Zero-energy vortices in gated graphene

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Quantum transport in 2D systems
Bagnères-de-Luchon, May 2015

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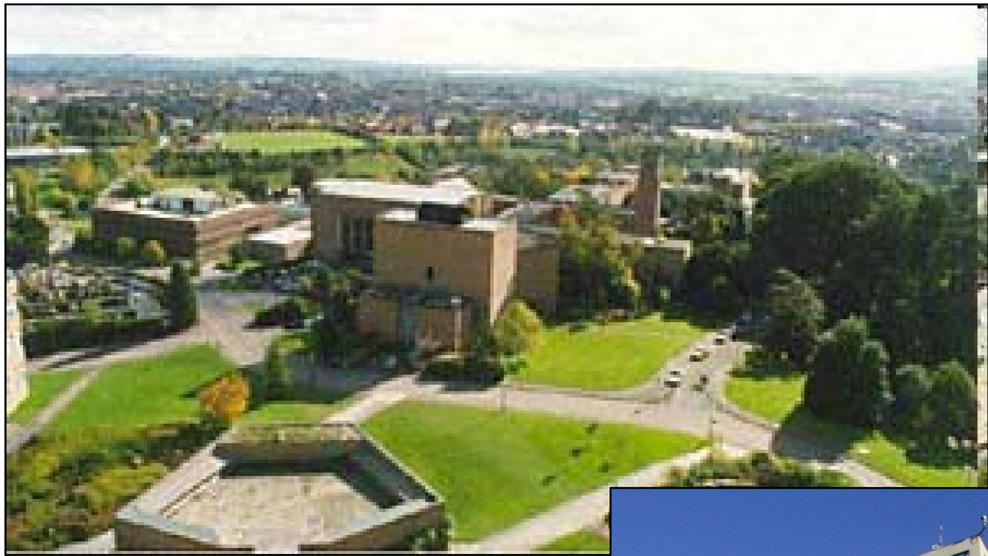
Universidade Federal do Rio Grande do Norte

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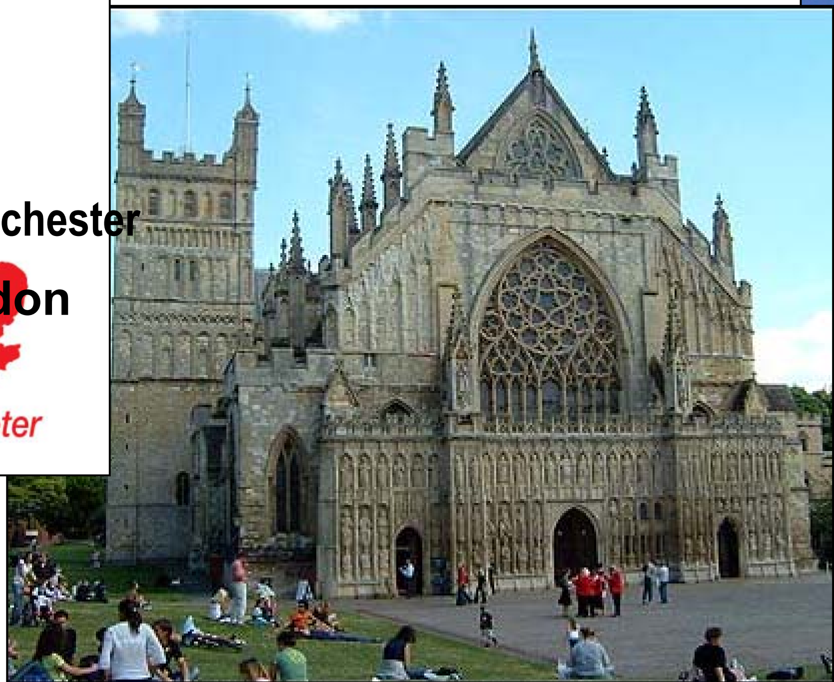
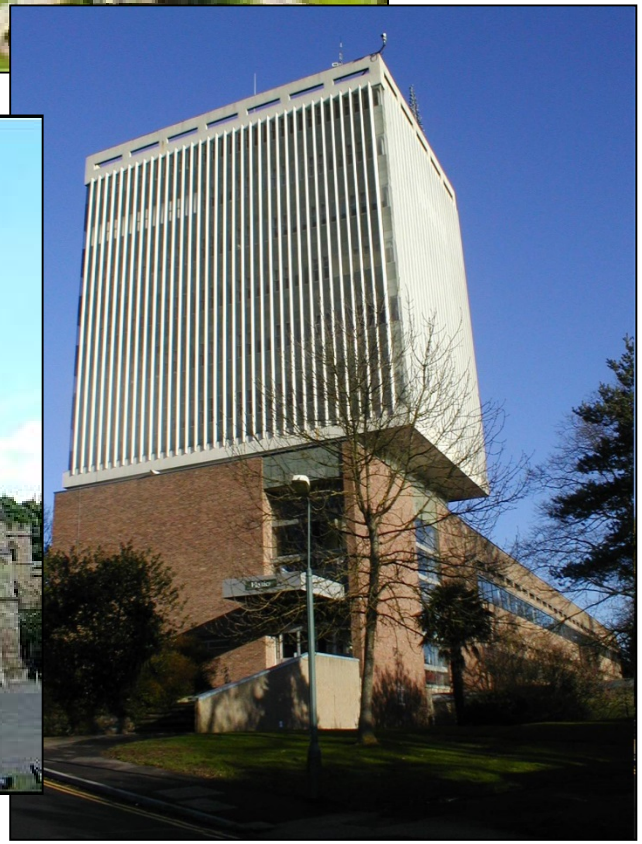
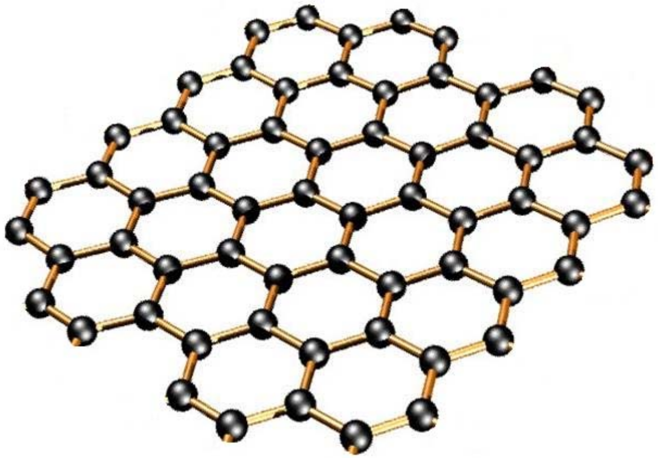
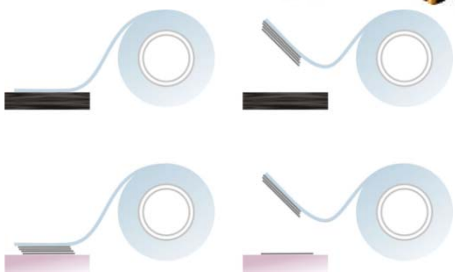
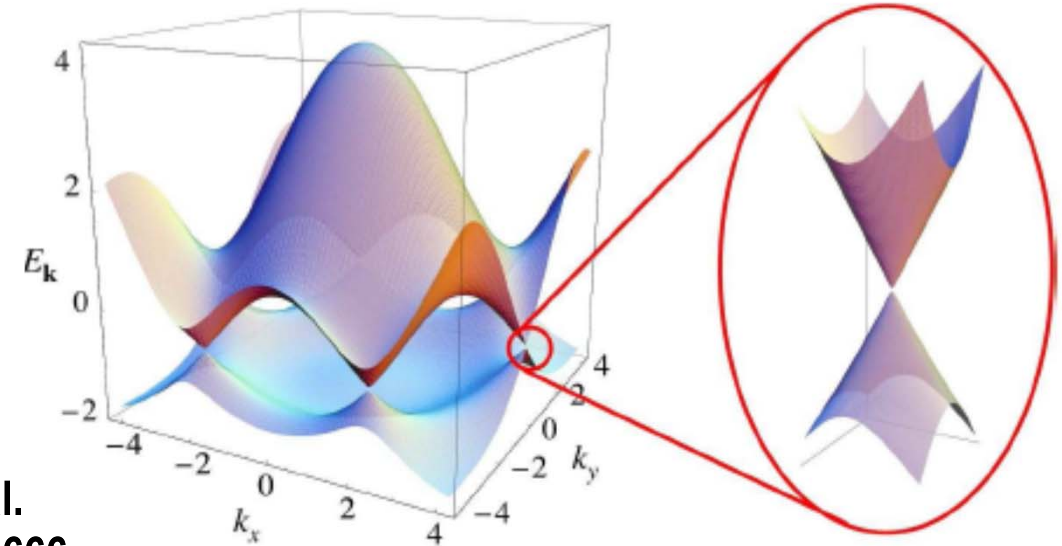








Novoselov et al.
Science 306, 666
(2004)



Graphene dispersion

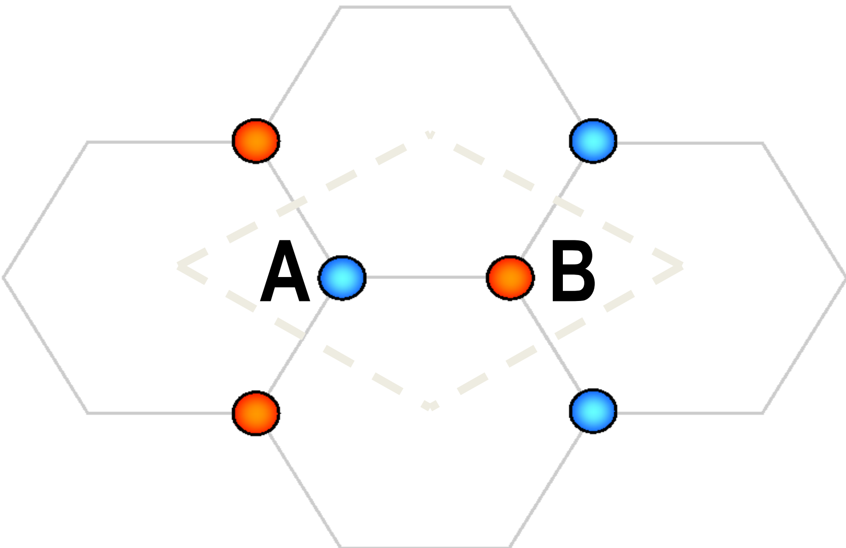
P.R. Wallace, 'The band theory of graphite', Phys. Rev. 71, 622 (1947).

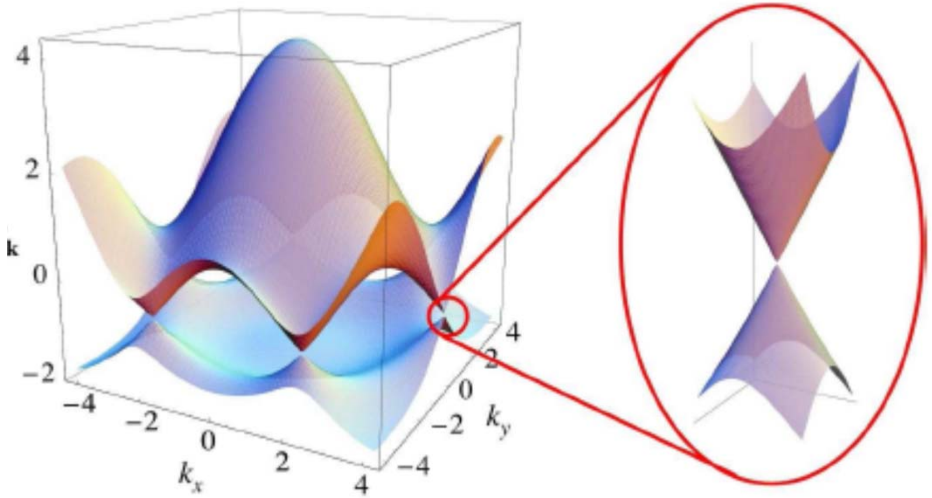
Unconventional QHE; huge mobility (suppression of backscattering), universal optical absorption…

Theory: use of 2D relativistic QM, optical analogies, Klein paradox, valleytronics…

Dispersion Relation $E = \pm\gamma_0\sqrt{|f(\underline{k})|}$

$$f(\underline{k}) = e^{i\left(\frac{a}{\sqrt{3}}k_x\right)} + 2e^{i\left(-\frac{a}{2\sqrt{3}}k_x\right)}\cos\left(k_y\frac{a}{2}\right)$$



