Thermalization-controlled electron transport

Dmitry POLYAKOV
Research Center, Karlsruhe Institute of Technology, Germany

Alexander DMITRIEV (loffe)
Igor GORNYI (KIT, loffe)
Thermalization in nonlinear transport: An example of MIRO

MIRO at order $O(P)$:

$$P = \text{microwave power}$$

“quantum MIRO”: \[ \sigma_{\text{osc}} \propto \frac{\tau_{ee}}{\tau} \left( + \frac{\tau}{4\tau_*} \right) \]

Dmitriev, Mirlin, Polyakov ’03

Dmitriev, Vavilov, Aleiner, Mirlin, Polyakov ’05

Khodas, Vavilov ’08

Dmitriev, Khodas, Mirlin, Polyakov, Vavilov ’09

“quasiclassical MIRO”: \[ \sigma_{\text{osc}} \propto \frac{\tau_{in}}{\tau} \]

Dmitriev, Mirlin, Polyakov ’04

Review: Dmitriev, Mirlin, Polyakov, Zudov, Rev. Mod. Phys. ’12

$\tau_{ee}$ — thermalization of electrons among themselves

$\tau_{in}$ — thermalization with the external bath

$\tau, \tau_*$ — disorder-induced scattering
Thermalization-controlled linear transport

resistivity $\rho = \left(\frac{m}{e^2 n}\right) \times \frac{1}{\tau}$ (Ohm’s law)

$\tau$ — momentum relaxation

$\rho = \left(\frac{m}{e^2 n}\right) \times \frac{1}{\tau_{ee}}$ ?

$\tau_{ee}$ — thermalization of electrons among themselves
(by itself) momentum-energy conserving

(anomalously) slow thermalization $\Rightarrow$ class of linear transport phenomena
in which $\rho \propto \frac{1}{\tau_{ee}}$

“thermalization-controlled transport” : $\tau \rightarrow \tau_{ee}$

“disorder-controlled thermalization” : $\tau \leftarrow \tau_{ee}$

also nontrivial, but different!
Disorder-controlled thermalization

- Most prominent example: Energy relaxation in a single-channel quantum wire

▶ Luttinger liquid with backscattering disorder (impurities)

Bagrets, Gornyi, Polyakov ’08–’09

nonequilibrium functional bosonization, kinetic equations for plasmons and electrons

energy relaxation rate $\tau_E^{-1} = \tau^{-1}$ \quad $T \gg 1/\alpha^2 \tau$

($\tau$: elastic scattering off disorder, $\alpha$: interaction constant)

interaction independent

up to a renormalization of the strength of disorder

double-step electron distribution function in the middle of a biased quantum wire

experiment (tunneling spectroscopy) on C nanotubes: Chen et al. ’09
Thermalization-controlled transport

This talk —→ two examples:

- **without disorder**: Coulomb drag resistivity for a double quantum wire
- **with disorder**: interaction-induced resistivity of a single quantum wire with smooth inhomogeneities

Both examples are for single-channel 1D systems (nanowires) —in which the effect is the strongest

*Dmitriev,Gornyi,Polyakov, PRB ’12 and to be published*
Semiconductor nanowires: 2D → 1D

- CEO, V-groove, … nanowires
- Quantum-Hall line junctions
- Double quantum wires
- …

$R \sim 10 \text{ nm}$

Atomic-precision “cleaved-edge” single-channel GaAs wires at the intersection of two quantum wells

From Auslaender et al., Science ’02

V-groove nanowire

From Levy et al., PRL ’06

Semiconductor nanowires: Mean free path $l \sim 10 \mu\text{m}$
Semiconductor nanowires: 2D → 1D

- CEO, V-groove, … nanowires
- Quantum-Hall line junctions
- Double quantum wires
- …

Quantum-Hall line junctions:
longest ($L \sim 1\text{ cm}$) single-channel GaAs quantum wires

**backscattering disorder = random interedge tunneling**

1D barrier in 2D: Kang et al., Nature ’00; Yang et al., PRL ’04
L-shaped quantum wells: Grayson et al., APL ’05, PRB ’07

Mean free path in 1D controlled continuously by magnetic field

*From* Grayson et al., PRB ’07
Semiconductor nanowires: 2D $\rightarrow$ 1D

- CEO, V-groove, ... nanowires
- Quantum-Hall line junctions
- Double quantum wires
- ...

