Chaotic capture of (dark) matter by binary systems

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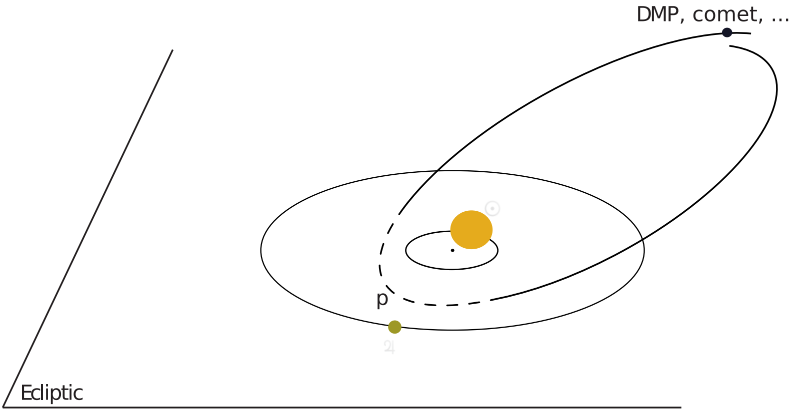
– Dynamics and chaos in astronomy and physics, Luchon 2016 –

Papers :
J. L., D. Shepelyansky, **Dark matter chaos in the Solar system**, MNRAS Letters 430, L25-L29 (2013)
G. Rollin, J. L., D. Shepelyansky, **Chaotic enhancement of dark matter density in binary systems**, A&A 576, A40 (2015)
P. Haag, G. Rollin, J. L., **Symplectic map description of Halley's comet dynamics**, Physics Letters A 379 (2015) 1017-1022

(Dark) matter capture – Three-body problem



Possible DMP capture (or comet capture) due to Jupiter and Sun rotations around the SS barycenter.

Dark matter capture – Restricted circular three-body problem

$$m_{\mathrm{DMP}} \ll m_{♃} \ll m_{\odot}$$

Dark matter capture – Restricted circular three-body problem

Newton's equations $m_{\text{DMP}} \ll m_{♃} \ll m_{\odot}$

$$\ddot{\mathbf{r}} = \frac{1 - m_{♃}}{\left\|\mathbf{r}_{\odot}(t) - \mathbf{r}\right\|^3}\left(\mathbf{r}_{\odot}(t) - \mathbf{r}\right) + \frac{m_{♃}}{\left\|\mathbf{r}_{♃}(t) - \mathbf{r}\right\|^3}\left(\mathbf{r}_{♃}(t) - \mathbf{r}\right)$$

$$G = 1, \quad m_{♃} + m_{\odot} = 1, \quad \left\|\dot{\mathbf{r}}_{♃}\right\| \simeq 13\text{km.s}^{-1} = 1, \quad \left\|\mathbf{r}_{♃}\right\| = 1$$

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Energy change after a passage at perihelion (wide encounter)

$$F \sim \frac{m_{♃}}{m_{\odot}}\left\|\dot{\mathbf{r}}_{♃}\right\|^2 \simeq 10^{-3}$$

Dark matter capture – Restricted circular three-body problem

Newton's equations

$$m_{\text{DMP}} \ll m_{\jupiter} \ll m_{\odot}$$

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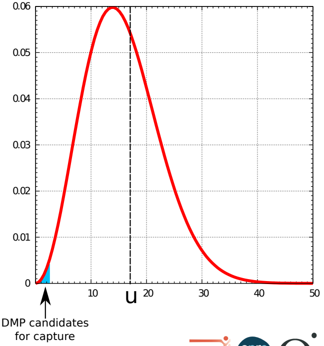
Assuming a Maxwellian distribution of Galactic DMP velocities

$$f(v)dv \sim v^2 \exp\left(-3v^2/2u^2\right) dv$$

with $u \simeq 220\text{km.s}^{-1} \sim 17$ (mean DMP velocity)



