



# Electrostatic control of spin polarization in a quantum Hall ferromagnet: a platform to realize high order non- Abelian excitations

Aleksander Kazakov & Leonid Rokhinson

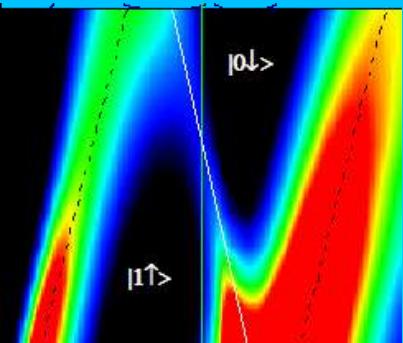
*Department of Physics, Purdue University*

V. Kolkovsky, Z. Adamus, & Tomasz Wojtowicz

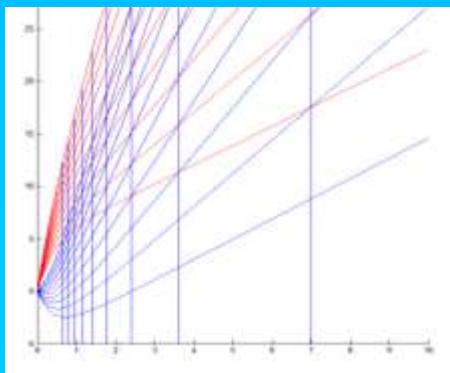
*Institute of Physics, Polish Academy of Science, Warsaw, Poland*

George Simion & Yuli Lyanda-Geller

*Department of Physics, Purdue University*



Luchon, France  
May 24 - 29, 2015



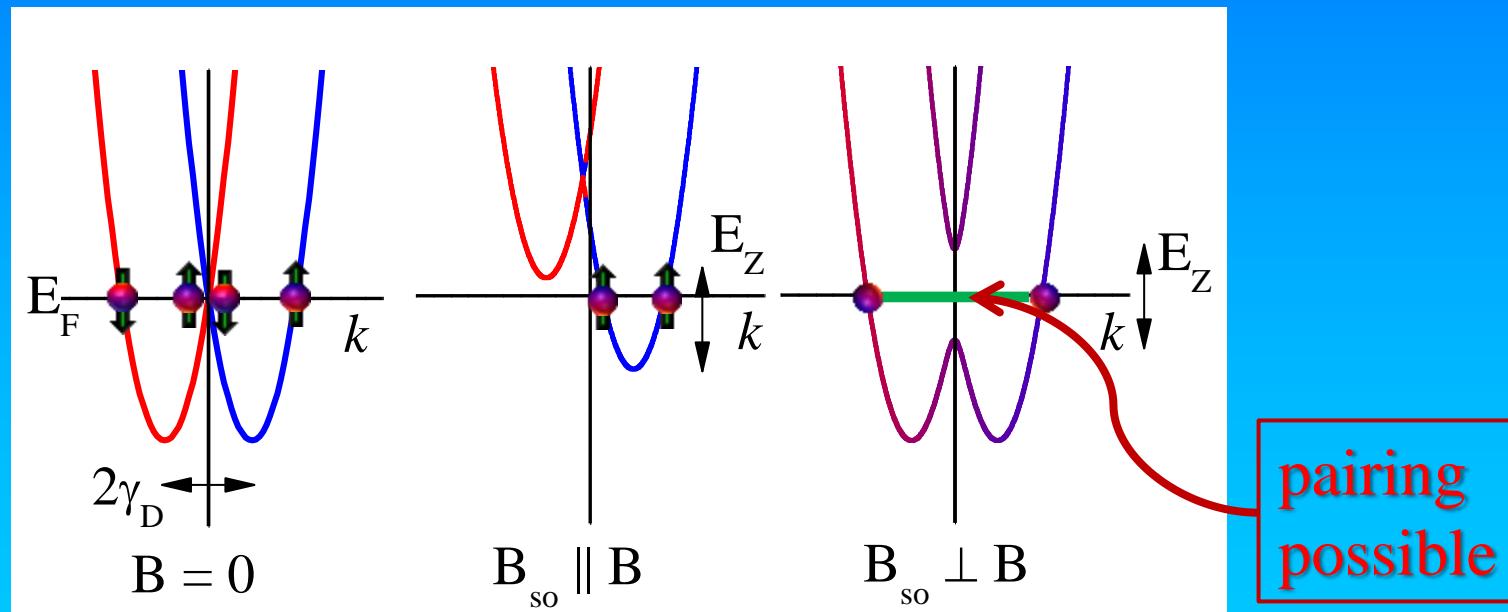
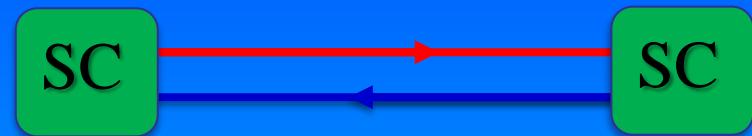
# *Engineering Majorana fermions*

requirements:

1D  
spinless (one mode)  
superconductor

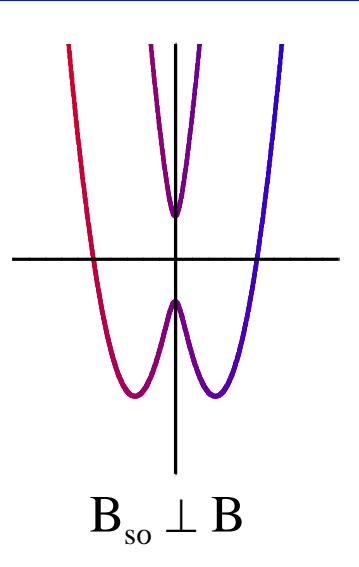


topological superconductor



Sau, et al '10, Alicea, et al '10

# *parameter space*



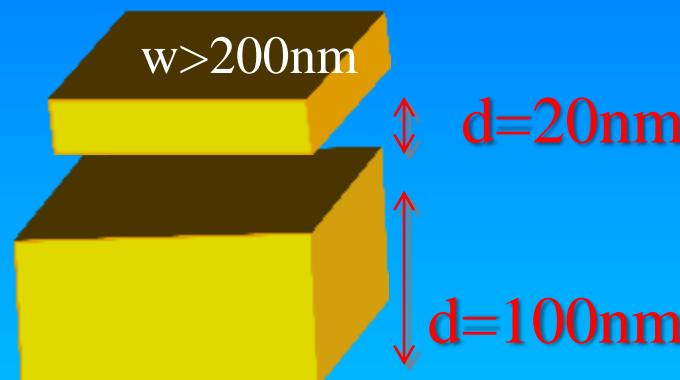
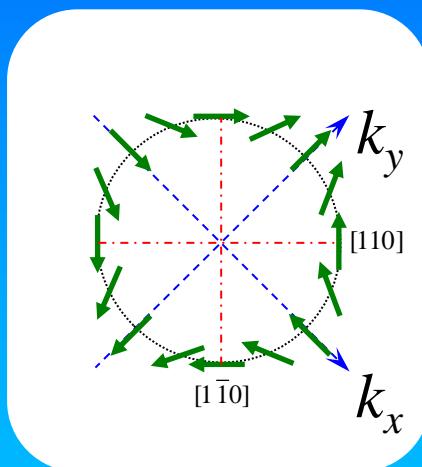
single-spin condition:

$$E_Z > \sqrt{\Delta^2 + E_F^2}$$

to protect superconductivity:

$$E_Z \sim E_{SO}$$

$$E_{SO} = \sqrt{2} \gamma_D \langle k_z^2 \rangle k = \sqrt{2} \gamma_D (\pi / d)^2 k$$



$$E_{SO} \approx 2.6 \cdot k \text{ [meV]}, \quad k[10^6 \text{ cm}^{-1}]$$

$$E_{SO} \approx 0.1 \cdot k \text{ [meV]}, \quad k[10^6 \text{ cm}^{-1}]$$

smallest dimension defines  $E_{so}$ :

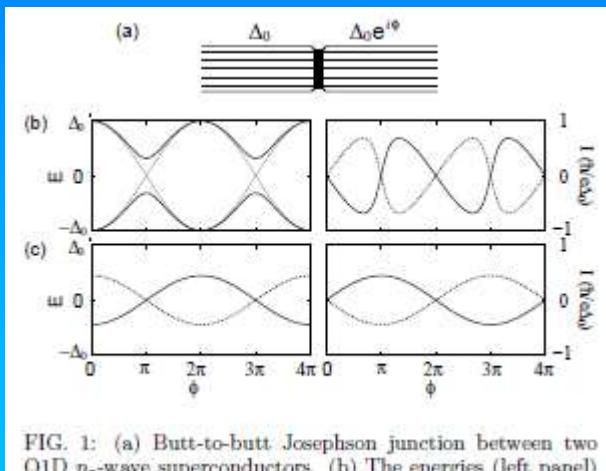
small d  $\Rightarrow$  large  $E_{so}$   $\Rightarrow$  large  $E_F$   $\Rightarrow$  less localization

# Characteristic $4\pi$ energy-flux relation

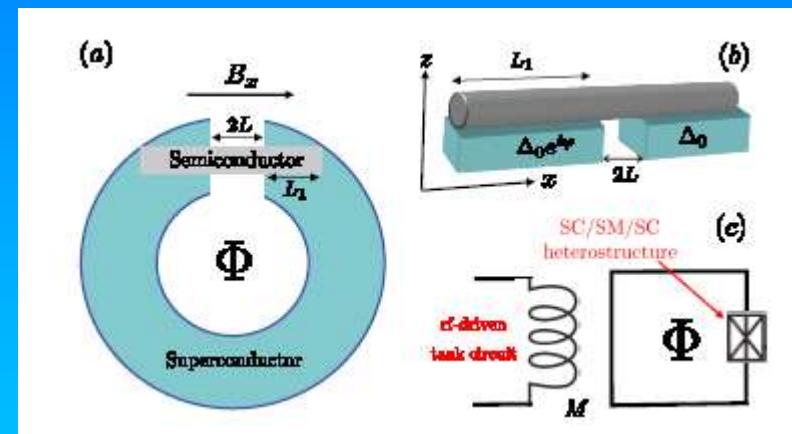
modification of the Josephson phase

trivial superconductor       $2\pi$   
 topological superconductor     $4\pi$

Cooper pairs,       $I \propto \sin(\phi)$   
 Majorana particles,  $I \propto \sin(\phi/2)$

$$b^\dagger = (\gamma_l - i\gamma_m) \quad \text{Kitaev '01}$$


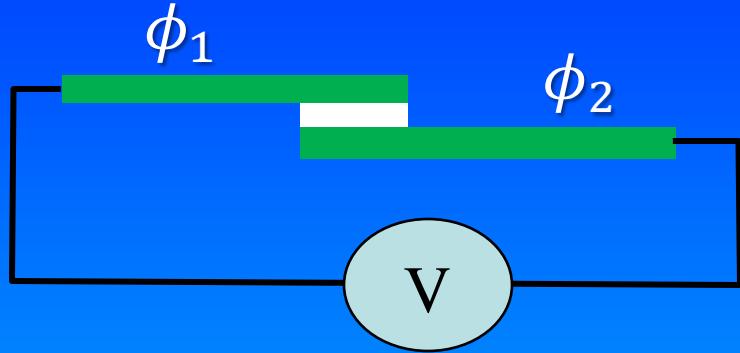
Kwon '04



Lutchyn '10

# *ac Josephson effect*

direct

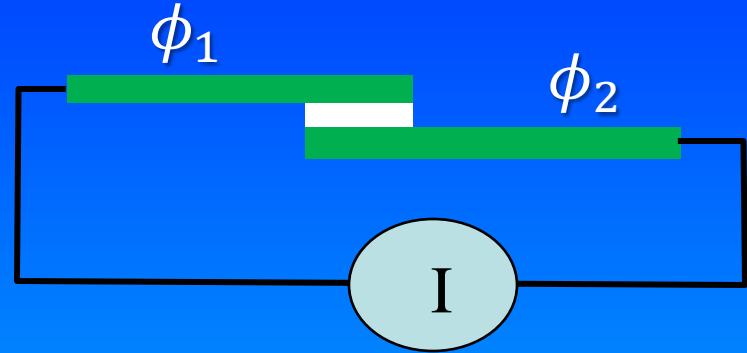


$$\frac{d(\Delta\phi)}{dt} = \frac{2eV}{\hbar}$$

$$I_s = I_c \sin(\omega_J t) = I_c \sin\left(\frac{2eV}{\hbar} t\right)$$

Current oscillates with frequency  $\propto V$

inverse

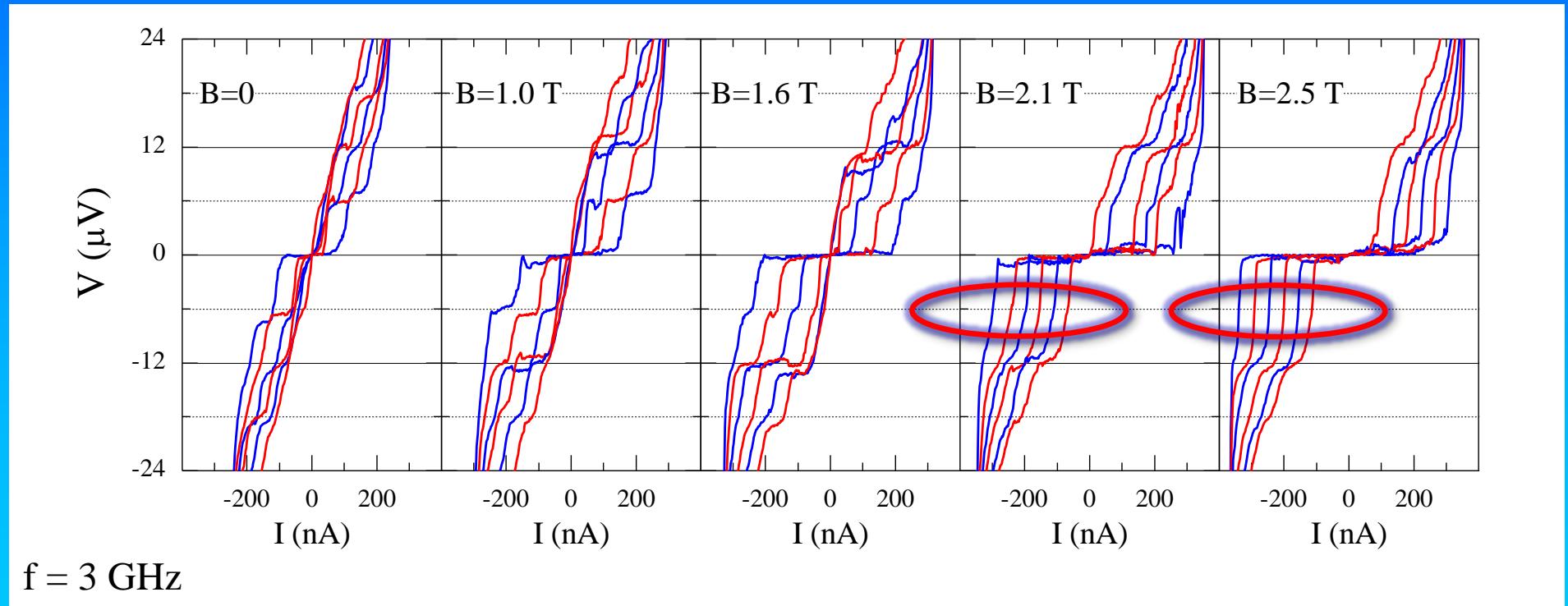
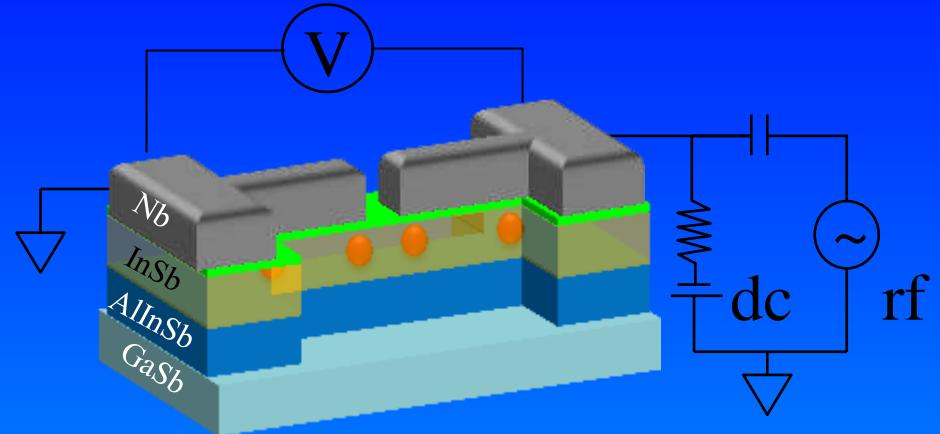
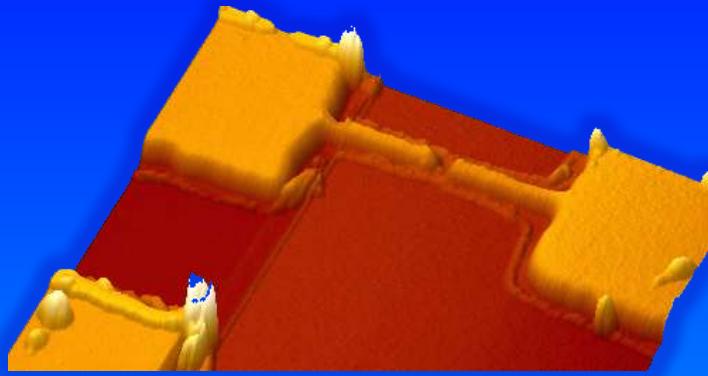


