Tales of Hierarchical Three-body Systems

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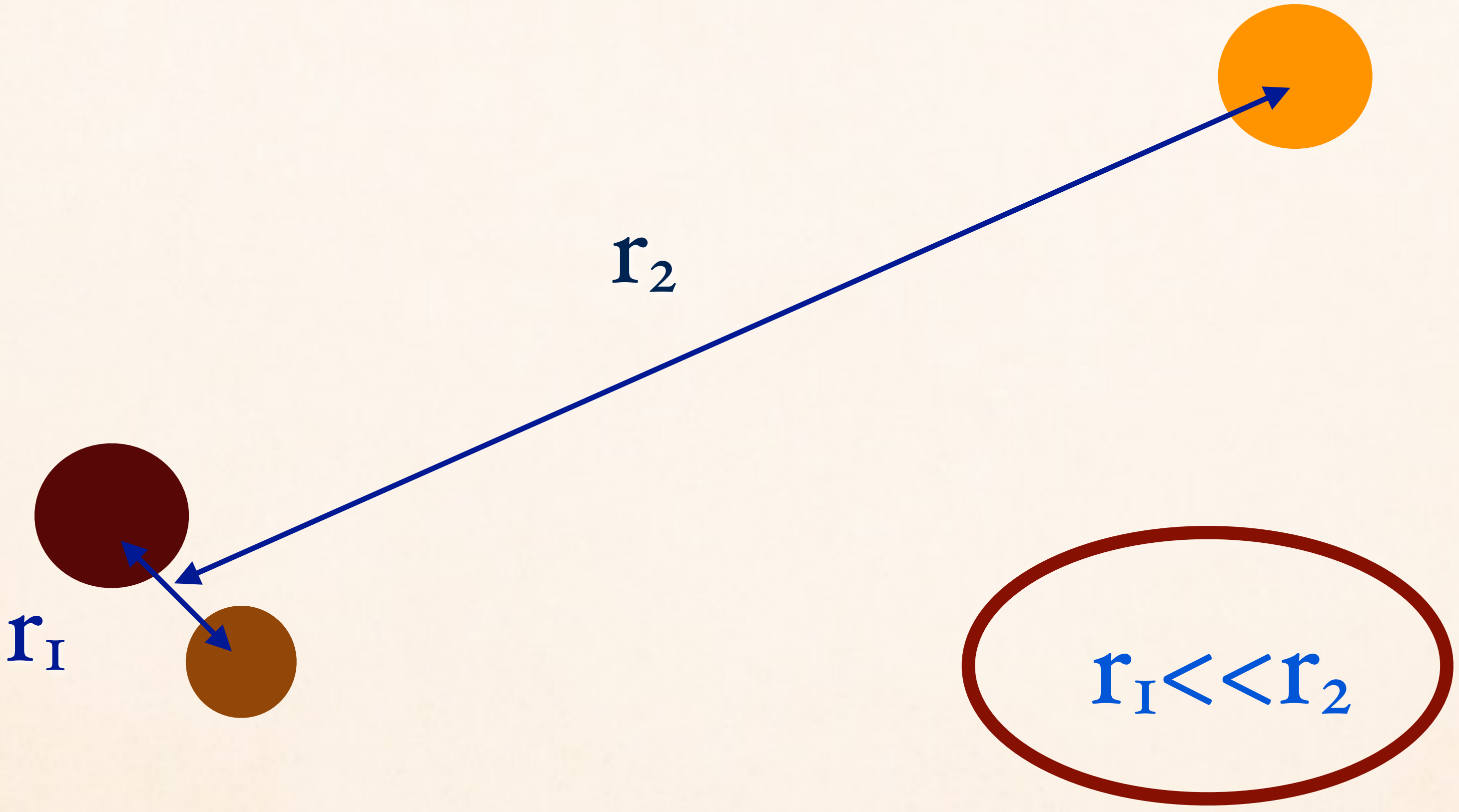
Image credit: “The Three-body Problem”, by Xinci Liu

HIERARCHICAL THREE-BODY SYSTEMS

- Configuration:



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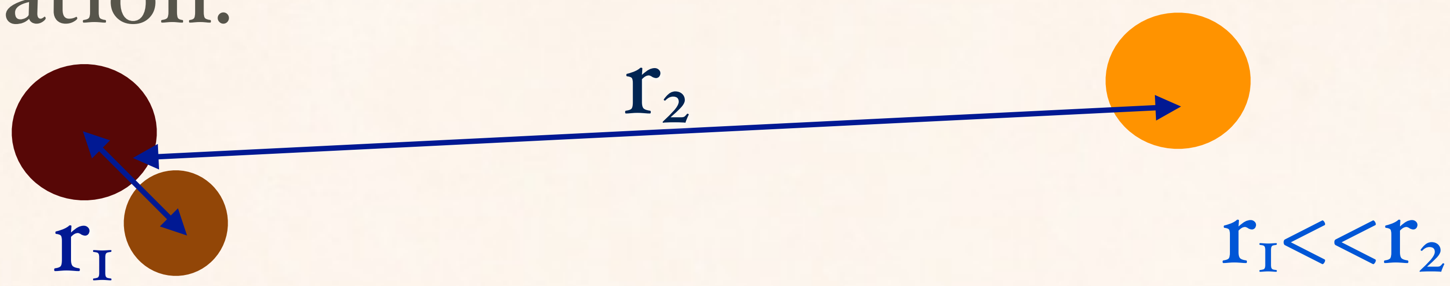


- Hierarchical configurations are COMMON:

- For binaries with periods shorter than 10 days, >40% of them are in systems with multiplicity ≥ 3. (*Tokovinin 1997*)
- For binaries with period < 3 days, ≥96% are in systems with multiplicity ≥3. (*Tokovinin et al. 2006*)
- 282 of the 299 triple systems (~ 94.3%) are hierarchical. (*Eggleton et al. 2007*)

- Hierarchical 3-body dynamics gives insight for hierarchical multiple systems.

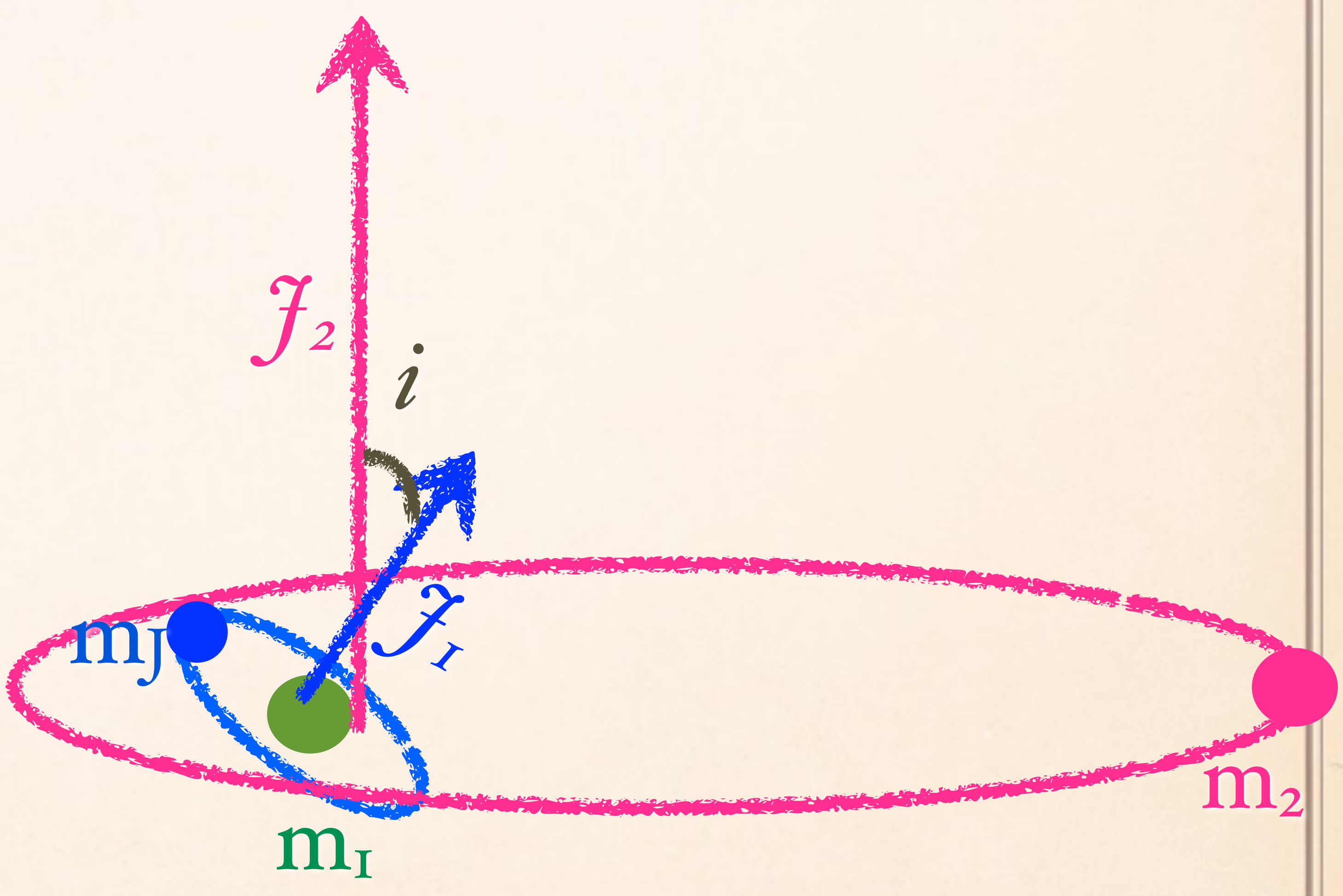
OUTLINE

- Overview of Hierarchical Three Body Dynamics
- Examples:
  - Formation of misaligned hot Jupiters
  - Enhancement of tidal disruption rates

CONFIGURATION OF HIERARCHICAL 3-BODY SYSTEM

System is stationary and can be thought of as interaction between two orbital wires (secular approximation):

- Inner wires (1): formed by m_1 and m_J.
- Outer wires (2): m_2 orbits the center mass of m_1 and m_J.
- $\mathcal{J}_{1/2}$: Specific orbital angular momentum of inner/outer wire.
- i: inclination between the two orbits.



KOZAI-LIDOV MECHANISM

Kozai-Lidov Mechanism

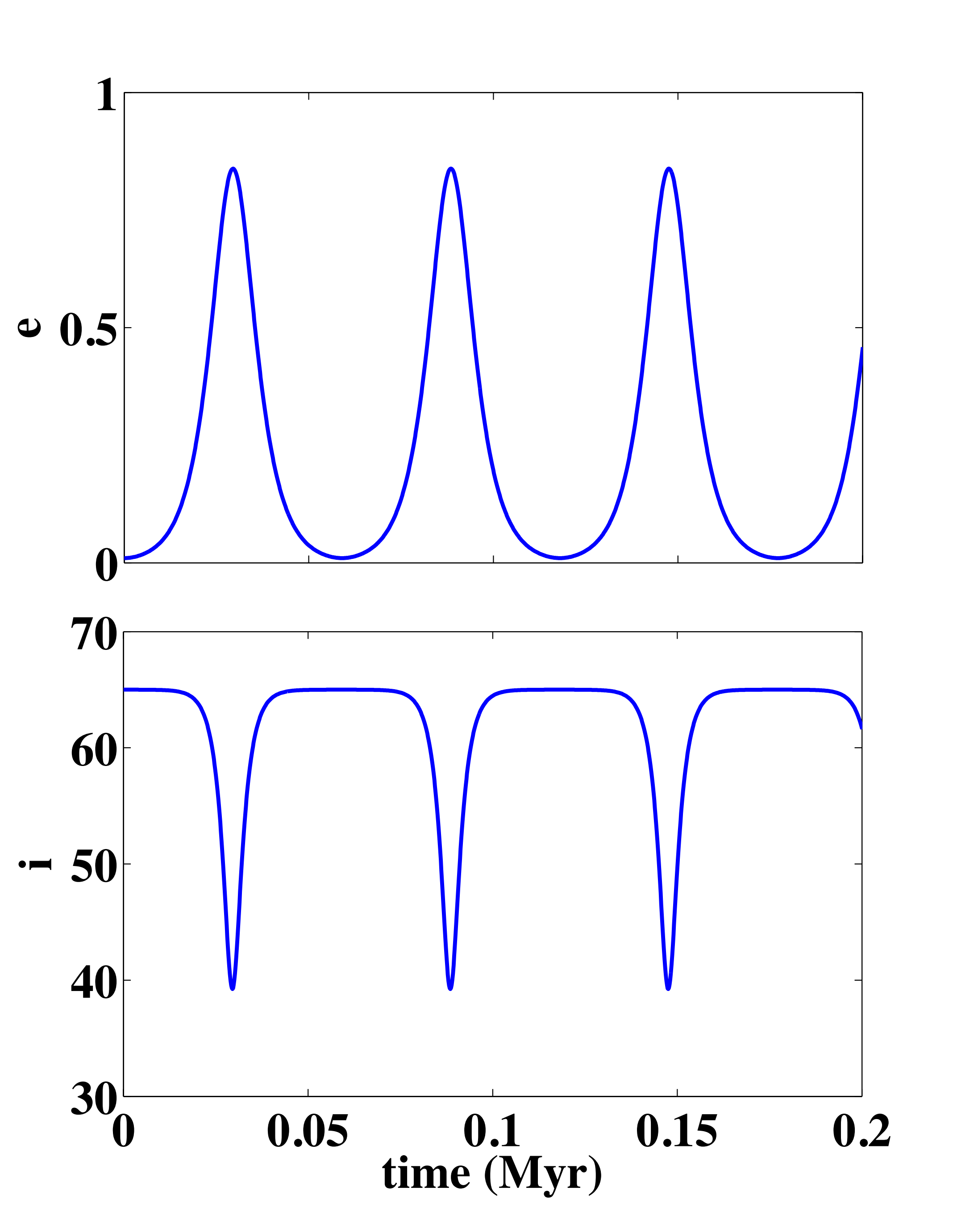
(e_2 = 0, $m_J \rightarrow 0$)

(Kozai 1962; Lidov 1962: Solar system objects)

- Octupole level $O((a_1/a_2)^3)$ is zero.
- Quadrupole level $O((a_1/a_2)^2)$:

=> $Jz = \sqrt{1 - e_1^2}\cos i_1$ conserved (axi-symmetric potential).

=> when i>40°, e_1 and i oscillate with large amplitude.



Example of Kozai-Lidov Oscillation.

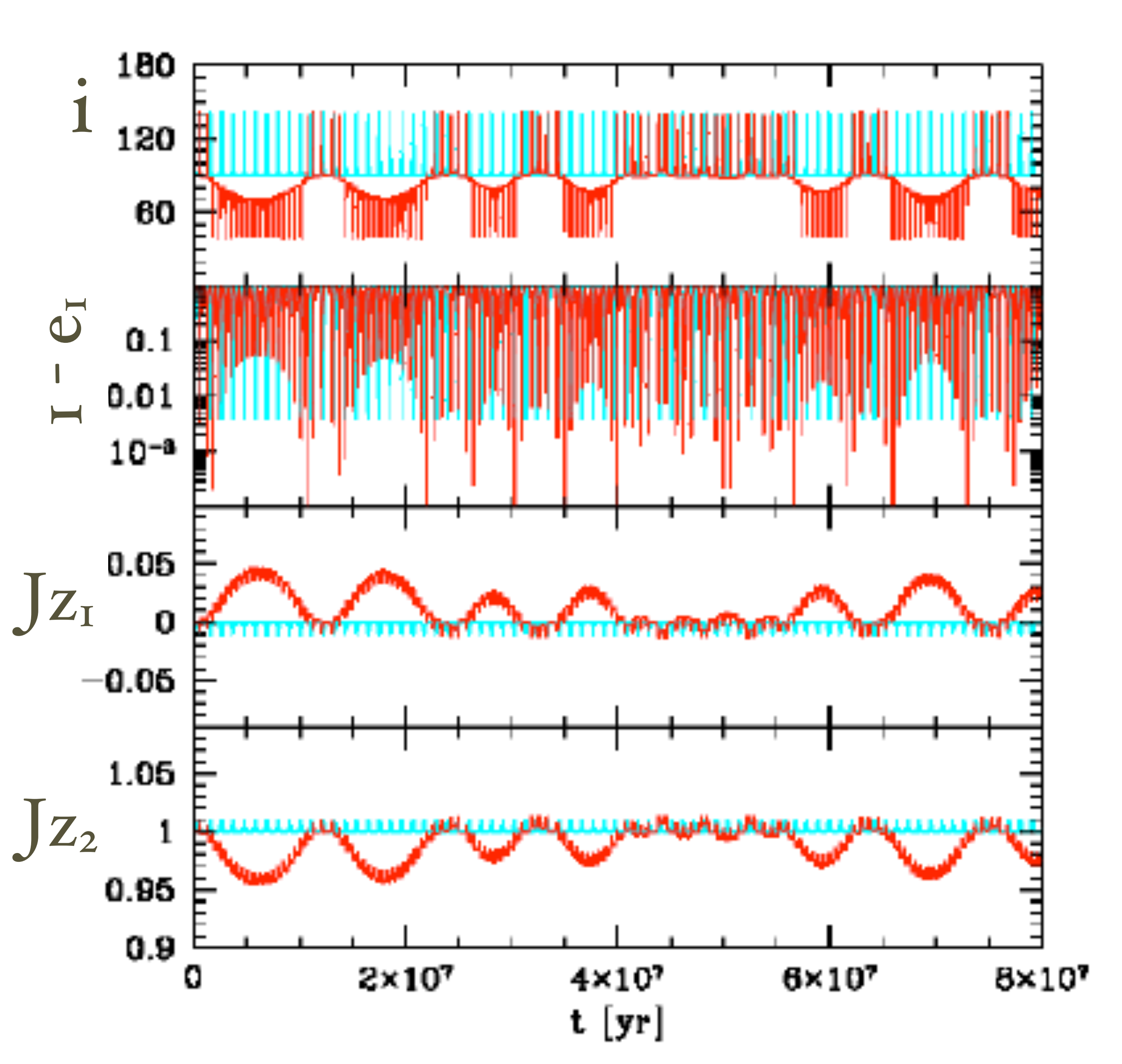
OCTUPOLE KOZAI-LIDOV MECHANISM

$e_2 \neq 0$ (Eccentric Kozai-Lidov Mechanism):

(e.g., *Naoz et al. 2011, 2013, test particle case: Katz et al. 2011, Lithwick & Naoz 2011*):

- Jz NOT constant, octupole ≠ 0.