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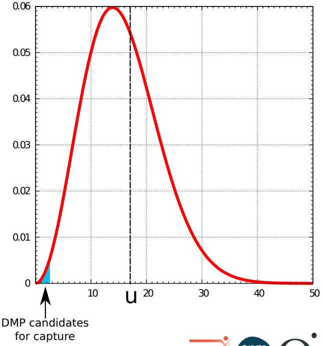
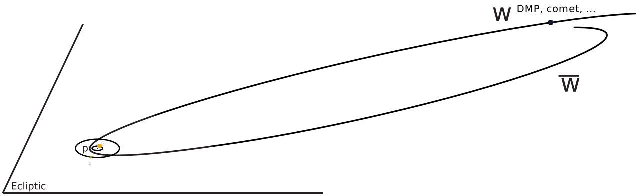


As $F \ll u^2$, not many candidates for capture among Galactic DMPs

Most of the capturable DMPs have close to parabolic approaching trajectories ($E \sim 0$)

Direct simulation of Newton's equations is difficult : very elongated ellipses, not many particles can be simulated, CPU time consuming (Peter 2009)

Kepler map



x : Jupiter's phase when particle at pericenter ($x = \varphi/2\pi \mod 1$)
w : particle energy at apocenter ($w = -2E/m_{\text{DMP}}$)

Symplectic Kepler map

$$\begin{aligned} \bar{w} &= w + F(x) &&= w + W\sin(2\pi x) && \leftarrow \text{energy change after a kick} \\ \bar{x} &= x + \bar{w}^{-3/2} &&&& \leftarrow \text{third Kepler's law} \end{aligned}$$

Map already used in the study of :

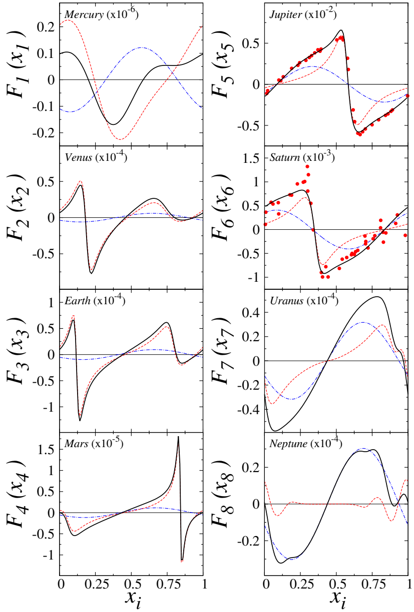
- Cometary clouds in Solar systems (Petrosky 1986)
- Chaotic dynamics of Halley's comet (Chirikov & Vecheslavov 1986)
- Microwave ionization of hydrogen atoms (see e.g. Shepelyansky, scholarpedia)

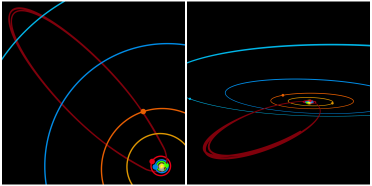
Advantage : if the kick function $F(x)$ is known the dynamics of a huge number of particles can simulated.

Let's make a digression ...

Halley map – Cometary case

Kick functions of SS planets





Rollin, Haag, J. L., Phys. Lett. A 379 (2015) 1017-1022

Our calculations match direct observation data and previous numerical data (Yeomans & Kiang 1981)

$$F(x_1, \dots, x_8) \simeq \sum_{i=1}^{8} F_i(x_i)$$

$$F_i(x_i) = -2\mu_i \int_{-\infty}^{+\infty} \nabla \left(\frac{\mathbf{r} \cdot \mathbf{r}_i}{r^3} - \frac{1}{\|\mathbf{r} - \mathbf{r}_i\|} \right) .\dot{\mathbf{r}}\ dt$$

Two main contributions

- Direct planetary Keplerian potential
- Rotating gravitational dipole potential due to the Sun movement around Solar System barycenter

