Thermalization-controlled electron transport

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Luchon, France, May '15

Thermalization in nonlinear transport : An example of MIRO

MIRO at order $\mathcal{O}(P)$:

P = *microwave power*

"quantum MIRO" : $\sigma_{\rm osc} \propto \tau_{ee}/\tau \quad (+\tau/4\tau_*)$

Dmitriev,Mirlin,Polyakov '03

Dmitriev,Vavilov,Aleiner,Mirlin,Polyakov '05

Khodas,Vavilov '08

Dmitriev,Khodas,Mirlin,Polyakov,Vavilov '09

"quasiclassical MIRO" : $\sigma_{\rm osc} \propto \tau_{in}/\tau$

Dmitriev,Mirlin,Polyakov '04

Review : Dmitriev,Mirlin,Polyakov,Zudov, Rev. Mod. Phys. '12

τ_{ee} — thermalization of electrons among themselves
τ_{in} — thermalization with the external bath
τ, τ_* — disorder-induced scattering

Thermalization-controlled linear transport

resistivity $\rho = (m/e^2 n) \times 1/\tau$ (Ohm's law)

τ – momentum relaxation

$$\rho = (m/e^2 n) \times 1/\tau_{ee} \quad ?$$

τ_{ee} – thermalization of electrons among themselves

(by itself) momentum-energy conserving

(anomalously) slow thermalization $\Rightarrow$ class of linear transport phenomena in which $\rho \propto 1/\tau_{ee}$

"thermalization-controlled transport" : $\tau \longrightarrow \tau_{ee}$

"disorder-controlled thermalization" : $\tau \longleftarrow \tau_{ee}$

also nontrivial, but different !

Disorder-controlled thermalization

- ***Most prominent example :*** **Energy relaxation in a single-channel quantum wire**
  - ▷ **Luttinger liquid with backscattering disorder (impurities)**

Bagrets,Gornyi,Polyakov '08-'09

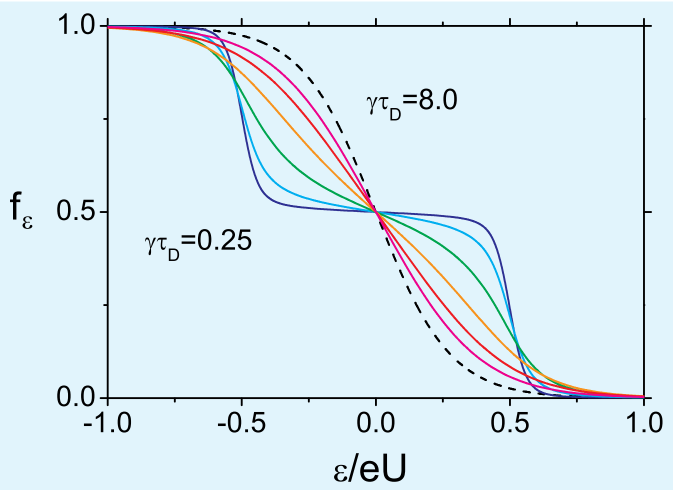
nonequilibrium functional bosonization, kinetic equations for plasmons and electrons

energy relaxation rate $\tau_E^{-1} = \tau^{-1}$ $\quad T \gg 1/\alpha^2\tau$

(τ **: elastic scattering off disorder,** α **: interaction constant)**

interaction independent

up to a renormalization of the strength of disorder



double-step electron distribution function in the middle of a biased quantum wire

experiment (tunneling spectroscopy) on C nanotubes : *Chen et al. '09*

Thermalization-controlled transport

This talk $\longrightarrow$ two examples :

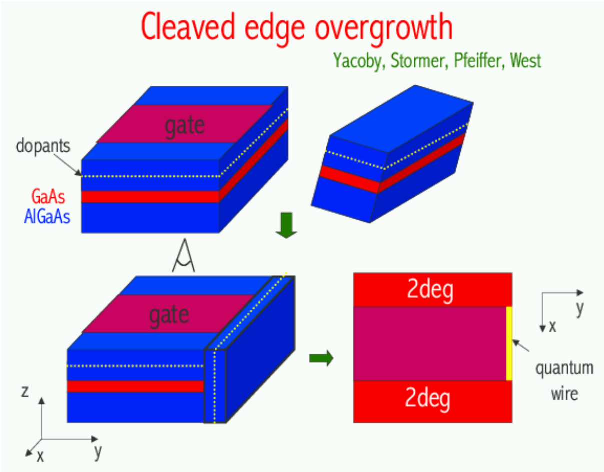
- ***without disorder* :** **Coulomb drag resistivity for a double quantum wire**

- ***with disorder* :** **interaction-induced resistivity of a single quantum wire with smooth inhomogeneities**

Both examples are for single-channel 1D systems (nanowires) —in which the effect is the strongest

Dmitriev,Gornyi,Polyakov, PRB '12 and to be published

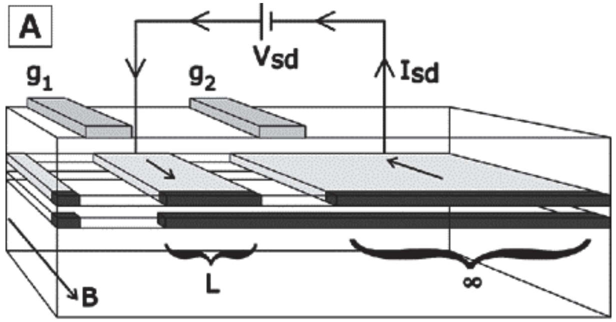
Semiconductor nanowires : 2D → 1D



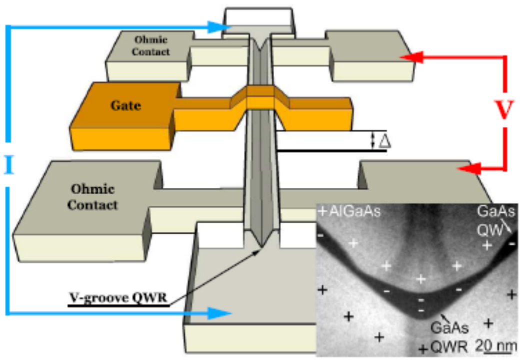
Atomic-precision "cleaved-edge"
single-channel GaAs wires
at the intersection of two
quantum wells

$R \sim \underline{10\,\mathrm{nm}}$

- CEO, V-groove, ... nanowires
- Quantum-Hall line junctions
- Double quantum wires
- ...