- when i>40º: $e_1 \rightarrow 1$.
- when i>40º: i crosses 90º



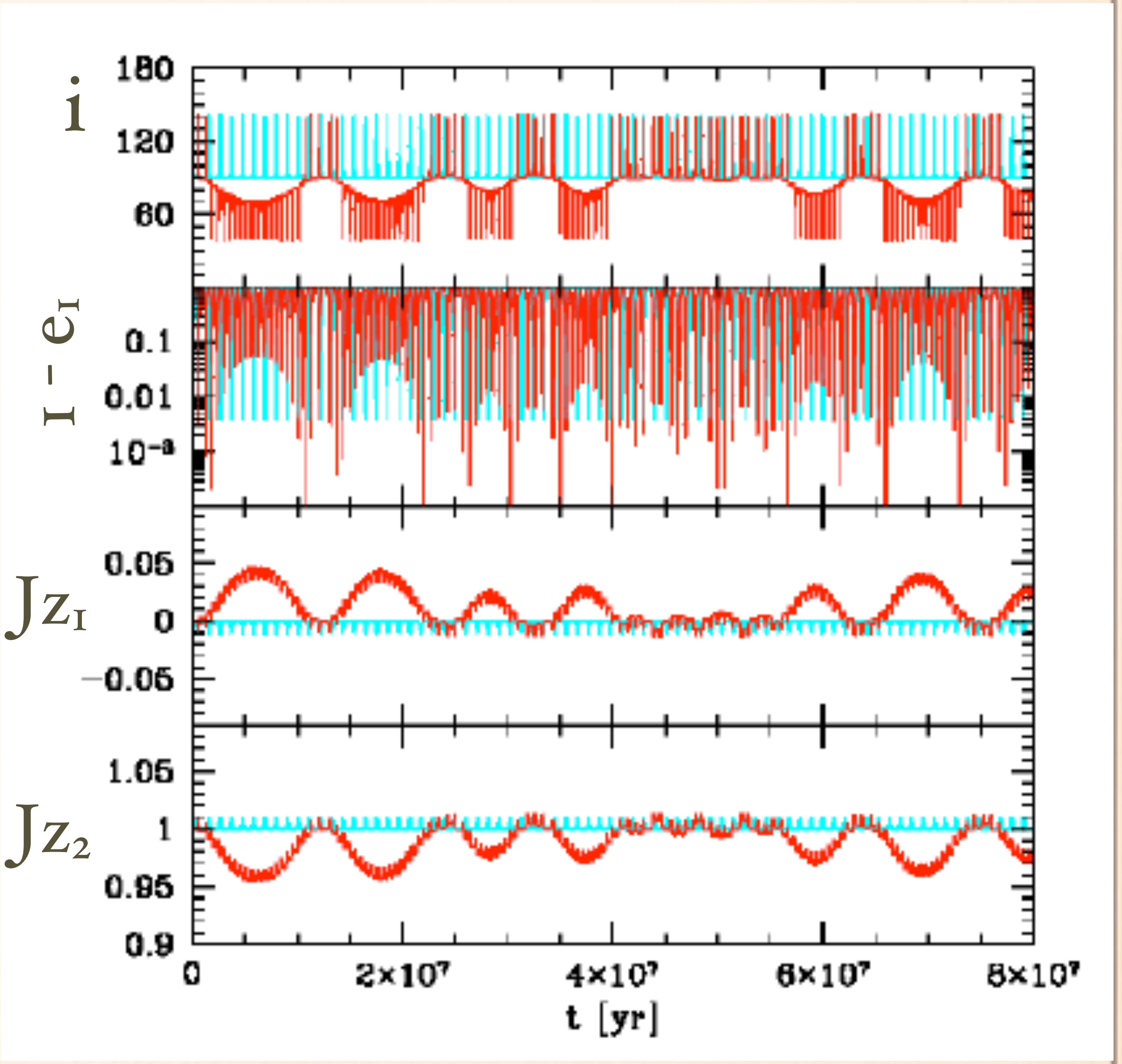
Cyan: quadrupole only.
Red: quadrupole + octupole. Naoz et al 2013

OCTUPOLE KOZAI-LIDOV MECHANISM

$e_2 \neq 0$ (Eccentric Kozai-Lidov Mechanism) or $m_J \neq 0$:

(e.g., *Naoz et al. 2011, 2013, test particle case: Katz et al. 2011, Lithwick & Naoz 2011*):

- Consequence:
  - Produces retrograde objects (i>90°)(e.g., Naoz et al. 2011)
  - Tidal disruption rate enhancement
    (e.g., Li et al. 2015)

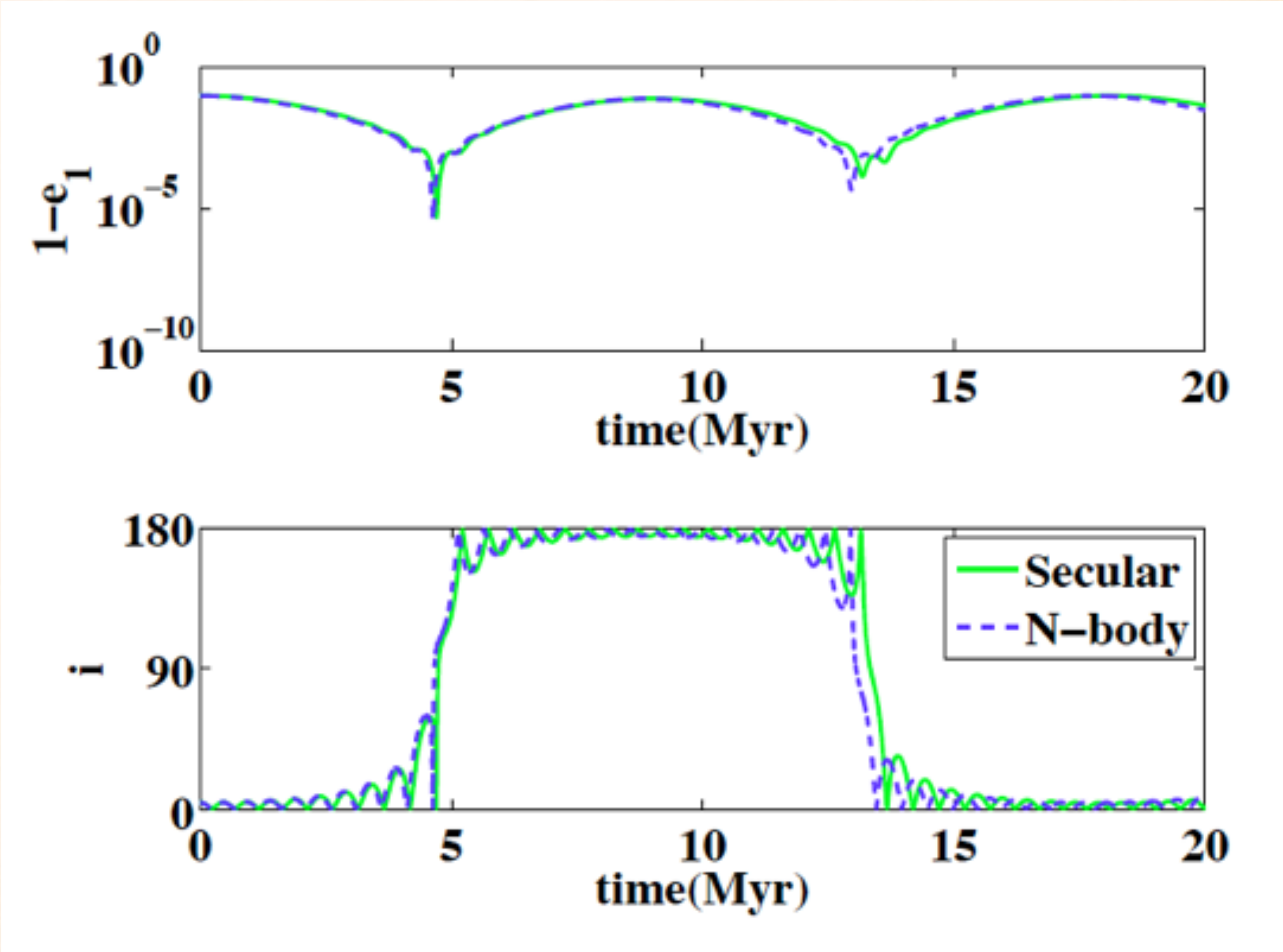


Cyan: quadrupole only.
Red: quadrupole + octupole. Naoz et al 2013

COPLANAR FLIP

- Starting with $i \approx 0$, $e_1 \geq 0.6$, $e_2 \neq 0$:

  $e_1 \rightarrow 1$, i flips by ≈180° (*Li et al. 2014a*).



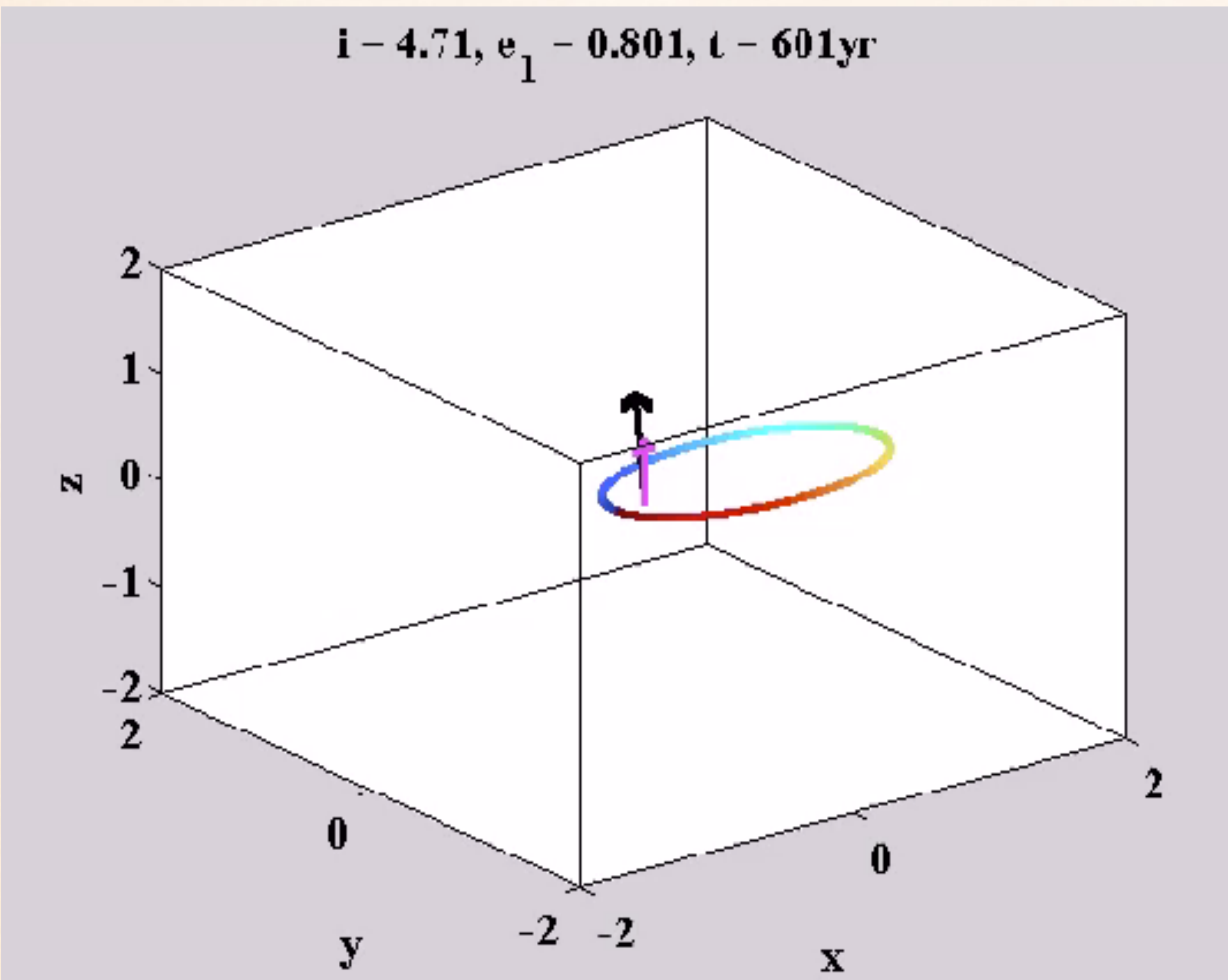
=> Produces counter orbiting objects.

=> Enhance tidal disruption rates (*Li et al. 2015*).

(Li et al. 2014a)

DIFFERENCES BETWEEN HIGH/LOW I FLIP

- Low inclination flip



- For simplicity:

take $m_j \rightarrow 0$ => outer orbit stationary.

- z direction: angular momentum of the outer orbit.

- ⬆ : direction of J_1.
- ⬆ : Jz_1 => indicates flip.
- Colored ring: inner orbit. Color: mean anomaly.

Li et al. 2014a

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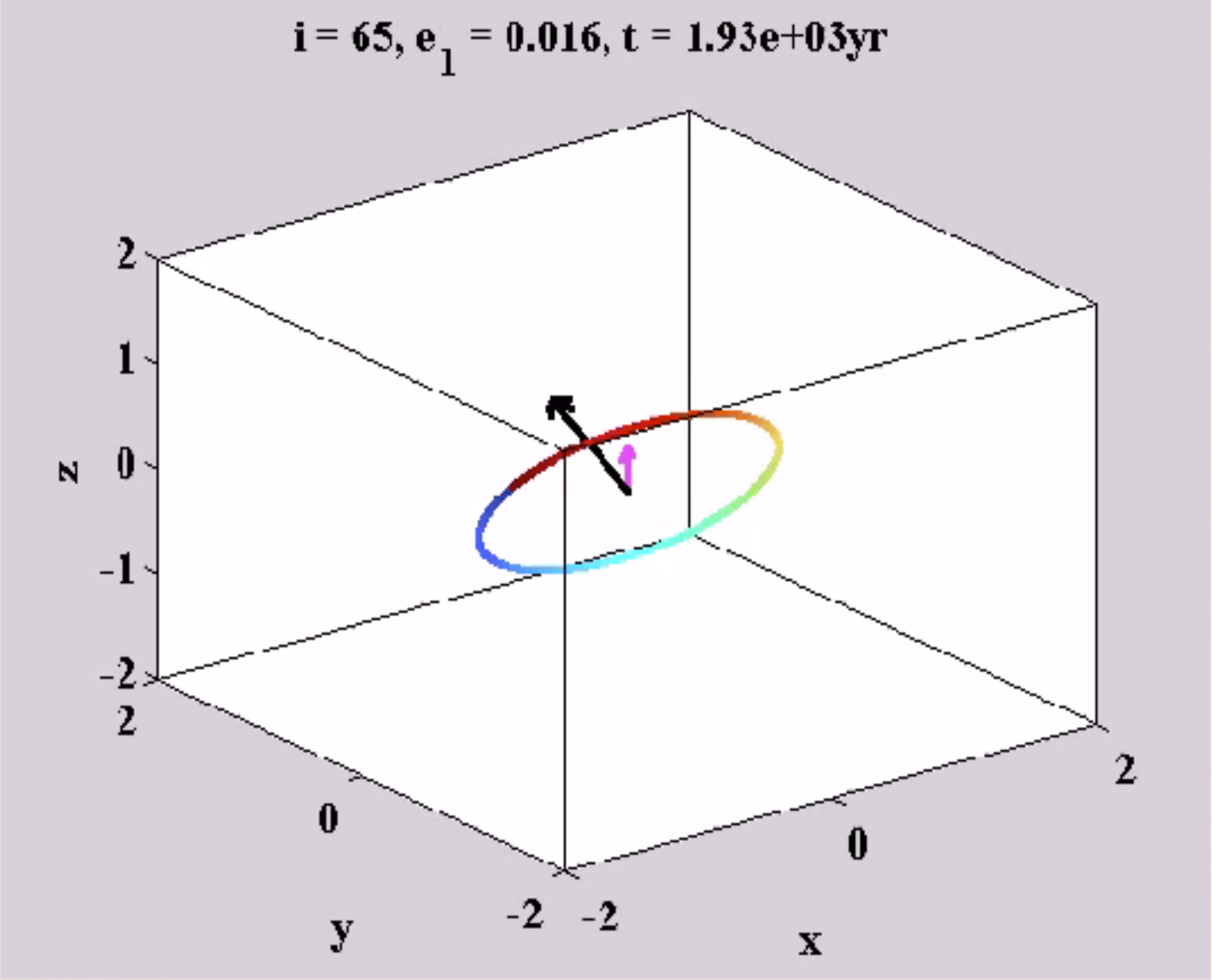

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Li et al. 2014a

ANALYTICAL OVERVIEW

- Hamiltonian has two degrees of freedom in test particle limit:

$$(J = \sqrt{1-e_1^2},\ Jz = \sqrt{1-e_1^2}\cos i_1,\ \omega,\ \Omega\ )$$

2 conjugate pairs: J & ω, Jz & Ω

$$H = -F_{quad} - \varepsilon\, F_{oct}$$

hierarchical parameter: $\epsilon = \frac{a_1}{a_2}\frac{e_2}{1-e_2^2}$

ANALYTICAL OVERVIEW

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  2 conjugate pairs: J & ω, Jz & Ω

$H = -F_{quad} - \varepsilon\, F_{oct}$

$$
\begin{aligned}
F_{quad} &= -(e_1^2/2) + \theta^2 + 3/2 e_1^2 \theta^2 \\
&\quad + 5/2 e_1^2 (1-\theta^2)\cos(2\omega_1), \\
F_{oct} &= \frac{5}{16}(e_1 + (3e_1^3)/4) \\
&\quad \times ((1 - 11\theta - 5\theta^2 + 15\theta^3)\cos(\omega_1 - \Omega_1) \\
&\quad + (1 + 11\theta - 5\theta^2 - 15\theta^3)\cos(\omega_1 + \Omega_1)) \\
&\quad - \frac{175}{64} e_1^3((1 - \theta - \theta^2 + \theta^3)\cos(3\omega_1 - \Omega_1) \\
&\quad + (1 + \theta - \theta^2 - \theta^3)\cos(3\omega_1 + \Omega_1)),
\end{aligned}
$$

Independent of Ω_1, J_z const.

