Quantum Mechanics of Mixed Systems

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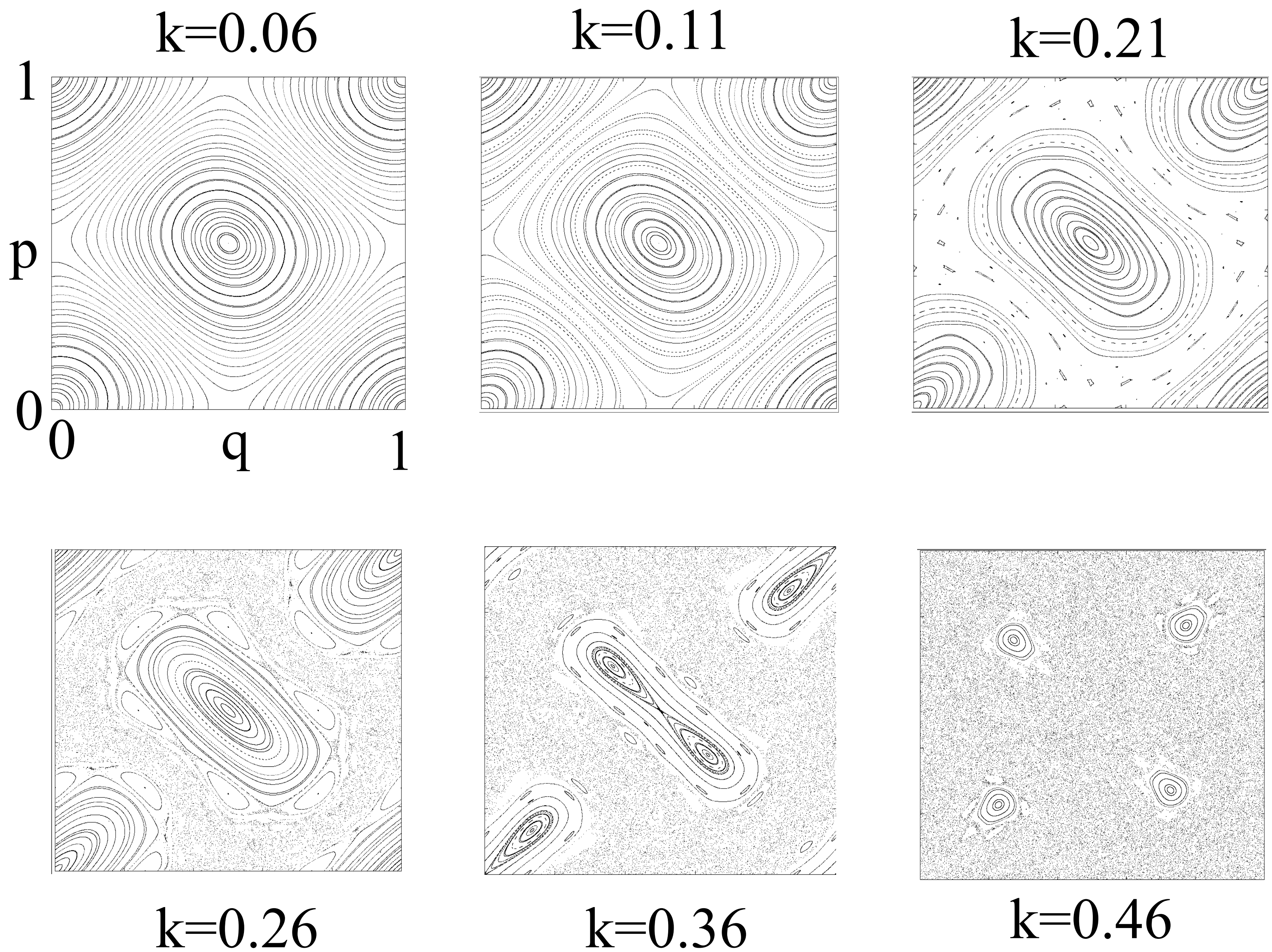
Luchon 2015

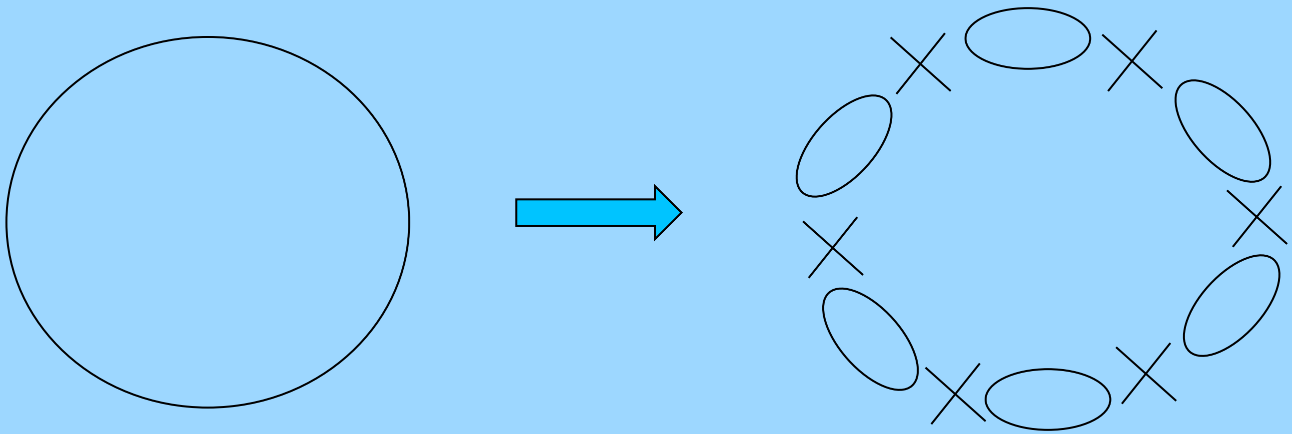
Let us start with an example…..

$$H(p,q,t) = \frac{1}{2\pi}\cos(2\pi p) + \frac{k}{2\pi}\cos(2\pi q)\sum_n \delta(t - nk)$$

$$\begin{aligned} p_{n+1} &= p_n + k\sin(2\pi q_n) \qquad &&(\mathrm{mod}\ 1), \\ q_{n+1} &= q_n - k\sin(2\pi p_{n+1}) \qquad &&(\mathrm{mod}\ 1), \end{aligned}$$


k=0.06
k=0.11
k=0.21
1
p
0
0
q
1
k=0.26
k=0.36
k=0.46

This transition is dictated by the KAM and the Poincare-Birkhoff theorems



I am interested the quantum manifestation of this transition

In the 80'

Resonances ⟺ AC

Uzer, Noid, Marcus JCP 83
Ozorio de Almeida JPC 84

Tunneling and the Semiclassical Spectrum for an Isolated Classical Resonance

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The island tori, surrounding a stable periodic orbit in a classical resonance, are not quantized by the Bohr–Sommerfeld rules, since these take no account of tunneling. The approximate four-dimensional Hamiltonian, generating the motion near the resonance, is integrable and can be separated by a canonical transformation. The resulting single degree of freedom problem has no momentum (action) symmetry, but it can still be mapped onto the ordinary Schrödinger equation with Bloch (Floquet) boundary conditions. The canonical invariance of the real and imaginary classical actions, which determine the semiclassical tunneling transmission coefficient, results in an eigenenergy condition which is uniformly valid for both the island tori and the tori which envelop the resonance. Tunneling between the latter is seen to be the cause of "avoided crossings" of pairs of energy levels. Different degrees of approximation are computationally verified for a model Hamiltonian whose eigenvalues can be calculated exactly.

In the 00'

Resonances ⟺ Tunneling

Brodier, Schlagheck, Ullmo PRL' 01
Annals of Phys. '02
Eltschka, Schlagheck PRL '05
Lock, Backer, Ketzmerik,
Schlagheck PRL '10
Hanada, Shudo, Ikeda arXiv '15

Resonance-Assisted Tunneling in Near-Integrable Systems

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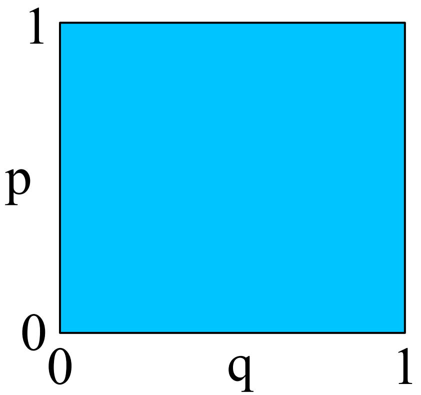
(Received 4 April 2001; published 19 July 2001)

Dynamical tunneling between symmetry related invariant tori is studied in the near-integrable regime. Using the kicked Harper model as an illustration, we show that the exponential decay of the wave functions in the classically forbidden region is modified due to coupling processes that are mediated by classical resonances. This mechanism leads to a substantial deviation of the splitting between quasidegenerate eigenvalues from the purely exponential decrease with $1/\hbar$ obtained for the integrable system. A simple semiclassical framework, which takes into account the effect of the resonance substructure on the invariant tori, allows one to quantitatively reproduce the behavior of the eigenvalue splittings.

