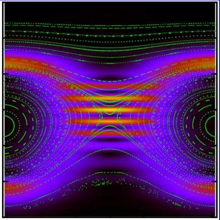
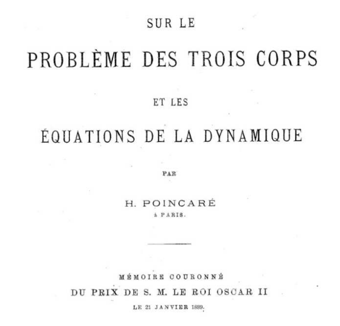
Poincaré sections and zero-resistance states





SUR LE

PROBLÈME DES TROIS CORPS

ET LES

ÉQUATIONS DE LA DYNAMIQUE

PAR

H. POINCARÉ

À PARIS.

MÉMOIRE COURONNÉ

DU PRIX DE S. M. LE ROI OSCAR II

LE 21 JANVIER 1889.

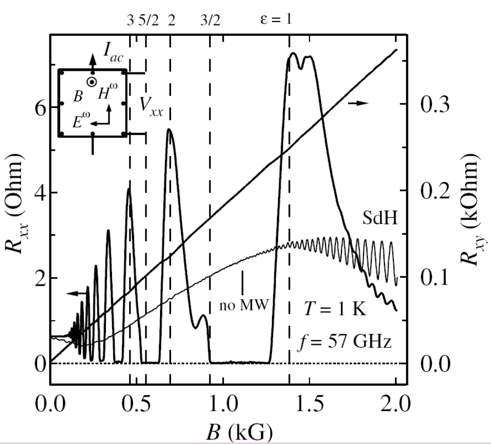
H.Poincaré Acta Math. **13**, 1 (1890)

Alexei Chepelianskii (CNRS Orsay)
and
Dima Shepelyansky (CNRS Toulouse)

discussions: O.V.Zhirov (BINP Novosibirsk)

continuation of arxiv:0905.0593 (2009); arXiv:1302.2778 (2013)

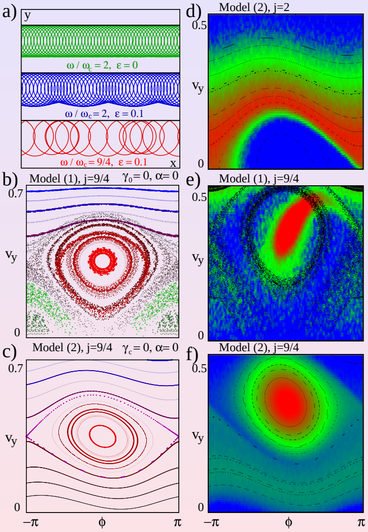
Zero resistance states discovery in 2002



- $R_{xx} \rightarrow$ zeros at high $j = \omega/\omega_c \approx integer + 1/4$
- $R_{xx}/R_{xx}(0) = 1$ at $j = \omega/\omega_c = integer$
- weak microwave field $\sim 1V/cm \rightarrow \epsilon = v_{osc}/v_F = eE/(m\omega v_F) = 0.003$
- High mobility 2DEG $\ell \approx 140\mu m$, Landau level $\nu = 62$, $\omega/2\pi = 50GHz$, $n_c = 3.5 \cdot 10^{11} cm^{-2}$, $B = 0.1T$, $r_c = v_F/\omega_c = 0.9\mu m$
- Temperature of about 1K

- R.G. Mani, J.H. Smet, K. von Klitzing, V. Narayanamurti, W.B. Johnson and V. Umansky, Nature **420**, 646 (2002).
- M.A.Zudov, R.R.Du, L. N. Pfeiffer and K. W. West PRL **90**, 046807 (2003).