From Auslaender et al., Science '02

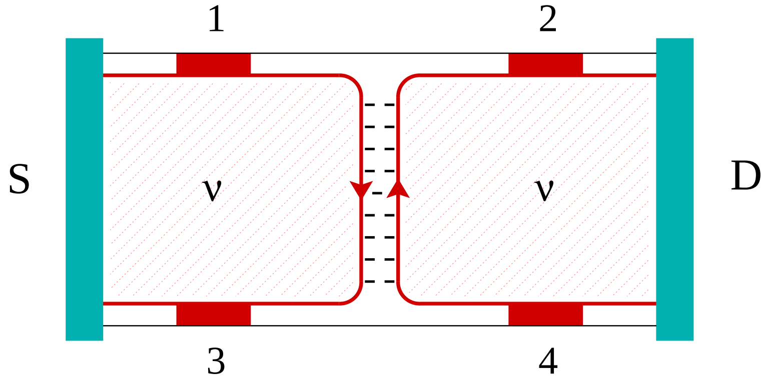


V-groove
nanowire

From Levy et al., PRL '06

Semiconductor nanowires :
Mean free path $l \sim \underline{10\,\mu\mathrm{m}}$

Semiconductor nanowires : 2D → 1D



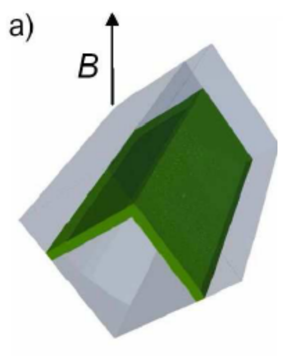
Quantum-Hall line junctions :
longest ($L \sim$ 1 cm) single-channel
GaAs quantum wires

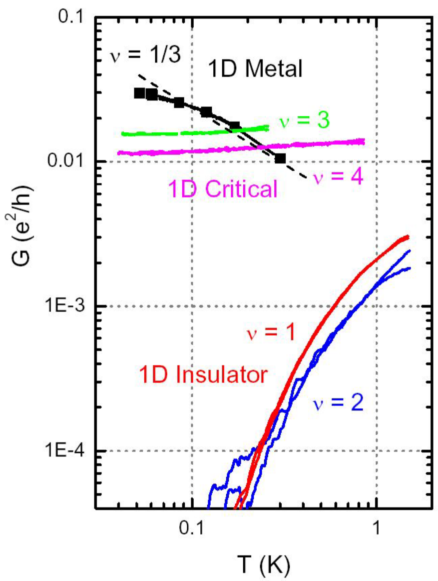
- CEO, V-groove, . . . nanowires
- Quantum-Hall line junctions
- Double quantum wires
- . . .

backscattering disorder = random interedge tunneling

1D barrier in 2D : Kang et al., Nature '00; Yang et al., PRL '04

L-shaped quantum wells : Grayson et al., APL '05, PRB '07



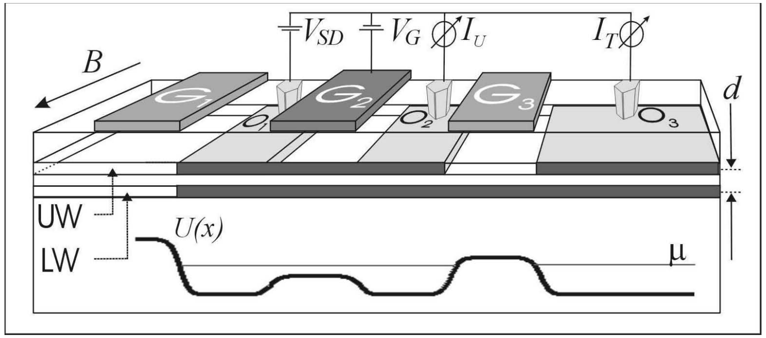


Mean free path in 1D controlled continuously by magnetic field

From Grayson et al., PRB '07

Semiconductor nanowires: 2D → 1D

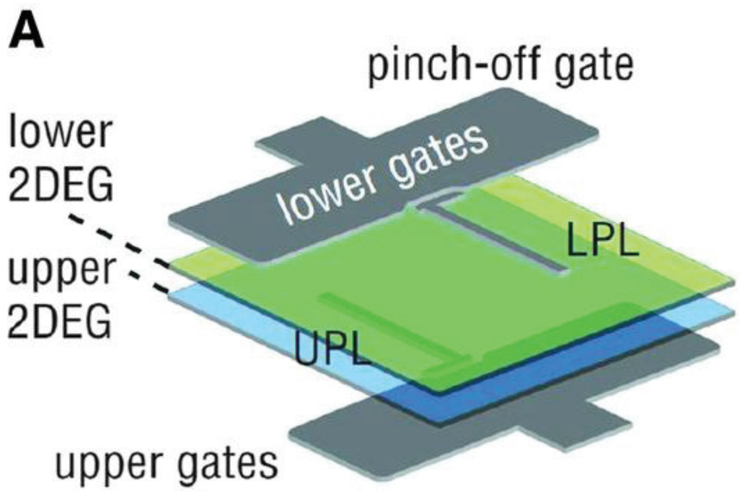
- CEO, V-groove, ... nanowires
- Quantum-Hall line junctions
- Double quantum wires
- ...



