Ages of Asteroid Families Affected by Secular Resonances

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Asteroid proper elements

Definition:

Proper elements are quasi-integrals of the equations of motion in the N-body problem.

In practice:

Integrals of simplified dynamics.

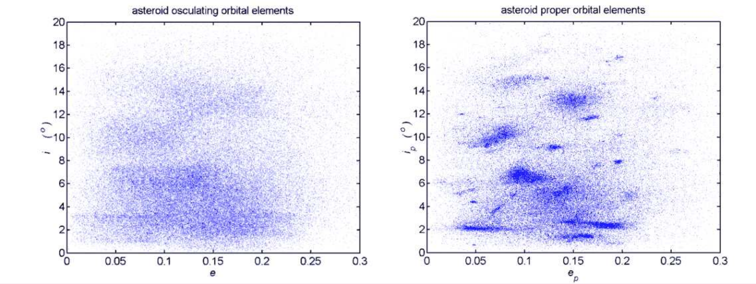
Nearly constant in time.
Deviation from constancy $\Rightarrow$ measure of accuracy.

Analytical proper elements:
Milani, A., and Z. Knežević: 1990, Secular perturbation theory and computation of asteroid proper elements. *Celestial Mechanics* **49**, 347–411.

Synthetic proper elements:
Knežević, Z., and A. Milani: 2000, Synthetic proper elements for outer main belt asteroids. *Celest. Mech. Dyn. Astron.* **78**, 17–46.

Secular resonant proper elements:
Morbidelli, A.: 1993, Asteroid Secular Resonant Proper Elements.*Icarus* **105**, 48–66.

Osculating vs. proper elements



Hierarchical Clustering Method (HCM)

Gauss equations;

Nearest neighbor selection;

Standard metrics:

$$\Delta = na_p \sqrt{k_1 (\frac{\delta a_p}{a_p})^2 + k_2 \delta e_p^2 + k_3 \delta I_p^2}$$

where Δ is the distance function and the values of the coefficients are $k_1 = 5/4$, $k_2 = k_3 = 2$. The distance has the dimension of velocity (expressed in m/s).

Minimum number of members N_{min};

Quasy Random Level (QRL) and/or d_{cutoff}.

Problems: mean motion (A.Milani's talk) and secular resonances.

Definition:

Secular resonances are locations in the phase space where linear combinations of fundamental frequencies allowed by D'Alembert rules equal zero.

Fundamental frequencies g, s are the average rates of secular progression of the longitude of perihelion ϖ, and the longitude of node Ω.

Fundamental frequencies in asteroid theory: $g, s, g_5, g_6, s_6, \dots$

Linear resonances (degree 2): $g - g_5; g - g_6, s - s_6;$
Nonlinear resonances (degree ≥ 4) $g + s - g_6 - s_6; g + g_5 - 2g_6; \dots$

In practice:

Resonant terms give rise to small divisors - large oscillations of proper elements.

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Computation of frequencies: analytical theory

Analytical theory of the second order in m'_j and up to degree 4(6) in $e/sinI$:

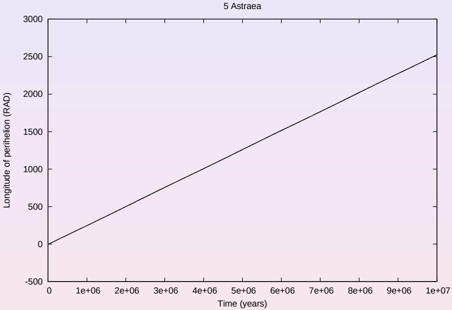
$$g = g_0 + \frac{2}{L^*}\frac{\delta F_1^{**}}{\delta \nu^2} \qquad s = s_0 + \frac{2}{L^*}\frac{\delta F_1^{**}}{\delta \mu^2}$$

$$g_0 = \sum_j 2\frac{m'_j A_{1j} + m'^2_j D_{1j}}{L^*} \qquad s_0 = \sum_j 2\frac{m'_j A_{3j} + m'^2_j D_{3j}}{L^*}$$

Knežević, Z., A. Milani, P. Farinella, Ch. Froeschle and Cl. Froeschle: 1991, Secular Resonances from 2 to 50 AU. *Icarus* **93**, 316–330.