Quantum Harper map

Map in a unit cell

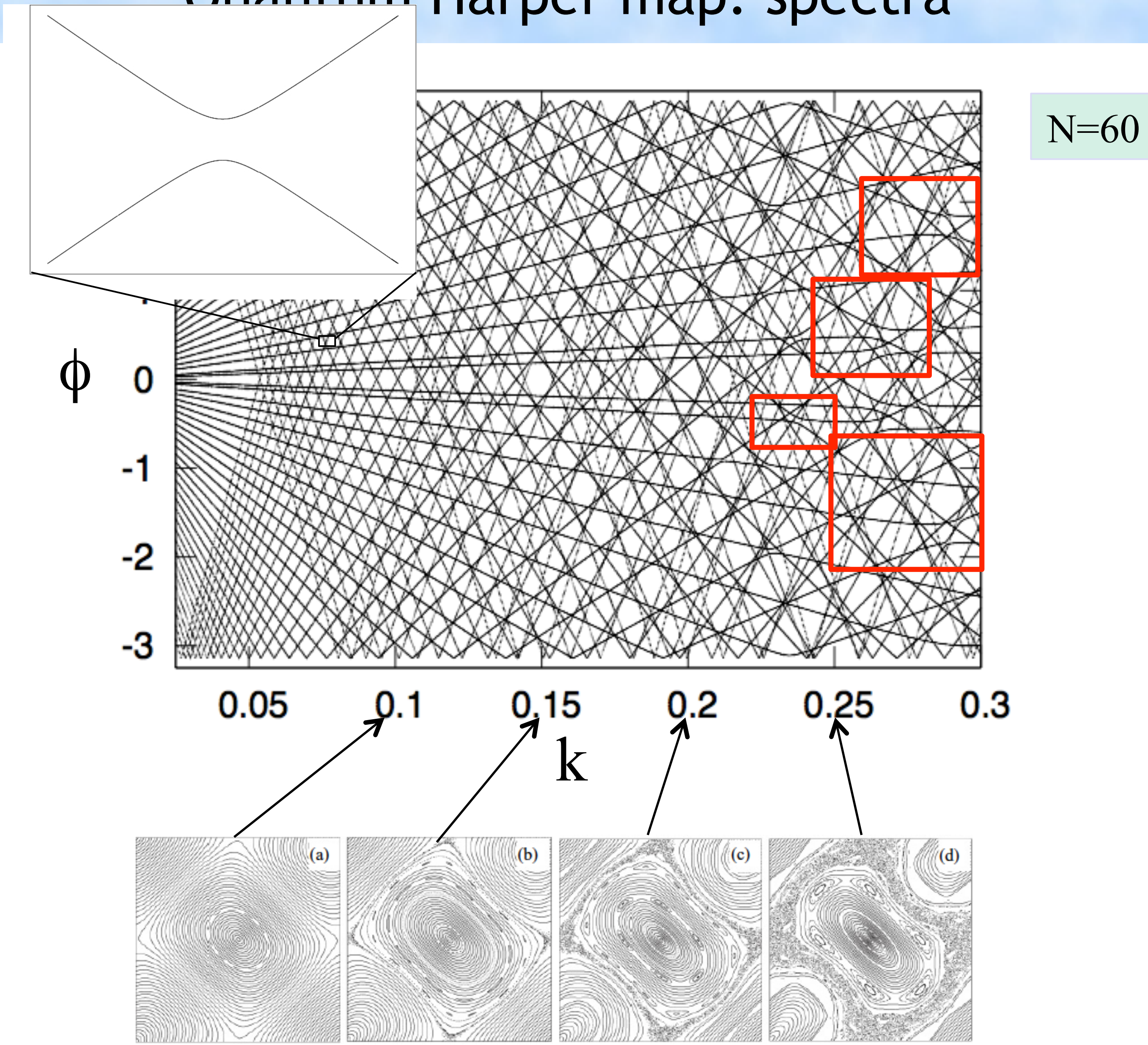
$$\hbar = \frac{1}{2\pi N}$$



Evolution operator

$$\hat{U}_k = \exp[\mathrm{i}Nk\cos(2\pi\hat{q})]\;\exp[\mathrm{i}Nk\cos(2\pi\hat{p})]$$

Quantum Harper map: spectra



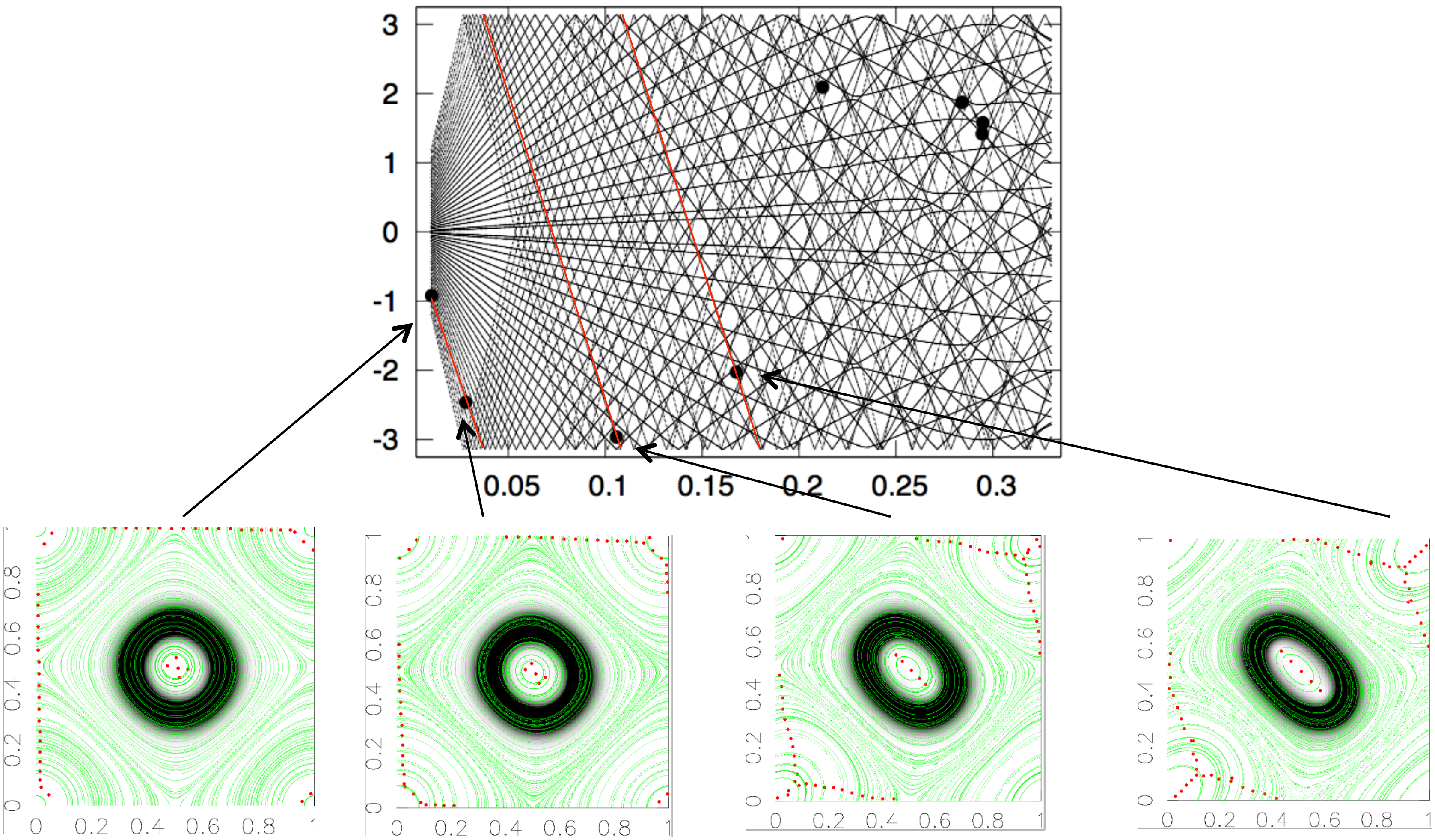
Quantum Harper map: eigenfunctions

Husimi distribution

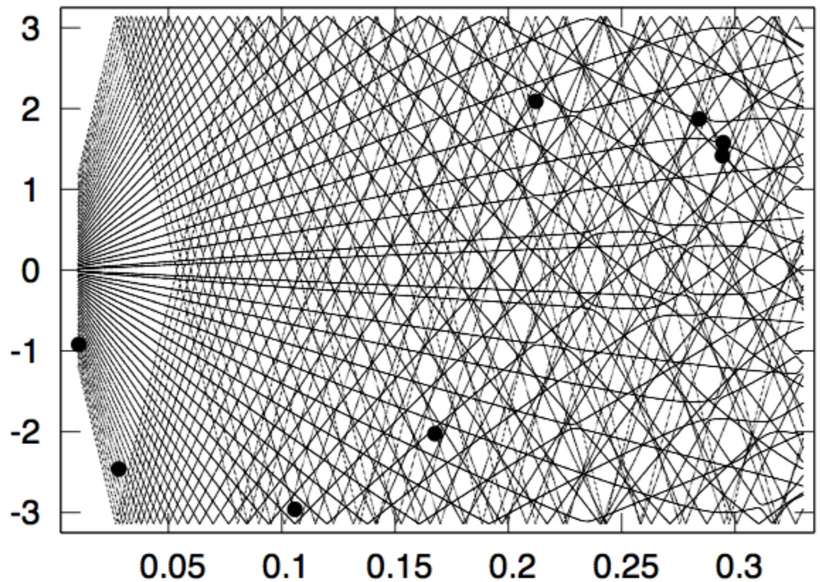
$$H_\phi(q,p) = |\langle q,p|\phi\rangle|^2$$

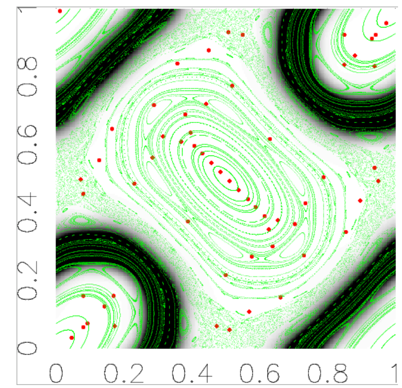
- Positive real function
- For maps in a torus has exact N zeros
- The set of zeros of H encodes the full quantum information of the state

