Transmission phase of a quantum dot and statistical fluctuations of partial-width amplitudes

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Transmission phase

Quantum scatterer connected to monochannel leads

$$e^{i k x}$$

$$r e^{-i k x}$$

$$t e^{i k x}$$

$$G = \frac{I}{V} = \frac{2e^2}{h} |t|^2$$

Transmission amplitude

$$t = |t| e^{i \alpha}$$

$$\alpha \quad \text{transmission phase}$$
AB interferometer containing a quantum dot

Schuster et al., Nature ’97

Experiments
Parity rule
Mean wave-function correlations
Fluctuations
Conclusions

\[ \sim 200 \text{ electrons} \]

continuous phase evolution in resonances (Friedel sum rule)

\[ \propto \delta \alpha \]

abrupt drops of \( \pi \) in valleys

subsequent peaks in phase
Crossover mesoscopic ↔ universal

“Mesoscopic”: $N \lesssim 10$; irregular phase evolution

“Universal”: $N > 14$; subsequent peaks in phase

Avinum-Kalish et al., Nature ’05

“Mesoscopic”: $N \lesssim 10$; irregular phase evolution

“Universal”: $N > 14$; subsequent peaks in phase

Avinum-Kalish et al., Nature ’05
Between resonances: Two-level model

\[ t(\epsilon) \sim \frac{\gamma_1^l \gamma_1^r}{\epsilon - \epsilon_1 + i\Gamma_1/2} + \frac{\gamma_2^l \gamma_2^r}{\epsilon - \epsilon_2 + i\Gamma_2/2} \]

\[ \Gamma_n = |\gamma_n^l|^2 + |\gamma_n^r|^2 \]

\[ \epsilon_n = \epsilon_n^{(0)} - V_g \]

\[ \gamma_1^l \gamma_1^r = -\gamma_2^l \gamma_2^r \quad \text{opposite parity} \]

\[ \gamma_1^l \gamma_1^r = \gamma_2^l \gamma_2^r \quad \text{same parity} \]

\[ |t|^2 > 0 \leftrightarrow \text{smooth phase increase} \]

\[ |t| = 0 \leftrightarrow \text{phase jump} \]

[Lee PRL '99; Taniguchi & Büttiker PRB '99, Levy-Yeyati & Büttiker PRB '00, Aharony et al. PRB '02]
Continuous evolution of $t$

\[ \gamma_1^l \gamma_1^r = -\gamma_2^l \gamma_2^r \] opposite parity

\[ \gamma_1^l \gamma_1^r = \gamma_2^l \gamma_2^r \] same parity
**Parity rule**

\[
D_n = \gamma_n^l \gamma_n^r \gamma_{n+1}^l \gamma_{n+1}^r \\
\text{sgn}(\gamma_n^l \gamma_n^r) = \pm \text{sgn}(\gamma_{n+1}^l \gamma_{n+1}^r) \rightarrow D_n \geq 0
\]

\[
D_n < 0 \\
\rightarrow \text{no transmission zero} \\
\text{no phase lapse}
\]

\[
D_n > 0 \\
\rightarrow |t| = 0 \\
\text{phase lapse}
\]

[Lee PRL ’99; Taniguchi & Büttiker PRB ’99, Levy-Yeyati & Büttiker PRB ’00, Aharony et al. PRB ’02]

Disordered dots: \( \mathcal{P}(D_n < 0) = 1/2 \) \( \leadsto \) irregular phase evolution

Experiment: \( \mathcal{P}(D_n < 0) = 0 \) \( \leadsto \) correlations between \( \gamma_n \) and \( \gamma_{n+1} \)?
Wave-function correlations in chaotic dots

\[ \gamma_n^{1(r)} \propto \int_0^W dy \Phi_1(y) \psi_n(x^{1(r)}, y) \sim \psi_n(x^{1(r)}, 0) \]

\[ D_n = \gamma_n^{1} \gamma_n^{r} \gamma_n^{1+1} \gamma_n^{r+1} = \psi_n(x^1, 0) \psi_n(x^r, 0) \psi_{n+1}(x^1, 0) \psi_{n+1}(x^r, 0) \]

Random wave model: [M.V. Berry, J. Phys. A ’77]

\[ \psi_n^{RWM}(r) = \frac{1}{N} \sum_{j=1}^{N} \cos[k_j r + \delta_j] \]

random \( \delta_j \)

\[ E_n = \frac{\hbar^2 k_n^2}{2m} \sim \text{randomly oriented } k_j \text{ with } |k_j| = k_n \]

