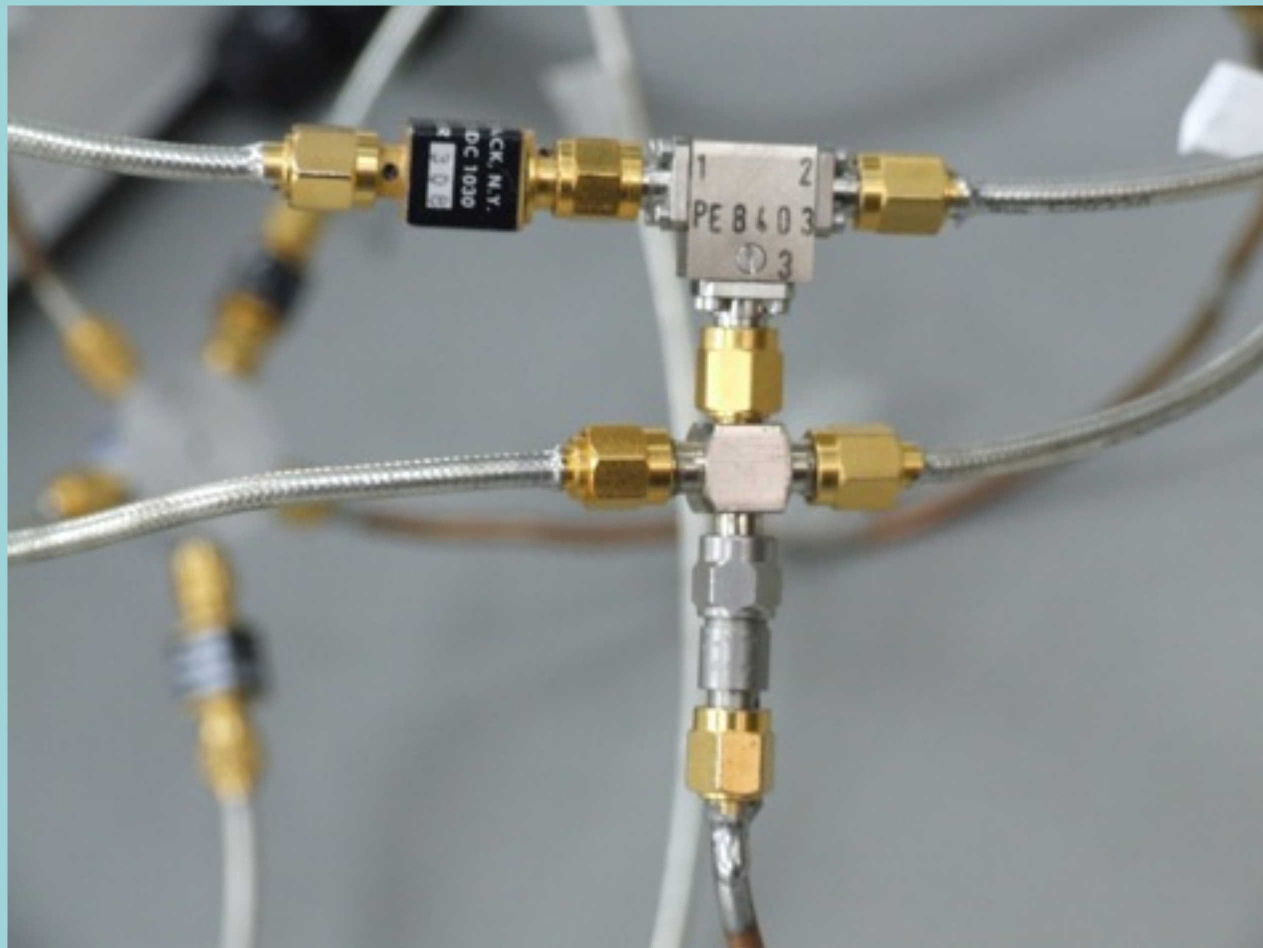
$$\frac{d^2}{dx^2} \Psi_{ij}(x) + k^2 \Psi_{ij}(x) = 0 \qquad (4)$$



Experimental set-up for measuring of hexagonal graphs with time reversal symmetry



An example of a hexagonal microwave network with a microwave circulator which was used to simulate quantum graphs with broken time reversal symmetry




Microwave elements used to build a network with broken time reversal symmetry

# The elastic enhancement factor

The enhancement factor is the ratio of variances in reflection ( $a = b$) to that in transmission ( $a \neq b$):

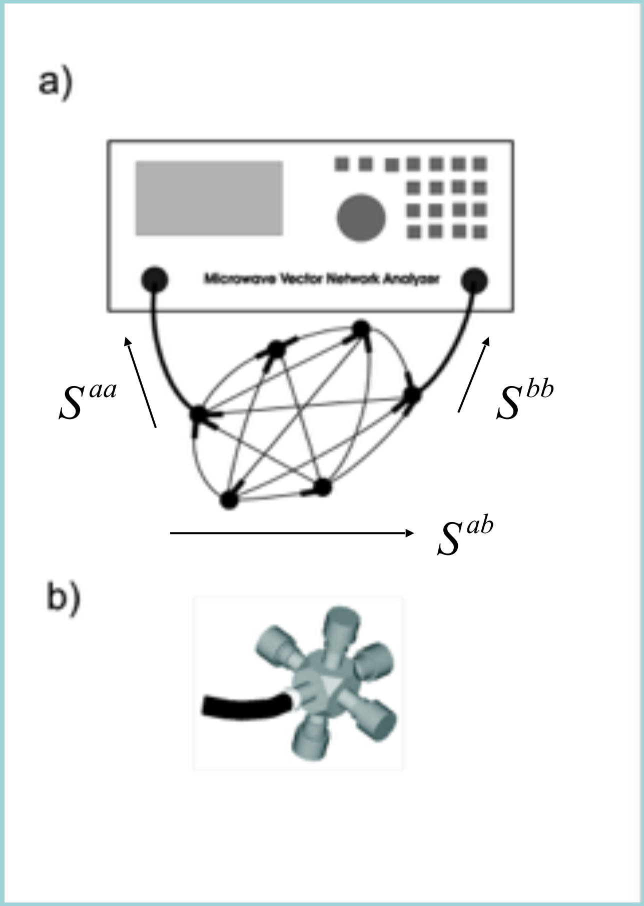
$$W_{S,\beta} = \frac{\sqrt{\mathrm{var}(S^{aa})\,\mathrm{var}(S^{bb})}}{\mathrm{var}(S^{ab})} , \qquad (11)$$

where $\mathrm{var}(S^{ab}) = \left\langle \left| S^{ab} \right|^2 \right\rangle - \left| \left\langle S^{ab} \right\rangle \right|^2 .$