Depend on both ω_1 and Ω_1 → both J and J_z are not const.

CO-PLANAR FLIP CRITERION

- Hamiltonian (at O(i)):
  - Evolution of e_1 only due to octupole terms:

    => e_1 does not oscillate before flip

  - Depend on only J_1 and $\varpi_1=\omega_1+\Omega_1$

    => System is integrable.

    => e_1(t) can be solved.

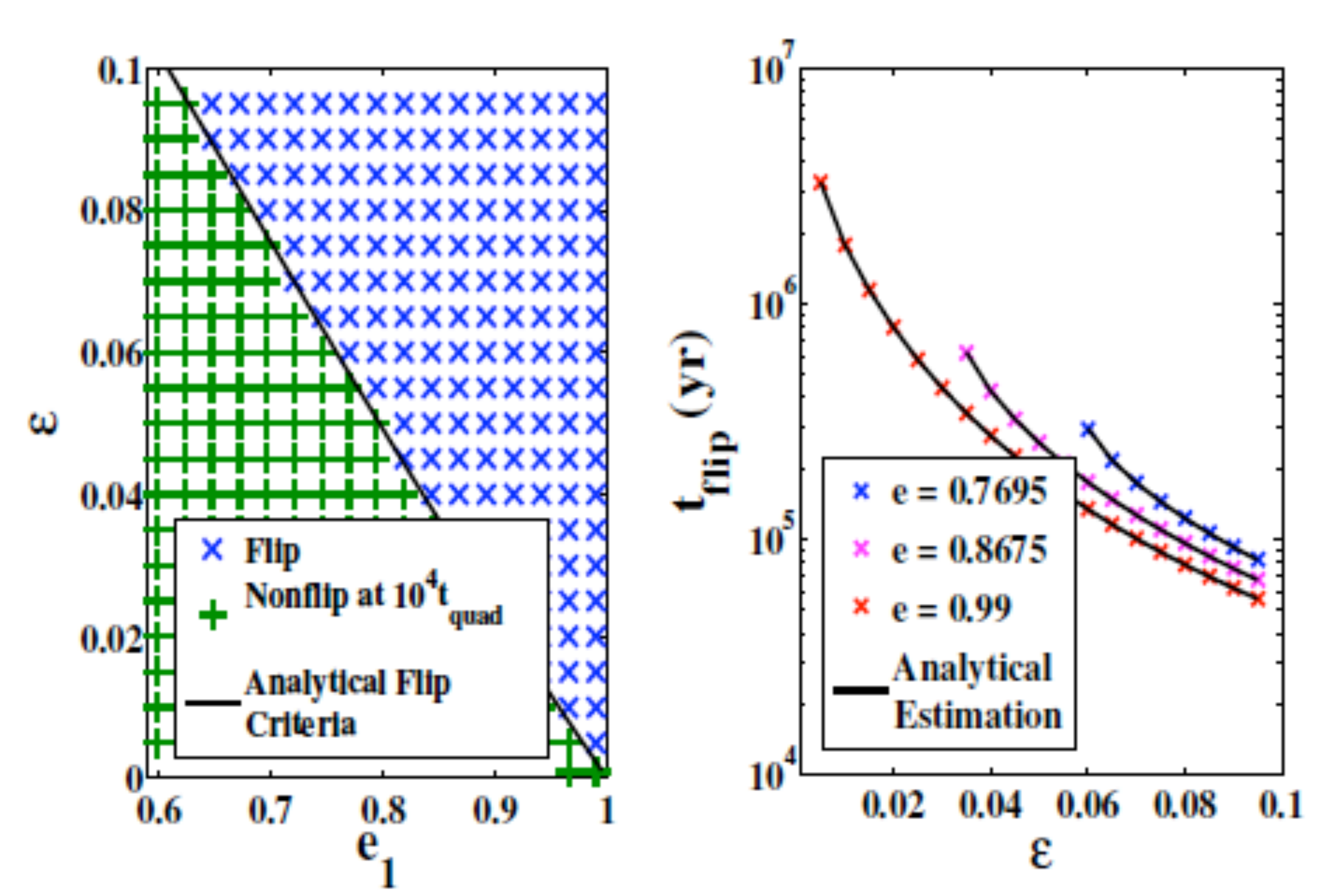
    => The flip timescale can be derived.

    => The flip criterion can be derived.

$$\varepsilon > \frac{8}{5}\frac{1-e_1^{2}}{7-e_1\left(4+3e_1^{2}\right)\cos\left(\omega_1+\Omega_1\right)}$$

Li et al. 2014a

ANALYTICAL RESULTS V.S. NUMERICAL RESULTS

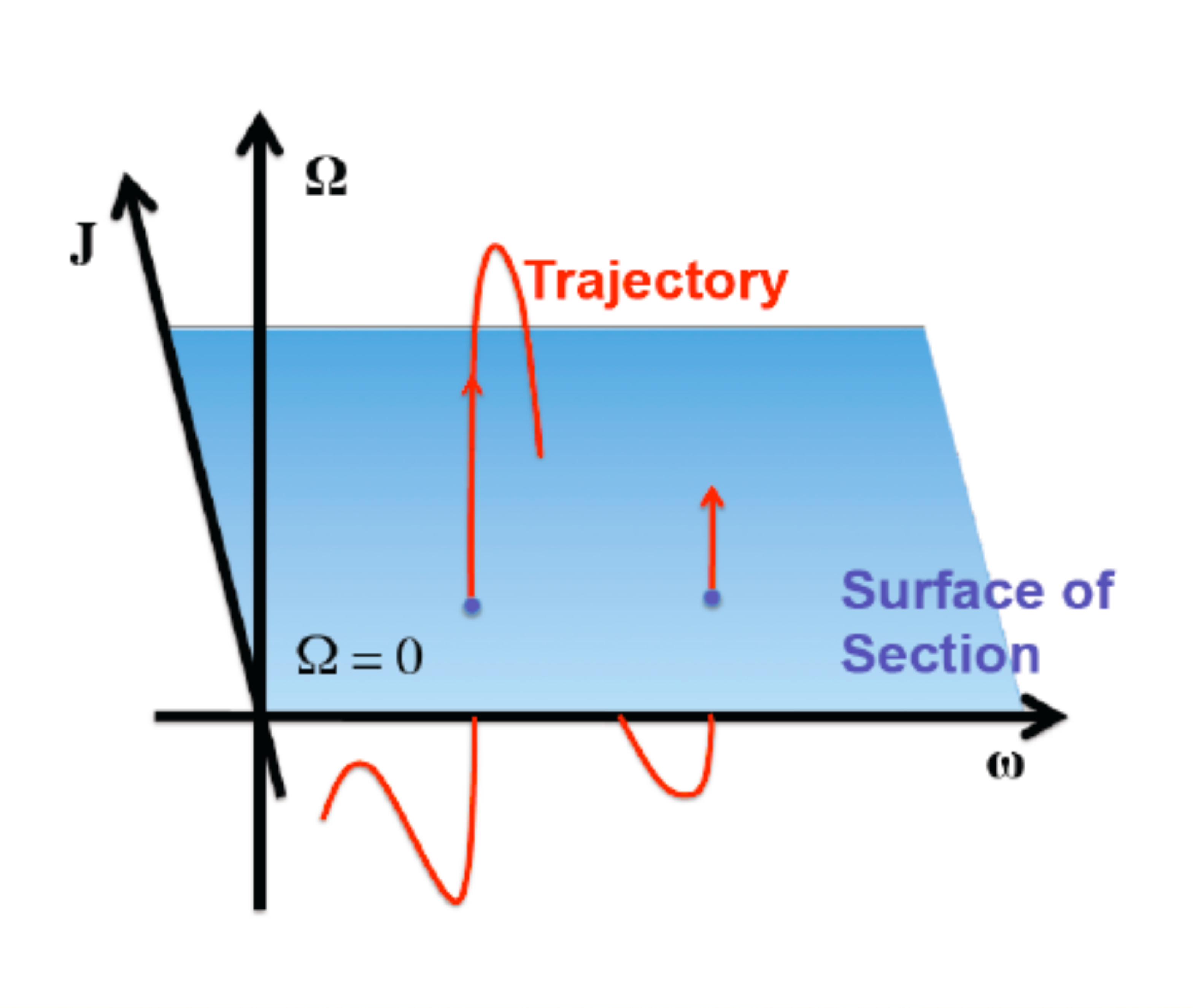


IC: i=5º.

- The flip criterion and the flip timescale from secular integration are consistent with the analytical results.

Li et al. 2014a

SURFACE OF SECTION

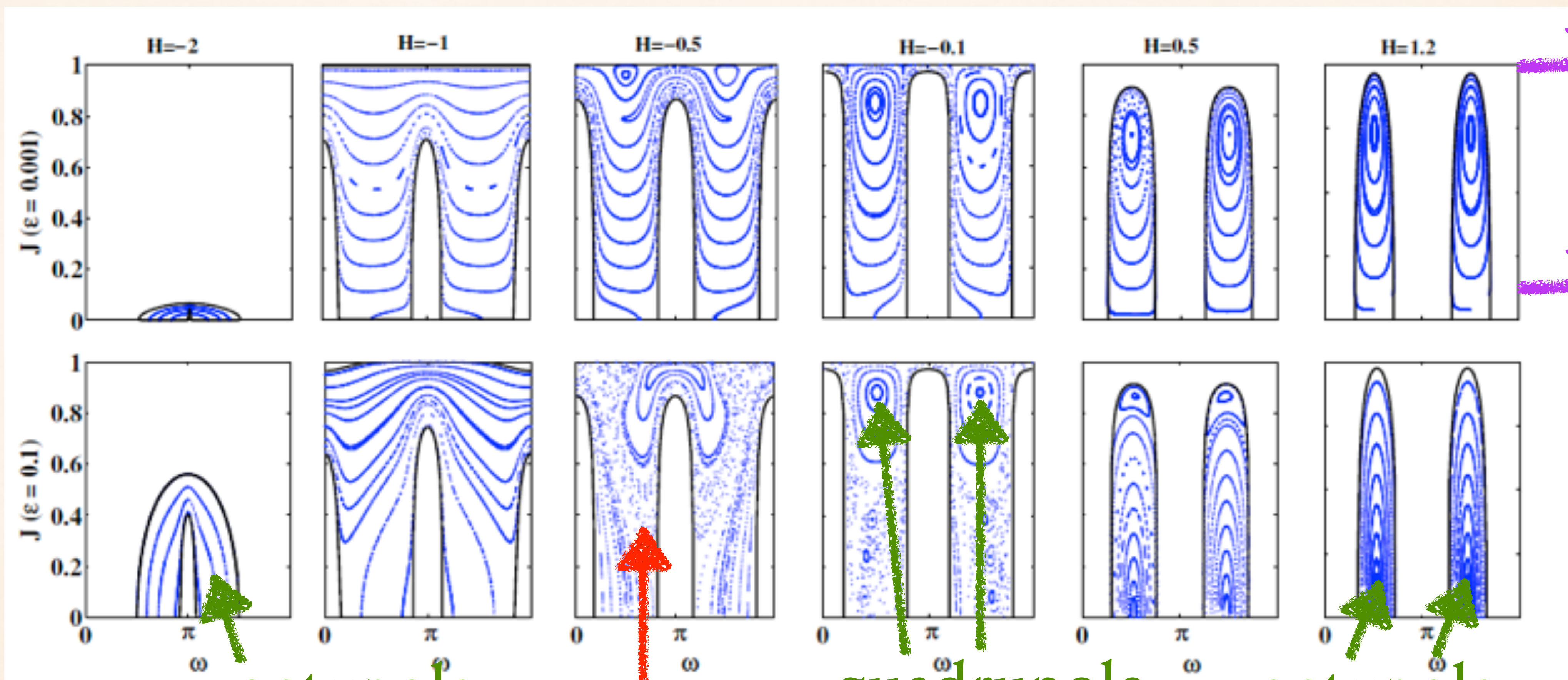


SURFACE OF SECTION

Low i ⟷ High i (40-60°) ⟷ i~90°

Quadrupole order dominates

Octupole order stronger



low e

high e

octupole resonances

chaos

quadrupole resonances

octupole resonances

Quadrupole resonances:

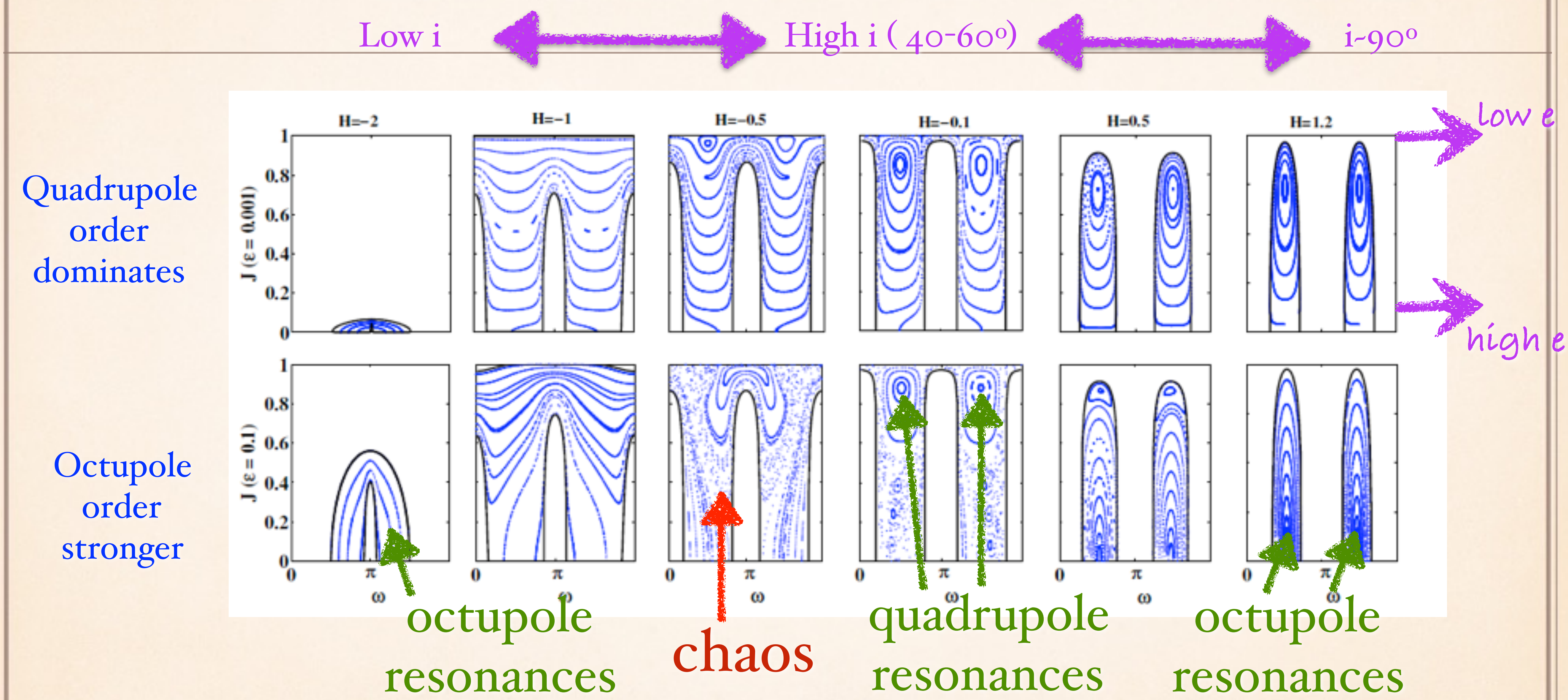
centers at low e_1, $\omega=\pi/2$ and $3\pi/2$ (*e.g., Kozai 1962*)

Octupole resonances:

centers at high e_1, $\omega=\pi$ or $\pi/2$ and $3\pi/2$

Li et al. 2014b

SURFACE OF SECTION



Low inclination clip regular

High inclination chaotic.