Edge transport and Chirikov standard map



Electron motion along a rigid wall + microwave, Newton equations (model 1), map description

Approximate description of a nonlinear resonance, $j \approx integer + 1/4$

velocity change at wall collision: double wall velocity

small angles near wall: time between collisions $\Delta t = 2(\pi - v_y)/\omega_c$

this leads to the Chirikov standard map :

$$\left\{ \begin{array}{c} v_y(n+1) = v_y(n) + 2\epsilon \sin\phi(n) + I_{cc} \\ \phi(n+1) = \phi(n) + 2(\pi - v_y(n+1))\omega_/\omega_c \end{array} \right.$$

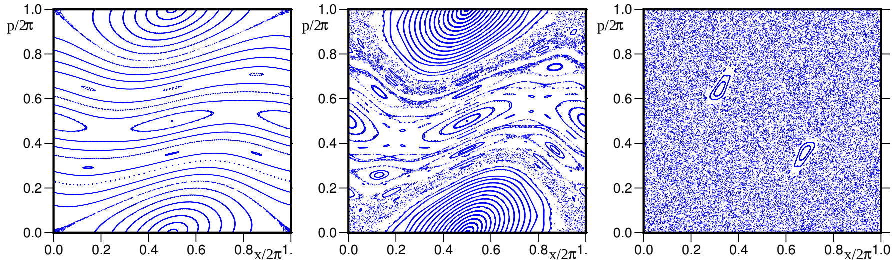
model 2, I_{cc} describes noise and dissipation

experimental confirmation by A.D.Levin *et al.* PRB **89**, 161304(R) (2014)

Chirikov standard map: integrability and chaos

$\bar{y} = y + K \sin x, \quad \bar{x} = x + \bar{y}, \qquad K \to 4\epsilon j;$

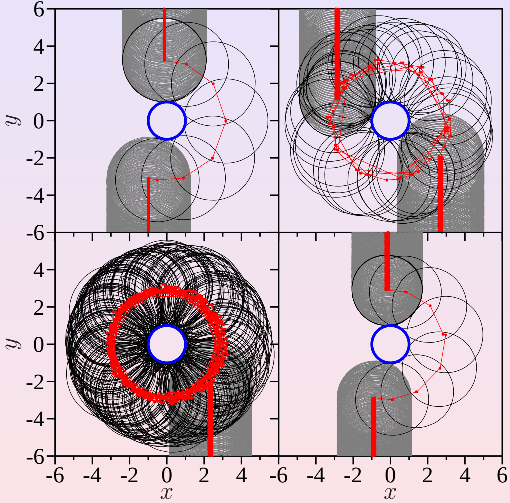
$K = 0.5; \; 0.971635; \; 5$



B.V.Chirikov Preprint N 267, Institute of Nuclear Physics, Novosibirsk (1969); Phys. Reports **52**, 263 (1979)

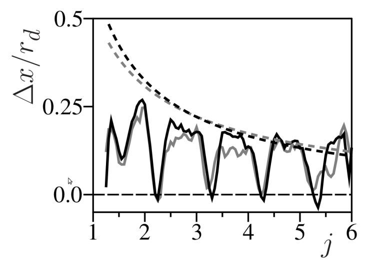
appears in plasma confinement, accelerator beams, cold atoms in optical lattices, comets in Solar System, Frenkel-Kontorova model of CDW, ...
classical and quantum evolution

Microwave induced scattering on disk



Disk radius $r_d \sim r_c$
Top left: $\epsilon = 0$, $j = 9/4$;
top right: temporary captured path at $\epsilon = 0.04$, $j = 9/4$;
bottom left: path captured forever at $\epsilon = 0.04$, $j = 9/4$;
bottom right: no capture at $\epsilon = 0.04$, $j = 2$;
dissipation at disk collisions $\gamma_d = 0.01$.

Classical conductivity calculations



Δx displacement of the cylcotron orbit after collision with the disc

$\Delta x \propto R_{xx}$, minimum at $j = n + 1/4$.