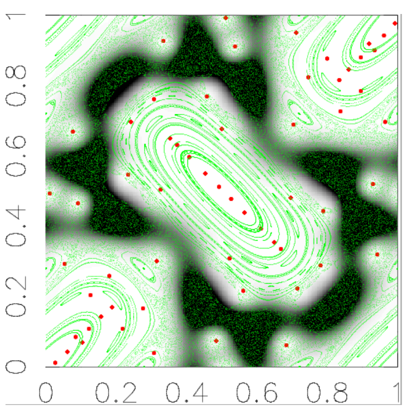
Quantum Harper map: eigenfunctions

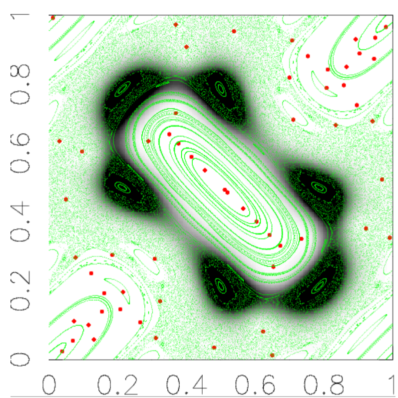


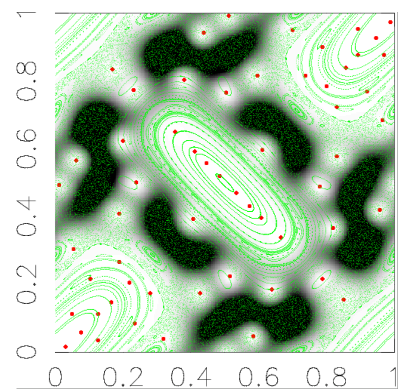
Quantum Harper map: eigenfunctions



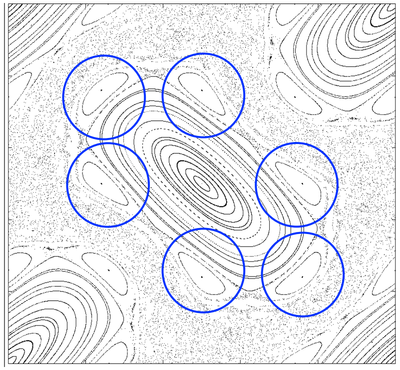




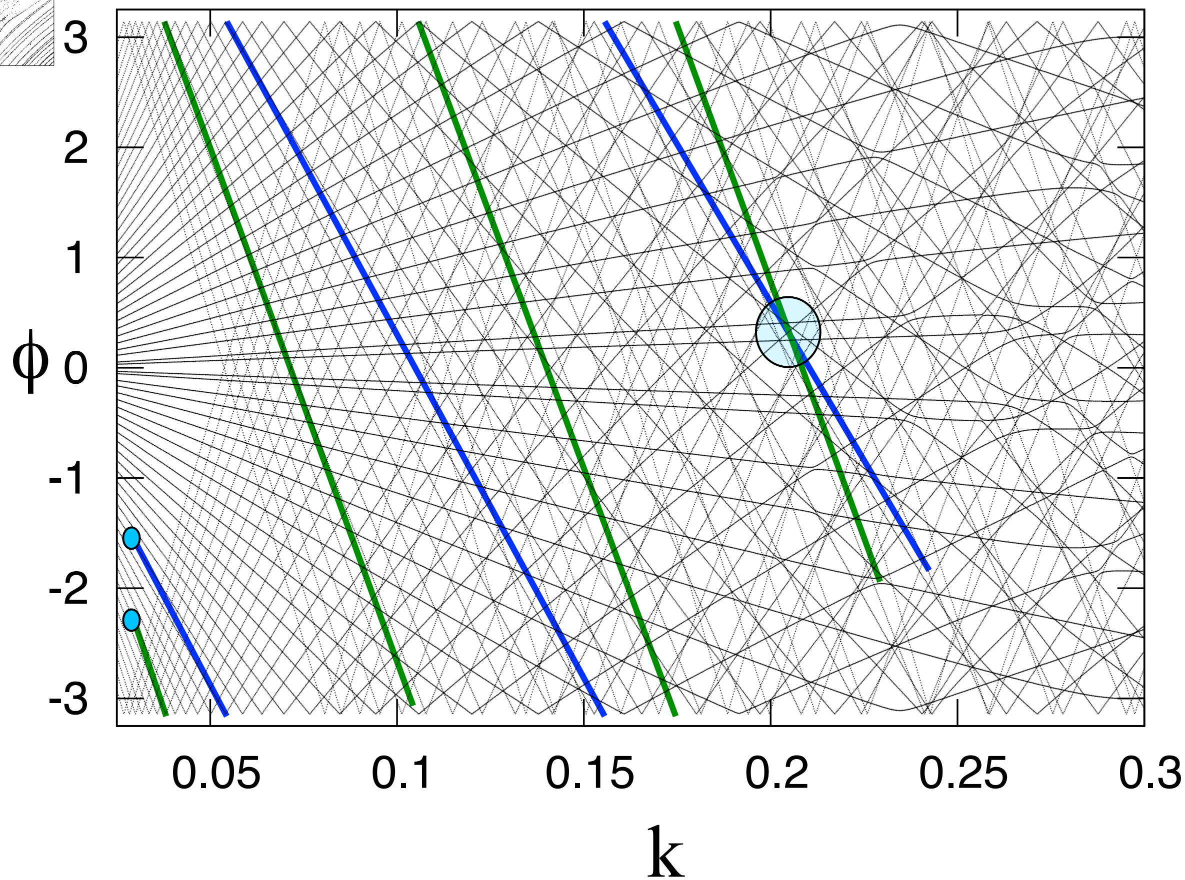




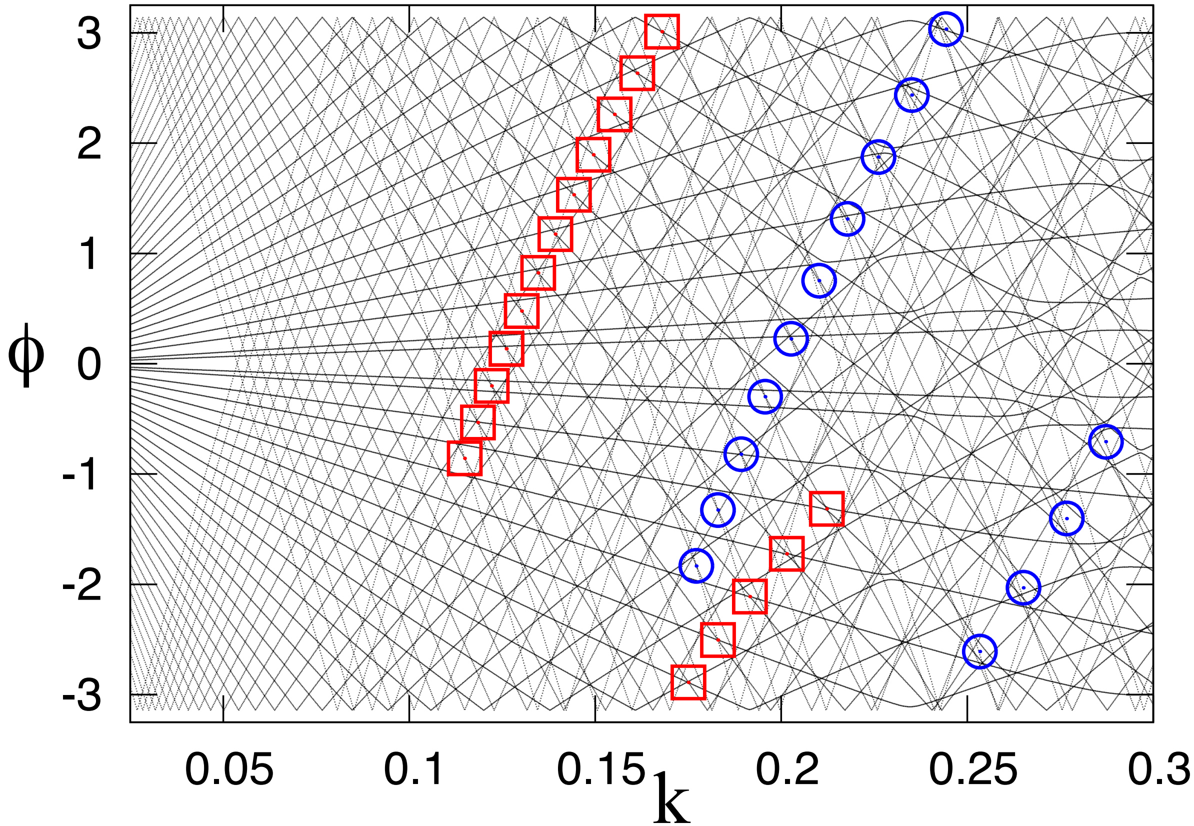
I want to study the ACs generated by a resonance