From Auslaender et al., Science ’05
- barrier width $\sim$ 6 nm
- wire width $\sim$ 20-30 nm

From Laroche et al., Science ’14
- barrier width $\sim$ 15 nm
- distance between the wires $\sim$ 35 nm
Coulomb drag: Current induced by current

Two conductors (quantum wells, quantum wires, ...) coupled by Coulomb interaction:

No tunneling between the wires, only coupling by e-e interactions

Coulomb drag = response of electrons in the passive conductor to a current in the active conductor, mediated by Coulomb interaction
Coulomb drag: Current induced by current

Two conductors (quantum wells, quantum wires, ...) coupled by Coulomb interaction:

\[ V_2 \]

1D drag

\[ \sigma_D = \frac{j_2}{E_1} \] (response to \( E_1 \))

Passive wire: no current if biased by \( V_2 \) to compensate for the drag

\[ \rho_D = \frac{-E_2}{j_1} \] (response to \( j_1 \))

No tunneling between the wires, only coupling by e-e interactions
Coulomb drag: Experiment

- **Discovery 2D-2D**: Gramila, Eisenstein, MacDonald, Pfeiffer & West, PRL ’91
- **Prediction**: Pogrebinskii, Sov. Phys. Semicond. ’77
- **“Orthodox theory”**: Zheng & MacDonald, PRB ’93; Jauho & Smith, PRB ’93
  Kamenev & Oreg, PRB ’95; Flensberg, Hu, Jauho & Kinaret, PRB ’95
- **Double-layer semiconductor structures**: Sivan et al. ’92; Kellogg et al. ’02
  Pillarisetty et al. ’02-’05; Price et al. ’07; Seamons et al. ’09
- **Drag in the QH regime**: Rubel et al. ’97; Lilly et al. ’98
  Kellogg et al. ’02-’03; Tutuc et al. ’09
- **Oscillatory magnetodrag**: Hill et al. ’96; Feng et al. ’98
  Lok et al. ’01; Muraki et al. ’03
- **Double graphene layers**: Kim et al. ’11-’12; Gorbachev et al. ’12
  Titov et al. ’13
- **Double quantum-point contacts**: Khrapai et al. ’07
- **Double quantum wires**: Debray et al. ’00-’02; Yamamoto et al. ’02-’06
  Laroche et al. ’11-’14
Coulomb drag: Experiment

- **Discovery 2D-2D**: Gramila, Eisenstein, MacDonald, Pfeiffer & West, PRL ’91
  
- **Prediction**: Pogrebinskii, Sov. Phys. Semicond. ’77
  
- **“Orthodox theory”**: Zheng & MacDonald, PRB ’93; Jauho & Smith, PRB ’93
  
  Kamenev & Oreg, PRB ’95; Flensberg, Hu, Jauho & Kinaret, PRB ’95

- **Double-layer semiconductor structures**: Sivan et al. ’92; Kellogg et al. ’02
  
  Pillarisetty et al. ’02–’05; Price et al. ’07; Seamons et al. ’09

- **Drag in the QH regime**: Rubel et al. ’97; Lilly et al. ’98
  
  Kellogg et al. ’02–’03; Tutuc et al. ’09

- **Oscillatory magnetodrag**: Hill et al. ’96; Feng et al. ’98
  
  Lok et al. ’01; Muraki et al. ’03

- **Double graphene layers**: Kim et al. ’11–’12; Gorbachev et al. ’12
  
  Titov et al. ’13

- **Double quantum-point contacts**: Khrapai et al. ’07

- **Double quantum wires**: Debray et al. ’00–’02; Yamamoto et al. ’02–’06
  
  Laroche et al. ’11–’14
Coulomb drag between quantum wires: Setup

**Planar geometry, GaAlAs**
- Soft barriers, width $\sim 80$ nm
- Distance between the wires $d \sim 200$ nm
- Length $\sim 4 \mu$m

**Vertical geometry, GaAlAs**
- Hard barriers, width $\sim 15$ nm
- Distance between the wires $d \sim 35$ nm
Coulomb drag between quantum wires: Experiment

Laroche et al., Science ’14

Drag effect up to 25% in closely packed nanowires on the 10 nm scale

Debray et al., JPCM ’01

Yamamoto et al., Science ’06
Coulomb drag: “Orthodox theory”