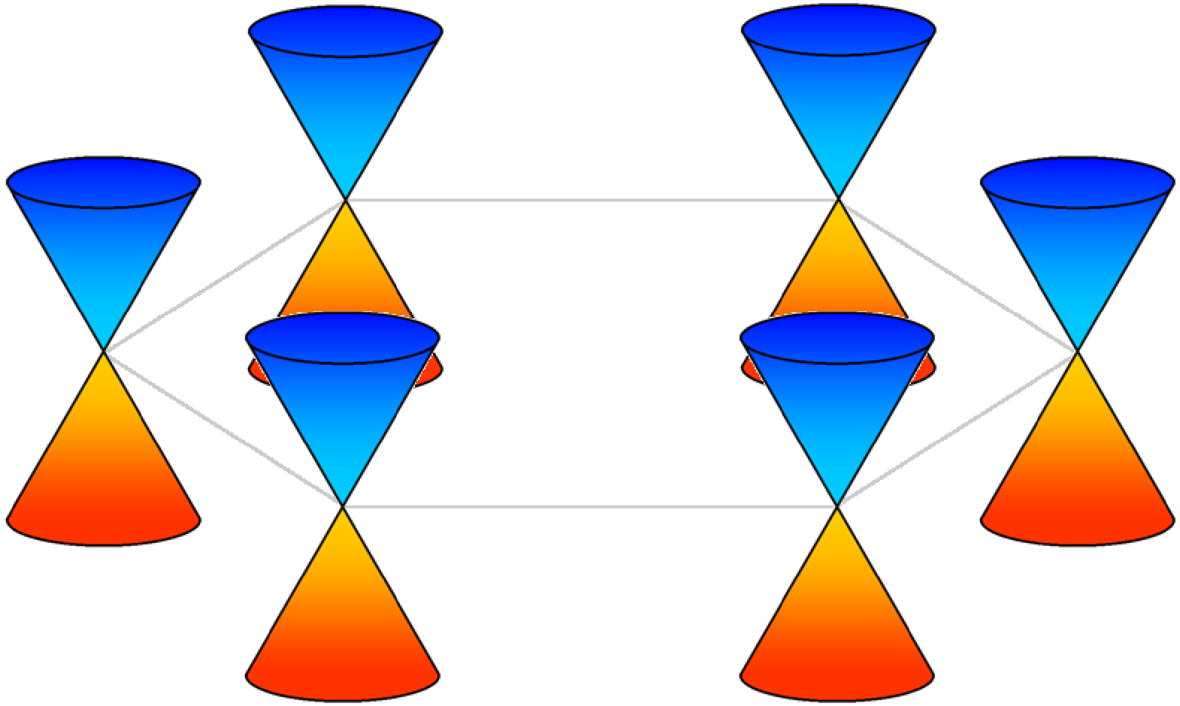


$$E = \pm\hbar v_F|\mathbf{q}| \qquad c/v_F \approx 300$$

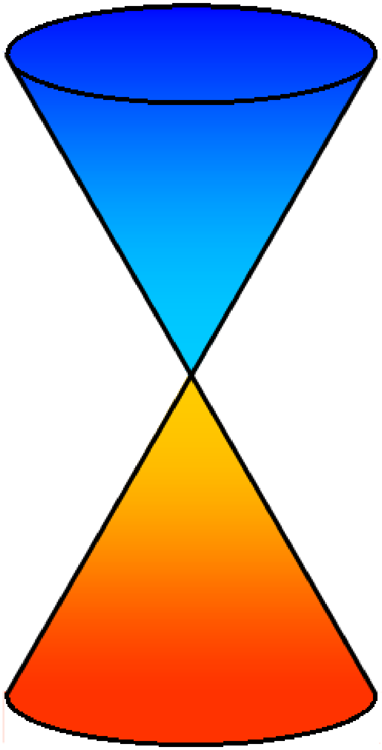
P. R. Wallace “The band theory of graphite”, Phys. Rev. **71**, 622 (1947)

“Dirac Points”

Expanding around the K points in terms of small q



$$\widehat{H} = \hbar v_{\mathrm{F}} \begin{pmatrix} 0 & \widehat{q}_x - i\widehat{q}_y \\ \widehat{q}_x + i\widehat{q}_y & 0 \end{pmatrix}$$



Light-like Dispersion: $E = \pm\hbar v_{\mathrm{F}}|\mathbf{q}|$

Graphene's charge carriers behave in an ultra-relativistic manner.

Optical Analogies

Veselago lens
Goos–Hänchen effect
Fabry-Pérot etalons
Waveguides
Whispering-gallery modes

Fully-confined states in quantum dots and rings

Circularly-symmetric potential $V(r)$

$$\psi(r,\vartheta) = \begin{pmatrix} \mathrm{e}^{im\vartheta}\chi_A(r) \\ \mathrm{e}^{i(m+1)\vartheta}\chi_B(r) \end{pmatrix}$$

$$\begin{pmatrix} V(r) & -i\frac{\partial}{\partial r} - \frac{i(m+1)}{r} \\ -i\frac{\partial}{\partial r} + \frac{im}{r} & V(r) \end{pmatrix} [\chi] = E\,[\chi]$$

$E \neq 0$ -- confinement is not possible for any fast-decaying potential…

“Theorem” - no bound states

$$\hat{H} = v_{\mathrm{F}} \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}} + V(r)$$

$$\left\{-\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{n^2}{r^2} - (E + v_0)^2\right\}\chi_1 = 0, \quad r < a,$$

$$\left\{-\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{n^2}{r^2} - E^2\right\}\chi_1 = 0, \quad r \geq a.$$

Inside well

$$J_n(|E + v_0|r)$$

Outside well

$J_n(|E|r)$ and $N_n(|E|r)$

Equation (20) involves only the square of energy. Therefore, its solutions do not depend on the sign of E. These solutions are equivalent to the scattering states for the ordinary radial Schrödinger equation with $E > 0$. Hence, it follows that, in such a potential well, bound states are absent. We stress that this conclusion does not depend on the depth and width of the well; i.e., a two-dimensional localization of quasiparticles in graphene (quantum dot) is fundamentally impossible (evidently,

With asymptotics

$$J_\alpha(z) = \sqrt{\frac{2}{\pi z}}\left(\cos\left(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + e^{|\mathrm{Im}(z)|}O(|z|^{-1})\right)$$

$$Y_\alpha(z) = \sqrt{\frac{2}{\pi z}}\left(\sin\left(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + e^{|\mathrm{Im}(z)|}O(|z|^{-1})\right)$$

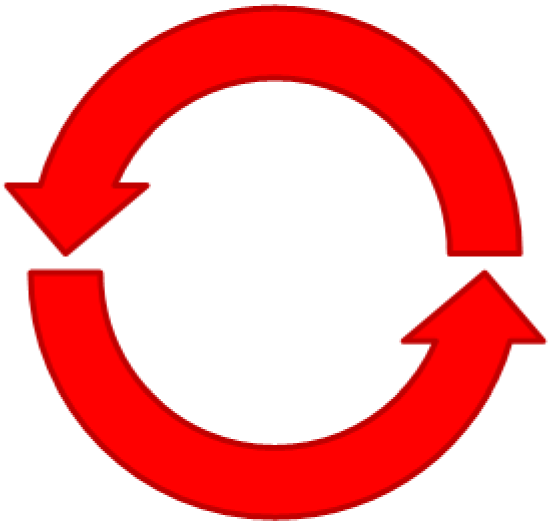
Tudorovskiy and Chaplik, JETP Lett. **84**, 619 (2006)

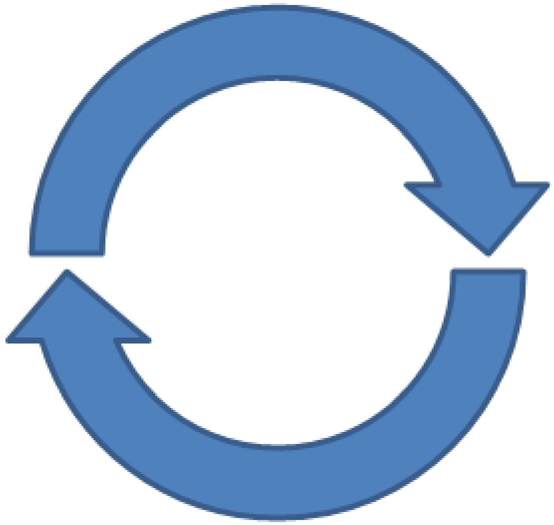
Fully-confined states for $E = 0$

=> DoS(0)≠0

Square integrable solutions require $m > 0$ or $m < -1$

=> vortices!





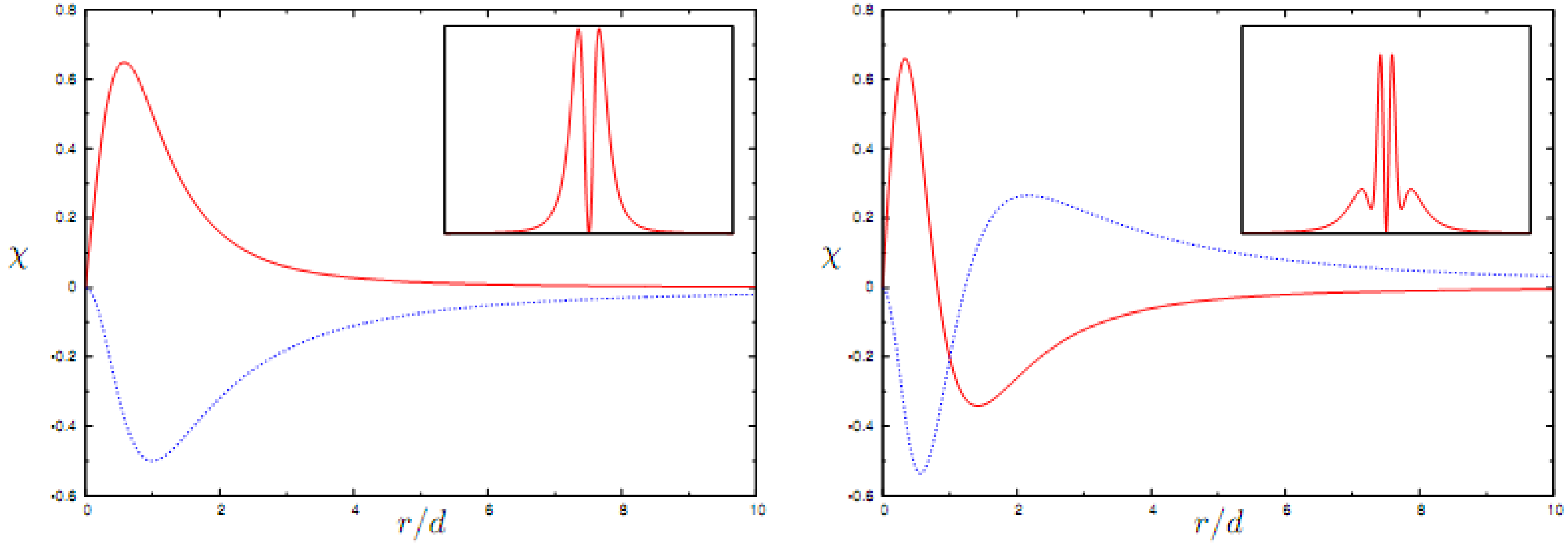
Exactly-solvable potential for $E = 0$

$$V(r) = \frac{V_0}{1 + (r/d)^2}$$

Condition for zero-energy states:

$$V_0 d = \begin{cases} 2(n + m + 1) & m > 0 \\ 2(n - m) & m < -1 \end{cases}$$

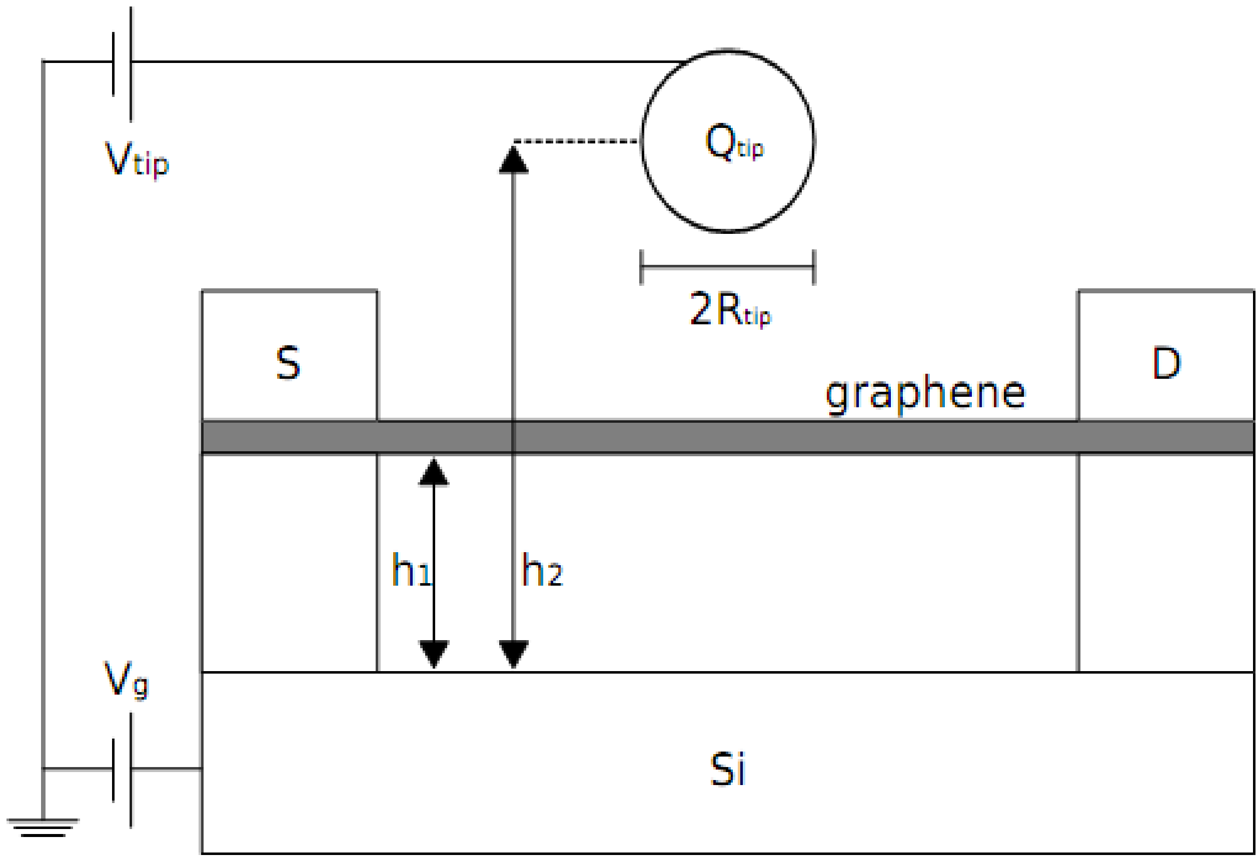
C.A.Downing, D.A.Stone & MEP, PRB **84**, 155437 (2011)