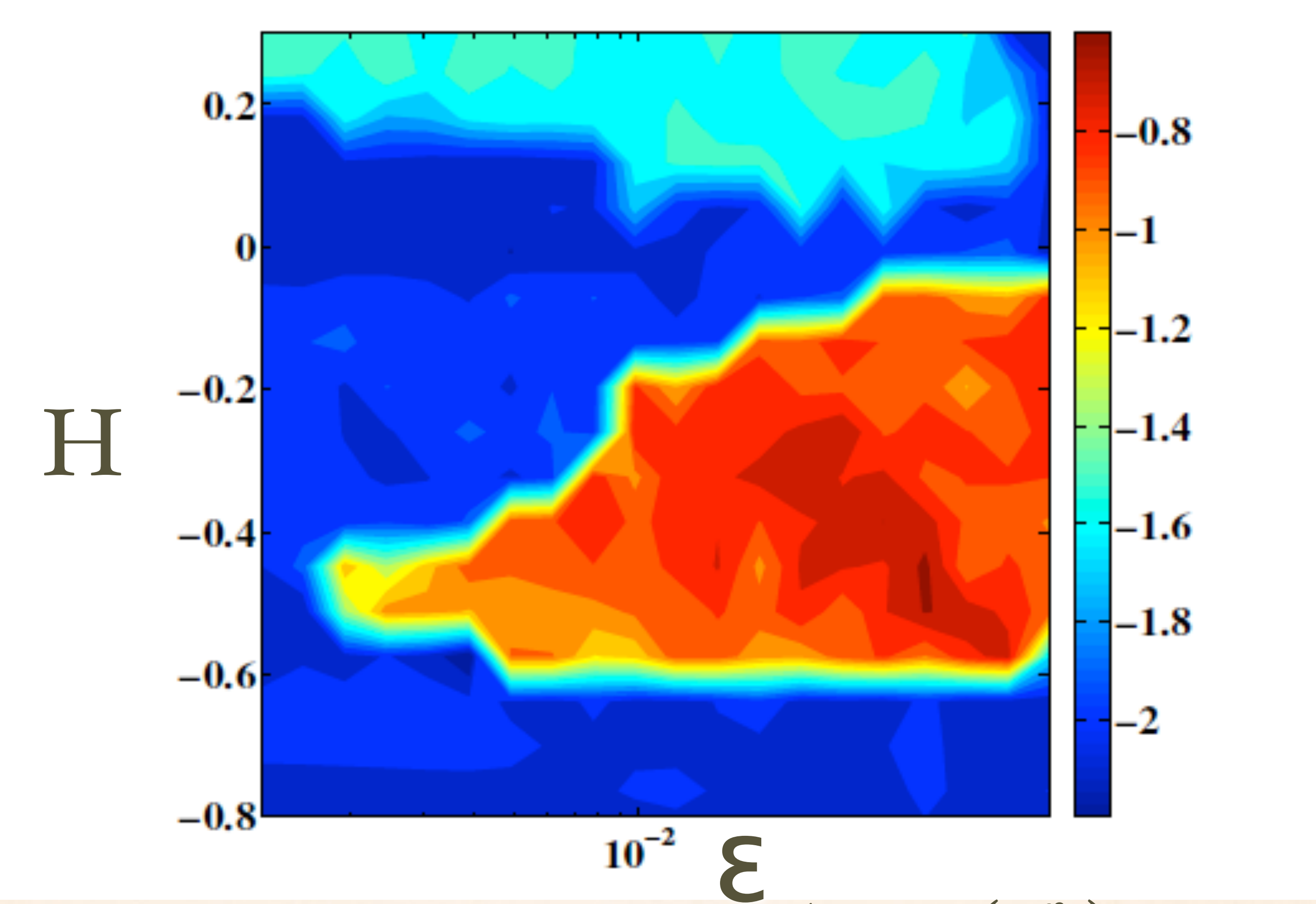
Li et al. 2014b

CHARACTERIZATION OF CHAOS

- Chaotic when H≤0 (correspond to high i cases).

Lyapunov Exponent: Log(λ)

H



ε

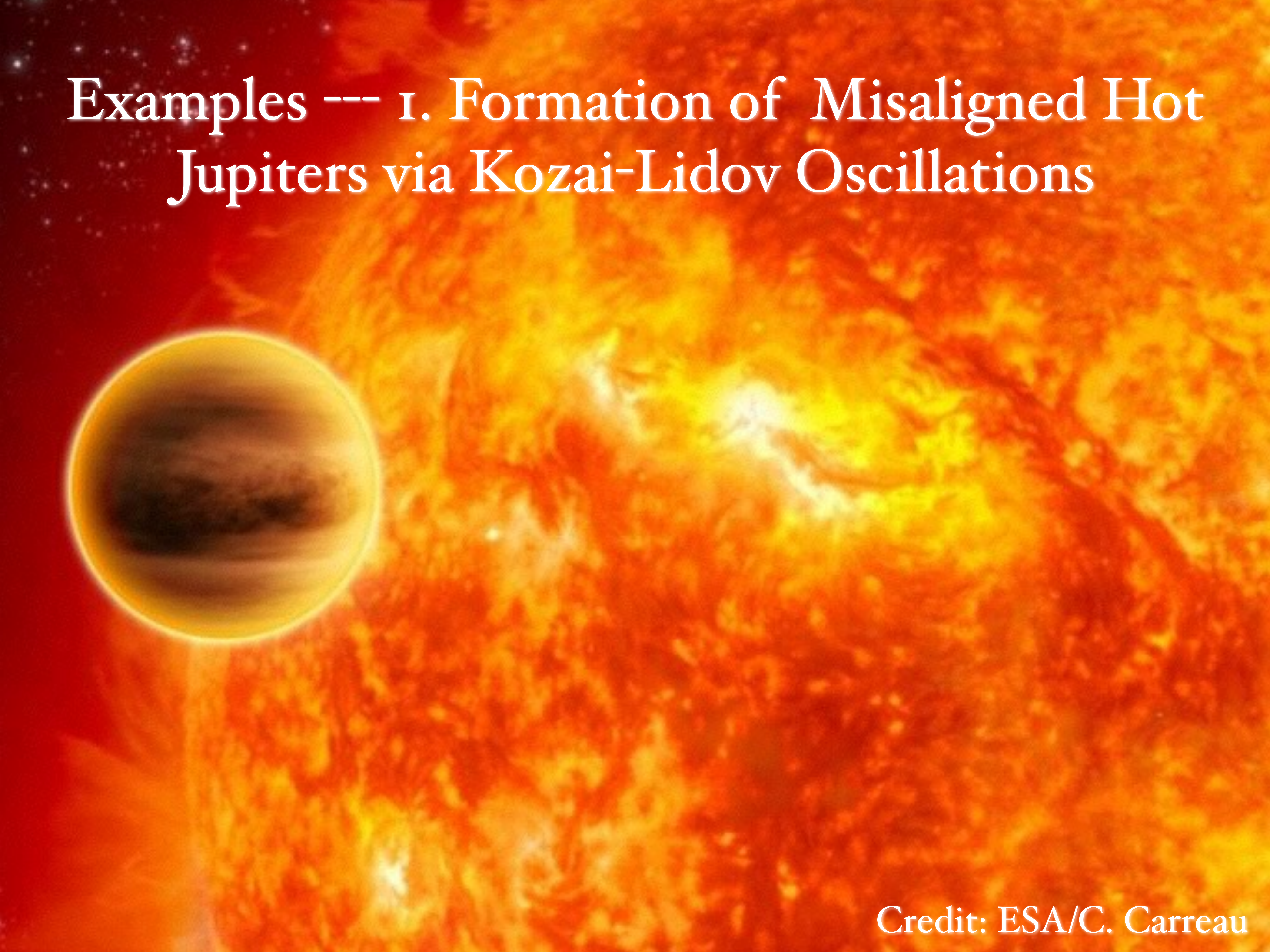
- In chaotic region, Lyapunov timescale $t_L=(1/\lambda) \approx 6t_K$.

  (t_K corresponds to the oscillation timescale of e_1 and i)

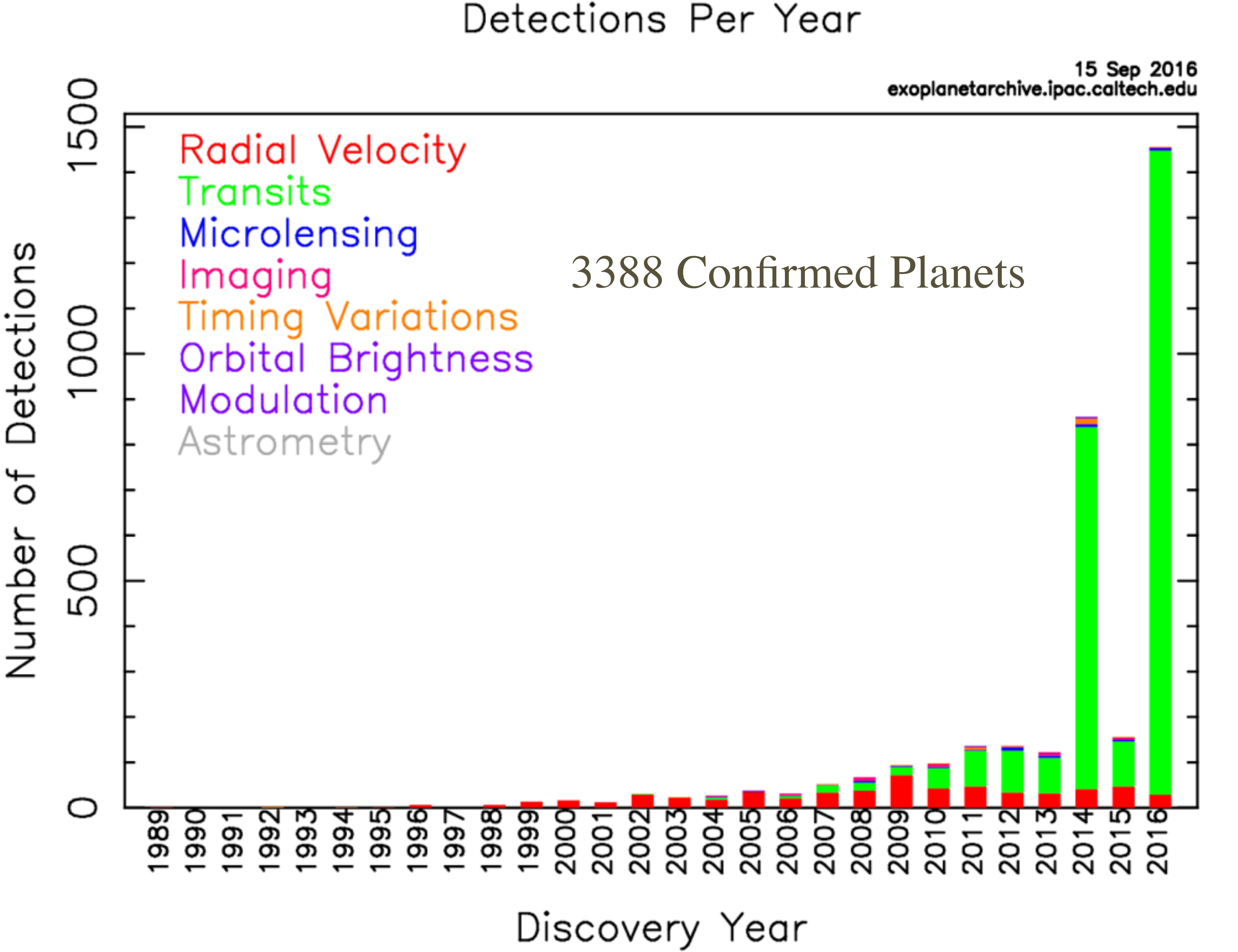
$$t_K = \frac{8}{3} P_{in} \frac{m_1}{m_2} \left(\frac{a_2}{a_1}\right)^3 (1-e_2^2)^{3/2}$$

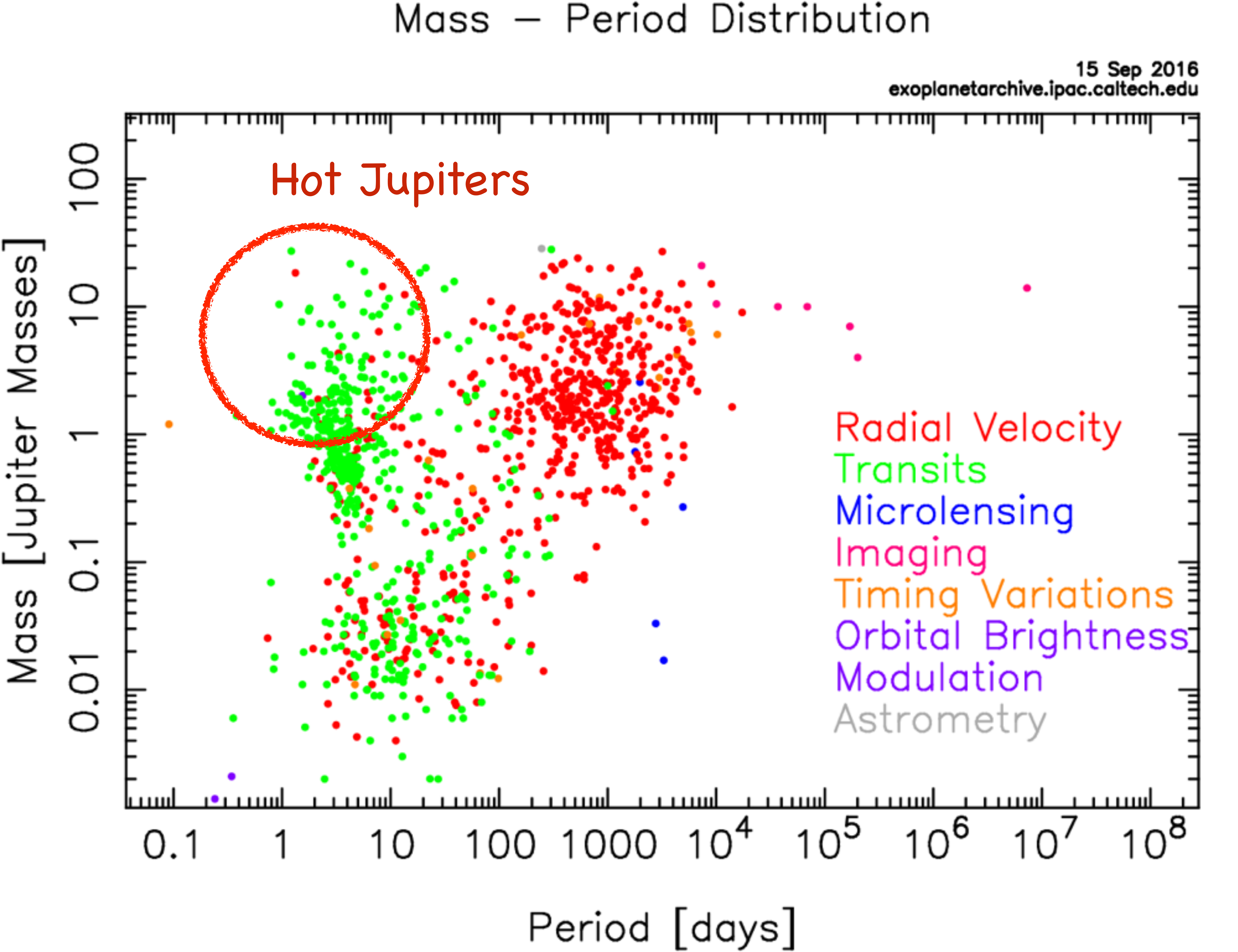
Li et al. 2014b

Examples --- 1. Formation of Misaligned Hot Jupiters via Kozai-Lidov Oscillations



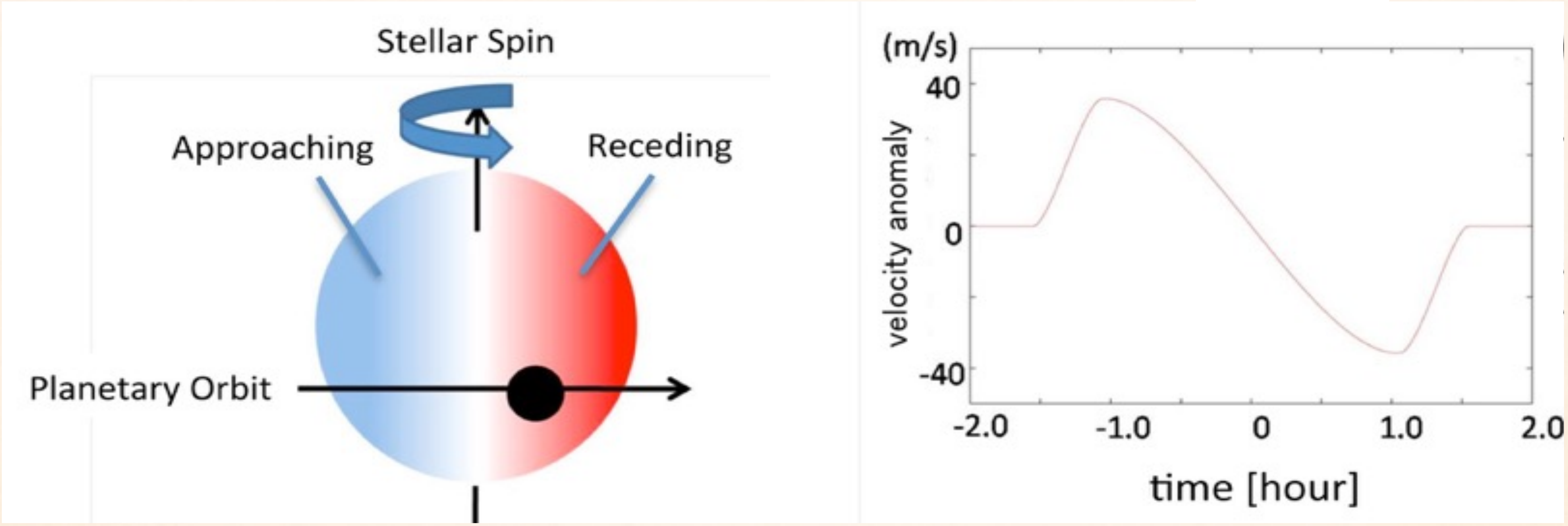
Credit: ESA/C. Carreau


Detections Per Year
15 Sep 2016
exoplanetarchive.ipac.caltech.edu
Radial Velocity
Transits
Microlensing
Imaging
Timing Variations
Orbital Brightness
Modulation
Astrometry
3388 Confirmed Planets
Number of Detections
0
500
1000
1500
1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016
Discovery Year



