Statistical Theory of Random (and Chaotic) lasers

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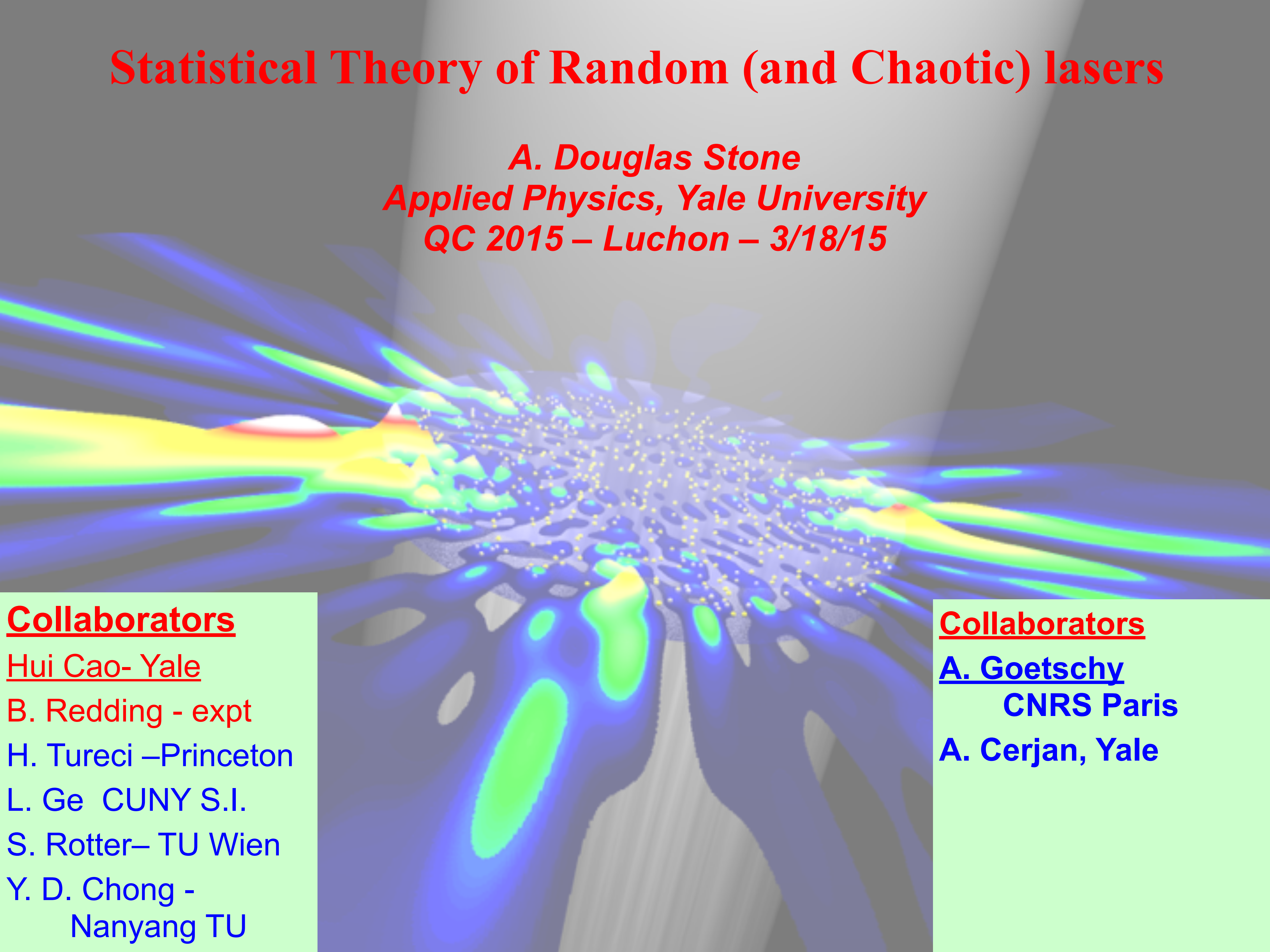
QC 2015 – Luchon – 3/18/15



Collaborators

- Hui Cao- Yale
- B. Redding - expt
- H. Tureci –Princeton
- L. Ge CUNY S.I.
- S. Rotter– TU Wien
- Y. D. Chong - Nanyang TU

Collaborators

- **A. Goetschy CNRS Paris**
- **A. Cerjan, Yale**

RMT in Optics/E&M

- ☐ Analysis of multiple scattering problems (including wave chaos) => Extremal eigenvalue problems for non-hermitian or non-unitary matrices

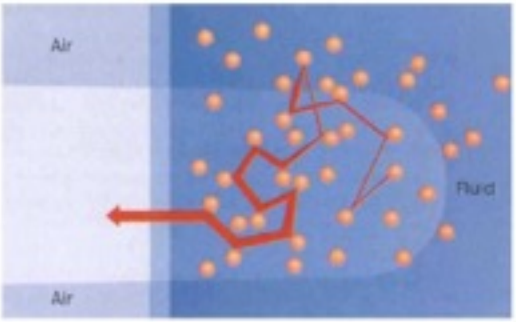
- ☐ Random/QC lasing: Non-unitary (non-linear) S-matrix
  - Realistic technological application for theory

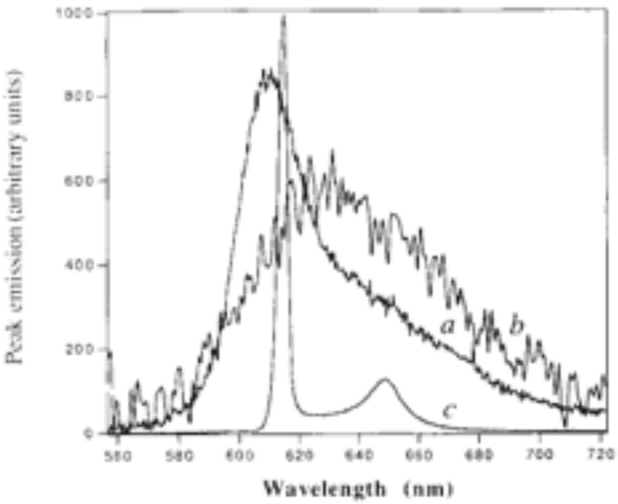
- ☐ Control of transmission/absorption/focusing in diffusive scattering media (another talk)

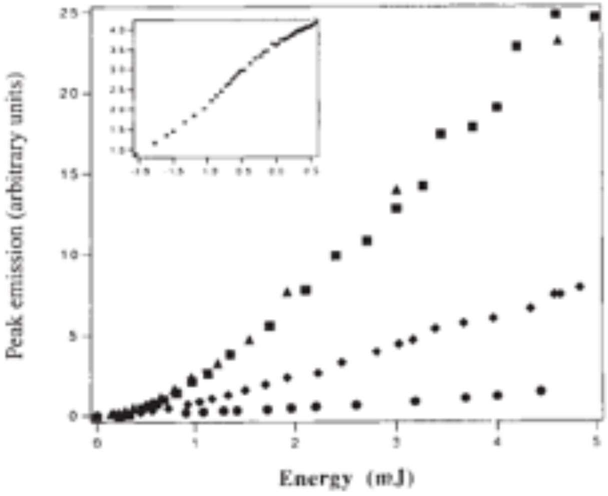
  Open Channels, correlations: DMPK (1984,1987),Imry (1986),Kane (1988)
  SLM-based Focusing thru opaque white media: Mosk et al. PRL 2007
  "Hidden Black", Y.D. Chong and ADS, PRL **107**, 163901, 2011
  "Filtered Random Matrices", A. Goetschy and ADS, PRL **111**, 063901 (2013);
  effect of incomplete "channel" control => Free probability theory
  "Control of Total Transmission", Popoff, Cao et al. PRL **112**, 133903 (2014):
  <T> = 5% => T_{max}= 18%

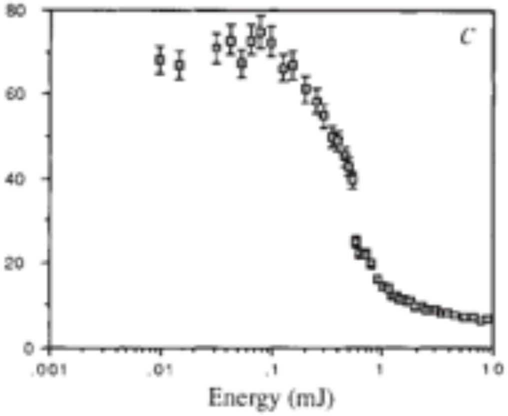
Pioneering Random Lasers

Lawandy, Balachandran, Gomes & Sauvain, Nature **368**, 436 (1994)
(following early ideas from Letokhov)



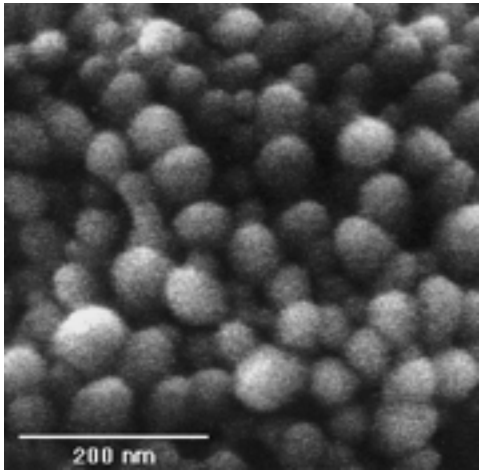


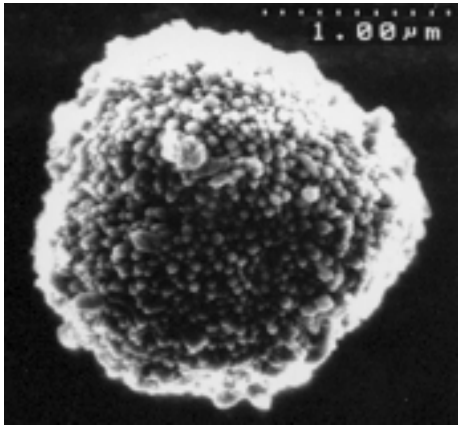


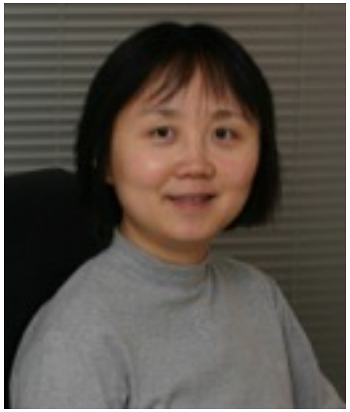


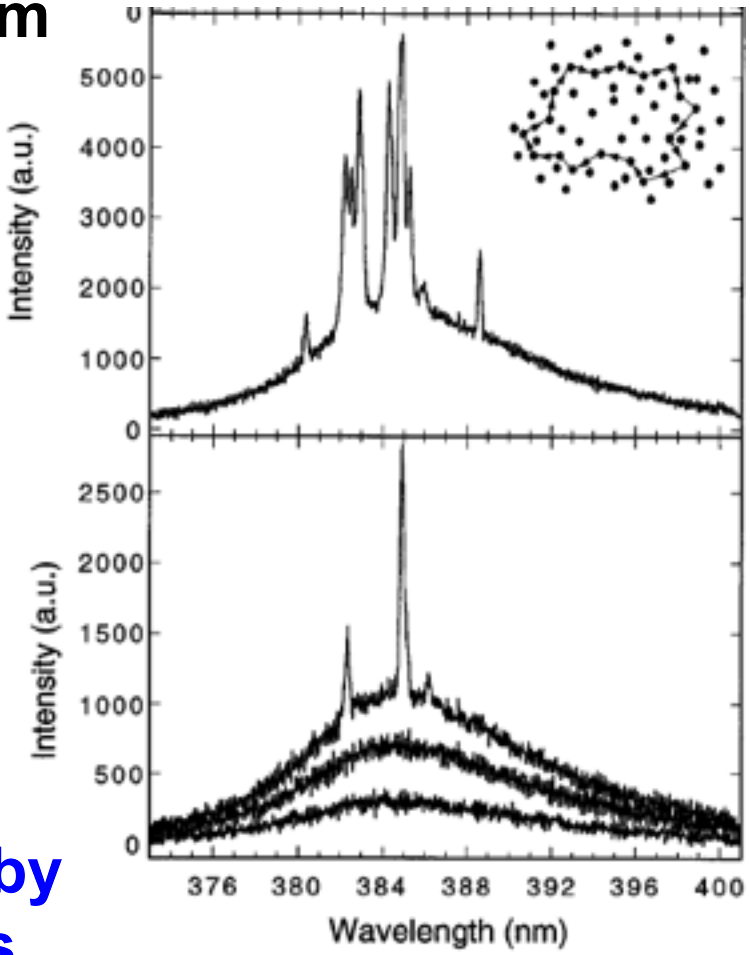
ZnO Nanorods and Powders

Average particle diameter ~ 100 nm









Also confirmed by photon statistics

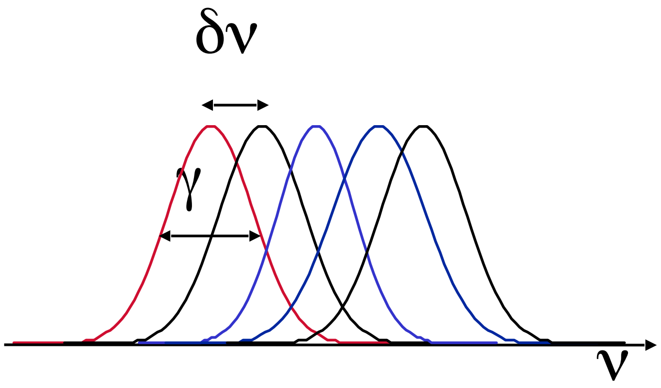
HC et al, Appl. Phys. Lett. **73**, 3656 (1998); Phys. Rev. Lett. **82**, 2278 (1999)

Why Interesting? Not due to Anderson Localized High Q modes – Diffusive regime

Thouless # $N_T = \frac{\gamma}{\delta\nu} >> 1$

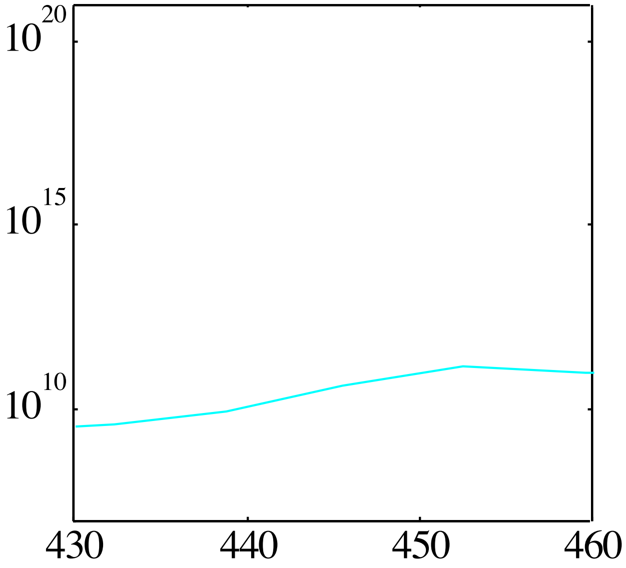
$N_T = g = 1/f$

DRL has $f << 1$

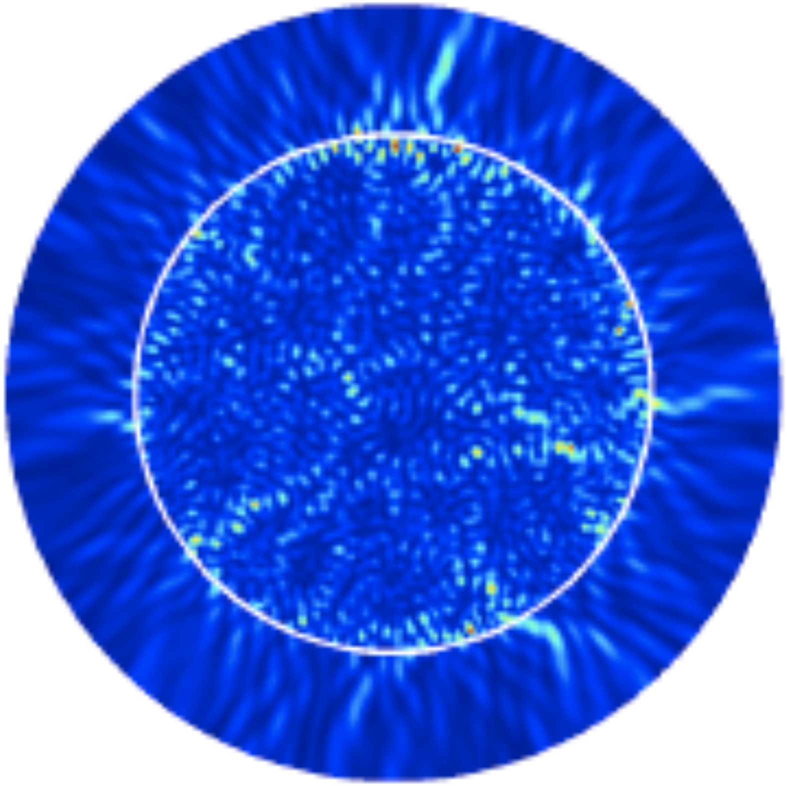


Resonances are strongly overlapping <u>spatially</u> and <u>spectrally</u>.