Wavefunction components and probability densities for the first two confined *m*=1 states in the Lorentzian potential

C.A.Downing, D.A.Stone & MEP, PRB **84**, 155437 (2011)

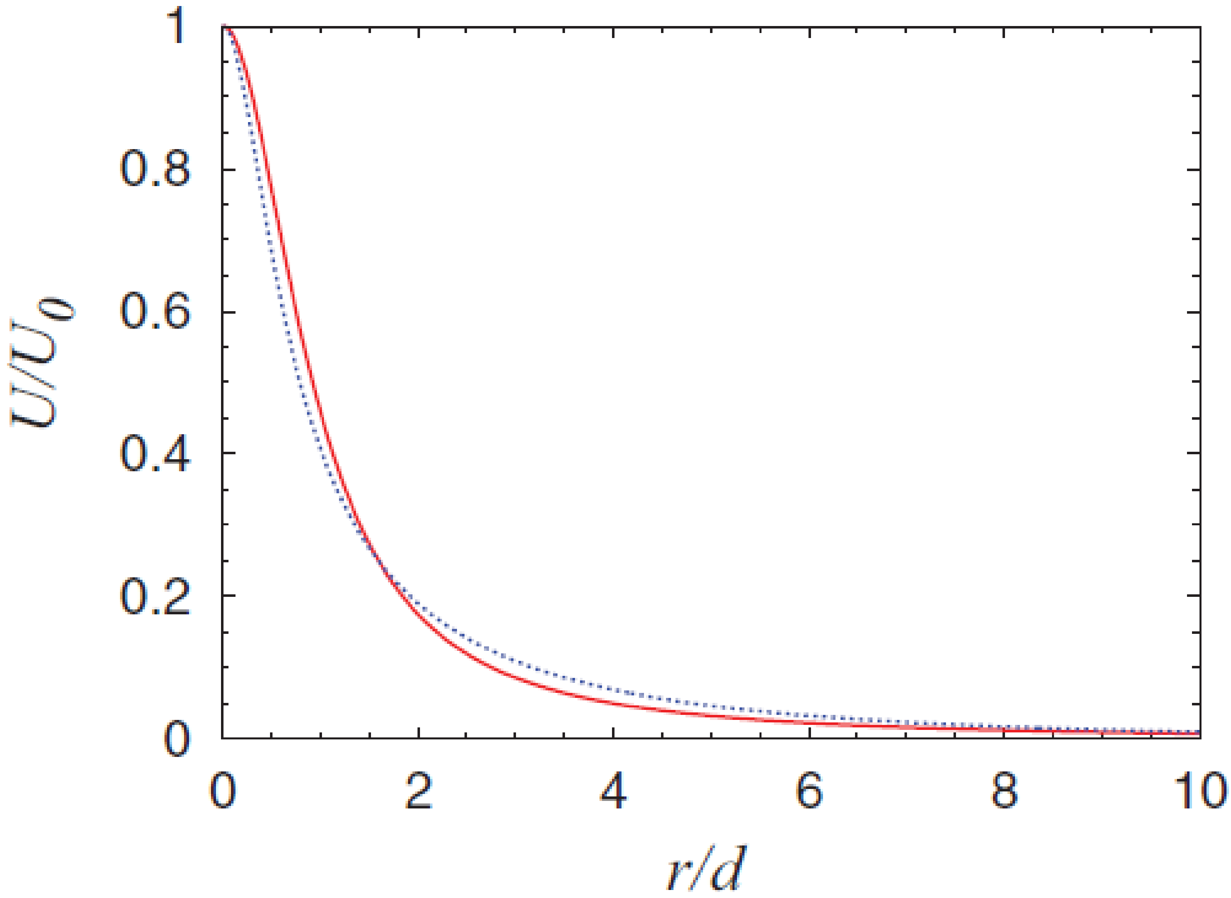
Relevance of the Lorentzian potential



STM tip above the graphene surface

STM tip above the graphene surface

$$U_{\rm STM}(r) \approx \frac{eQ_{\rm tip}}{4\pi\kappa}\left(\frac{1}{\sqrt{r^2+(h_1-h_2)^2}} - \frac{1}{\sqrt{r^2+(h_1+h_2)^2}}\right)$$



$$V_0 = \frac{e\,Q_{\rm tip}}{4\pi\,\kappa\hbar v_{\rm F}}\,\frac{2h_1}{h_2^2-h_1^2}$$

$$d = \frac{h_2^2-h_1^2}{\pi h_1}\ln\left(\frac{h_2+h_1}{h_2-h_1}\right)$$

Coulomb impurity + image charge in a back-gated structure

Exactly-solvable smooth quantum rings

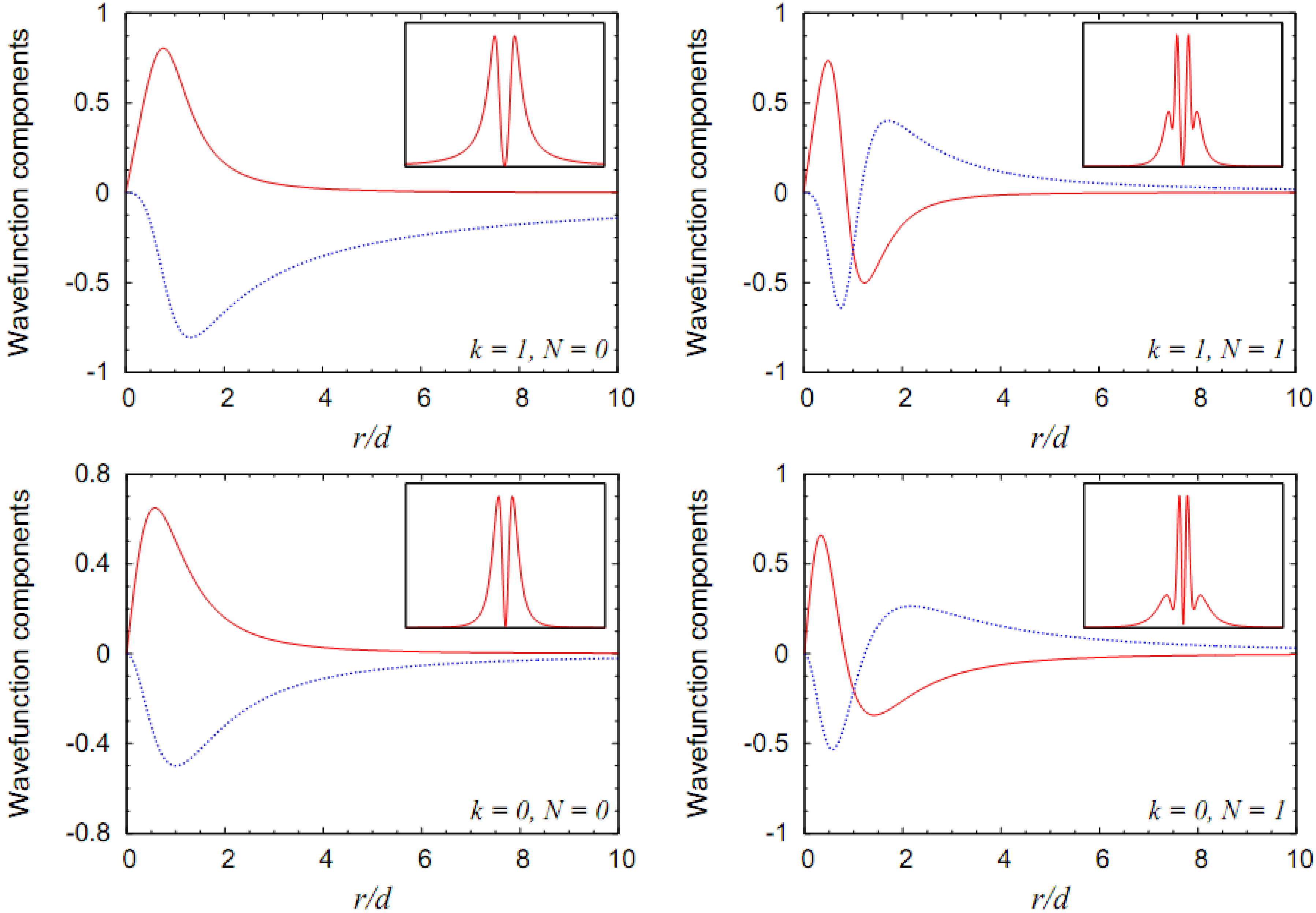
$$V(r) = \frac{V_0 (r/d)^k}{1 + (r/d)^{2(k+1)}}$$

$$\eta(r) = c_1 \; {}_2F_1\left(p_1, -p_1; q_1; \frac{1}{1+\xi^{-1}}\right) + c_2 \; {}_2F_1\left(p_2, -p_2; q_2; \frac{1}{1+\xi}\right),$$

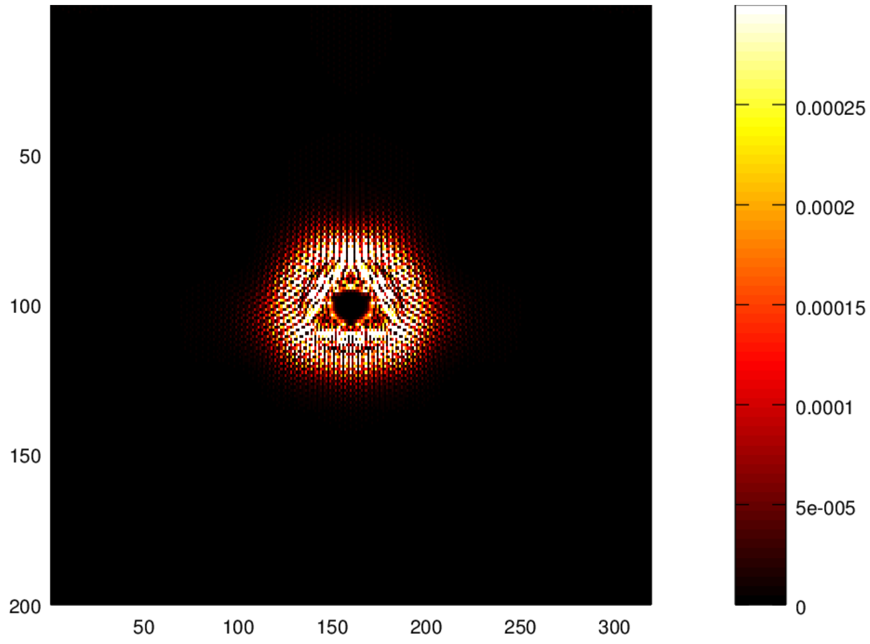
$$q_1 = \frac{k+2m+2}{2k+2}, \qquad q_2 = \frac{k-2m}{2k+2}, \qquad \xi = \left(\frac{r}{d}\right)^{2k+2}.$$

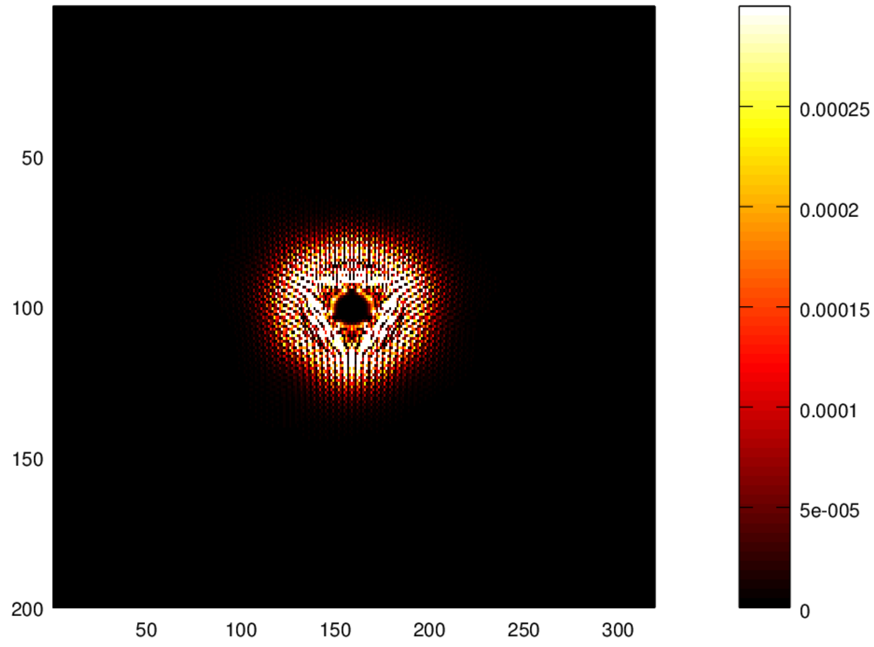
$$p_{1,2} = \frac{V_0 d}{2k+2} = N + q_{1,2}$$ - should be integer.