Computation of frequencies: synthetic theory

Linear fit to the time series of the corresponding angle



$$g \simeq 2500/10^7 * 180/\pi * 3600 \simeq 52.21 \text{ arcsec/y}$$

$$g_5 - 2g_6 = 52.24 \text{ arcsec/y} \rightarrow \text{RESONANCE!}$$

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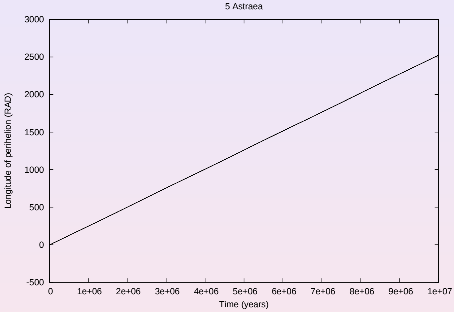

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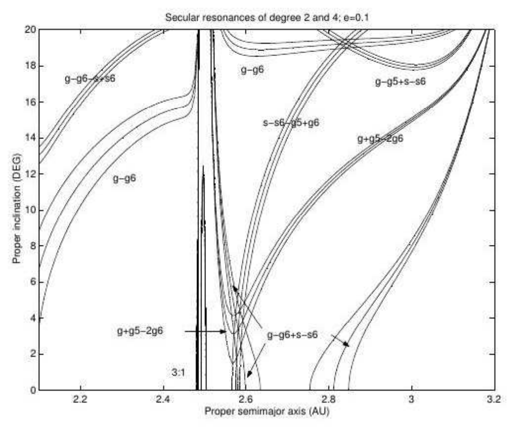
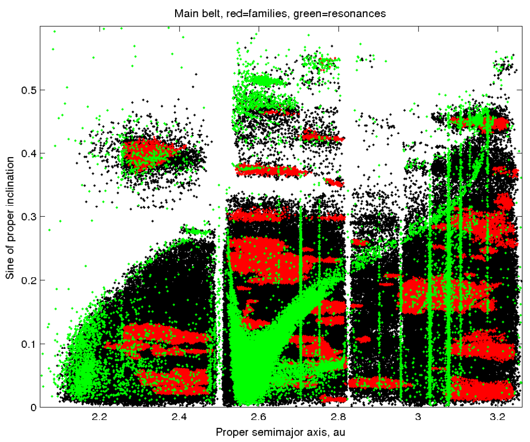
Secular resonances of degree 2 and 4 in the asteroid main belt

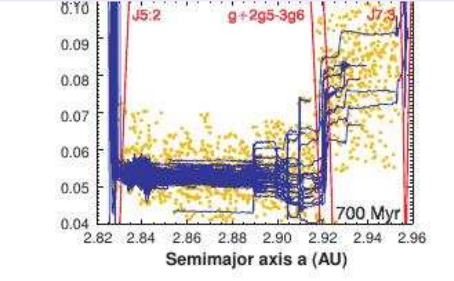


Asteroids affected by resonances

green: large errors; red: families; black: background



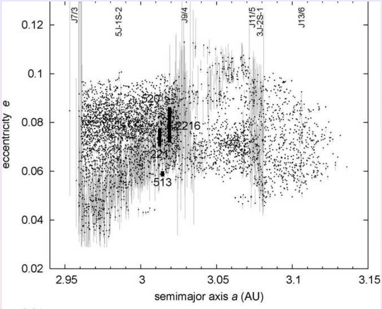
Bottke, W.F., D. Vokrouhlický, M. Brož, D. Nesvorný, A. Morbidelli,: 2001, Dynamical spreading of asteroid families via the Yarkovsky effect: The Koronis family and beyond. *Science* **294**, 1693–1695.