Zheng & MacDonald ’93
Kamenev & Oreg ’95; Flensberg, Hu, Jauho & Kinaret ’95

\[ \rho_D = \frac{1}{e^2 n_1 n_2} \int \frac{d\omega}{2\pi} \int \frac{d^Dq}{(2\pi)^D} \frac{(q^2/D)|V_{12}(\omega,q)|^2}{2T \sinh^2(\frac{\omega}{2T})} \text{Im}\Pi_1(\omega, q) \text{Im}\Pi_2(\omega, q) \]

\( V_{12} \) – interwire (screened) interaction, \( \Pi_{1,2} \) – density-density correlators

2D: \( \rho_D \propto (T/\Lambda)^2 \) \( \Lambda \) – UV cutoff (Fermi energy)

“Golden rule approach” \( \rho_D \propto V_{12}^2 \)

\( \rho_D = 0 \) in a particle-hole symmetric \( (\Lambda \to \infty) \) 2D system

(electron drag current) = \( -(\text{hole drag current}) \)
Drag between clean quantum wires: Electron-hole symmetry

Nazarov & Averin ’98
Klesse & Stern ’00; Fiete, Le Hur & Balents ’06

Golden rule in 1D Fermi liquid (linear dispersion):

\[ \rho_D \sim g_1^2 \frac{\hbar}{e^2} \frac{\Lambda}{v_F} \frac{T}{\Lambda} \]

Hu & Flensberg ’96

\( g_1 \) – interwire e-e backscattering

Linearized dispersion in 1D: No drag due to forward scattering
but backward scattering does contribute
Drag between clean quantum wires: Electron-hole symmetry

Nazarov & Averin ’98
Klesse & Stern ’00; Fiete, Le Hur & Balents ’06

Golden rule in 1D Fermi liquid (linear dispersion):
\[ \rho_D \sim g_1^2 \frac{\hbar}{e^2} \frac{\Lambda}{v_F} \frac{T}{\Lambda} \]
Hu & Flensberg ’96

\( g_1 \) – interwire e-e backscattering

Linearized dispersion in 1D: No drag due to forward scattering but backward scattering does contribute

Luttinger-liquid renormalization: \[ \rho_D \propto g_1^2 (T/\Lambda)^\kappa \quad \kappa < 1 \]

“Pseudospin gap”: \[ \rho_D \propto \exp(\Delta/T) \quad T \to 0 \]

Zigzag CDW ordering \[ T \ll \Delta \sim (g_1)^{\frac{2}{1-\kappa}} \Lambda \propto \exp\left(-\frac{4kFd}{1-\kappa}\right) \]

“Absolute drag” \((j_1 \simeq j_2)\) in long Luttinger constrictions
Drag between clean quantum wires: Curvature

Beyond the Luttinger model: Nonlinear dispersion of the (bare) electron spectrum

\[ \rho_D \sim \beta^2 \frac{h}{e^2} \frac{\Lambda}{v_F} \left( \frac{T}{\Lambda} \right)^2 \propto k_F/m^2 \]

\( \beta \) – interwire e-e forward scattering

\( T \gg v_F/d \rightarrow \rho_D = \text{const}(T) \)

\( d \) – distance between the wires

\( T \ll \beta \Lambda \rightarrow \rho_D \sim \frac{1}{\beta} \frac{h}{e^2} \frac{\Lambda}{v_F} \left( \frac{T}{\Lambda} \right)^5 \)

Aristov ’07
strength of interwire backscattering \( g_1^2 \propto \exp(-4k_Fd) \)

\( d \) - distance between the wires

\( \rightarrow \) for \( k_Fd \gg 1 \), drag at not too low \( T \)

\( \rightarrow \) by forward scattering with small-momentum transfer \( \ll k_F \)
Drag due to forward scattering

\[ \rho_D \sim \beta^2 \frac{h}{e^2} \frac{T^2}{\epsilon_F v_F} \]

obtained by means of:

- \textbf{Pustilnik et al.:} drift ansatz = “orthodox” formula
- \textbf{Aristov:} bosonization for large \( \omega \) + “Loretzian ansatz”
- \textbf{Rozhkov:} bosonization/refermionization + “orthodox” formula

**Drift ansatz**: Electrons are at thermal equilibrium in the moving (with the drift velocity) frame
Drag due to forward scattering

\[ \rho_D \sim \beta^2 \frac{h}{e^2} \frac{T^2}{\epsilon_F v_F} \]

obtained by means of:

- Pustilnik et al. → drift ansatz = “orthodox” formula
- Aristov → bosonization for large \( \omega \) + “Lorentzian ansatz”
- Rozhkov → bosonization/refermionization + “orthodox” formula

**Drift ansatz**: Electrons are at thermal equilibrium in the moving (with the drift velocity) frame

\[
j_2 = 0 \quad \rightarrow \quad e n_2 E_2 = \int \frac{d\omega}{2\pi} \int \frac{dq}{2\pi} q V_{12}(q) S_1(\omega, q) S_2(-\omega, -q)
\]

\( S_{1,2}(\omega, q) \) — dynamic structure factors (intrawire int’s incl.)