Exactly-solvable smooth quantum rings



Numerical experiment: 300×200 atoms graphene flake, Lorentzian potential is decaying from the flake center (on-site energy is changing in space)

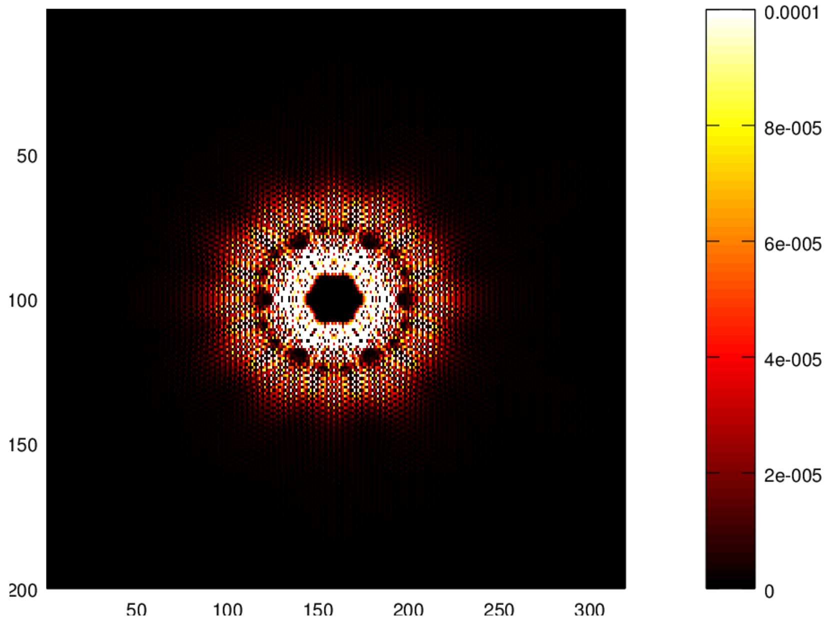




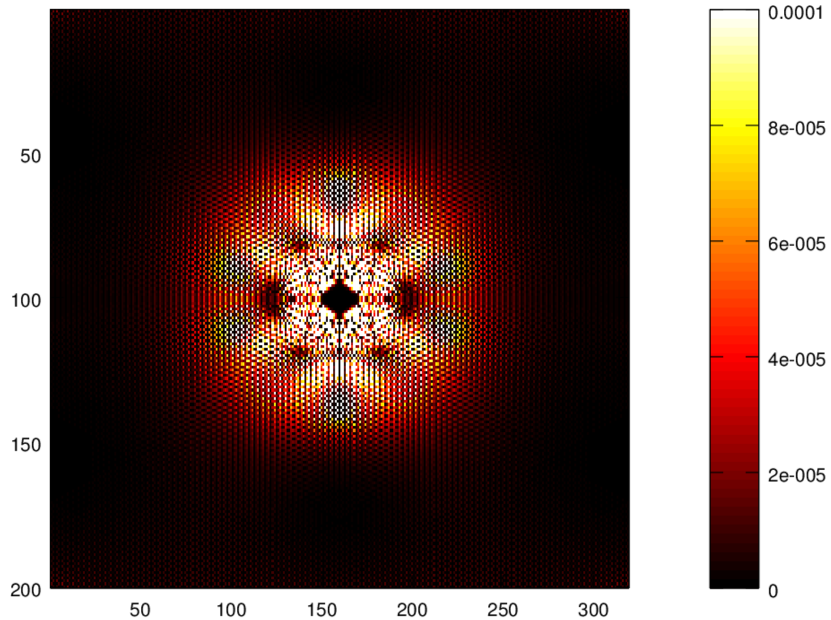
Potential is centred at an "A" atom.

Potential is centred at a "B" atom.

Numerical experiment: 300×200 atoms graphene flake, Lorentzian potential is decaying from the flake center (on-site energy is changing in space)

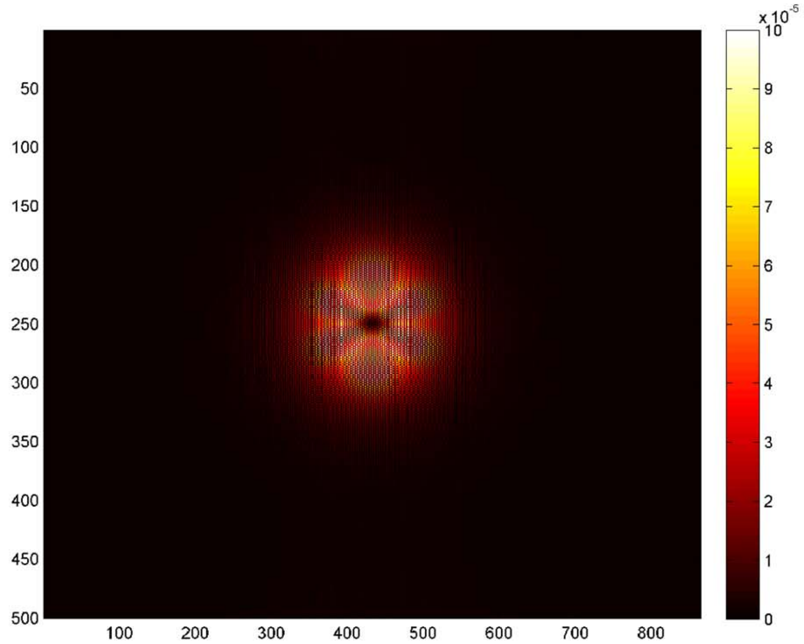


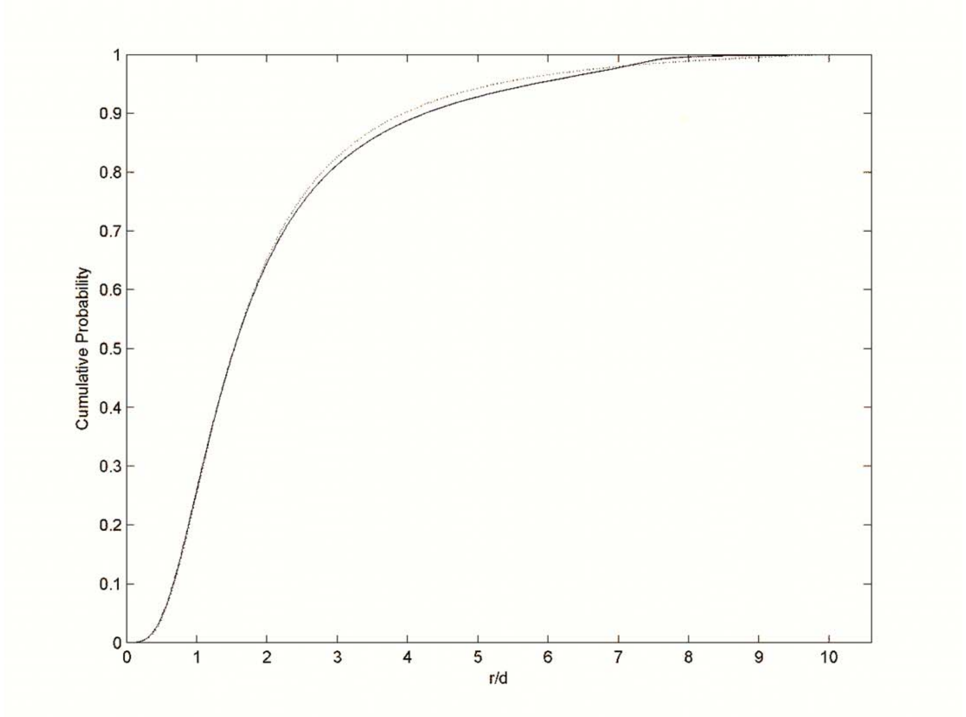
Potential is centred at the hexagon centre.



Potential is centred in the centre of a bond.

Numerical experiment: 300×200 atoms graphene flake, Lorentzian potential is decaying from the flake center (on-site energy is changing in space)





C.A.Downing, D.A.Stone & MEP, arXiv: 1503.08200

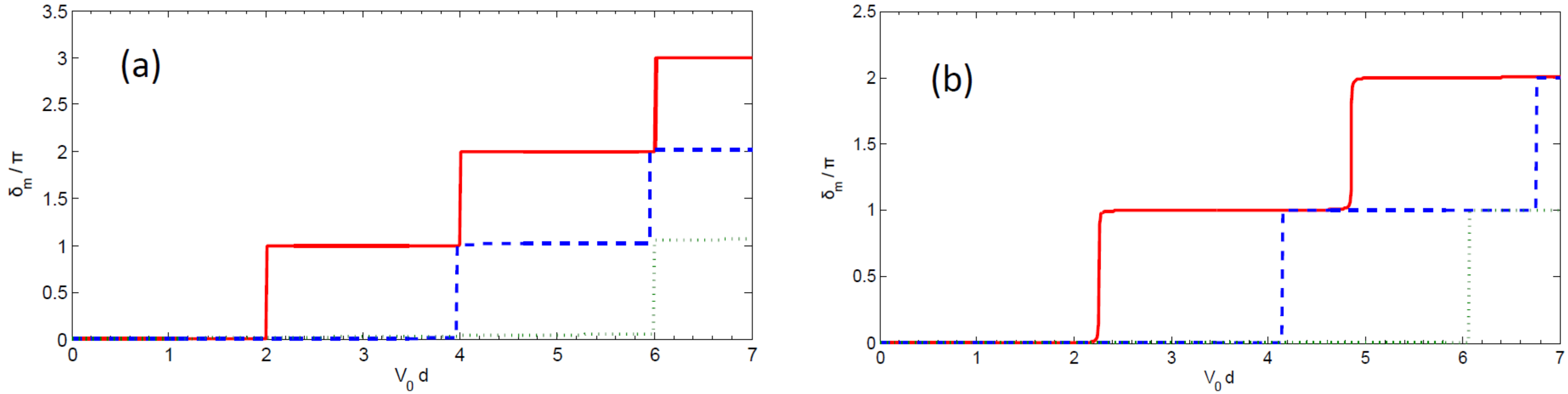
Variable-phase method + Levinson's theorem can be used to find "optimal strength" for any short-range potential

$$\frac{d}{dr}\delta_m(r) = \frac{\pi r}{2}p(r)\left[\frac{1}{k - V(r)}\frac{dV(r)}{dr}\left(q(r) - \frac{m}{r}p(r)\right) + \left(V(r)^2 - 2kV(r)\right)p(r)\right],$$

$$p(r) = J_m(kr)\cos(\delta_m(r)) - N_m(kr)\sin(\delta_m(r)), \qquad \delta_m(0) = 0$$

$$q(r) = J'_m(kr)\cos(\delta_m(r)) - N'_m(kr)\sin(\delta_m(r)), \qquad \delta_m = \lim_{r\to\infty}\delta_m(r)$$

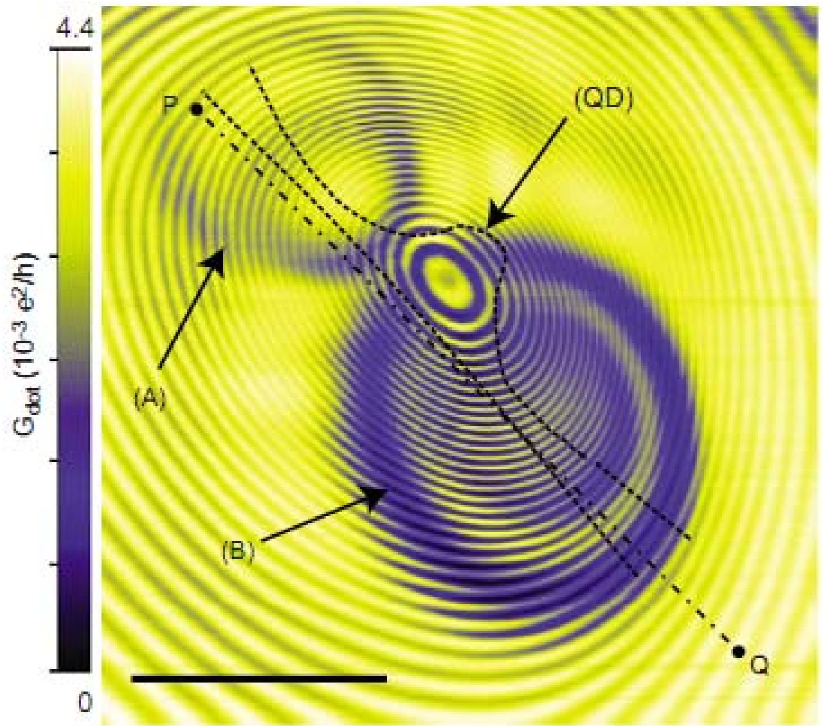
Zero-energy states ($k \to 0$): $\delta_m = n\pi, \qquad n = 1, 2, 3...$



$V(r) = -V_0/[1 + (r/d)^2]$ $\qquad$ $V(r) = -V_0/[1 + (r/d)^3]$

C.A. Downing, A. R. Pearce, R. J. Churchill, and MEP, arXiv:1503.08200

Experimental manifestations



PRB 82, 165445 (2010)
Klaus Ensslin & Co

Experimental manifestations

Creating and probing electron whispering-gallery modes in graphene

Yue Zhao,[1,2*] Jonathan Wyrick,[1*] Fabian D. Natterer,[1*] Joaquin F. Rodriguez-Nieva,[3*] Cyprian Lewandowski,[4] Kenji Watanabe,[5] Takashi Taniguchi,[5] Leonid S. Levitov,[3] Nikolai B. Zhitenev,[1†] Joseph A. Stroscio[1†]