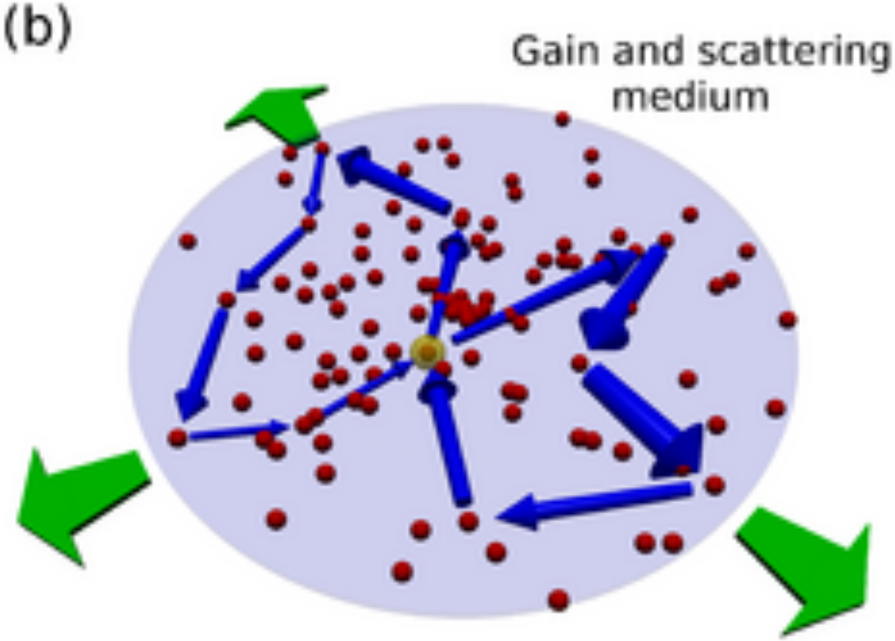
Passive cavity scattering spectrum shows no isolated resonances – not within standard laser theory



Modes are pseudo-random in space – not based on periodic orbits



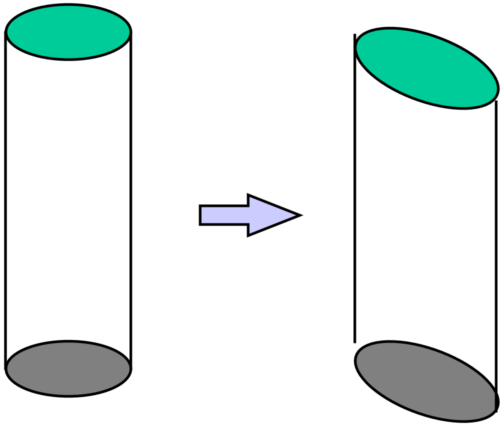
Tureci, Ge, Rotter, ADS, Science **320**, 643 (2008)
SALT-based calculations



Vanneste, Sebbah & H. Cao, Phys. Rev. Lett. **98**,143902 (2007).

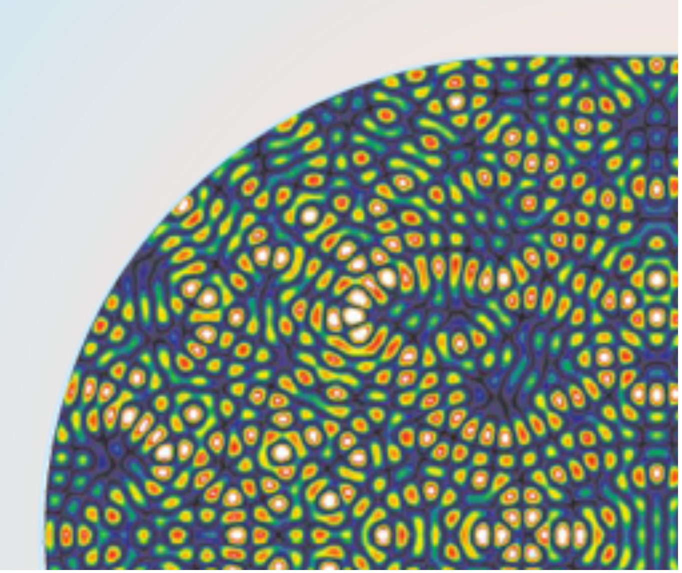
Similar to Wave-chaotic Lasers

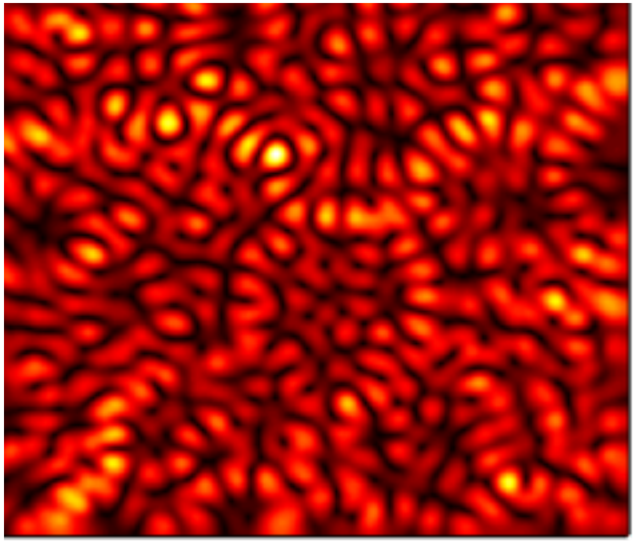
“Ray and Wave Chaos in Asymmetric Resonant Optical Cavities”,
J. U. Nöckel, A. D. Stone,
Nature, 385, 45 (1997).
Open wave-chaotic systems



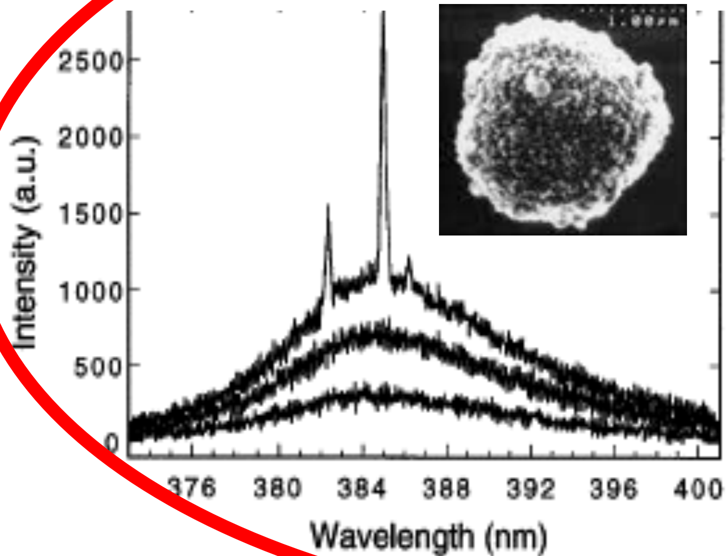
Hard Chaos

KAM Transition to ray chaos

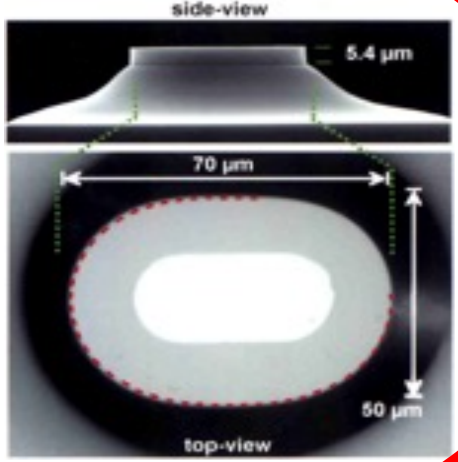




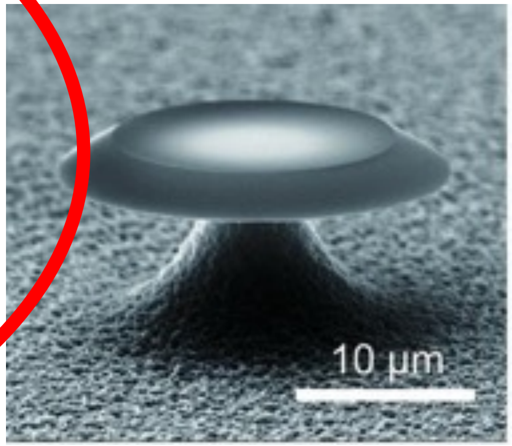
Theory for lasers with complex geometry

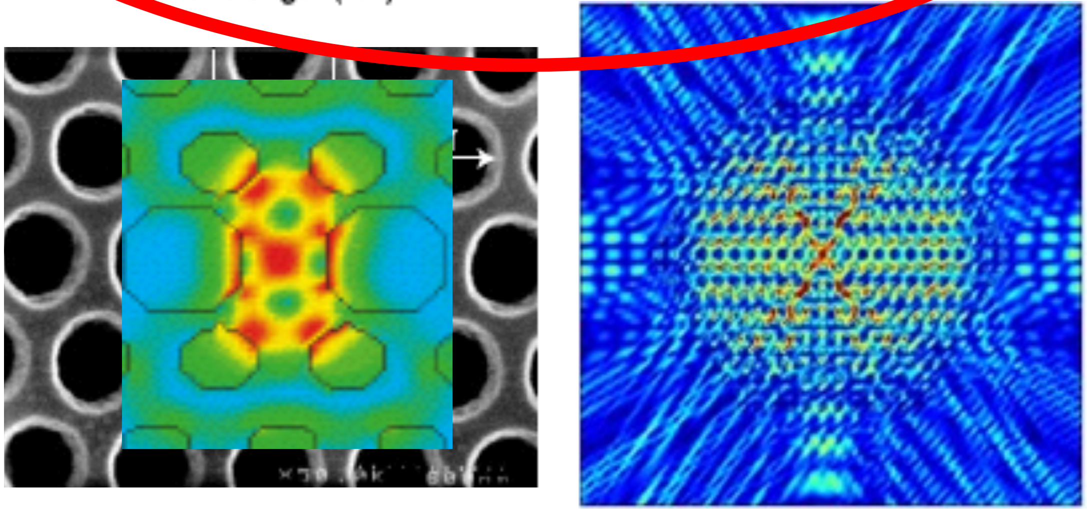


Chaotic-ARC



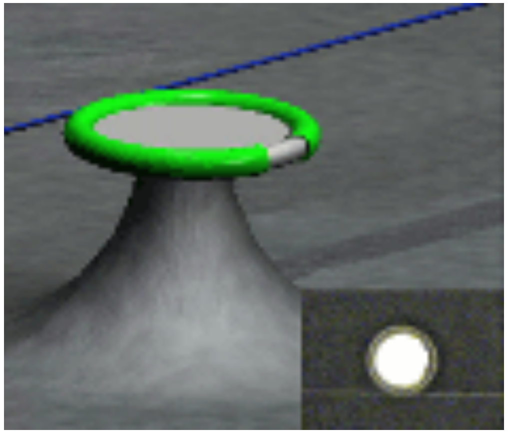
microdisk





Photonic Crystal Lasers

microtoroid



Universal: Lasers as scattering systems

Non-hermitian Eq.
Flux not conserved

$$[\nabla^2 + n^2(\mathbf{r})k^2]\mathbf{E}(\mathbf{r}) = 0$$

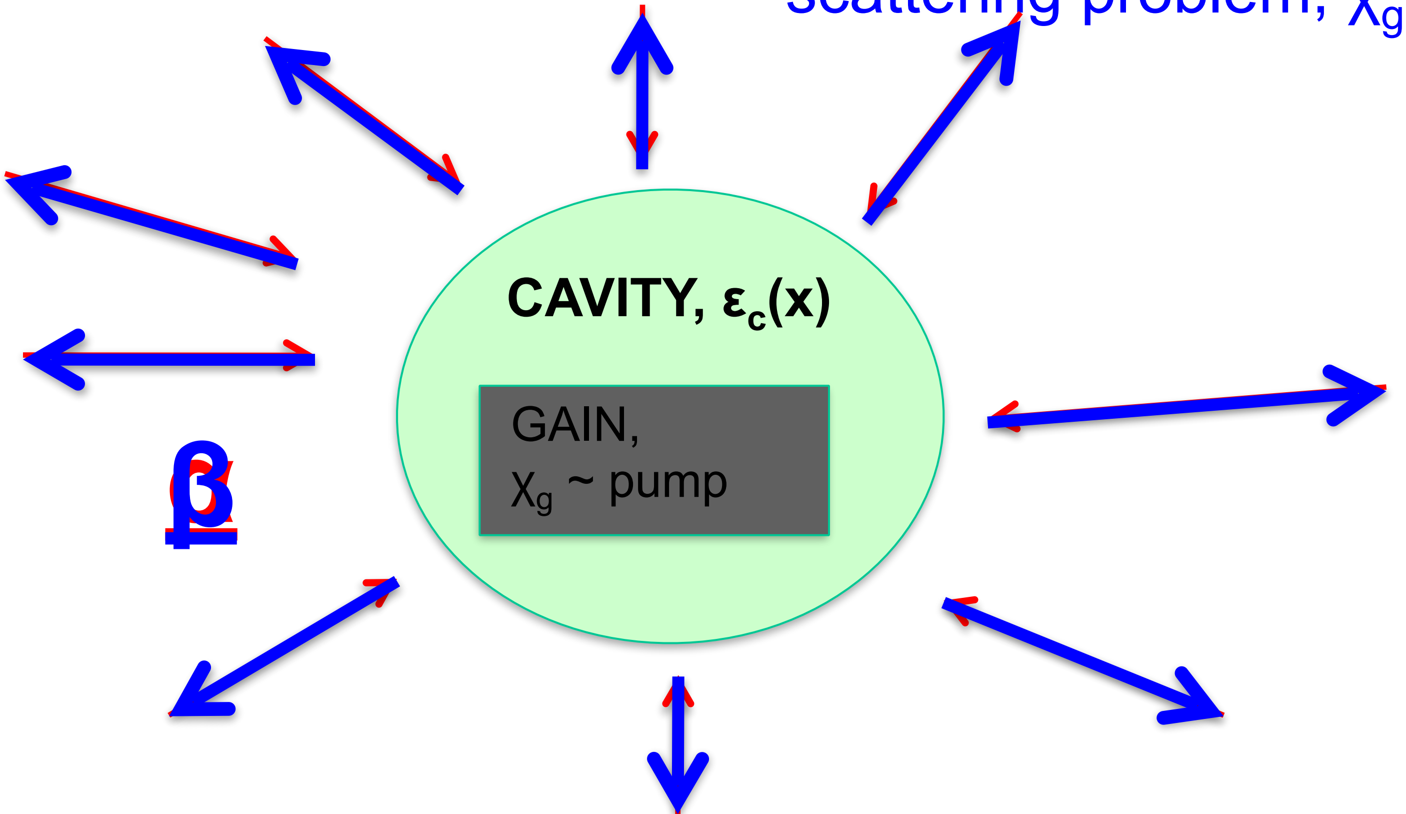
$$k \equiv \omega / c$$

$$n(\vec{r}) = \sqrt{\epsilon_c(\vec{r}) + 4\pi\chi_g(\vec{r})}$$

$$\mathbf{S}(n(\vec{r})k) \cdot \underline{\alpha} = \underline{\beta}$$

X_g is complex => n(r) complex, n_2<0 (amplifying)

Non-unitary non-linear scattering problem, $\chi_g = \chi_g(E)$



CAVITY, ε_c(x)

GAIN,
χ_g ~ pump

$\underline{\beta}$

Threshold lasing modes

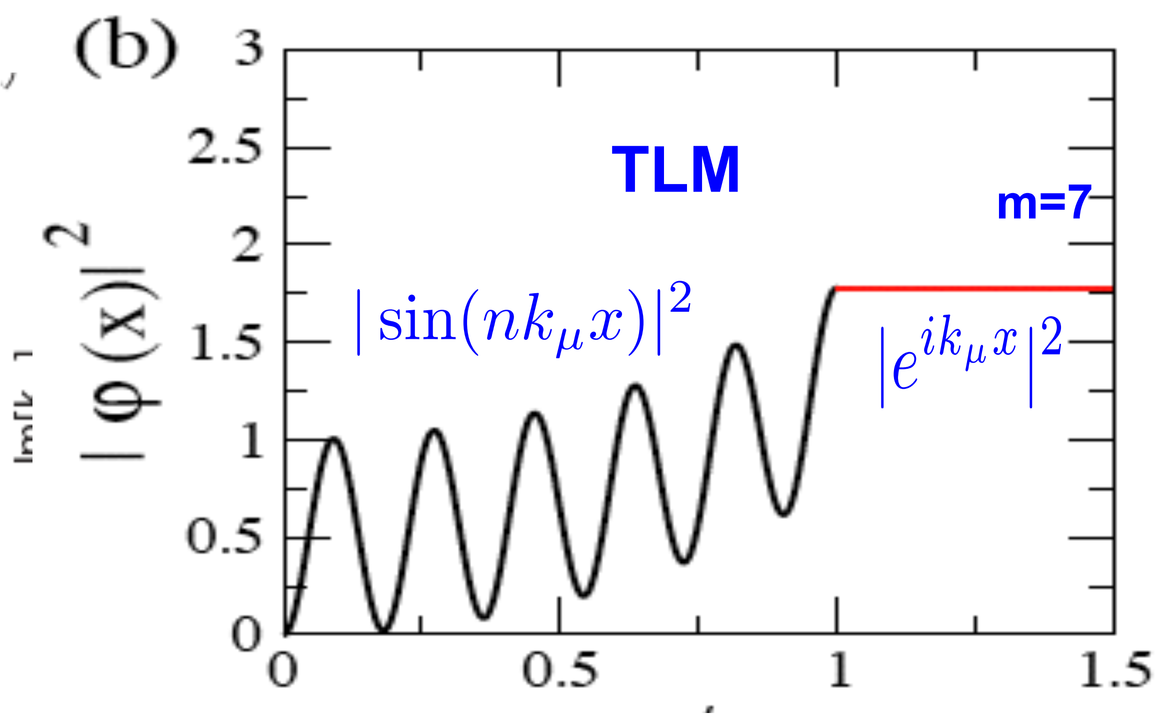
$$\mathbf{S}(n(\vec{r})k)\cdot\cancel{\underline{\alpha}} = \underline{\beta}$$

