Three Body Mean Motion Resonances

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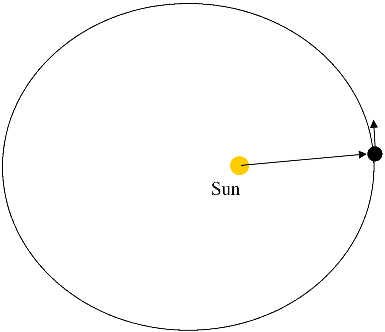
- preliminaries
- types of three body resonances (3BRs)
- semi analytical method
- numerical studies
  - dynamical maps
  - induced migration

Preliminaries

Preliminaries

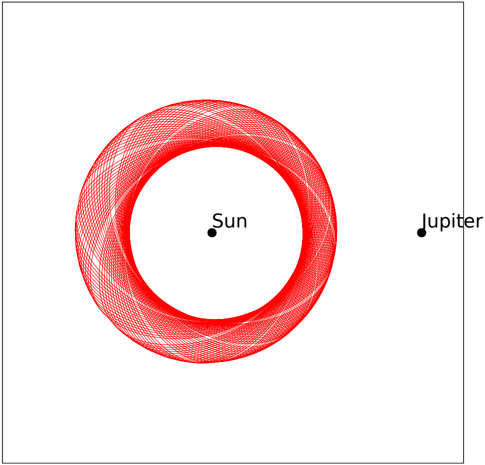
e = eccentricity
a = semimajor axis (in astronomical units)
n = mean motion = mean angular velocity = $\frac{2\pi}{\text{period}} \propto \frac{1}{a^{3/2}}$



Two body resonance: $k_0 n_0 + k_1 n_1 \simeq 0$
with k_0, k_1 integers.

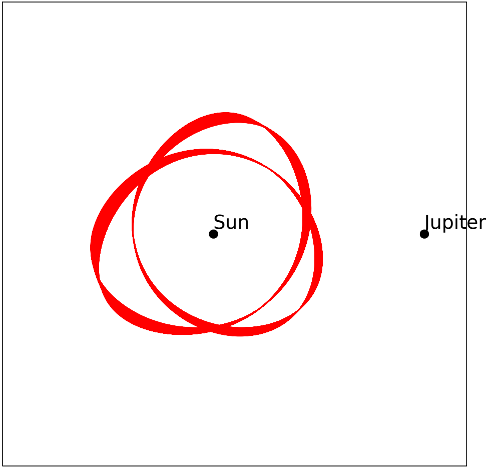
Non resonant asteroid: relative positions

Mean perturbation is radial: Sun-Jupiter

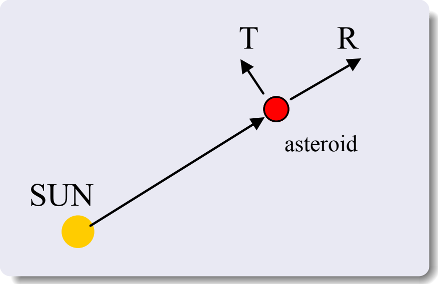


Resonant asteroid

Mean perturbation has a **transverse** component.



from Gauss equations



$$F_{perturb} = (R, T, N)$$

$$\frac{da}{dt} \propto (R, T)$$

$$< \frac{da}{dt} > \propto T$$

Non resonant

$T = 0 \Rightarrow a =$ constant

Resonant

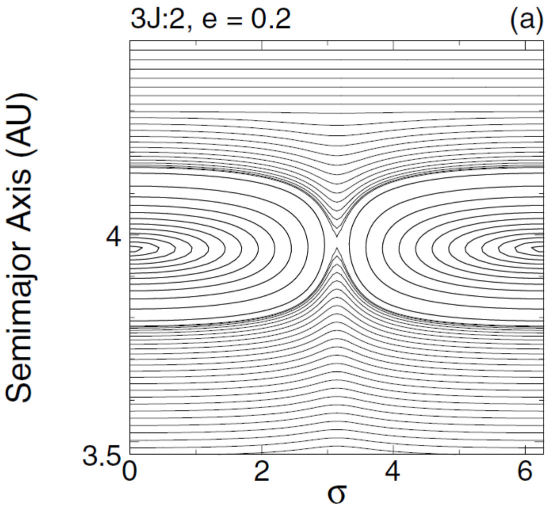
$T \neq 0 \Rightarrow a =$ oscillating

Critical angle σ

For resonance $k_0 n_0 + k_1 n_1 \simeq 0$, is defined:

- $\sigma = k_0\lambda_0 + k_1\lambda_1 + \gamma(\varpi_0, \varpi_1)$
- the λ's are quick varying angles (mean longitudes)
- $\gamma(\varpi_0, \varpi_1)$ is a linear combination of slow varying angles
- $\sigma(t)$ indicates if the motion is resonant or not:
  - $\sigma(t)$ oscillating means resonance
  - $\sigma(t)$ circulating means NO resonance
- resonant motion: $a(t)$ is correlated with $\sigma(t)$

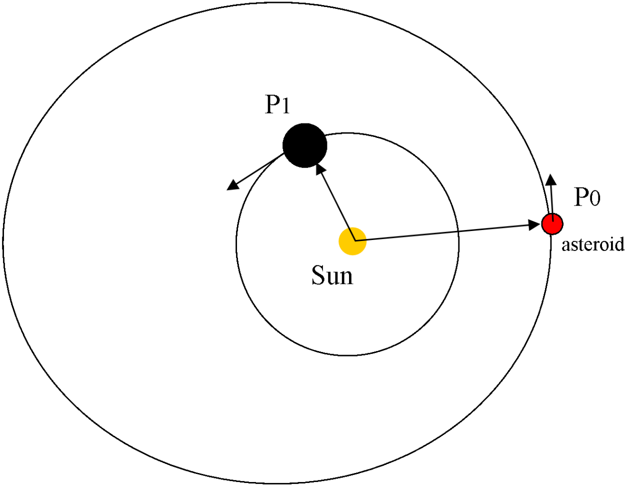
Semimajor axis: width



Nesvorny et al. in Asteroids III

Two body resonance, restricted case $m_0 = 0$

$k_0 n_0 + k_1 n_1 \simeq 0$ P_1 does not feel the resonance, only P_0



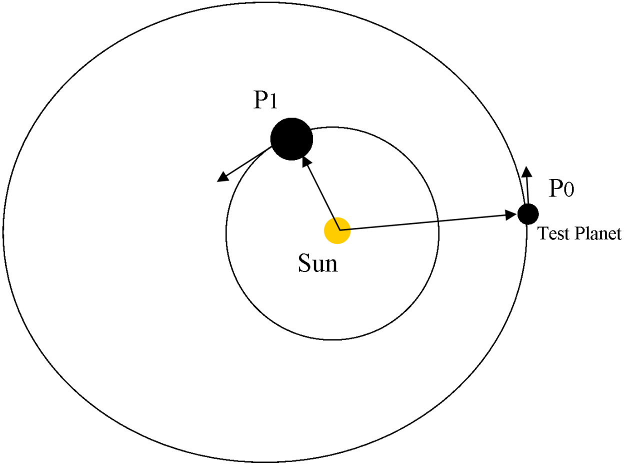
Two body resonance

$k_0 n_0 + k_1 n_1 \simeq 0$

- Order: $q = |k_0 + k_1|$
- Strength of resonance is approximately $\propto C m_1 e^q$
- Theories try to obtain expressions for coefficients C
- Strength is related with amplitude of $a(t)$

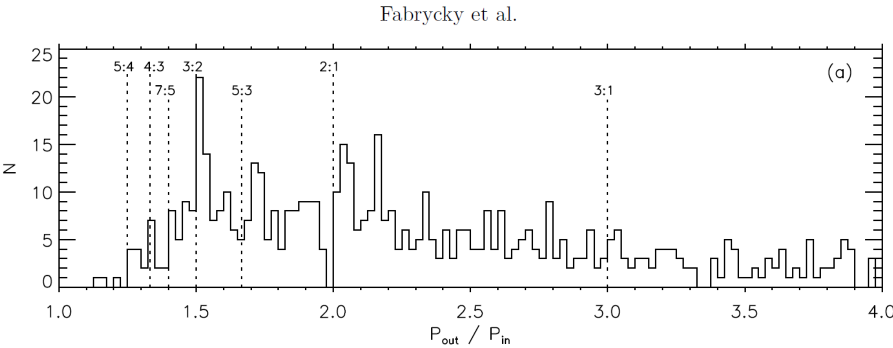
Two body resonance, planetary case $m_0 \neq 0$