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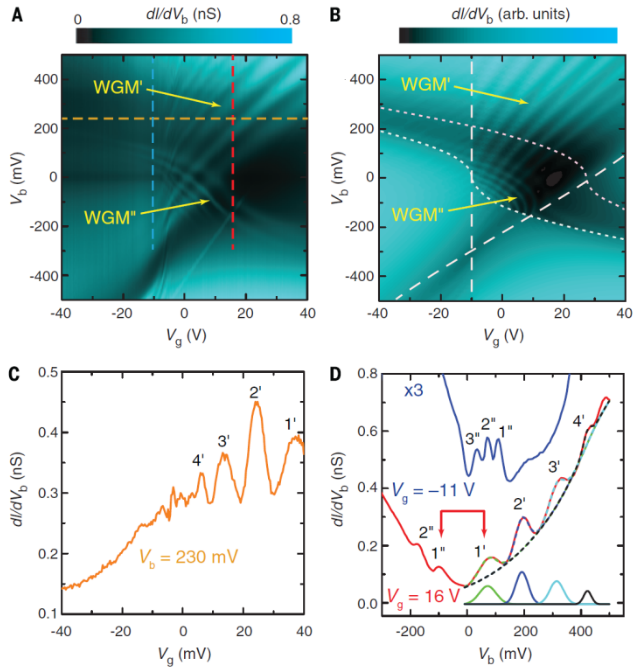
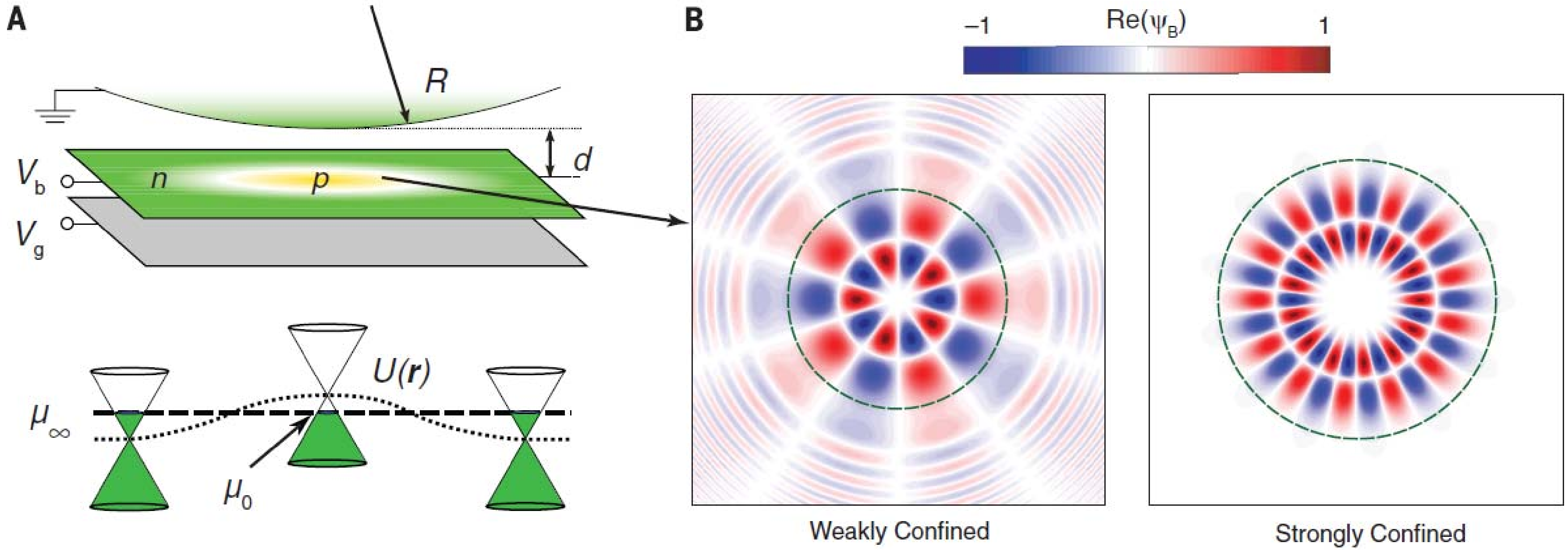




?? $\rho(r) \propto 1/(d + r^2/2R)$

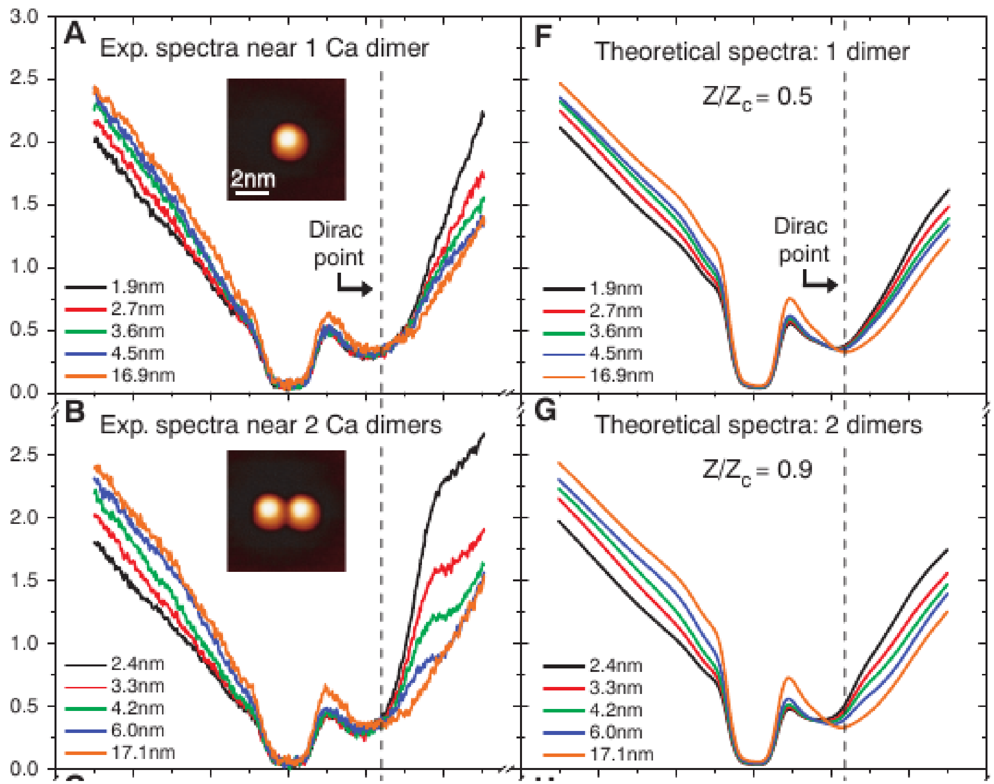
$$\varepsilon u(r) = \left(-i\sigma_x\partial_r + \frac{m+1/2}{r}\sigma_y + \kappa r^2\right)u(r)$$

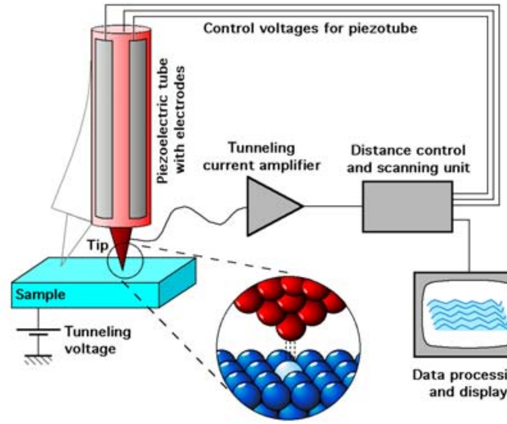
Crommie experiments

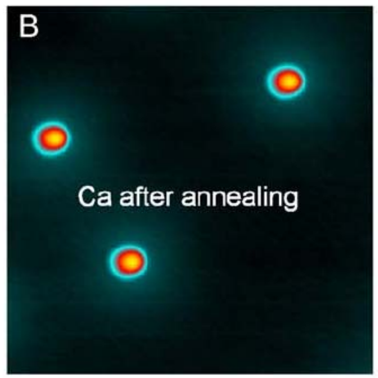
– Ca dimers on graphene have two states, charged and uncharged
– They can be moved around by STM tip, and the charge states can be manipulated
– Thus, one can make artificial atoms and study them via tunneling spectroscopy

Tunneling conductance (DOS)

$$\frac{dI}{dV}$$





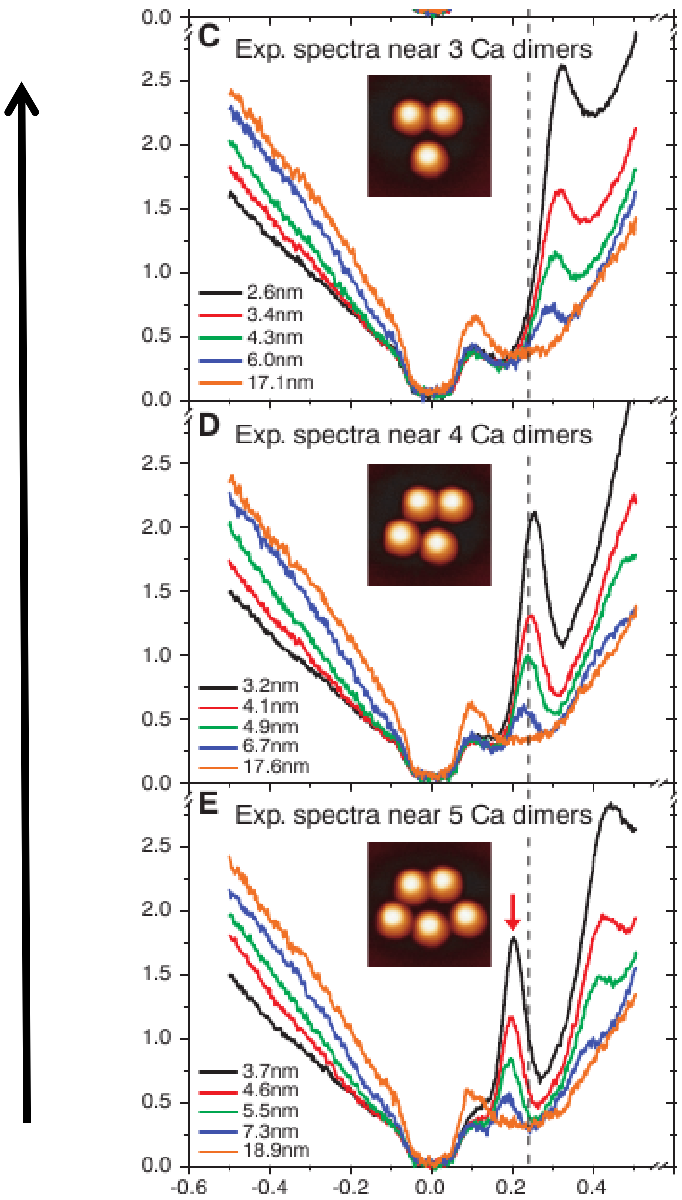


V

Tip-to-sample bias (electron energy)

Crommie group, Science **340**, 734 (2013) + “collapse” theory by Shytov and Levitov

Crommie experiments – atomic collapse theory



Electron density (Gate voltage)

Features **not** explained by the atomic collapse theory

– The resonance is sensitive to doping.

– Sometimes, it occurs on the wrong side with respect to the Dirac point.

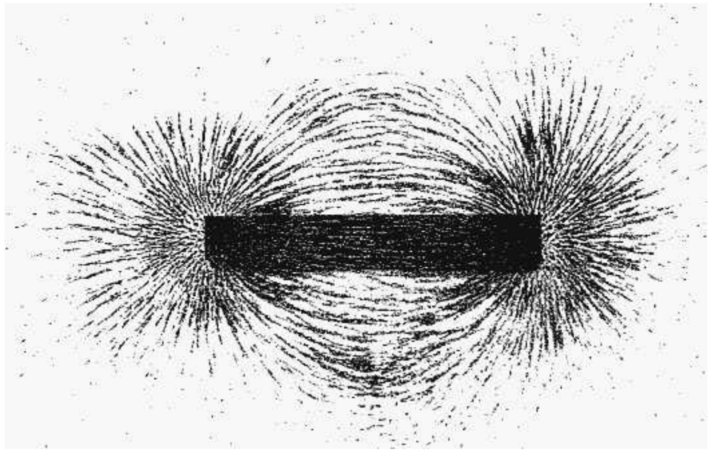
– Distance dependence of peak intensity.

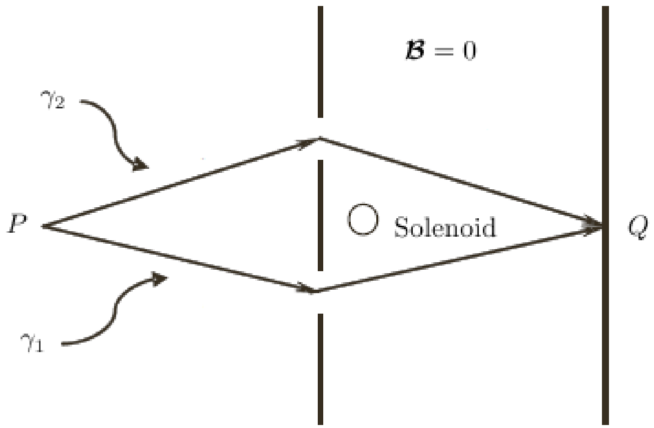
How to combat precise tailoring of potential?