$$I = I_0 + I_\omega \sin(\omega t)$$

$$V = \left(\frac{\hbar\omega}{2e}\right)$$

Constant voltage steps  $\propto \omega$

# *Disappearance of the first Shapiro step*

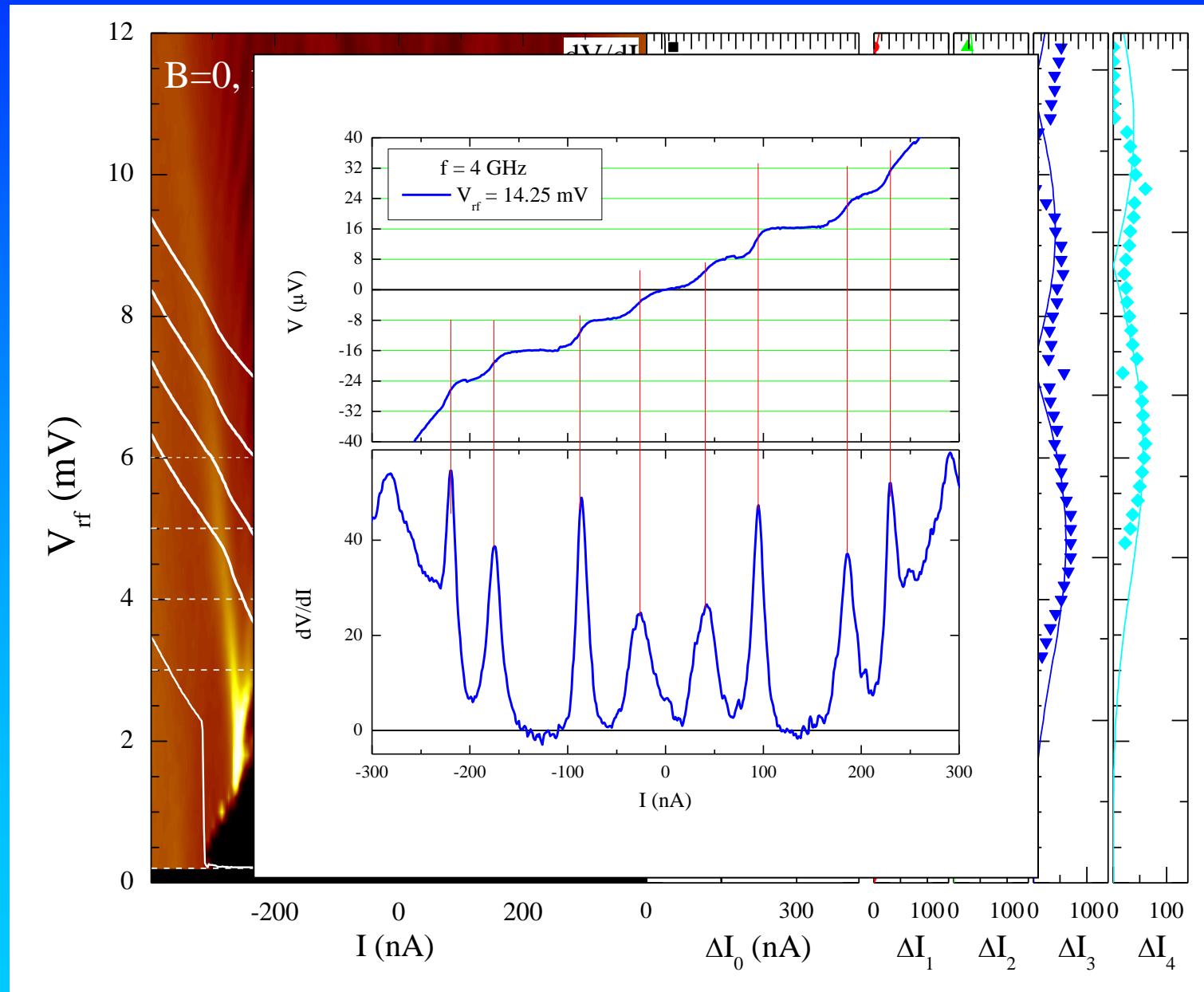


$f = 3 \text{ GHz}$

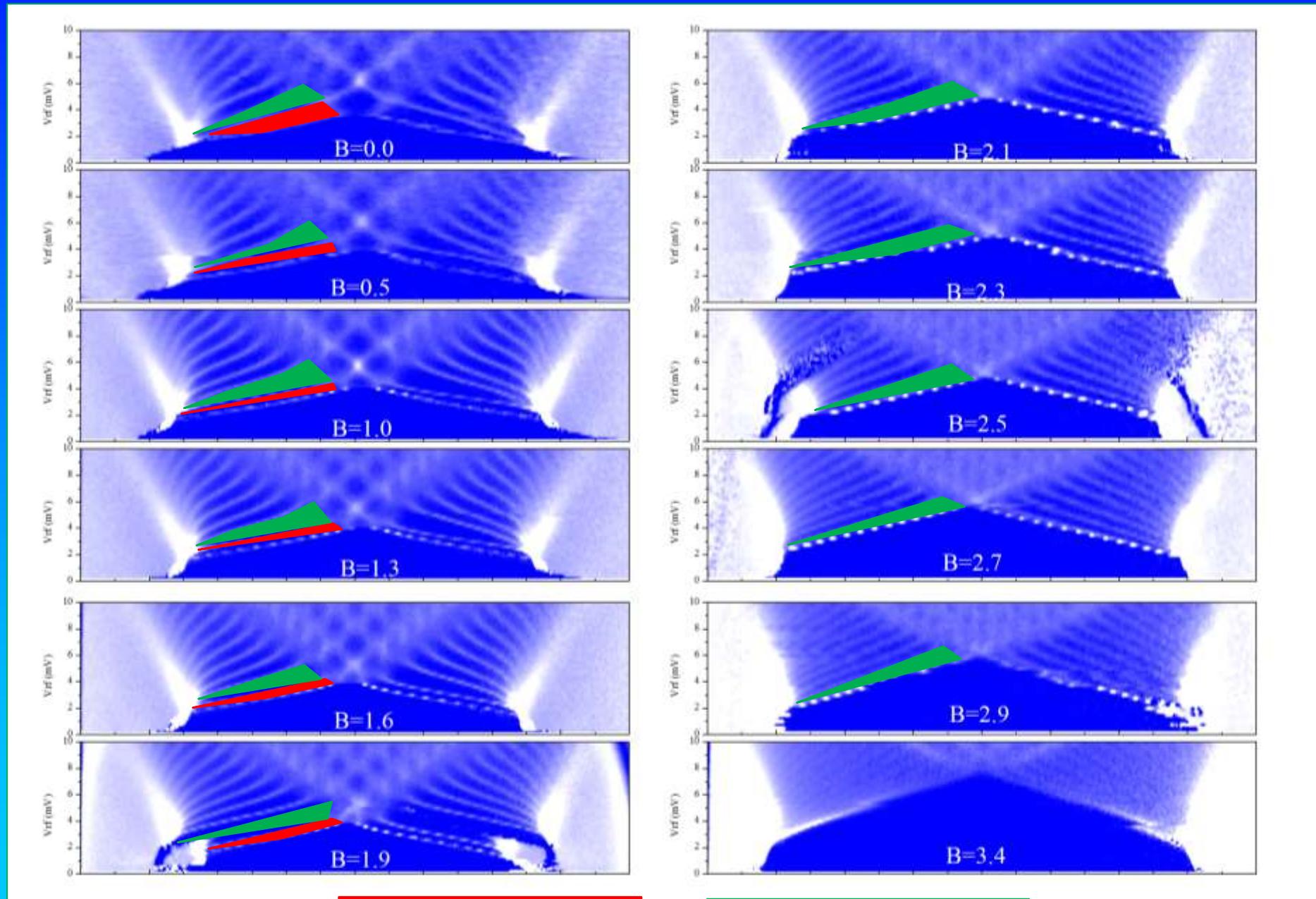
LR, X. Liu, J. Furdyna, Nature Physics 8, 795 (2012)

# *Shapiro steps*

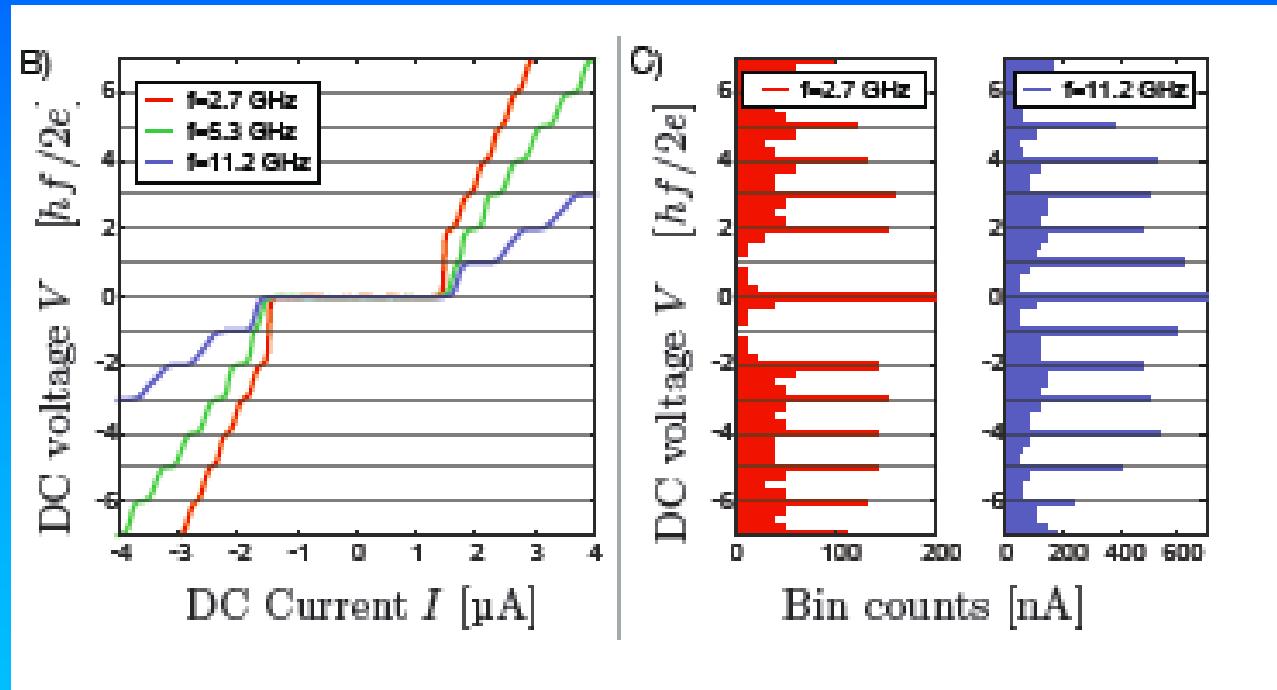
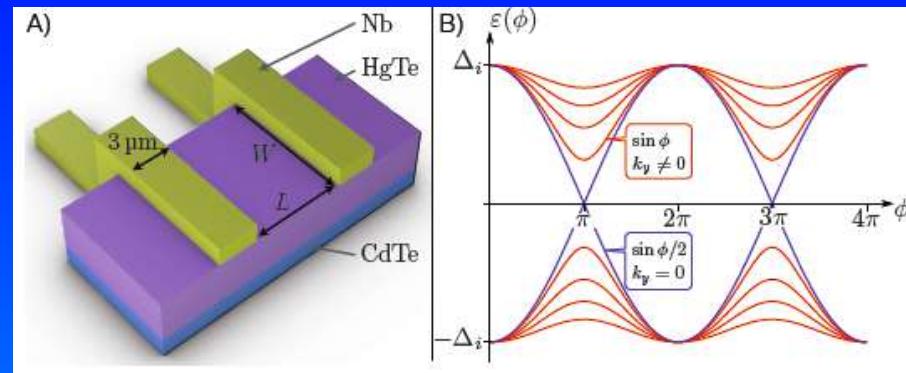
$$\Delta I_n = A |J_n \left( \frac{2eV_{rf}}{\hbar\omega_{rf}} \right)|$$