$k_0 n_0 + k_1 n_1 \simeq 0$ both P_0 and P_1 feel the resonance



Two body resonance, planetary case

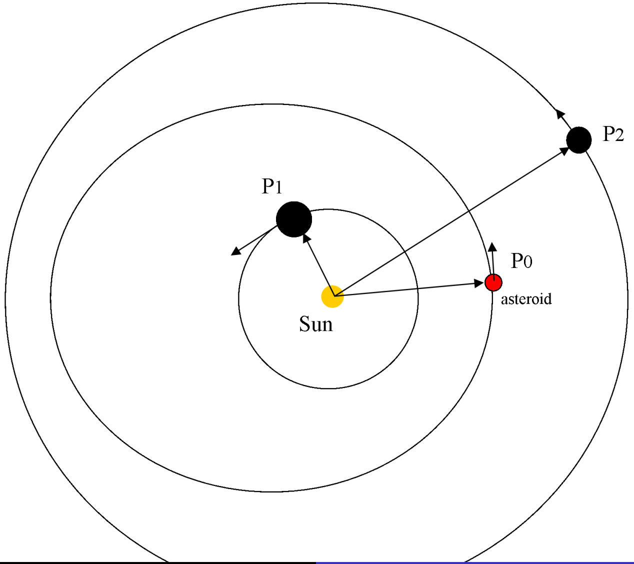
Observational evidence in extrasolar systems



Three body resonances

Three body resonance, restricted case $m_0 = 0$

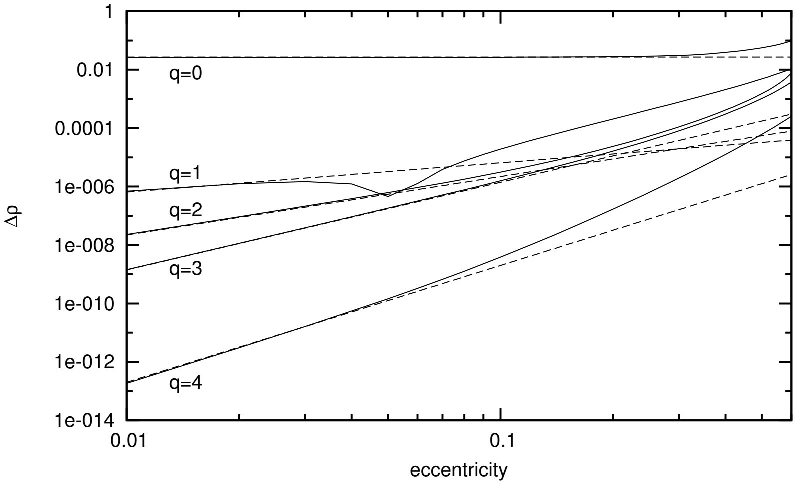
$k_0 n_0 + k_1 n_1 + k_2 n_2 \simeq 0$ only P_0 feels the resonance



Three body resonance

- Order: $q = |k_0 + k_1 + k_2|$
- Strength of resonance is approximately $\propto Cm_1m_2e^q$
- 3BRs are weaker than 2BRs ($m_1m_2 << m_1$)
- Theories try to obtain expressions for coefficients C
- Only planar theories have been developed

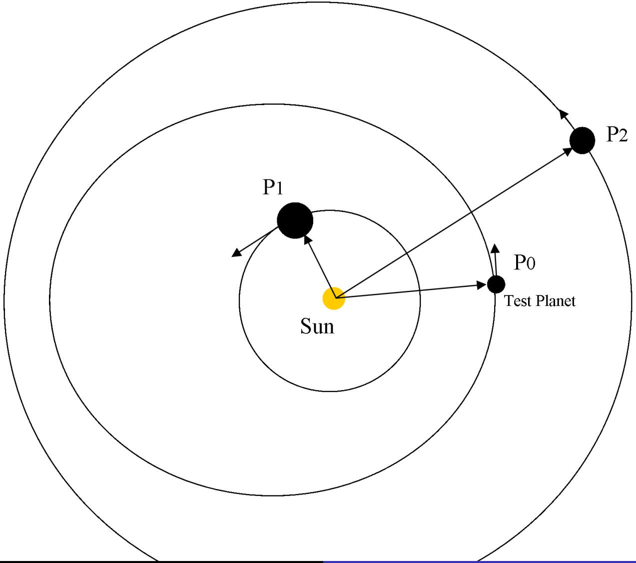
Strength and eccentricity



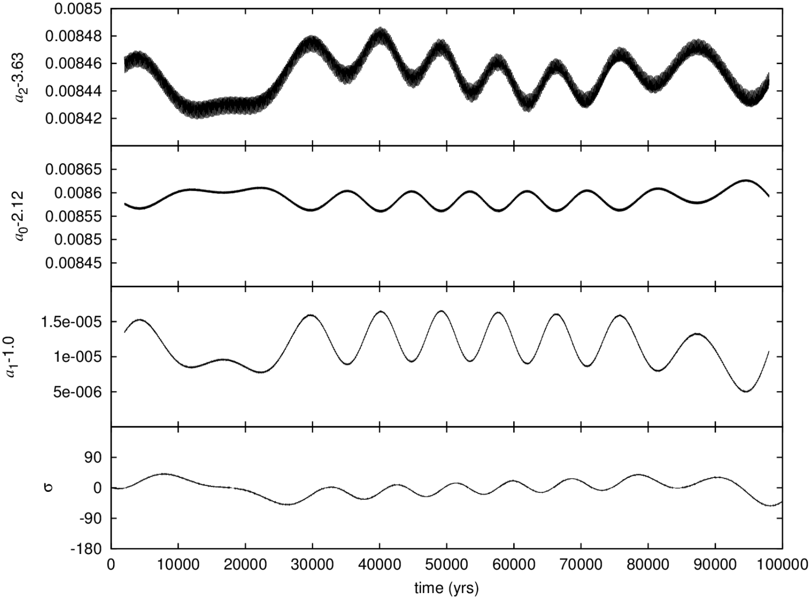
for low e strength $\propto e^q$

Three body resonance, planetary case $m_0 \neq 0$

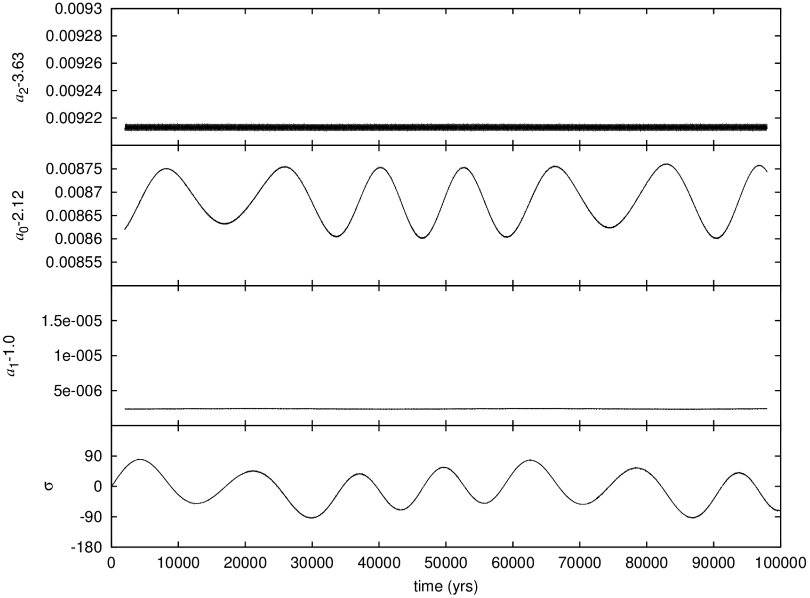
$k_0 n_0 + k_1 n_1 + k_2 n_2 \simeq 0$ all three bodies feel the resonance



3BR $4n_0 - 1n_1 - 2n_2$, planetary case $m_0 \neq 0$



3BR $4n_0 - 1n_1 - 2n_2$, restricted case $m_0 = 0$



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Three body resonance

$$k_0 n_0 + k_1 n_1 + k_2 n_2 \simeq 0$$

It is not necessary to have a chain of 2BRs:

- P_0 and P_1 not in two body resonance
- P_0 and P_2 not in two body resonance
- P_2 and P_1 not in two body resonance

but...

1784: Laplacian resonance

$$3\lambda_{Europa} - \lambda_{Io} - 2\lambda_{Ganymede} \simeq 180^\circ$$

$$3n_{Europa} - n_{Io} - 2n_{Ganymede} \simeq 0$$

They are also in commensurability by pairs:

$$2n_{Europa} - n_{Io} \simeq 0$$

$$2n_{Ganymede} - n_{Europa} \simeq 0$$

$$\Downarrow$$

It must be the consequence of some physical mechanism.

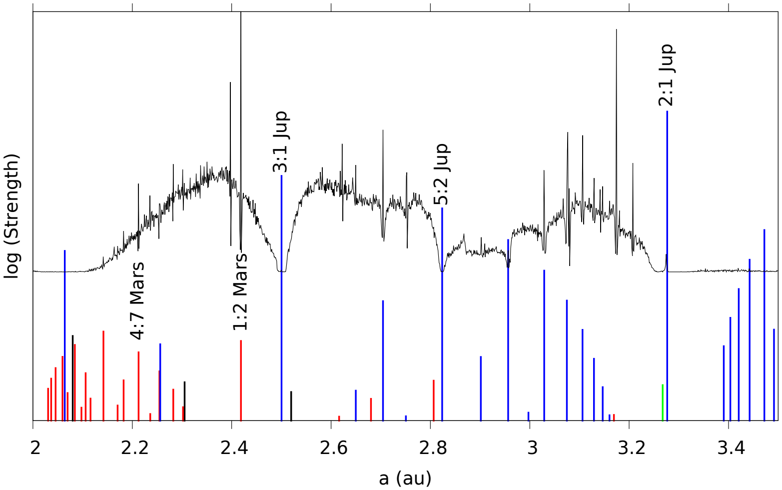
Two types of 3BRs

Three body resonance as...