Small impurity $\rightarrow r_d \ll r_c$

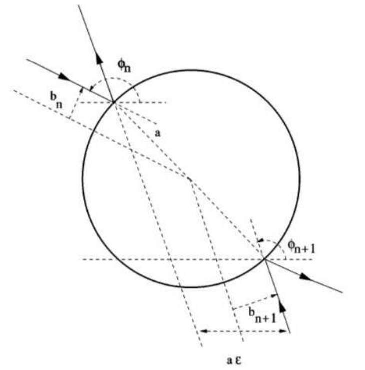
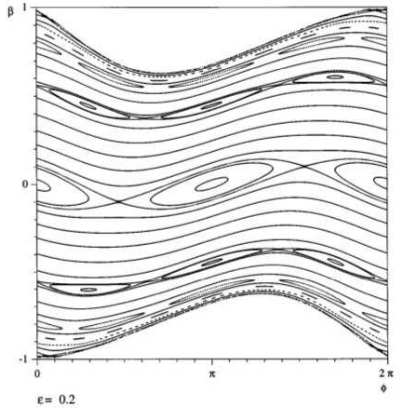


FIG. 1. Geometry of two successive collisions with the hard disk. The impact parameter b_n and incoming angle ϕ_n ($\phi_n > 3\pi/2$ in this example) define collision number n.

static electric field, small disk, magnetic field
N.Berglund *et al.*
PRL **77**, 2149 (1996)



Larmor center shift during one Larmor period: $\epsilon = v_d 2\pi/\omega_c$; β is impact parameter

$$\phi_{n+1} = \phi_n + \pi - 2\sin^{-1}\beta_n$$
$$\beta_{n+1} = \beta_n - \epsilon \sin\phi_{n+1}$$

Small disk and circular polarized microwave

Hamiltonian in the rotating frame ($e = m = 1$):
$H = {p_r}^2/2m + {p_\phi}^2/2mr^2 + p_\phi \omega_c/2 + m{\omega_c}^2 r^2/8 + U_d(r) - Er\cos\phi - \omega p_\phi$

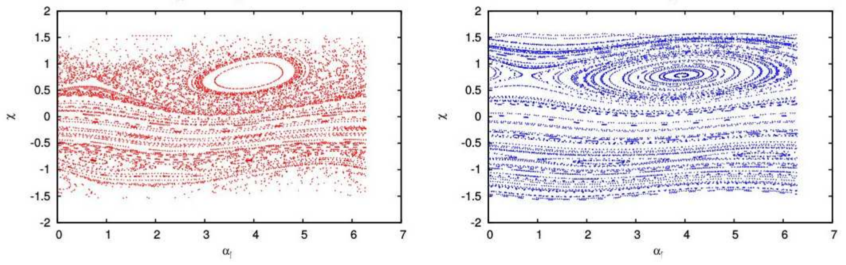
Approximate symplectic map (bar marks new value):
$\bar{\chi} = \chi + R_J(\alpha)$
$\bar{\alpha} = \alpha + 2\bar{\chi} - 2\pi j + \pi$
$R_J(\alpha) = \arg[1 + \eta \exp(-i\alpha)[\exp(2\pi i|j|) - 1]] \propto \mu$
$\eta = eE/(m\omega^2 r_d)$

velocity angle α is analog of impact angle β
$p_\phi \sim \sin\chi$ is analog of impact parameter b

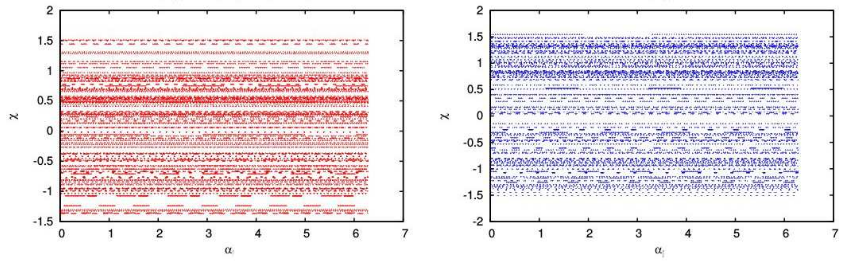
New features:
* kick amplitude is proportional to $\eta = [eE/(m\omega^2 r_c)][r_c/r_d]$
  with the enhancement factor $r_c/r_d \gg 1$
* kick amplitude is zero at $j = integer$