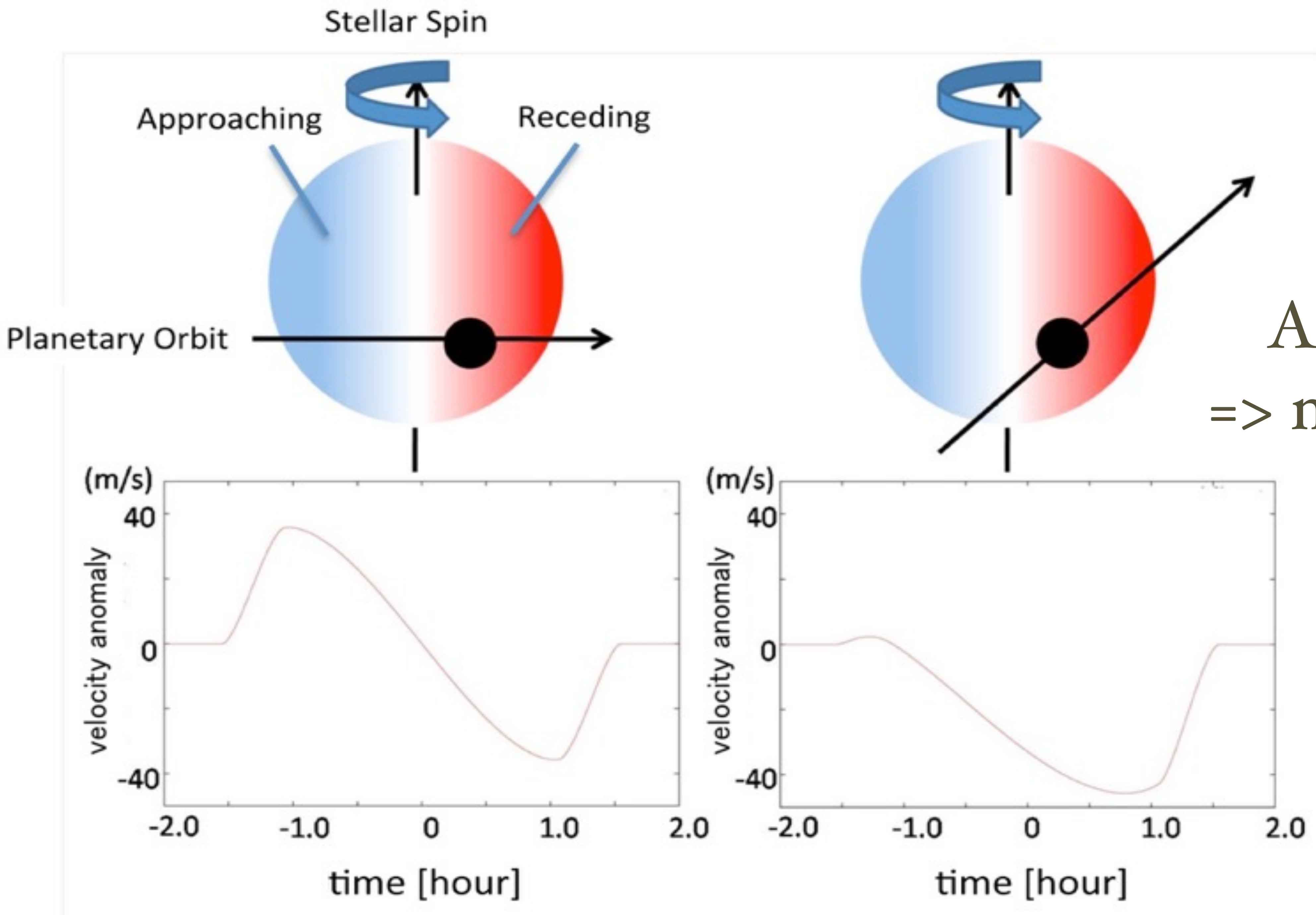
Mass — Period Distribution
15 Sep 2016
exoplanetarchive.ipac.caltech.edu
Hot Jupiters
Mass [Jupiter Masses]
100
10
1
0.1
0.01
Radial Velocity
Transits
Microlensing
Imaging
Timing Variations
Orbital Brightness
Modulation
Astrometry
0.1
1
10
100
1000
10^4
10^5
10^6
10^7
10^8
Period [days]

ROSSITER-MCLAUGHLIN METHOD (SPIN-ORBIT MISALIGNMENT)



e.g., Ohta et al. 2005, Winn 2006

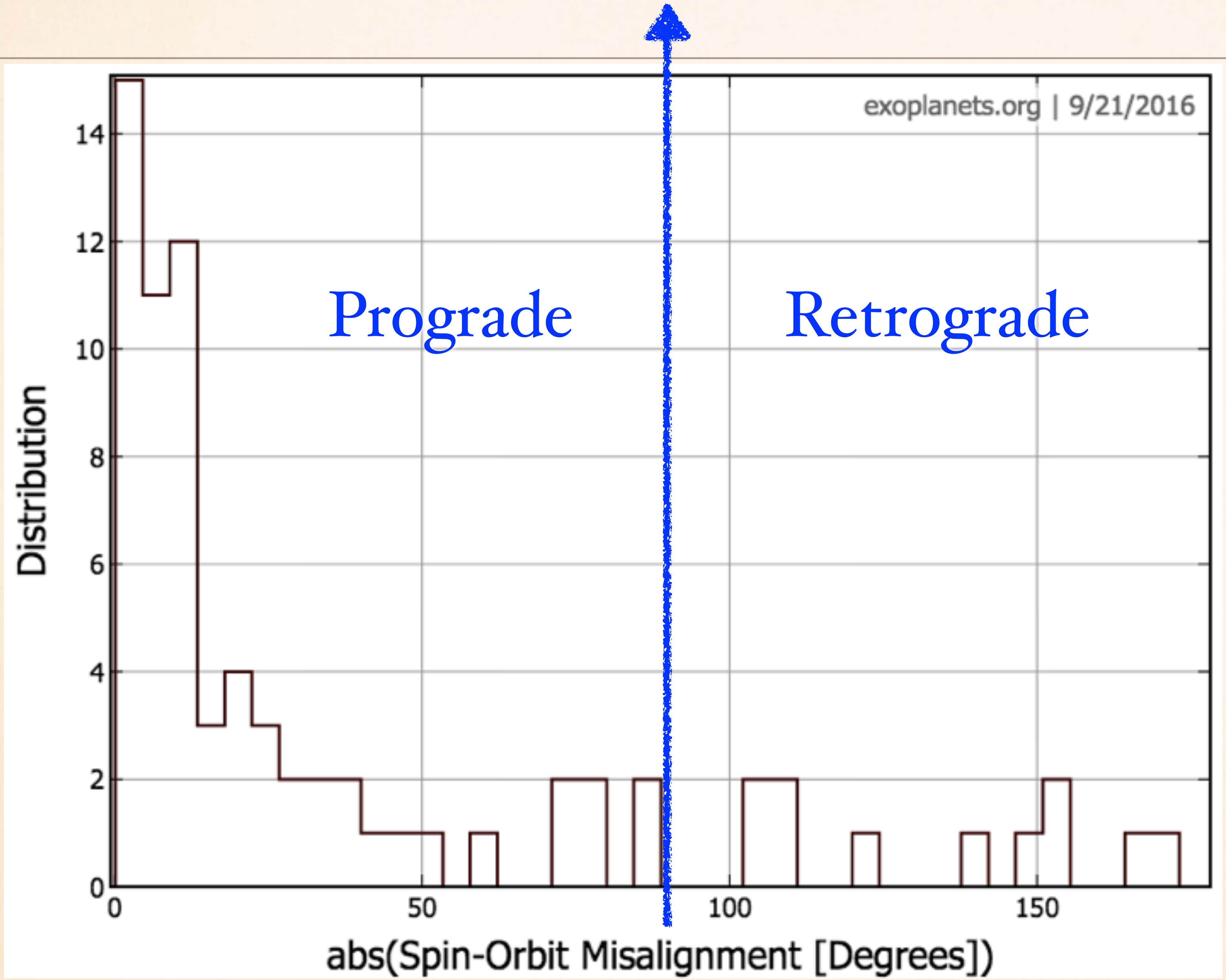
ROSSITER-MCLAUGHLIN METHOD (SPIN-ORBIT MISALIGNMENT)



Asymmetric
=> misalignment

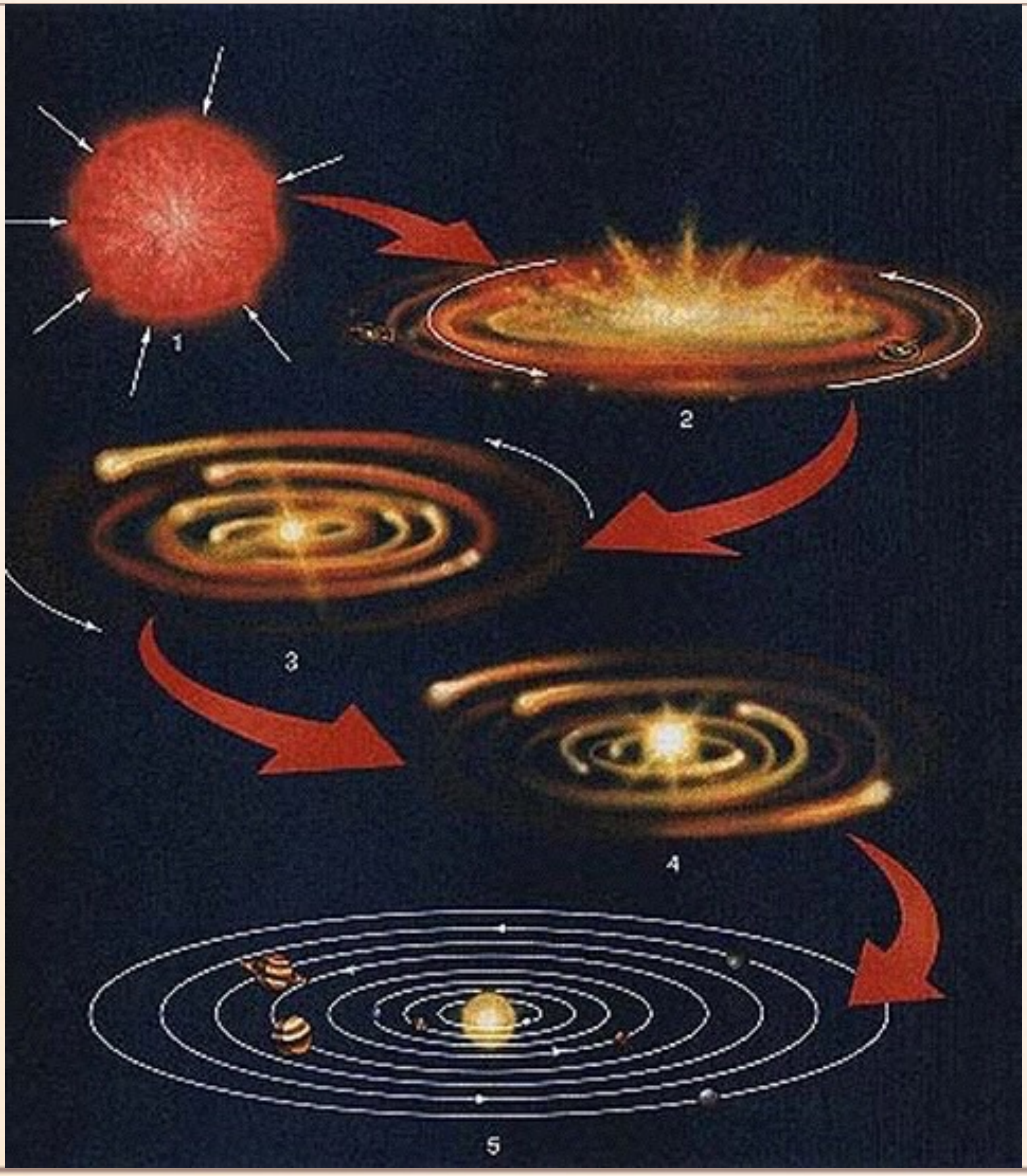
e.g., Ohta et al. 2005, Winn 2006

OBSERVED SPIN-ORBIT MISALIGNMENT



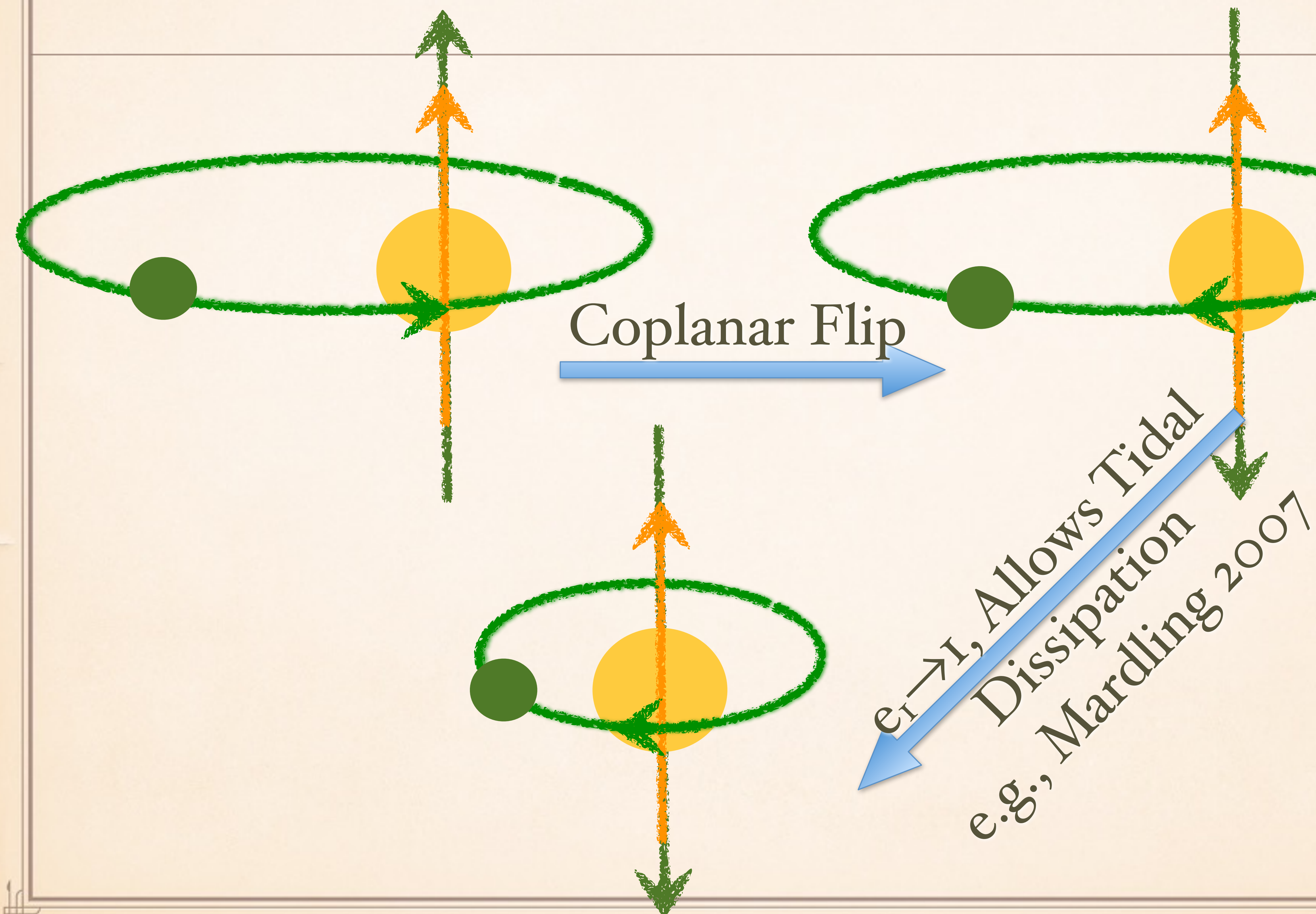
Solar system spin-orbit misalignment $\lesssim 7^{o}$ (Lissauer 1993)