- passive wire: \( S_2(\omega, q) = S_{\text{eq}}(\omega, q) \) ← equilibrium
- active wire: \( S_1(\omega, q) = S_{\text{eq}}(\omega - q v_d, q) \) ← Galilean transform

+ FDT: \( S_{\text{eq}}(\omega, q) = 2 \text{Im} \Pi(\omega, q)/(1 - e^{-\omega/T}) \) → “orthodox” formula
Drag vs. thermalization

“Orthodox theory” $\equiv$ electrons at equilibrium in the moving frame

This (innocently looking) assumption is only justified for “perturbative” drag with $1/\tau_D \ll 1/\tau_{eq}$

$1/\tau_D$ – “drag rate” $\rho_D = m/e^2 n \tau_D$ (for $n_1 = n_2 = n$)

$1/\tau_{eq}$ – (smallest) thermalization rate

“Orthodox theory” $\leftarrow$ totally wrong for forward scattering in 1D (right-left thermalization rate $1/\tau_{eq} = 0$)

In fact, for forward scattering $\rho_D = 0$

No dc friction between chiral electrons!
Kinetic theory approach to drag

Quadratic dispersion: pair collisions in 1D $\rightarrow$ momentum exchange

$$\delta \varepsilon \delta k = \frac{m}{|k - k'|} \delta(k_1 - k'_2) \delta(k_2 - k'_1)$$

no change in the distribution function $f(k)$ in a single wire

double wire: center-of-mass distribution $f_1(k) + f_2(k) = \text{const.}$

kinetic equation for the relative distribution $f_1(k) - f_2(k)$

chiral equilibration

backscattering at the bottom $\Rightarrow$ right-left equilibration
Drag with small momentum transfer: Diffusion in energy space

\[ T \gg v/d \rightarrow \text{Fokker-Planck equation} \]

for the relative distribution \( f_-(k) \):

current in momentum space \( J = -D \partial_k f_- + f_- \partial_k D \)

with the diffusion coefficient \( D(k) \propto 1/\cosh^2[(k^2 - k_F^2)/4mT] \)

energy space: diffusion of an electron-hole pair (electron in wire 1, hole in wire 2)

at the bottom: \( D(0) \propto \exp(-\epsilon_F/T) \)
Drag with small-momentum transfer: Pair collisions

\[ \rho_D = \frac{24\pi}{e^2 v_F \tau_D(\infty)} \left( \frac{2\epsilon_F^3}{\pi T^3} \right)^{1/2} e^{-2\epsilon_F/T} \]

for \( v_F/d \ll T \ll \epsilon_F \)

Friction due to effective backscattering by diffusion in energy space through the point \( k = 0 \)

No drag without R-L thermalization

For \( T \ll v_F/d \): backscattering dominates with \( 1/\tau_D \propto e^{-4kFd} \)
Thermalization in the moving or stationary frame:
The difference matters!

$g_-(k) \leftarrow$ measure of nonequilibrium: $f_-(k) = f^T + g_-(k)T \partial_\epsilon f^T$

“step” in $g_-(k)$, weak drag $\rightarrow$ split chiral chemical potentials, thermal in the stationary frame

linear $g_-(k)$, strong drag $\rightarrow$ drift ansatz, thermal in the moving frame

In which frame the system chooses to thermalize (how strong drag is) depends crucially on intrawire equilibration (triple collisions)!
Enhancement of drag by intrawire thermalization

\[ \mathcal{D}_2 \leftarrow \text{pair inter wire} \quad \mathcal{D}_3 \leftarrow \text{triple intra wire} \]

\[ \rho_\mathcal{D} \exp(-\epsilon_F/T) \]

\[ \exp(\epsilon_F/T) \]

\[ \mathcal{D}_3/\mathcal{D}_2 \]

\[ (\mathcal{D}_3/\mathcal{D}_2 \text{ for fixed } \mathcal{D}_3) = \text{(distance between the wires)} \]