Asteroid migrating due to Yarkovsky effect and encountering nonlinear secular resonance $g + 2g_5 - 3g_6$ interacts with the resonance: it undergoes jumps which explain the Prometheus surge.

Previous work

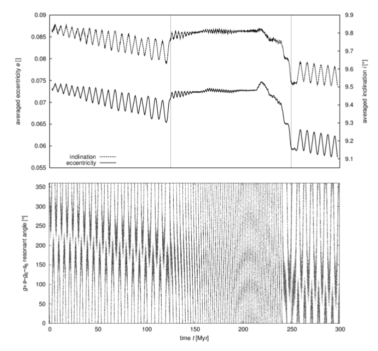
Vokrouhlický, D., M. Brož, A. Morbidelli, W.F. Bottke, D. Nesvorný, D. Lazzaro, A.S. Rivkin: 2006, Yarkovsky footprints in the Eos family. *Icarus* **182**, 92–117.



Asteroid migrating due to Yarkovsky effect and encountering nonlinear secular resonance $z_1 = g + s - g_6 - s_6$ can become captured for tens to hundreds of My. During this time, its orbital elements slide along the z_1 resonance while its semimajor axis changes.

Previous work

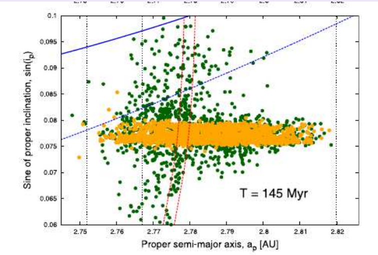
Vokrouhlický, D., M. Brož: 2002, Interaction of the Yarkovsky-drifting orbits with weak resonances: Numerical evidence and challenges. In: Celletti, A., Ferraz-Mello, S., Henrard, J. (Eds.), Modern Celestial Mechanics: From Theory to Applications. Kluwer Academic, Dordrecht, pp. 46–472.

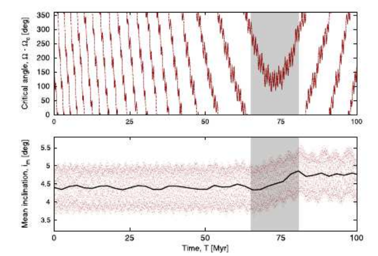


Orbital evolution of an Eos family member. Except for the $\simeq 100$ My interruption due to interaction with mean motion resonance, orbit captured in $z_1 = g + s - g_6 - s_6$ secular resonance. The accumulated change $\Delta e = 0.015$; $\Delta I = 0.3^\circ$; $\Delta a = 0.025$ au.

Recent work

Novaković, B., C. Maurel, G. Tsirvoulis, Z. Knežević: 2015, Asteroid secular dynamics: Ceres' fingerprint identified. *Astroph. J. Letters*, 807:L5, 5pp





The observed spread of Hoffmeister family in proper inclination is best explained by the interaction with $s - s_C$ linear secular resonance with (1) Ceres.

Change of inclination due to the resonance + Yarkovsky drift + time = observed spread.

Age determination for asteroid families affected by secular resonances

Milani et al. 2016: difficult cases.

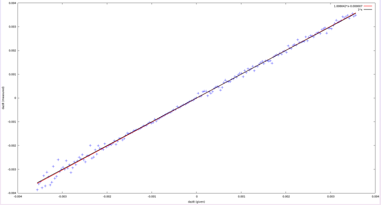
Three families with almost all or good portion of the members locked in the resonance: 5 Astraea, 363 Padua, 945 Barcelona;

Two families with only a minor portion of members affected by a secular resonance: 283 Emma, 686 Gersuind.

One-sided family with $\sim 1/3$ of members affected by a secular resonance: 25 Phocaea.

Calibration for the Yarkovsky effect: (5) Astraea

Calibration inside vs. outside the resonance.