CHALLENGES CLASSICAL PLANETARY FORMATION THEORIES

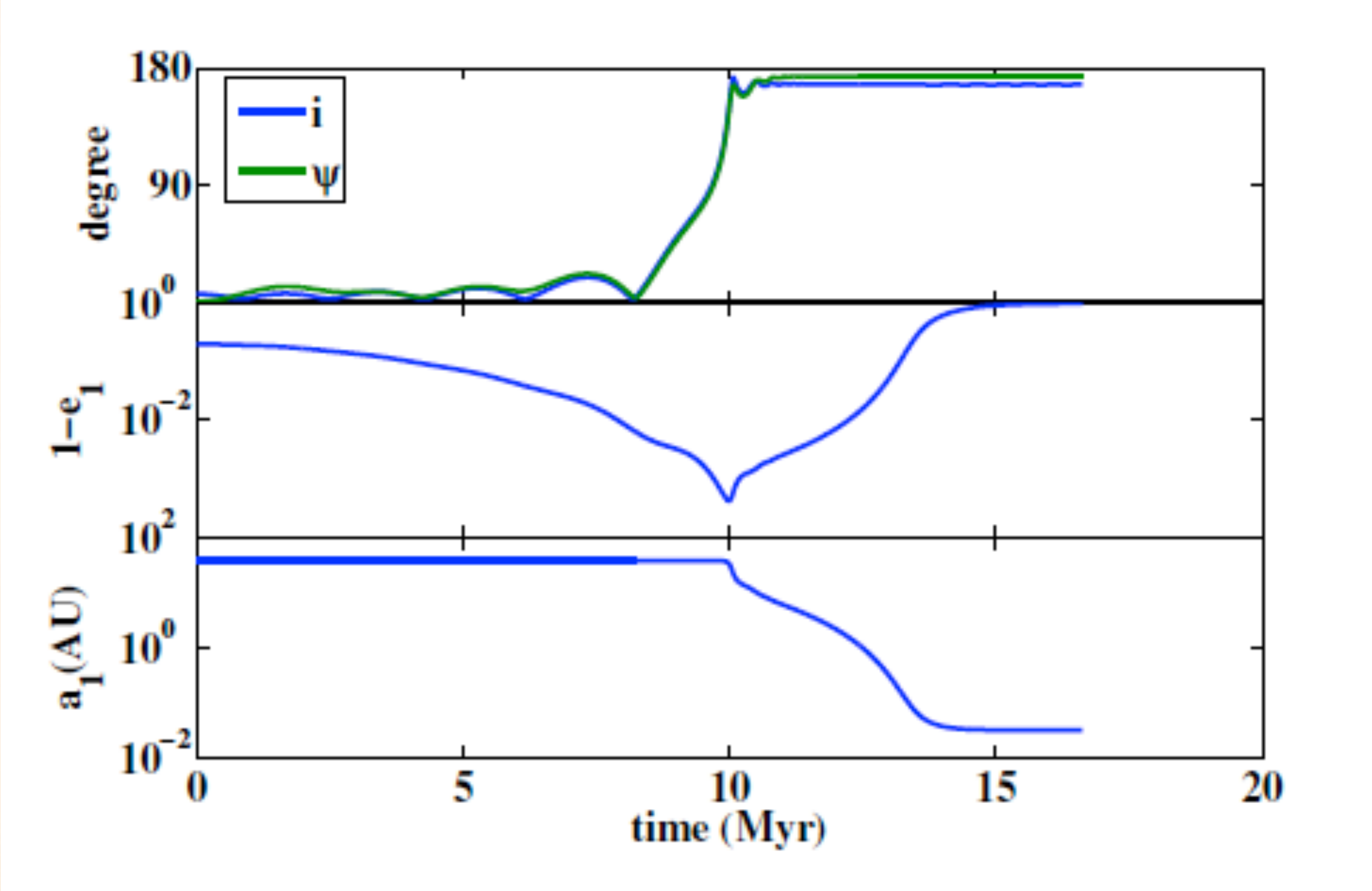


Classical planetary formation theory: Star and planets form in a molecular cloud, and share the same direction of rotation.

FORMATION OF COUNTER ORBITING HOT JUPITERS (KL + TIDE)



FORMATION OF COUNTER ORBITING HOT JUPITERS (KL + TIDE)



e_1 → 1 during the flip

=> r_p↓, tide dominates.

=> e_1→0, a_1↓, i, ψ ≈ 180°.

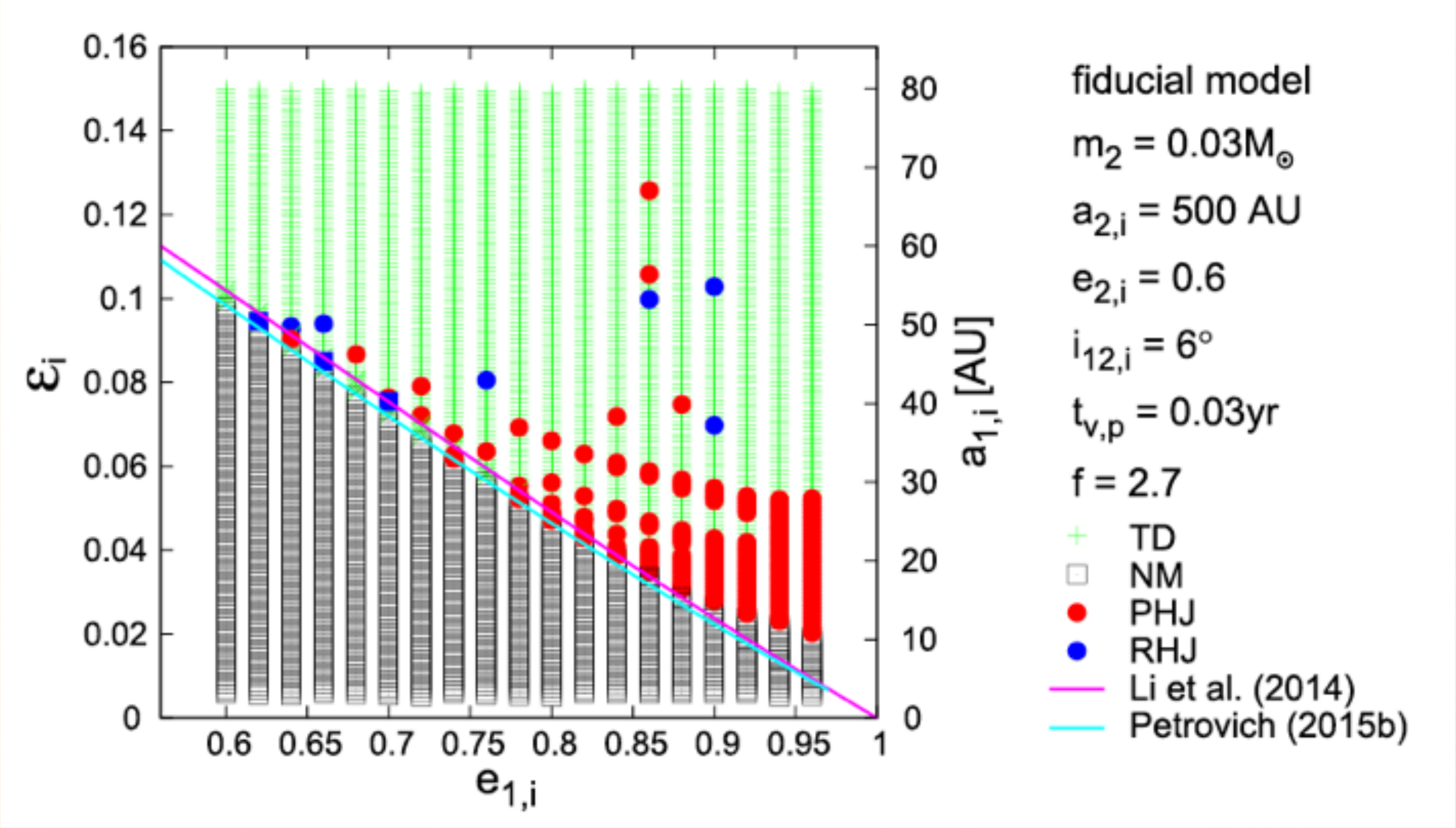
Li et al. 2014a

DIFFICULTY IN THE FORMATION OF COUNTER-ORBITING HOT JUPITERS

Numerical simulations including short range forces.

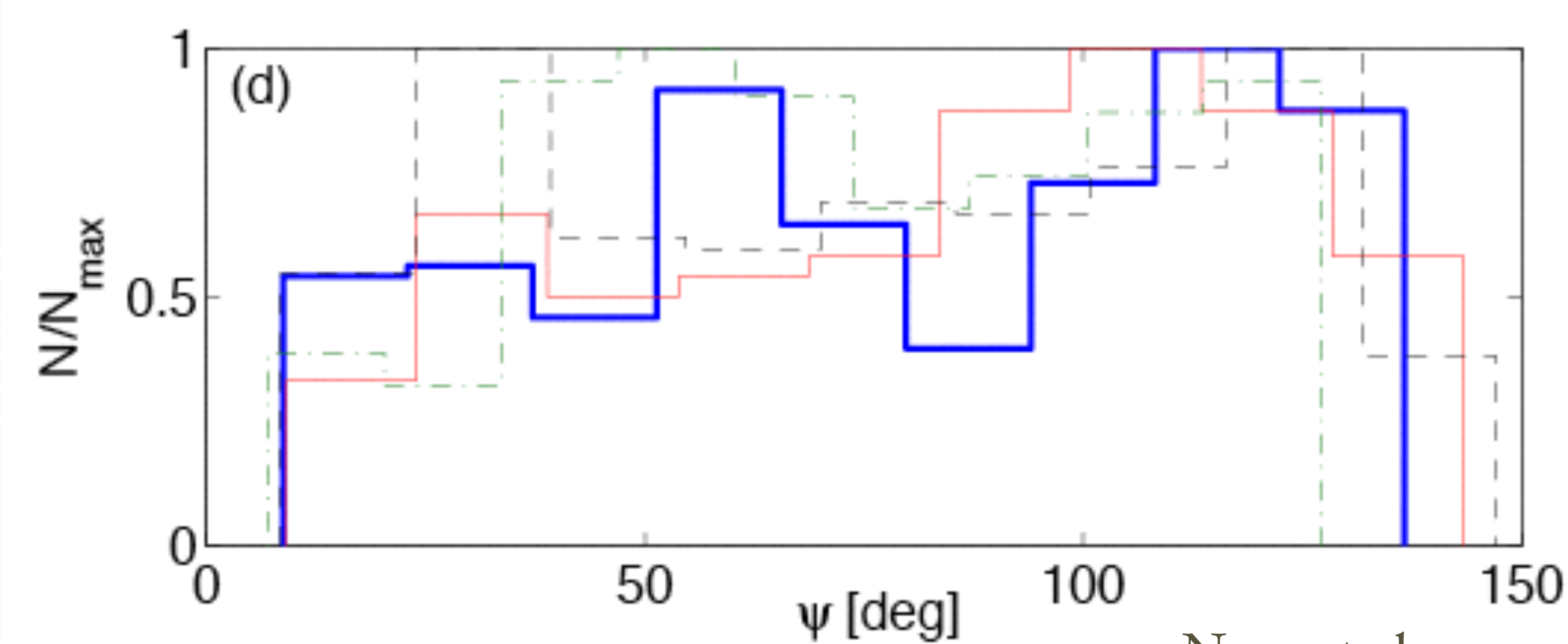
Most systems are tidally disrupted and a small fraction turn out to be prograde.

The formation of counter-orbiting HJs in a very restricted parameter region.



Xue & Suto 2016

FORMATION OF MISALIGNED HOT JUPITERS (KL + TIDE) BY POPULATION SYNTHESIS

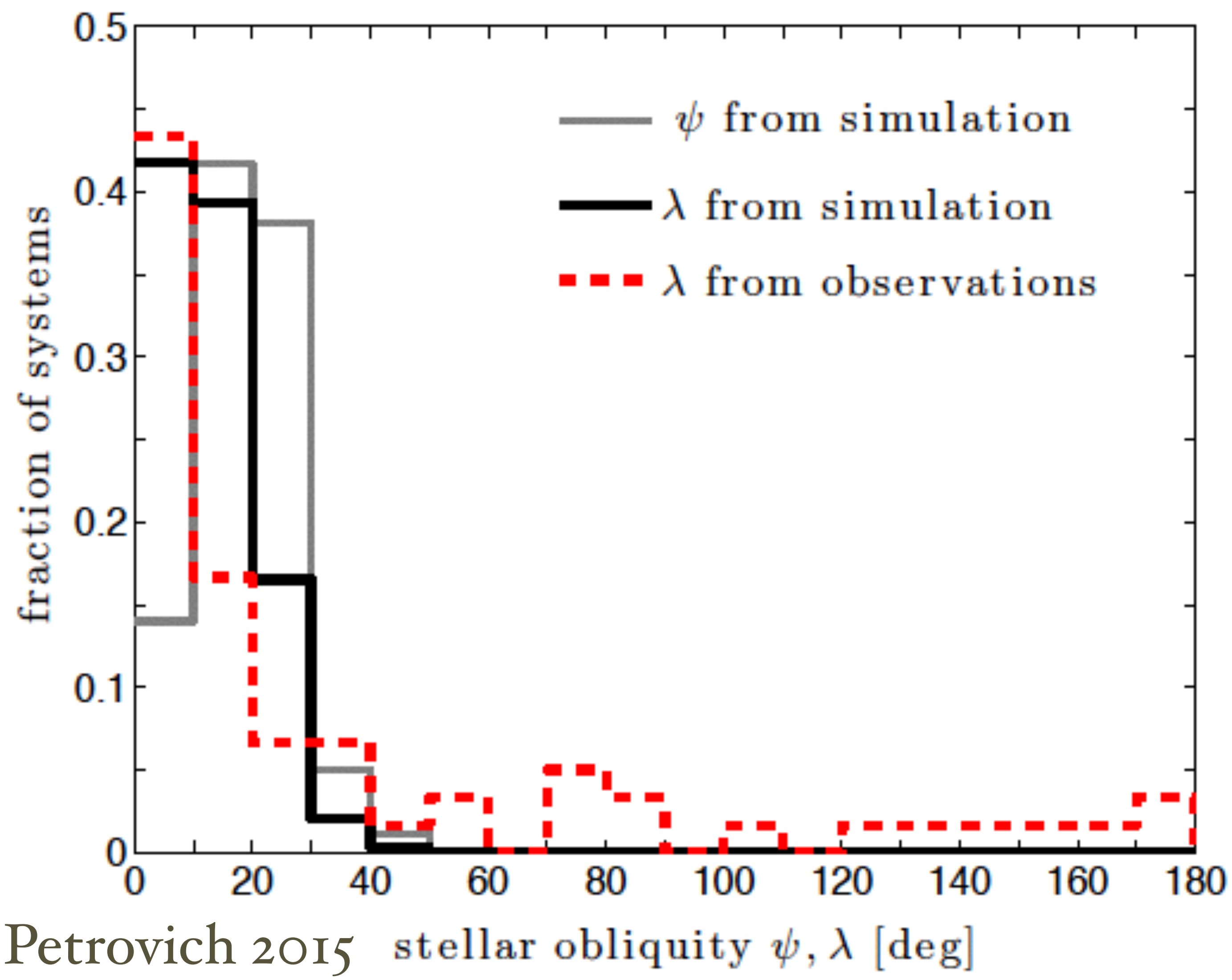


Naoz et al. 2011

- 15% of systems produce hot Jupiters
- EKL may account for about 30% of hot Jupiters

(Naoz et al. 2011)

FORMATION OF MISALIGNED HOT JUPITERS (KL + TIDE) BY POPULATION SYNTHESIS



Petrovich 2015

Population synthesis study of interaction of two giant planets.