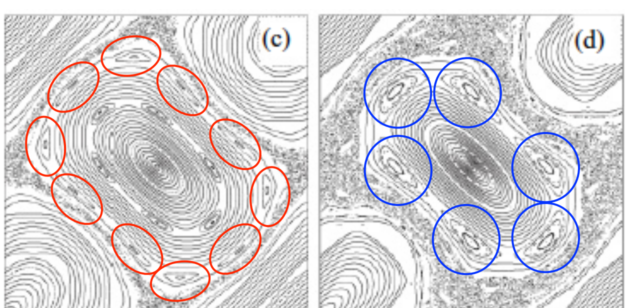
N=60



n_1=5 n_2=11

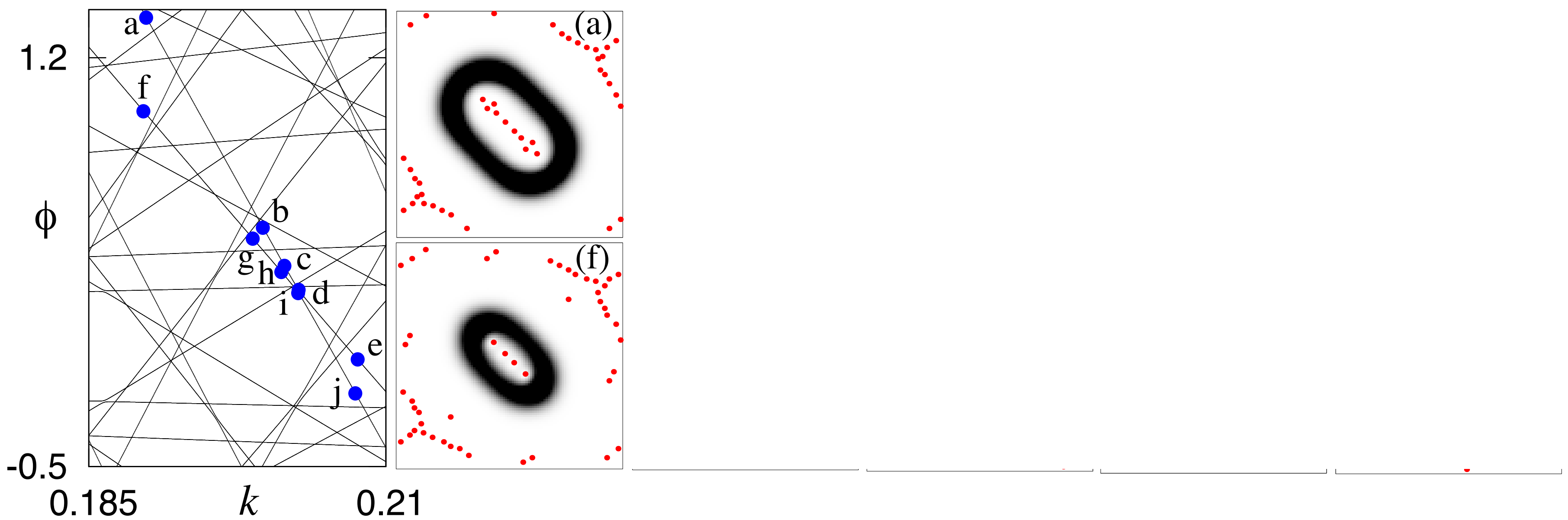


Δn=10



Δn=6

$N_1=5 \quad n_2=11$



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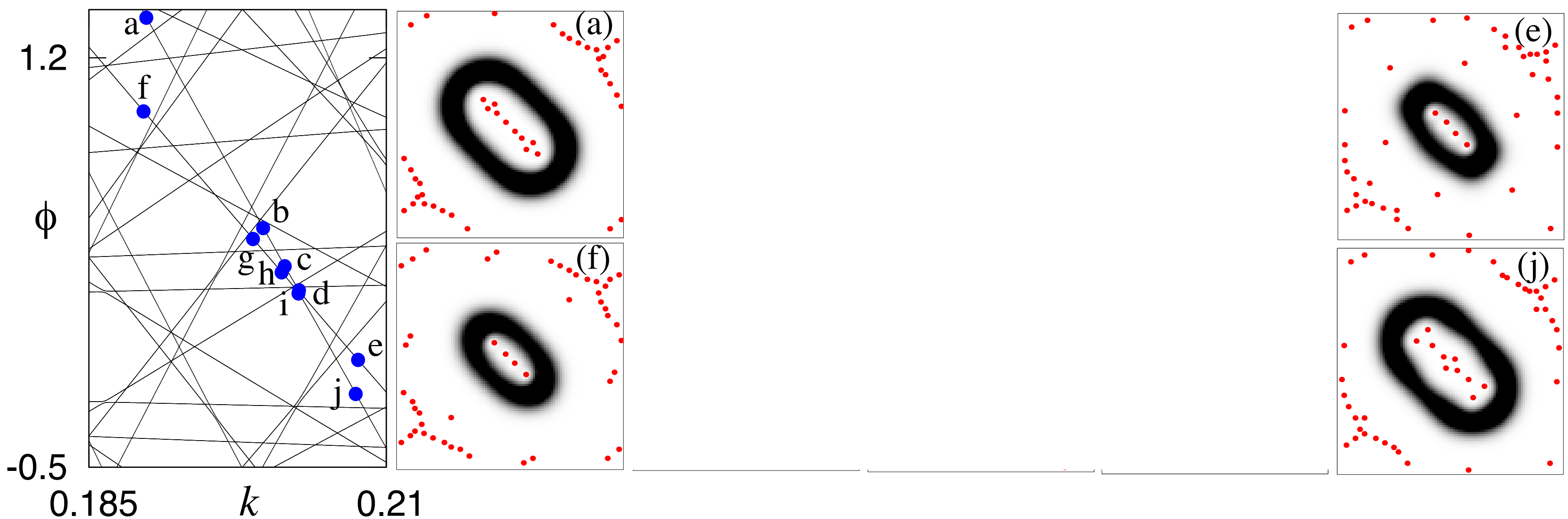
φ
0.185
k
0.21
(a)
(f)



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ϕ
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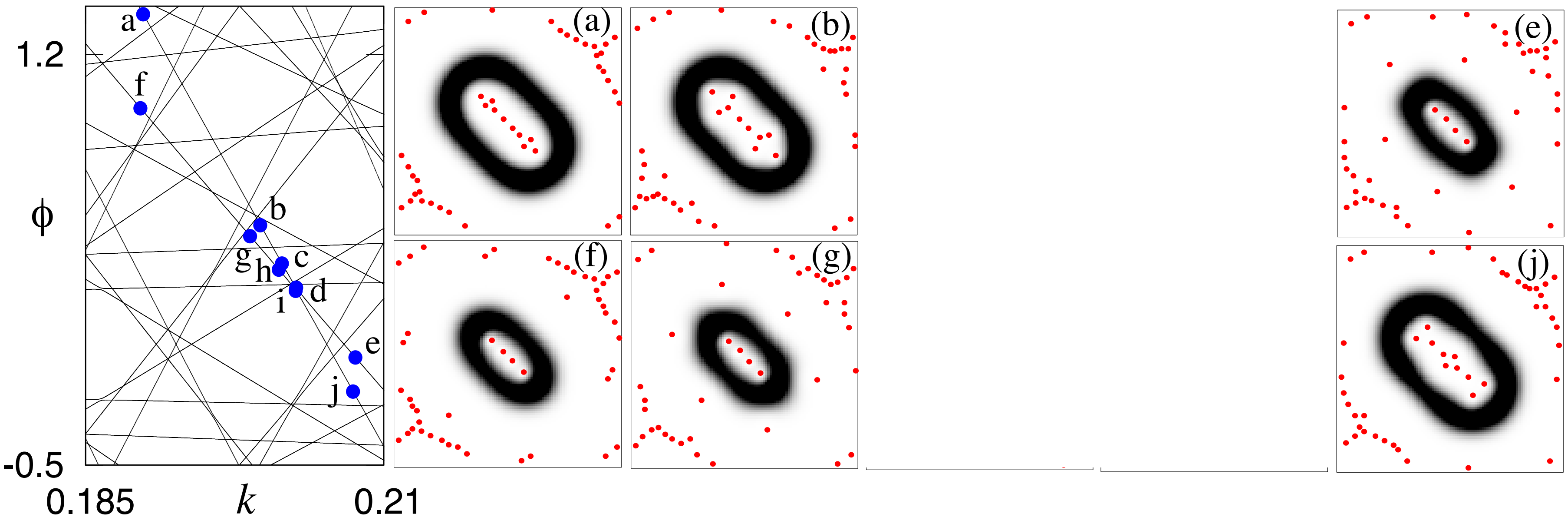

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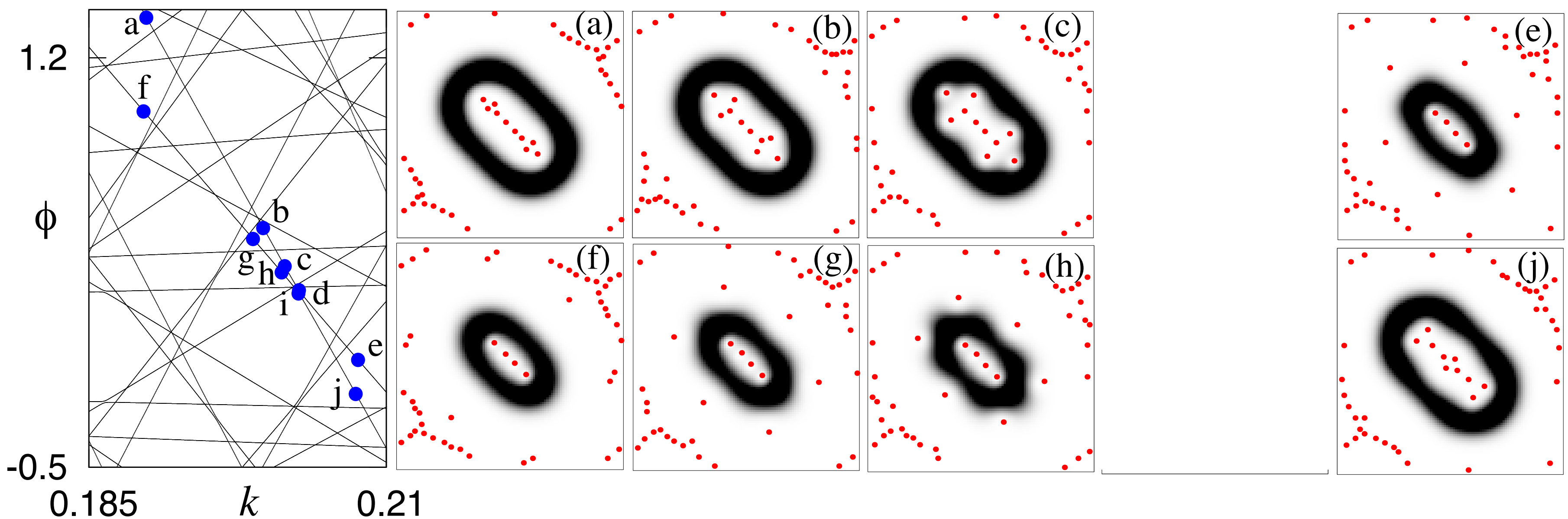