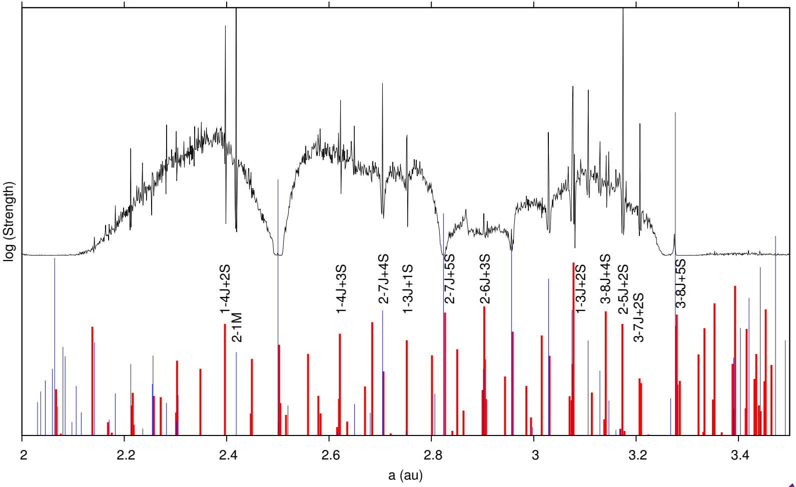
- **superposition** or **chain** of 2 two-body resonances

  - $n_I - 2n_E \sim 0$
  - $n_E - 2n_G \sim 0$
  - adding: $n_I - n_E - 2n_G \sim 0 \Rightarrow$ 3BR order 2
  - substraction: $n_I - 3n_E + 2n_G \sim 0 \Rightarrow$ 3BR order 0
- **pure**: 3BR that are NOT due to 2BR + 2BR.
  - asteroids + Jupiter + Saturn

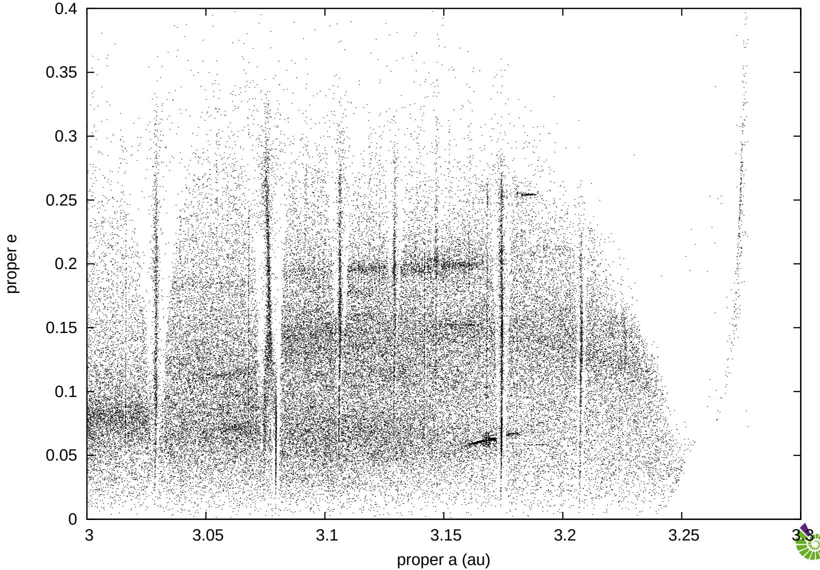
Asteroids: histogram of a + 2BRs



Asteroids: histogram of a + 3BRs



Dynamical evidence from AstDyS



Thousands of asteroids in 3BRs with Jupiter and Saturn

Icarus 222 (2013) 220–228

Contents lists available at SciVerse ScienceDirect

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Icarus

journal homepage: www.elsevier.com/locate/icarus

Massive identification of asteroids in three-body resonances

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Smirnov and Shevchenko (2013)

See next talk!

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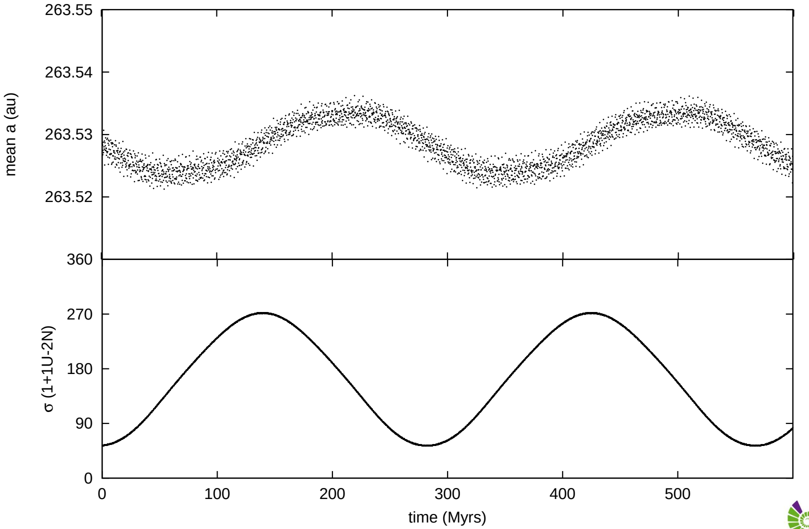
Resonance 1 + 1U−2N, a weird case



3BRs are WEAK and numerous

- Given two planets P_1 and P_2, an **infinite** family of 3BRs is defined:

$$n_0 = \frac{-k_1 n_1 - k_2 n_2}{k_0}$$

- Don't miss the "TBR Locator" for Android!
- Each resonance is defined by (k_0, k_1, k_2)
- The question is: **how strong** are they?
- They are **weak** because the perturbation that drives the resonant motion is factorized by $m_1 m_2$.
- There is a huge number of 3BRs: superposition generates **chaotic diffusion**.

Multiplet resonances and chaos

$$\sigma = k_0\lambda_0 + k_1\lambda_1 + k_2\lambda_2 + k_4\varpi_0 + k_5\Omega_0$$

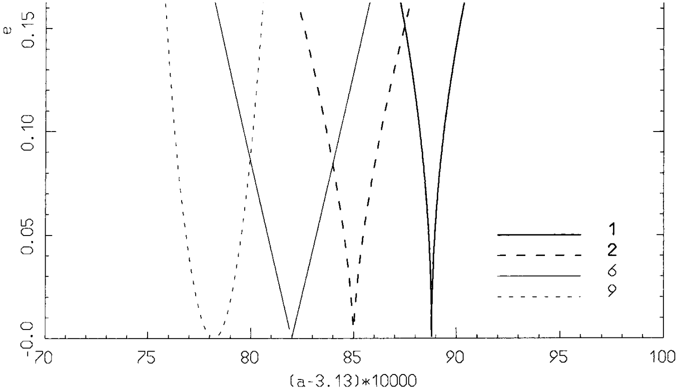
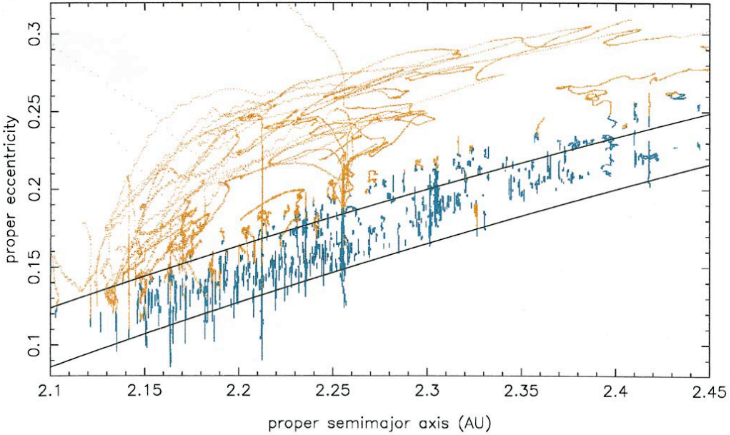


Figure 8. Separatrices of four multiplet resonances of the 6 1 − 3 three-body resonance.