Halley map – Cometary case

Renormalized kick function

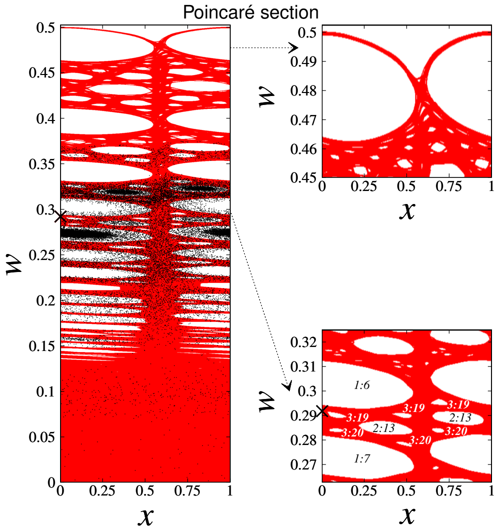
$$f_i(x_i) = F_i(x_i)/v_i^2/\mu_i$$

Exponential decay with q for $q > 1.5a_i$
More precisely

$$f_i \simeq 2^{1/4}\pi^{1/2}\left(\frac{q}{a_i}\right)^{-1/4}\exp\left(-\frac{2^{3/2}}{3}\left(\frac{q}{a_i}\right)^{3/2}\right)$$

(Heggie 1975, Petrosky 1986, Petrosky & Broucke 1988, Roy & Haddow 2003, Shevchenko 2011, see also Rollin's talk yesterday)

Halley map – Dynamical chaos



Symplectic Halley map

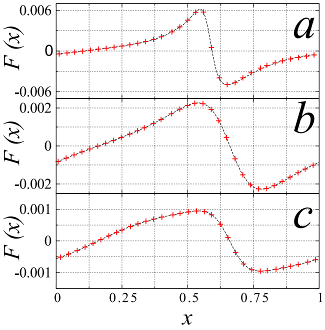
$$\bar{x} = x + \bar{w}^{-3/2}$$
$$\bar{w} = w + F(x)$$

Chaotic dynamics of 1P/Halley

“Lifetime” $\sim 10^7$ years

Let's come back to (dark) matter ...

Dark matter capture – Dark map



a : Halley's comet
b : $q = 1.5, \omega = 0.7, i = 0$
c : $q = 0.5, \omega = 0., i = \pi/2$

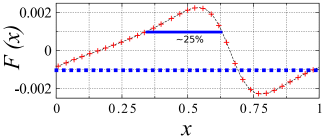
Kick function determination

We determine numerically the kick function for any parabolic orbit (q, i, ω)

$$F(x) = F_{q,i,\omega}(x)$$

By nonlinear fit we obtain analytical functions.

Dark matter capture – Dark map



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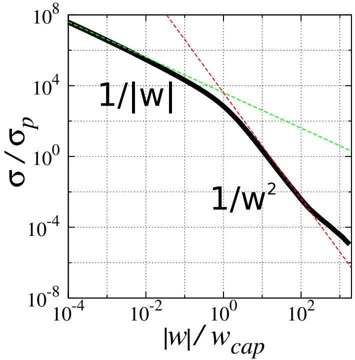
By nonlinear fit we obtain analytical functions.

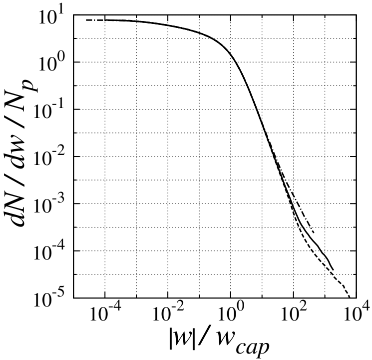
Chance to be captured with a given energy ($w < 0 \leftrightarrow E > 0$)

$$h_{q,i,\omega}(w)$$

Dark matter – Capture cross section

$w_{cap} = \frac{m_{♃}}{m_{\odot}} \left\|\dot{\mathbf{r}}_{♃}\right\|^2 \simeq 10^{-3}$ $\qquad$ $\sigma_p = \pi \left\|\mathbf{r}_{♃}\right\|^2$ area enclosed by Jupiter's orbit



