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The Statistical Signature of BosonSampling

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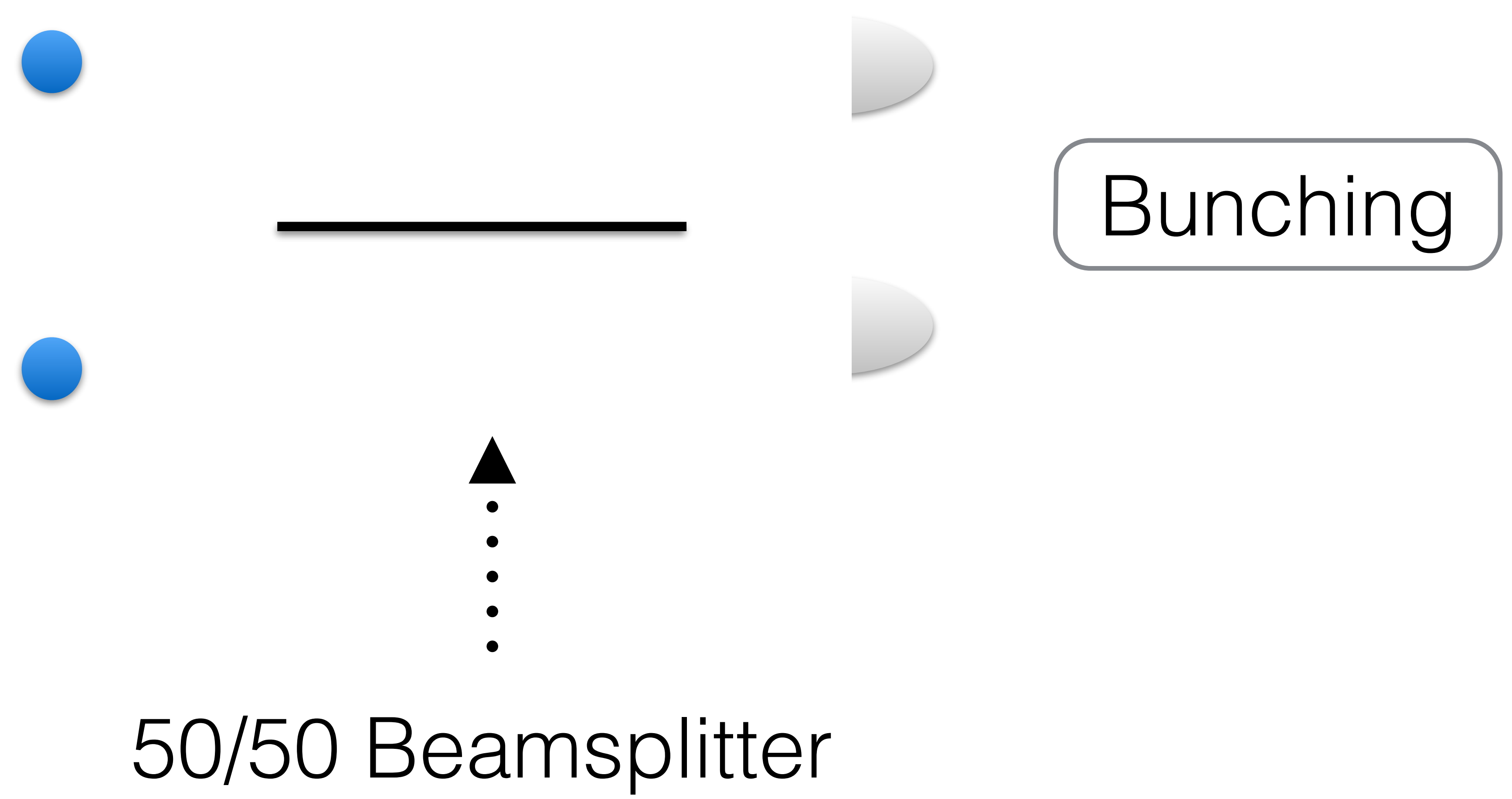
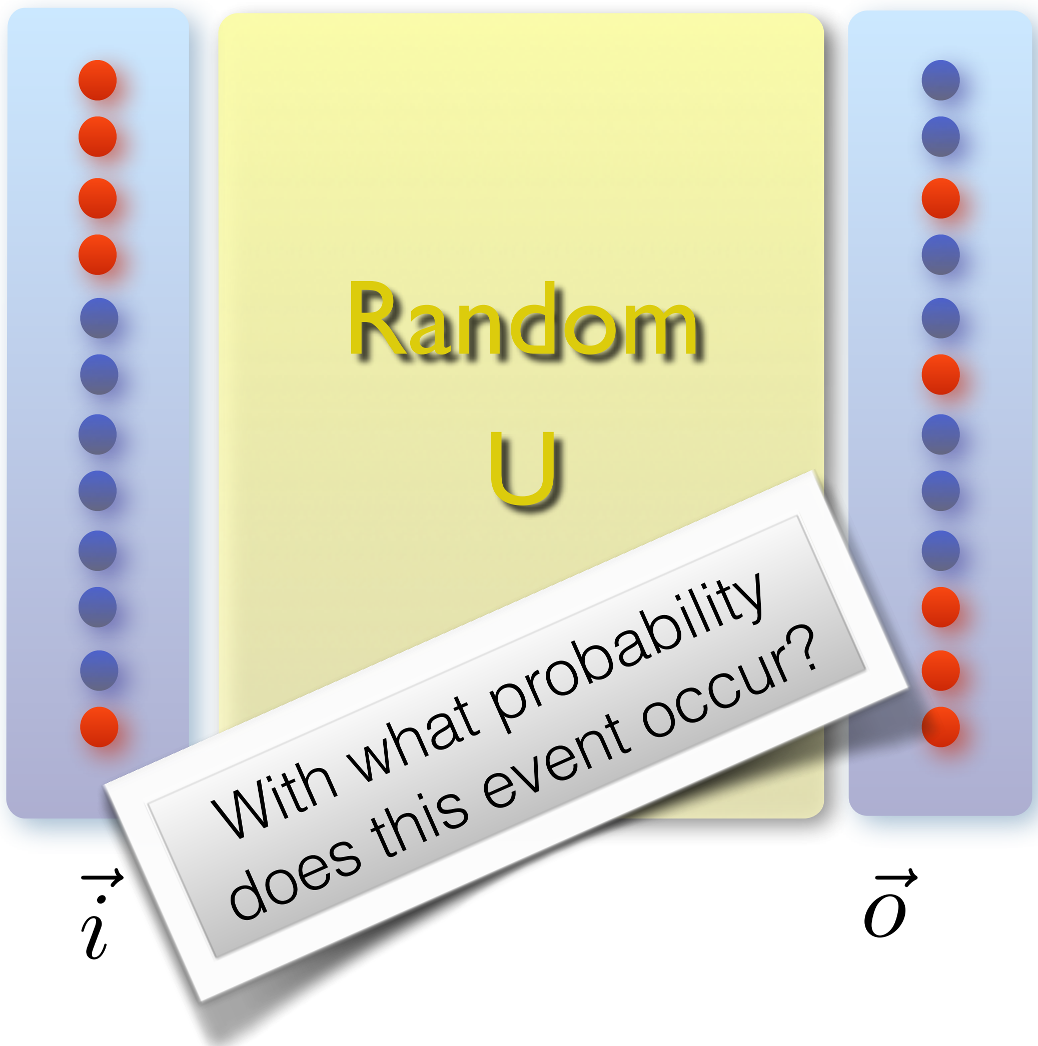
Luchon, March 2015