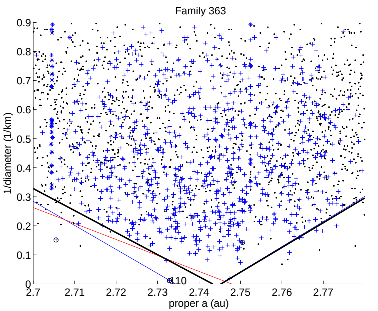
200 clones in the range $-3.6 \times 10^{-9} < da/dt(\mathrm{NR}) < 3.6 \times 10^{-9}$ au/y.

$$\frac{da}{dt}(\mathrm{RES}) = 1.008 \cdot \frac{\mathrm{da}}{\mathrm{dt}}(\mathrm{NORES})$$

The amount of secular change in proper *a* due to Yarkovsky is not significantly affected by the secular resonance.

Family of (363) Padua

A dynamical family of (110) Lydia has almost all members locked in the $z_1 = g + s - g6 - s6$ resonance. (110) has WISE albedo $= 0.17 \pm 0.04$ while 90% of the members having WISE data (with $S/N > 3$) have albedo < 0.1. Thus (110) Lydia is a likely interloper and the family namesake should be (363) Padua.



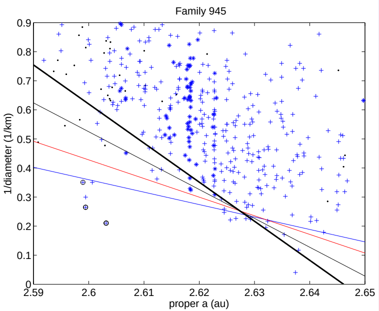
Calibration coefficient 0.985.

The ages indicate a family at the low end of the "old" range: 284 ± 73 for the IN, and 219 ± 48 for the OUT side, in My.

Compatible with a single collisional origin of fragmentation type, although with a largest remnant (363) containing as much as 75% of the family volume.

Family of (945) Barcelona

The family of (945) Barcelona has about $2/3$ of the members (the portion of the family with proper $e > 0.23$) strongly affected by the linear secular resonance $|g - g_5| < 2$ arcsec/y. Another unidentified nonlinear secular resonance is also relevant for the dynamics of most members.



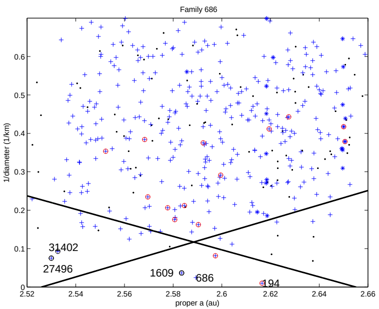
Calibration coefficient 1.046.

One-sided V-shape of the family of (945) Barcelona. The gap on the right, where the family terminates on the OUT side, corresponds to the $11/4J$ resonance at $a \simeq 2.649$ au. The IN side fit is good, with some outliers.

The age of 203 ± 56 My is at the low end of the "old" range. The family is of fragmentation type with large remnant (945) comprising 73% of the family volume.

Family of (686) Gersuind

Our classification contains a dynamical family 194, with 408 members. However, the family namesake (194) Prokne has albedo 0.052 ± 0.015 (WISE) thus it does not belong to the family, which, after removal of members with albedo < 0.08 and > 0.25, has a mean albedo 0.145 ± 0.037 and becomes family of (686) Gersuind.



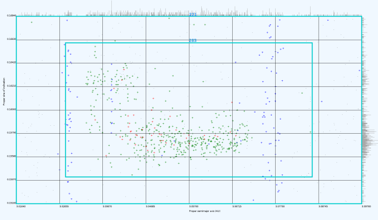
Calibration coefficient 1.003.

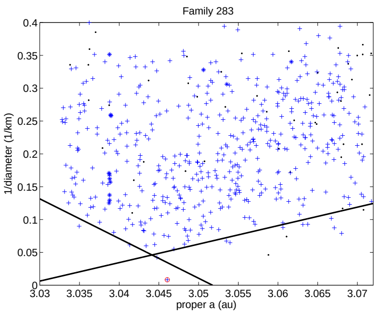
The family is only partially affected by the $s - s_6 - g_5 + g_6$ nonlinear secular resonance, with only some 14% of members in the OUT side having the corresponding frequency < 0.5 arcsec/y.