From **Auslaender et al., Science '05**

barrier width ∼ 6 nm

wire width ∼ 20-30 nm



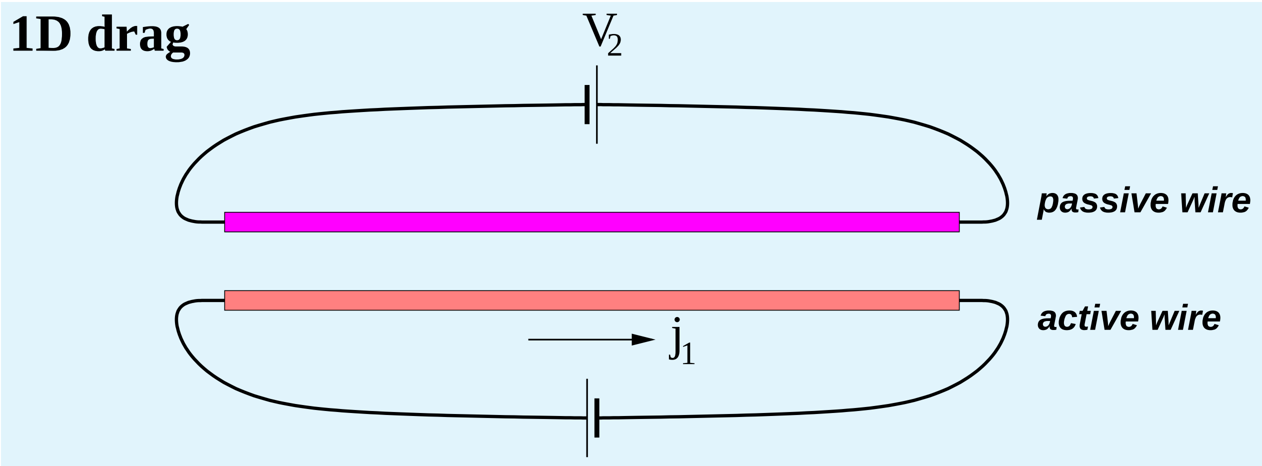
From **Laroche et al., Science '14**

barrier width ∼ 15 nm

distance between the wires ∼ 35 nm

Coulomb drag: Current induced by current

Two conductors (quantum wells, quantum wires, ...) coupled by Coulomb interaction:

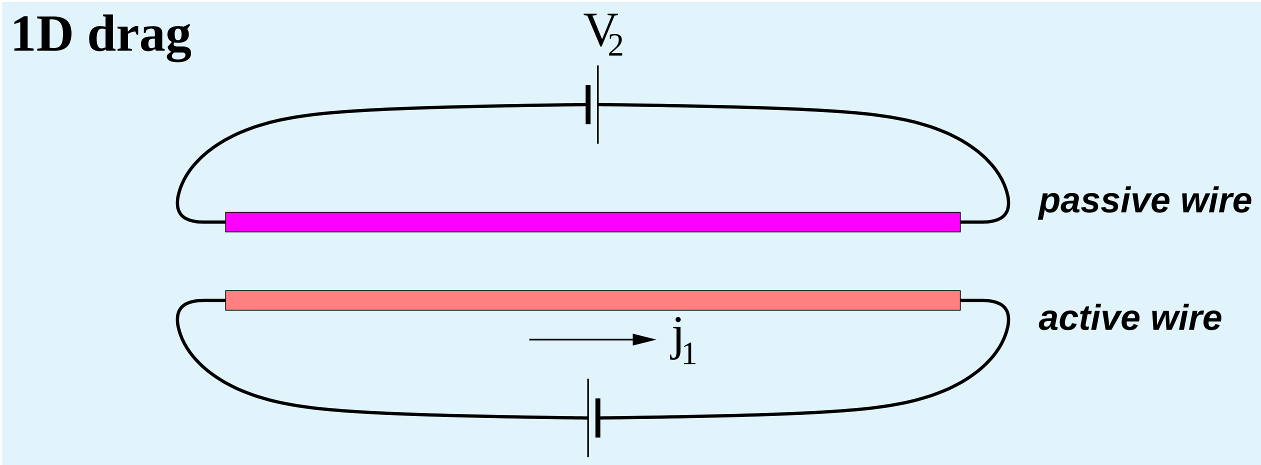


No tunneling between the wires, only coupling by e-e interactions

Coulomb drag = response of electrons in the passive conductor to a current in the active conductor, mediated by Coulomb interaction

Coulomb drag: Current induced by current

Two conductors (quantum wells, quantum wires, ...) coupled by Coulomb interaction:



No tunneling between the wires, only coupling by e-e interactions

Passive wire: no current if biased by V_2 to compensate for the drag

Transresistivity $\rho_D = -E_2/j_1$ **(response to j_1)**

Transconductivity $\sigma_D = j_2/E_1$ **(response to E_1)**

Coulomb drag : Experiment

- **Discovery 2D-2D :** *Gramila, Eisenstein, MacDonald, Pfeiffer & West, PRL '91*

  - **Prediction :** *Pogrebinskii, Sov. Phys. Semicond. '77*
  - **"Orthodox theory" :** *Zheng & MacDonald, PRB '93; Jauho & Smith, PRB '93*
    Kamenev & Oreg, PRB '95; Flensberg, Hu, Jauho & Kinaret, PRB '95

- **Double-layer semiconductor structures :** *Sivan et al. '92; Kellogg et al. '02*
  Pillarisetty et al. '02-'05; Price et al. '07; Seamons et al. '09
- **Drag in the QH regime :** *Rubel et al. '97; Lilly et al. '98*
  Kellogg et al. '02-'03; Tutuc et al. '09
- **Oscillatory magnetodrag :** *Hill et al. '96; Feng et al. '98*
  Lok et al. '01; Muraki et al. '03
- **Double graphene layers :** *Kim et al. '11-'12; Gorbachev et al. '12*
  Titov et al. '13
- **Double quantum-point contacts :** *Khrapai et al. '07*
- **Double quantum wires :** *Debray et al. '00-'02; Yamamoto et al. '02-'06*
  Laroche et al. '11-'14

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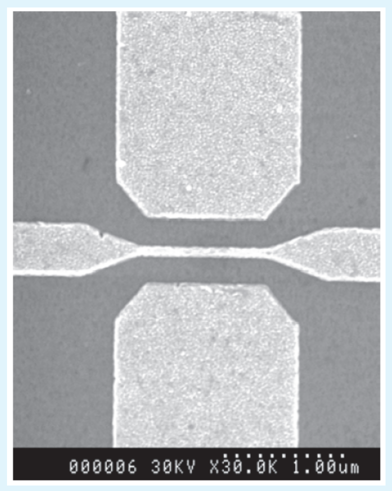
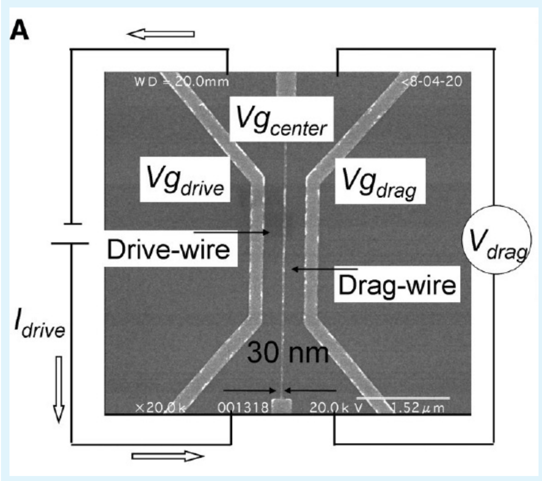
Laroche et al. '11-'14