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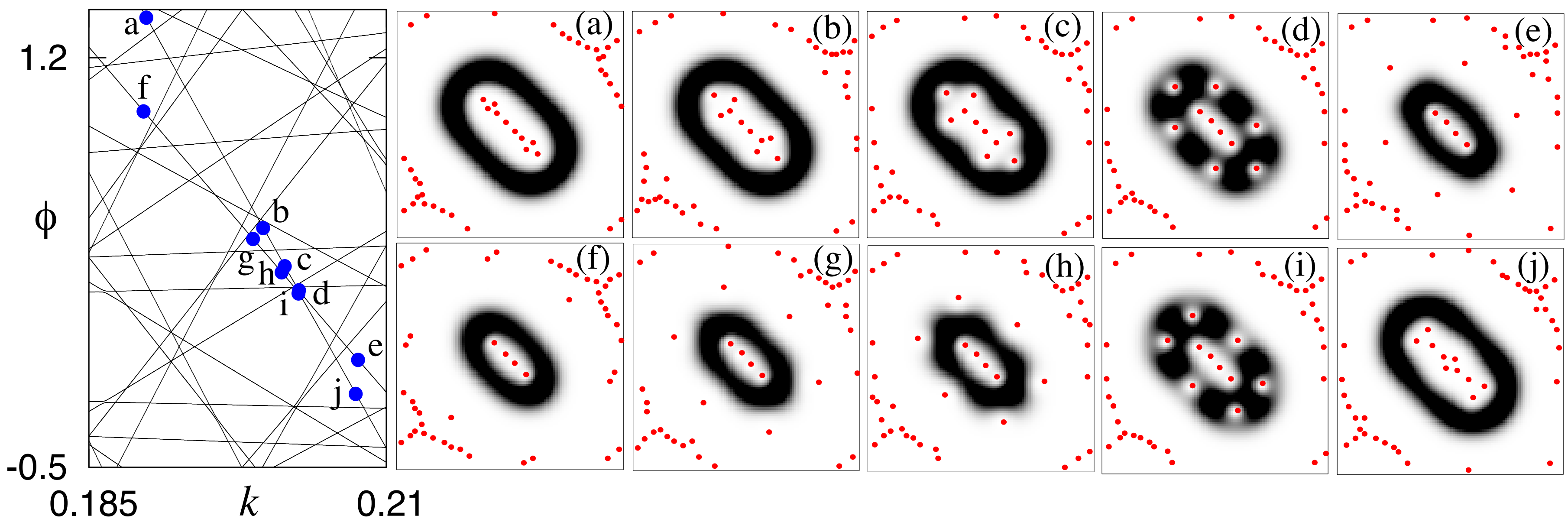

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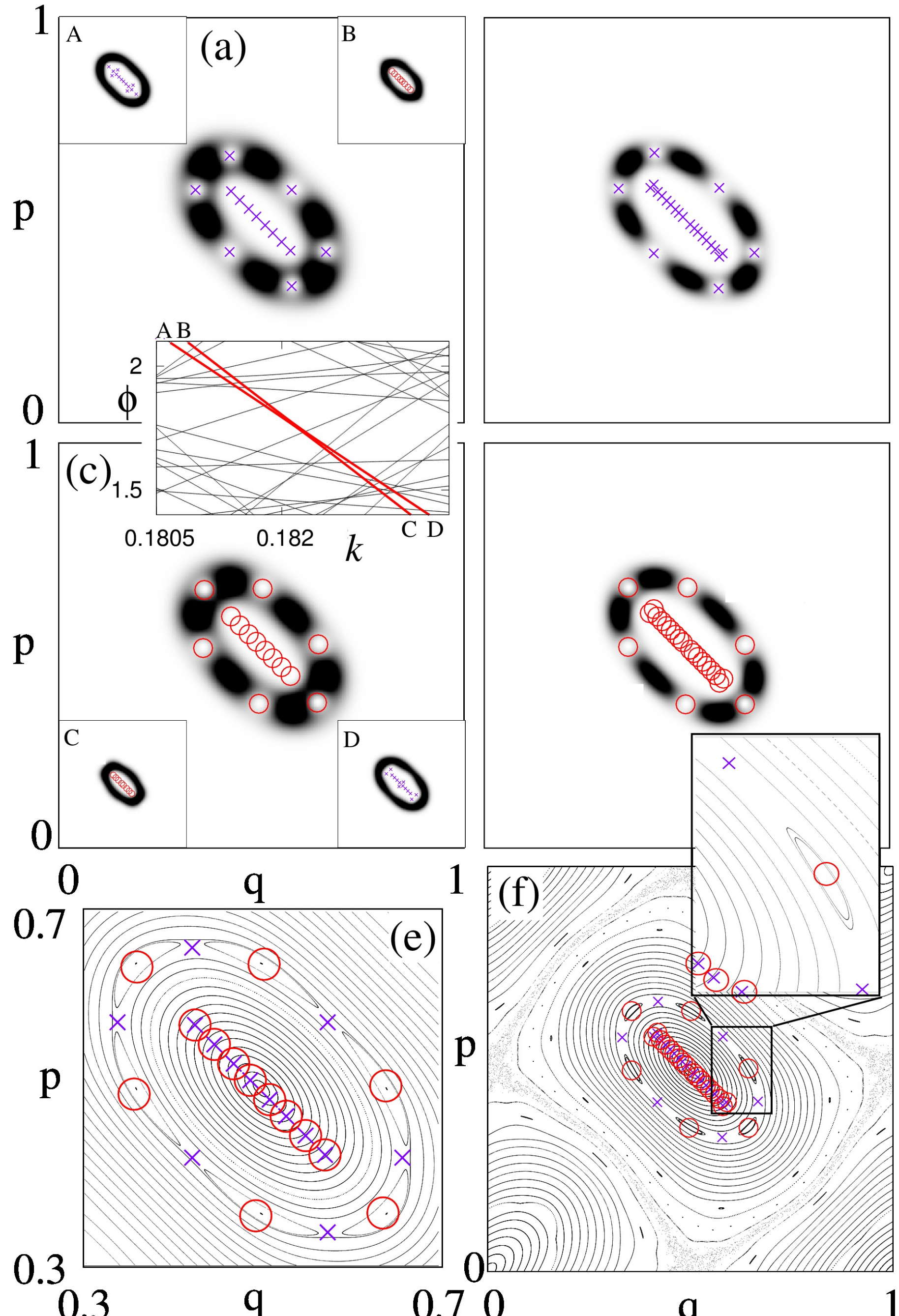
N=160