Poincaré sections for circular polarization

Poincaré sections: left - Newton equations; right - symplectic disk map



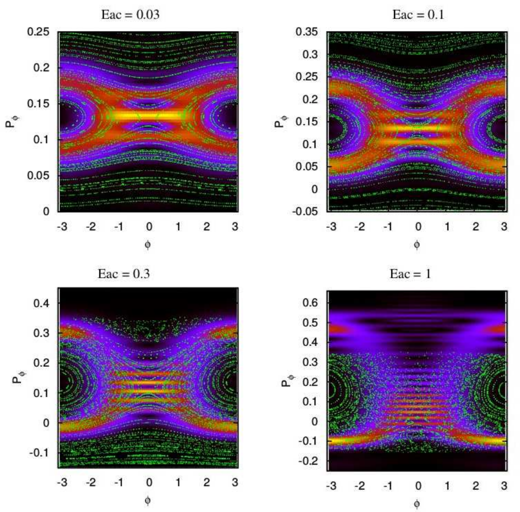
$j = 4.75$ (top) and $j = 3$ (bottom), $E = 0.01$, $r_d/r_c = 0.1$



Quantum impurity and quantum Poincaré sections

Numerical solution of stationary Schödinger equation in rotation frame

Husimi function (Gaussian smoothing of Wigner function) relation between classical and quantum variables:
$p_\phi \omega_c / E_F = \hbar \omega_c l_z / E_F$;
$r_d / r_c = r_d / (\ell_B \sqrt{2\nu})$;
$E_{ac} \ell_B / \hbar \omega_c = E r_c \sqrt{2\nu} / 2 E_F$
In numerics $\ell_B = 1$, $r_d / \ell_B = 2$,
$j = 2.75$, $\nu = 45$,
$E_{ac} = 0.3 \rightarrow E/(\omega v_F) = 1\%$

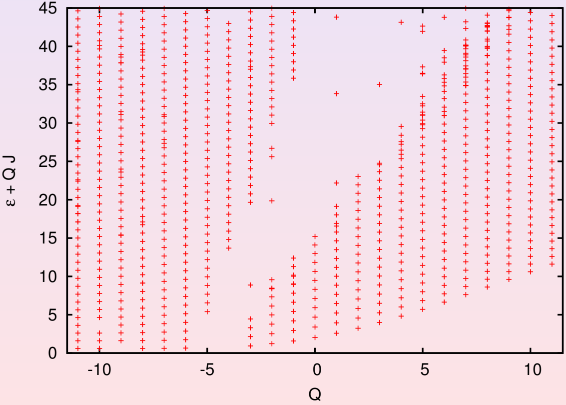


Formation of a topological charge gap ?

A winding number to generalize the orbital momentum quantum number:

$$Q = \lim_{r \to r_d^+} \frac{1}{2\pi i} \int \frac{1}{\psi(r,\theta)} \frac{d\psi(r,\theta)}{d\theta} d\theta$$

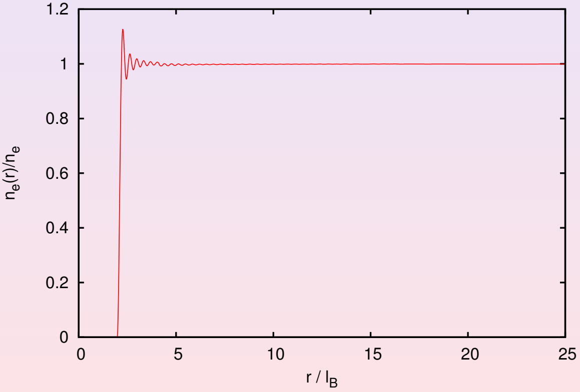
Eigenenergies as function of Q at $J = 2.75$