Coulomb drag between quantum wires: Setup



Debray et al., JPCM '01



Yamamoto et al., Science '06
(Tarucha group)

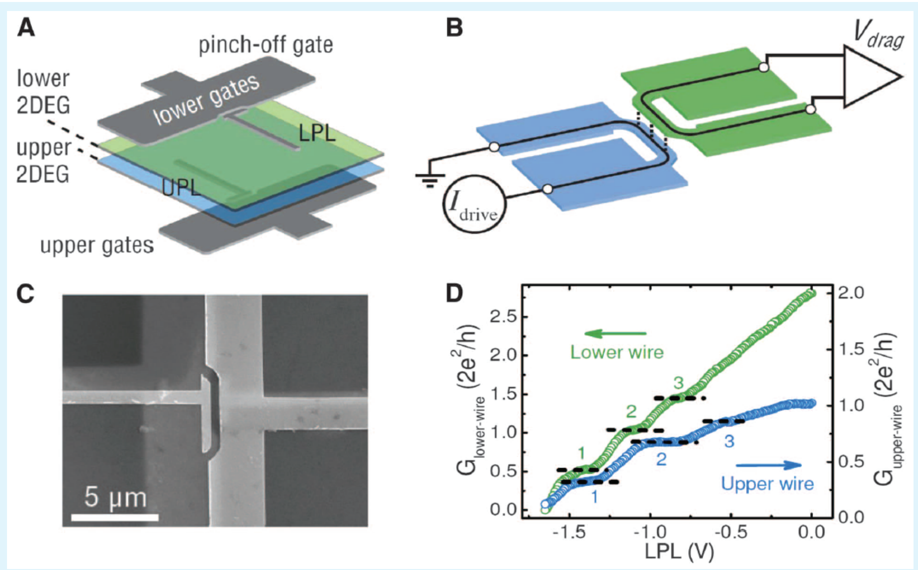
planar geometry, GaAlAs

soft barriers, width $\sim 80\,\mathrm{nm}$

distance between the wires

$d \sim 200\,\mathrm{nm}$

length $\sim 4\,\mu\mathrm{m}$



Laroche et al., Nature Nanotech. '11, Science '14
(Gervais group)

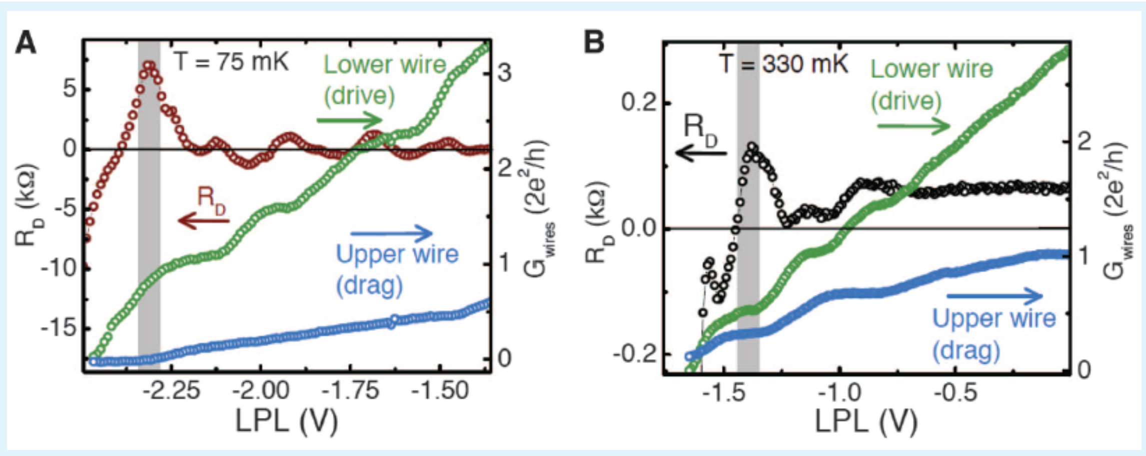
vertical geometry, GaAlAs

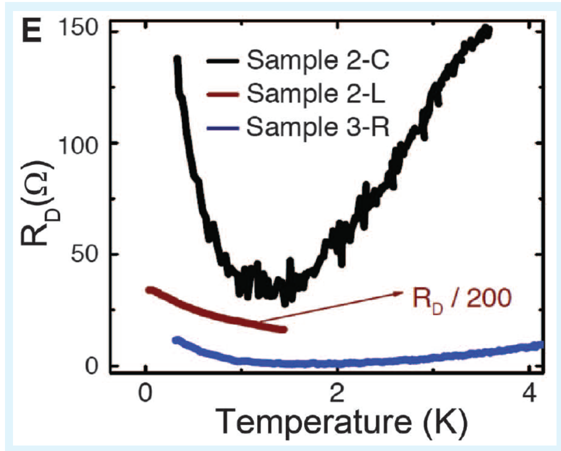
hard barriers, width $\sim 15\,\mathrm{nm}$

distance between the wires

$d \sim 35\,\mathrm{nm}$

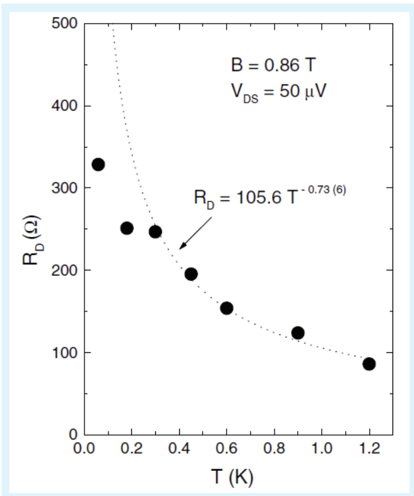
Coulomb drag between quantum wires: Experiment