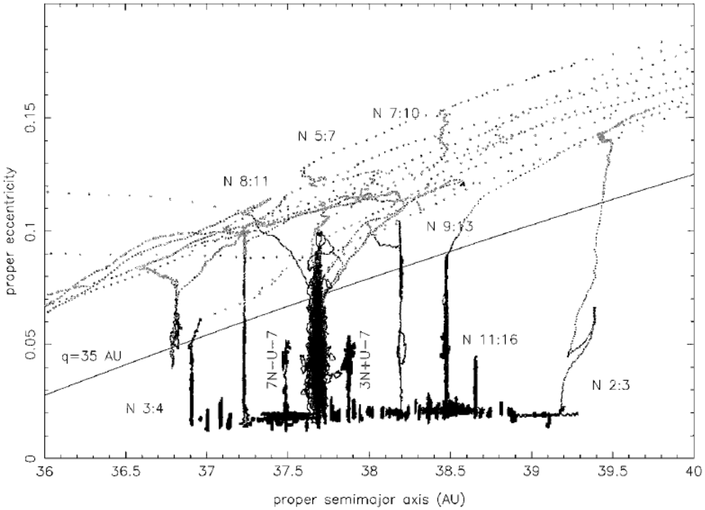
Nesvorny and Morbidelli (1999)

Chaotic diffusion: growing e



Morbidelli and Nesvorny (1999)

Chaotic diffusion in the TNR: growing *e*



Nesvorny and Roig (2001)

Three body resonances as...

- **Chains** of two body resonances
  - Galilean satellites (Sinclair 1975, Ferraz-Mello, Malhotra, Showman, Peale, Lainey...)
  - Callegari and Yokoyama (2010): satellites of Saturn
  - Extrasolar systems (Libert and Tsiganis 2011; Martí, Batygin, Morbidelli, Papaloizou, Quillen...)
- **Pure** three body resonances
  - Lazzaro et al. (1984): satellites of Uranus
  - Aksnes (1988): zero order asteroidal resonances
  - Nesvorny y Morbidelli (1999): theory Jupiter-Saturn-asteroid
  - Cachucho et al. (2010): diffusion in 5J -2S -2.
  - Quillen (2011): zero order extrasolar systems
  - Gallardo (2014), Gallardo et al. (2016): semianalytic
  - Showalter and Hamilton (2015): Pluto satellites $3n_S - 5n_N + 2n_H \sim 0$

Disturbing function

Disturbing function for resonance $k_0 + k_1 + k_2$:

$$R = k^2 m_1 m_2 \sum_j \mathcal{P}_j \cos(\sigma_j)$$

$$\sigma_j = k_0\lambda_0 + k_1\lambda_1 + k_2\lambda_2 + \gamma_j$$

$$\gamma_j = k_3\varpi_0 + k_4\varpi_1 + k_5\varpi_2 + k_6\Omega_0 + k_7\Omega_1 + k_8\Omega_2$$

$\mathcal{P}_j$ is a polynomial function depending on the eccentricities and inclinations which its lowest order term is

$$Ce_0^{|k_3|} e_1^{|k_4|} e_2^{|k_5|} \sin(i_0)^{|k_6|} \sin(i_1)^{|k_7|} \sin(i_2)^{|k_8|}$$

Theories are complicated...

- it is necessary to consider several $\mathcal{P}_j \cos(\sigma_j)$
- with several terms in $\mathcal{P}_j$
- calculation of the Cs is not trivial
- only planar theories exist

To avoid the difficulties of the analytical methods we proposed to calculate R numerically.

Semi analytical method

Icarus

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Atlas of three body mean motion resonances in the Solar System

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Icarus

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Planetary and satellite three body mean motion resonances

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Method

- Disturbing function is a mean over all possible resonant configurations.
- **The point:** the disturbing function R must be calculated with the **perturbed** positions.
- We cannot assume unperturbed ellipses for the three orbits.

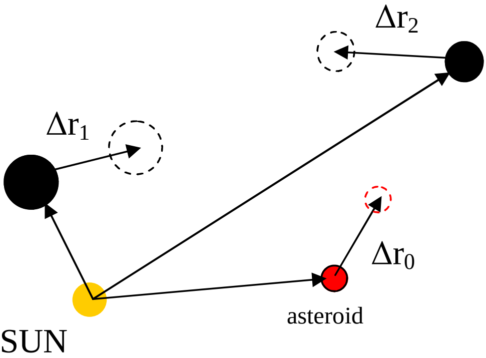
Method

For a given resonance:

- consider a large sample of configurations verifying the resonant condition ($\sigma = $ constant)
- calculate the mutual perturbations $\Delta r_0, \Delta r_1, \Delta r_2$
- calculate the effect ΔR due to $(\Delta r_0, \Delta r_1, \Delta r_2)$
- integrate all ΔR and obtain $\rho(\sigma)$
- repeat for several $\sigma \in (0, 360)$ obtaining $\rho(\sigma)$

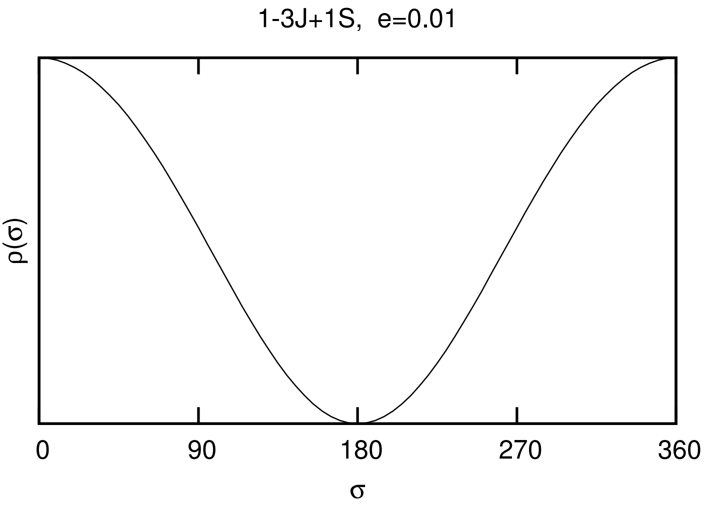
Method

Then, being in a resonant configuration

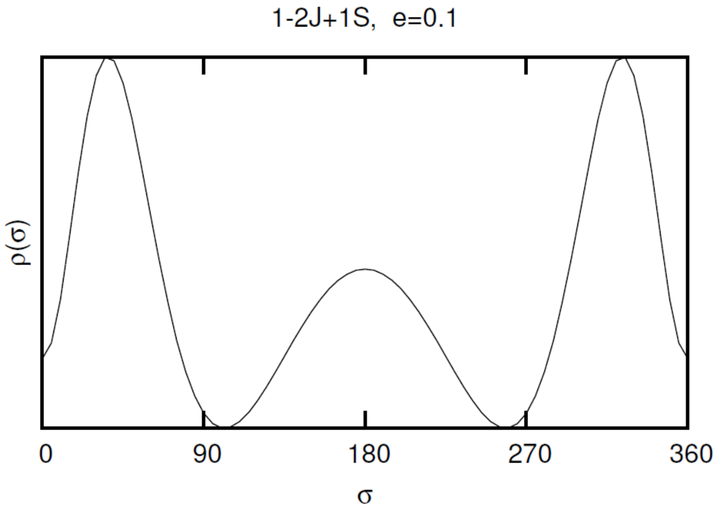


and these Δr generate the ΔR

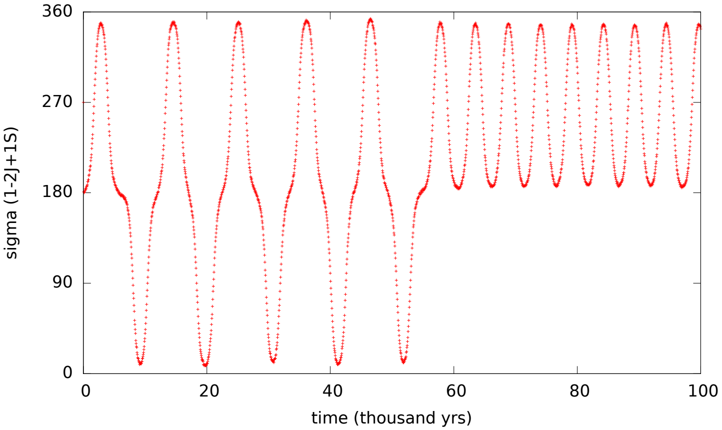
Disturbing function $\sim \rho(\sigma)$



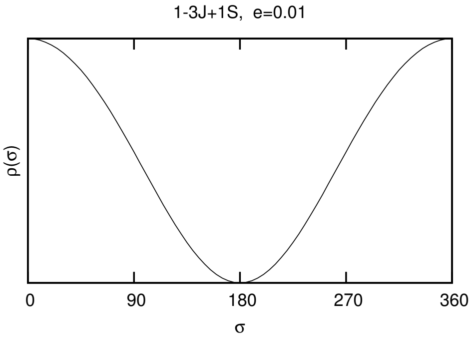
Asymmetric equilibrium points



Zero order resonance, $e \simeq 0{,}05$, $a \simeq 3{,}8$



Numerical integration of full equations of motion.