Bosons are Remarkable



BosonSampling



$$U_{\vec{i},\vec{o}} = \begin{pmatrix} U_{3,1} & U_{3,2} & U_{3,3} & U_{3,4} & U_{3,12} \\ U_{6,1} & U_{6,2} & U_{6,3} & U_{6,4} & U_{6,12} \\ U_{10,1} & U_{10,2} & U_{10,3} & U_{10,4} & U_{10,12} \\ U_{11,1} & U_{11,2} & U_{11,3} & U_{11,4} & U_{11,12} \\ U_{12,1} & U_{12,2} & U_{12,3} & U_{12,4} & U_{12,12} \end{pmatrix}$$

Distinguishable

$$p_{\vec{i}\to\vec{o}} = \text{perm} \left|U_{\vec{i},\vec{o}}\right|^2_{\text{comp}}$$

Fermions

$$p_{\vec{i}\to\vec{o}} = \left|\det U_{\vec{i},\vec{o}}\right|^2$$

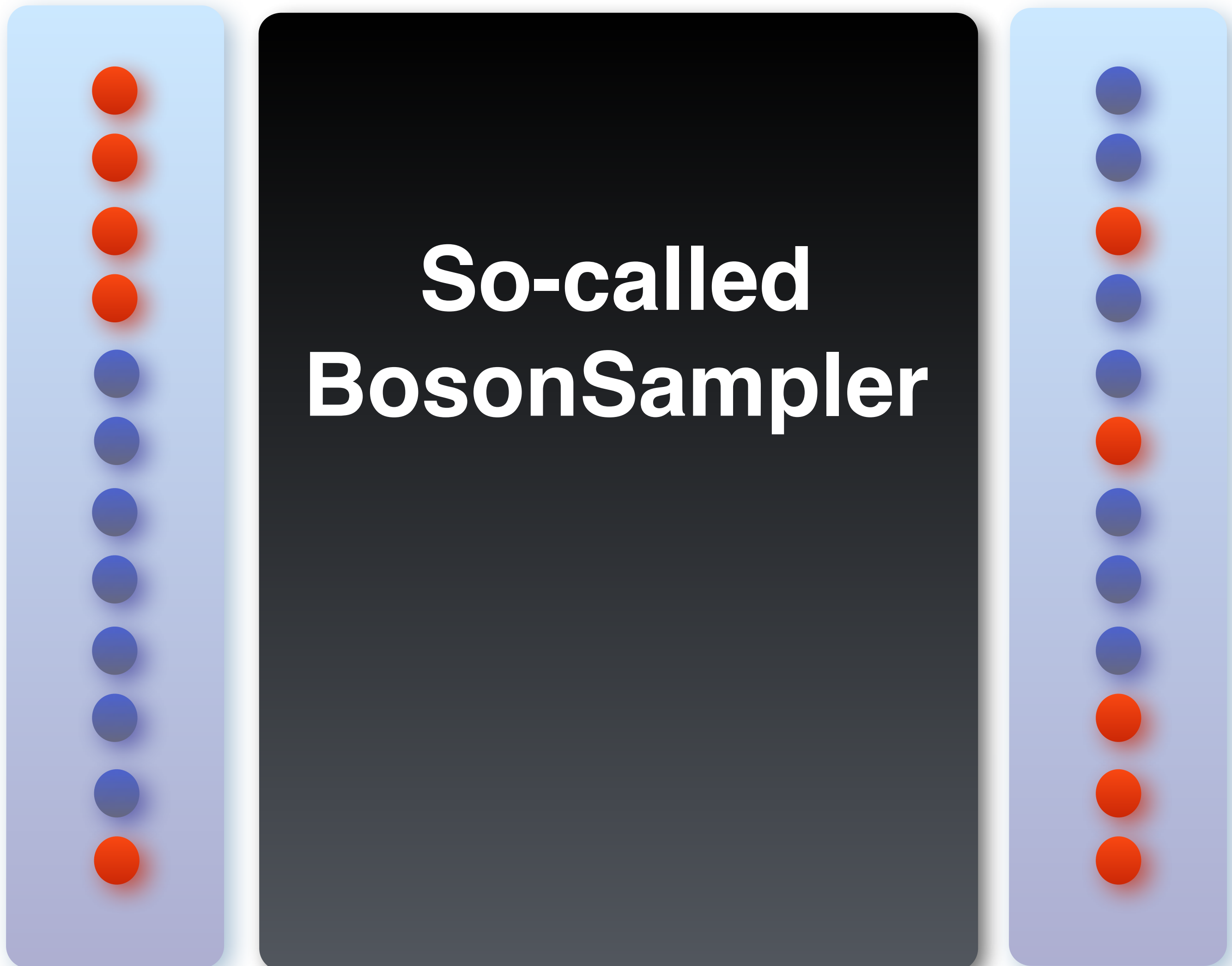
Bosons

$$p_{\vec{i}\to\vec{o}} = \left|\text{perm}\ U_{\vec{i},\vec{o}}\right|^2$$

Computationally Complex

Aaronson and Arkhipov, Theory of Computing **4**, 143 (2013)