# $dV/dI$ vs $B$



# 4-periodic Josephson supercurrent in HgTe-based 3D TI



Wiedenmann, ... M. Klapwijk, ..., Seigo Tarucha, L. W. Molenkamp

arXiv:1503.05591

**Advantage of 1D wires: Majorana modes are localized**  
easy to perform spectroscopy

**Disadvantage of 1D wires: Majorana modes are localized**  
almost impossible to perform exchange

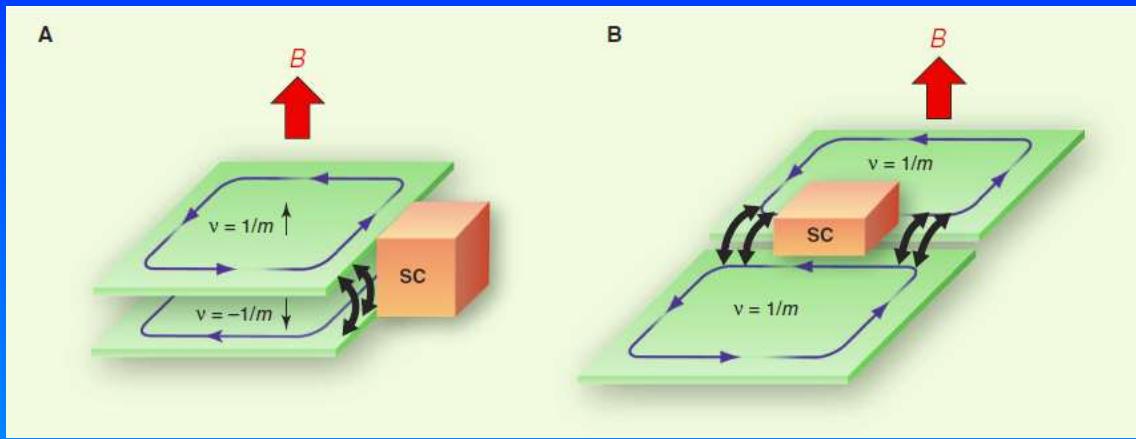
magnetic  
semiconductors

quantum Hall  
effect

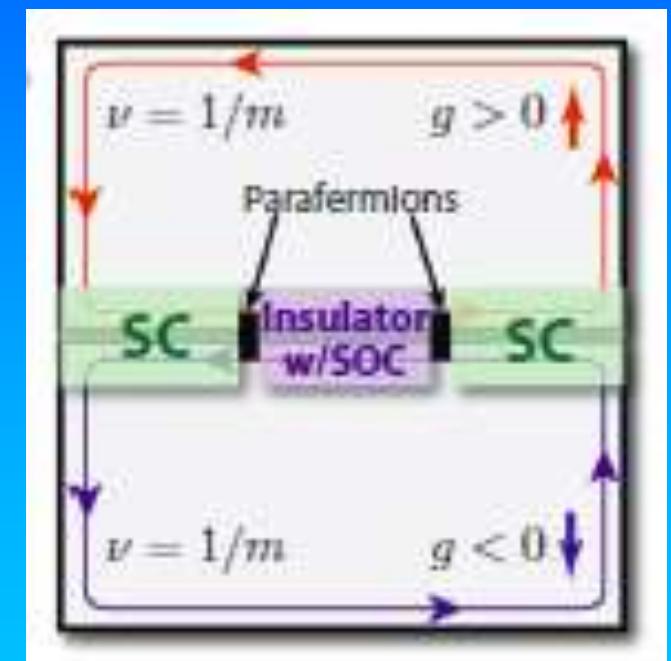
superconductivity

new materials to support exotic non-Abelian excitations  
reconfigurable 1D topological superconductors in 2D systems

# Motivation and inspiration

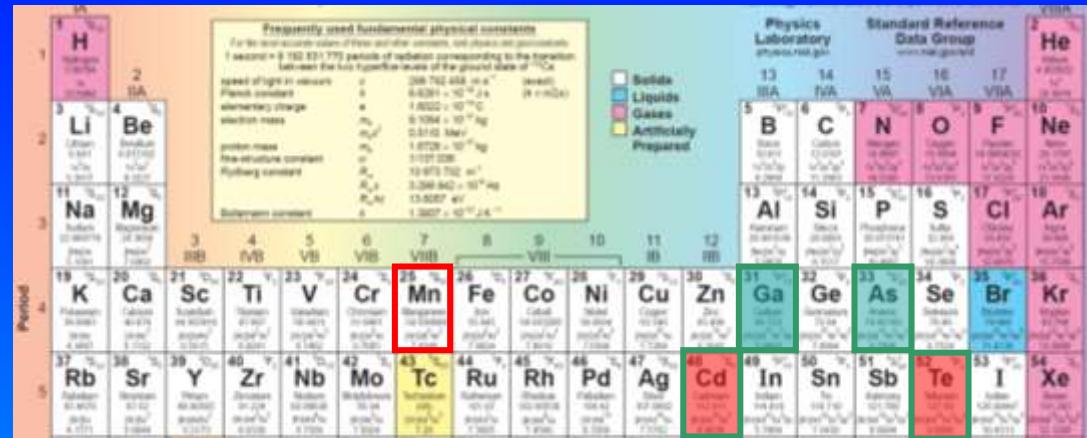
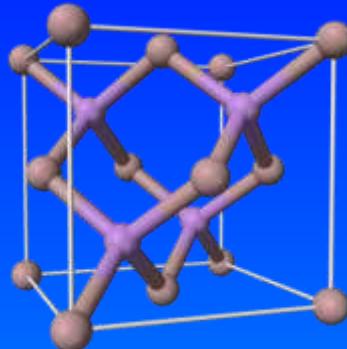


*Topological Quantum Computation - From Basic Concepts to First Experiments*  
Ady Stern & Netanel Lindner  
*Science*, 2013, 339, 1179



*Exotic non-Abelian anyons from conventional fractional quantum Hall states*  
David J. Clarke, Jason Alicea, and Kirill Shtengel  
*Nature Commun.*, 2012, 4, 1348

# *Development of a new system CdTe:Mn QW*



GaAs:Mn

Ga [Ar]3d<sup>10</sup>4s<sup>2</sup>4p<sup>1</sup>      As [Ar]3d<sup>10</sup>4s<sup>2</sup>4p<sup>3</sup>

S=5/2

Mn [Ar]3d<sup>5</sup>4s<sup>2</sup>

p-doping

large s-d exchange (ferromagnetic)

exchange split ~3 eV (Hunds rule), 1/2 filled

CdTe:Mn

Cd [Kr]4d<sup>10</sup>5s<sup>2</sup>      Te [Kr]4d<sup>10</sup>5s<sup>2</sup>5p<sup>4</sup>

neutral impurity, large s-d exchange

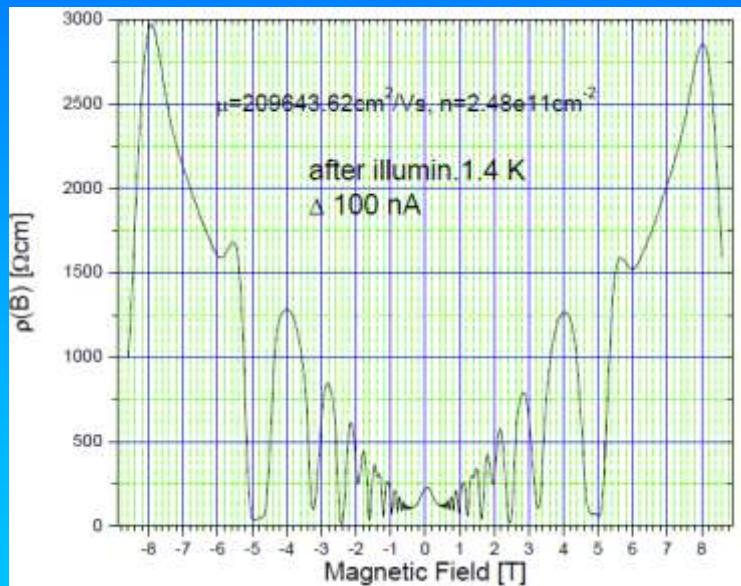
# *Development of a new system*

High mobility 2D gas in CdTe/CdMgTe QW

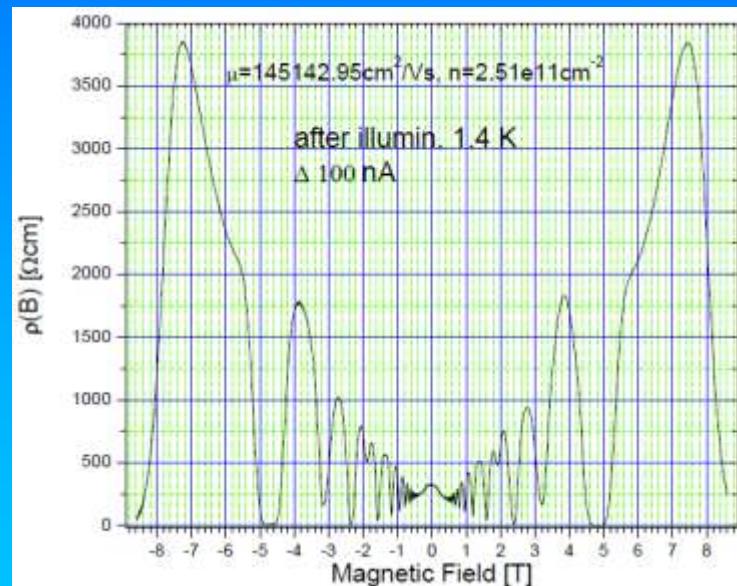
$$m^*=0.11, E_g=1.44 \text{ eV}$$

add Mn into CdTe (neutral impurity with 5/2 spin)

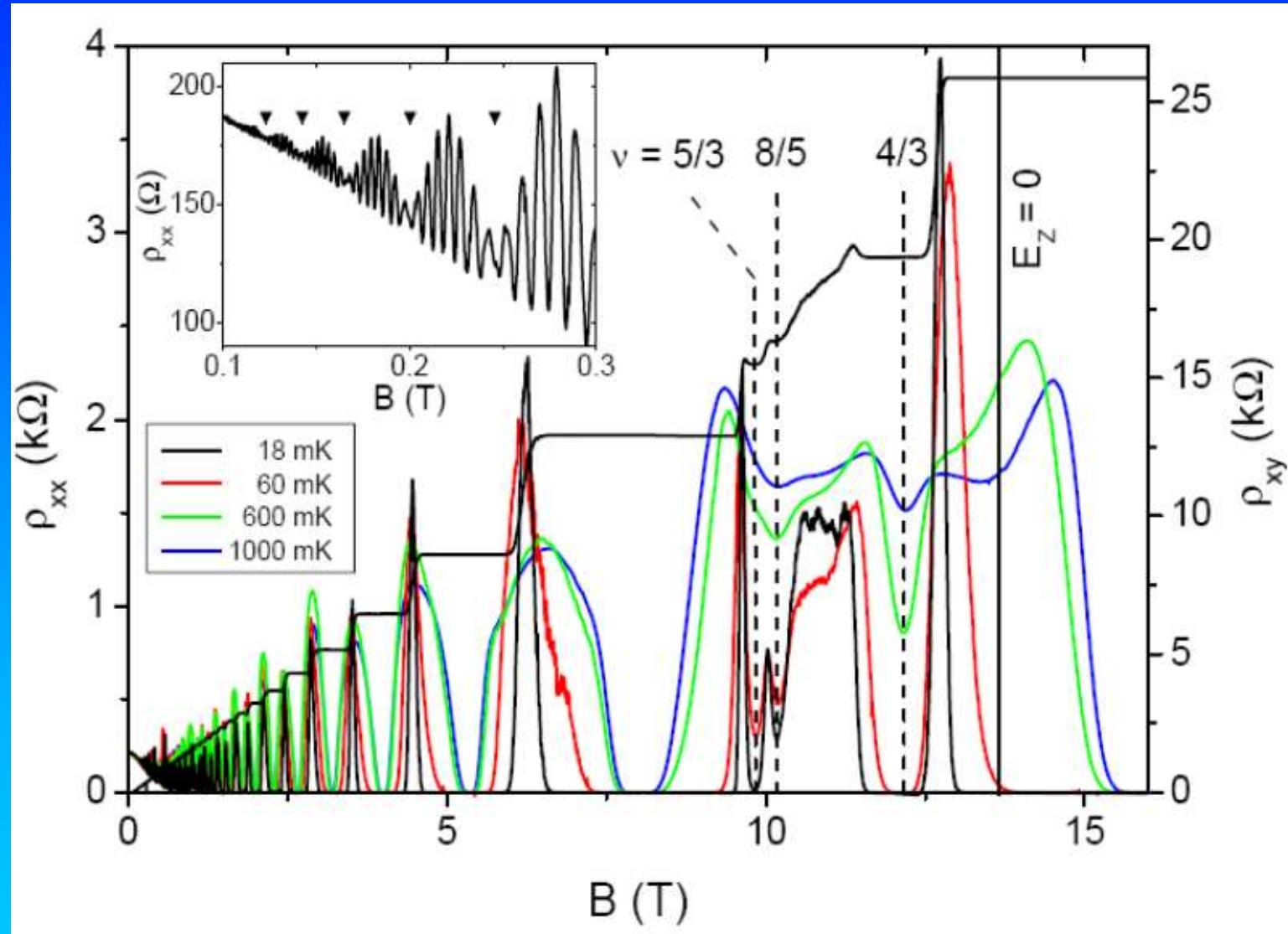
no Mn



~1% Mn



# *FQHE in CdTe:Mn*



T. Wojtowicz

Betthausen, et al, Phys. Rev. B **90**, 115302 (2014)

# Anomalous Zeeman splitting in CdTe:Mn

$$E_{n,\uparrow\downarrow} = (n + \frac{1}{2})\hbar\omega_c \pm \frac{1}{2} \left[ g^* \mu_B B + x_{Mn} E_{sd} \mathfrak{B}_S \left( \frac{g\mu_B SB}{k_B T} \right) \right]$$



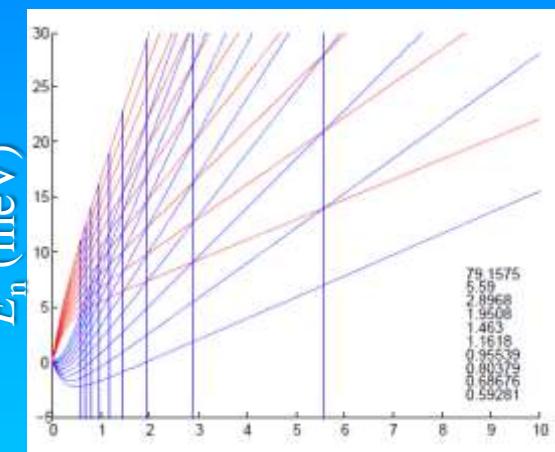
cyclotron

Zeeman

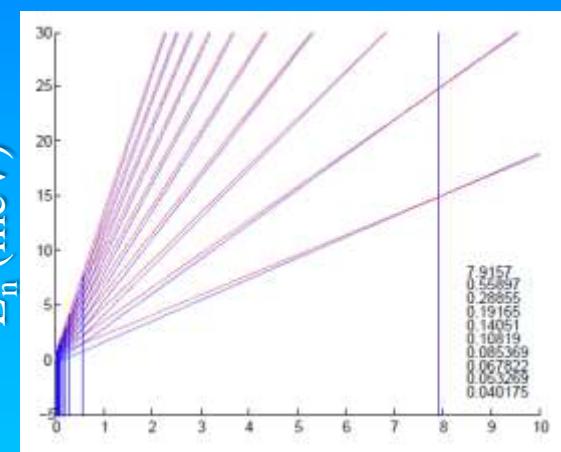
$g^* = -1.6$

s-d exchange (>0)