- **large variations** of ρ with σ is indicative of a **strong** resonance
- **small variations** of ρ with σ is indicative of a **weak** resonance
- an **extreme** of $\rho(\sigma)$ at some σ means there is an **equilibrium point**

Strength, S

- We numerically obtain $\rho(\sigma)$
- We define Strength

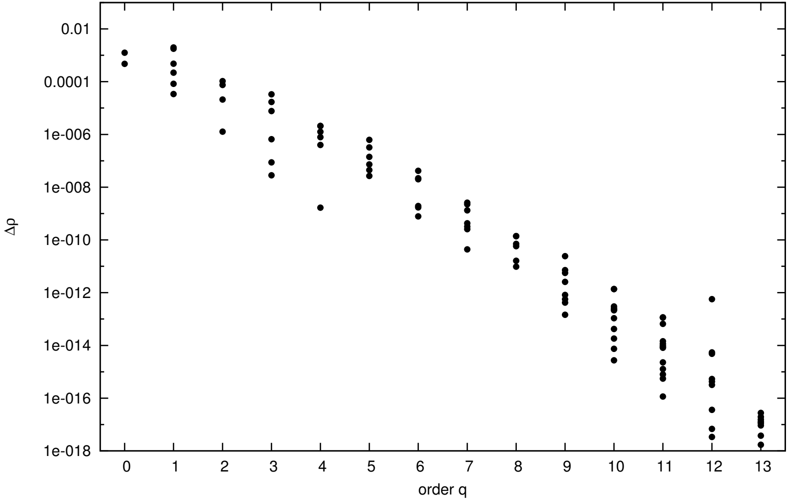
$$S = \frac{1}{2}\Delta\rho(\sigma)$$

- For planetary case we have 3 strengths

$$S_i = \frac{1}{2}\Delta\rho_i(\sigma)$$

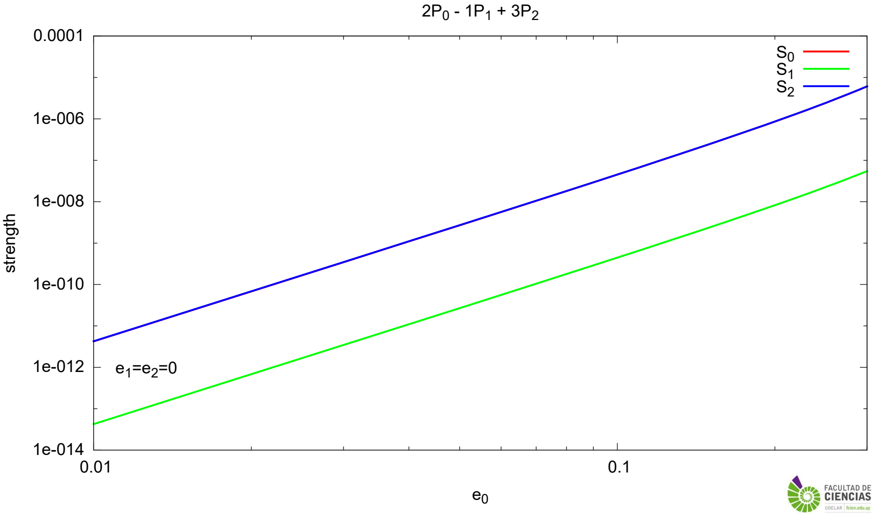
Codes: `www.fisica.edu.uy/~gallardo/atlas`

Strength and order: 3BRs with Jupiter and Saturn

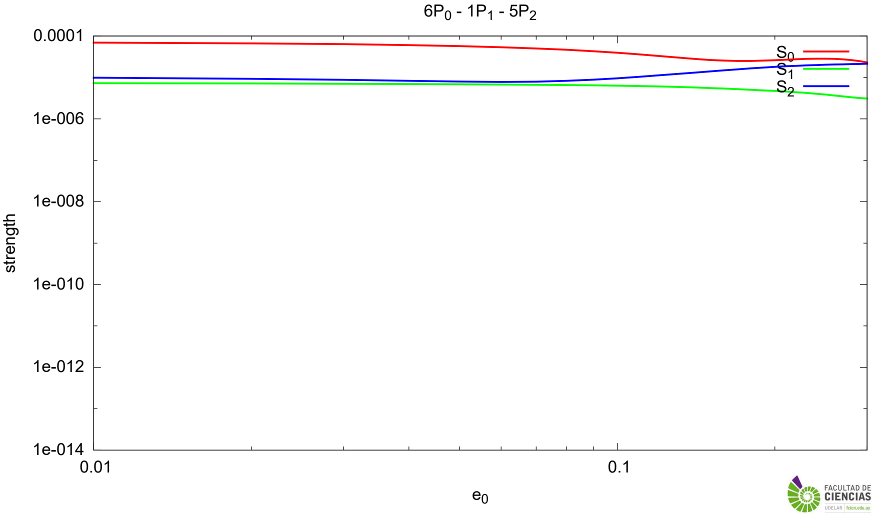


$$\log(\Delta\rho) \propto -q$$

Planetary case. Dependence on e_0. Case $q = 4$.

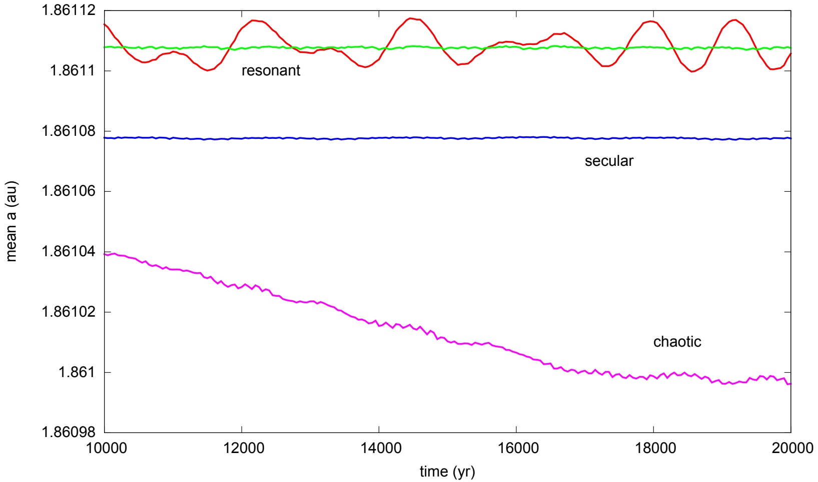


Planetary case. Dependence on e_0. Case $q = 0$.

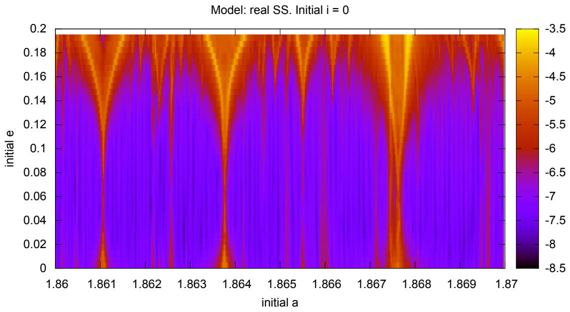


Dynamical maps

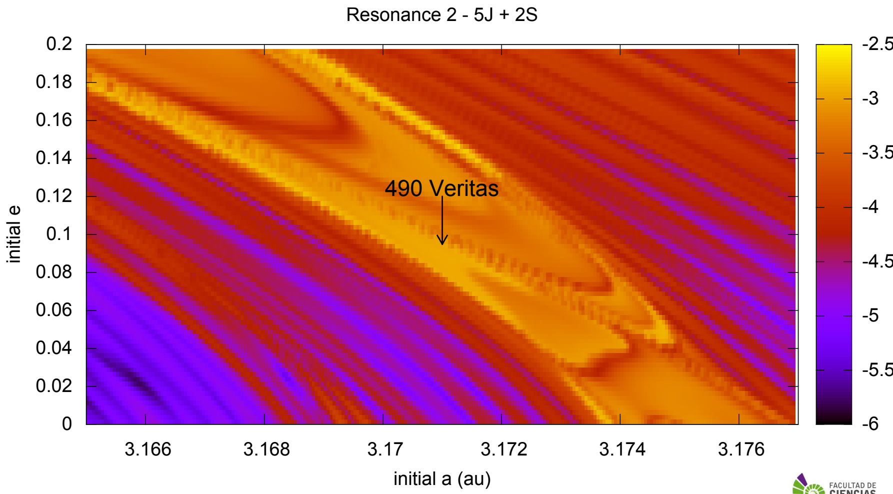
Variations in mean a



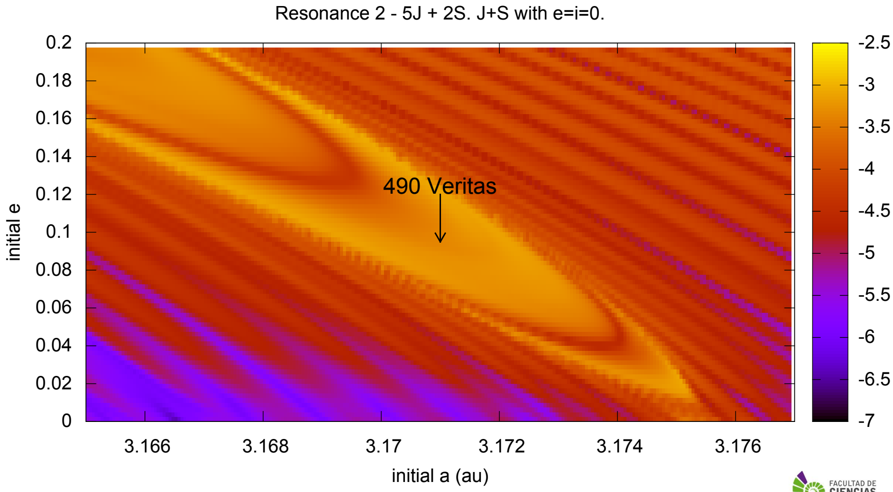
- take set of initial values (a, e)
- integrate for some 10.000 yrs
- calculate the mean $< a >$ in some interval
- calculate the variation $\Delta < a >$ (running window)
- surface plot of $\Delta < a > (a, e)$