Certification

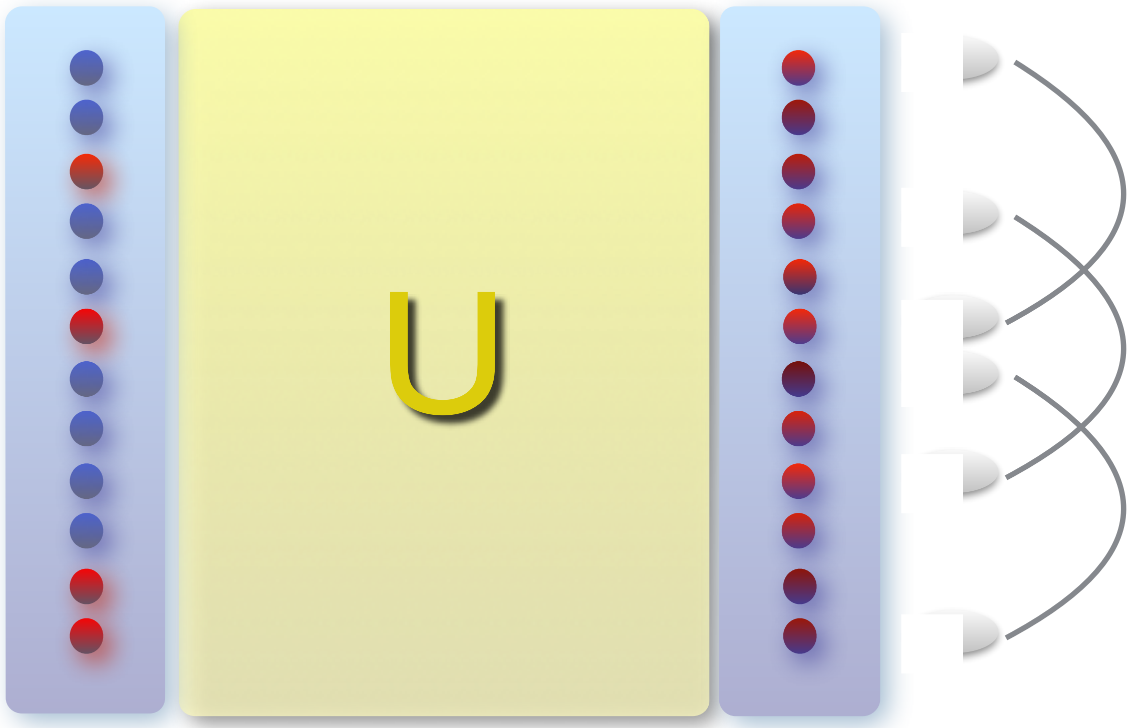


Does the machine work?

Let's calculate the result!

The reason why we like BosonSampling is also the reason why we cannot **directly** certify it

Obtaining the Statistical Signature



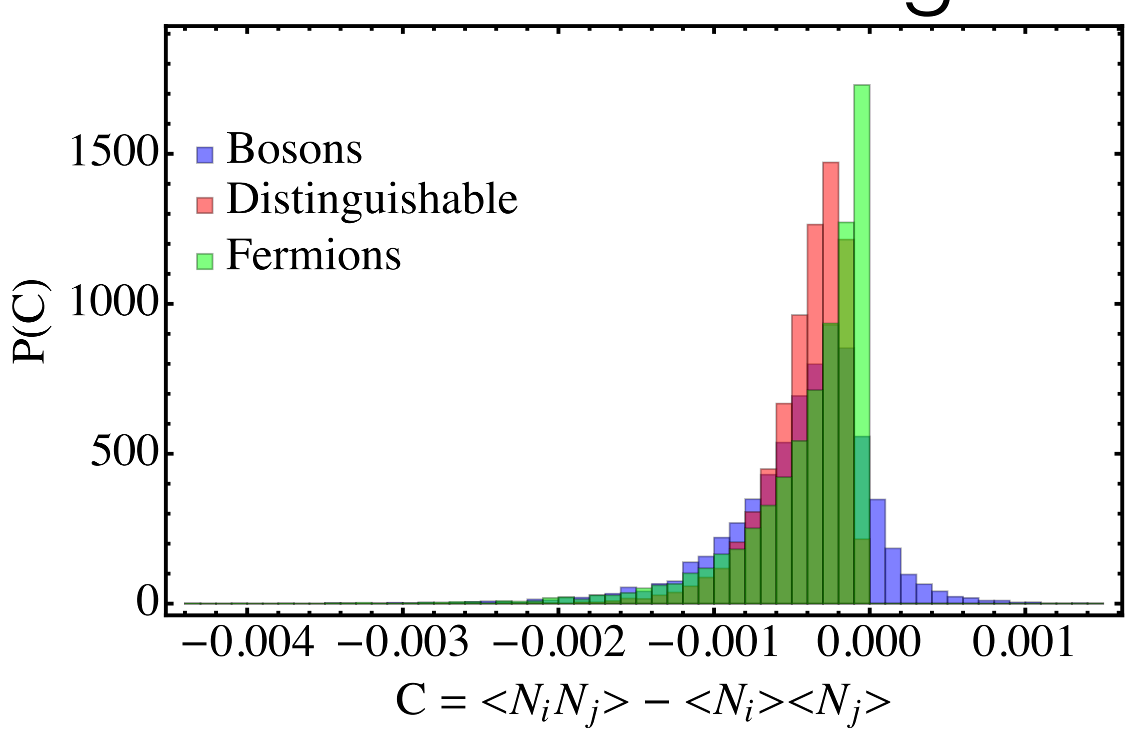
$$C_{ij} = \langle \hat{n}_i \hat{n}_j \rangle - \langle \hat{n}_i \rangle \langle \hat{n}_j \rangle$$