Laser: lasing mode β goes out, nothing in

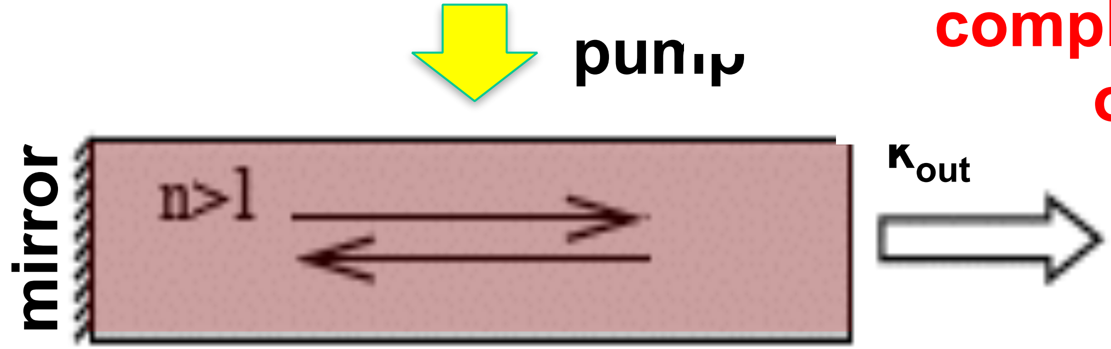
⇒Poles of the S-matrix

Passive cavity: $n = (\varepsilon_c)^{1/2}$, S unitary, poles complex.

Simple example: 1D uniform dielectric cavity:



complex sine inside, purely outgoing outside



Now add gain medium + pump,
$n = n_c + \Delta n_g$

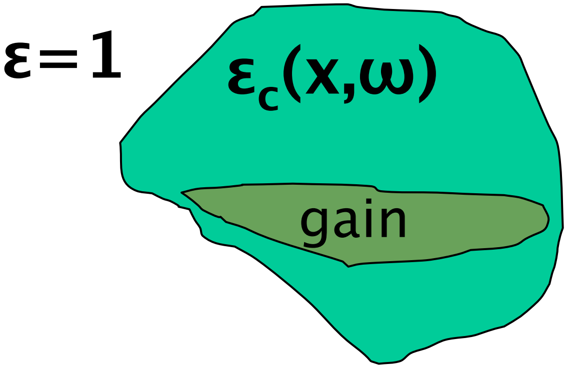
TLM stabilized by non-linearity!

Pump harder => multimode lasing

Semiclassical lasing theory + SALT

no spont emission
no laser linewidth

Cavity arbitrary



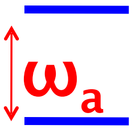
$$\nabla^2 E(\mathbf{x},t) - \frac{\epsilon_c(\mathbf{x})}{c^2}\frac{\partial^2}{\partial t^2}E(\mathbf{x},t) = \frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}P_g(\mathbf{x},t)$$

$$P_g = \chi_g E$$

Any cavity, gain medium, N-levels, M ind. transitions, non-uniform pumping

Simplest case:2-level atoms

$$\epsilon_c \ddot{E}^+ = \nabla^2 E^+ - 4\pi \ddot{P}^+$$



$$P = n_a \mathrm{Tr}\, p\rho$$

$$D = n_a(\rho_{22} - \rho_{11})$$

$$\dot{P}^+ = (-i\omega_a - \gamma_\perp)P^+ - i|g|^2 \boxed{E^+ D}$$

$$\dot{D} = \gamma_\parallel(D_0 - D) + 2i(\boxed{E^+ P^{+*}} - E^{+*}P^+)$$

$$\gamma_\perp = 1/T_2, \quad \gamma_\parallel = 1/T_1$$

Not studying dynamical chaos
Look for non-linear steady-state, with purely outgoing BC

$$D_0 = pump\ strength$$
$$g^2 = dipole\ coupling$$

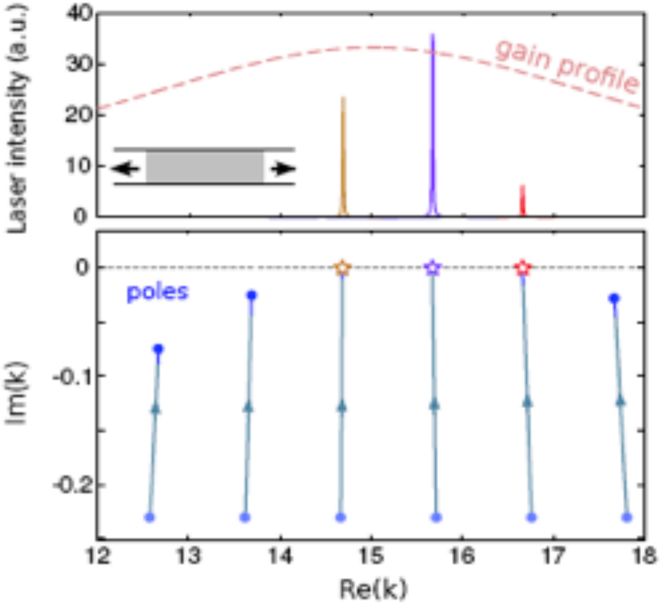
Maxwell-Bloch equations

Haken(1963), Lamb (1963) – the standard model

- ☐ dD/dt≈0, in steady-state => SALT Eqs
- ☐ $\gamma_{perp} << \Delta, \gamma_{par}$ => good approx for microlasers

$$[\nabla^2 + (\epsilon_c + \tilde{D}_0\gamma_\mu)k_\mu^2]\Psi_\mu = 0$$

$$\tilde{D}_0(\mathbf{r}) \equiv \frac{D_0(\mathbf{r})}{1+\sum_\nu^N |\gamma_\nu \Psi_\nu(\mathbf{x})|^2}$$



Non-linear coupled time-independent wave equations with outgoing BC

$$\mu = 1, 2, \ldots N$$
$$\Psi_\mu \sim e^{+ikr}/r, \quad r \to \infty$$

Saline Solution

❑ Specialized TLM/TCF (non-hermitian biorthogonal) basis set method(Tureci,ADS,Ge,Rotter,Chong)

$$\Psi_\mu(\vec{r}) = \sum_n a_n^\mu\, u_n(\vec{r}, k_\mu) \xrightarrow{\text{Lasing Map}} \boxed{D_0 \sum_{n'} T^\mu_{nn'} a^\mu_{n'} = a^\mu_n}$$

TCF basis

$$\left[\nabla^2 + \Big(\epsilon_c(\vec{r}) + \eta_n(k)\, F(\vec{r})\Big)k^2\right] u_n(\vec{r}, k) = 0$$

❑ Solve by iterative method (rapidly convergent).

❑SALT for DRL: approx sum by a single term, soln in terms of evalues of Green fcn for this non-herm eq.

$$\eta_n(k_\mu) = \eta_1 - i\eta_2$$

Freq shift

gain

$$\Lambda_n \equiv \frac{1}{\eta_n} \approx \frac{+i}{\eta_2} \quad (RL)$$

Why SALT it is good for you

- ☐ **General theory of CW steady-state lasing, partially analytic and analytic approx => physical insight**

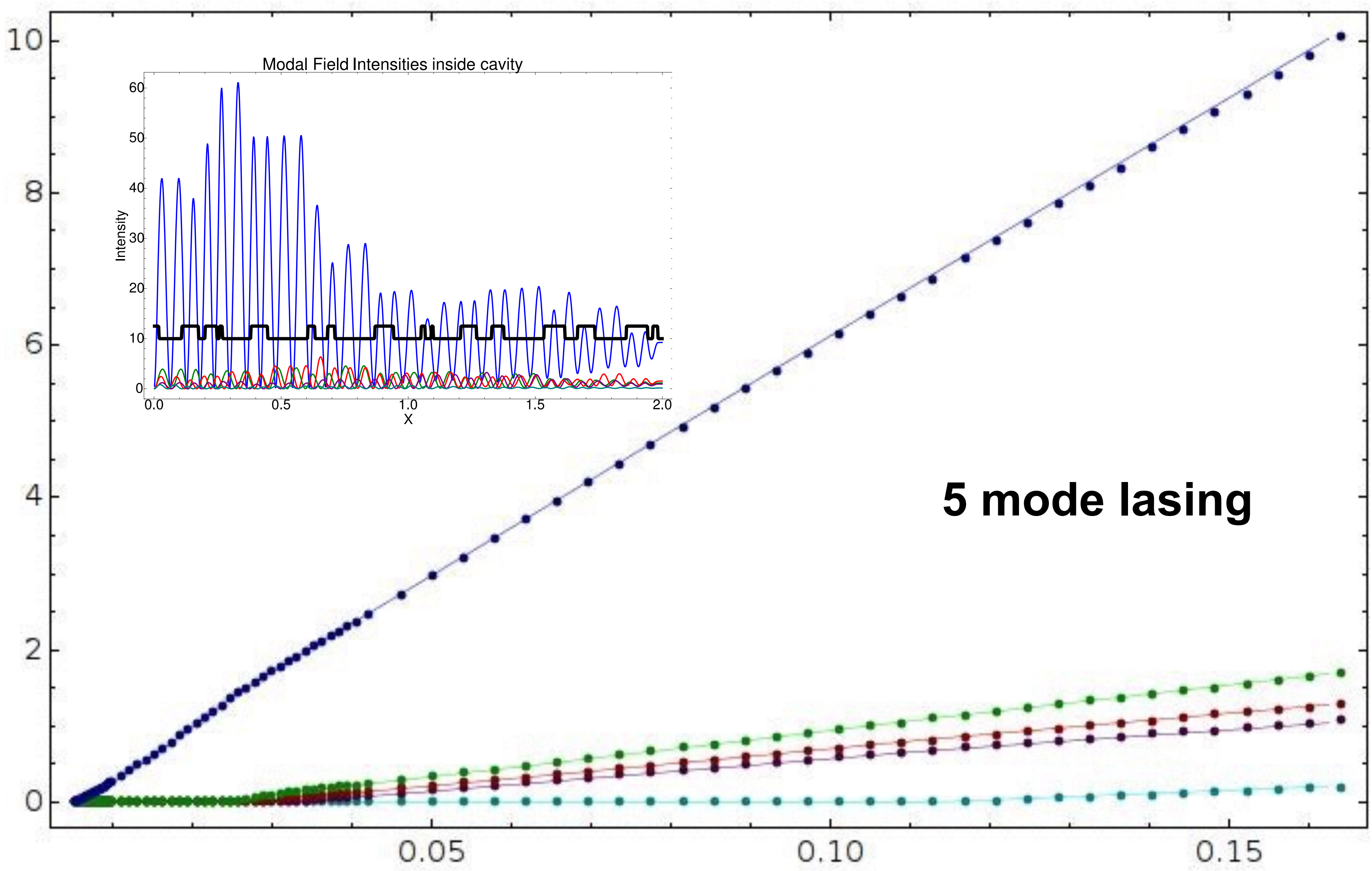
- ☐ **Computationally tractable, no time integration**

- ☐ **Cavities/modes of arbitrary complexity and openness.**

- ☐ **Non-linear hole-burning interactions to infinite order**

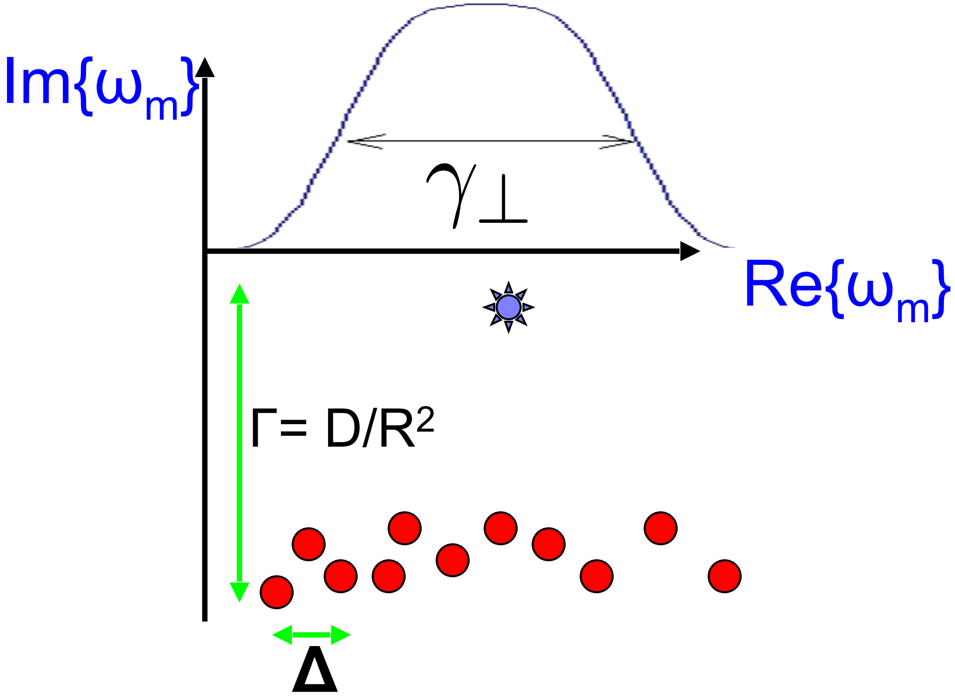
- ☐ **How well does it work? (it has an approximation)**

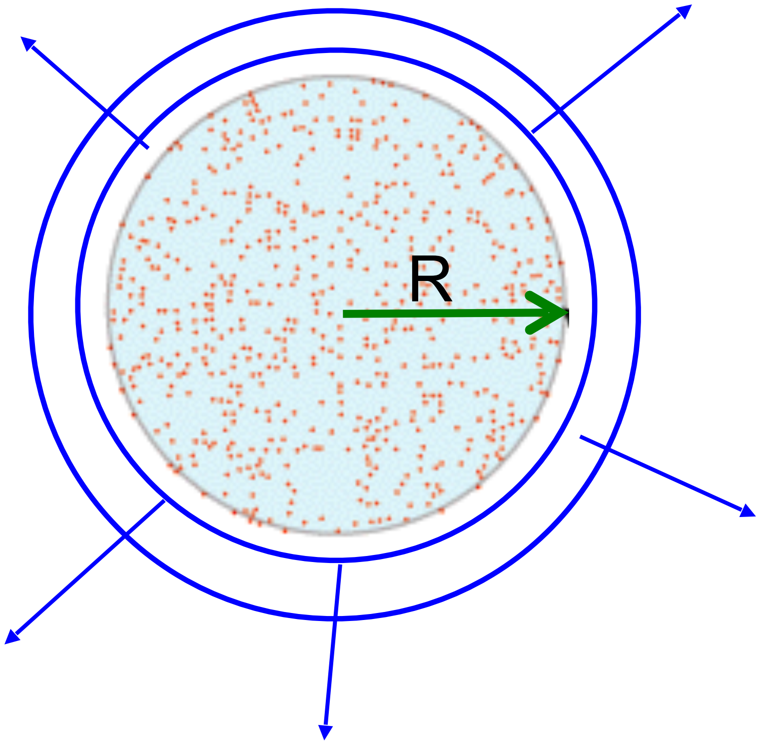
Test: SALT and FDTD agree for 1D random laser



5 mode lasing

- Other FDTD tests of SALT: 2D PCSEL and 3D PC defect mode laser, coupled cavities; also multiple transitions, and injected signals
- No FDTD on 2D RLs (yet), SALT studies:



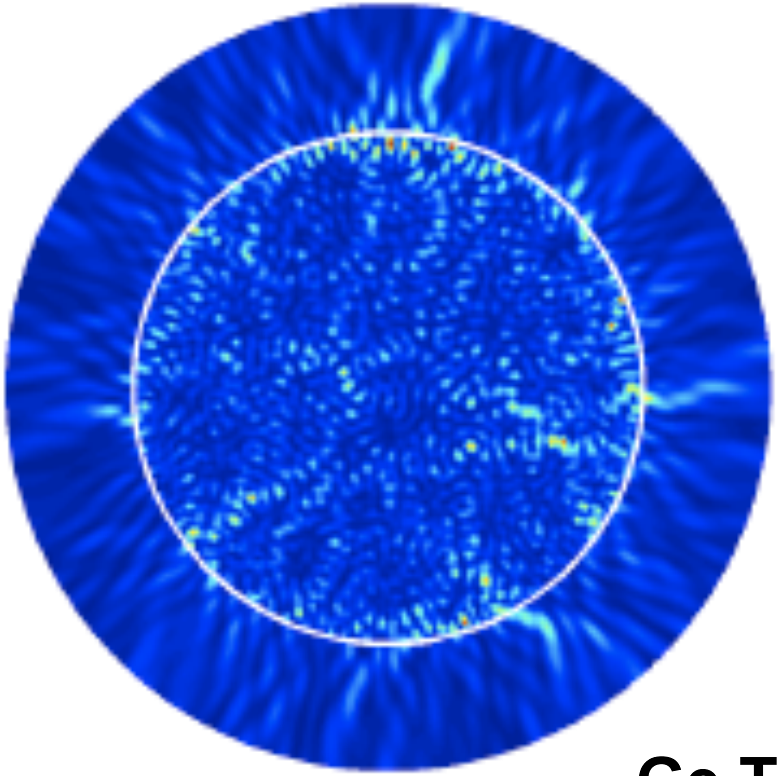


Many modes with similar thresholds as kR gets large

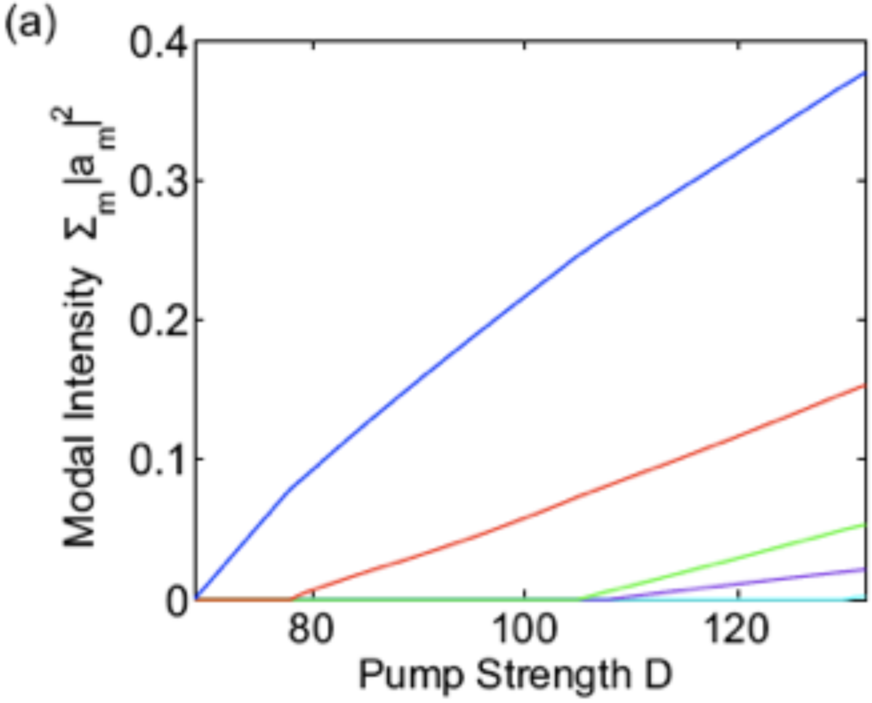
SALT for 2D random laser

“Strong interactions in multimode random lasers”,
H. Tureci, L. Ge, S. Rotter, ADS; Science, 320,643 (2008)

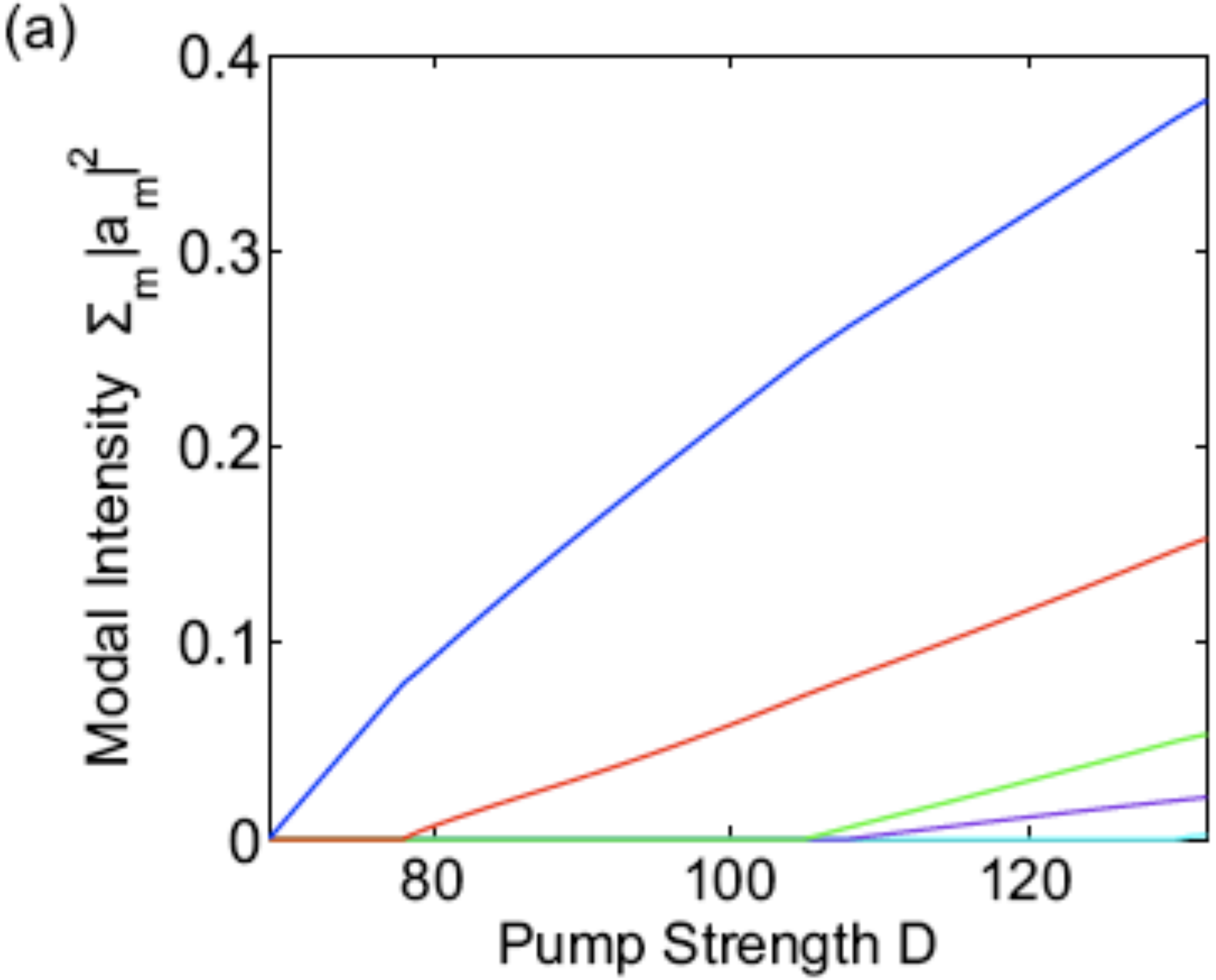
Also, L. Ge, PhD thesis (diffusive regime), and A. Cerjan and A. Goetschy (in preparation) – focus on diffusive results



Ge Thesis



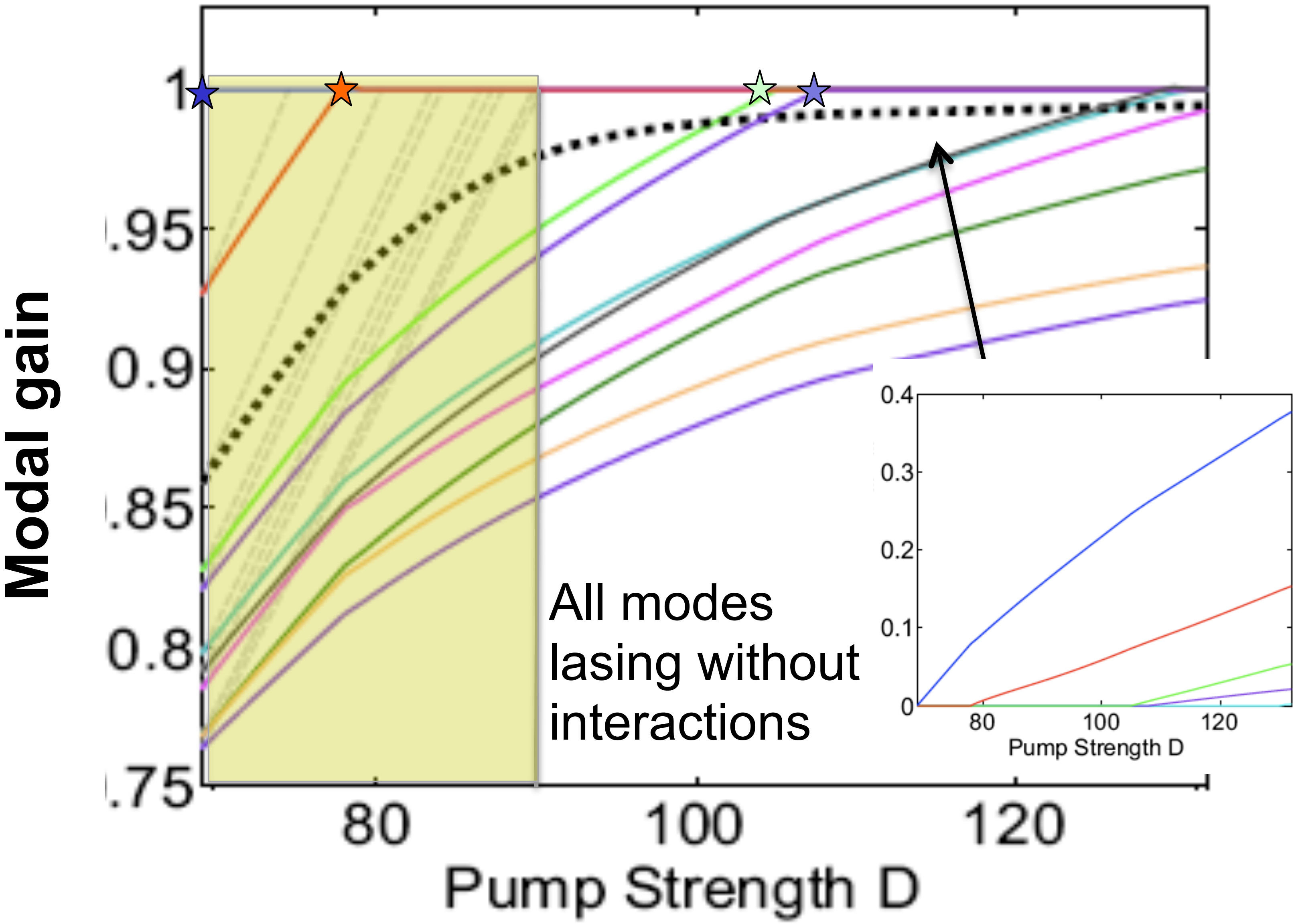
Diffusive Random Laser



4 mode lasing

Note: decreasing power slope

Non-linear interactions



All modes lasing without interactions

Analytic Theory for DRL

Goetschy, Cerjan, ADS, in preparation

Linearized dynamics: $\{\nabla^2 + k^2[1 + \delta\epsilon_c(\mathbf{r},\omega) + \delta\epsilon_g(\omega)]\} E(\mathbf{r},\omega) = 0$

$(k = \omega/c)$

Disorder

Gain curve

Pump profile

Outgoing solution: $\boxed{E(\omega) = G(\omega)F(\mathbf{r})\delta\epsilon_g(\omega)E(\omega)}$

$$G(\omega) = \frac{-k^2}{\nabla^2 + k^2\epsilon_c(\mathbf{r}) + i0^+}$$

Green's function of the disorder

Lasing threshold: $\boxed{\Lambda_n = 1/\delta\epsilon_g(\omega)}$ $\Rightarrow Im\{\Lambda_n\} = 1/\tilde{D}_0$

Eigenvalue of G(ω)

$$G(\omega) = \frac{-k^2}{\nabla^2 + k^2\epsilon_c(\mathbf{r}) + i0^+}$$

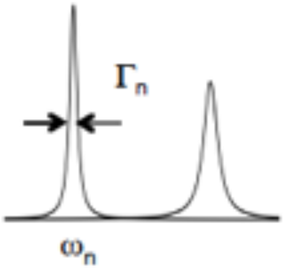
$$\Lambda_n = \frac{\omega^2}{(\omega_n - \mathrm{i}\Gamma_n/2)^2 - \omega^2}$$

Effective H similar to open chaotic cavities

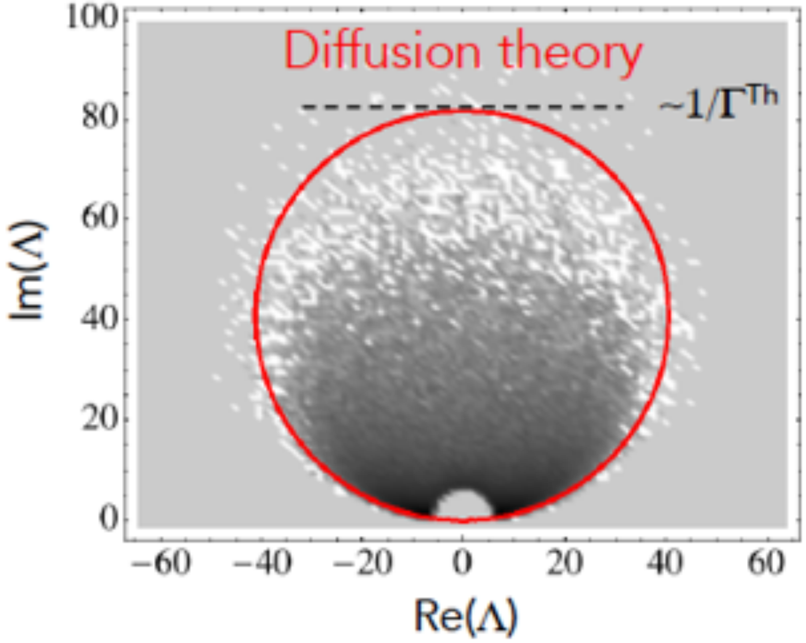
$$\downarrow$$

$$G(\omega) = \frac{-k^2}{k^2 - H^e(\omega)}$$

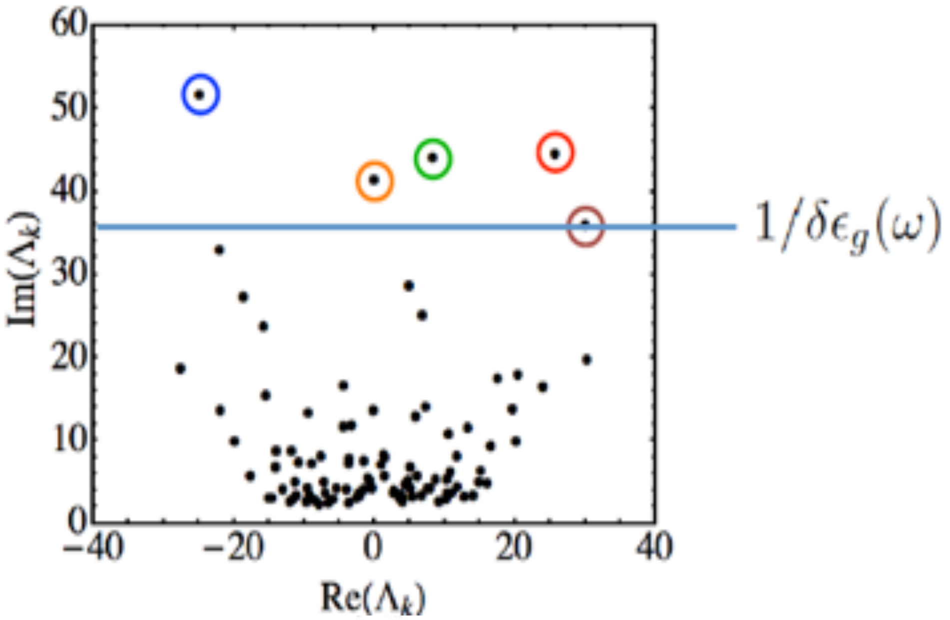
$$H^e = -\nabla^2 - k^2\delta\epsilon_c(\mathbf{r}) + H^{\mathrm{BC}} \longrightarrow$$ Eigenvalue $\omega_n - \mathrm{i}\Gamma_n/2$



Spectrum of the Green's operator



Lasing threshold



Numerical solution obtained from the wave equation in open 2D disk (1 disorder realization)

Statistical averages

Single pole approx to SALT:

$$\Psi_\mu(\vec{r}) = \sum_n a_n^\mu u_n(\vec{r}, k_\mu) \approx a_\mu u_\mu(\vec{r})$$