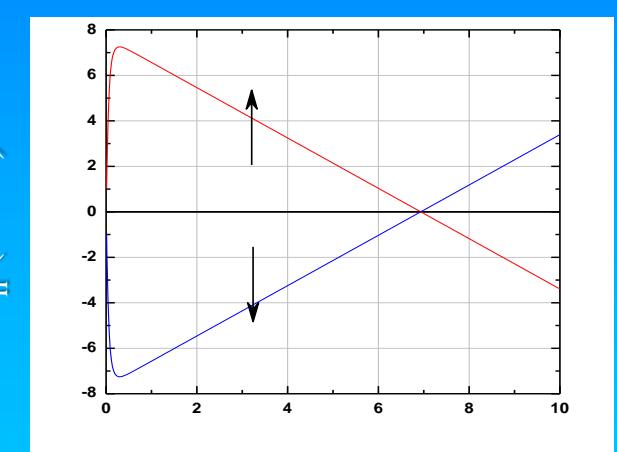
1.3% Mn



0.13% Mn



for n=1



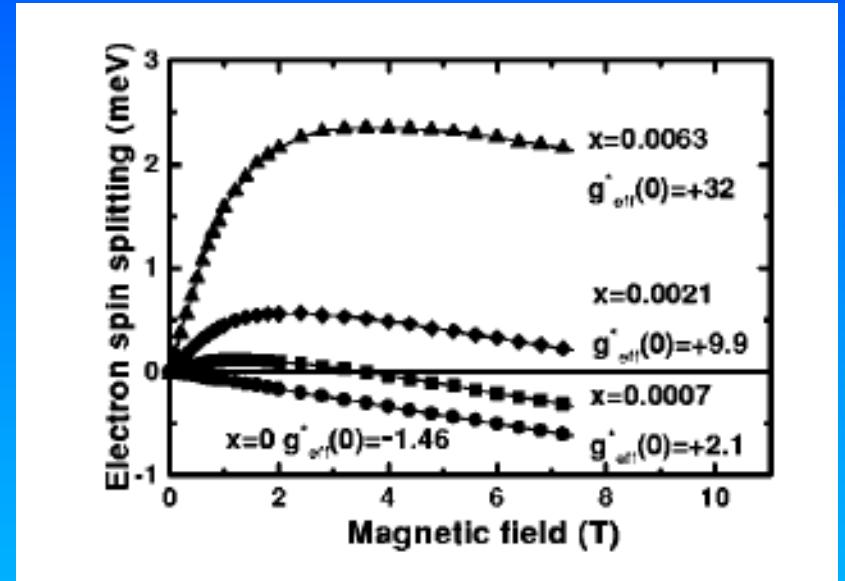
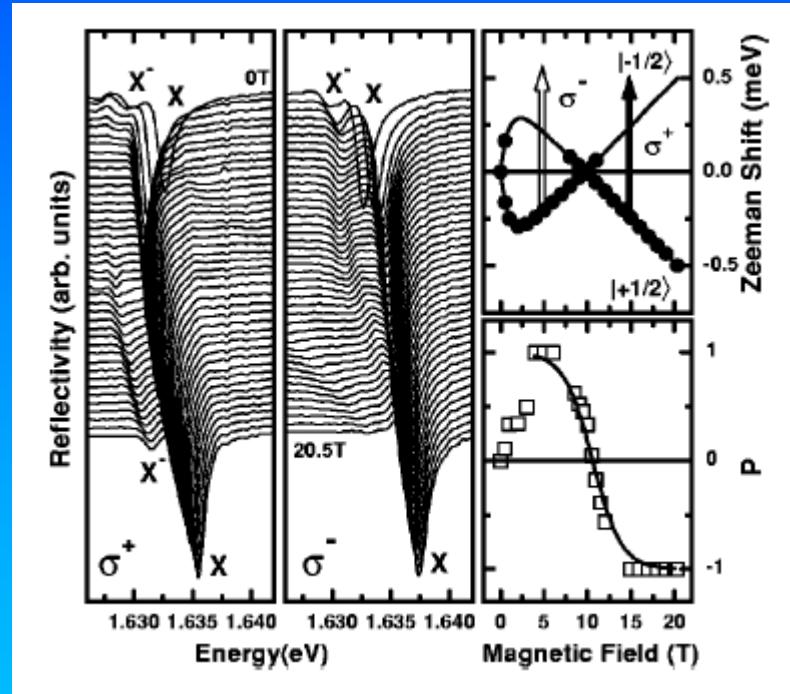
$B$  (Tesla)

$B$  (Tesla)

$B$  (Tesla)

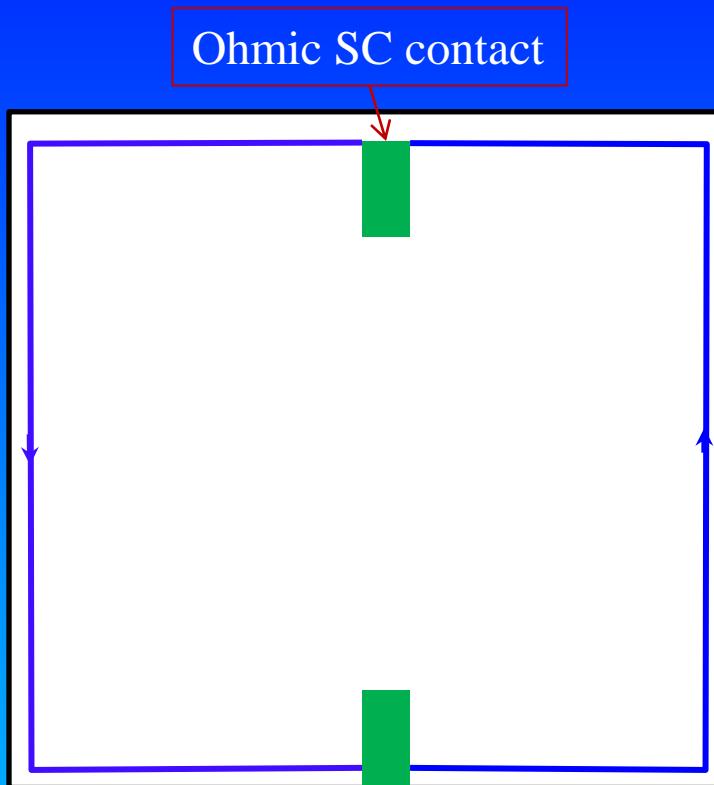
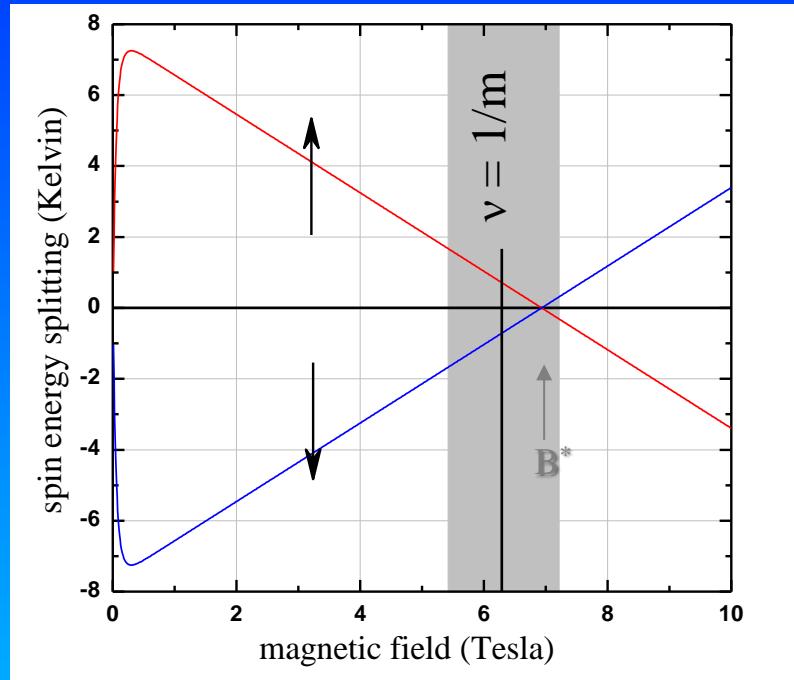
# *Magnetoreflectivity studies*

negatively charged exciton complex  $X^-$  (trion) to singlet  $X$  transition under polarized  $\sigma^+/\sigma^-$  light

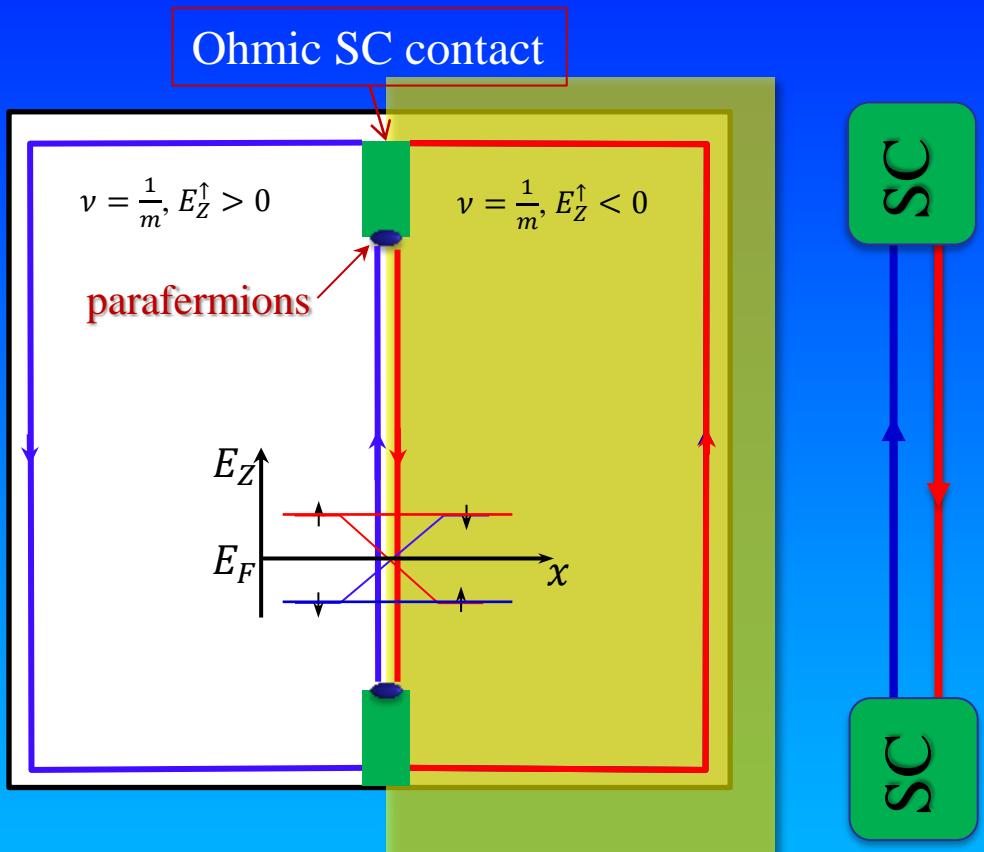
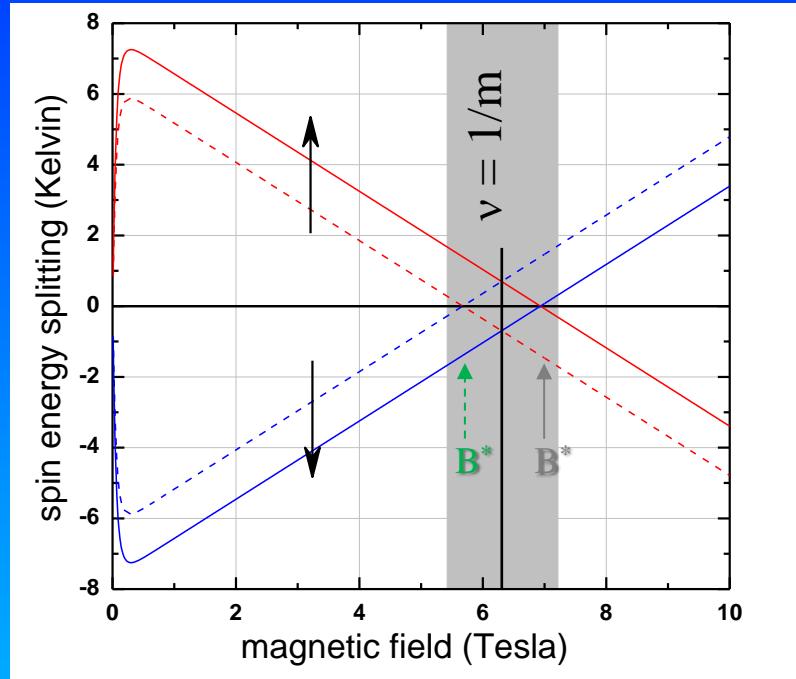


Wojtowicz, et al, PRB 59, R10437 (1999)

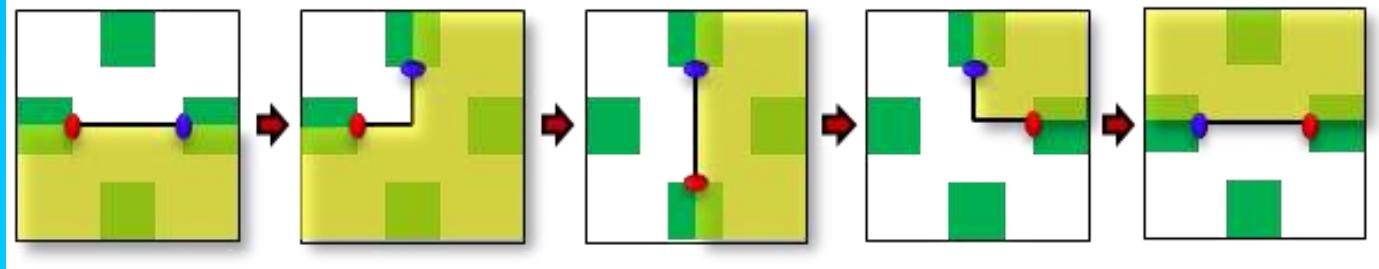
# *new platform for non-Abelian excitations*