Calculate this quantity
for all modes i and j

C-dataset

C_{12} C_{13} C_{14} … C_{m-1m}

Histogram



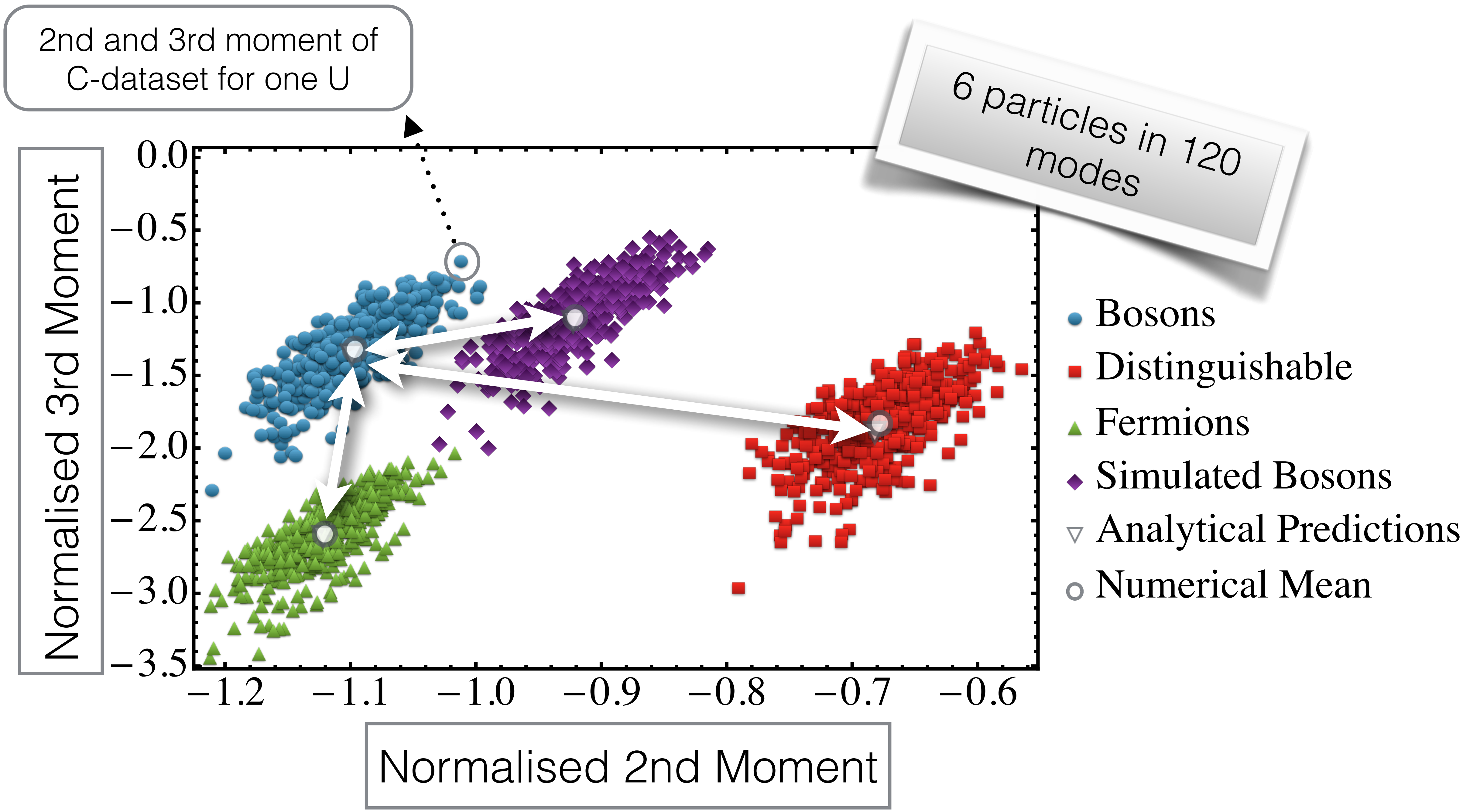
Benchmarking the Statistical Signature

Random Matrix Theory

Averaging over the Unitary group allows to analytically estimate the first moments of the C-dataset

$$\begin{aligned} \mathbb{E}_U(C_{i,j}) \\ \mathbb{E}_U(C^2_{i,j}) \\ \mathbb{E}_U(C^3_{i,j}) \end{aligned} \approx \begin{aligned} \frac{2}{m(m-1)} \sum_{i>j} C_{i,j} \\ \frac{2}{m(m-1)} \sum_{i>j} {C_{i,j}}^2 \\ \frac{2}{m(m-1)} \sum_{i>j} {C_{i,j}}^3 \end{aligned}$$

Statistical Certification