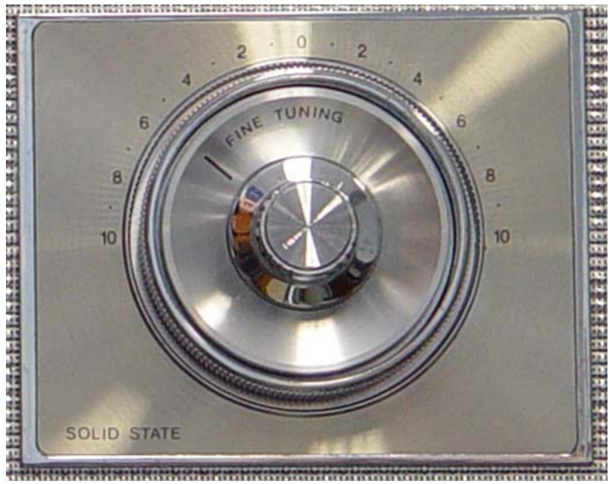
-- What happens to massless Dirac fermions when you add a magnetic flux?

-- Can we get better control of zero-energy bound states?

-- Any interesting physical or mathematical effects?







Adding a magnetic flux

2D-DE $$\hat{H} = v_{\mathrm{F}} \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}} + V(r)$$

Introduce vector potential via modification of momentum $$\hat{\boldsymbol{p}} \rightarrow \hat{\boldsymbol{p}} + e\boldsymbol{A}$$

Choose flux $$A_\theta(r) = \frac{\hbar}{e}\frac{f}{r}, \qquad f = \Phi/\Phi_0 \qquad (\Phi_0 = h/e)$$

Resulting in a relabeling of quantum number

$$m \rightarrow \tilde{m} = m + f$$

Quantum dots with a magnetic flux

$$U(r) = \frac{-U_0}{1 + r^2/d^2}$$

Solutions with short-range asymptotics

$$\chi_A \sim r^{|\tilde{m}|}$$

$$U_0 d = 2n + p, \quad n = 0, 1, 2 \ldots \qquad p = 1 + |\tilde{m}| + |1 + \tilde{m}|.$$

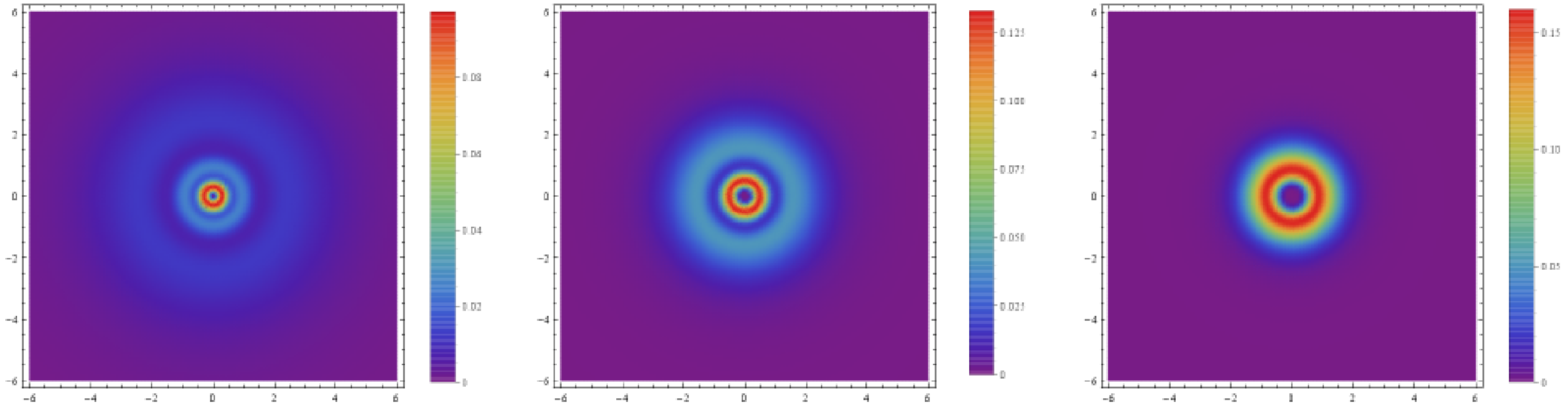
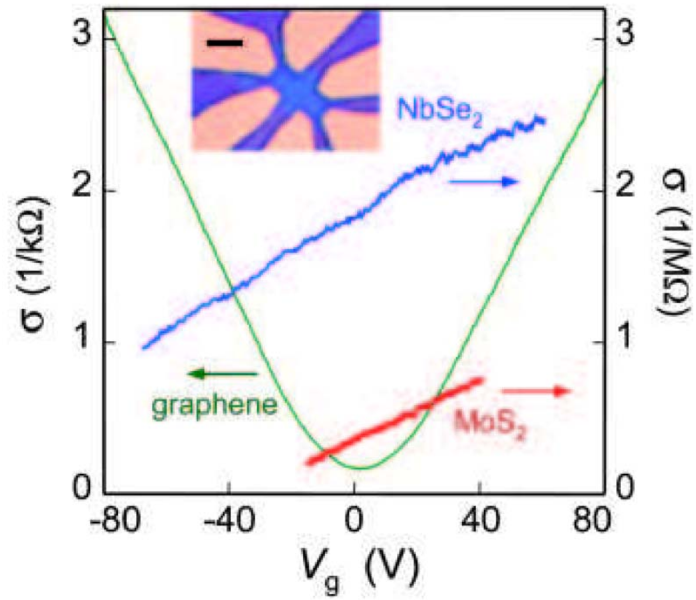


FIG. 3: Plot of density $|\Psi|^2 d^2$ as a function of distance measured in units of d, for a dot with $U_0 d = 8$ and (left-to-right) parameters $(\tilde{m}, n) = (1, 2)$, $(2, 1)$ and $(3, 0)$.

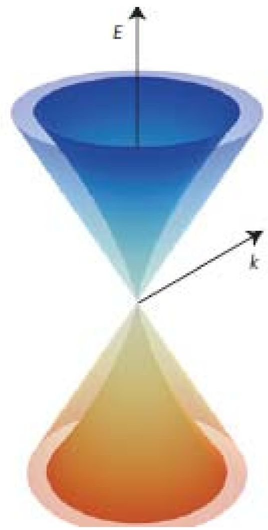
C.A.Downing, K. Gupta & MEP (2014)

Zero-energy states – So what?

- Non-linear screening favors zero-energy states. Could they be a source of minimal conductivity in graphene for a certain type of disorder?
- Could the BEC of zero-energy bi-electron vortices provide an explanation for the Fermi velocity renormalization in gated graphene?
- Where do electrons come from in low-density QHE experiments?



Novoselov et al., PNAS 102, 10451 (2005)



Elias, …, Geim, Nature Physics 7, 201 (2011)

QHE experiments

PRL **111**, 096601 (2013) PHYSICAL REVIEW LETTERS week endi 30 AUGUST

Phase Space for the Breakdown of the Quantum Hall Effect in Epitaxial Graphene

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We report the phase space defined by the quantum Hall effect breakdown in polymer gated epitaxial graphene on SiC (SiC/G) as a function of temperature, current, carrier density, and magnetic fields up to 30 T. At 2 K, breakdown currents (I_c) almost 2 orders of magnitude greater than in GaAs devices are observed. The phase boundary of the dissipationless state ($\rho_{xx} = 0$) shows a $[1 - (T/T_c)^2]$ dependence and persists up to $T_c > 45$ K at 29 T. With magnetic field I_c was found to increase $\propto B^{3/2}$ and $T_c \propto B^2$. As the Fermi energy approaches the Dirac point, the $\nu = 2$ quantized Hall plateau appears continuously from fields as low as 1 T up to at least 19 T due to a strong magnetic field dependence of the carrier density.

Nicholas group, PRL **111**, 096601 (2013)

Also seen by many other groups:
Janssen et. al, PRB **83**, 233402 (2011)
R. Ribeiro-Palau, Nature Comm. (2015)
Benoit Jouault (2011-2015)
A. Lebedev etc…

Apparent difference in carrier densities without B and in a strong magnetic field.
Reservoir of "silent" carriers?

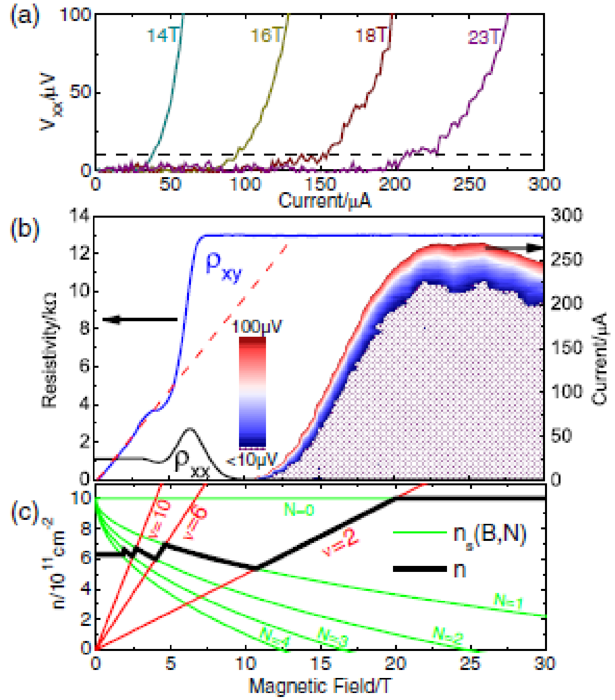


FIG. 1 (color online). (a) $I - V_{xx}$ characteristics of sample 1 at 2.0 K, with a breakdown condition of $V_{xx} = 10\ \mu$V, giving a maximum critical current density $j_c = 43$ A/m at 23 T. (b) Combined magnetotransport [ρ_{xy} and ρ_{xx}] data and $I - V_{xx} - B$ contour plot; the hashed region represents $V_{xx} < 10\ \mu$V, the dissipationless quantum Hall regime. Extrapolating the low field Hall coefficient to $\rho_{xy} = h/2e^2$ (dashed red line) gives the expected field for the peak breakdown of $\nu = 2$ without a field dependent n. (c) Magnetic field dependence of the carrier density (thick black line), following lines of constant filling factor (thin red lines) while E_F lies between Landau levels and then the charge transferred from surface donors in SiC, $n(B, N)$ (thin green curves), while the Landau levels fill, from the model in Ref. [7].

The puzzle of the mass of an exciton in graphene

Excitonic gap & gost insulator (selected papers):

D. V. Khveshchenko, PRL. 87, 246802 (2001).

J.E. Drut & T.A. Lähde, PRL. 102, 026802 (2009).

T. Stroucken, J.H.Grönqvist & S.W.Koch, PRB 84, 205445 (2011).

and many-many others (Guinea, Lozovik, Berman etc…)

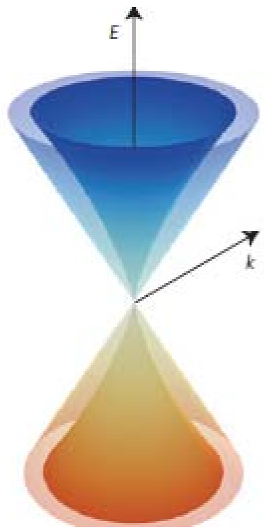
Warping => angular mass: Entin (e-h, K≠0), Shytov (e-e, K=0)

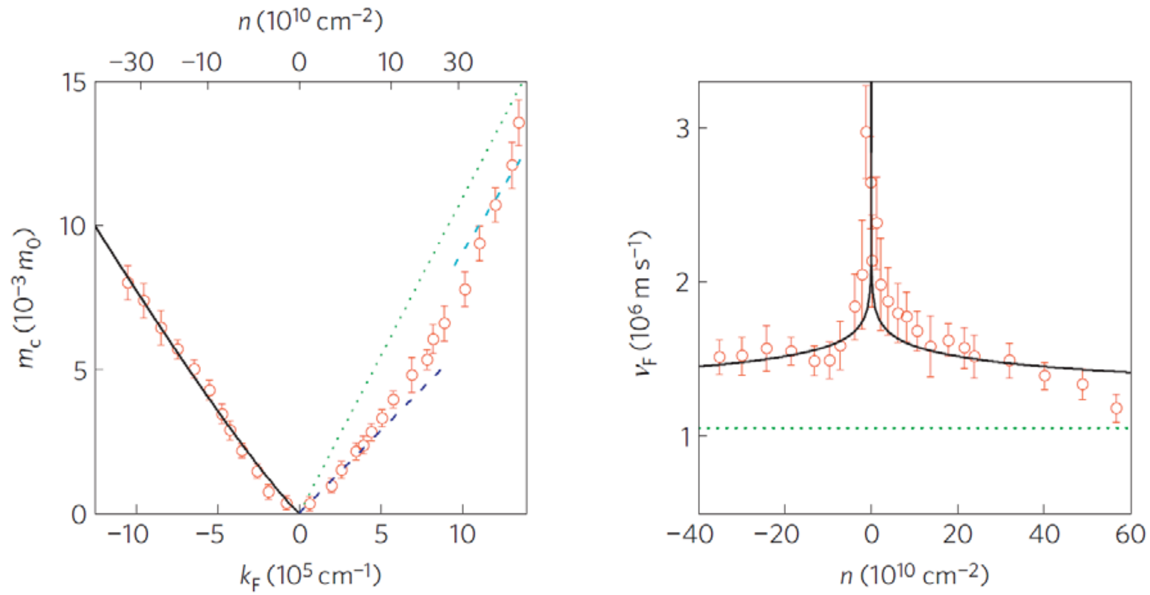
Massless particles do not bind!

Or do they?

Excitonic gap has never been observed!