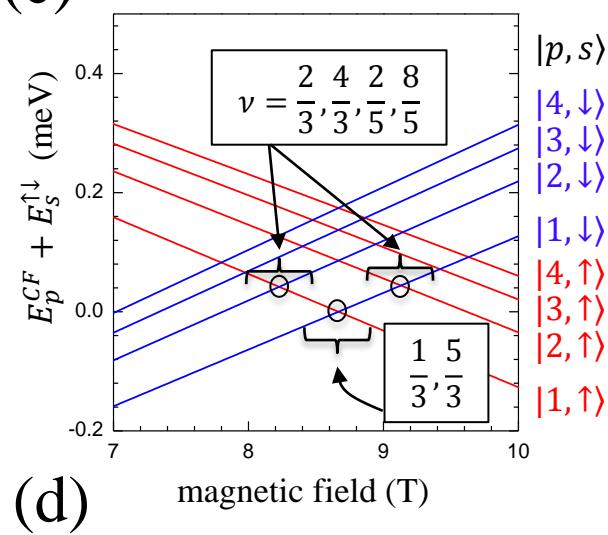
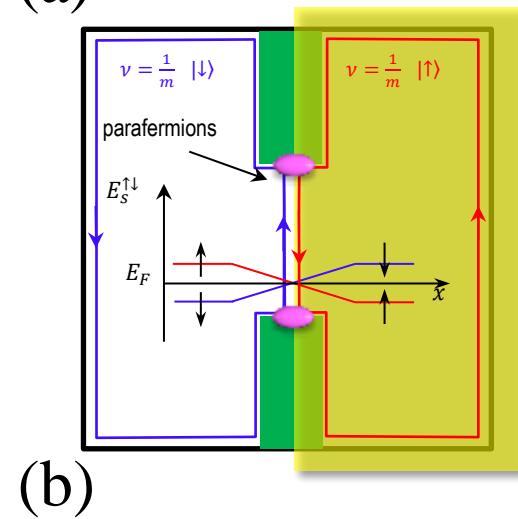
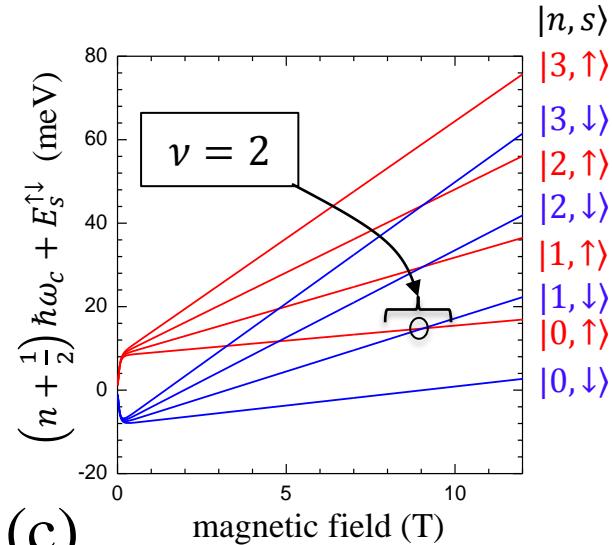
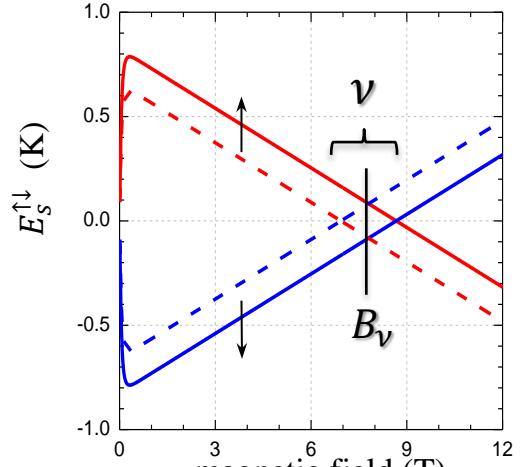
# *new platform for non-Abelian excitations*



braiding sequence

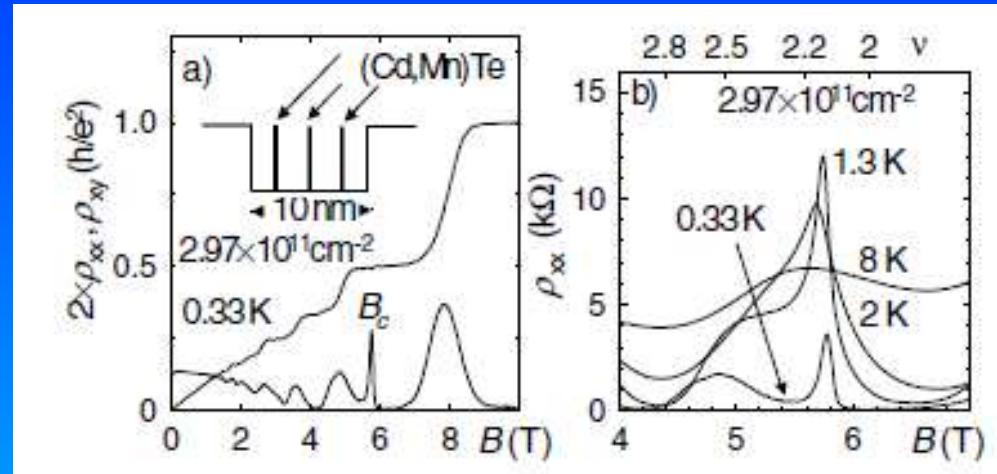


# Crossing of neighboring LLs

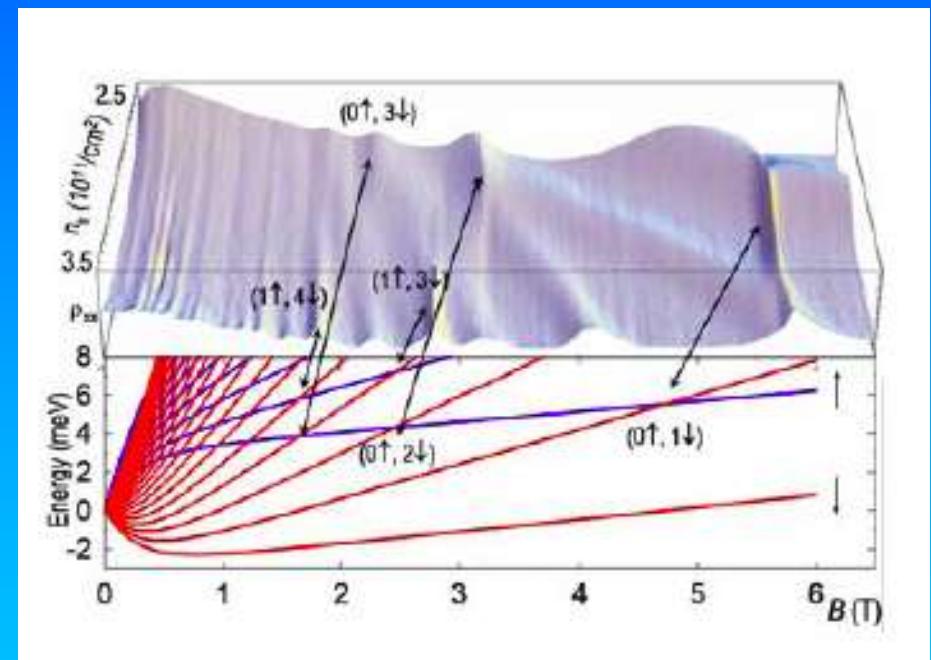


# *Quantum Hall ferromagnet & level crossing*

uniformly Mn-doped quantum well



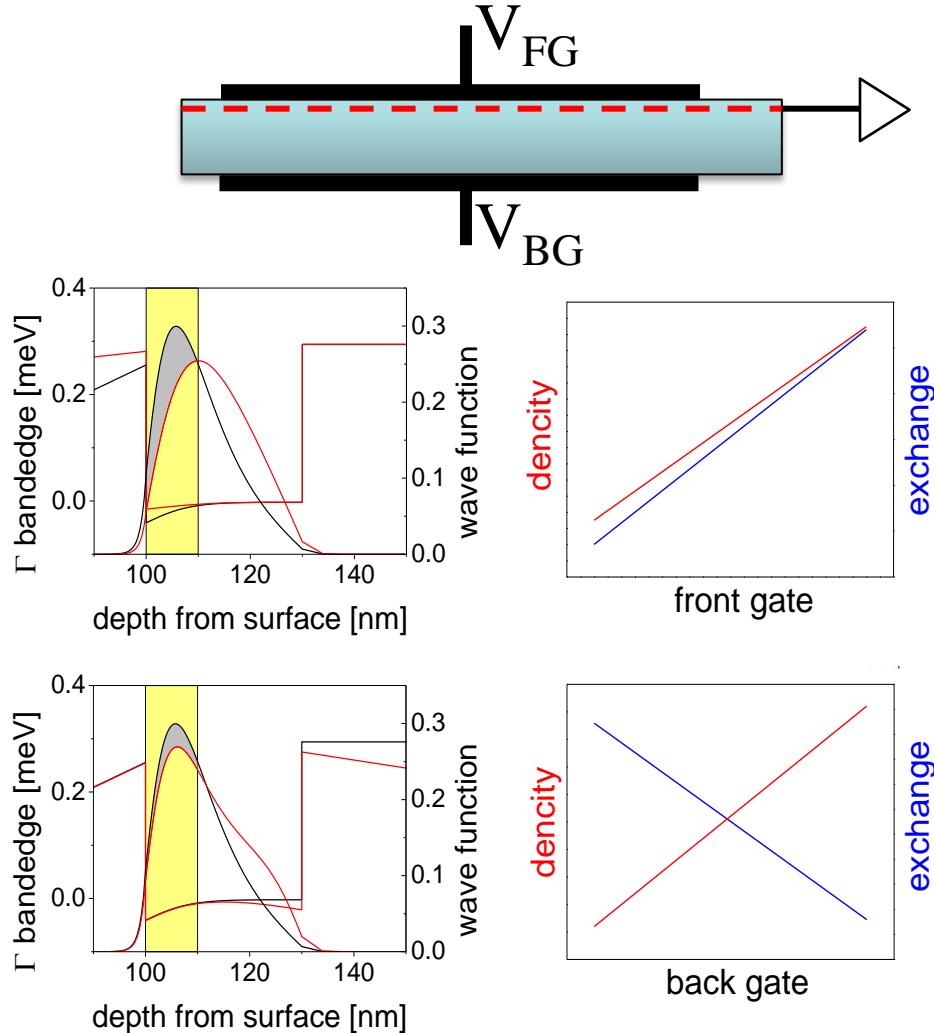
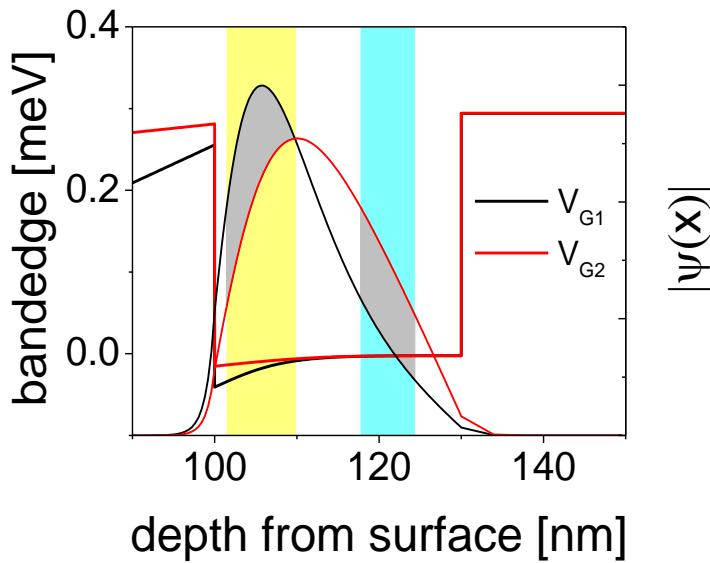
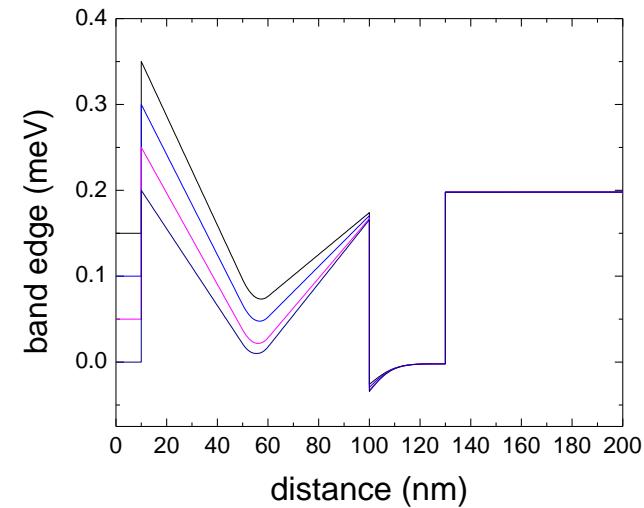
Jaroszynski, et al, PRL **89**, 266802 (2002)



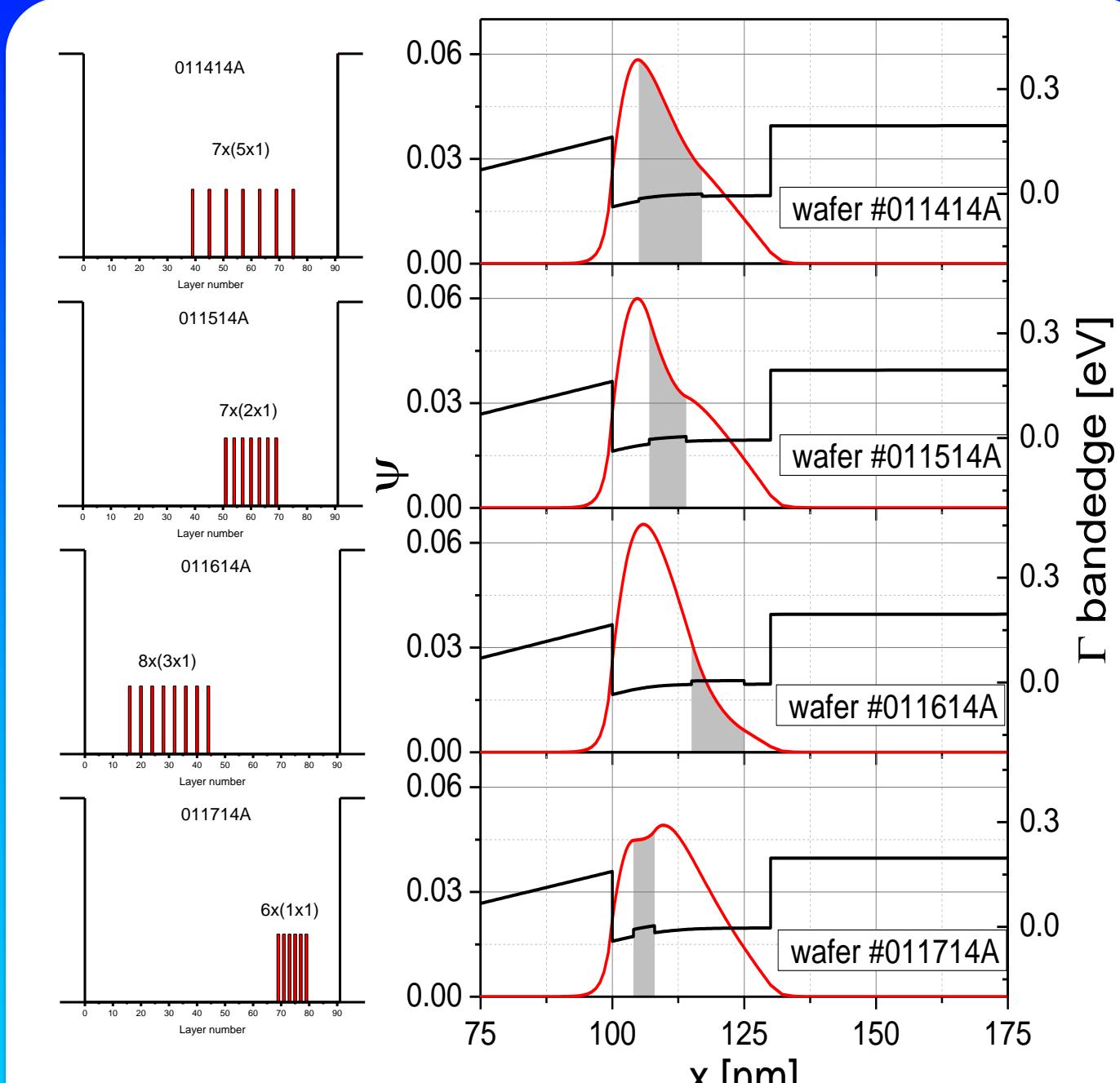
Jaroszynski, et al, AIP conference proceedings (2005)