Different data points obtained by either changing the circuit or varying the input state

Take Home Message

Complex systems require a statistical treatment

Two-point correlation functions contain a significant amount of information on many-body interference

Doing statistics on all possible two-point correlation functions -“C-dataset”- allows us to certify that the sampled particles are bosons

Interested? **arXiv:1410.8547**

Extra Slide: Bosons Bunch…

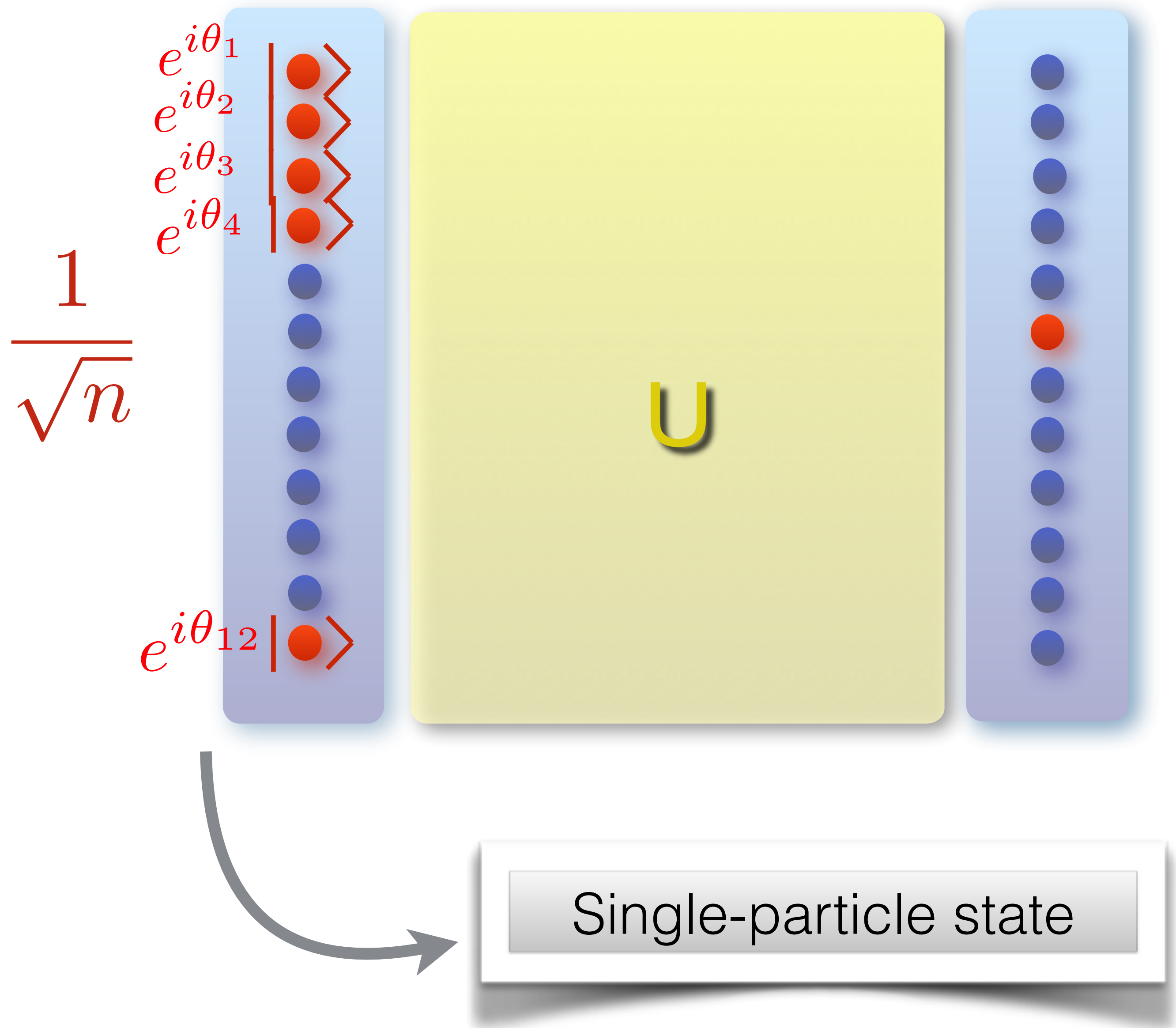
Idea: Bosonic quantum statistics enhances the probability of events with *multiple particles per output mode*.