The estimated ages are 1490 ± 843 and 1436 ± 469 My; OUT value more significant.

Family of (283) Emma

A small fraction (about 8%) of members are affected by the nonlinear secular resonance $g + s - g_6 - s_6$, opening a gap between two parts of the family.





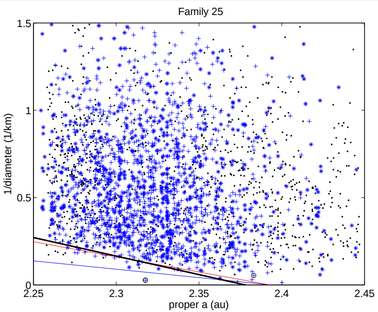
The two slopes of the V-shape are ostensibly different. The family is of cratering type, thus the parent body (283) Emma (red cross) is excluded from the fit: calibration performed on the second largest family member (32931) Ferioli (coefficient 1.010).

The age estimates are 290 ± 67 on IN side, and 628 ± 234 My on the OUT side.

Two cratering events consistent with a double jet shape in the proper $(a, \sin I)$ plane.

Family of (25) Phocaea

A one-sided V-shape in the (a, 1/D) plane, with (25) at the high *a* end. The missing part due to the resonances bordering the stable region, including the $3/1J$. The region contains about $4,000$ asteroids, while the dynamical family 25 contains only $1,405$: a substructure with distinctive number density inside the stability region.



Calibration coefficient 0.977.

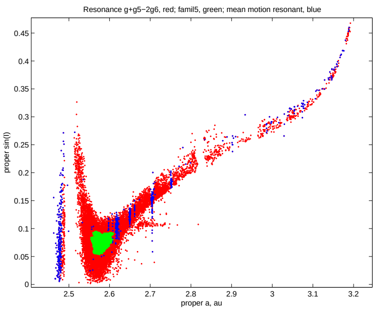
Lack of family members near the value of proper a of (25) Phocea: possible YORP induced central gap (Paolicchi and Knežević, 2016).

$1/3$ of the family members affected by $g - s - g_6 + s_6$. Cratering family: calibration by using clones of (323) Brucia, but only with negative Yarkovsky drift in proper *a*.

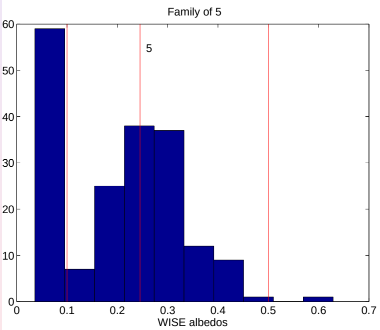
The age of 1187 ± 319 My indicates an ancient family.

Family 5 Astraea. Beginning of Saga

Almost all members locked in the $g + g_5 - 2g_6$ nonlinear secular resonance (with values $|g + g_5 - 2g_6| < 0.5$ arcsec/y); the proper e has a large spread, up to 0.236 (MMR with inner planets). 77% of the members have $D < 2$ km $\rightarrow$ cratering. Good albedo data for 6.5% of members: (5) Astraea 0.227 ± 0.027, 31% with albedo < 0.1.



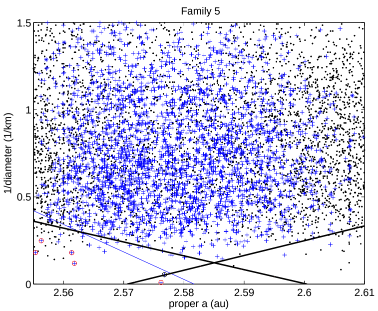
Synthetic proper elements



Family 5 Astraea. Nothing suspicious.

The existence of a usable V-shape is a good argument for claiming that a family must exist: the inaccuracy of the proper elements due to the secular resonance does not affect at all the semimajor axis, thus the V-shape in *a*.

The IN side of the V-shape affected by a group of four members too large for their position: e.g., (4700) Carusi D $>$ 8 km. Near the center of the family, (1044) Teutonia D = 15 km, incompatible with the cratering origin of the family. Removed from the fit.