μ -> m, u_μ -> R_m

$$\Phi_m = |a_m|^2 \int \mathrm{d}\mathbf{r} |R_m(\mathbf{r})|^2,$$

$$\omega_m = \omega_a + \frac{\mathrm{Re}\Lambda_m(\omega_m)}{\mathrm{Im}\Lambda_m(\omega_m)} \gamma_\perp,$$

➡

$$\sum_{p=1}^{N_L} \alpha_{mp} \Gamma_p \Phi_p = \tilde{D}_0 \mathrm{Im}\Lambda_m(\omega_m) - 1,$$

Constrained linear Eq. for modal intensities

$$\alpha_{mp} = \frac{\int \mathrm{d}\mathbf{r} F(\mathbf{r}) R_m(\mathbf{r})^2 |R_p(\mathbf{r})|^2}{\int \mathrm{d}\mathbf{r} |R_m(\mathbf{r})|^2}.$$

Approx real

$$\alpha_{mp} \simeq \langle \alpha_{mp} \rangle \simeq \frac{1 + 2\delta_{mn}}{V},$$

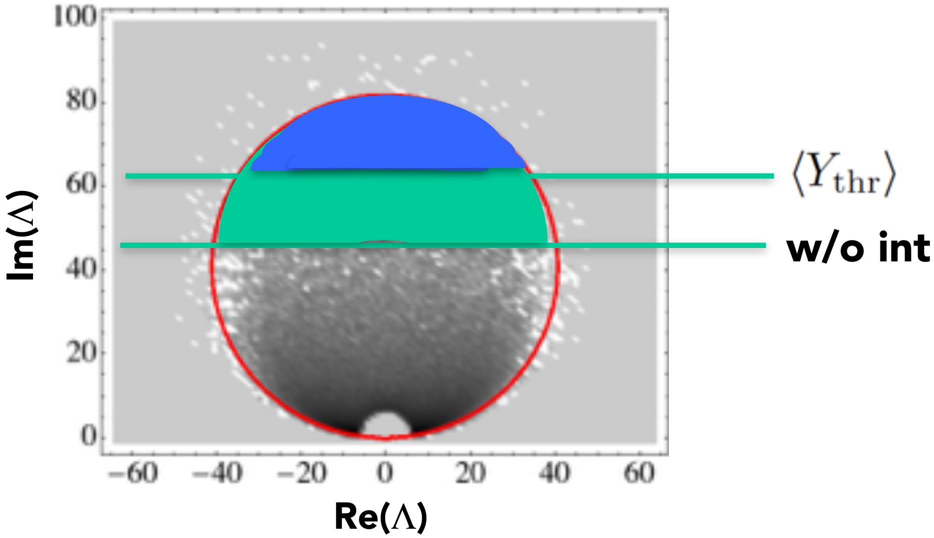
Self- averaging gaussian approx

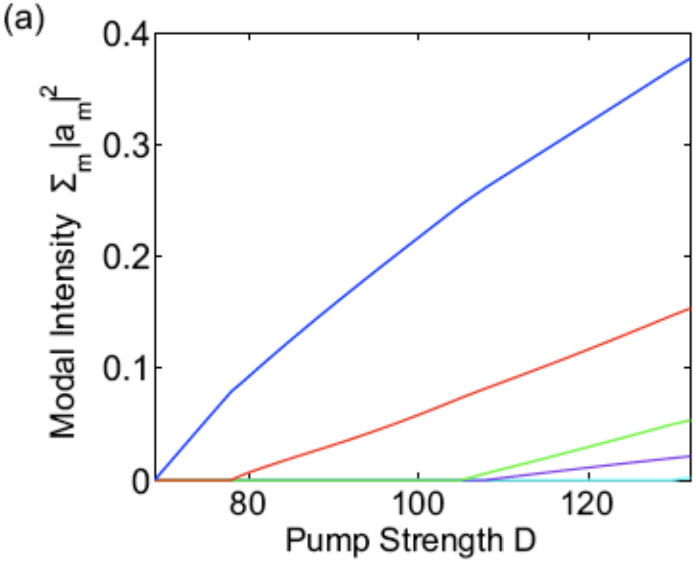
$$\Phi_m = \frac{V}{2\Gamma_m}\left[\tilde{D}_0 \mathrm{Im}\Lambda_m(\omega_m) - \qquad Y_{thr} \qquad \right].$$

Modal interactions

Express all properties of interest in terms of P(Im{Λ})

Predicts monotonically decreasing modal slopes

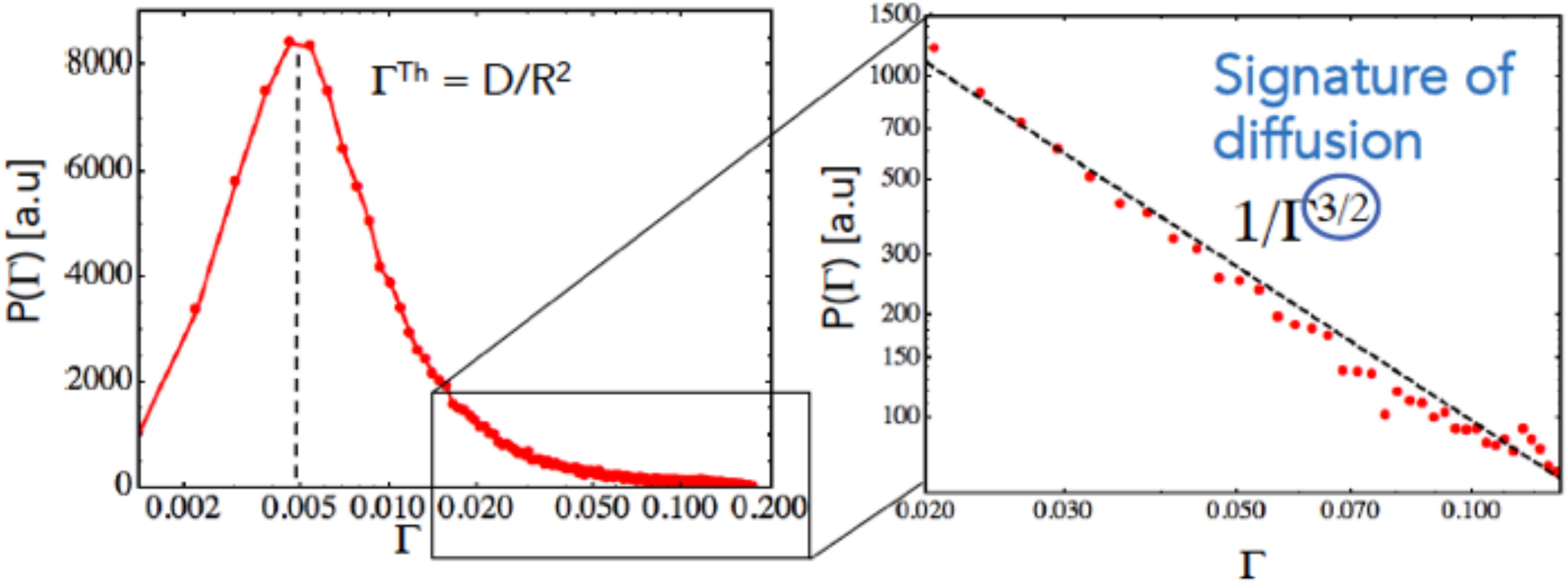




$\langle Y_{\mathrm{thr}} \rangle$ Eventually saturates with pump

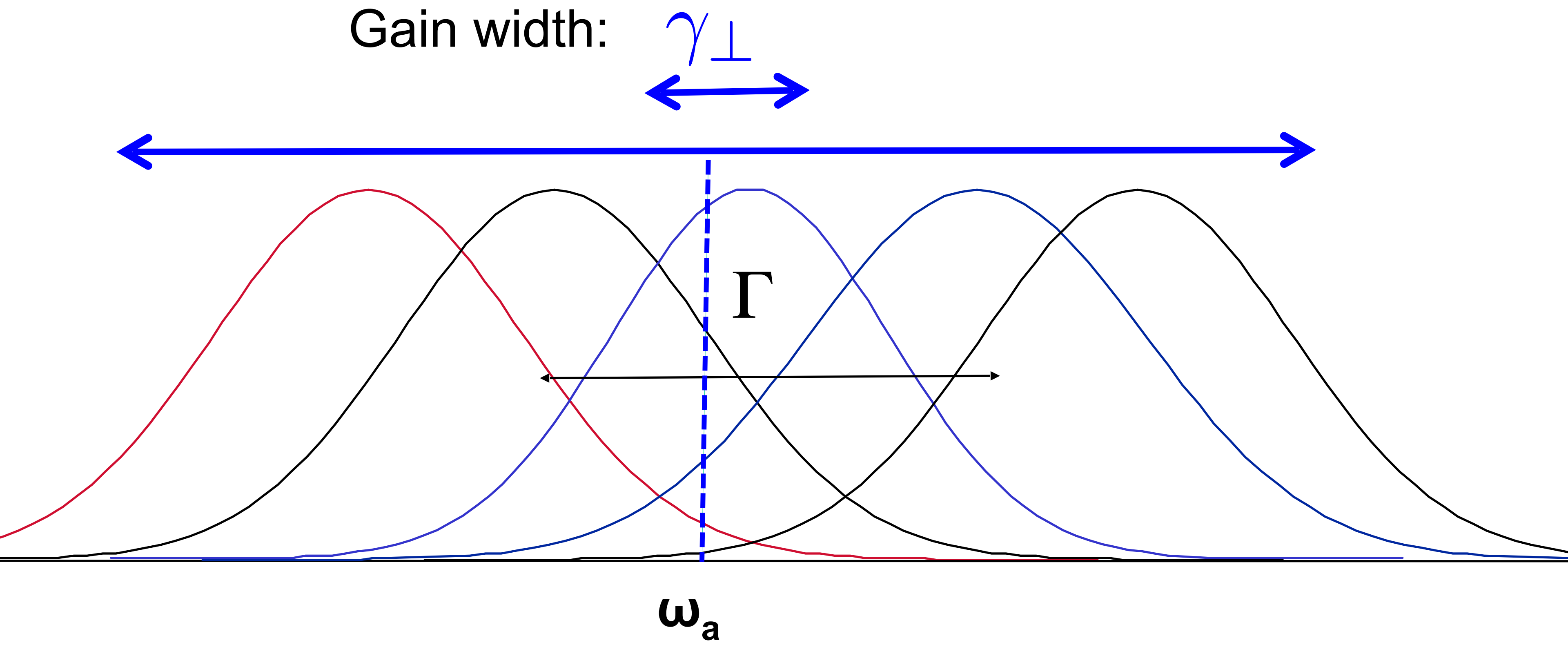
Need prob dist of Im{Λ} ~ Γ

Quasi-modes statistics
(simulations in open 2D disk)



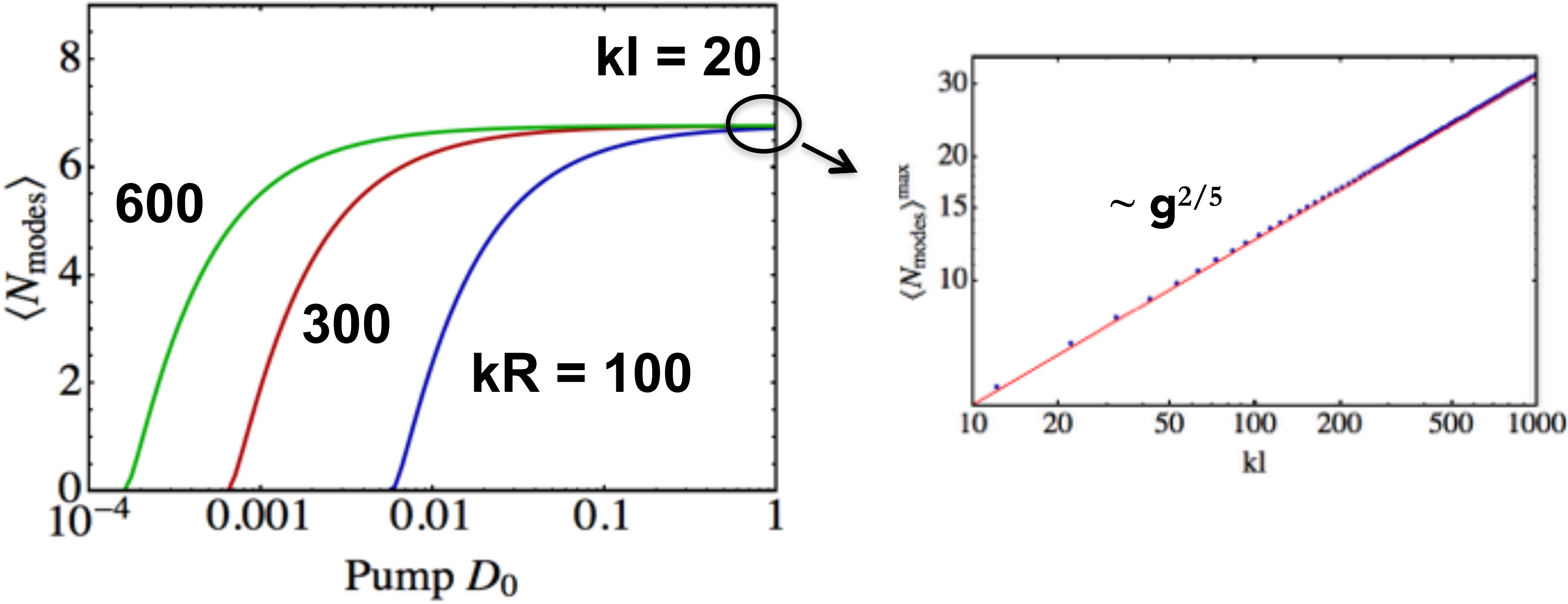
$$\boxed{P(\mathrm{Im}\Lambda) \simeq \sqrt{2}\frac{kR\sqrt{k\ell}}{(\mathrm{Im}\Lambda)^{3/2}} \ln\left(\sqrt{\frac{\omega/\Gamma^{\mathrm{Th}}}{\mathrm{Im}\Lambda} - 1} + \sqrt{\frac{\omega/\Gamma^{\mathrm{Th}}}{\mathrm{Im}\Lambda}}\right)}$$