For strong absorption

$$W_{S,\beta} = \frac{2}{\beta} . \qquad (12)$$

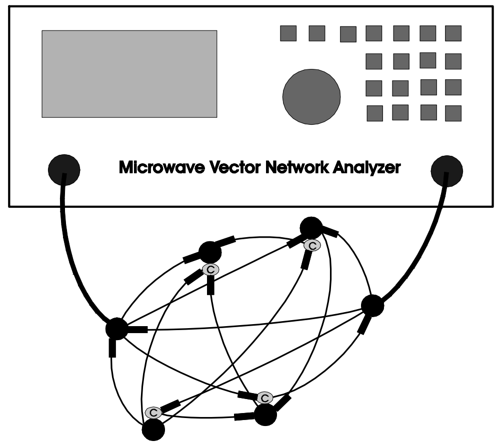
Savin *et al.,* Acta Physica Polonica A **109**, 53 (2006).
Kharkov and Sokolov, Phys. Lett. B **718,** 1562 (2013).



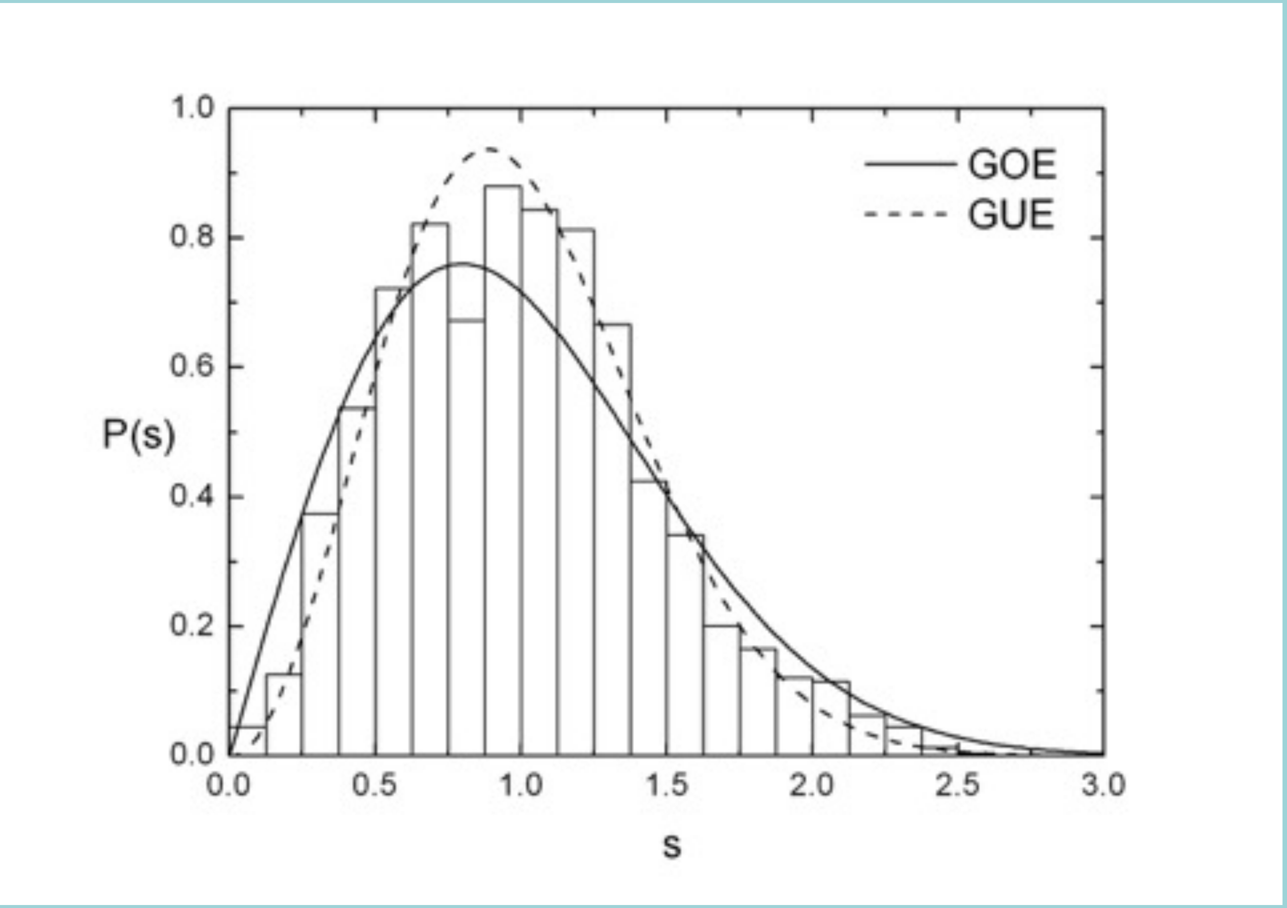

$$\hat{S} = \begin{pmatrix} S^{aa} & S^{ab} \\ S^{ba} & S^{bb} \end{pmatrix}$$

(a) The experimental setup for measuring the two-port scattering matrix $\hat{S}$ of fully connected hexagon microwave networks.
The measurements were performed in the frequency window: 0.5--14 GHz.