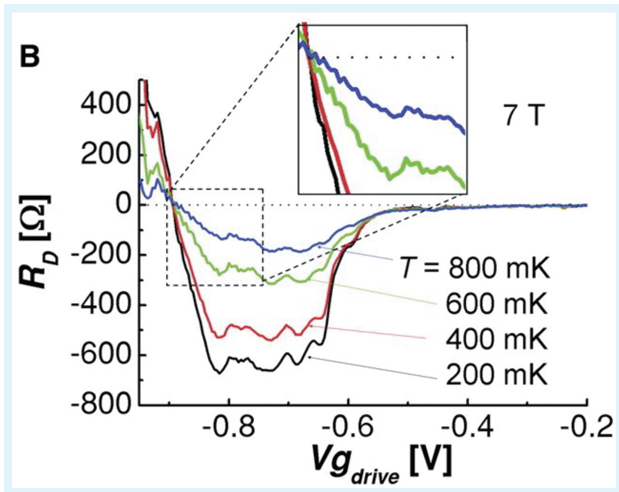


Laroche et al., Science '14

Drag effect up to 25% in closely packed nanowires on the 10 nm scale



Debray et al., JPCM '01



Yamamoto et al., Science '06

Coulomb drag: "Orthodox theory"

Zheng & MacDonald '93
Kamenev & Oreg '95; Flensberg, Hu, Jauho & Kinaret '95

$$\rho_D = \frac{1}{e^2 n_1 n_2} \int \frac{d\omega}{2\pi} \int \frac{d^D q}{(2\pi)^D} \frac{(q^2/D)|V_{12}(\omega,q)|^2}{2T\sinh^2(\frac{\omega}{2T})} \; \mathrm{Im}\Pi_1(\omega,q)\; \mathrm{Im}\Pi_2(\omega,q)$$

V_{12} – interwire (screened) interaction, $\Pi_{1,2}$ – density-density correlators

2D: $\rho_D \propto (T/\Lambda)^2$ Λ – UV cutoff (Fermi energy)

"Golden rule approach" $(\rho_D \propto V_{12}^2)$

$\rho_D = 0$ in a particle-hole symmetric $(\Lambda \to \infty)$ 2D system

(electron drag current) $= -$(hole drag current)

Drag between clean quantum wires : Electron-hole symmetry

Nazarov & Averin '98

Klesse & Stern '00; Fiete, Le Hur & Balents '06

Golden rule in 1D Fermi liquid (linear dispersion) : $\rho_{\mathrm{D}} \sim g_1^2 \, \frac{h}{e^2} \, \frac{\Lambda}{v_F} \, \frac{T}{\Lambda}$

Hu & Flensberg '96

g_1 – **interwire e-e backscattering**

Linearized dispersion in 1D : **No drag due to forward scattering**
but backward scattering does contribute

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Luttinger-liquid renormalization: $\rho_{\mathrm{D}} \propto g_1^2 \, (T/\Lambda)^{\kappa}$ $\quad \kappa < 1$

"Pseudospin gap": $\rho_{\mathrm{D}} \propto \exp(\Delta/T)$ $\quad T \to 0$

Zigzag CDW ordering $\quad T \ll \Delta \sim (g_1)^{\frac{2}{1-\kappa}} \Lambda \propto \exp\left(-\frac{4k_F d}{1-\kappa}\right)$

"Absolute drag" ($j_1 \simeq j_2$) in long Luttinger constrictions

Drag between clean quantum wires: Curvature

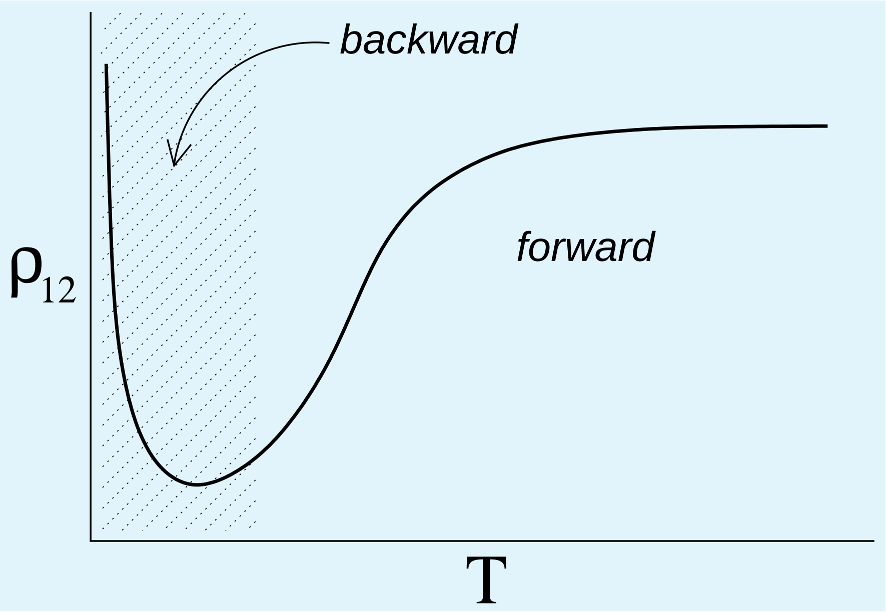
Pustilnik, Mishchenko, Glazman & Andreev '03

Aristov '07; Rozhkov '08

Beyond the Luttinger model: **Nonlinear dispersion of the (bare) electron spectrum**

$$\rho_{\mathrm{D}} \sim \beta^2 \frac{h}{e^2} \frac{\Lambda}{v_F} \left(\frac{T}{\Lambda}\right)^2 \propto k_F/m^2$$

β – **interwire e-e forward scattering**



$T \gg v_F/d \quad \rightarrow \quad \rho_{\mathrm{D}} = \mathrm{const}(T)$

d – **distance between the wires**

$T \ll \beta\Lambda \quad \rightarrow \quad \rho_{\mathrm{D}} \sim \frac{1}{\beta} \frac{h}{e^2} \frac{\Lambda}{v_F} \left(\frac{T}{\Lambda}\right)^5$

Aristov '07

identical wires

strength of interwire backscattering $g_1^2 \propto \exp(-4k_F d)$

d - distance between the wires

$\Longrightarrow$ *for $k_F d \gg 1$, drag at not too low T*

$\longrightarrow$ *by forward scattering with small-momentum transfer $\ll k_F$*

Drag due to forward scattering

$$\rho_{\mathrm{D}} \sim \beta^2 \, \frac{h}{e^2} \, \frac{T^2}{\epsilon_F v_F}$$

Pustilnik, Mishchenko, Glazman & Andreev '03

Aristov '07; Rozhkov '08

obtained by means of:

Pustilnik et al. $\rightarrow$ drift ansatz = "orthodox" formula

Aristov $\rightarrow$ bosonization for large ω + "Loretzian ansatz"

Rozhkov $\rightarrow$ bosonization/refermionization + "orthodox" formula

Drift ansatz : Electrons are at thermal equilibrium in the moving (with the drift velocity) frame

Drag due to forward scattering

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Drift ansatz : Electrons are at thermal equilibrium in the moving (with the drift velocity) frame

passive wire

$$j_2 = 0 \;\rightarrow\; e n_2 E_2 = \int \frac{d\omega}{2\pi} \int \frac{dq}{2\pi}\, q V_{12}(q) S_1(\omega, q) S_2(-\omega, -q)$$

$S_{1,2}(\omega, q)$ − dynamic structure factors (intrawire int's incl.)

passive wire: $S_2(\omega, q) = S^{\mathrm{eq}}(\omega, q)$ $\leftarrow$ ***equilibrium***

active wire: $S_1(\omega, q) = S^{\mathrm{eq}}(\omega - q v_d, q)$ $\leftarrow$ ***Galilean transform***

+ FDT: $S^{\mathrm{eq}}(\omega, q) = 2\mathrm{Im}\Pi(\omega, q)/(1 - e^{-\omega/T})$ $\longrightarrow$ "orthodox" formula

Drag vs. thermalization

"Orthodox theory" $\equiv$ electrons at equilibrium
in the moving frame

this (innocently looking) assumption is only justified for "perturbative" drag with

$$1/\tau_D \ll 1/\tau_{eq}$$

$1/\tau_D$ – "drag rate" $\qquad \rho_D = m/e^2 n \tau_D$ (for $n_1 = n_2 = n$)

$1/\tau_{eq}$ – (smallest) thermalization rate

"***Orthodox theory***" $\leftarrow$ ***totally wrong for forward scattering in 1D***

(right-left thermalization rate $1/\tau_{eq} = 0$***)***

In fact, for forward scattering

$$\rho_D = 0$$

No dc friction between chiral electrons!

Kinetic theory approach to drag

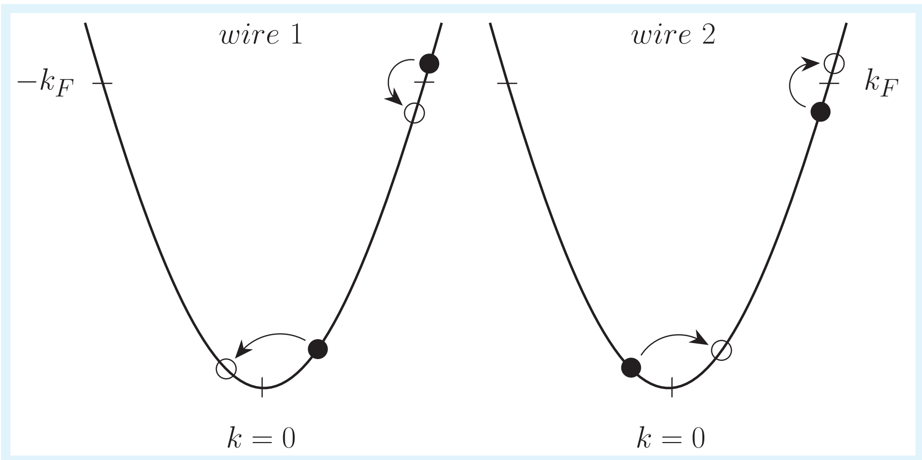
Quadratic dispersion: pair collisions in 1D $\to$ momentum exchange

$$\delta_\epsilon \delta_k = \frac{m}{|k-k'|}\, \delta(k_1 - k_2')\delta(k_2 - k_1')$$

no change in the distribution function $f(k)$ in a single wire

double wire: center-of-mass distribution $f_1(k) + f_2(k) = \text{const.}$

kinetic equation for the relative distribution $f_1(k) - f_2(k)$



$\longleftarrow$ *chiral equilibration*

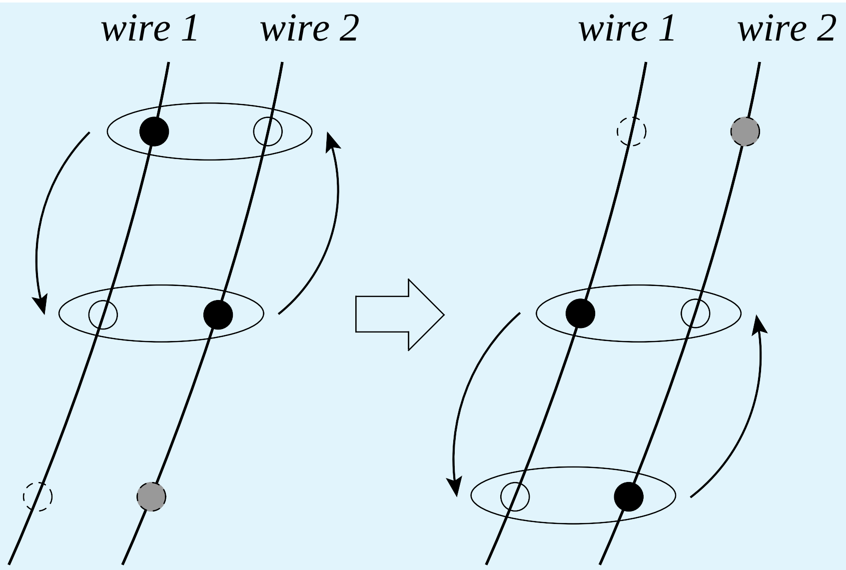
$\longleftarrow$ *backscattering at the bottom*
$\Rightarrow$ *right-left equilibration*

Drag with small momentum transfer: Diffusion in energy space

$T \gg v/d$ $\rightarrow$ Fokker-Planck equation
for the relative distribution $f_{-}(k)$:

current in momentum space $J = -D\partial_k f_{-} + f_{-}\partial_k D$

with the diffusion coefficient $D(k) \propto 1/\cosh^2[(k^2 - k_F^2)/4mT]$



energy space: diffusion of an electron-hole pair (electron in wire 1, hole in wire 2)

at the bottom: $D(0) \propto \exp\left(-\epsilon_F/T\right)$

Drag with small-momentum transfer: Pair collisions

$$\rho_{\mathrm{D}} = \frac{24\pi}{e^2 v_F \tau_{\mathrm{D}}(\infty)} \left(\frac{2\epsilon_F^3}{\pi T^3}\right)^{1/2} e^{-2\epsilon_F/T}$$