n_1=9 n_2=15

Δn=6



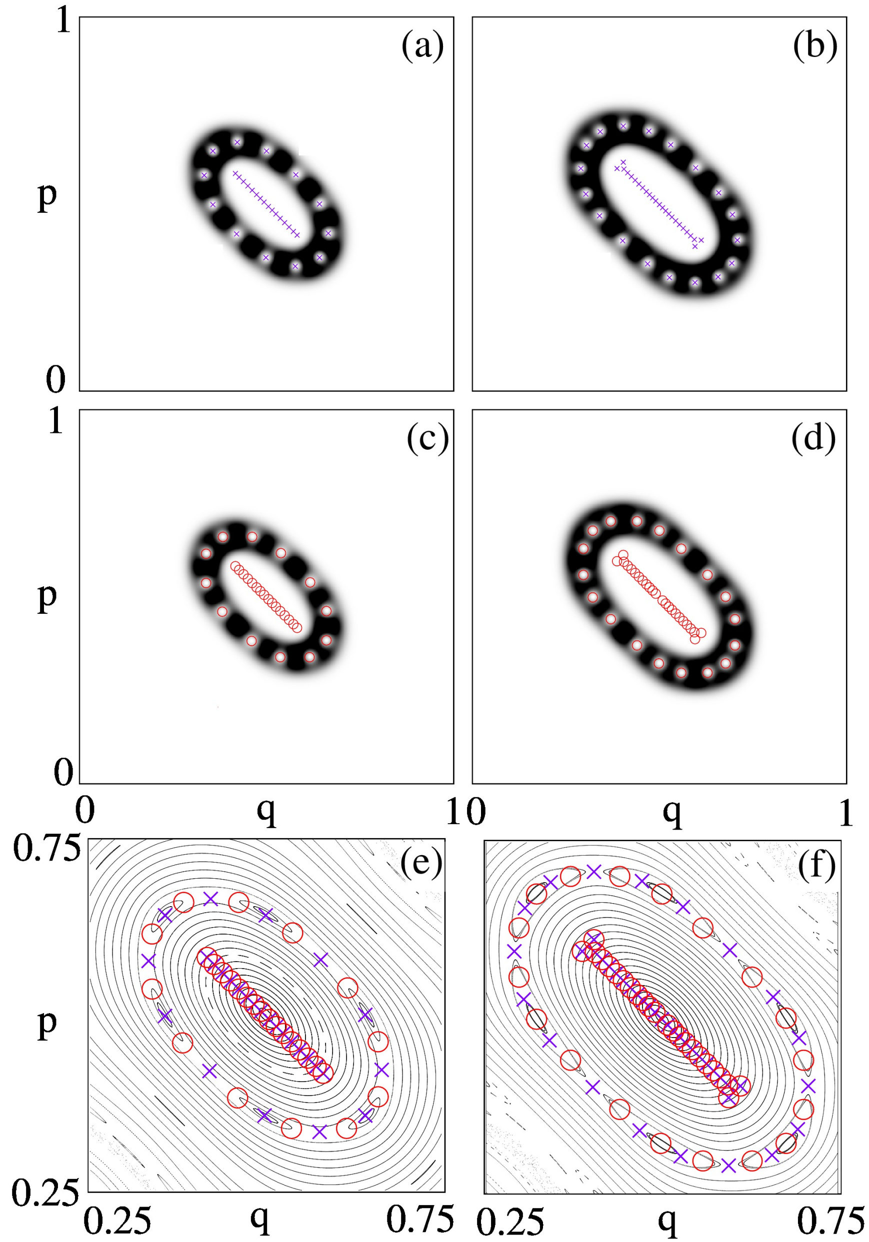
N=300

n_1=18 n_2=24

N=300

n_1=15 n_2=27

Δn=6*2=12

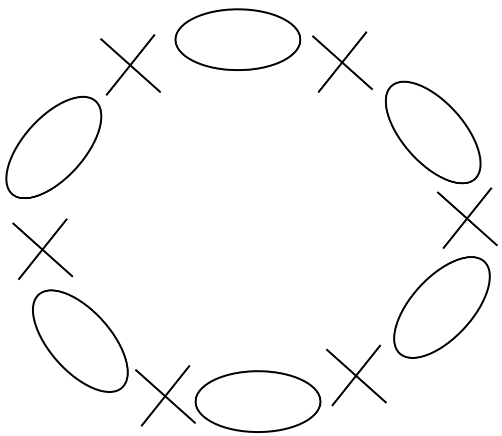


n_1=26 n_2=44

Δn=6*3=18

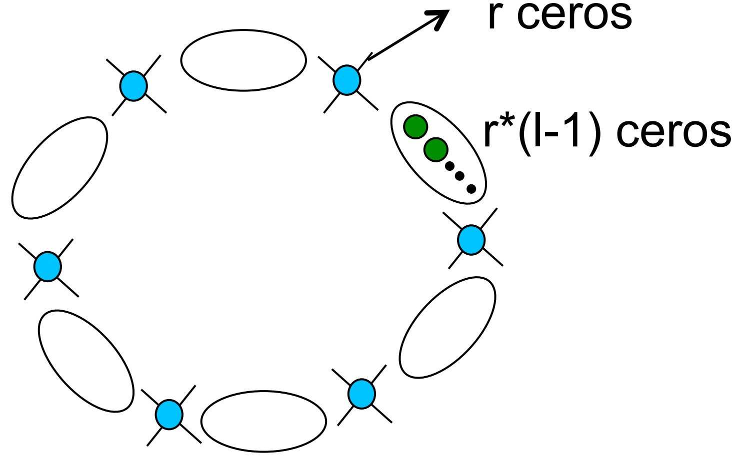
Classical

Resonance r islands



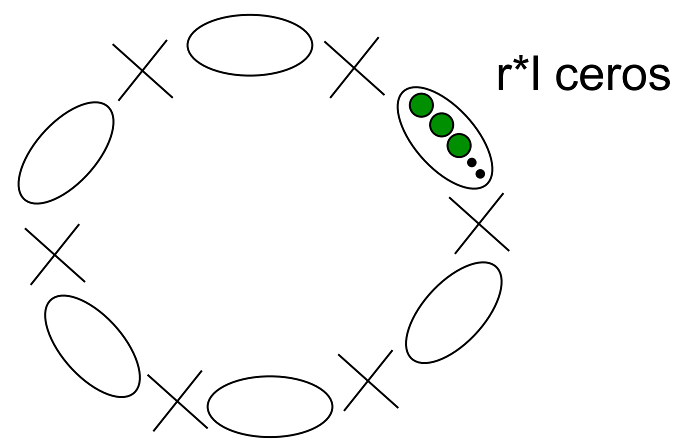
Quantum

State localized in the islands



$\Delta n = r*l$

AC



State localized in PO (Scaring???)

Semiclassical analysis of the gaps

Annals of Physics **300**, 88–136 (2002)
doi:10.1006/aphy.2002.6281

Resonance-Assisted Tunneling

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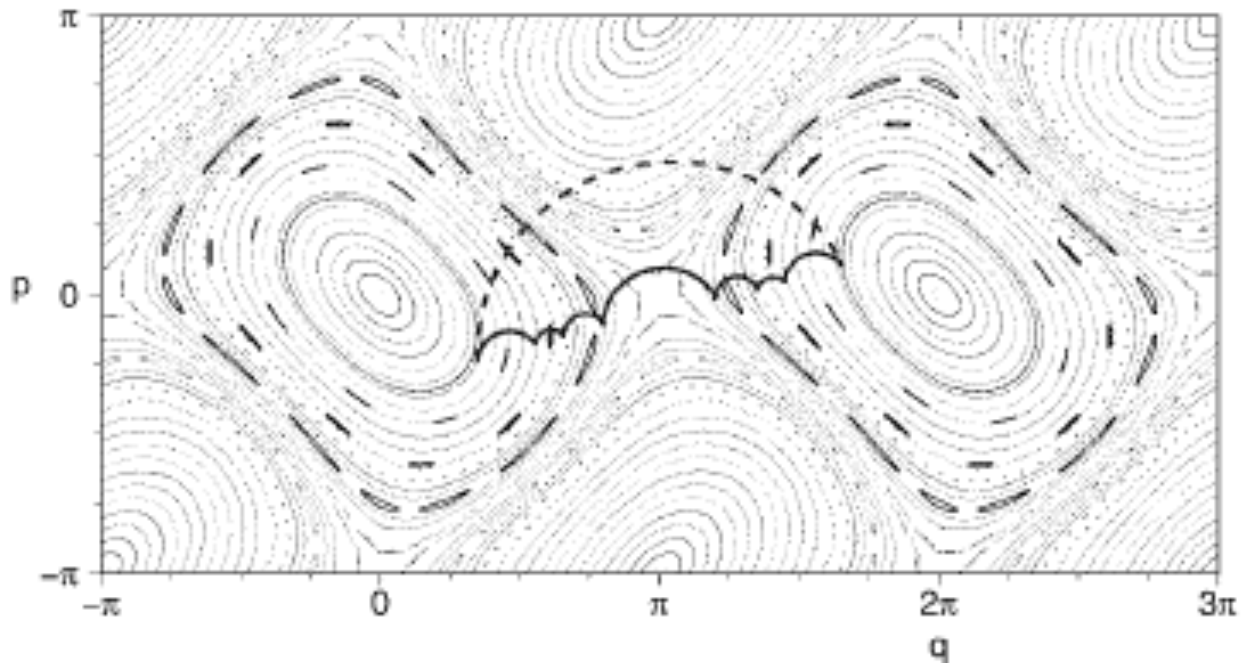
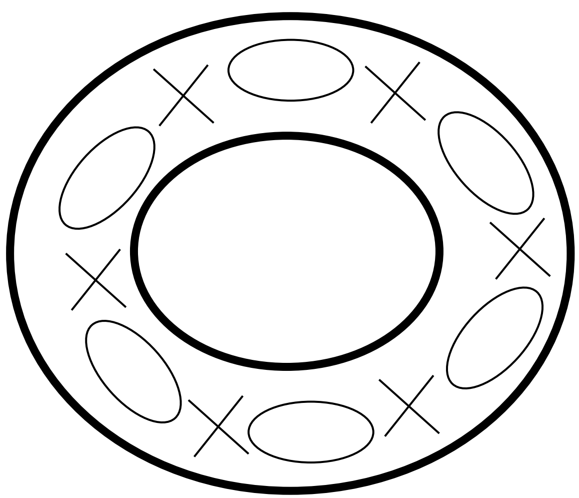
Received May 24, 2002





$$\langle T_{n_1} | \, H \, | T_{n_2} \rangle$$

Semiclassical analysis of the gaps

$$H_{r:s} \simeq H_0(I_{r:s}) + \sum_{l=1}^{\infty} V_{r,l}(I_{r:s}) \cos(lr\theta + \phi_l)$$