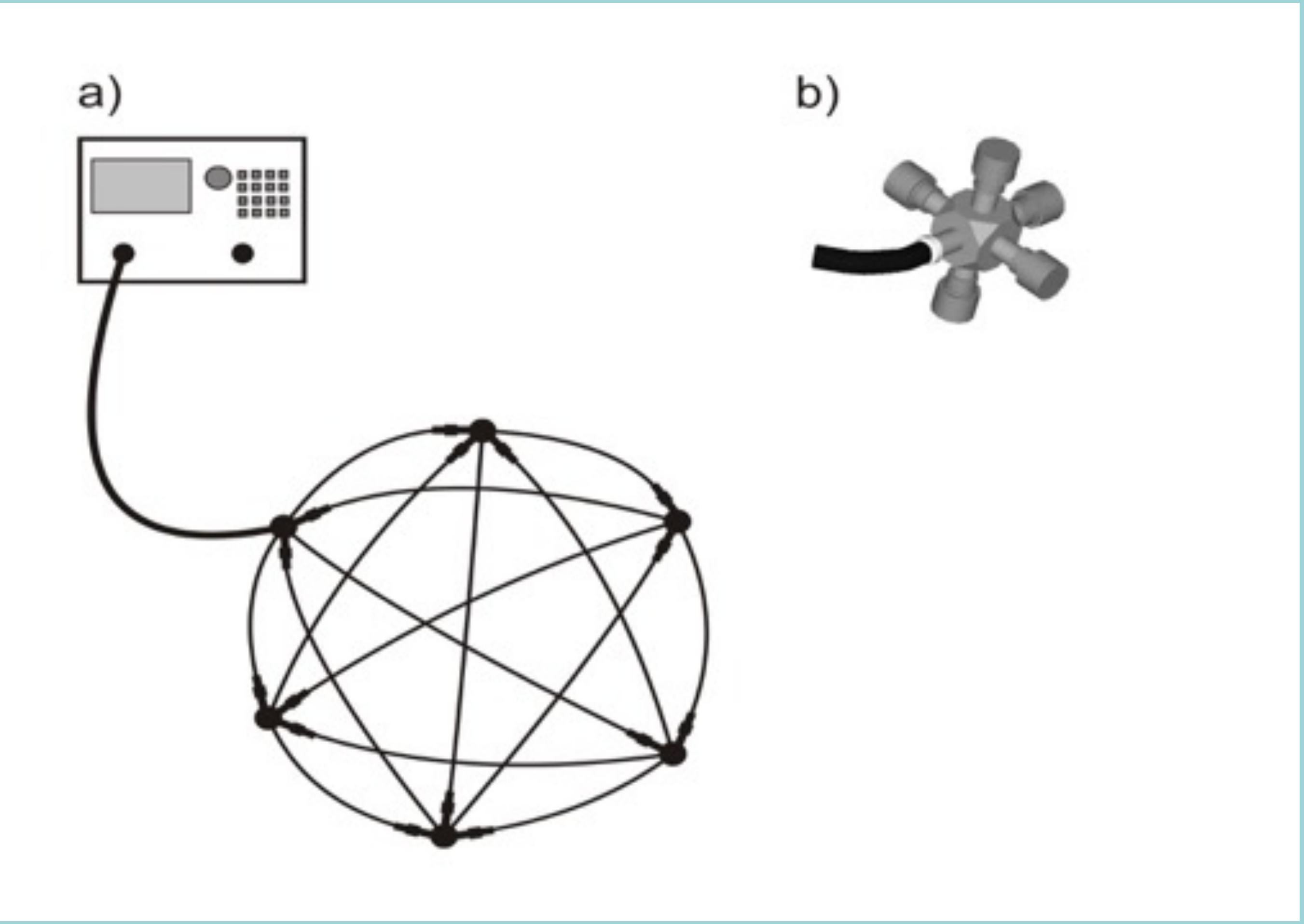
(b) The scheme of the setup used to measure the radiation scattering matrix $S^{aa}$ of the 6-joint connector.