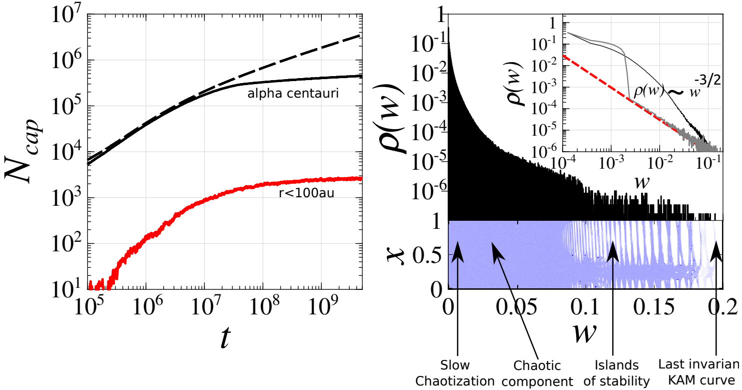


$\sigma/\sigma_p \simeq \pi \frac{m_{\odot}}{m_{♃}} \frac{w_{cap}}{|w|}$ in agreement with Khriplovich & Shepelyansky 2009

- Predominance of wide encounters as suggested by Peter 2009
- Very small contribution from close encounters invalidating previous numerical results (Gould & Alam 2001 and Lundberg & Edsjö 2004)

Dark map – Dark matter capture

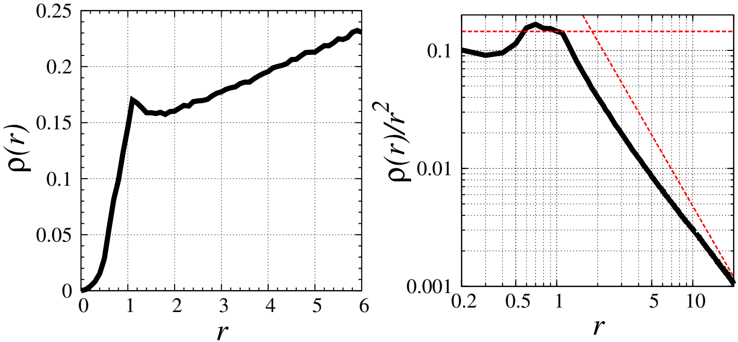
Simulation of the (isotropic) injection, the capture and the escape of DMPs during the whole lifetime of the Solar system.
Injection of $N_{tot} \simeq 1.5 \times 10^{14}$ DMPs with energy $|w|$ in the range $[0, \infty]$ with $N_H = 4 \times 10^9$ DMPs in the Halley's comet energy interval $[0, w_H]$.



- Equilibrium reached after a time $t_d \sim 10^7$yr similar to the diffusive escape time scale of the Halley's comet (Chirikov & Vecheslavov 1989) $\longrightarrow$ Equilibrium energy distribution $\rho(w)$
- The dynamics of dark matter particles in the Solar system is essentially chaotic

Back to real space – Density distribution of captured DMPs

Nowadays equilibrium density distribution ($t_S = 4.5 \times 10^9$yr)



- **The profile of the radial density $\rho(r) \propto dN/dr$ is similar to those observed for galaxies where DMP mass is dominant.** Indeed $\rho(r)$ is almost flat (increases slowly) right after Jupiter orbit $(r = 1) \longrightarrow$ according to virial theorem the circular velocity of visible matter is consequently constant as observed e.g. in Rubin 1980

  Virial theorem : $v_m^2 \sim \int_0^r dr' \rho(r')/r \sim \rho(r) \sim r^2 \left(\rho(r)/r^2\right) \underset{here}{\sim} r^2 r^{-1.53} \sim r^{1/2}$

  Ergodicity along radial dynamics : $d\mu \sim dN \sim \rho(r)dr \sim dt \sim dr/v_r \sim r^{1/2}dr$

  Consequently, $v_m \propto r^{0.25}$ (Dark map) to compare to $v_m \propto r^{0.35}$ (Rubin 1980)