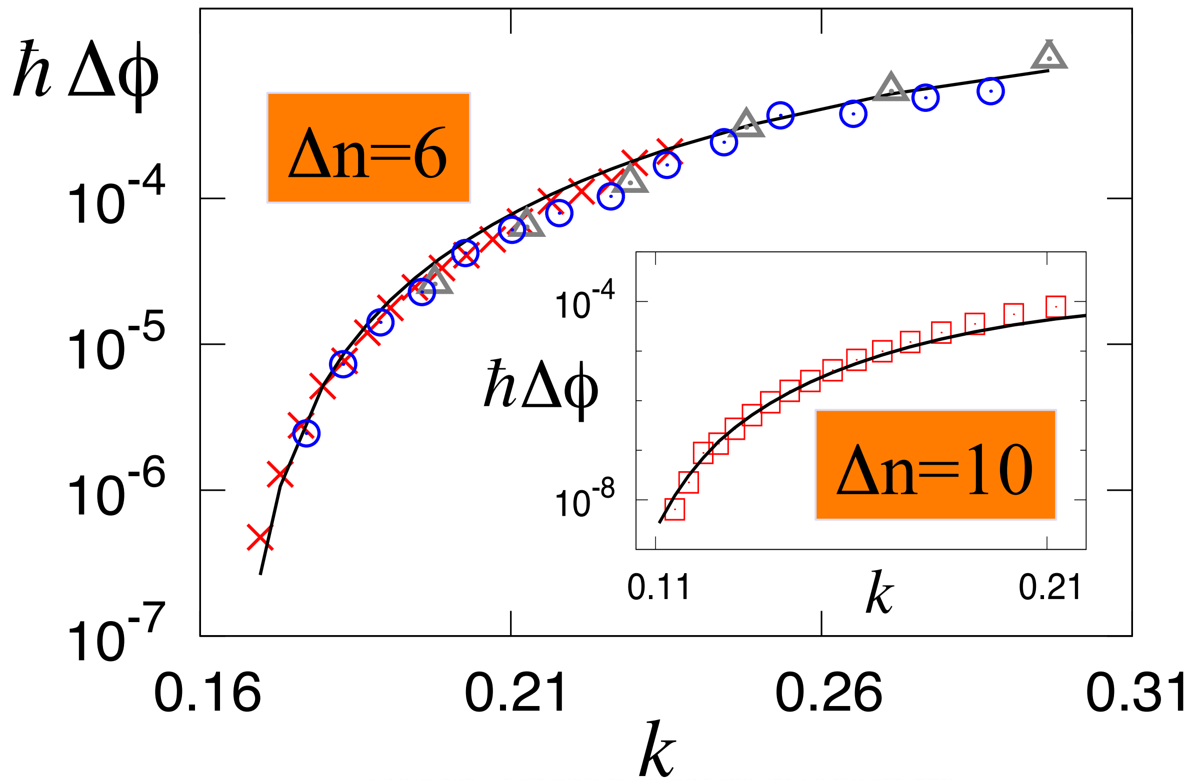
$$\langle n| H_{res} |n + rl\rangle = V_{sc}^{r,l}$$

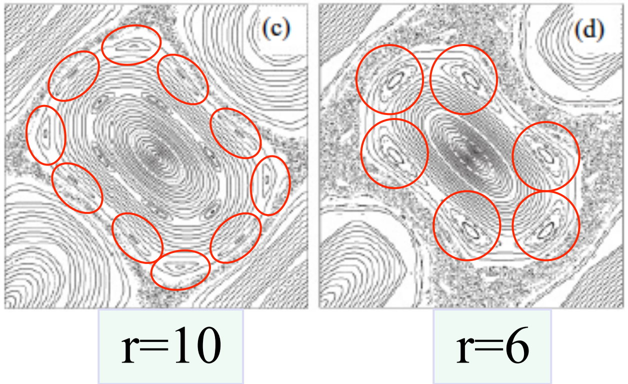
$$V_{sc}^{r,l} = \frac{e^{-i\varphi_l}}{i\pi rlk} \int_0^{2\pi} \exp(-irl\theta)\, \delta I_{r:s}(\theta) d\theta$$