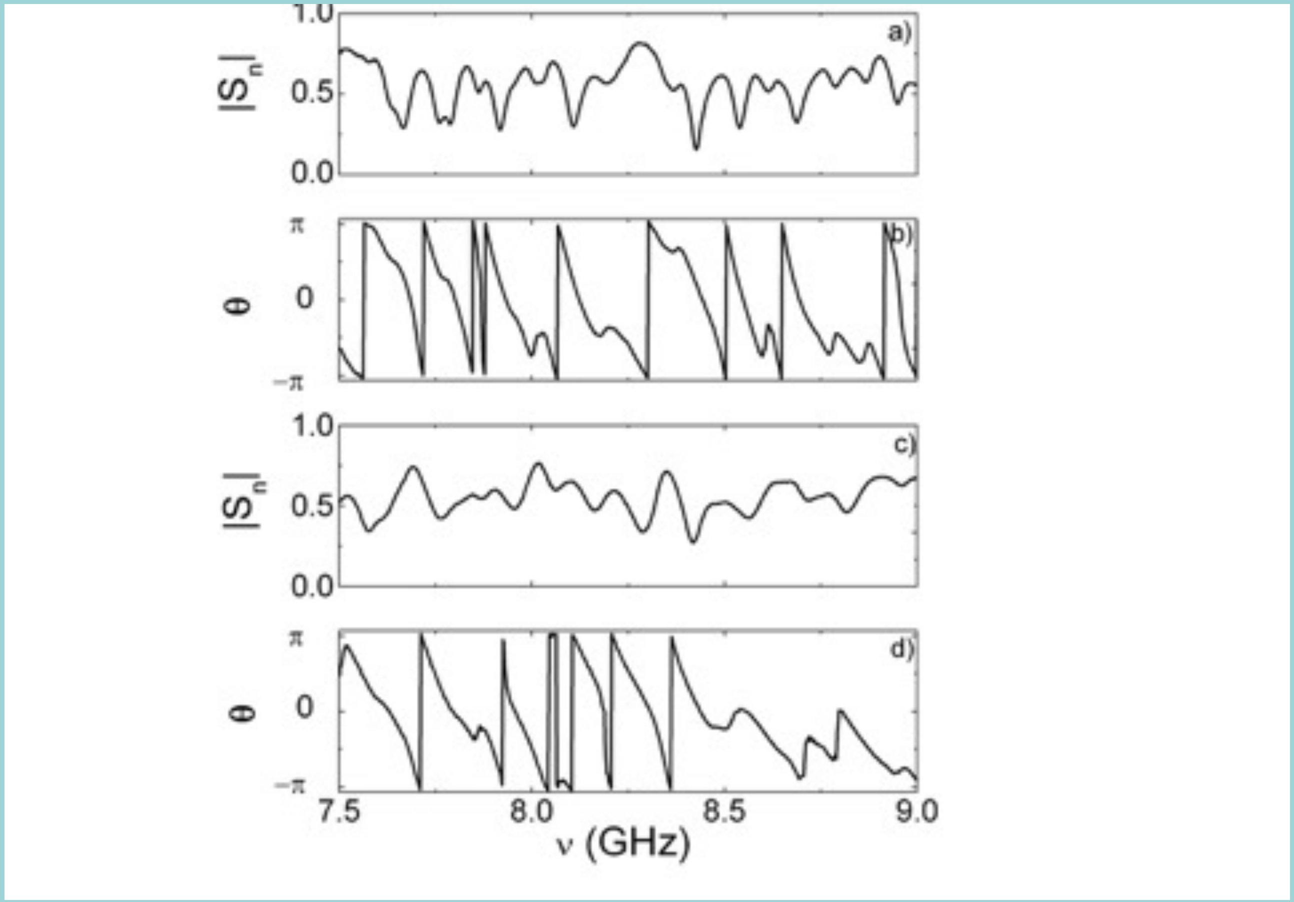

The experimental setup for measuring the two-port scattering matrix $\hat{S}$
of microwave networks with broken time reversal symmetry.
The network additionally to the attenuators contains four microwave circulators.
The measurements were performed in the frequency window: 7--14 GHz.



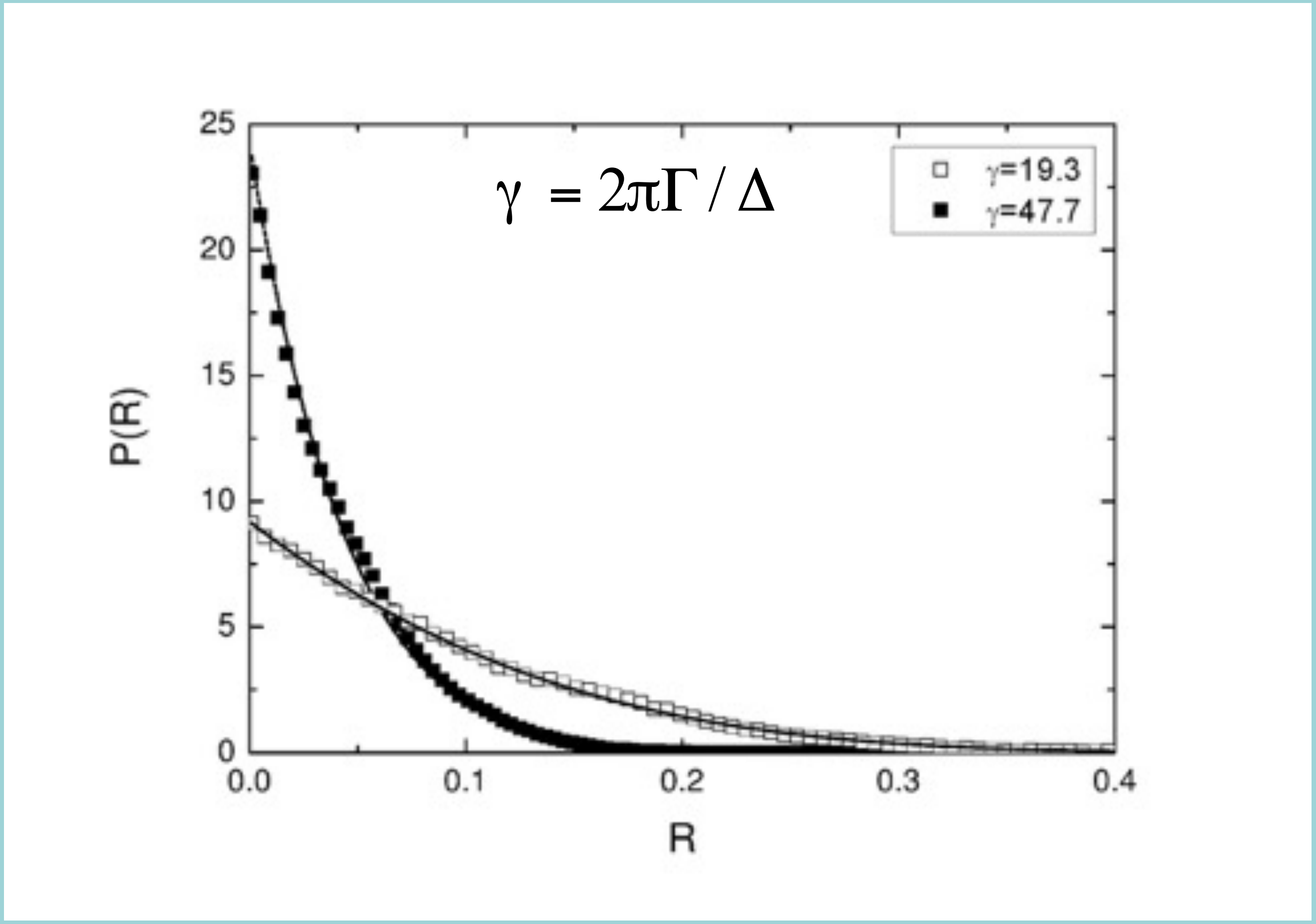

The nearest neighbor spacing distribution averaged for five realization of the directed microwave networks with 4 circulators. Frequency range: 7-9 GHz.