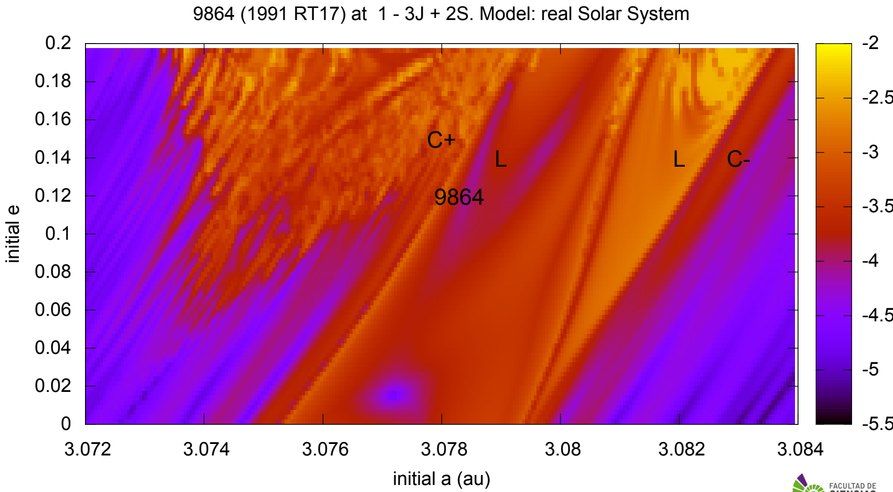


Resonance 2 - 5J + 2S. Model: real Solar System

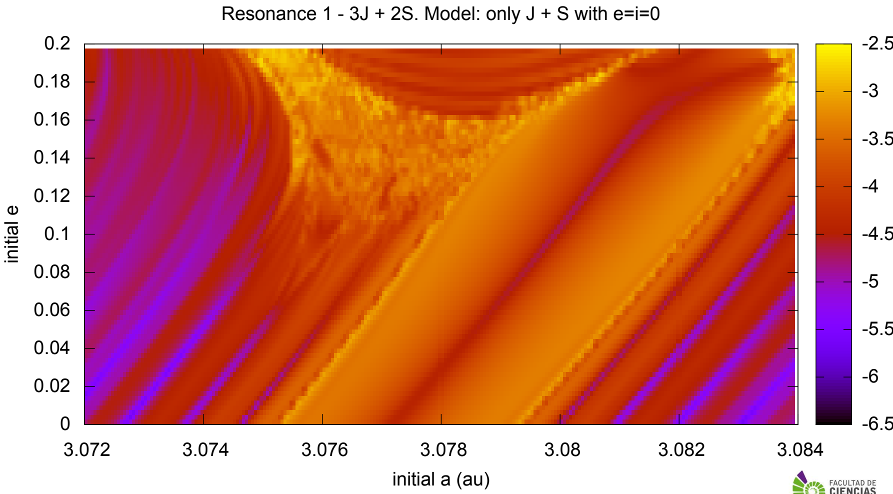


Resonance 2 - 5J + 2S. Model: J+S with circular orbits

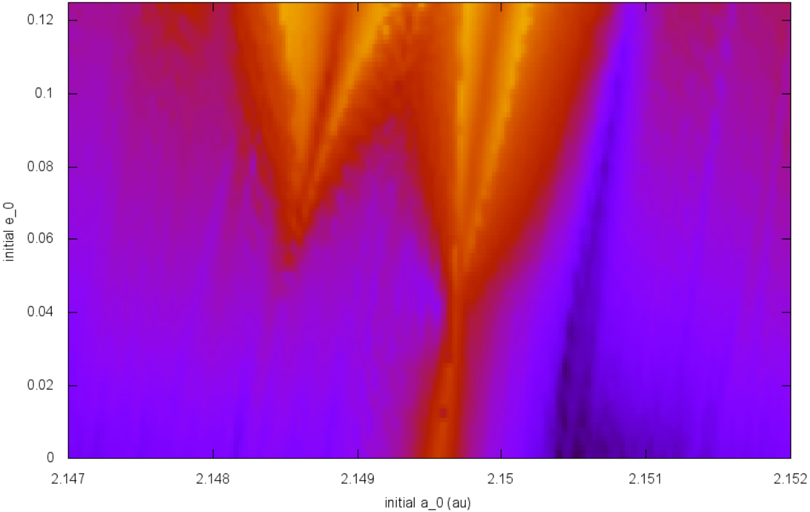



9864 (1991 RT17) at 1 - 3J + 2S. Model: real Solar System
initial e
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
initial a (au)
3.072
3.074
3.076
3.078
3.08
3.082
3.084
C+
L
9864
L
C-
-2
-2.5
-3
-3.5
-4
-4.5
-5
-5.5

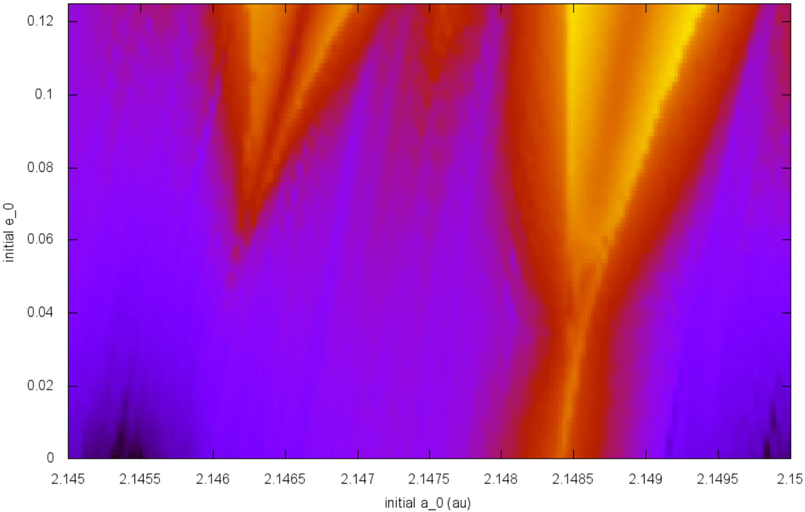
Resonance 1 - 3J + 2S. Model: J+S with circular orbits



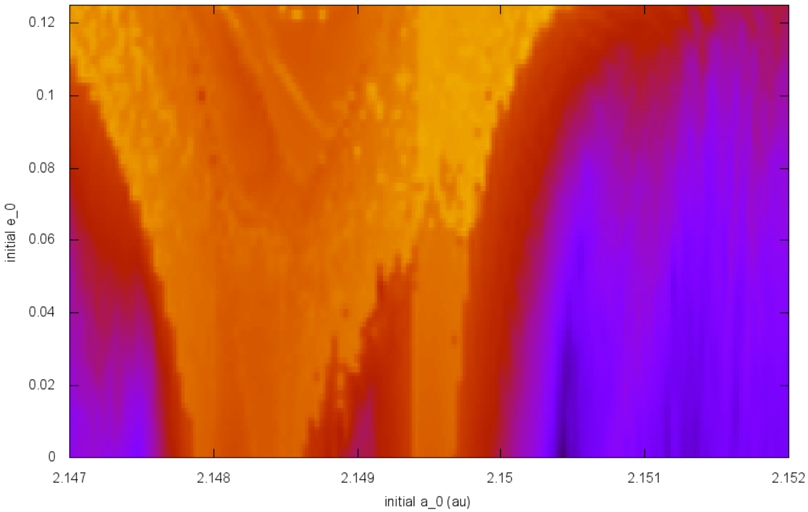
2BR $6P_0 - 13P_2$ and 3BR $5P_0 - 1P_1 - 4P_2$.