# Gate control of exchange

$$E_{sd} \propto \int \psi_e(x) \chi_{Mn}(x) dx$$



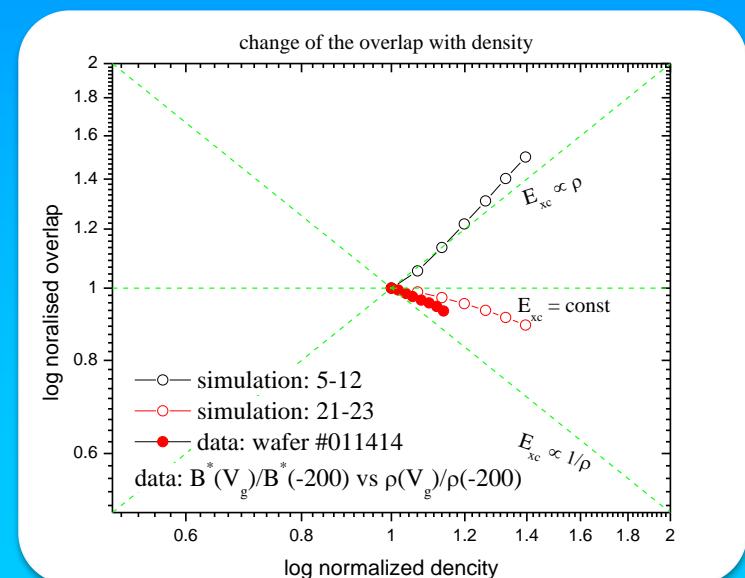
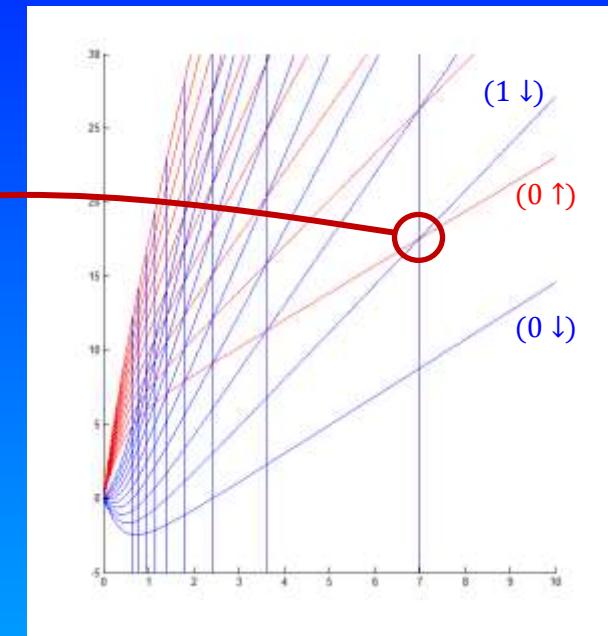
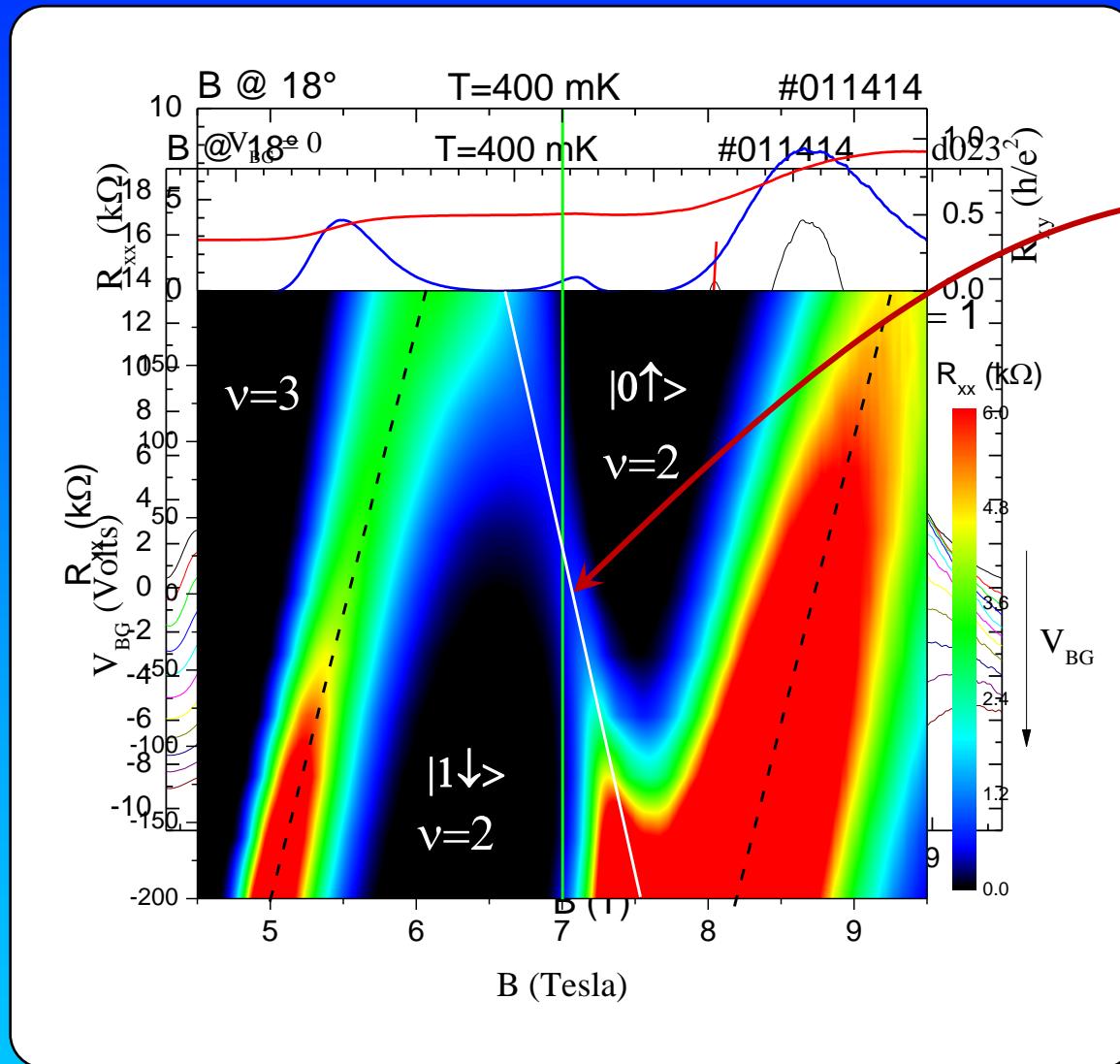
# *Structures with asymmetric doping*



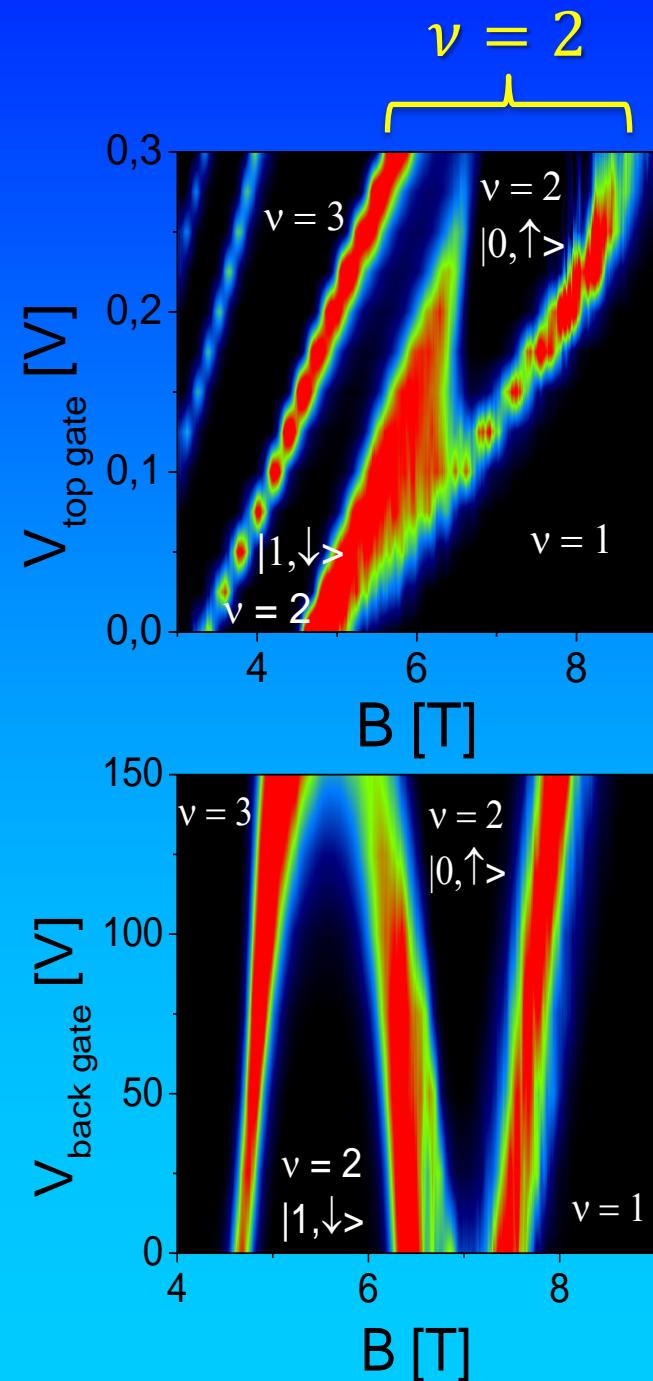
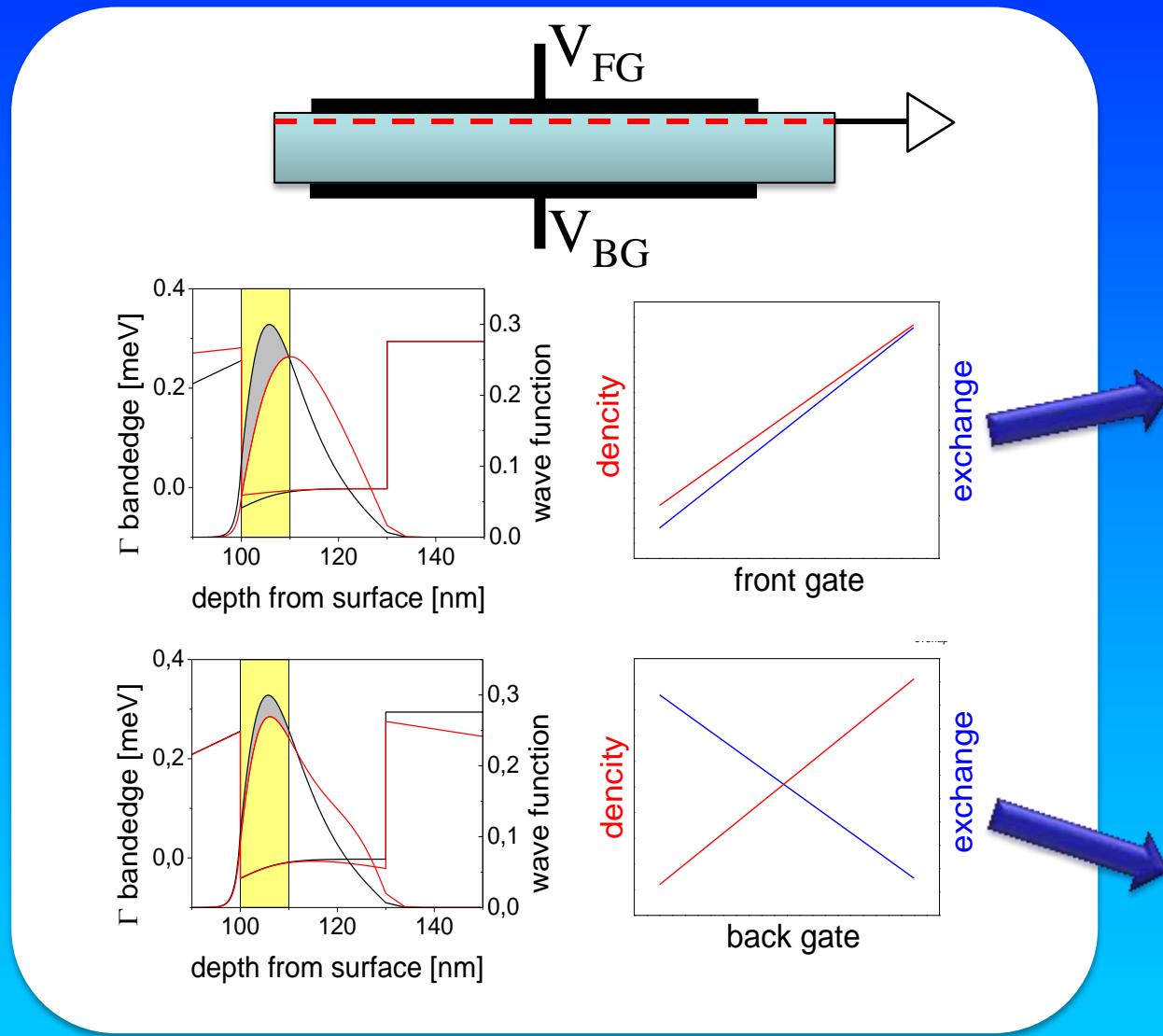
# Gate control of s-d exchange