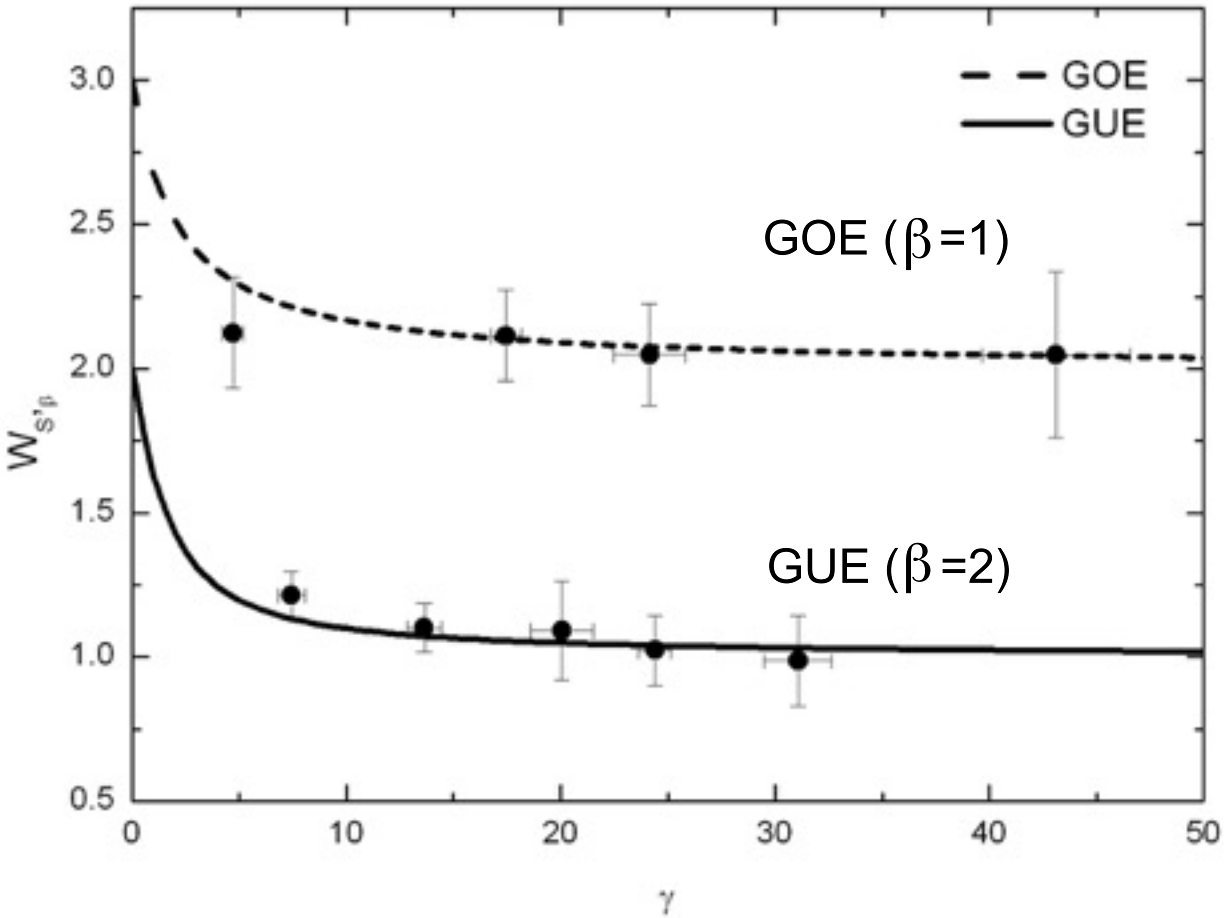

(a) Experimental set-up used to measure the scattering matrix $S_n$ of fully connected irregular hexagon microwave graphs with absorption.
(b) Scheme of the setup used to measure the radiation scattering matrix $S_r$.




Panels (a) and (b): the modulus $\left|S_n\right|$ and the phase $\theta$ of the scattering matrix $S_n$ measured for the graph with absorption parameter $\gamma$ = 19.9 with use of 1 dB attenuators. Panels (c) and (d): measurements for the graph with $\gamma$ = 47.9 with use of 2 dB attenuators.




Experimental distribution $P(R)$ of the reflection coefficient $R$ at different values of the absorption strength parameter: $\gamma$ = 19.3 (open squares) and $\gamma$ = 47.7 (full squares).
$\Gamma$ is the absorption width and $\Delta$ is the mean level spacing.




The enhancement factor $W_{S,\beta}$ for microwave networks with preserved and broken time reversal symmetry.

# Quantum billiards modelled by  microwave thin cavities

An electric field inside a thin microwave cavity is described by the Helmholtz equation

$$\frac{d^2}{dx^2}E_z(x)+\frac{\omega^2}{c^2}E_z(x)=0 \qquad (13)$$

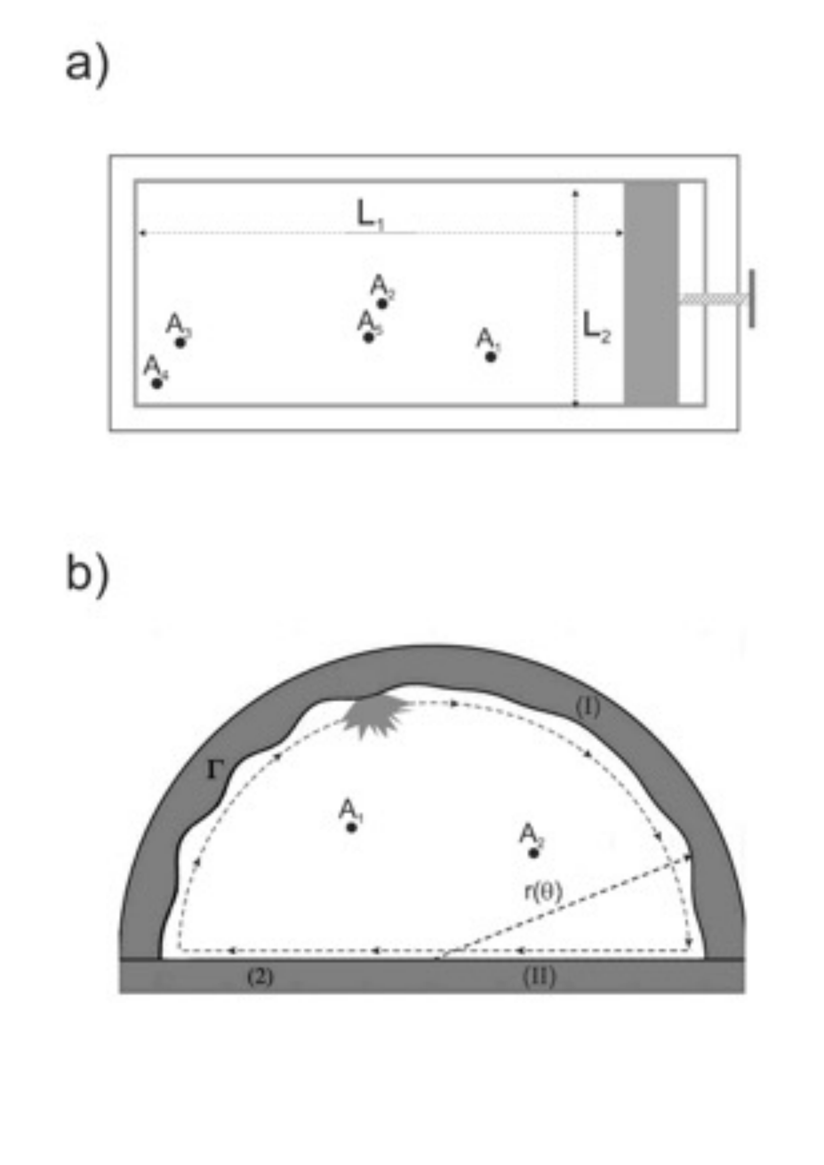
Assuming the following correspondence $\Psi(x) \Leftrightarrow E_z(x)$ and $k^2 \Leftrightarrow \dfrac{\omega^2}{c^2}$ ,