Two limits

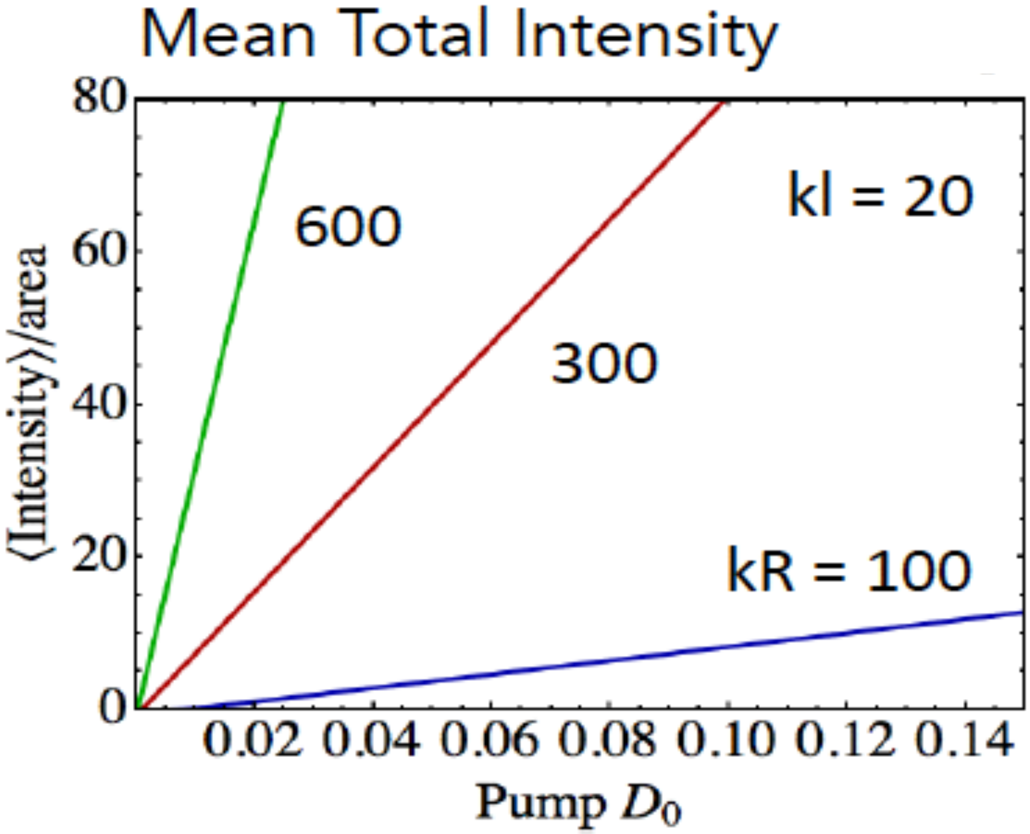


Results $\gamma_\perp < \bar{\Gamma}$

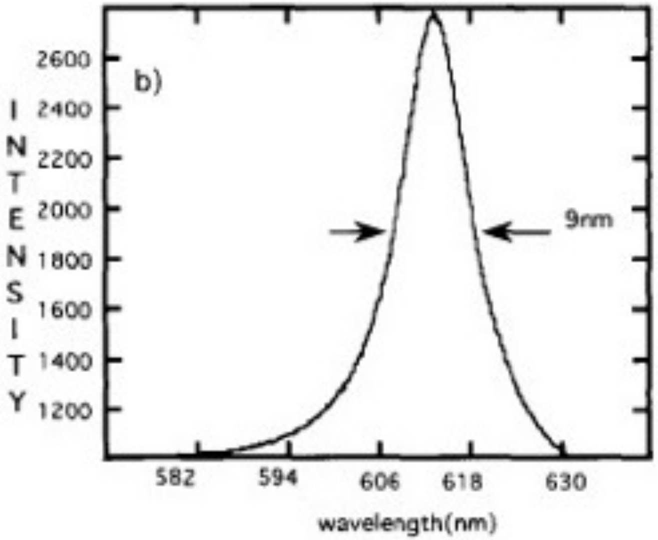
In 2D



Total Intensity $\gamma_\perp < \bar{\Gamma}$

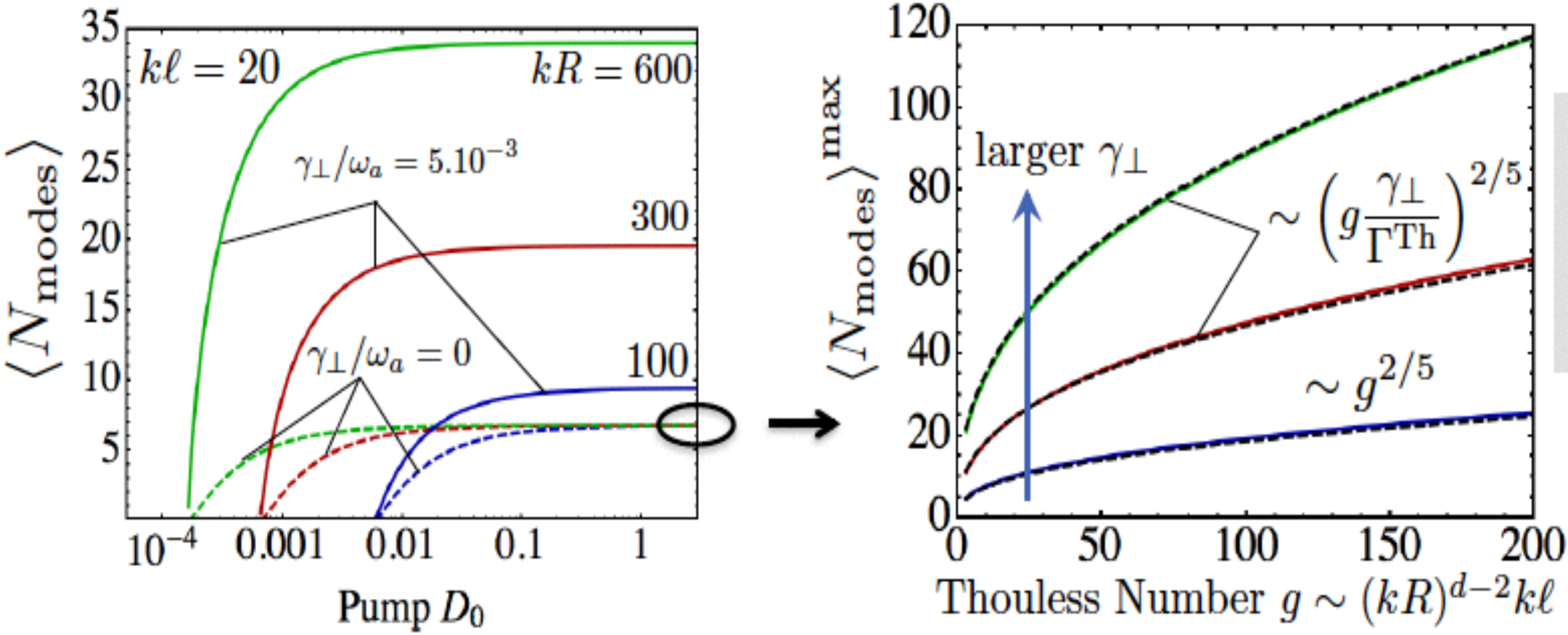


$$slope \sim (kR)^2$$



Useful for unresolved random laser emission

Results: $\gamma_\perp > \bar{\Gamma}$

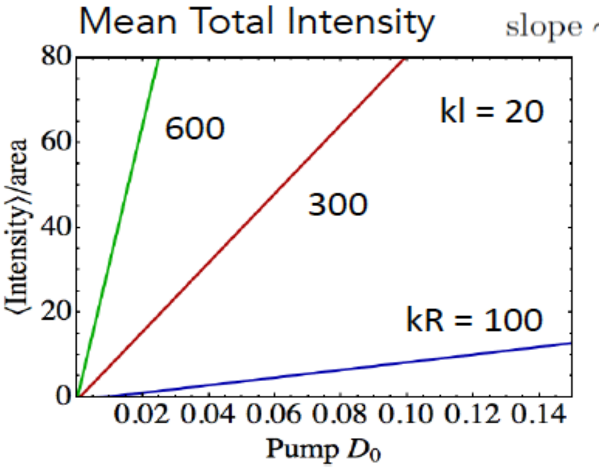


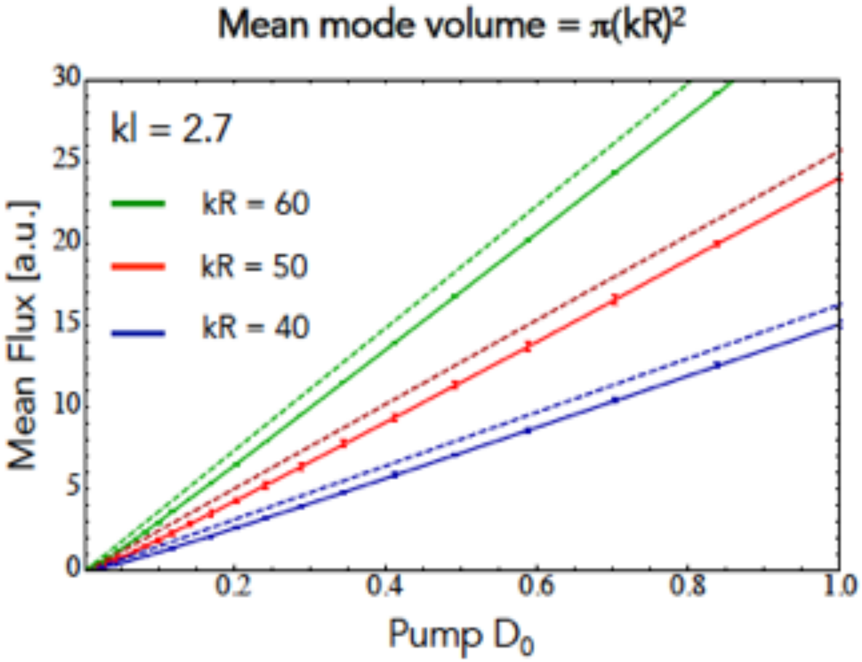
Scaling parameter is:

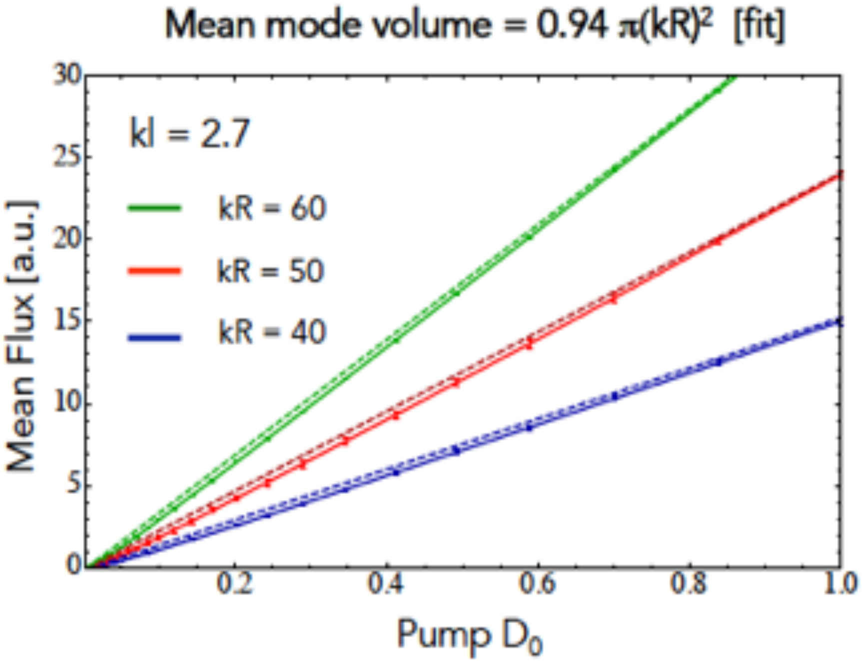
$$g = \longleftrightarrow \gamma_\perp < \bar{\Gamma}$$

$$g' = g\frac{\gamma_\perp}{\bar{\Gamma}} \longleftrightarrow \gamma_\perp > \bar{\Gamma}$$

Comparison of total intensities

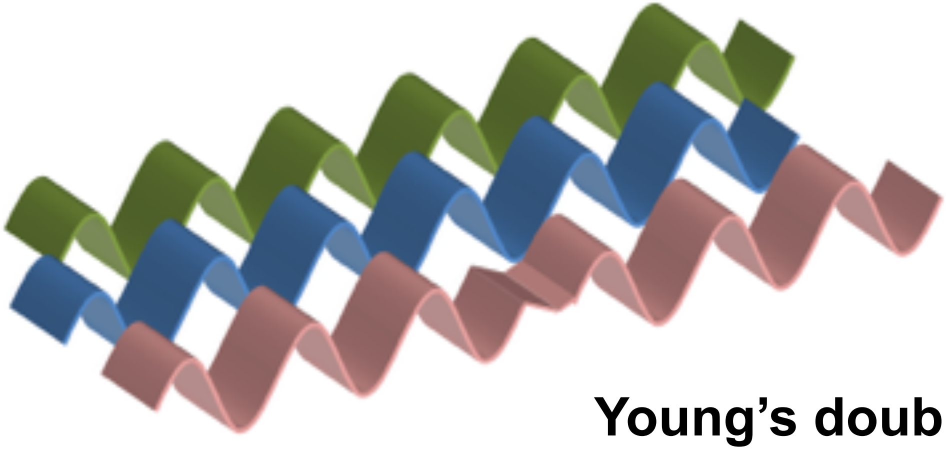

Mean Total Intensity


Mean mode volume = π(kR)²


Mean mode volume = 0.94 π(kR)² [fit]

Savior er App for Random Lasers: Exploiting spatial incoherence

Spatial Coherence



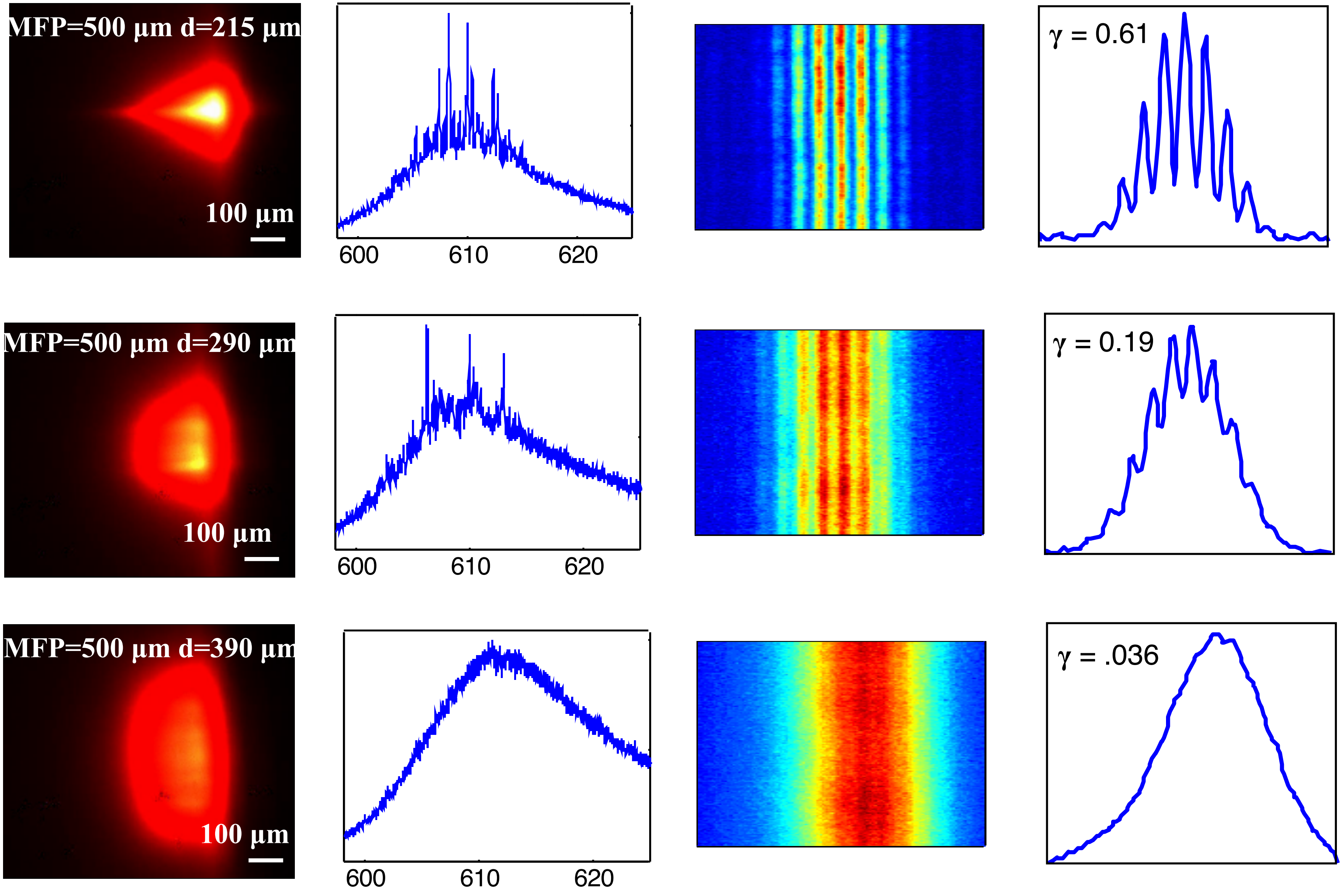
Young's double slit experiment

If wavefronts at different points have a stable phase relationship there will be interference fringes

⇒ Always true of single mode lasing

⇒ Not true of multimode

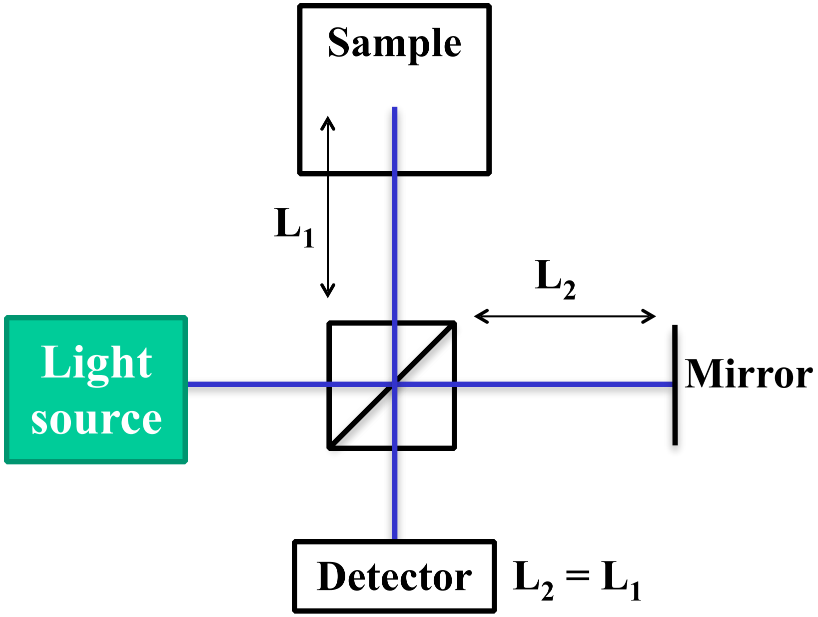
Controlling Spatial Coherence in RL by Varying Pump Volume



Imaging applications of RL?