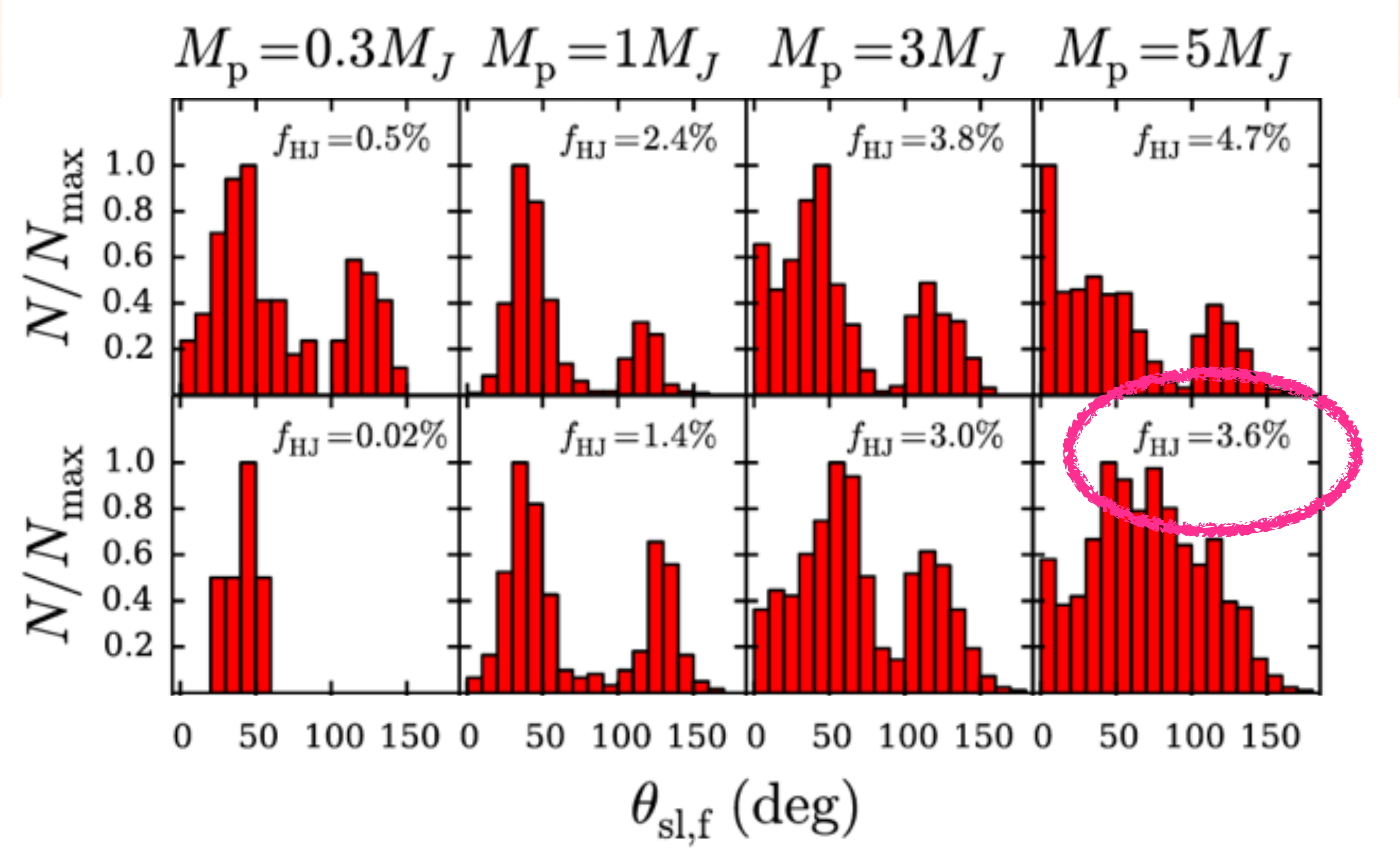
=> a different mechanism is needed (Petrovich 2015)

FORMATION OF MISALIGNED HOT JUPITERS (KL + STELLAR OBLATENESS + TIDE)

Anderson et al. 2016:

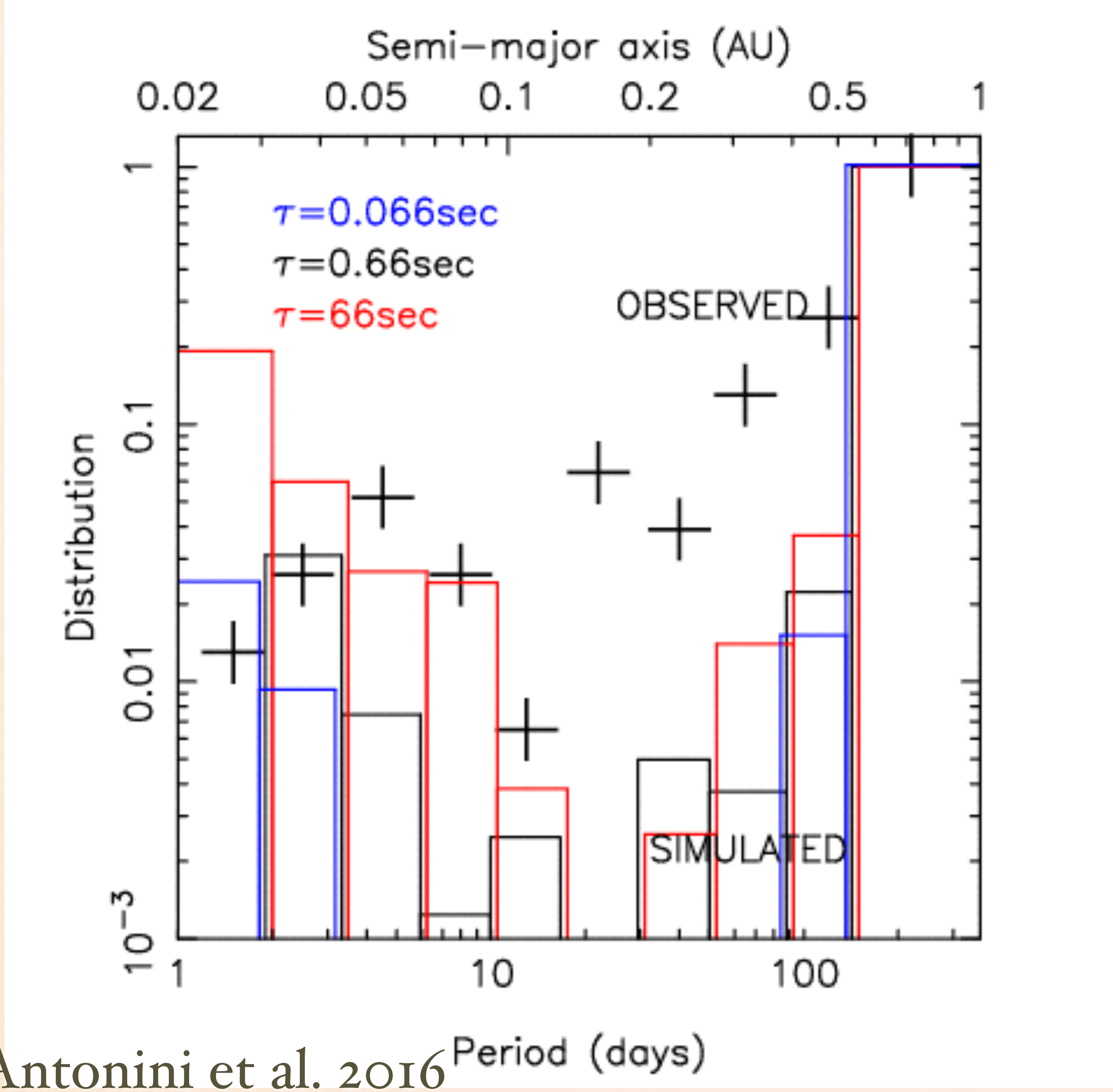
Mp < 3 MJ
=> bimodal

Mp ~ 5MJ
=> low misalignment (solar-type stars)
=> higher misalignment (more massive stars)



Anderson et al. 2016

FORMATION OF WARM JUPITERS

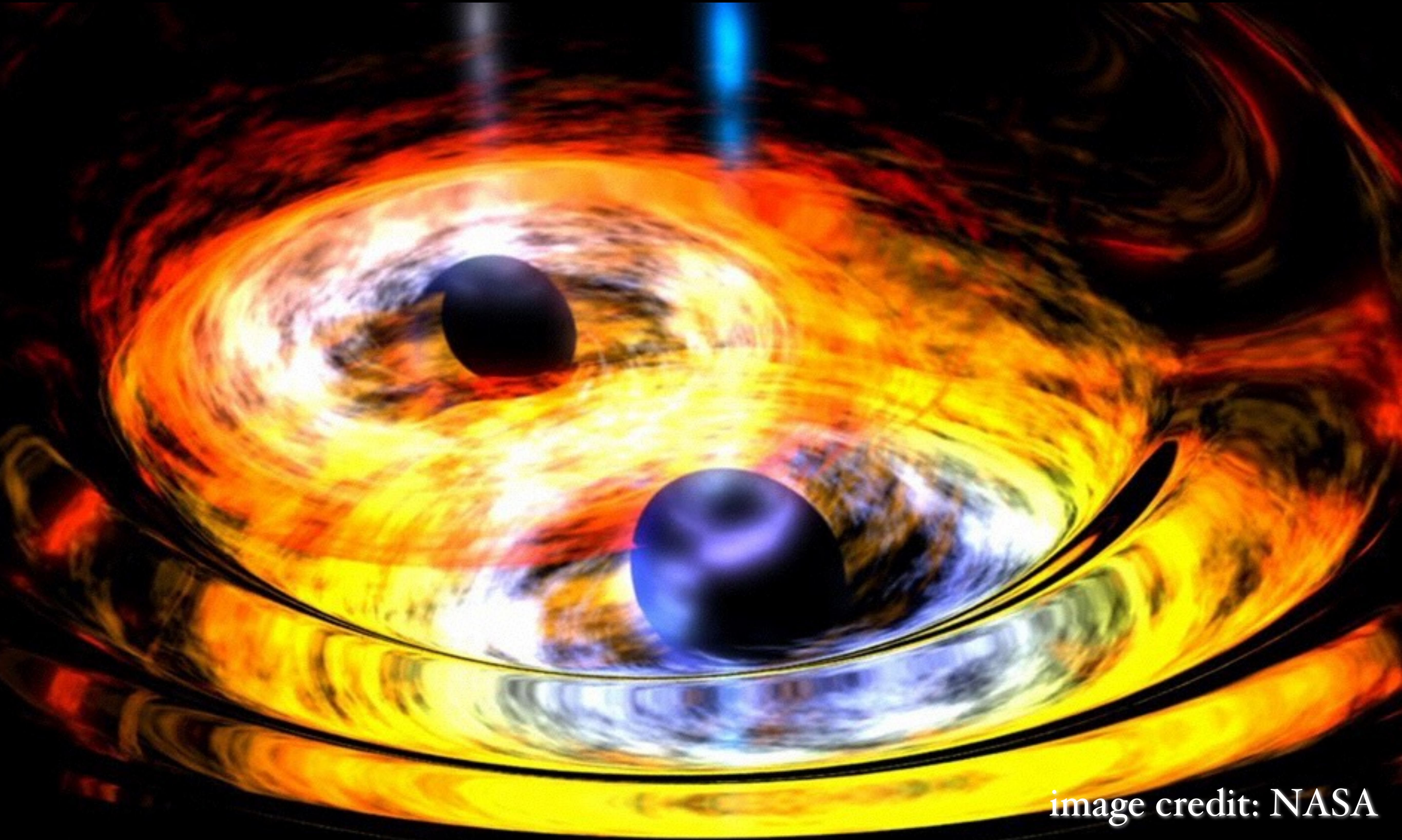


Antonini et al. 2016

EKL produces warm Jupiters (Dawson & Chiang 2014)

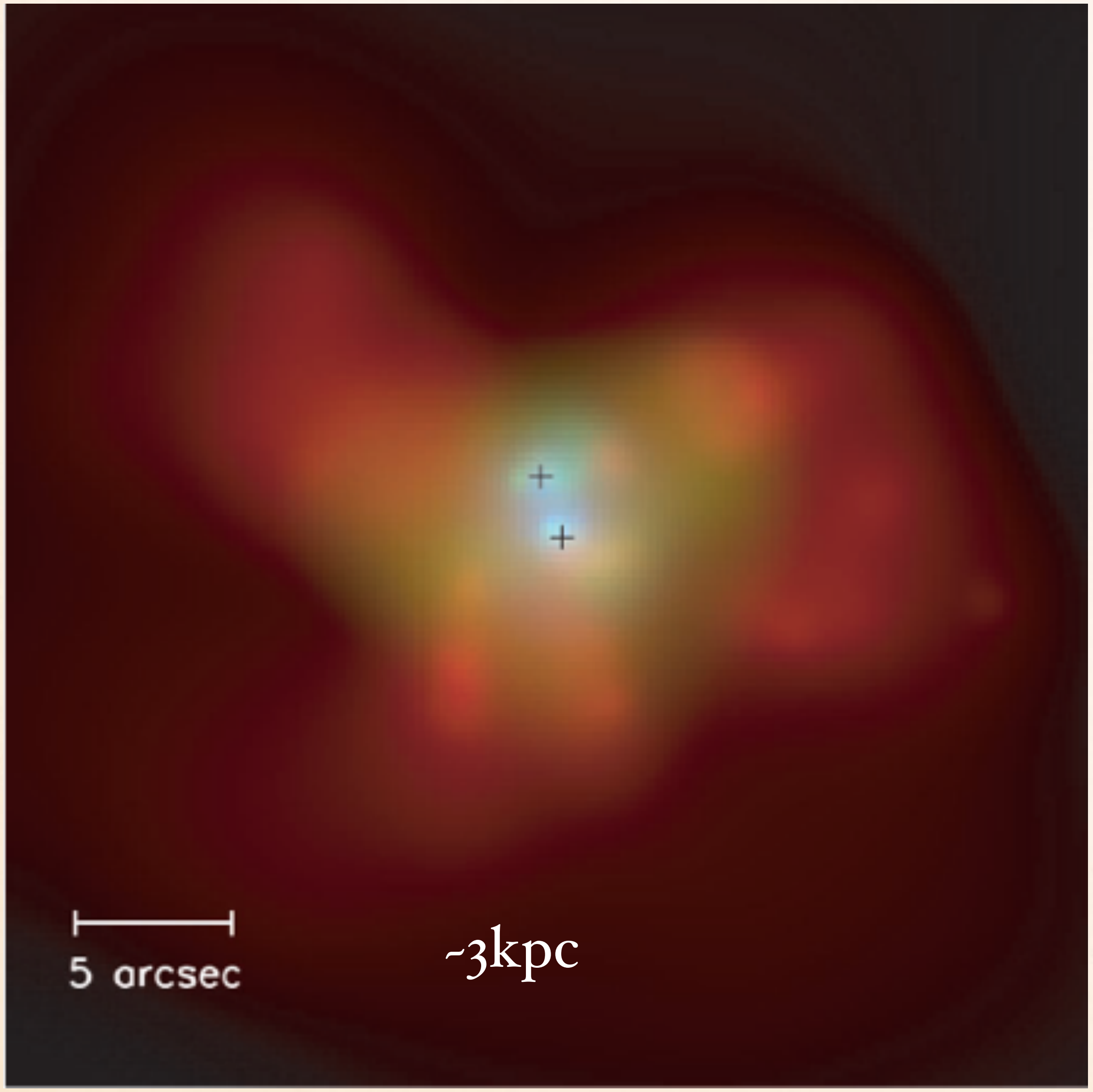
EKL accounts for <10-20% of the observed warm Jupiters (Antonini et al. 2016, Petrovich & Tremaine 2016)

EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB



EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB

- SMBHBs originate from mergers between galaxies.



- SMBHBs with mostly ~kpc separation have been observed with direct imagine.

  (e.g., Woo et al. 2014; Komossa et al. 2013, Fabbiano et al. 2011, Green et al. 2010, Civano et al. 2010, Rodriguez et al. 2006, Komossa et al. 2003, Hutchings & Neff 1989)

Multicolor image of NGC 6240. Red p soft (0.5–1.5 keV), green p medium (1.5–5 keV), and blue p hard (5–8 keV) X-ray band. (Komossa et al. 2003)

PERTURBATIONS ON STARS SURROUNDING SMBHB

- Identify SMBHB at ~1 pc separation by stellar features due to interactions with SMBHB.

  (e.g., Chen et al. 2009, 2011, Wegg & Bode 2011, Li et al. 2015)