the equation (13) is formally equivalent to the one-dimensional Schrödinger

equation (with $\hbar = 2m = 1$) in a quantum billiard

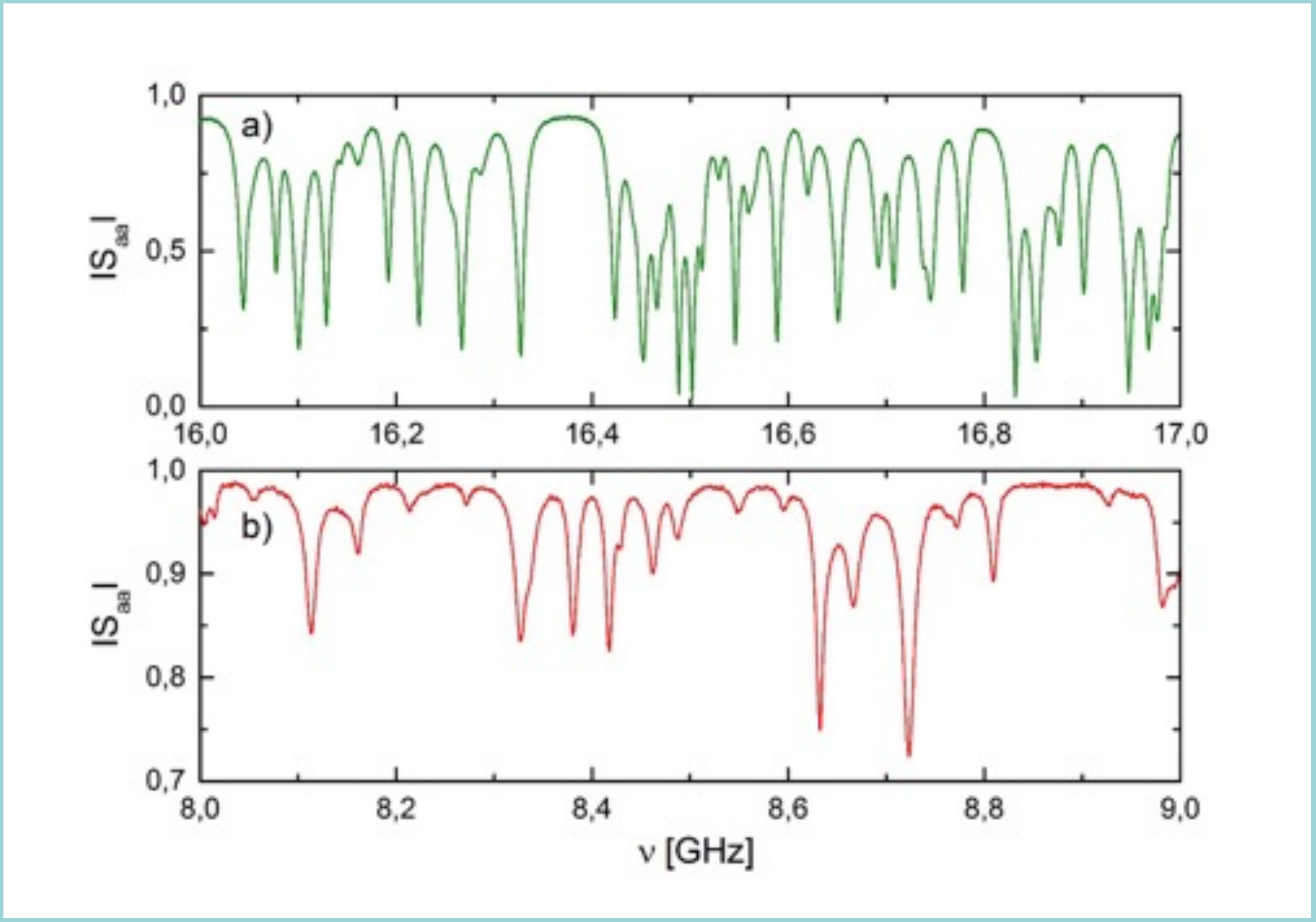
$$\frac{d^2}{dx^2}\Psi(x)+k^2\Psi(x)=0 \qquad (14)$$