Back to real space – Density distribution of captured DMPs

Surface density

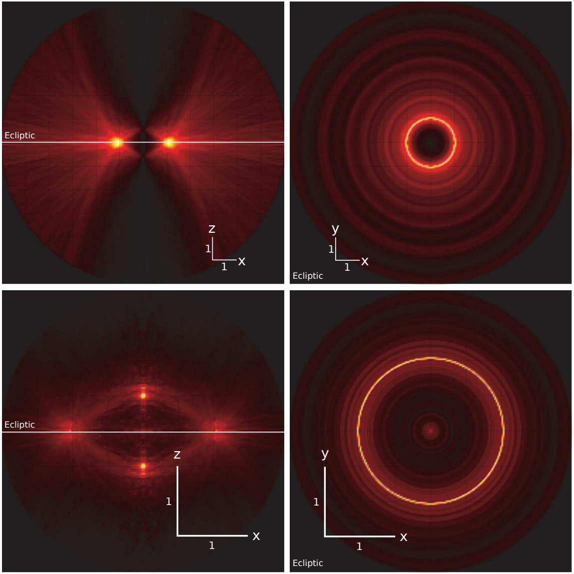
$$\rho_s(z, R) \propto dN/dzdR$$

where

$$R = \sqrt{x^2 + y^2}$$

Volume density

$$\rho_v(x, y, z) \propto dN/dxdydz$$



How much dark matter is present in the Solar system ?

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The total mass of DMP passed through the System solar during its lifetime $t_S = 4.5 \times 10^9$yr is

$$M_{\rm tot} = \rho_g t_S \int_0^\infty dv\, v f(v)\sigma(v) \approx 35 \rho_g t_S G \left\|\mathbf{r}_{♃}\right\| M_\odot / u \approx 0.9 \times 10^{-6} M_\odot \sim M_{♀}$$

At time t_S the mass of captured DMPs in the Solar system is

$$M_{AC} \approx \eta_{AC} M_{tot} \approx 2 \times 10^{-15} M_\odot \qquad \text{within } r < 0.5 \text{ distance}_{\text{Sun-}\alpha\text{Centauri}}$$
$$M_{100\text{au}} \approx \eta_{100\text{au}} M_{tot} \approx 1.3 \times 10^{-17} M_\odot \qquad \text{within } r < 100\text{au}$$

The captured DMP mass in the volume of the Neptune orbit radius is

$$M_{♆} \approx \eta_{♆} M_{AC} \approx 0.9 \times 10^{-18} M_\odot \approx 1.5 \times 10^{15}\text{g}$$

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The average volume density of captured dark matter inside the Jupiter orbit sphere is

$$\rho_♃ = \frac{3M_♃}{4\pi r_♃^3} \approx 5 \times 10^{-29}\text{g/cm}^3 \approx 1.2 \times 10^{-4} \rho_g \ll \rho_g \text{ (Galactic DMP density)}$$

Globally, not much dark matter captured by the Solar system, but ...

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Let's compare to the capturable DMP density

$$\rho_{gH} = \rho_g \int_0^{\sqrt{w_H}} dv\, v f(v) \approx 1.4 \times 10^{-32}\text{g/cm}^3$$

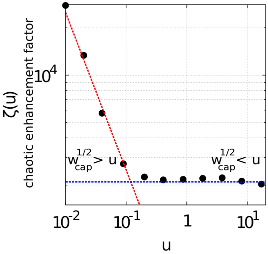
$\Longrightarrow$

The long range interaction capture mechanism is very efficient for binary systems (1+2) with $m_1 \gg m_2$

Huge chaotic enhancement $\zeta = \rho_{♃} / \rho_{gH} \approx 4 \times 10^3$ of the density of actually capturable DMPs.