Carolan *et al*, Nat. Photon. **8**, 621 (2014)

Tichy, J. Phys. B: At. Mol. Opt. Phys. **47**, 103001 (2014)

…but they are not alone

Idea: Mean-field theories have been effectively applied to mimic many-particle behaviour. → **“Simulated Bosons”**



Repeat n times for
one sampling event

Add random phase to each sampling event

BUNCHING
&
CLOUDING

Tichy *et al*, PRL **113**, 020502 (2014)

Extra Slide: Correlation Functions

$$C_{ij} = \langle\phi|\,\hat{n}_i\hat{n}_j\,|\phi\rangle - \langle\phi|\,\hat{n}_i\,|\phi\rangle\,\langle\phi|\,\hat{n}_j\,|\phi\rangle$$

$$|\phi\rangle = \sum_{i_1,\ldots,i_n=1}^{m} U_{q_1,i_1}a^*_{i_1}\ldots U_{q_n,i_n}a^*_{i_n}\,|\Omega\rangle$$

input mode $q_1,\ldots q_n$ ⟶ output mode $i,j=1,\ldots m$

$$C^B_{ij} = -\sum_{k=1}^{n} U_{q_k,i}U_{q_k,j}U^*_{q_k,i}U^*_{q_k,j} + \sum_{k\neq l=1}^{n} U_{q_k,i}U_{q_l,j}U^*_{q_l i}U^*_{q_k,j},$$

$$C^F_{ij} = -\sum_{k=1}^{n} U_{q_k,i}U_{q_k,j}U^*_{q_k,i}U^*_{q_k,j} - \sum_{k\neq l=1}^{n} U_{q_k,i}U_{q_l,j}U^*_{q_l,i}U^*_{q_k,j}$$

$$C^D_{ij} = -\sum_{k=1}^{n} U_{q_k,i}U_{q_k,j}U^*_{q_k,i}U^*_{q_k,j}$$

$$C^S_{ij} = \left(1-\frac{1}{n}\right)\sum_{r\neq s=1}^{n} U_{q_s,i}U_{q_r,j}U^*_{q_r,i}U^*_{q_s,j} - \frac{1}{n}\sum_{r,s=1}^{n} U_{q_r,i}U_{q_s,j}U^*_{q_r,i}U^*_{q_s,j}$$

Extra Slide: RMT averaging

$$\mathbb{E}_U(U_{a_1,b_1}\dots U_{a_n,b_n}U^*_{\alpha_1,\beta_1}\dots U^*_{\alpha_n,\beta_n})$$

$$= \sum_{\sigma,\pi\in S_n} V_N(\sigma^{-1}\pi)\prod_{k=1}^{n}\delta(a_k-\alpha_{\sigma(k)})\delta(b_k-\beta_{\pi(k)}),$$