# The microwave rectangular and chaotic rough cavities



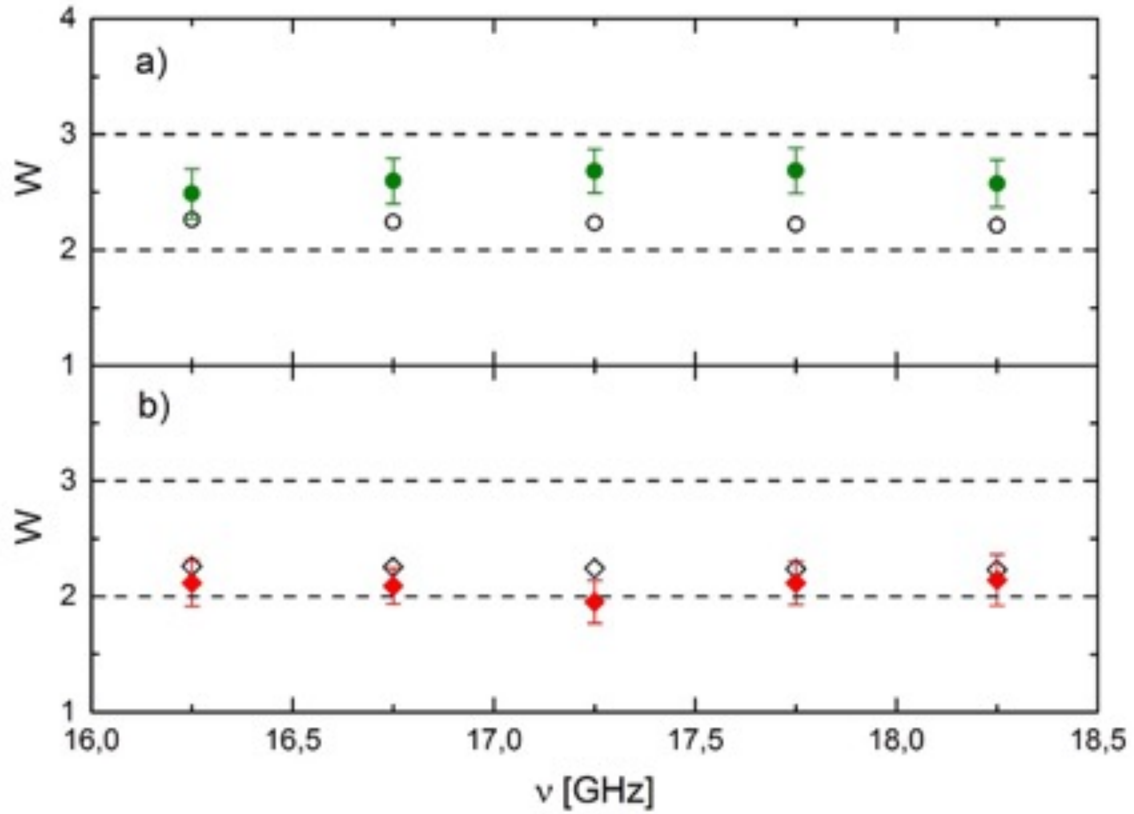
# The microwave rough cavity

# Typical spectra of the cavities




(a) The rectangular cavity in the frequency range 16-17 GHz
(b) The rough cavity in the frequency range 8-9 GHz

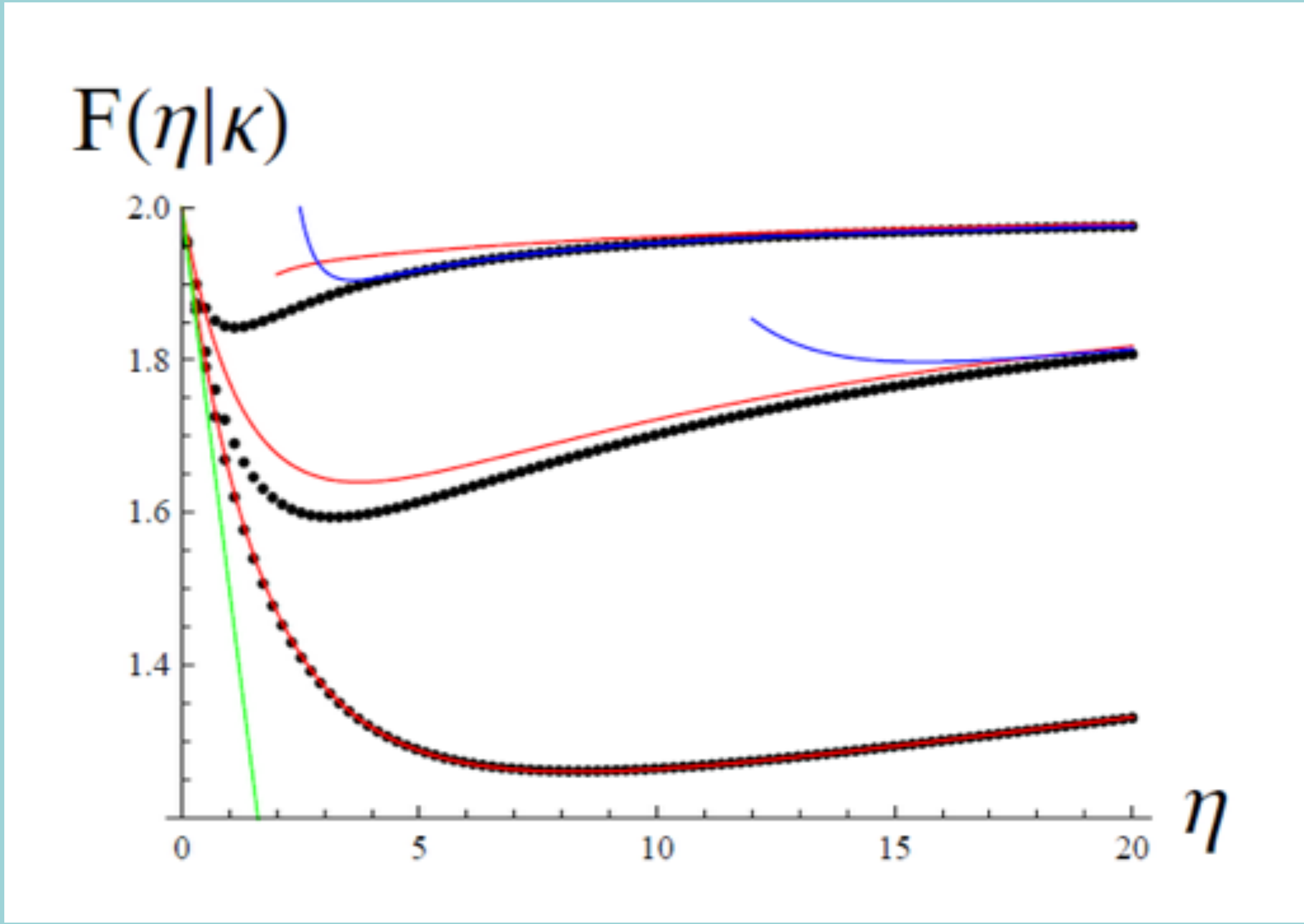
# The enhancement factor measured for microwave thin cavities