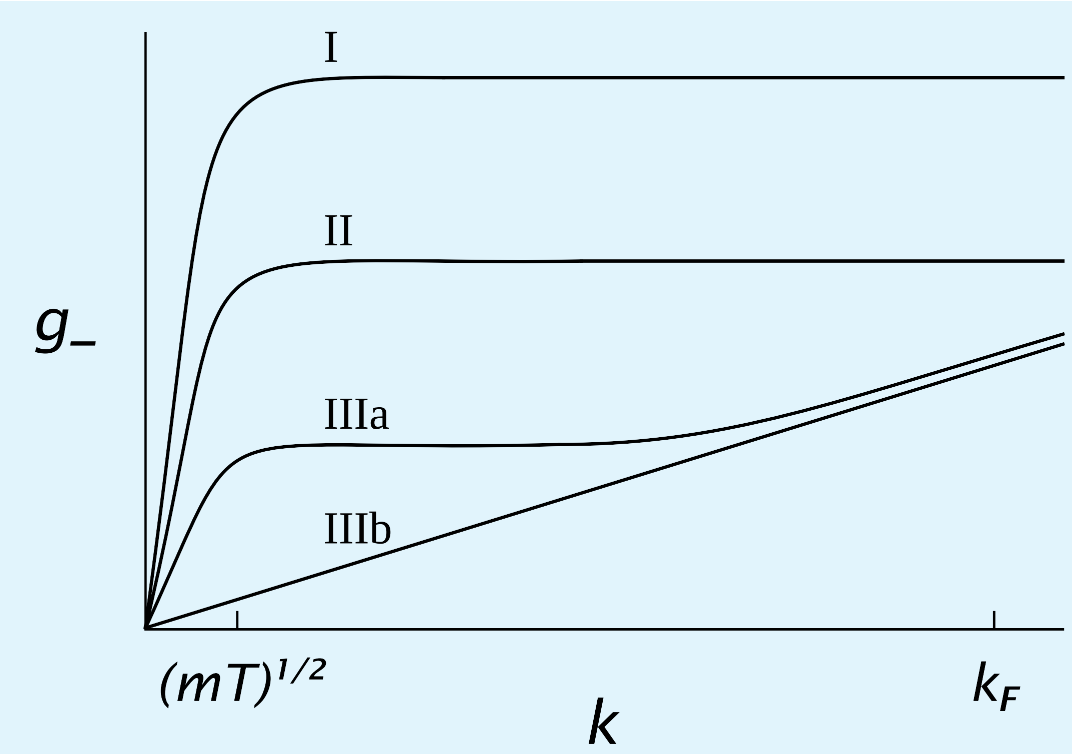
for $v_F/d \ll T \ll \epsilon_F$

Friction due to effective backscattering by diffusion in energy space through the point $k = 0$

No drag without R-L thermalization

For $T \ll v_F/d$: backscattering dominates with $1/\tau_{\mathrm{D}} \propto e^{-4k_F d}$

Thermalization in the moving or stationary frame: The difference matters!



***"step"* in $g_-(k)$, weak drag →**
split chiral chemical potentials,
thermal in the stationary frame

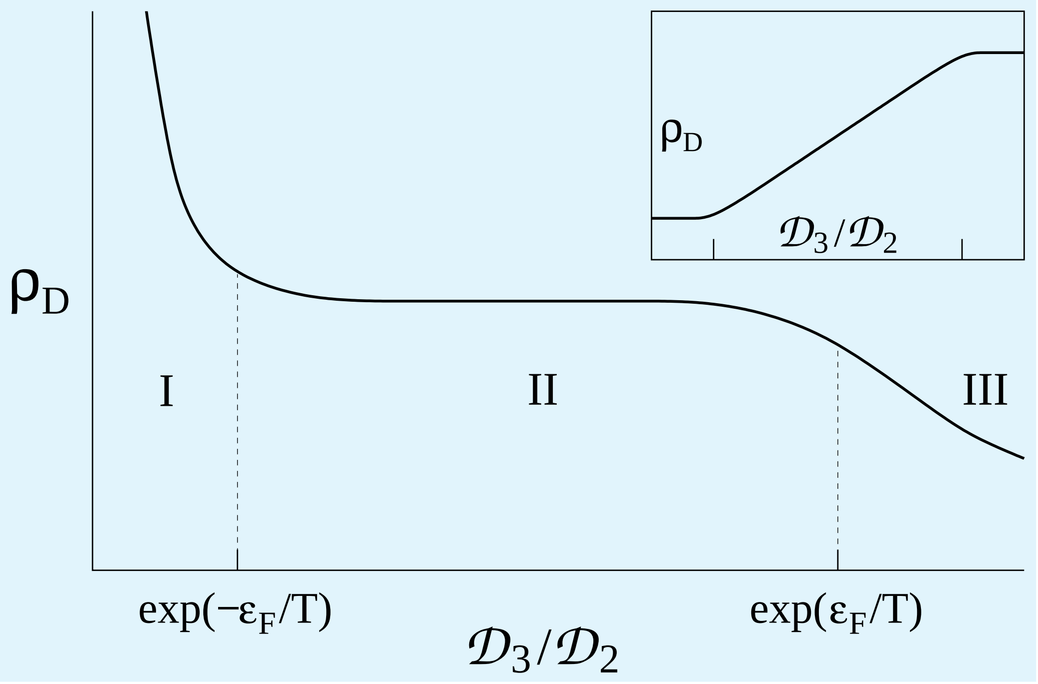
***linear* $g_-(k)$, strong drag →**
drift ansatz,
thermal in the moving frame

$$g_-(k) \;\leftarrow\; \text{measure of nonequilibrium}: \quad f_-(k) = \underbrace{f^T}_{\textit{thermal}} + g_-(k) T \partial_\epsilon f^T$$

In which frame the system chooses to thermalize (how strong drag is) depends crucially on intrawire equilibration (triple collisions) !