Friedel oscillations near an impurity

Friedel oscillations around an impurity without microwaves :

$$n_e(r) = \sum_{\epsilon_n < E_F} |\psi_n(r)|^2$$

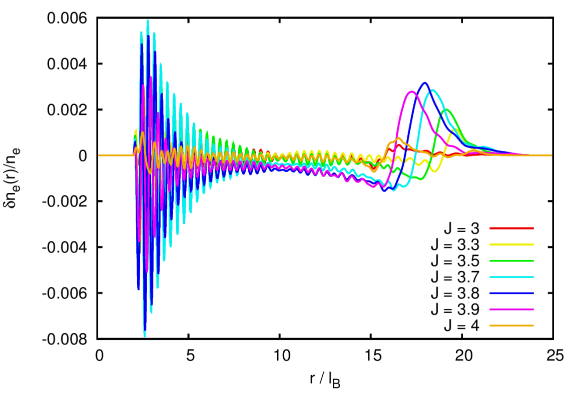


Microwave induced Friedel oscillations near impurity

Numerical solution of master equation for density matrix with relaxation:

$\partial\hat{\rho}/\partial t = -i[\hat{H(t)}, \hat{\rho}]/\hbar - (\hat{\rho} - \hat{\rho_{eq}})/\tau$

Here $\ell_B = 1$, $r_d/\ell_B = 2$, $\nu = 40$, $E_{ac} = 0.2 \to \epsilon = E/(\omega v_F) = 6 \times 10^{-3}$

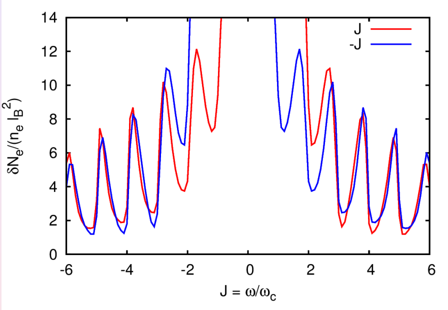


Electron density variation with distance to impurity: many-body screening ?

Total displaced charge around the impurity

$$\delta N_e = \int dr 2\pi r |\rho(r) - \rho_{eq}(r)|$$

Dependence on $J = \omega/\omega_c$ ($J > 0$ right-handed polarization, $J < 0$ left handed):



Slow decay with $J = \omega/\omega_c$
For $|J| > 2$ independent on polarization chirality (experiment by J. Smeth et.al.).
$\delta N_e \sim 10^3$: detectable experimentally

Theoretical conditions for ZRS samples

Upper bound on impurity density n_i:

$n_i \lambda_F v_F \tau_{scat} \sim 1; \quad n_i \sim 1/(\lambda_F v_F \tau_{scat}) \sim \mu m/(e \lambda_F v_F)$

Single impurity scattering takes place when Larmor diameter is smaller than impurity distance:

$r_c < 1/\sqrt{n_i} \sim \sqrt{\lambda_F \ell_e}\,; \quad \ell_e = v_F \tau_{scat}$

Samples like Mani (2002) - Zudov (2003)

$\mu = 2.5 \times 10^7 cm^2/Vs$, $n_e = 3.5 \times 10^{11} cm^{-2}$, $\sqrt{\lambda_F \ell_e} \approx 3 \mu m$

Single impurity scattering at $B > 0.03 Tesla$

Sample like Bykov (2006)

$\mu = 0.6 \times 10^6 cm^2/Vs$, $n_e = 8.5 \times 10^{11} cm^{-2}$, $\sqrt{\lambda_F \ell_e} \approx 0.6 \mu m$

Single impurity scattering at $B > 0.25 Tesla$

Conclusions

- Microwaves can stabilize edge trajectories against small angle disorder scattering
- Microwaves induce Friedel like density oscillations near impurity
- Nonlinear dynamics near impurity plays an important role: periodicity in j, no excitation at integer $j = \omega/\omega_c$
- Classical simulations suggest R_{xx} minimum at $j = n + 1/4$
- Quasi-classical effect : less sensitive to a short quantum lifetime
- Possible microwave topological protection against impurity scattering ?
- Further research: quantum computation of resistivity ...