Panel a): Rectangular cavity; Panel b): Chaotic rough cavity

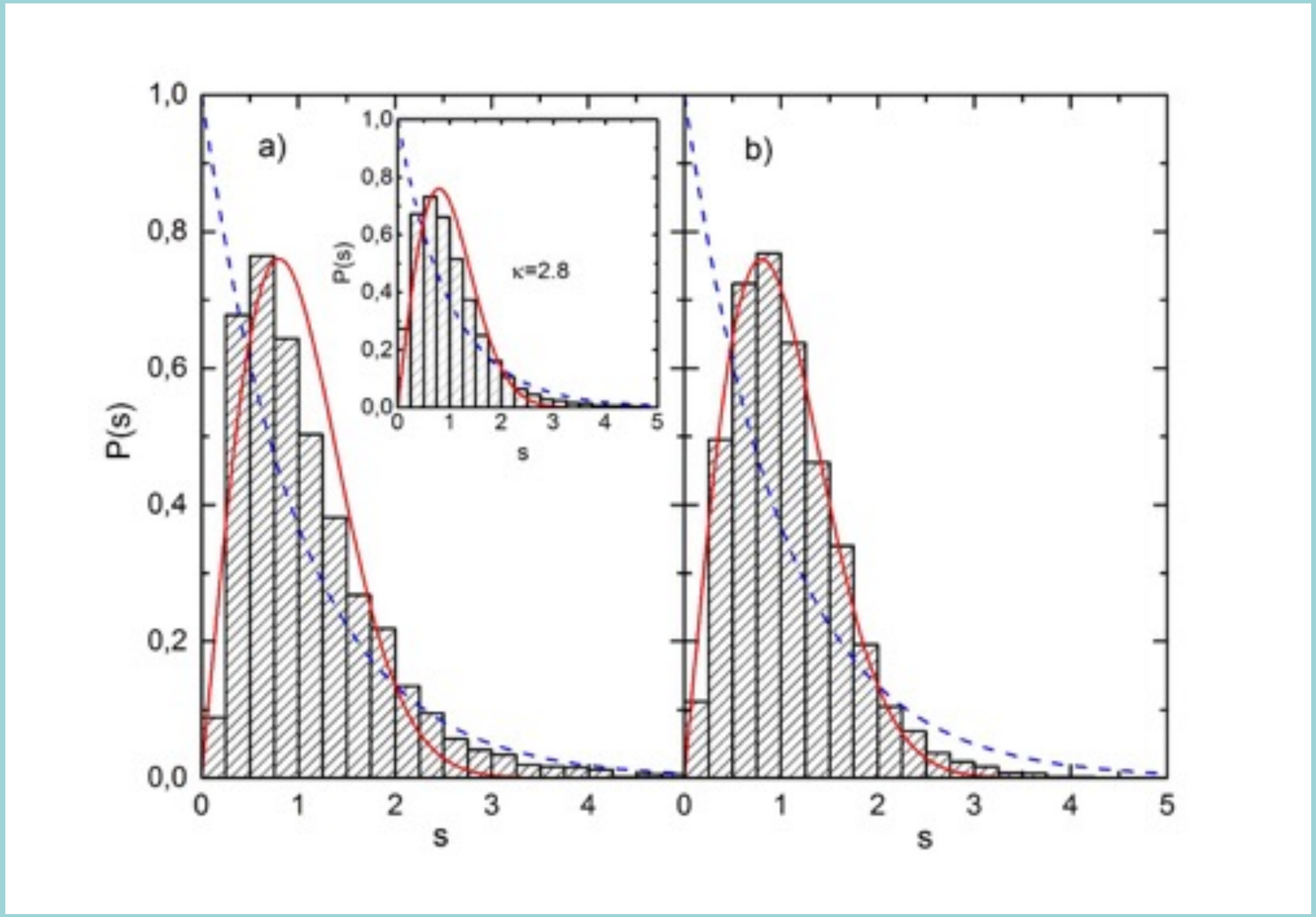
*M. Ławniczak, M. Białous, V. Yunko, S. Bauch, and L. Sirko, accepted for printing in PRE 2015*

# GUE systems – Kharkov & Sokolov




F versus $\eta$ for (from top to bottom) $\kappa = 0.5,\ 5,\ 50.$ Black points - exact numerics; blue lines - small $\kappa$ approximation; red lines - large $\kappa$ approximation; green line indicates the slope at the point $\eta = 0$.

# The nearest neighbor spacing distributions




Left panel: P(s) for coupled rectangular cavity;
The inset: numerically reconstructed P(s) for the chaoticity parameter $\kappa$=2.8
Right panel: P(s) for chaotic rough cavity

# Rosenzweig-Potter like model – from integrable to chaotic systems

$$a_{ij} = g_{ij}[\delta_{ij} + \lambda(1 - \delta_{ij})],$$

where $g_{ij}$ denotes a symmetric matrix which belongs to GOE matrices. As it is defined off-diagonal elements $g_{ij}$ are independently Gaussian distributed with the same variance=1 and the mean=0. The diagonal elements $g_{ii}$ are distributed independently with the variance=2. $\lambda$ is the transition parameter.

For the matrix $a_{ij}$ of size NxN, we found out that $\lambda = \kappa/N$

$\kappa$ - chaoticity parameter

# GOE and GUE systems – Sokolov & Zhirov

*arXiv:1411.6211v2[nucl-th], 12 Dec 2014*

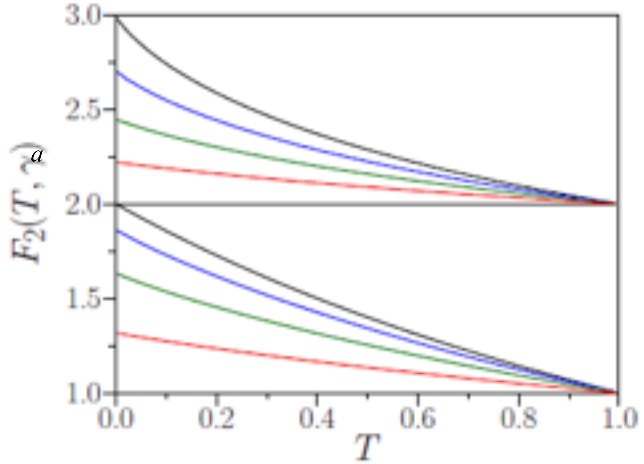



**Figure 6:** $\mathbf{F}_2(T, \gamma_a)$ **for** $\beta = 1$ **(up) and** $\beta = 2$ **(down):** $\gamma_a = 0, 0.03, 1, 3$ **(from top to bottom. The influence of absorption is stronger in the case of T-invariant systems.)**

$$\gamma \;\; = T_a + T_b + \gamma_a$$

# Conclusions

1. We showed that the experimental and theoretical results for the elastic enhancement factor for irregular networks with preserved and broken time reversal symmetry are in good overall agreement with the theoretical predictions.

2. Our experimental results suggest that the elastic enhancement factor can be used as a measure of internal chaos that can be especially useful for systems with significant absorption or openness.

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