(Dark) matter capture in binary systems

$w_{cap} \sim w_H \sim 0.005$
$u = 220 km/s$



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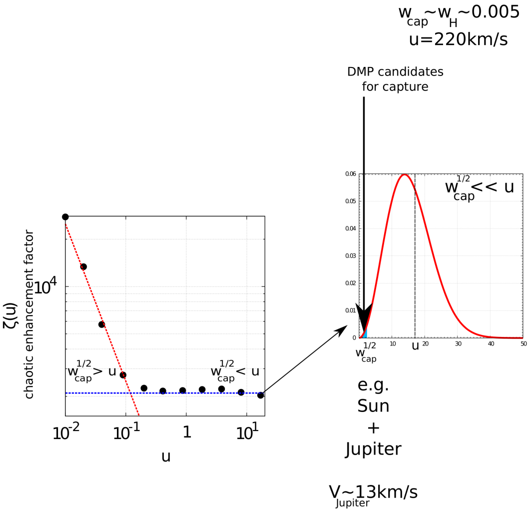

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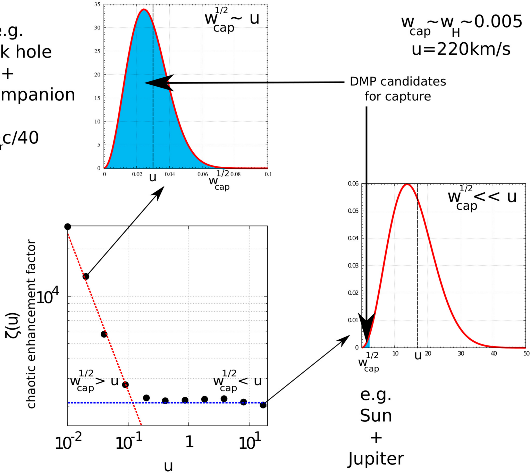
(Dark) matter capture in binary systems

e.g.
Black hole
+
Star companion

V_{star}~c/40



w_{cap}~w_H~0.005
u=220km/s

e.g.
Sun
+
Jupiter

$V_{Jupiter}$~13km/s

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(Dark) matter capture in binary systems

Global
volume density
enhancement
x10^4

all the galactic DMPs are captured

e.g.
Black hole
+
Star companion

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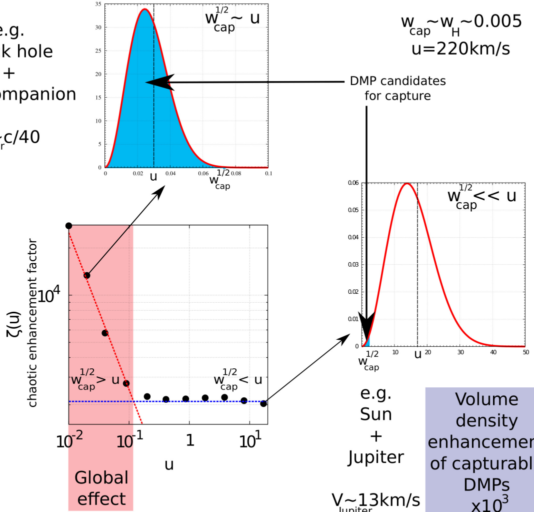

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$V_{Jupiter} \sim 13$km/s

Volume
density
enhancement
of capturable
DMPs
x10^3

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(Dark) matter capture in binary systems

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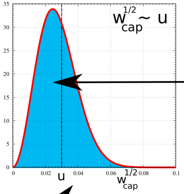

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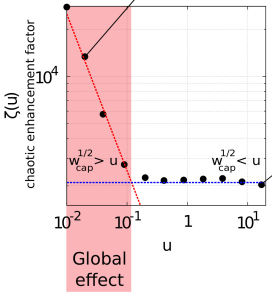
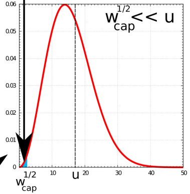
u=220km/s

DMP candidates
for capture





Ionization process:
- DMPs acceleration
- wandering black holes
- ...

e.g.
Sun
+
Jupiter

$V_{Jupiter} \sim 13$km/s

Volume
density
enhancement
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$\times 10^3$

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Thank You !