Can be obtained recursively

In practice you look them up in tables

Extra Slide: Results written out

Fermions

$$\mathbb{E}_U\left(C_F\right) = \frac{n(n-m)}{m\left(m^2-1\right)},$$

$$\mathbb{E}_U\left({C_F}^2\right) = \frac{2n(n+1)(m-n)(m-n+1)}{m^2(m+2)(m+3)\left(m^2-1\right)},$$

$$\mathbb{E}_U\left({C_F}^3\right) = -\frac{6n(n+1)(n+2)(m-n)(m-n+1)(m-n+2)}{m^2(m+1)(m+2)(m+3)(m+4)(m+5)\left(m^2-1\right)},$$

Extra Slide: Results written out
Distinguishable Particles

$$\mathbb{E}_U\left(C_D\right) = -\frac{n}{m(m+1)}, \tag{1}$$

$$\mathbb{E}_U\left({C_D}^2\right) = \frac{n\left(m^2n + 3m^2 + mn - 5m + 2n - 2\right)}{m^2(m+2)(m+3)\left(m^2-1\right)}, \tag{2}$$

$$\mathbb{E}_U\left({C_D}^3\right) = -\frac{n\left(m^2n^2 + 9m^2n + 26m^2 + 5mn^2 + 21mn - 62m + 12n^2 + 60n - 72\right)}{m^2(m+2)(m+3)(m+4)(m+5)\left(m^2-1\right)}, \tag{3}$$

Extra Slide: Results written out
Bosons

$$\mathbb{E}_U\left(C_B\right) = \frac{n(-m-n+2)}{m\left(m^2-1\right)}, \tag{1}$$

$$\mathbb{E}_U\left({C_B}^2\right) = \frac{2n\left(m^2n+m^2+9mn-11m+n^3-2n^2+5n-4\right)}{m^2(m+2)(m+3)\left(m^2-1\right)}, \tag{2}$$

$$\mathbb{E}_U\left({C_B}^3\right) = -2n\left(\frac{m^3n^2+15m^3n+2m^3+3m^2n^3+6m^2n^2+213m^2n-222m^2-3mn^4}{m^2(m+1)(m+2)(m+3)(m+4)(m+5)\left(m^2-1\right)} + \frac{45mn^3+32mn^2+372mn-464m+3n^5-6n^4+45n^3+78n^2+168n-288}{m^2(m+1)(m+2)(m+3)(m+4)(m+5)\left(m^2-1\right)}\right), \tag{3}$$

Extra Slide: Results written out *Simulated Bosons*

$$\mathbb{E}_U\left(C_S\right) = -\frac{n(m+n-2)}{m\left(m^2-1\right)}, \tag{1}$$

$$\mathbb{E}_U\left({C_S}^2\right) = \frac{4mn - m - 14n^2 + 8n - 2}{m^2(m+2)(m+3)\left(m^2-1\right)n} + \frac{2m^2n^3 - m^2n^2 + 4m^2n - m^2 + 18mn^3 - 25mn^2 + 2n^5 - 4n^4 + 10n^3}{m^2(m+2)(m+3)\left(m^2-1\right)n}, \tag{2}$$

$$\mathbb{E}_U\left({C_S}^3\right) = \left(\frac{-2m^3n^5 - 21m^3n^4 + 30m^3n^3 - 41m^3n^2 - 10m^3n + 8m^3 - 6m^2n^6 - 3m^2n^5}{(m-1)m^2(m+1)^2(m+2)(m+3)(m+4)(m+5)n^2} + \frac{-285m^2n^4 + 261m^2n^3 + 75m^2n^2 - 66m^2n + 24m^2 + 6mn^7 - 90mn^6 - 55mn^5}{(m-1)m^2(m+1)^2(m+2)(m+3)(m+4)(m+5)n^2} + \frac{-360mn^4 + 591mn^3 + 8mn^2 - 128mn + 64m}{(m-1)m^2(m+1)^2(m+2)(m+3)(m+4)(m+5)n^2} + \frac{-6n^8 + 12n^7 - 90n^6 - 120n^5 - 24n^4 + 396n^3 - 168n^2 - 48(n-1)}{(m-1)m^2(m+1)^2(m+2)(m+3)(m+4)(m+5)n^2}\right). \tag{3}$$