1.3% Mn

crossing  $|1\downarrow\rangle$  and  $|0\uparrow\rangle$



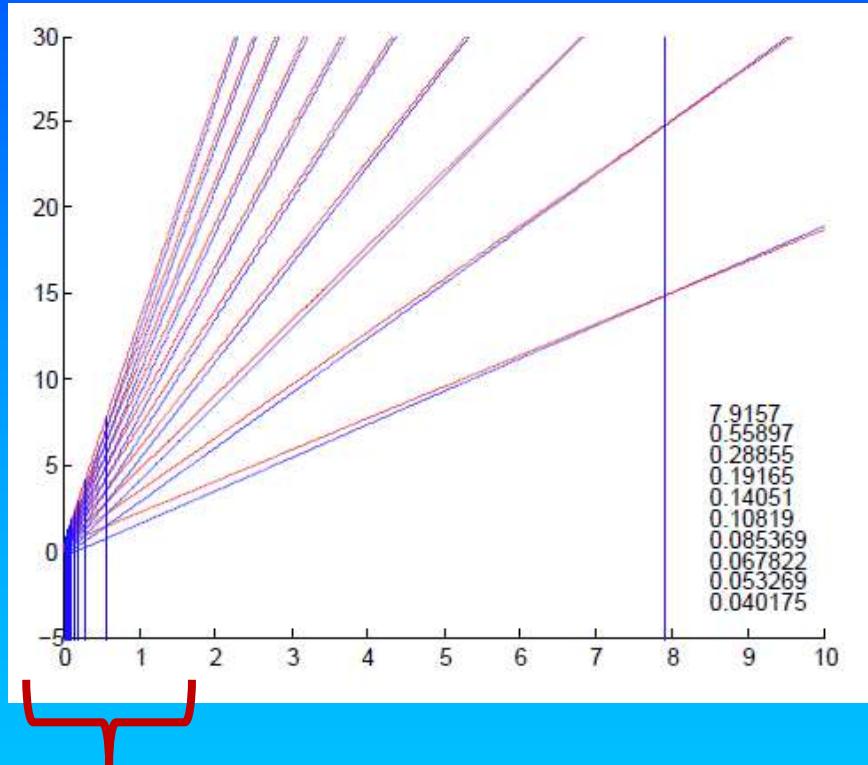
# Gate control of the crossing



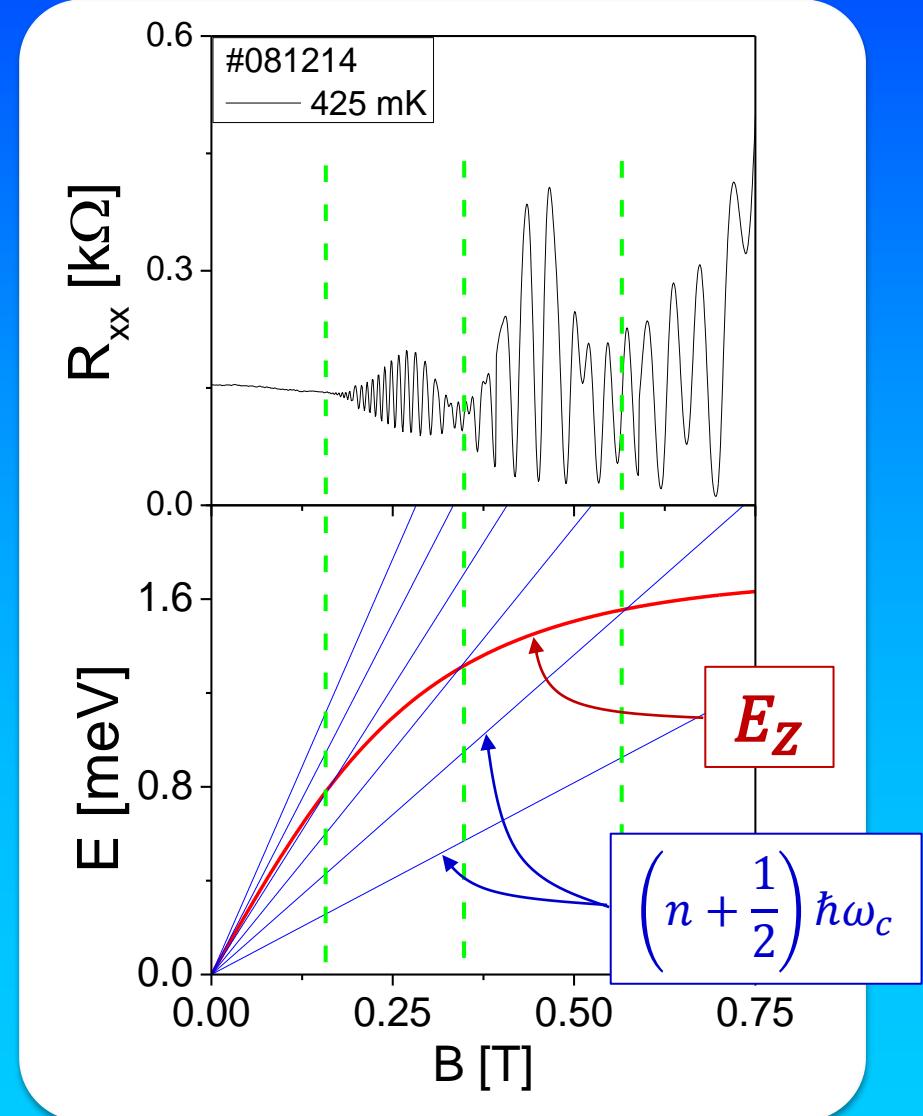
# *low Mn concentration*

Node position:

$$E_Z = \left( N + \frac{1}{2} \right) \hbar \omega_c = g^* \mu_B B + \alpha x_{eff} S B_S \left( \frac{g \mu_B S B}{k_B T_0} \right)$$

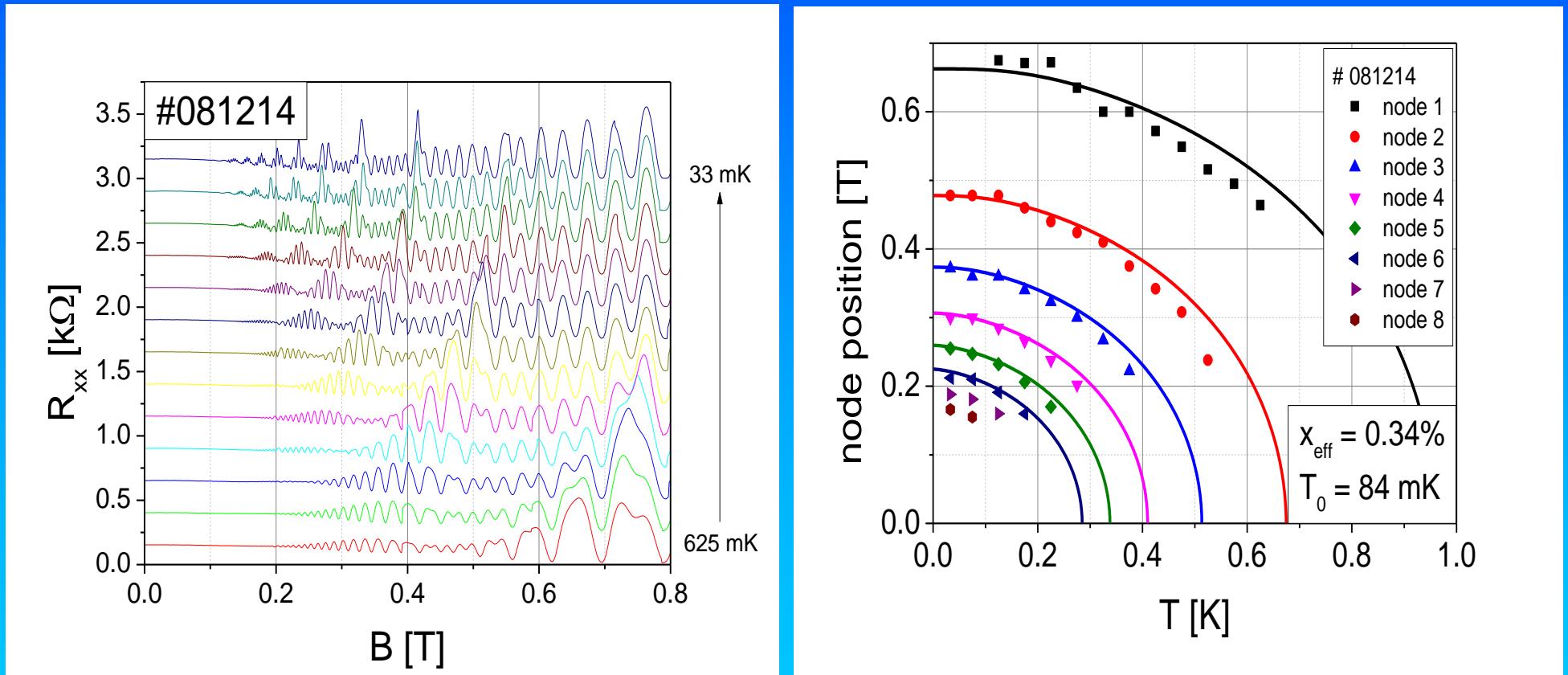


beating in SdH



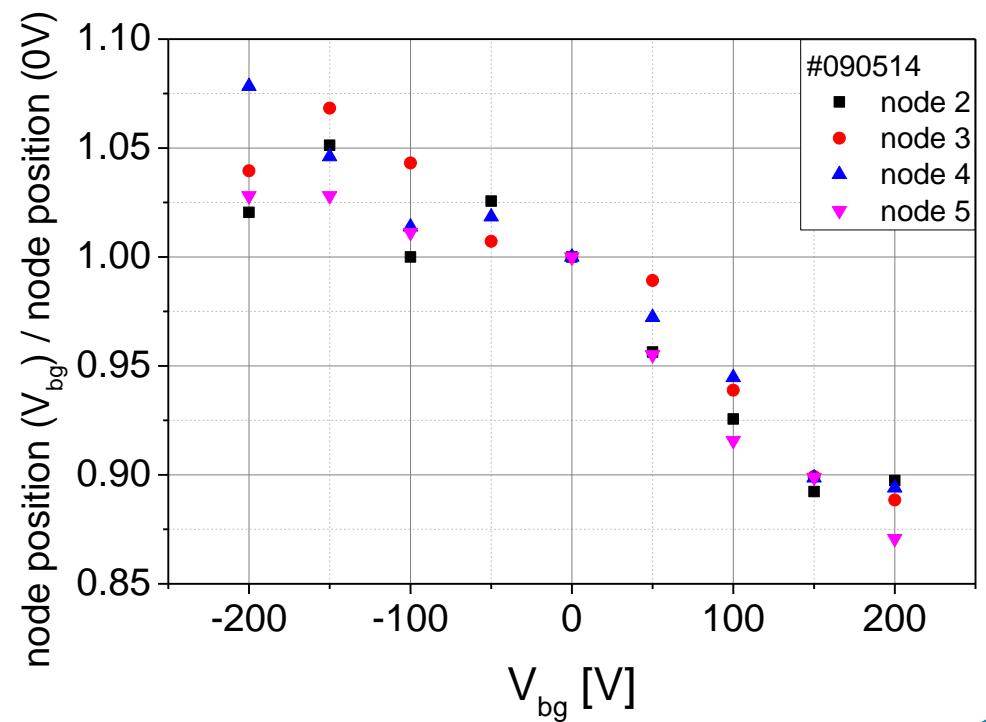
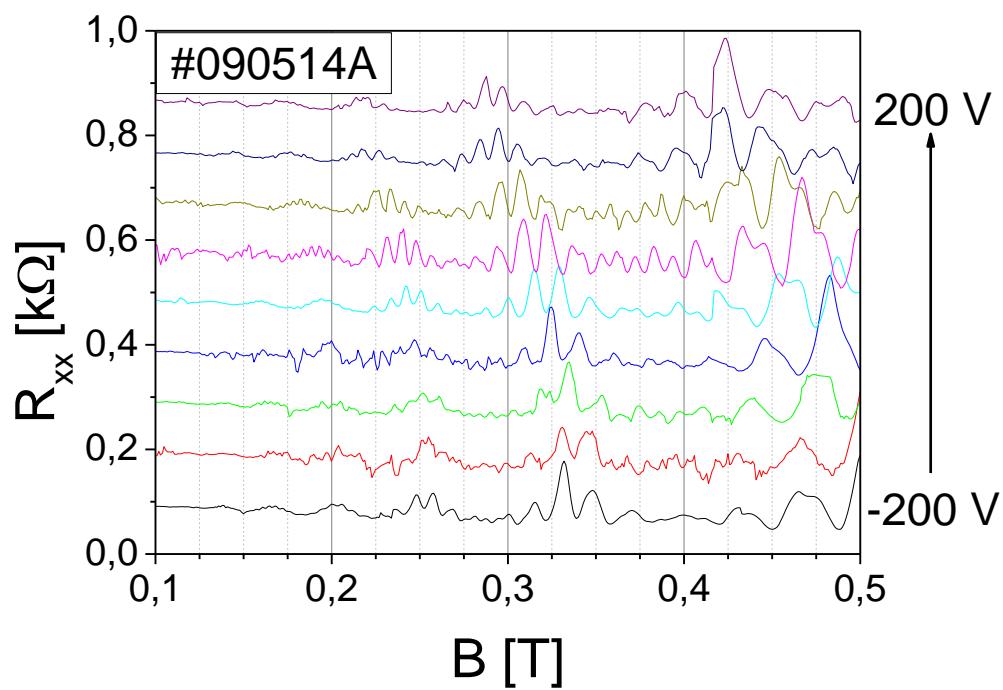
# *SdH beating, $x_{eff}$ and $T_{FM}$*

$$\left(N + \frac{1}{2}\right) \hbar\omega_C = g^* \mu_B B + \alpha x_{eff} S B_S \left( \frac{g \mu_B S B}{k_B (T_{AF} + T)} \right)$$