Correlations over a distance \( L = x^r - x^1 \)

\[ \langle \psi_n(r) \psi_n(r') \rangle \sim \frac{1}{A} J_0(k|r - r'|) \sim \langle D_n \rangle \sim J_0(k_n L) J_0(k_{n+1} L) \]
Universal behavior in the semiclassical limit

\[ \langle D_n \rangle \sim J_0(k_n L) J_0(k_{n+1} L) \]

\[ \Delta k L = k_{n+1} L - k_n L \approx \frac{\pi}{k L} \]

Average-based probability of having no phase lapse (\( \langle D_n \rangle < 0 \))

\[ P(\langle D_n \rangle < 0) \sim \frac{1}{k L} \]

Tendency towards the universal regime at large \( k L \)

Numerics for the transmission amplitude

\[ t \]

\[ \text{Re}[t] \]

\[ \text{Im}[t] \]

\[ kL \]

\[ 86 \quad 88 \quad 90 \quad 92 \quad 94 \]

\[ 0 \quad 0.5 \quad 1 \]

\[ -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1 \]

\[ 86 \quad 88 \quad 90 \quad 92 \quad 94 \]

\[ V \]

\[ V_g \]

\[ E_F \]

\[ x \]

\[ y \]

\[ W \]

\[ L \]

circles ↔ Breit-Wigner peaks
Counting zeros and resonances

“universal” regime at very large $kL$
Are averaged correlations sufficient?

Conclusions were drawn from the sign of the average

\[ \langle D_n \rangle = \langle \gamma_n^l \gamma_n^r \gamma_{n+1}^l \gamma_{n+1}^r \rangle \]

and assuming narrow leads

\[ \gamma_n^{1/r} \sim \psi_n(x^{1/r}, 0) \]

Questions:
- What happens in the case of wider leads?
- Do statistical fluctuations of the \( \gamma_n^{1/r} \) change the results?
Dots with chaotic classical dynamics $\nearrow$ Gaussian distribution of $\psi_n$

$\gamma_n^{l(r)} \propto \int_{0}^{W} dy \Phi_{l(r)}(y) \psi_n(x^{l(r)}, y)$

$\Rightarrow$ Gaussian distribution of the PWAs $\gamma_{n}^{l/r}$ with joint density

$$p(\gamma_n^L, \gamma_n^R) = \frac{1}{2\pi\sigma_n^2\sqrt{1-\rho_n^2}} \exp \left( -\frac{(\gamma_n^L)^2 + (\gamma_n^R)^2 - 2\rho_n\gamma_n^L\gamma_n^R}{2\sigma_n^2(1-\rho_n^2)} \right)$$

Variance and correlator with LR symmetry:

$$\sigma_n^2 = \langle \gamma_n^L \gamma_n^L \rangle = \langle \gamma_n^R \gamma_n^R \rangle \quad \rho_n = \frac{1}{\sigma_n^2} \langle \gamma_n^L \gamma_n^R \rangle$$

$\Rightarrow$ Probability for positive parity

$$P(\gamma_n^L \gamma_n^R > 0) = \frac{1}{2} + \frac{1}{\pi} \arcsin(\rho_n)$$
Probability for no phase lapse

\[ P(D_n < 0) = P(\gamma^l_n \gamma^r_n > 0) \left[ 1 - P(\gamma^l_{n+1} \gamma^r_{n+1} > 0) \right] + P(\gamma^l_{n+1} \gamma^r_{n+1} > 0) \left[ 1 - P(\gamma^l_n \gamma^r_n > 0) \right] \]

With mean wave-number spacing \( \Delta k_n = \pi / k_n L^2 \):

\[ P(D_n < 0) \simeq 2f(k_n) + \Delta k_n f'(k_n) \quad \text{with} \quad f(k) = \frac{1}{4} - \frac{1}{\pi^2} \arcsin^2(\rho(k)) \]
Numerics for $\mathcal{P}(D_n < 0)$ averaging over 14 cavities

Blue: $\mathcal{P}(D_n < 0)$; smoothing $k_n$ interval of $\delta/L$ with $\delta = \pi/4$ and $\pi$

Red: from the statistical model; same smoothing
Conclusions

- Wave-function correlations: probability for non-universal evolution $\sim 1/kL$ in chaotic dots
  - Tendency towards universal behavior at large $N$
- Gaussian fluctuations of partial-width amplitudes
  - Reduced tendency towards universal behavior
- Numerics for ballistic cavities
  - Intermediate tendency towards universal behavior

Outlook: More realistic dot models? Beyond Gaussian fluctuations?
Correlations $n \leftrightarrow n + 1$?

R.A. Jalabert et al., PRE 89, 052911 (2014)