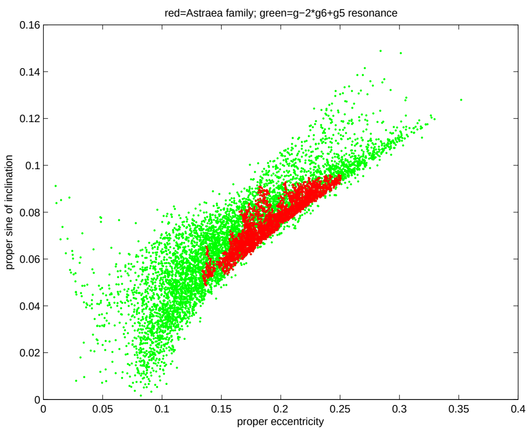


Calibration coefficient 1.008.

The age: 339 ± 104 My for the IN side, and 319 ± 98 My for OUT.
Family 5 is old, but far from ancient.
Another very good example of cratering, significantly younger than Vesta.

Family 5 Astraea. Surprise!

REFEREE: The family of (5) Astraea does not exist !!!

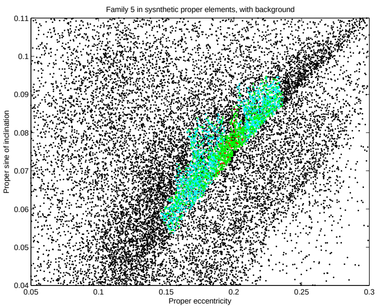


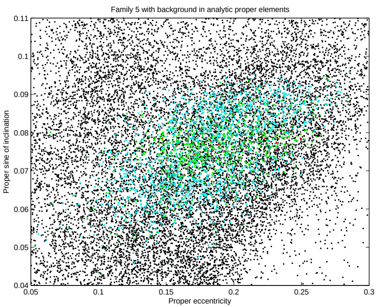
The "family" is only an artifact of the proper elements computation, which partially removes the oscillations related to the secular resonance and creates gaps above and below the libration center, which are indeed detected by the HCM, but there is no reason to claim this is a collisional family.

Family 5 Astraea. Detective story begins.

Let's resort to analytical proper elements. They are of lower accuracy but can be used to asses the existence of families and their membership. For secular resonant asteroids the adapted theory is used, that does not account for the term(s) with small divisor(s) (hence, no compression to libration center).

Rationale: if we find family 5 by using analytical proper elements, with reasonable overlap in membership with the synthetic one $\rightarrow$ proof of the existence of the family.





In the space of analytic proper elements, deteriorated by the secular resonance due to the low torsion, hence large libration amplitude, the family appears enormously more spread: that is, instead of a compression effect, there is a dilatation effect.

The existence of family 5 confirmed!!.

Family 5 Astraea. Towards the solution.

Family 5 in synthetic proper elements:
53% $\sigma(e) > 0.01$; 27% $\sigma(e) > 0.02$; 8% $\sigma(e) > 0.03$.

$\rightarrow$ better proper elements needed.

Resonant proper elements specifically adapted to the secular resonance $g + g_5 - 2g_6$.

To be used only for the resonant asteroids with the critical argument $\varpi + \varpi_5 - 2\varpi_6$ not circulating over extended period of time (10 My).

The proper $\sin I$ is little affected by a resonance containing only g frequencies, averaging over time gives small instabilities: $\sigma(\sin I) > 0.005$ in 1.2% cases.

The proper a is also unaffected: computed by the usual method.

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Family 5 Astraea. Definition and computation.

The proper element we use Δe is defined as the amplitude of libration in the eccentricity e associated with the libration of the critical argument.