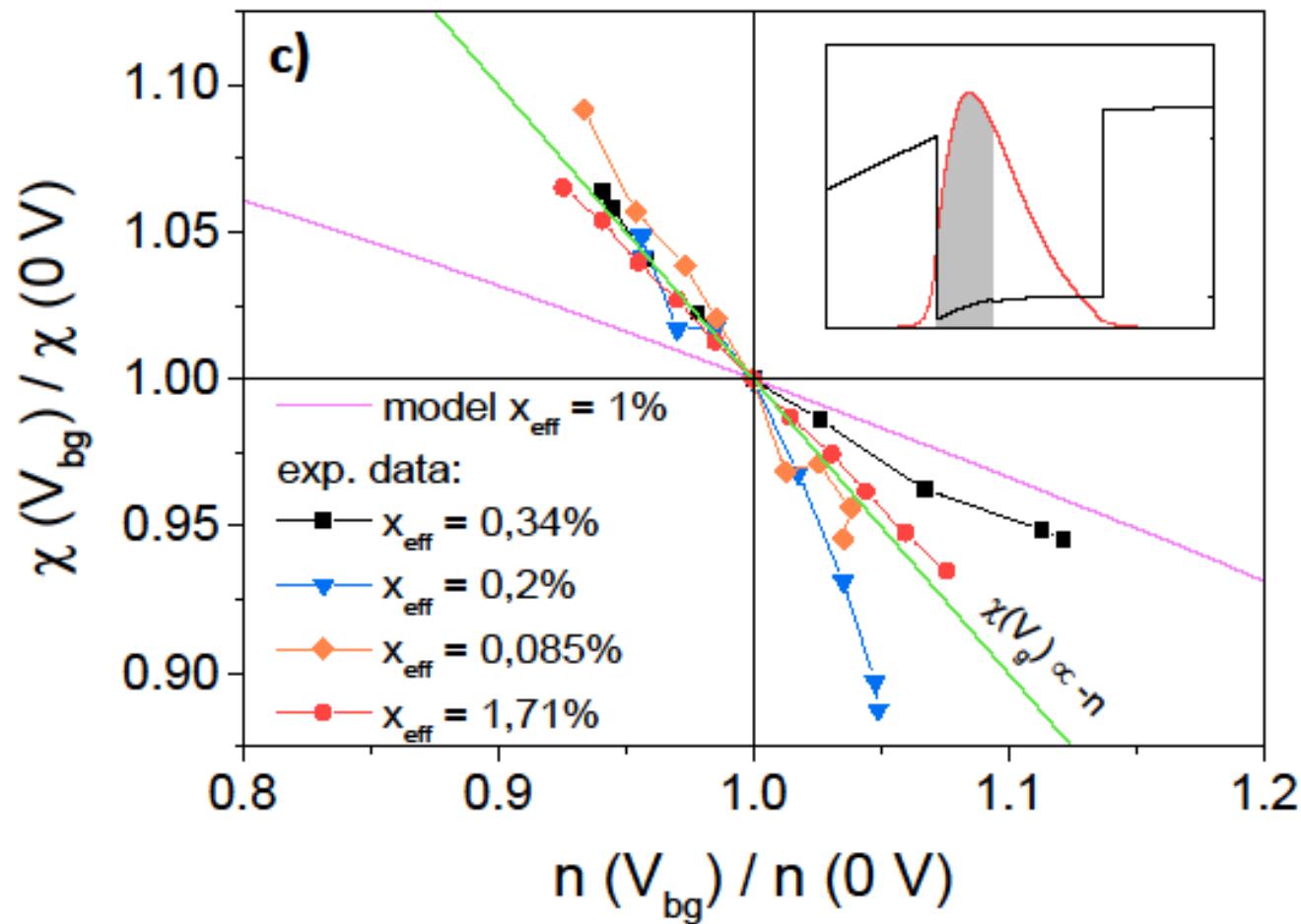
temperature dependence

# *Gate control of SdH beating*



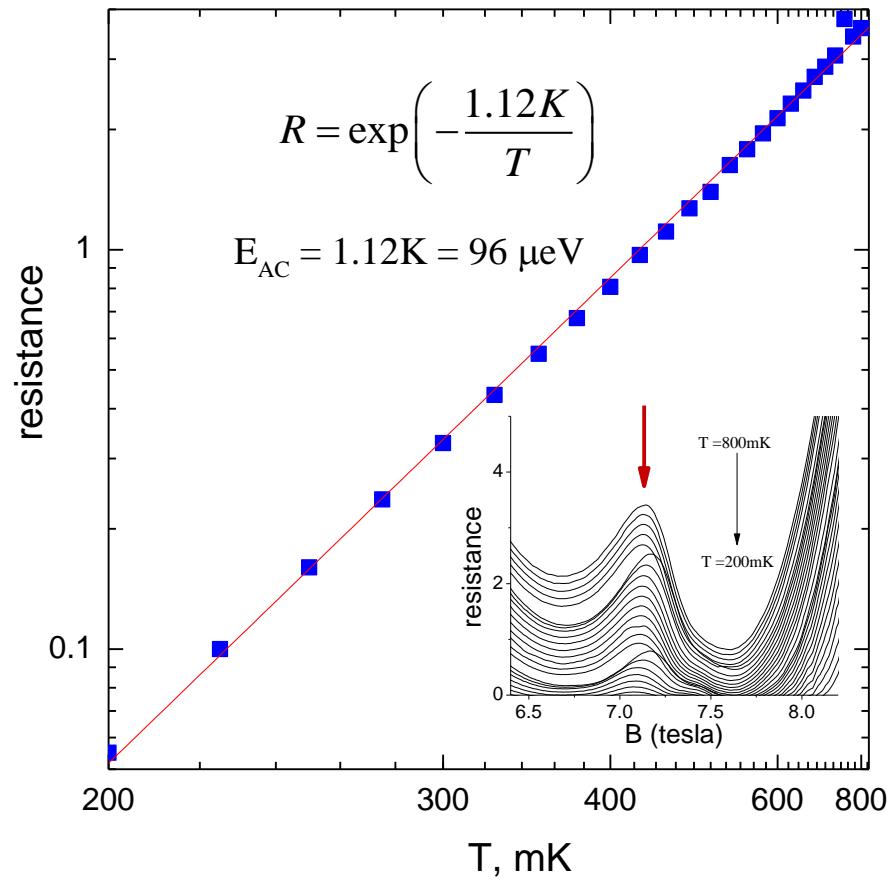
back gate dependence

# *Comparison of high and low Mn concentrations*



# *Anticrossing between 1<sup>st</sup> and 2<sup>nd</sup> LLs*

anticrossing gap between  $E_{|0\uparrow\rangle}$  and  $E_{|1\downarrow\rangle}$



# The role of SO interactions

$$H_{SO} = \gamma_D \boldsymbol{\sigma} \cdot \boldsymbol{\kappa} + \gamma_R (\boldsymbol{\sigma} \times \boldsymbol{k}) \cdot \boldsymbol{\mathcal{E}}$$

$$\kappa = (\{k_x, k_y^2 - k_z^2\}, \{k_y, k_z^2 - k_x^2\}, \{k_z, k_x^2 - k_y^2\})$$

$$\hat{a}^\dagger = \ell \frac{\hat{k}_x - i \hat{k}_y}{\sqrt{2}} \quad \quad \hat{a}^\dagger u_n \quad = \quad \sqrt{n+1} u_{n+1}$$

$$\hat{a} = \ell \frac{\hat{k}_x + i \hat{k}_y}{\sqrt{2}} \quad \quad \hat{a} u_n \quad = \quad \sqrt{n} u_{n-1}$$

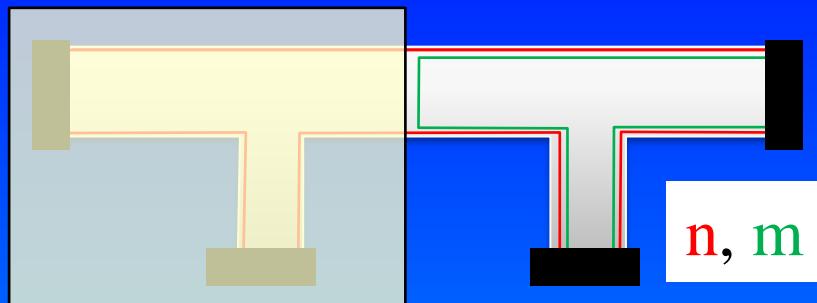
$$H_{SO} = \begin{pmatrix} \gamma_D k_z \left[ (a^\dagger)^2 + a^2 \right] & \frac{\gamma_D}{\sqrt{2}} \left[ aa^\dagger a - \left( a^\dagger \right)^3 - 2k_z^2 a \right] + i\sqrt{2}\gamma_R \mathcal{E} a^\dagger \\ \frac{\gamma_D}{\sqrt{2}} \left[ a^\dagger a a^\dagger - a^3 - 2k_z^2 a^\dagger \right] - i\sqrt{2}\gamma_R \mathcal{E} a & \gamma_D k_z \left[ (a^\dagger)^2 + a^2 \right] \end{pmatrix}$$

$$E_\pm = \frac{E_{1,\uparrow}^0 + E_{0,\downarrow}^0 \pm \sqrt{\left(E_{1,\uparrow}^0 - E_{0,\downarrow}^0\right)^2 + 8\gamma_R^2 \mathcal{E}^2}}{2}$$

$$\delta = 2\sqrt{2}\gamma_R \mathcal{E} \approx 100\mu eV$$

Only Rashba coupling contributes to N=1 and N+2 anticrossing

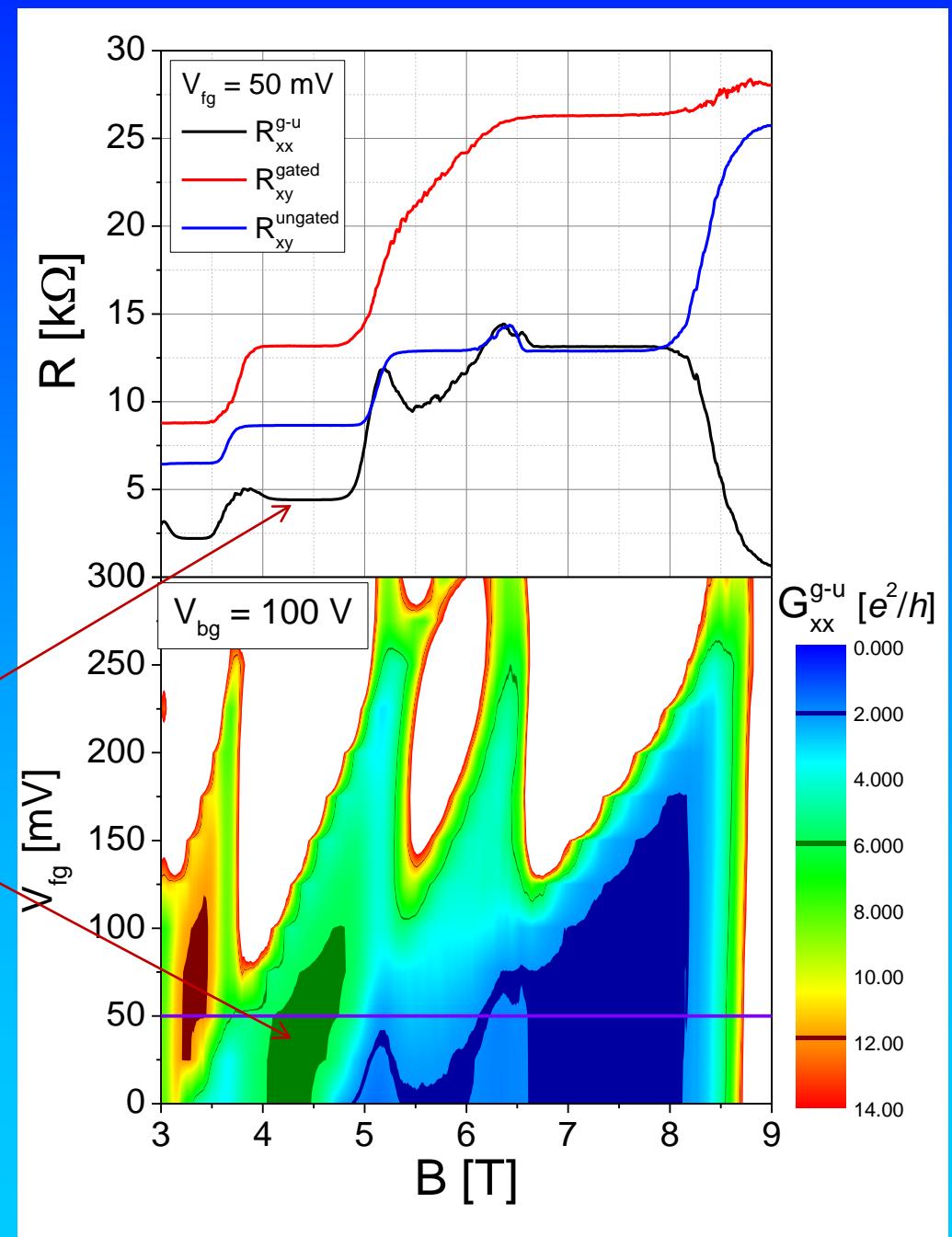
# Transport across a gate



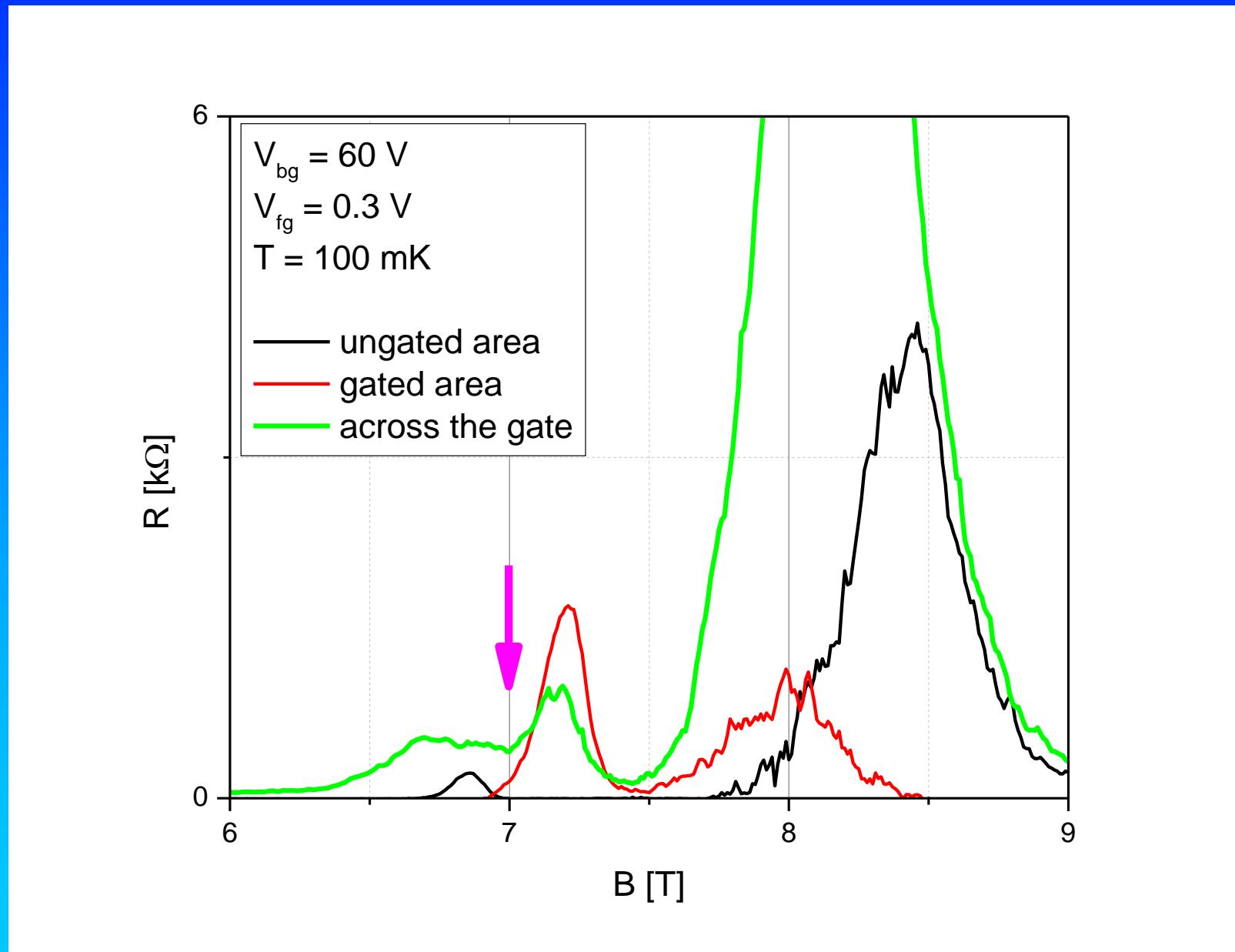
Landauer-Buttiker:

$$G = G_0 \frac{n(n + m)}{m}$$

$\nu = 2$  and  $\nu = 3$   
 $(n = 2, m = 1)$   
 $G = 6G_0$



# *Transport across QHFn domain wall*



# *people involved*



Aleksander Kazakov, Purdue University



Tomasz Wojtowicz  
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