Experiment: Fermi velocity renormalization...





$$m_c = \hbar(\pi n)^{1/2}/v_F^*$$

?? $$v_F(n) = v_F(n_0)\left[1 + \frac{\alpha}{8\varepsilon_G}\ln(n_0/n)\right]$$

Elias, …, Geim, Nature Physics 7, 201 (2011)
Mayorov et al., Nano Lett. 12, 4629 (2012)

Two-body problem – construction

Construct wavefunction

$$\begin{pmatrix} \psi_{A,A}(\mathbf{r_1},\mathbf{r_2}) \\ \psi_{A,B}(\mathbf{r_1},\mathbf{r_2}) \\ \psi_{B,A}(\mathbf{r_1},\mathbf{r_2}) \\ \psi_{B,B}(\mathbf{r_1},\mathbf{r_2}) \end{pmatrix}$$

Electron-hole

$$H_{e-h} = v_F \begin{bmatrix} V(r) & p_{x_e} - ip_{y_e} & -p_{x_h} + ip_{y_h} & 0 \\ p_{x_e} + ip_{y_e} & V(r) & 0 & -p_{x_h} + ip_{y_h} \\ -p_{x_h} - ip_{y_h} & 0 & V(r) & p_{x_e} - ip_{y_e} \\ 0 & -p_{x_h} - ip_{y_h} & p_{x_e} + ip_{y_e} & V(r) \end{bmatrix}$$

Electron-electron

$$H_{e1-e2} = v_F \begin{bmatrix} V(r) & p_{x_{e1}} - ip_{y_{e1}} & p_{x_{e2}} - ip_{y_{e2}} & 0 \\ p_{x_{e1}} + ip_{y_{e1}} & V(r) & 0 & p_{x_{e2}} - ip_{y_{e2}} \\ p_{x_{e2}} + ip_{y_{e2}} & 0 & V(r) & p_{x_{e1}} - ip_{y_{e1}} \\ 0 & p_{x_{e2}} + ip_{y_{e2}} & p_{x_{e1}} + ip_{y_{e1}} & V(r) \end{bmatrix}$$

Two body problem – free solutions

Diagonalize
$$E = \pm v_F \left(p_{x_1}^2 + p_{y_1}^2\right)^{1/2} \pm v_F \left(p_{x_2}^2 + p_{y_2}^2\right)^{1/2}$$

Centre-of-mass (COM) and relative coordinates
$$X = (x_1 + x_2)/2 \qquad Y = (y_1 + y_2)/2$$
$$x = x_1 - x_2 \qquad y = y_1 - y_2$$

So equivalently
$$E/\hbar v_F = \pm \left((K_X/2 + k_x)^2 + (K_Y/2 + k_y)^2\right)^{1/2}$$
$$\pm \left((K_X/2 - k_x)^2 + (K_Y/2 - k_y)^2\right)^{1/2}$$

COM and relative ansatz
$$\Psi_i(\boldsymbol{R}, \boldsymbol{r}) = \exp(i\boldsymbol{K} \cdot \boldsymbol{R})\psi_i(\mathbf{r})$$

COM momentum K=0,
system reduces to 3 by 3 matrix

$$\begin{bmatrix} \frac{U(r)-E}{\hbar v_F} & \partial_r + \frac{m}{r} & 0 \\ 2\left(-\partial_r + \frac{m-1}{r}\right) & \frac{U(r)-E}{\hbar v_F} & -2\left(\partial_r + \frac{m+1}{r}\right) \\ 0 & \partial_r - \frac{m}{r} & \frac{U(r)-E}{\hbar v_F} \end{bmatrix} \begin{bmatrix} \phi_1(r) \\ \phi_2(r) \\ \phi_4(r) \end{bmatrix} = 0, \quad (4)$$

Two body problem – bound states

$$\begin{bmatrix} \frac{U(r)-E}{\hbar v_F} & \partial_r + \frac{m}{r} & 0 \\ 2\left(-\partial_r + \frac{m-1}{r}\right) & \frac{U(r)-E}{\hbar v_F} & -2\left(\partial_r + \frac{m+1}{r}\right) \\ 0 & \partial_r - \frac{m}{r} & \frac{U(r)-E}{\hbar v_F} \end{bmatrix} \begin{bmatrix} \phi_1(r) \\ \phi_2(r) \\ \phi_4(r) \end{bmatrix} = 0, \quad (4)$$

Only binding at Dirac point energy E=0, consider interaction potential

$$U(r) = -U_0/(1 + (r/d)^2)$$

$$\phi_2(r) = \frac{A}{d} \times \frac{(r/d)^{|m|}}{(1+(r/d)^2)^\eta} f(r),$$

Angular momentum

$$m = 0, \pm 1, \pm 2, \ldots$$

Gauss hypergeometric

$$f(\xi) = {}_2F_1\left(-n, -n + \frac{1}{2}\frac{U_0 d}{\hbar v_F}; |m|+1; \frac{\xi}{1+\xi}\right)$$

useful to define

$$\eta = \frac{|m| + 1 + \sqrt{|m|^2 + 1}}{2},$$

Two-body problem – exactly solvable model

$$\frac{U_0 d}{\hbar v_F} = \frac{300}{137}\frac{1}{\epsilon}\frac{d}{r_0} = 4(n+\eta), \quad n = 0, 1, 2...$$

$$\eta = \frac{|m| + 1 + \sqrt{|m|^2 + 1}}{2}$$

1 - Length scale d of the order of 30 nm due to necessity of gate
2- Cut-off energy depends on geometry and differs strongly for monolayer graphene or interlayer exciton in spatially separated graphene layers
3-Results do not depend on the sign of the interaction potential

1. Monolayer vortex
Cut-off comes from Ohno strength of 11.3 eV, thus r0 = 0.04nm

$$U_0 = \frac{e^2}{4\pi\epsilon_0\epsilon} r_0^{-1}$$

2. Interlayer exciton
Cut-off comes from interlayer spacing of r0 = 1.4nm

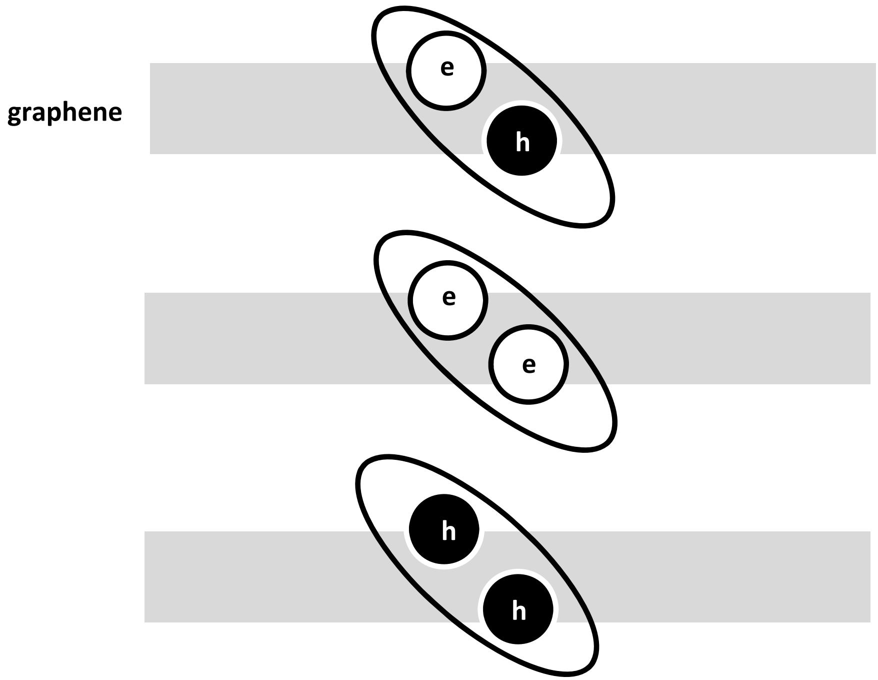
Nb assuming BN with relative permittivity of ϵ = 3.2

Exactly solvable model – two systems

1. Monolayer exciton or e-e pair

U0d = 515.39…

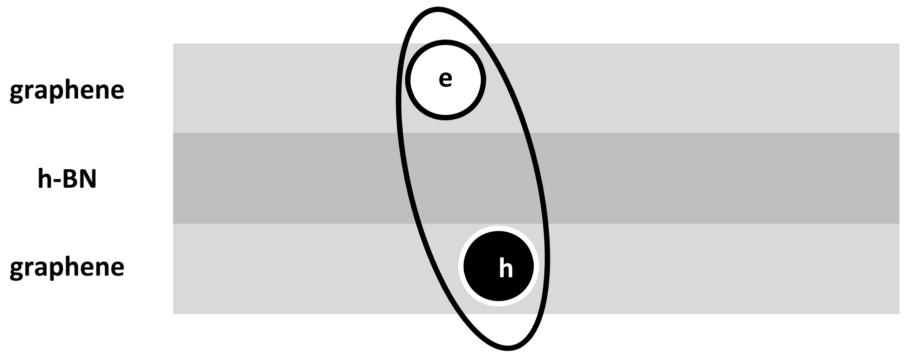
(m, n) = (128, 0), size <r> = 1.006 d
(m, n) = (127, 1), size <r> = 1.018 d
(m, n) = (126, 2), size <r> = 1.030 d



2. Interlayer exciton

U0d = 14.66…

(m, n) = (3, 0), size <r> = 1.433 d
(m, n) = (2, 1), size <r> = 2.639 d
(m, n) = (1, 2), size <r> = 4.415 d



Two-body problem – exactly solvable model

Results for d =100 nm, monolayer graphene, repulsive interaction

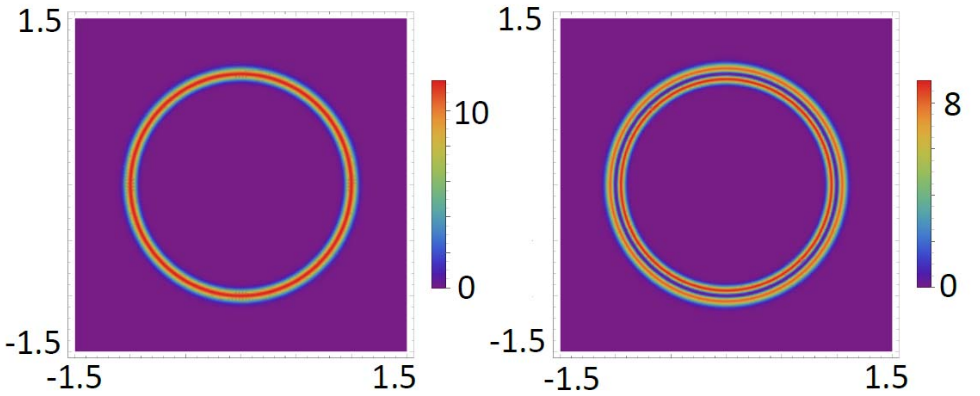
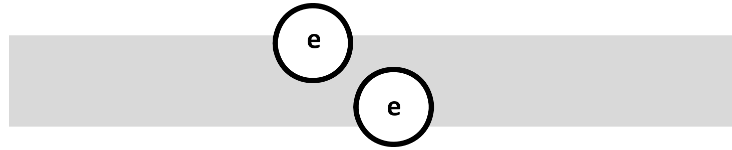
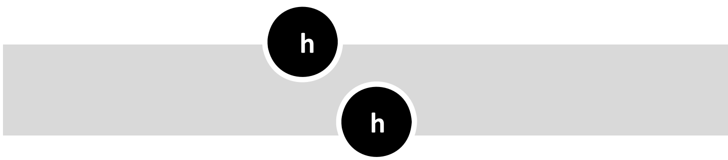


FIG. 1: Radial probability densities for the pair states with quantum numbers $(n, m) = (0, 429), (1.428)$ left-to-right.





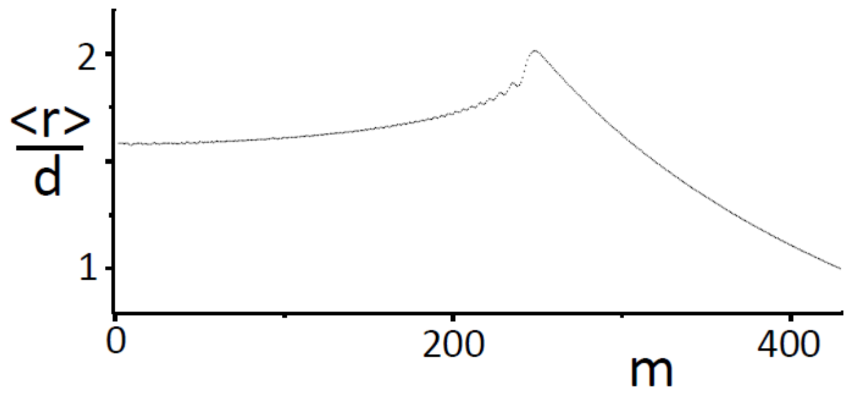


FIG. 2: A plot of the average size of the pair state as a function of quantum number m.