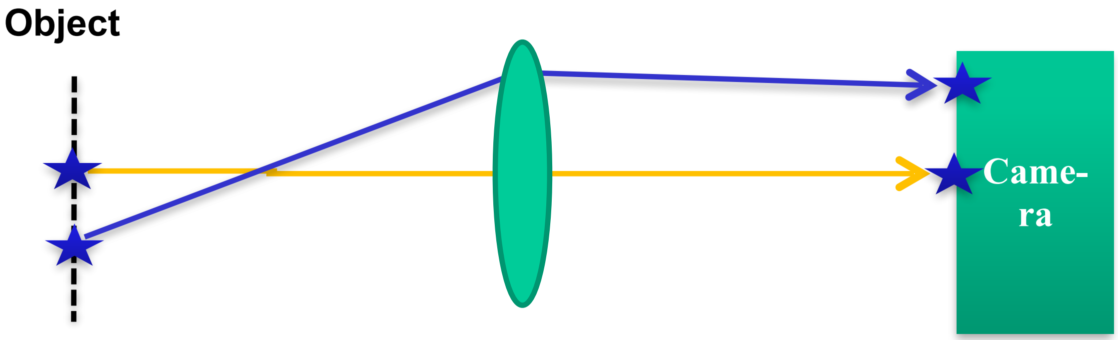
Prof. Michael Choma, MD. PhD, Yale Medicine

Optical coherence tomography (OCT)

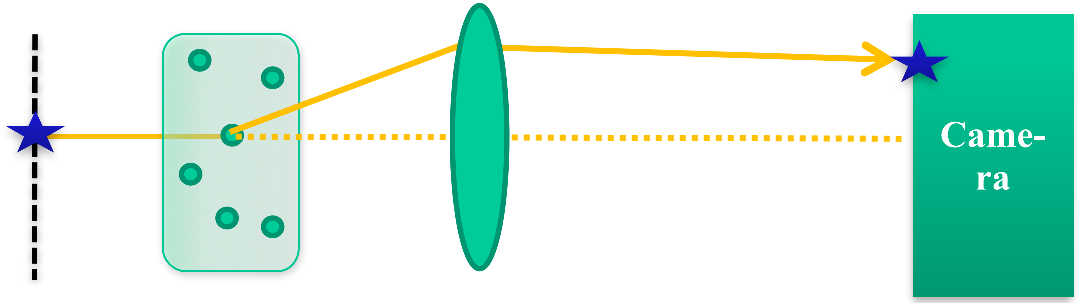


Brandon Redding,
Res. Scientist (Cao Group)

Spatial cross talk



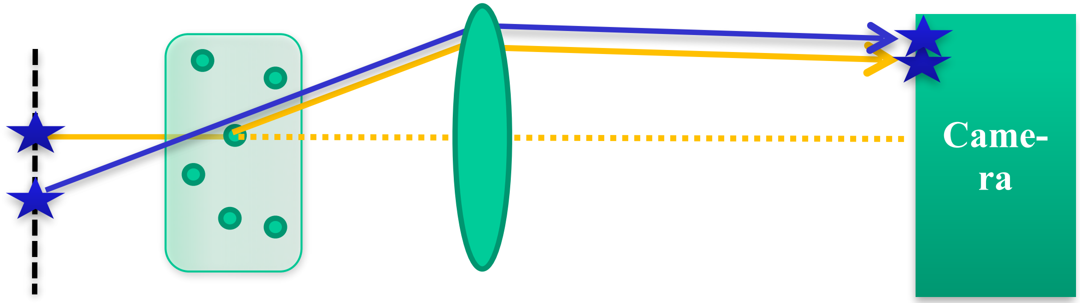
Coherent illumination



$$I = |E|^2 = |E_1 + E_2|^2$$

$$= |E_1|^2 + |E_2|^2 + \cancel{2E_1E_2\cos(\theta)}$$

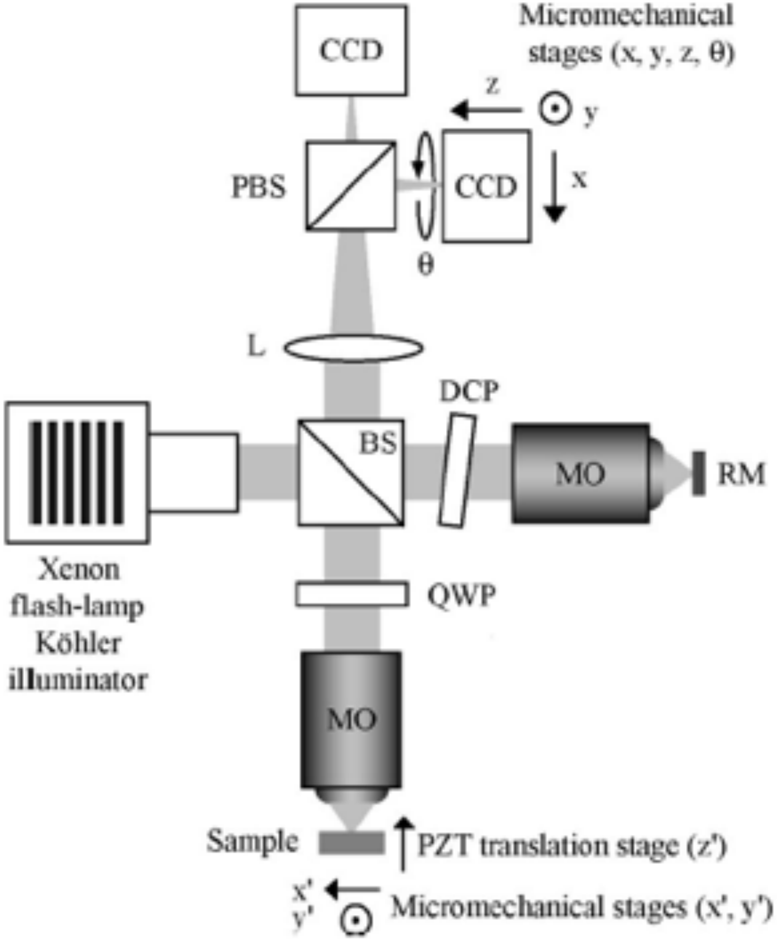
Incoherent illumination



$$I = I_1 + I_2$$

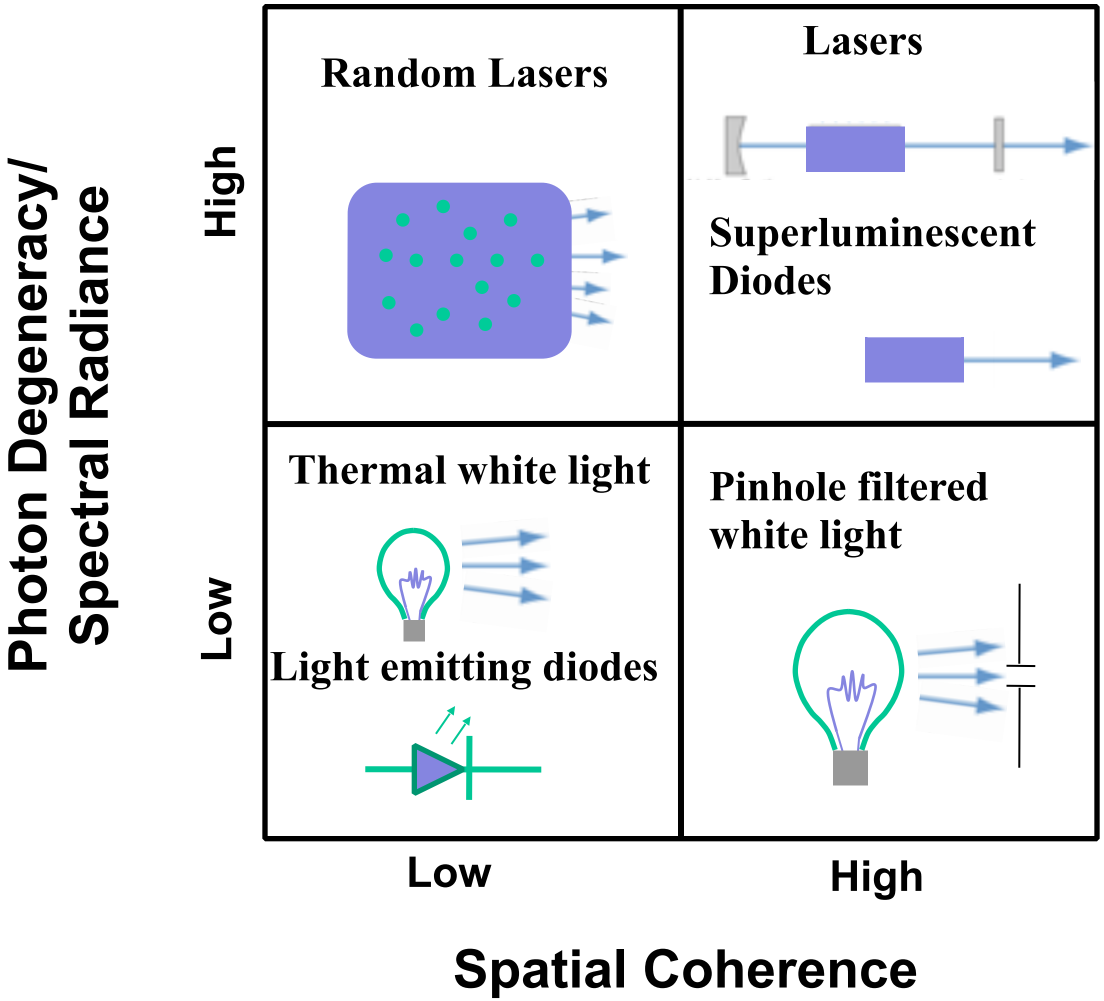
Much reduced artifacts

Full-Field OCT



Moneron, Boccara, & Dubois, Opt. Lett. **30**, 1351 (2005)

Ideal Illumination Source for Imaging

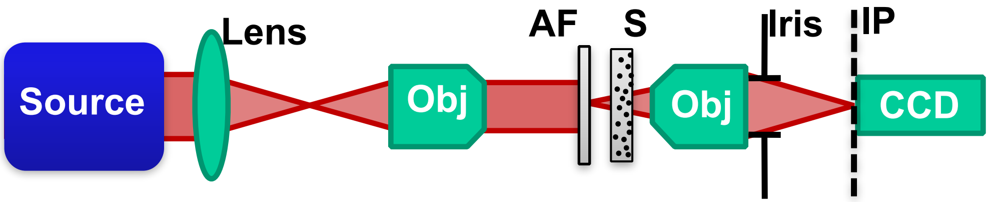


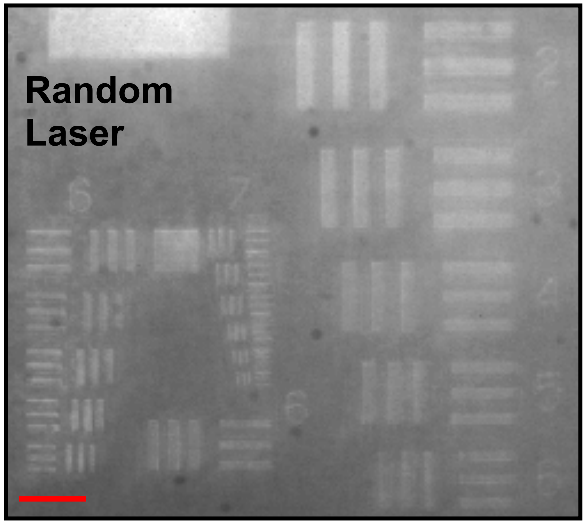
Photon Degeneracy/ Spectral Radiance

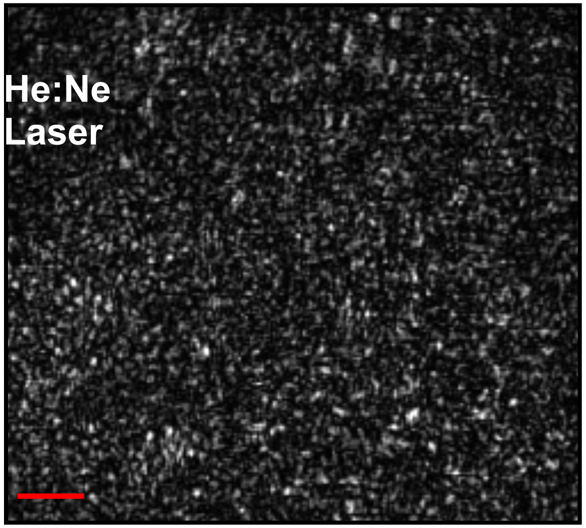
High	Random Lasers	Lasers Superluminescent Diodes
Low	Thermal white light Light emitting diodes	Pinhole filtered white light
	Low	High

Spatial Coherence

Speckle-free Laser Imaging

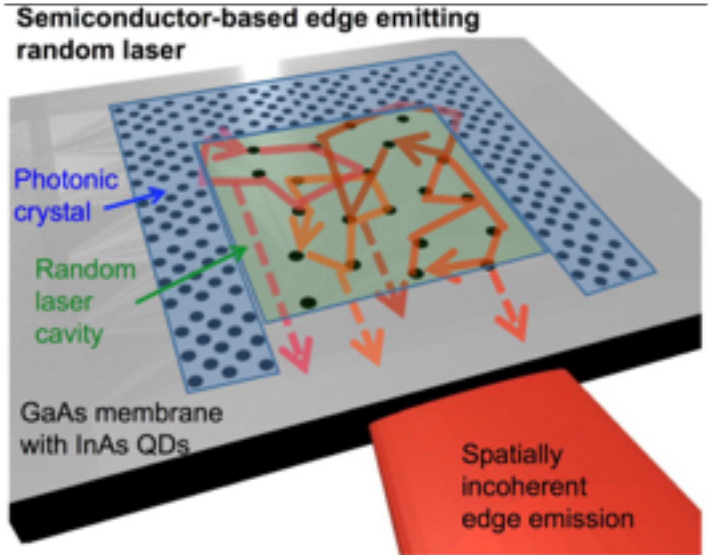






Redding, Choma & HC, Nature Photonics **6,** 355 (2012)

On-chip Electrically-Pumped Semiconductor Random laser

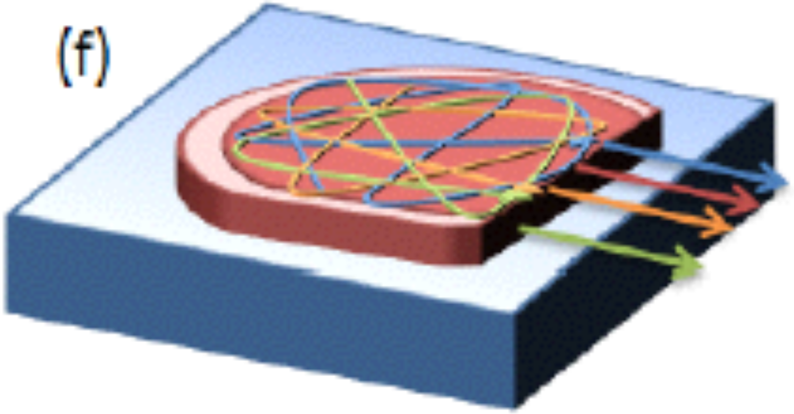


Development of a New Light Source for Massive Parallel Confocal Microscopy and Optical Coherence Tomography

NIH National Institutes of Health
Turning Discovery Into Health

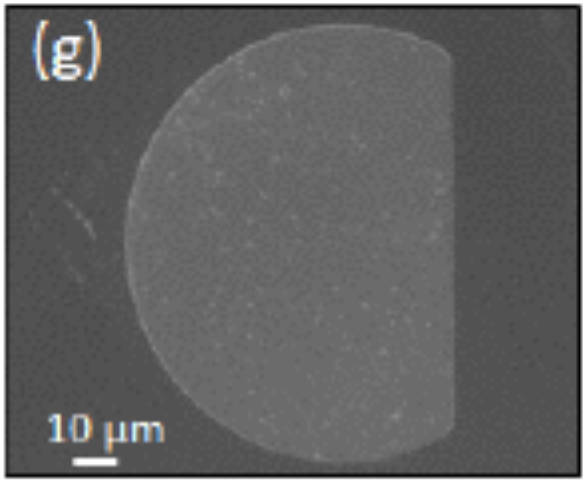
Do we really want to use a random laser?

No – a simpler chaotic shape is easier to fabricate



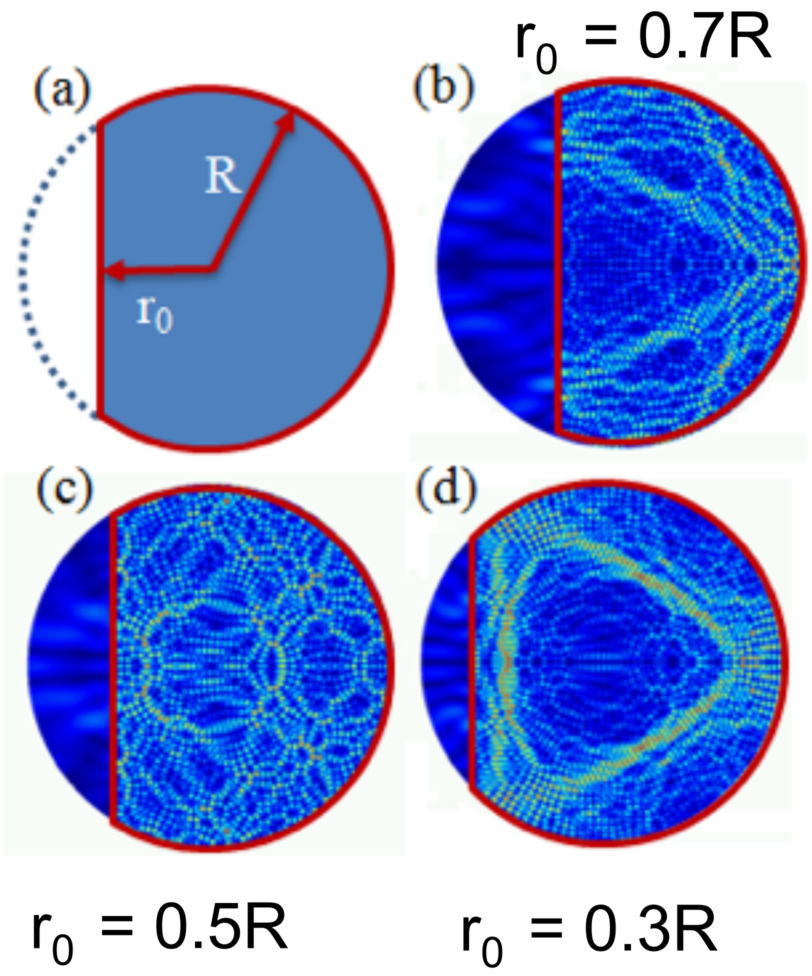
Fabricated on chip AlGaAs - GaAs QW structure.