Enhancement of drag by intrawire thermalization

$\mathcal{D}_2 \leftarrow$ pair **inter** wire $\quad \mathcal{D}_3 \leftarrow$ triple **intra** wire



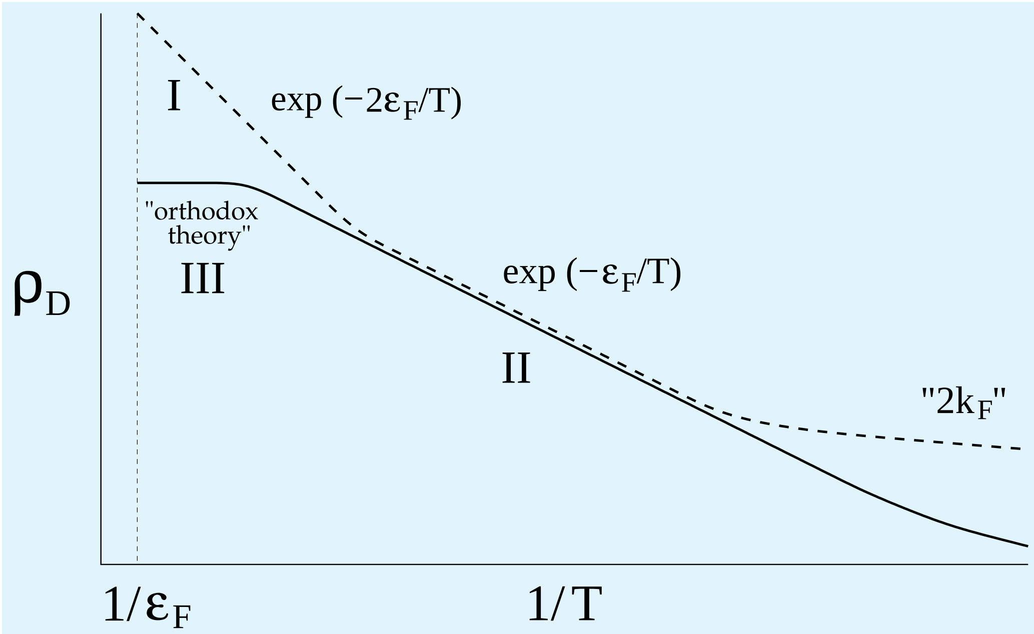
($\mathcal{D}_3/\mathcal{D}_2$ for fixed $\mathcal{D}_3$) = (distance between the wires)

Plateau:

$$\rho_{\mathrm{D}} = \frac{4\pi\mathcal{D}_3}{e^2 k_F}\left(\frac{\epsilon_F}{\pi T^3}\right)^{1/2} e^{-\epsilon_F/T} = \frac{m}{e^2 n} \times \frac{1}{\tau_{\mathrm{eq}}}$$

independent of $\mathcal{D}_2$, i.e., the distance between the wires!

Coulomb drag in quantum wires : Temperature dependence



"orthodox theory" only for a large distance between the wires (solid line) right below ϵ_F

Activation (regime II) : ***independent of the distance between the wires !***

Interaction-induced resistivity of a quantum wire with smooth disorder

- smooth (for electrons) disorder $\rightarrow$ *negligeable backscattering on the Fermi level*

- w/o e-e interactions $\rightarrow$ *zero-width Drude peak: dc* $\rho = 0$

$$\sigma(\omega) = \frac{e^2 v_F}{\pi}\left(1 - \frac{3\langle VV\rangle_{q=0}}{2m^2 v_F^4}\right)\delta(\omega) + \frac{\langle VV\rangle_{q=\omega/v_F}}{2\pi m^2 v_F^4}$$

$\langle VV\rangle_q$ - correlator (at momentum q) of the random potential

- e-e interactions (amplitude α) $\rightarrow$ *golden-rule (high-ω) momentum relaxation rate (friction in the moving frame)*

$$\frac{1}{\tau} = \frac{\alpha^2}{32 k_F^4 T}\int \frac{dq}{2\pi}\frac{q^4}{\sinh^2(v_F q/4T)}\langle VV\rangle_q$$

- dc ρ for pair collisions - ?

$$\rho \propto e^{-\epsilon_F/T} \ll m/e^2 n\tau$$

effective backscattering due to diffusion in energy space

Interaction-induced resistivity of a quantum wire with smooth disorder

disorder-mediated pair collisions $\rightarrow$ momentum relaxation

diffusion coefficient (in energy space) D_2

$$+$$

momentum-conserving triple collisions

diffusion coefficient D_3

nonequilibrium part of the distribution function $g(k) \simeq g_{\rm sf}(k) + g_{\rm mf}(k)$

$g_{\rm sf} \propto {\rm sgn}(k)$ $\leftarrow$ thermalization in the stationary frame

$g_{\rm mf} \propto k$ $\leftarrow$ thermalization in the moving frame

$$\rho^{-1} \simeq \rho_{\rm sf}^{-1} + \rho_{\rm mf}^{-1}$$

$$\rho_{\rm sf}^{-1} \sim e^2 n \epsilon_F \frac{1}{D_2+D_3} e^{\epsilon_F/T} \left(\frac{T}{\epsilon_F}\right)^{3/2} \qquad \rho_{\rm mf}^{-1} \sim e^2 n \epsilon_F \frac{D_3-D_2}{(D_2+D_3)D_2}$$

Interaction-induced resistivity of a quantum wire with smooth disorder

disorder-mediated pair collisions $\rightarrow$ momentum relaxation

diffusion coefficient (in energy space) D_2

$$+$$

momentum-conserving triple collisions

diffusion coefficient D_3

$$\rho \simeq m/e^2 n \tau_{\rm eq}$$

$$D_2 \ll D_3 \ll D_2 \, e^{\epsilon_F/T} \left(\frac{T}{\epsilon_F}\right)^{3/2}$$

ρ independent of disorder !

see also Levchenko,Micklitz,Rech,Matveev '10

Summary

- ***Thermalization-controlled linear transport:***
  Relaxation rate in Ohm's law $\rightarrow$ ***thermalization rate*** $1/\tau_{ee}$

  $$(\,\rho = m/e^2 n\,\tau_{ee}\,)$$

- ***Coulomb drag by small-momentum transfer between ballistic quantum wires:***
  - ***"Orthodox" theory not valid***
  - ρ_D ***independent of the strength of*** ***<u>inter</u>wire-*** ***but strongly dependent on the strength of <u>intra</u>wire interactions***

- ***Resistivity of a smoothly-inhomogeneous quantum wire:***
  - ***independent of the strength of disorder in the thermalization-controlled regime***