C.A.Downing & MEP (2015)

Numerics – expansion in Fourier-Bessel series

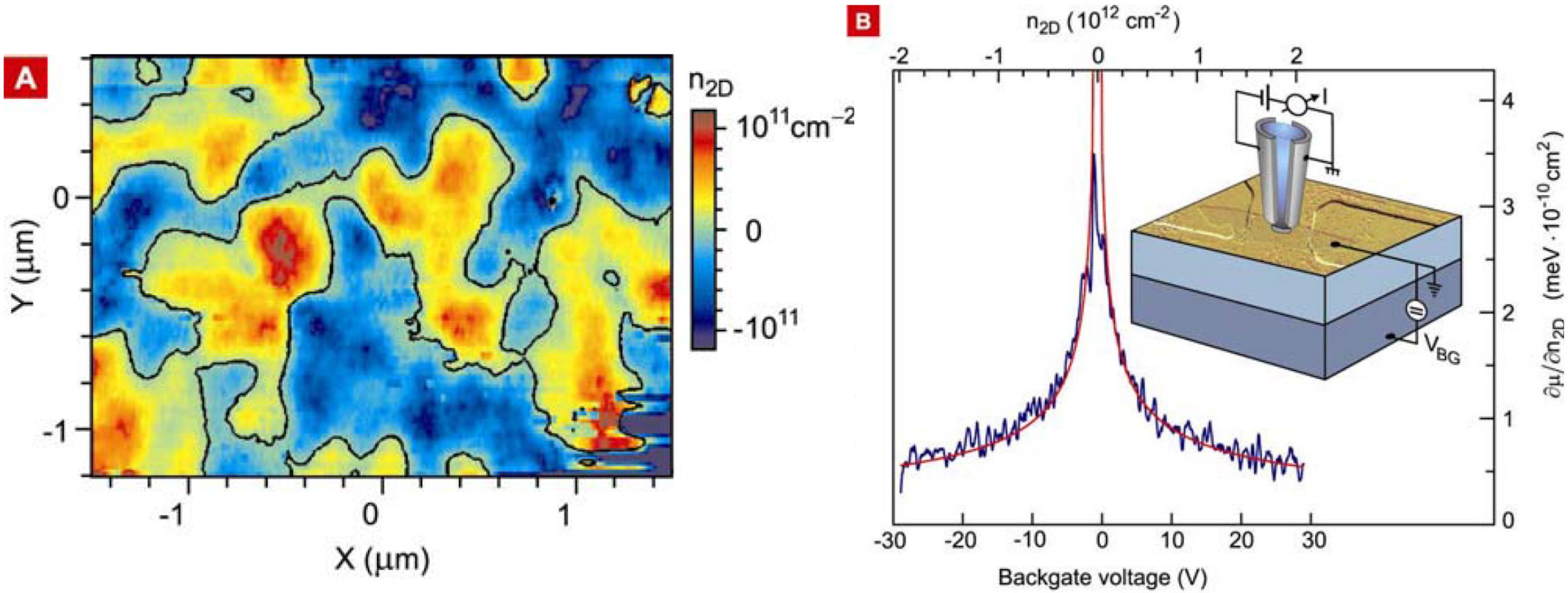
When K=0, E=0, one can reduce the problem to a single differential equation in one of the four wavefunction components, which can be solved by expanding in a Fourier-Bessel series

$$\phi_2(r) = \frac{\sqrt{2}}{L} \sum_{n=1}^{\infty} \frac{a_n}{J_{m+1}(\alpha_n)} J_m(\alpha_n \tfrac{r}{L}),$$

where α_n are roots of the first Bessel function and L is a large distance over which we satisfy orthonormality

To find the parameters of the potential required for the existence of zero-energy states, one needs to solve the resulting secular equation

Electron-hole puddles in disordered graphene or droplets of two-particle vortices?

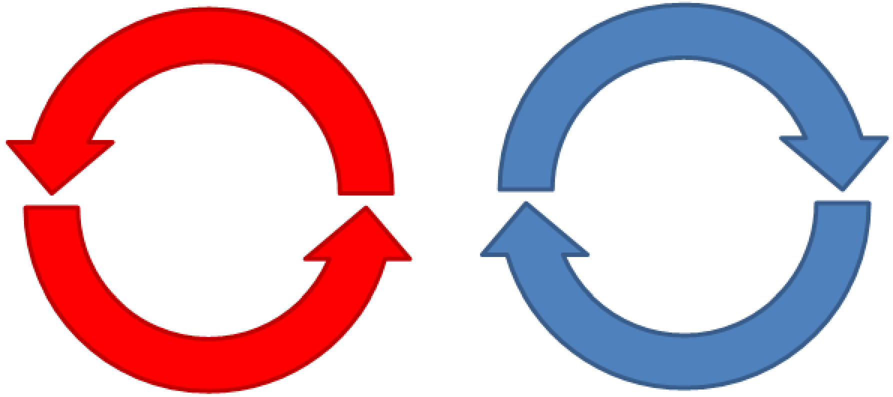


J. Martin, N. Akerman, G. Ulbricht, T. Lohmann, J. H. Smet, K. von Klitzing & A. Yakobi, *Observation of electron–hole puddles in graphene using a scanning single-electron transistor*, **Nature Physics** 4, **144 (2008)**
[cited by over a 1000]

Is it a step in the on-going search for Majorana fermions in condensed matter systems?

Ettore Majorana
1906 - ?

"Majorana had greater gifts than anyone else in the world. Unfortunately he lacked one quality which other men generally have: plain common sense." (E. Fermi)



Practical applications?

APPLIED PHYSICS LETTERS **104**, 161116 (2014)

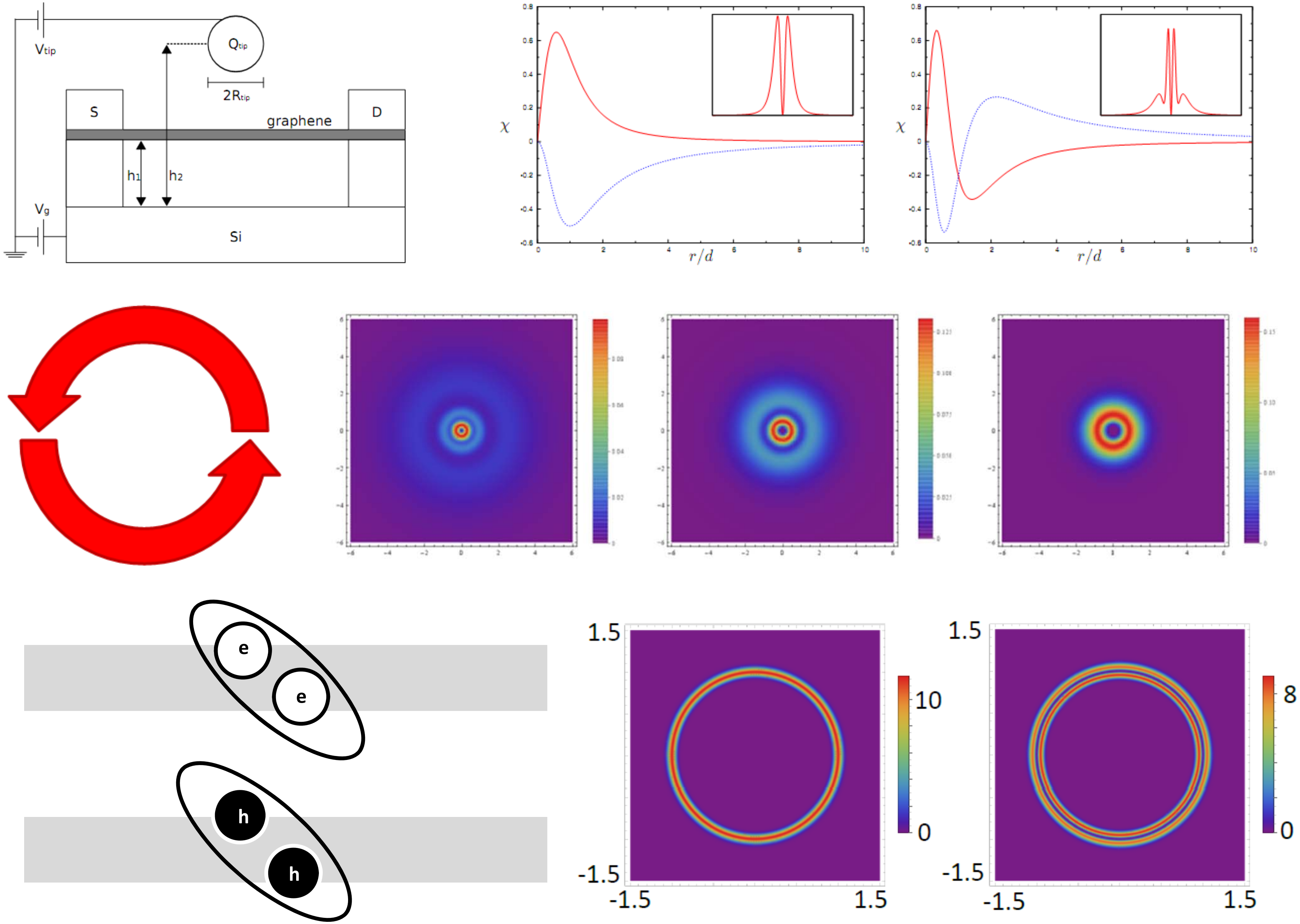
Graphene—A rather ordinary nonlinear optical material

J. B. Khurgin[a)]
Johns Hopkins University, Baltimore, Maryland 21218, USA

Essentially, all the electrons residing further away from the Dirac point hardly contribute to nonlinearity at all, but they are needed to keep the Fermi level high enough to mitigate the loss due to band-to-band absorption. Clearly, absence of a real bandgap (rather than one induced by blocking the absorption) is a handicap that seems to have no remedy.

Remedy – a reservoir of charged vortices at the Dirac point.

Highlights



V_tip
Q_tip
2R_tip
S
D
graphene
h1
h2
V_g
Si
χ
r/d
e
e
h
h
1.5
-1.5
1.5
-1.5
10
0
1.5
-1.5
1.5
-1.5
8
0

Highlights

Contrary to the widespread belief electrostatic confinement in graphene and other 2D Weyl semimetals is indeed possible!

Several smooth fast-decaying potential have been solved exactly for the 2D Dirac-Weyl Hamiltonian.

Precisely tailored potentials support zero-energy states with non-zero values of angular momentum (vortices). The threshold in the effective potential strength is needed for the vortex formation.

An electron and hole or two electrons (holes) can also bind into a zero-energy vortex reducing the total energy of the system.

The existence of zero-energy vortices explains several puzzling experimental results in gated graphene.

Confined modes might also play a part in minimum conductivity (puddles)?

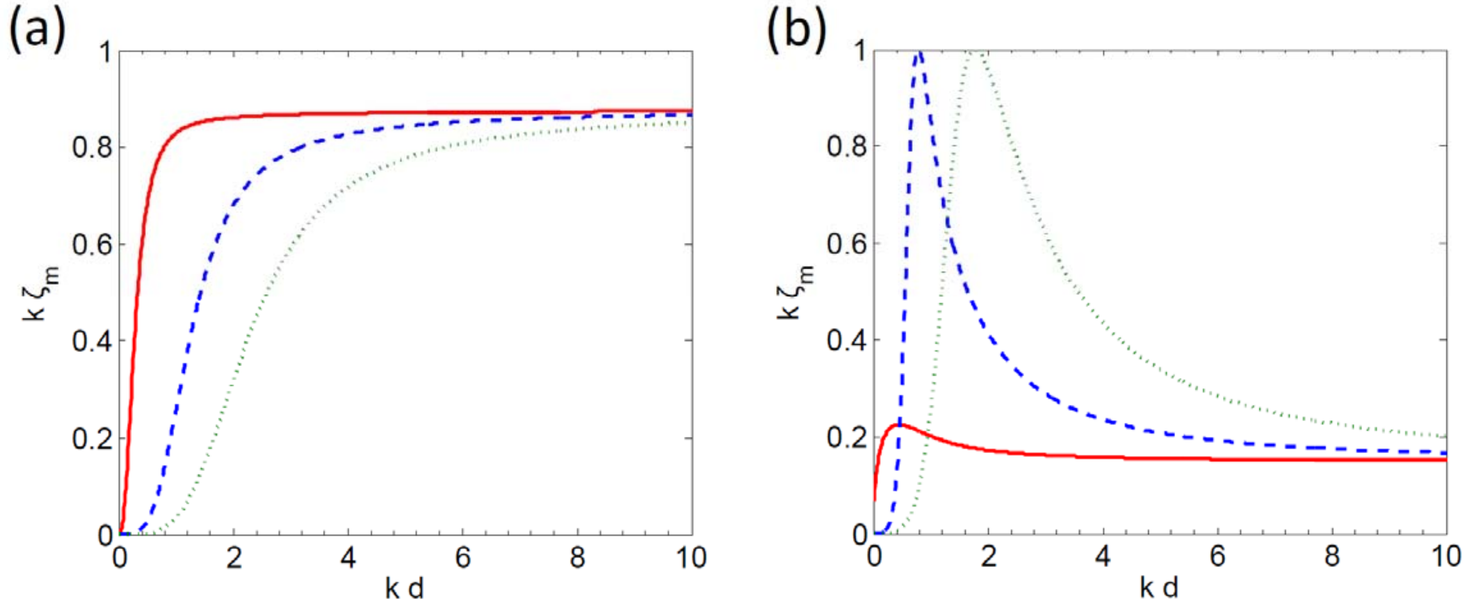
Charles Downing
& Dave Stone

Robin Churchill
& Drew Pearce

Variable-phase method: Scattering cross-sections

$$\zeta = \frac{4}{k} \sum_{m=-\infty}^{\infty} \sin^2(\delta_m), \quad \zeta_T = \frac{2}{k} \sum_{m=-\infty}^{\infty} \sin^2(\delta_{m+1} - \delta_m)$$

$$k\zeta_m = \sin^2(\delta_m) \quad \text{for} \quad V(r) = \frac{-V_0}{1 + (r/d)^3}$$



(a) $V_0 d = 1.00$ and critical (b) $V_0 d = 2.27$. We show results for $m = 0, 1, 2$, corresponding to the solid line (red), dashed line (blue) and dotted line (green) respectively.

C.A. Downing, A. R. Pearce, R. J. Churchill, and MEP, arXiv:1503.08200