Plateau:

\[ \rho_\mathcal{D} = \frac{4\pi \mathcal{D}_3}{e^2 k_F} \left( \frac{\epsilon_F}{\pi T^3} \right)^{1/2} e^{-\epsilon_F/T} = \frac{m}{e^2 n} \times \frac{1}{\tau_{eq}} \]

independent of \( \mathcal{D}_2 \), i.e., the distance between the wires!
Coulomb drag in quantum wires: Temperature dependence

“orthodox theory” only for a large distance between the wires (solid line) right below $\epsilon_F$

Activation (regime II): independent of the distance between the wires!
Interaction-induced resistivity of a quantum wire with smooth disorder

- smooth (for electrons) disorder $\rightarrow$ negligible backscattering on the Fermi level

- w/o e-e interactions $\rightarrow$ zero-width Drude peak: $dc \ \rho = 0$

$$
\sigma(\omega) = \frac{e^2 v_F}{\pi} \left( 1 - \frac{3\langle VV \rangle_{q=0}}{2m^2 v_F^4} \right) \delta(\omega) + \frac{\langle VV \rangle_{q=\omega/v_F}}{2\pi m^2 v_F^4}
$$

$\langle VV \rangle_q$ - correlator (at momentum $q$) of the random potential

- e-e interactions (amplitude $\alpha$) $\rightarrow$ golden-rule (high-$\omega$) momentum relaxation rate (friction in the moving frame)

$$
\frac{1}{\tau} = \frac{\alpha^2}{32k_F^4 T} \int dq \frac{q^4}{2\pi} \sinh^2(v_F q/4T) \langle VV \rangle_q
$$

- $dc \ \rho$ for pair collisions - ?

$$
\rho \propto e^{-\epsilon_F/T} \ll \frac{m}{e^2 n\tau}
$$

effective backscattering due to diffusion in energy space
Interaction-induced resistivity of a quantum wire with smooth disorder

disorder-mediated pair collisions $\rightarrow$ momentum relaxation

diffusion coefficient (in energy space) $D_2$

\[ + \]

momentum-conserving triple collisions

diffusion coefficient $D_3$

nonequilibrium part of the distribution function $g(k) \simeq g_{sf}(k) + g_{mf}(k)$

\[ g_{sf} \propto \text{sgn}(k) \quad \leftarrow \text{thermalization in the stationary frame} \]

\[ g_{mf} \propto k \quad \leftarrow \text{thermalization in the moving frame} \]

\[ \rho^{-1} \simeq \rho_{sf}^{-1} + \rho_{mf}^{-1} \]

\[ \rho_{sf}^{-1} \sim \frac{e^2 n \epsilon_F}{D_2 + D_3} \frac{1}{e_F/T} \left( \frac{T}{\epsilon_F} \right)^{3/2} \]

\[ \rho_{mf}^{-1} \sim \frac{e^2 n \epsilon_F}{(D_2 + D_3)D_2} \frac{D_3 - D_2}{D_2 + D_3} \]
Interaction-induced resistivity of a quantum wire with smooth disorder

disorder-mediated pair collisions $\rightarrow$ momentum relaxation

**diffusion coefficient** (in energy space) $D_2$

$+$

momentum-conserving triple collisions

**diffusion coefficient** $D_3$

$$\rho \simeq \frac{m}{e^2 n \tau_{eq}}$$

$$D_2 \ll D_3 \ll D_2 e^{\epsilon_F/T} \left( \frac{T}{\epsilon_F} \right)^{3/2}$$

$\rho$ independent of disorder!

see also Levchenko, Micklitz, Rech, Matveev ’10
Summary

● **Thermalization-controlled linear transport:**
  Relaxation rate in Ohm’s law $\rightarrow$ thermalization rate $1/\tau_{ee}$
  \[ \rho = \frac{m}{e^2 n \tau_{ee}} \]

● **Coulomb drag by small-momentum transfer between ballistic quantum wires:**
  - “Orthodox” theory not valid
  - $\rho_D$ independent of the strength of interwire-
    but strongly dependent on the strength of intrawire
    interactions

● **Resistivity of a smoothly-inhomogeneous quantum wire:**
  - independent of the strength of disorder
    in the thermalization-controlled regime