For larger m_1

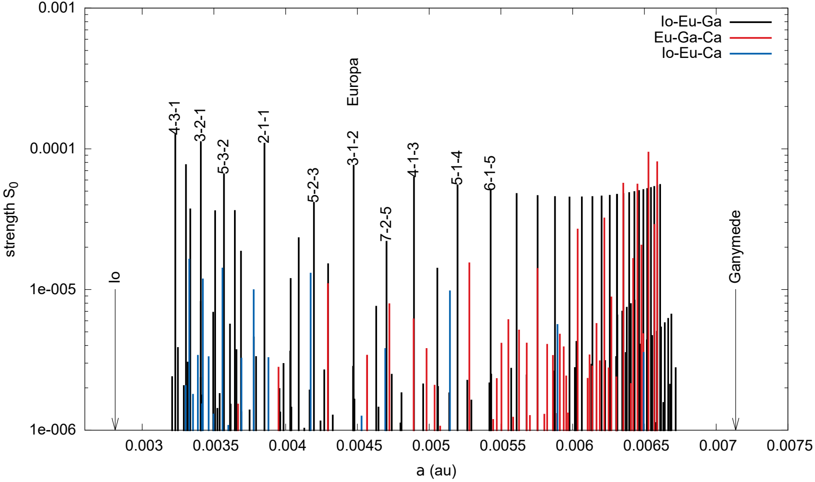


Excited orbits

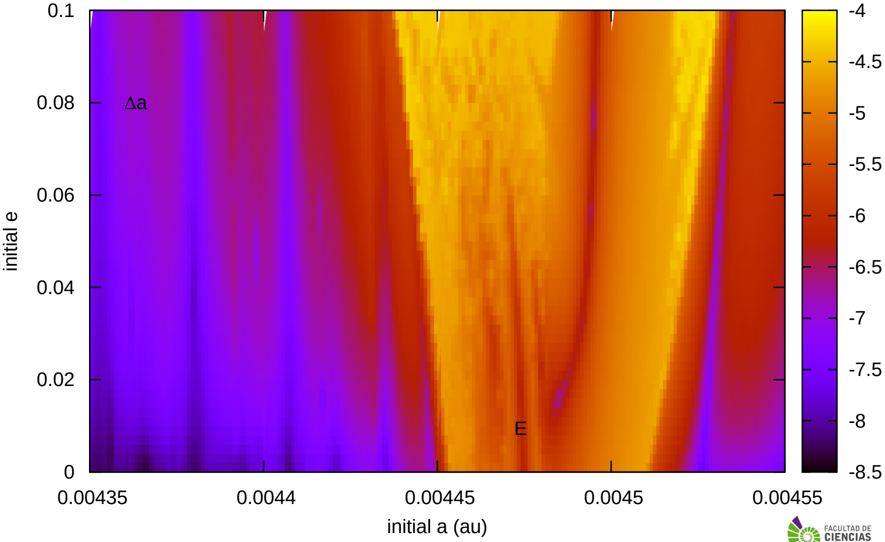


Galilean satellites

3BRs near Europa

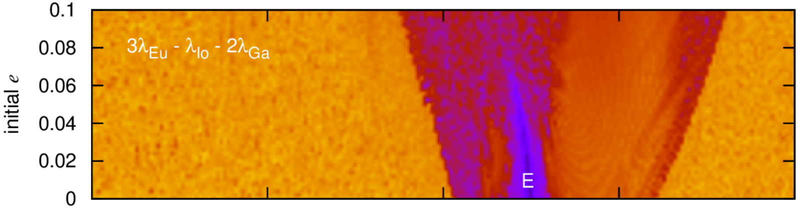


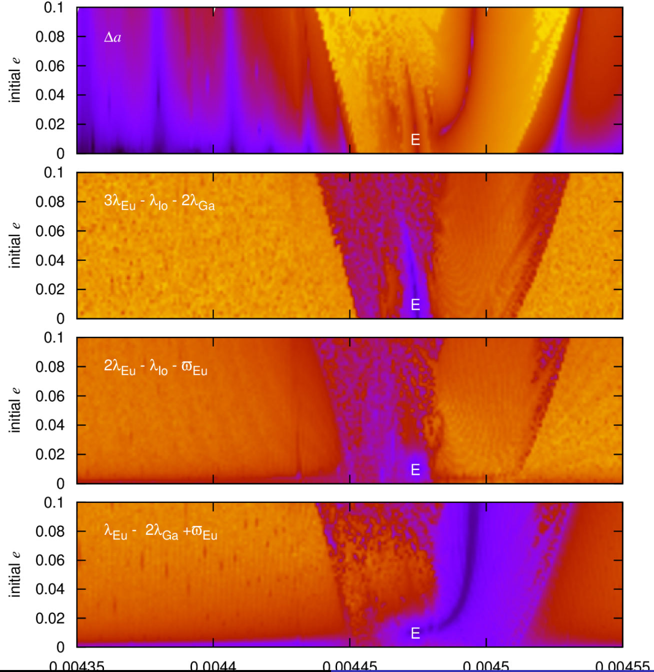
Dynamical map: $\Delta a(a, e)$



Maps for critical angles

- take set of initial values (a, e)
- integrate for some 1.000 yrs
- calculate the distribution of σ between 0 and 360
- uniform or wide distribution: circulation or large amplitude oscillations
- narrow distribution: small amplitude oscillations



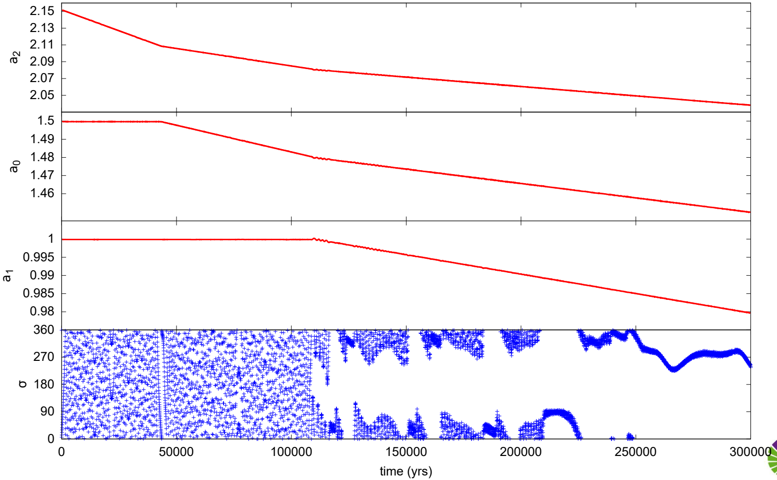


Δa
3λEu - λIo - 2λGa
2λEu - λIo - ϖEu
λEu - 2λGa +ϖEu
E
initial e
0.1
0.08
0.06
0.04
0.02
0
0.00435
0.0044
0.00445
0.0045
0.00455

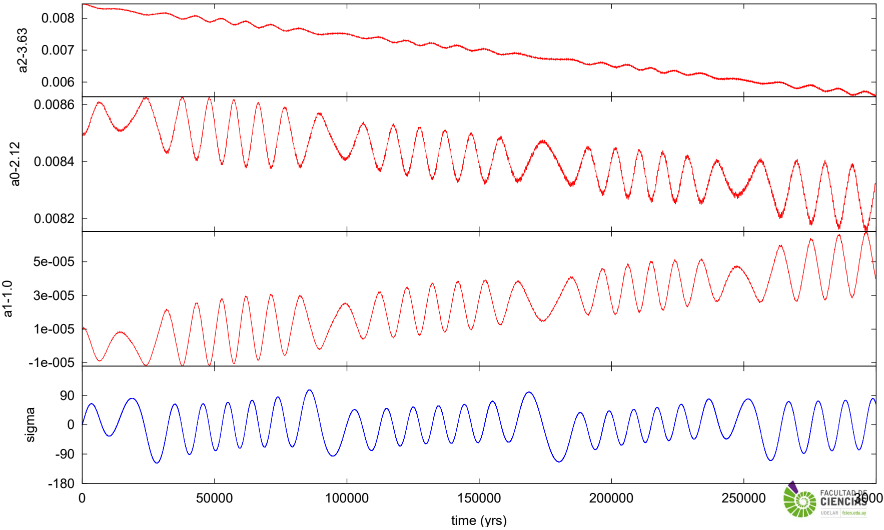
Inducing migration

Capture in a chain of 2BRs

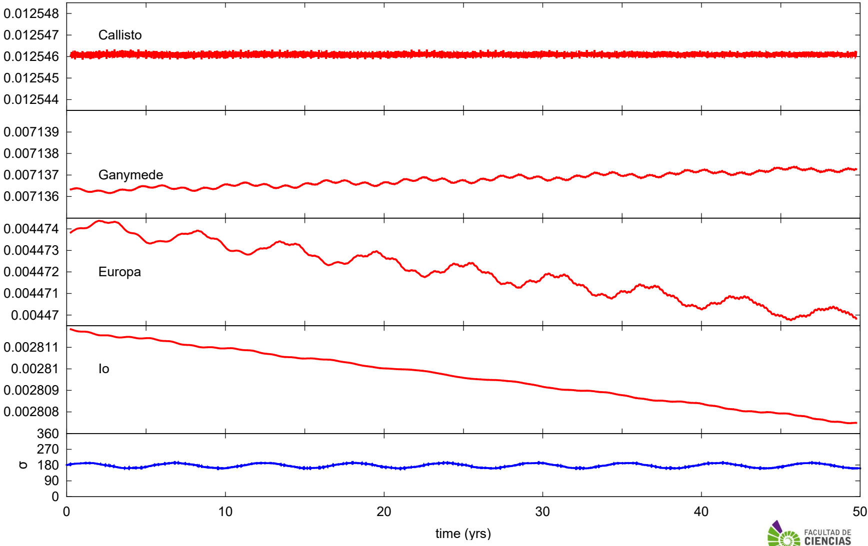
$$2\times(3P_0-5P_2)+(9P_0-5P_1) = 15P_0-5P_1-10P_2 = 3P_0-1P_1-2P_2$$