B. Redding, A. Cerjan, X. Huang, ADS, M. L. Lee, M. A. Choma, H. Cao, PNAS, in press

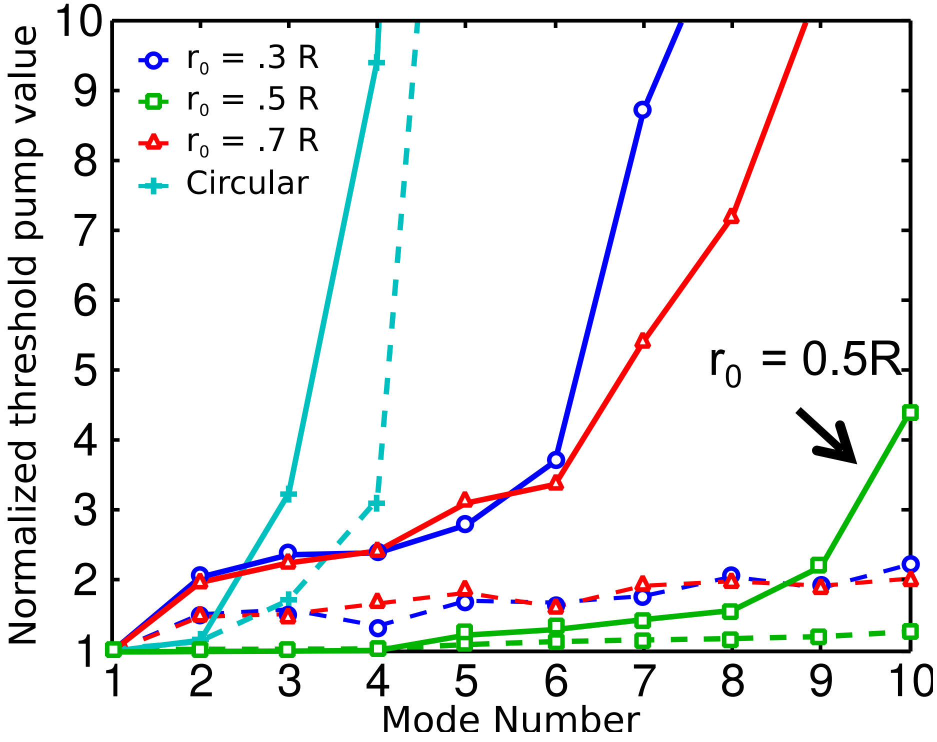


What Specific D-Shape is best?

Want max number of lasing modes at lowest power level – perfect for SALT



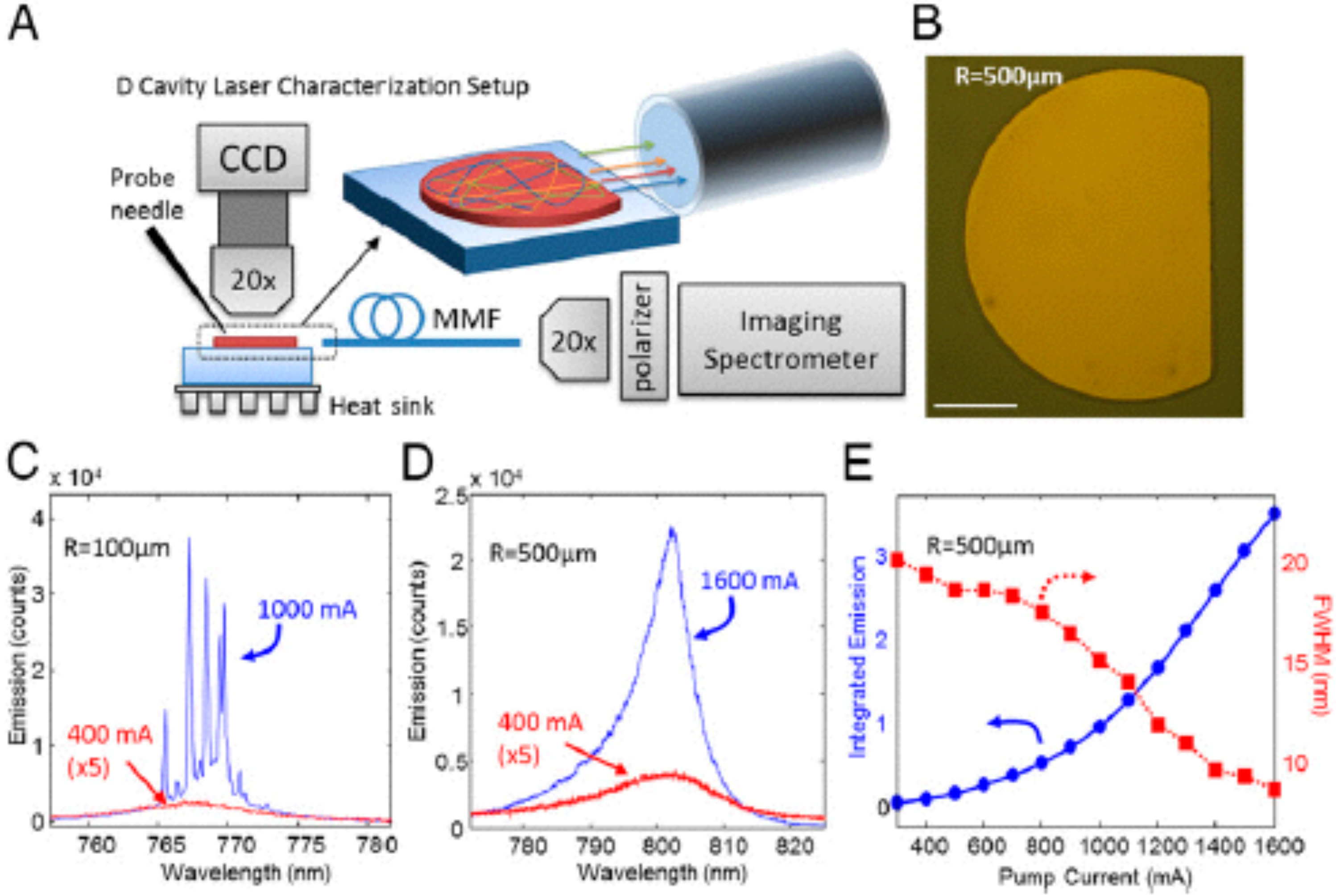
Which is best?



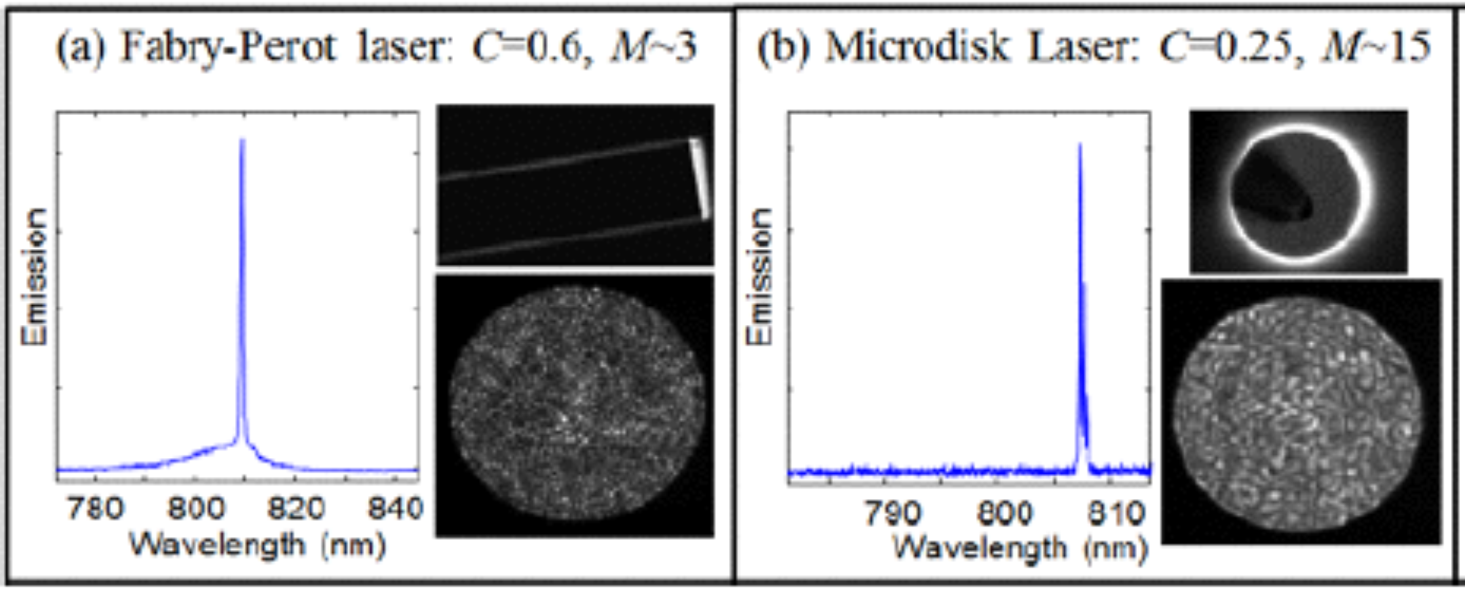
Effect comes from:

1) Flat dist of passive cavity Q
2) Weaker mode comp for more uniform chaotic states

D-laser characterization

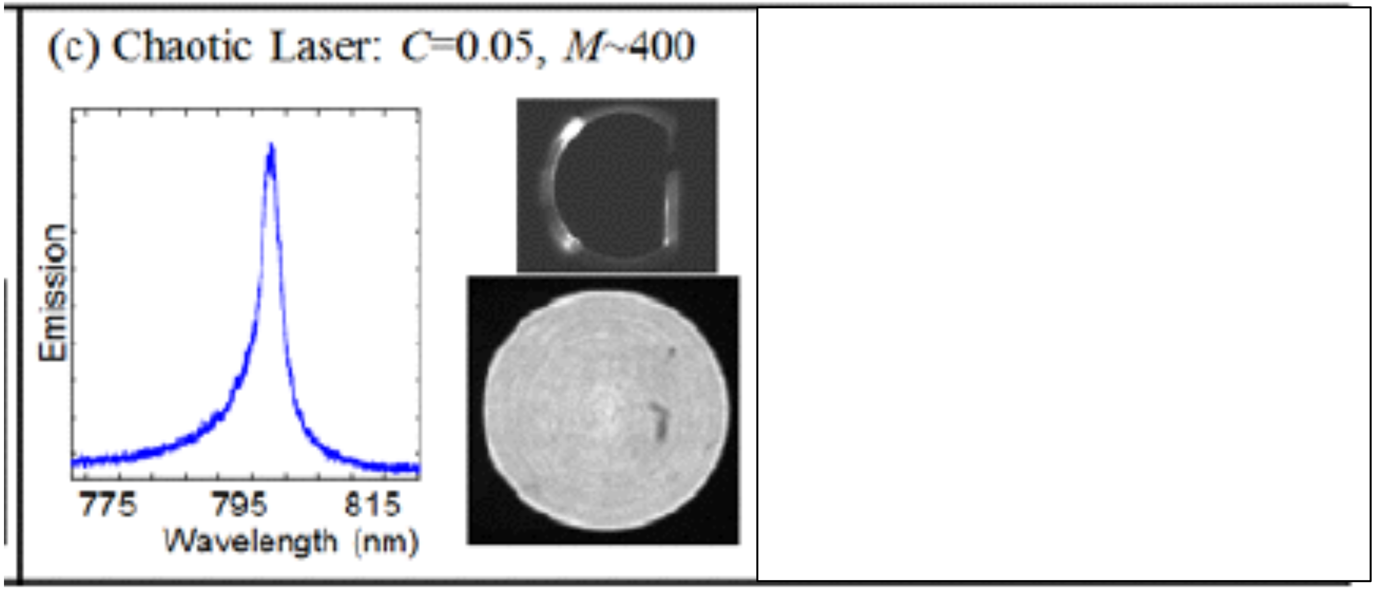


Results?



(a) Fabry-Perot laser: C=0.6, M~3

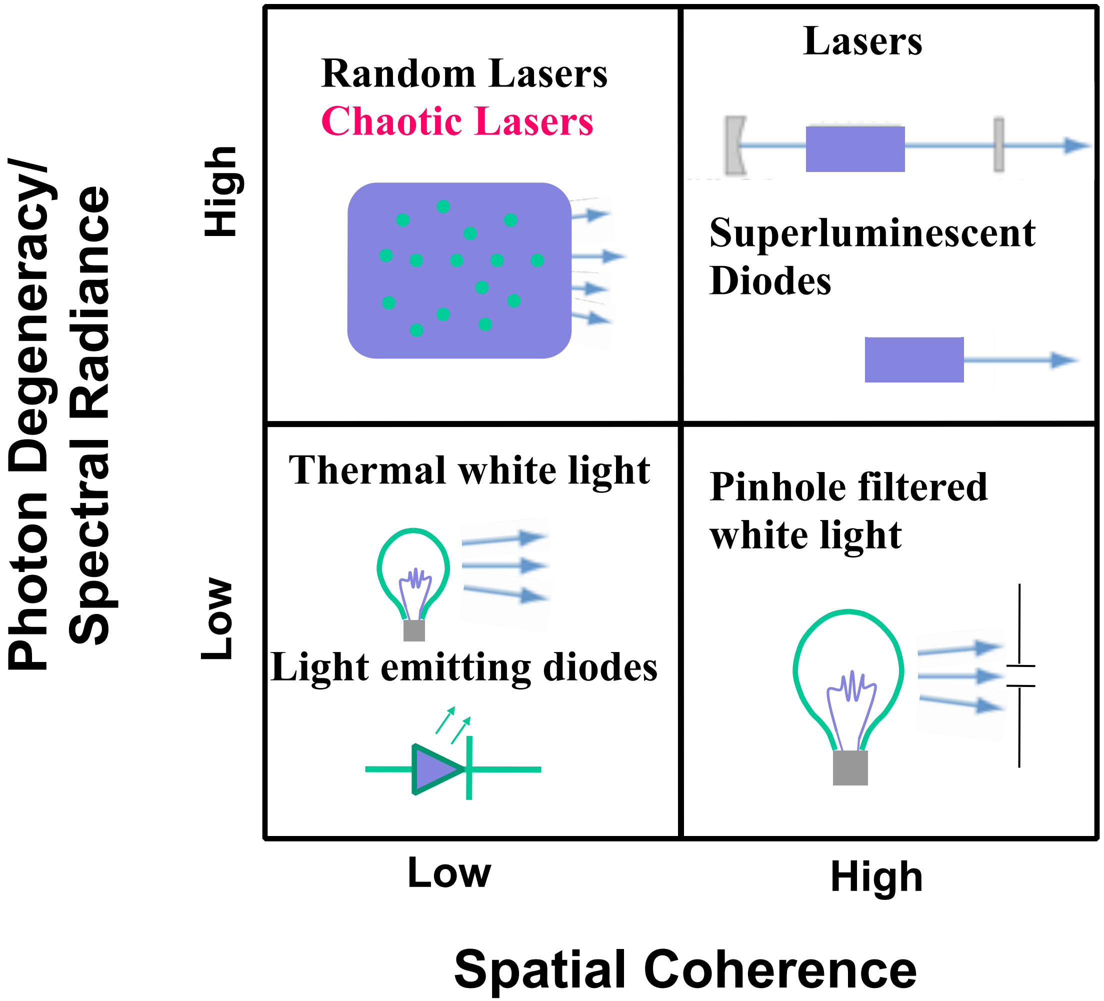
(b) Microdisk Laser: C=0.25, M~15



(c) Chaotic Laser: C=0.05, M~400

Success!

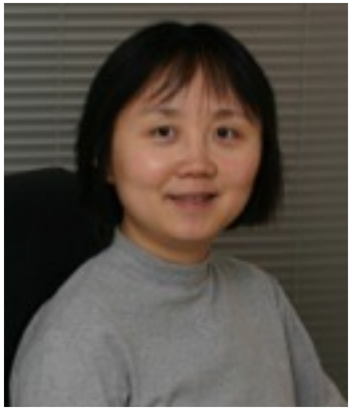
Ideal illumination Source for Imaging



Thanks!

Yidong

Li



Hui

Hakan

Stefan

Arthur

Alex

Brandon