$$\delta I_{r:s}(\theta) = I^{(-1)}(I_{r:s}, \theta) - I_{r:s},$$

$$\Delta\phi \approx \frac{|V_{sc}^{r,l} k|}{\hbar}$$

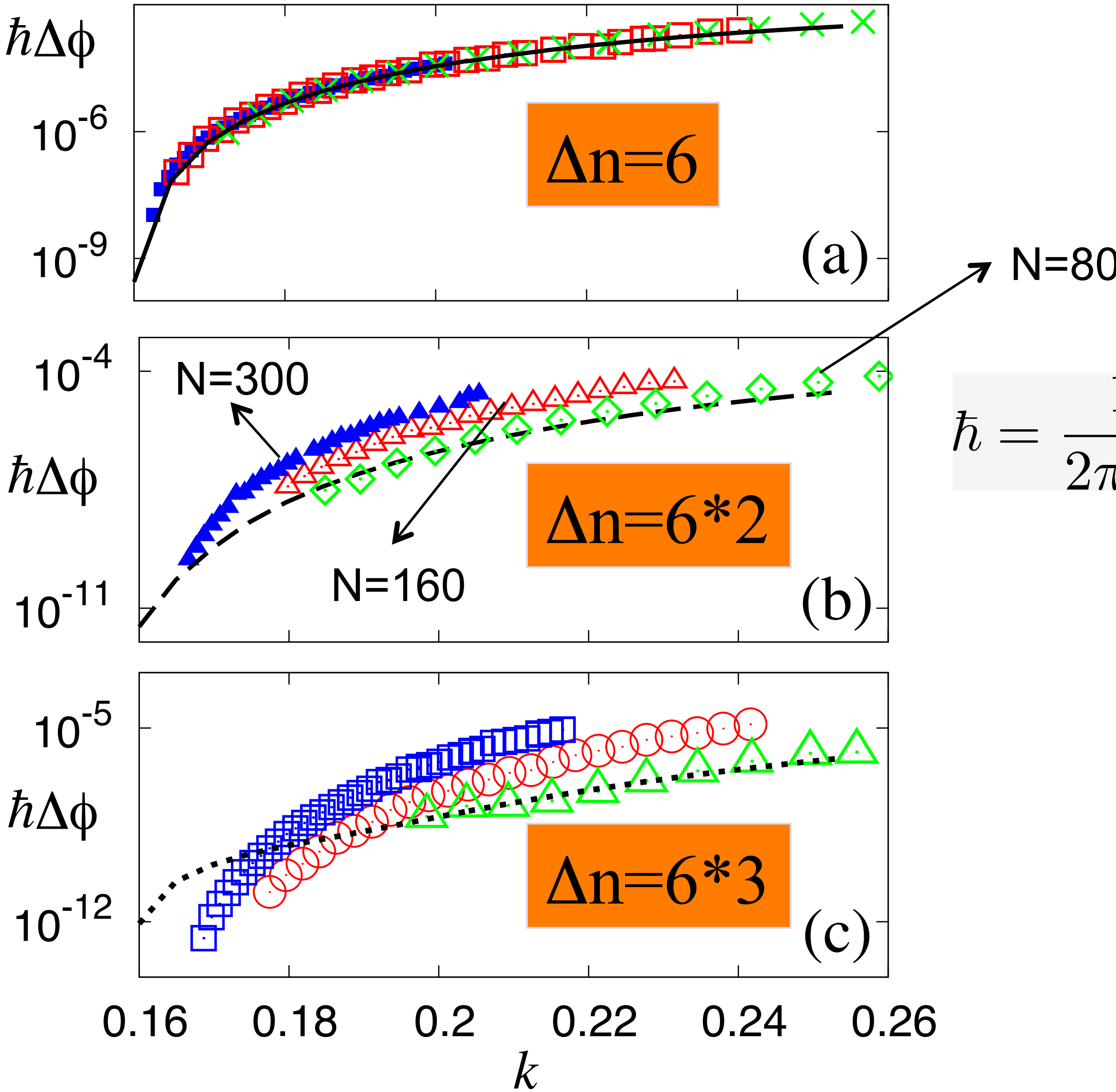
Semiclassical analysis of the gaps





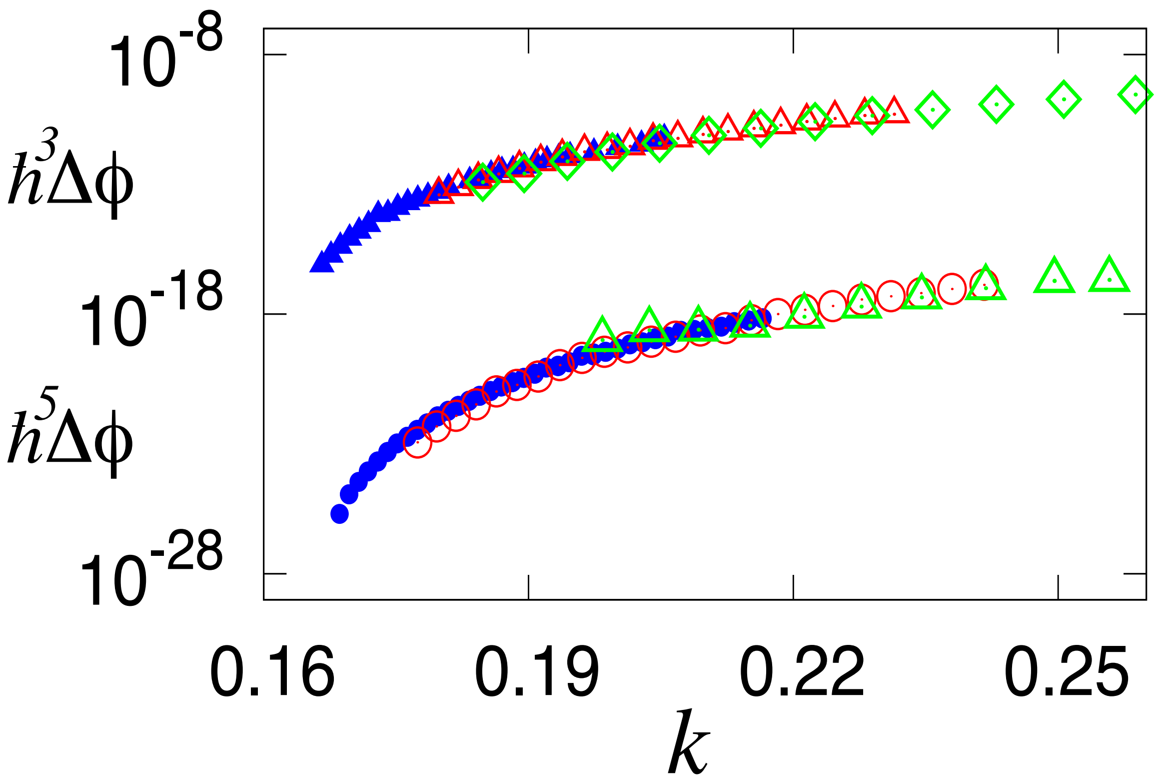
Semiclassical analysis of the gaps

r=6



$$\hbar = \frac{1}{2\pi N}$$

Semiclassical analysis of the gaps



Universality

Universality of Poincaré–Birkhoff structures in Quantum Mechanics

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[2] *Departamento de Física and IFIBA, FCEyN, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina*
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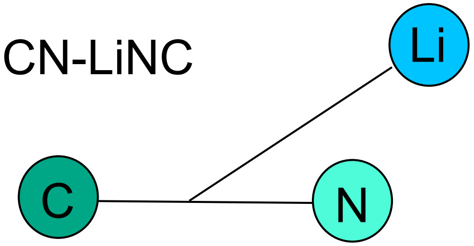
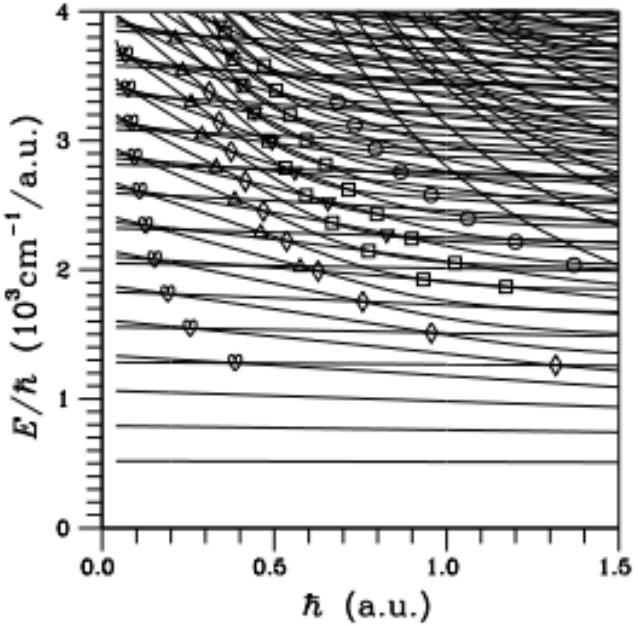
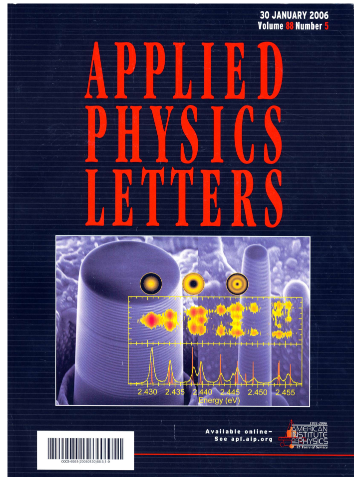
[5] *Instituto de Ciencias Matemáticas (ICMAT), Universidad Autónoma de Madrid, Cantoblanco, 28049–Madrid, Spain*
(Dated: September 2, 2013)

LiCN-LiNC





Applications



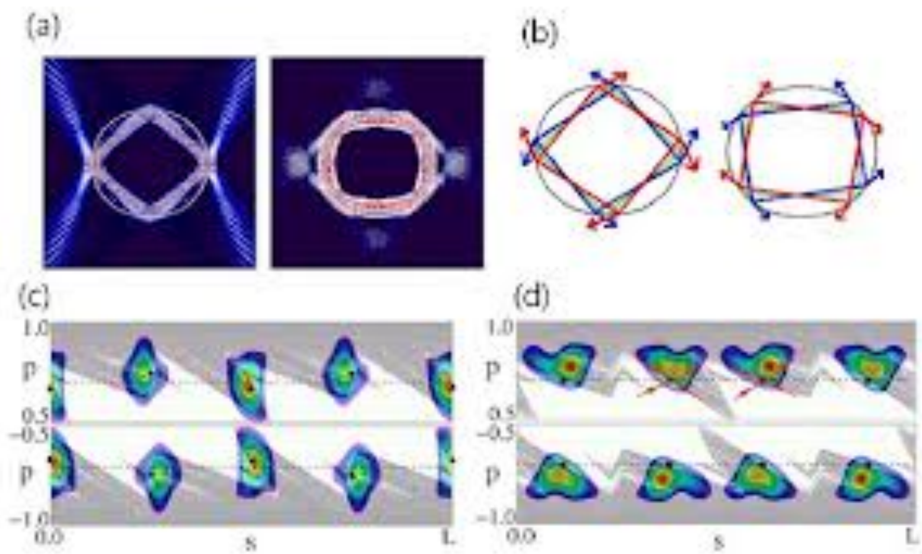
NEWS & VIEWS NATURE|Vol 465|10 June 2010

NONLINEAR DYNAMICS

Chaotic billiard lasers

A. Douglas Stone

The chaotic motion of light rays gives microlasers surprising emission properties, enhancing quantum tunnelling by many orders of magnitude and producing highly directional output beams.



Experimental observation

Observation of resonance-assisted dynamical tunneling in an asymmetric microcavity

Hojeong Kwak, Younghoon Shin, Songky Moon, and Kyungwon An*
School of Physics and Astronomy, Seoul National University, Seoul 151-742, Korea
(Dated: May 28, 2013)

We report the first experimental observation of the resonance-assisted dynamical tunneling (RADT) in the inter-mode interaction in an asymmetric-deformed microcavity. A selection rule for strong inter-mode coupling induced by RADT was observed on angular mode numbers as predicted by the RADT theory. In addition, the coupling strength was measured to be proportional to the square of the phase-space area associated with the nonlinear resonance involved in RADT. The proportionality constant was found to depend only on the nonlinear resonance, supporting the semiclassical nature of RADT.

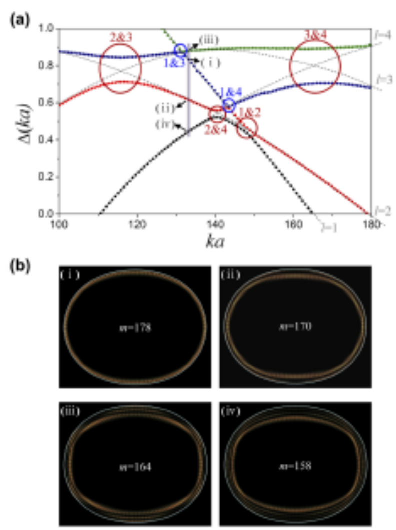


FIG. 1: (a) Mode dynamics diagram showing relative frequencies $\Delta(ka)$ of l=1, 2, 3 and 4 modes calculated with respect to a reference frequency in the range from ka ~100 to 180 when η=0.16. (b) Spatial mode-distribution intensity plots of the l=1, 2, 3 and 4 modes marked by arrows in (a). These modes are far from the interaction regions. The solid line indicates the cavity boundary.

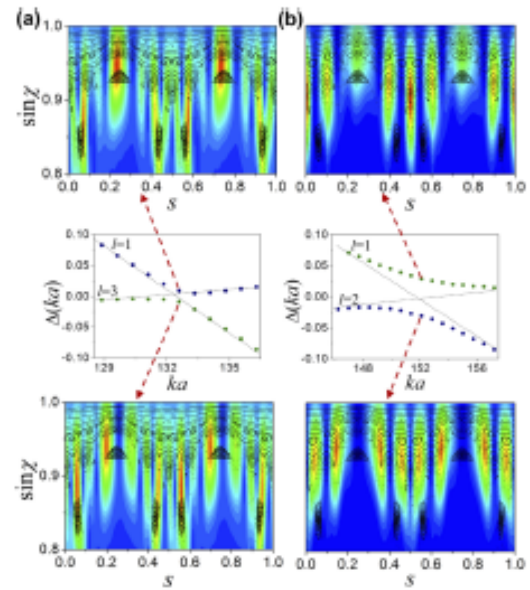


FIG. 2: (a) Relative frequencies of l=1 and 3 modes near the AC region and the Husimi plots of the two modes at the closest encounter, marked by dashed arrows, when $\eta = 0.19$. (b) The same plots for l=1 and 2 modes when $\eta = 0.19$.

- Classical nonlinear resonance imprints a systematic influence in the quantum eigenvalues and eigenfunctions of a mixed system.

- Universal structure embedded in the spectra: states localized in tori interact if the quantum numbers differ in a multiple of the order of the resonance.

- Eigenstates in the AC has a particular morphology. One state is localized in the vicinity of the unstable PO and the other state is localized on the island chain.

- These findings could be of importance in the design of optical microcavities.

PHYSICAL REVIEW E **84**, 026206 (2011)

Poincaré-Birkhoff theorem in quantum mechanics

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[1]*Departamento de Física and IFIBA, FCEyN, UBA Ciudad Universitaria, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina*

[2]*Departamento de Física, Comisión Nacional de Energía Atómica, Avenida del Libertador 8250, 1429 Buenos Aires, Argentina*

[3]*Grupo de Sistemas Complejos and Departamento de Física, Escuela Técnica Superior de Ingenieros Agrónomos, Universidad Politécnica de Madrid, 28040 Madrid, Spain*

[4]*Departamento de Química, and Instituto Mixto de Ciencias Matemáticas, CSIC-UAM-UC3M-UCM, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain*

(Received 6 May 2011; revised manuscript received 30 June 2011; published 22 August 2011)

Quantum manifestations of the dynamics around resonant tori in perturbed Hamiltonian systems, dictated by the Poincaré-Birkhoff theorem, are shown to exist. They are embedded in the interactions involving states which differ in a number of quanta equal to the order of the classical resonance. Moreover, the associated classical phase space structures are mimicked in the quasiprobability density functions and their zeros.

DOI: 10.1103/PhysRevE.84.026206 PACS number(s): 05.45.Mt, 03.65.Sq

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Universal wave functions structure in mixed systems

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PACS 05.45.Mt – Quantum chaos; semiclassical methods
PACS 03.65.Sq – Semiclassical theories and applications