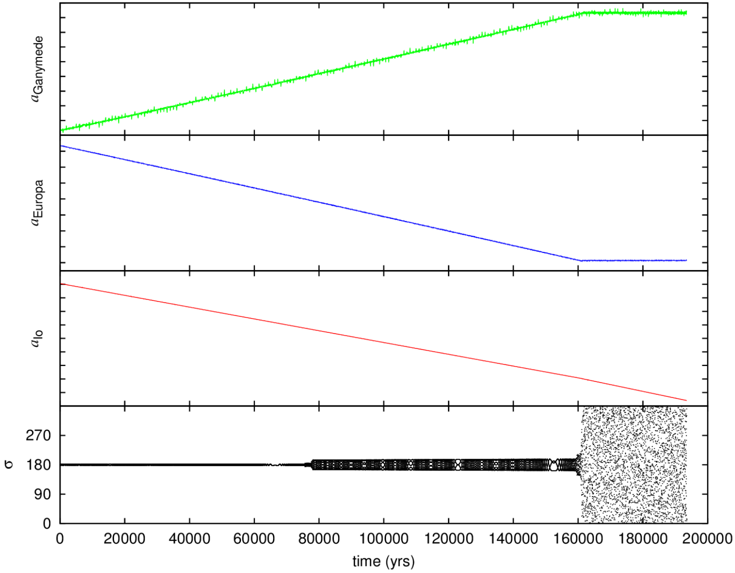
Migration while inside a pure 3BR (4-1-2)



Inducing migration on Io



Galilean migration



Migration: 2BRs versus 3BRs

two body resonances

$$n_I - 2n_E \simeq 0$$

$$\Delta n_E \simeq 0{,}5\Delta n_I$$

The two bodies migrate both inwards or both outwards.

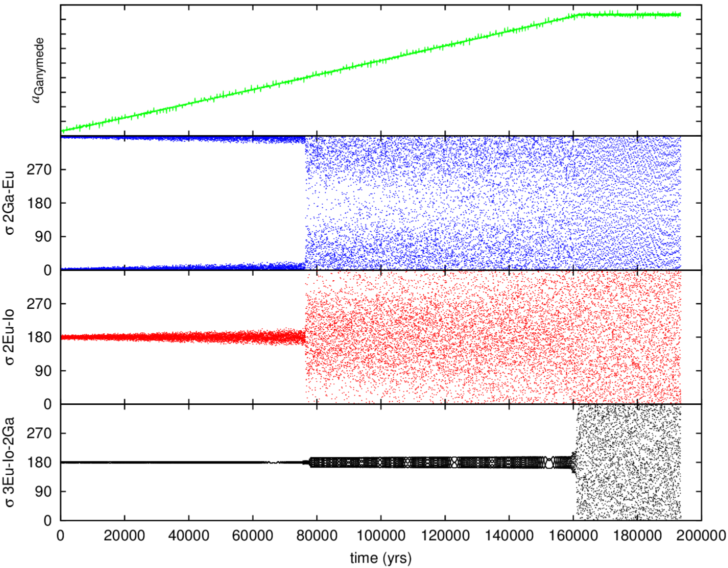
three body resonances

$$3n_E - n_I - 2n_G \simeq 0$$

$$3\Delta n_E - 2\Delta n_G \simeq \Delta n_I$$

In 3BRs bodies can migrate in different directions while trapped in resonance.

Galilean migration: critical angles



Conclusions

- restricted and planetary cases
- weak but numerous (chaotic diffusion)
- zero order resonances are the strongest, especially at $e \sim 0$
- for excited orbits high order 2BRs dominate
- there are pure 3BRs and chains of 2BRs
- is easiest to capture planetary (satellite) systems in a chain of 2BRs than in a pure 3BR
- migration in a 3BRs generates positive AND negative Δa
- lot of work must to be done to understand the structure in (a, e, i)

Thanks!

Merci!

See you at Montevideo!

Appendix

Disturbing function for the asteroid

$$R(\vec{r_0}, \vec{r_1}, \vec{r_2}) = R(\lambda_0, \lambda_1, \lambda_2) = R_{01} + R_{02}$$

being

$$R_{ij} = k^2 m_j \left(\frac{1}{r_{ij}} - \frac{\vec{r_i} \cdot \vec{r_j}}{r_j^3}\right)$$

resonance condition:

$$\lambda_0 = \left(\sigma - k_1 \lambda_1 - k_2 \lambda_2 + (k_0 + k_1 + k_2)\varpi_0\right)/k_0$$

$$\implies \lambda_0 = \lambda_0(\sigma, \lambda_1, \lambda_2, \varpi_0)$$

Averaging

$$\mathfrak{R}(\sigma) = \frac{1}{4\pi^2} \int_0^{2\pi} d\lambda_1 \int_0^{2\pi} R\Big(\lambda_0, \lambda_1, \lambda_2\Big) d\lambda_2$$

- $R = R_{01}(\lambda_0, \lambda_1) + R_{02}(\lambda_0, \lambda_2)$, both independent of σ !!
- **we cannot calculate $R_{01} + R_{02}$ using the unperturbed Keplerian positions**

The idea

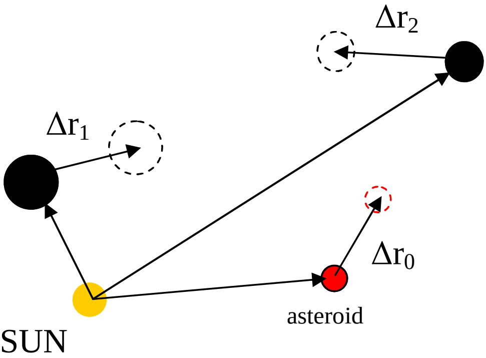
We adopt the following scheme:

$$R(\lambda_0, \lambda_1, \lambda_2) \simeq R_u + \Delta R$$

- R_u is R calculated at the unperturbed positions of the three bodies (useless!)
- ΔR stands from the variation in R_u generated by the perturbed displacements of the three bodies in a small interval Δt.

Method

Then, being in a resonant configuration



and these Δ generate the ΔR

Approximate mean resonant disturbing function $\rho(\sigma)$

The integral of $R_u = R_{01} + R_{02}$ is independent of σ, then we only need to calculate $\rho(\sigma)$ defined by

$$\rho(\sigma) = \frac{1}{4\pi^2} \int_0^{2\pi} d\lambda_1 \int_0^{2\pi} \Delta R \, d\lambda_2$$

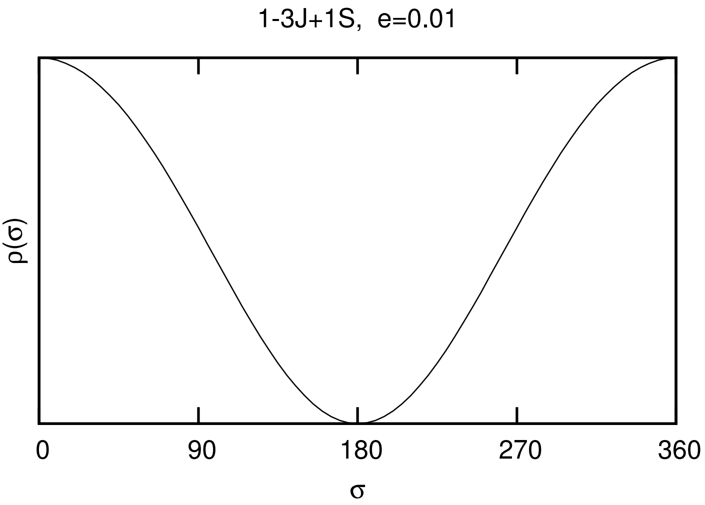
always satisfying the resonant condition $\lambda_0(\sigma, \lambda_1, \lambda_2, \varpi_0)$.

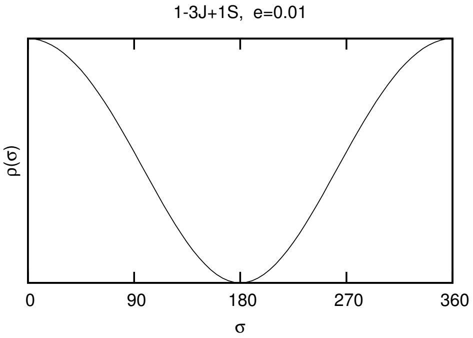
Method

For a given resonance:

- consider a large sample of configurations verifying the resonant condition ($\sigma =$ constant)
- calculate the mutual perturbations $\Delta r_0, \Delta r_1, \Delta r_2$
- calculate the effect ΔR due to $(\Delta r_0, \Delta r_1, \Delta r_2)$
- integrate all ΔR and obtain $\rho(\sigma)$
- repeat for several $\sigma \in (0, 360)$

Disturbing function $\sim \rho(\sigma)$





- **large variations** of ρ with σ is indicative of a **strong** resonance
- **small variations** of ρ with σ is indicative of a **weak** resonance
- an **extreme** of $\rho(\sigma)$ at some σ means there is an **equilibrium point**

Density of resonances versus density of asteroids