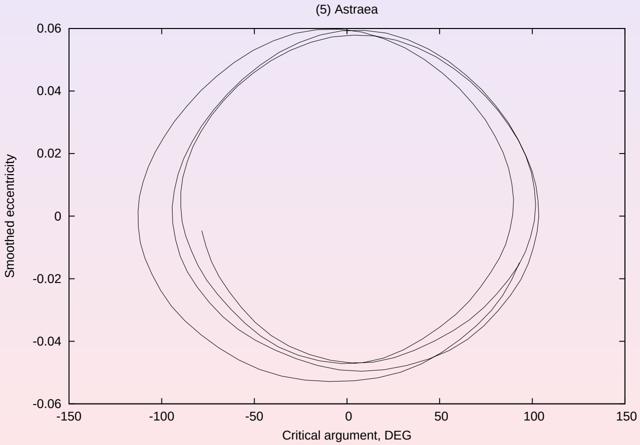
Start from the output of the numerical integration, filter to remove all oscillations with periods up to $30,000$ y.

Compute spectrum for periods in the interval between 1 and 6 My, then select the maximum spectral density, and use the corresponding amplitude as proper element.

Stability of proper eccentricity: 2.5% $\sigma(\Delta e) > 0.01$; 0.1% $\sigma(\Delta e) > 0.02$.

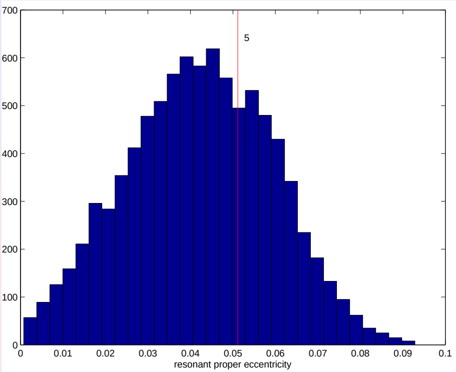
Family 5 Astraea. Librations.

Libration of eccentricity and the critical argument: the quantities displayed have been doubly filtered, removing all oscillations with period $< 30,000$ y. The amplitude in e is 0.051, in the critical angle is about 100°.



Family 5 Astraea. Results.

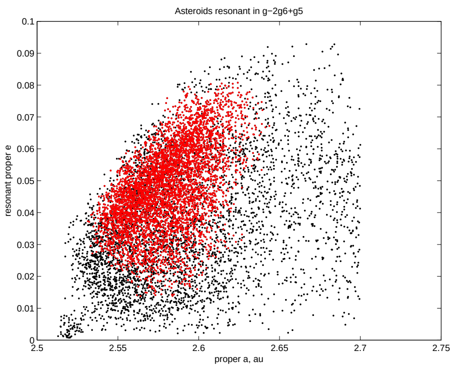
8,972 sets of resonant proper elements computed, including for now only numbered asteroids, with $2.5 < \text{proper } a < 2.7$ au and proper $\sin I < 0.3$.



Histogram of the resonant proper eccentricity Δe for 8,972 asteroids satisfying the resonance criterion. The peak number density is near, but not exactly at the location of (5) Astraea; the over-density sharply decreases for > 0.065 and < 0.015.

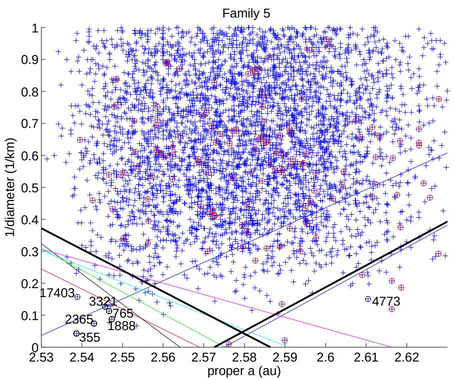
Family 5 Astraea. Family.

HCM: families with $\leq 41(30)$ members at 60(50) m/s.



We find a family including (5) Astraea with 3,369 members at 50 m/s and 5,192 members at 60 m/s. The previous family computed with "unstable" proper elements had only 2,477 numbered members. 1,808 or 3/4 of the numbered members of the old family, are found also in the new one.

Family 5 Astraea. Family age.



IN: 408 $\pm$ 269 My; OUT: 395 $\pm$ 91 My.

THANK YOU!