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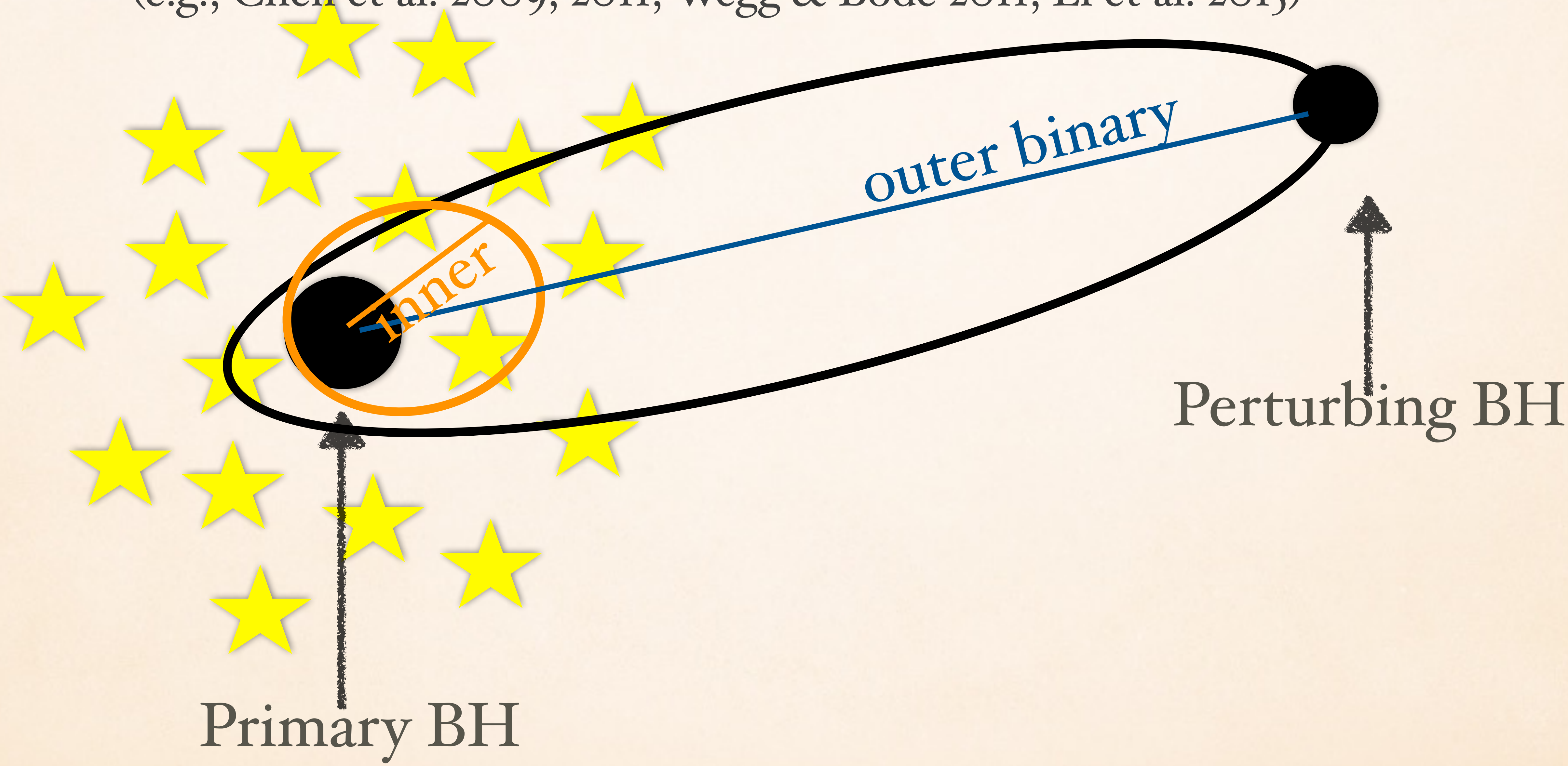
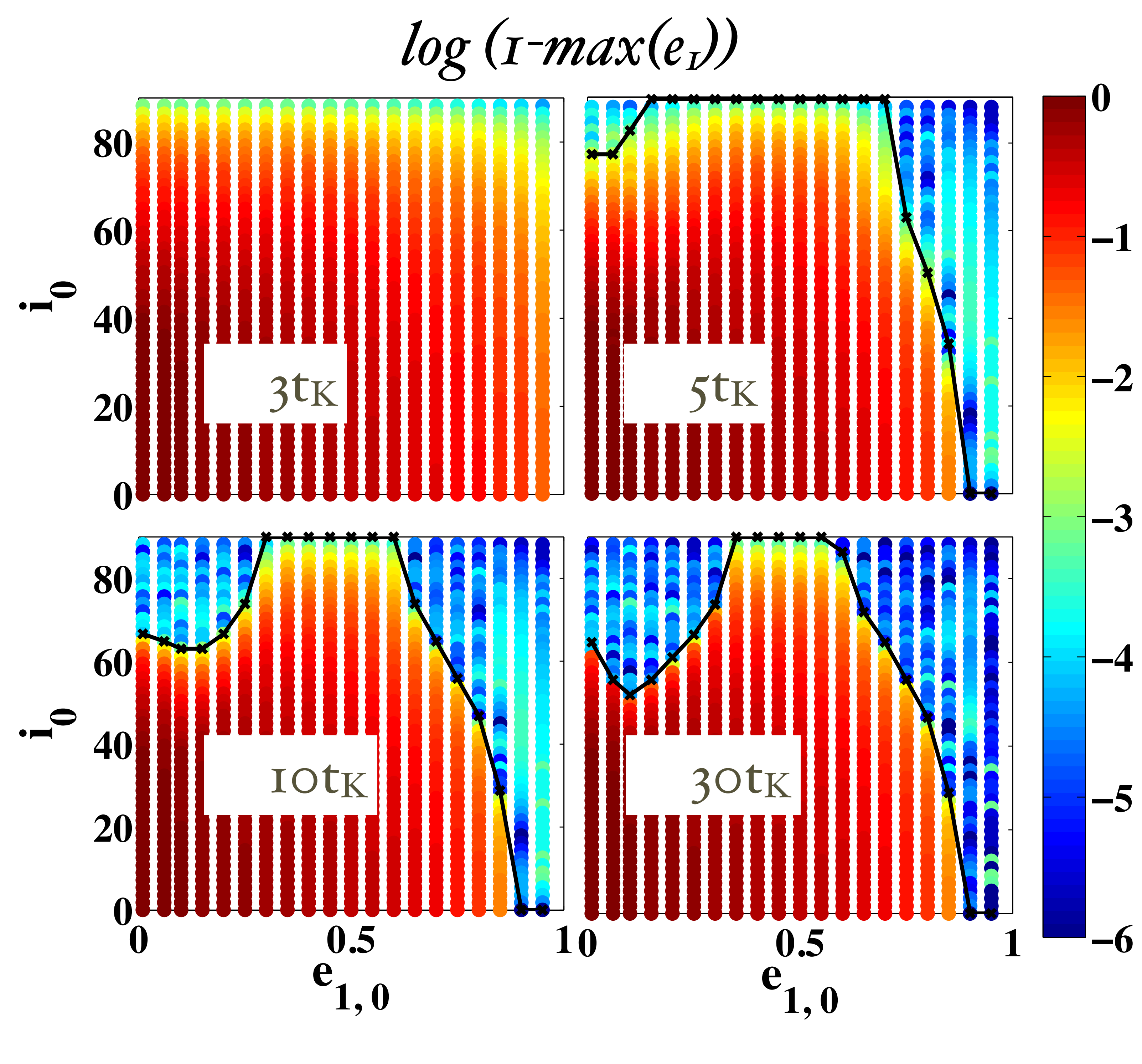


ENHANCEMENT OF TIDAL DISRUPTION RATES



$e_{1,\,max}$ determines the closest distance:

$r_p \propto (1-e_1)$

$$t_K = \frac{8}{3} P_{in} \frac{m_1}{m_2} \left(\frac{a_2}{a_1}\right)^3 (1-e_2^2)^{3/2}$$

e_{max} reaches $1-10^{-6}$ over ~$30t_K$

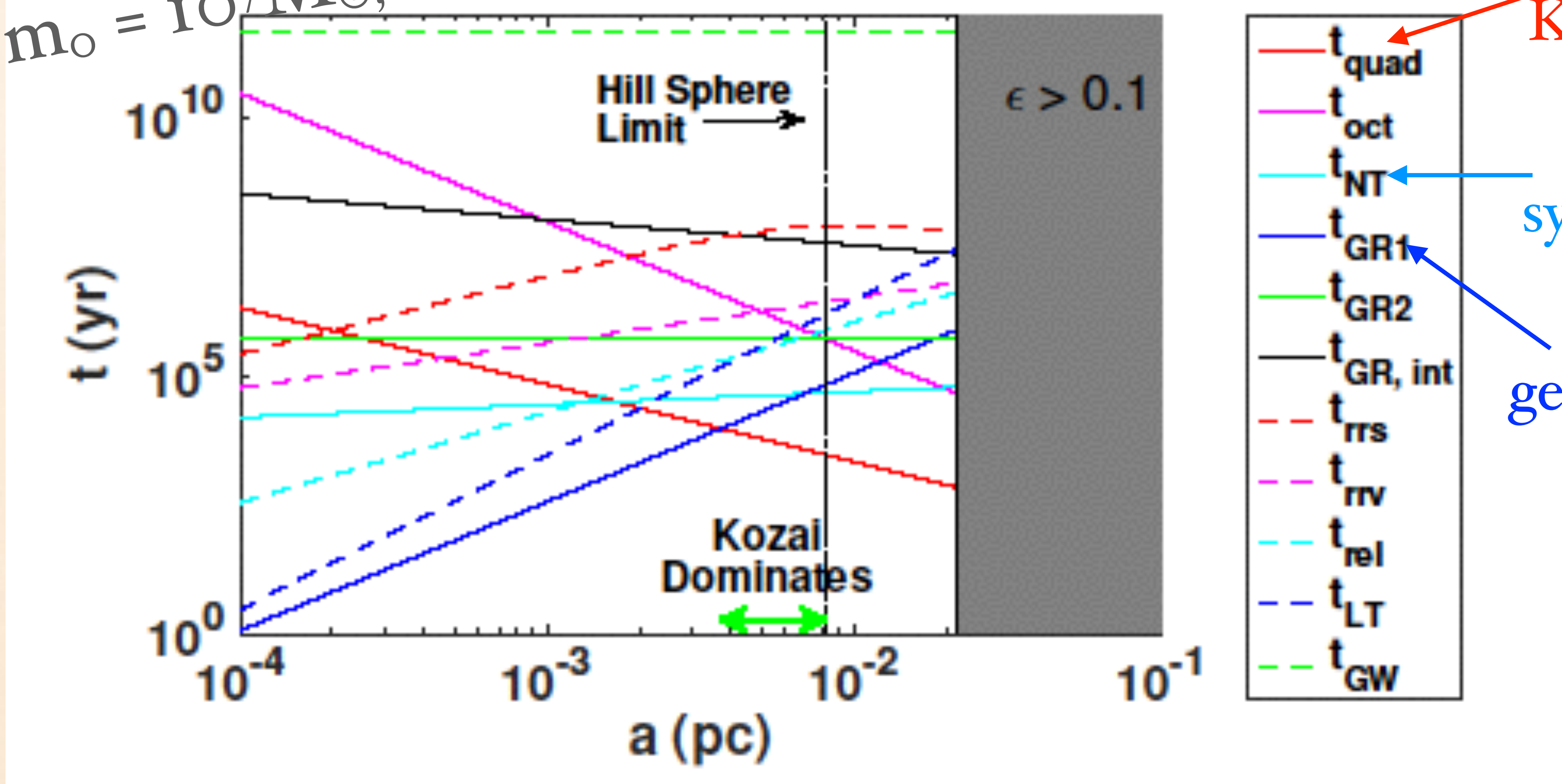
Starting at a~10^6R_t, it's still possible to be disrupted in ~$30t_K$!

Li et al. 2014a

SUPPRESSION OF EKL

- Eccentricity excitation suppressed when precession timescale < Kozai timescale.

$m_0 = 10^7 M_\odot$, $m_2 = 10^9 M_\odot$



Quadrupole Kozai timescale

Due to stellar system self-gravity

Due to general relativity

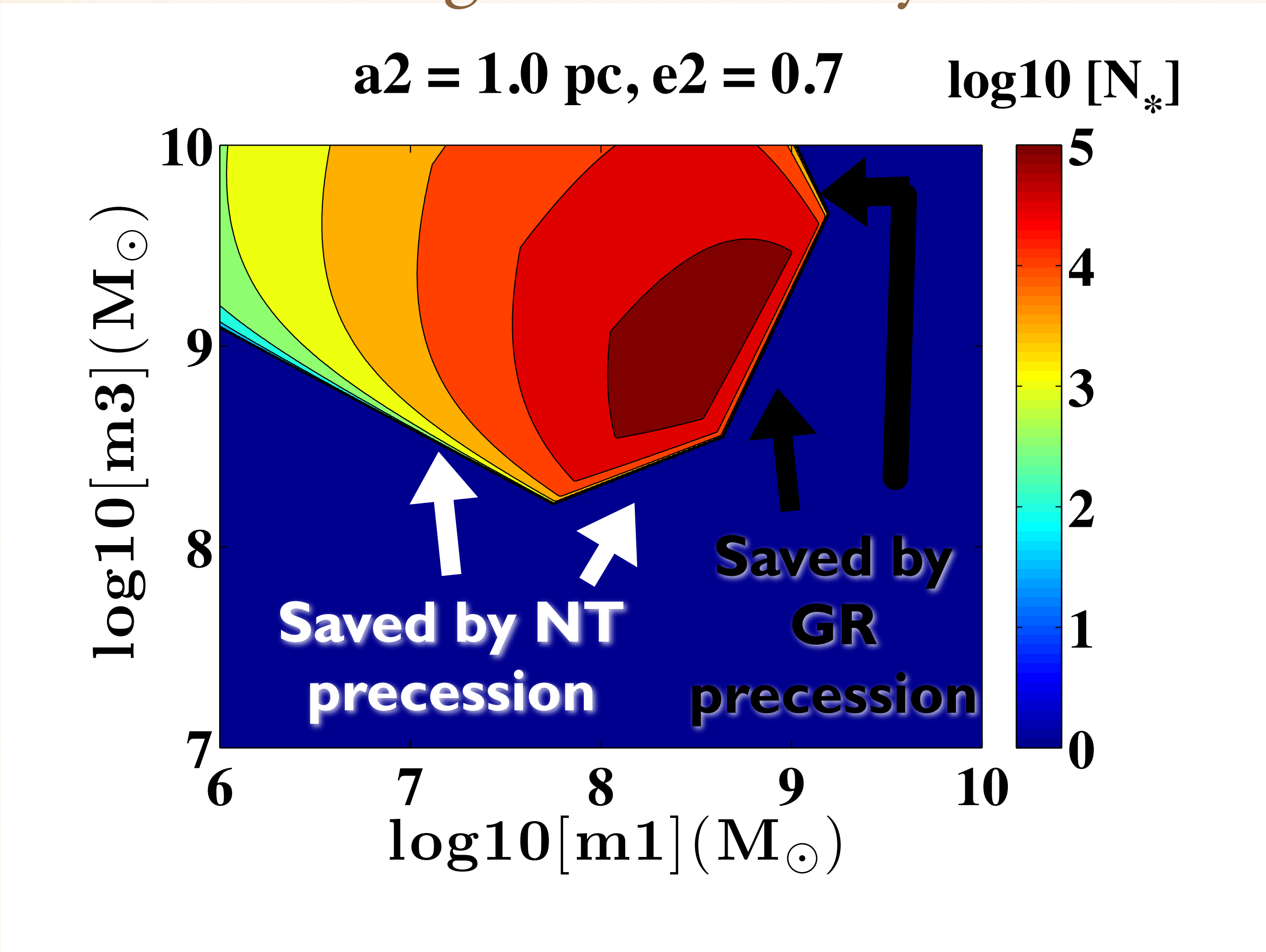
$e_1 = 2/3$, $a_2 = 0.3$ pc, $m_1 = 1 M_\odot$, $e_2 = 0.7$.

(Li et al. 2015)

EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB

- Eccentricity excitation suppressed when precession timescale < Kozai timescale.
- Stars around SMBHB: GR and NT precession.

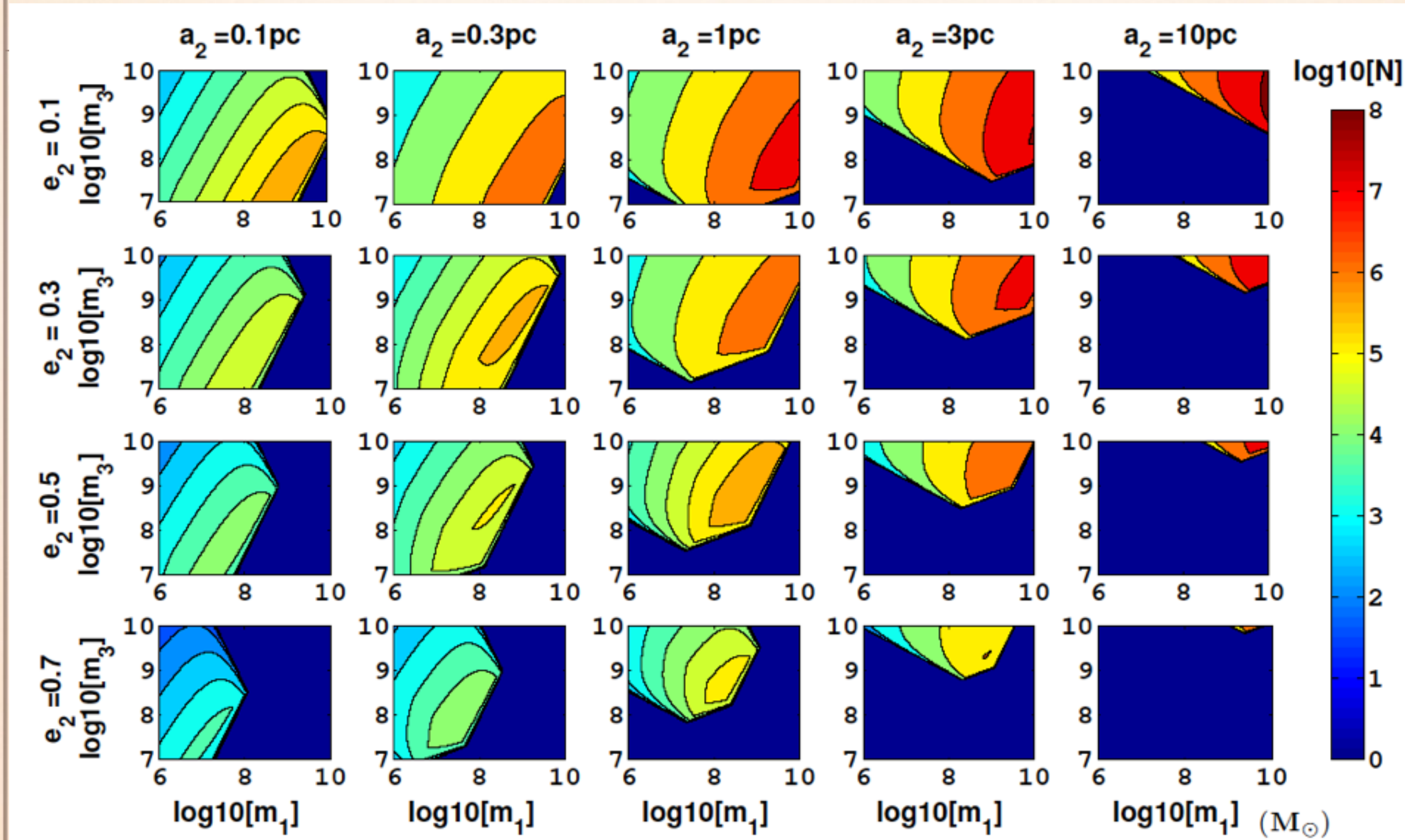
Due to general relativity   Due to stellar system self-gravity



More stars with $t_K < t_{GR/NT}$ when perturber more massive

(Li et al. 2015)

SUPPRESSION OF EKL



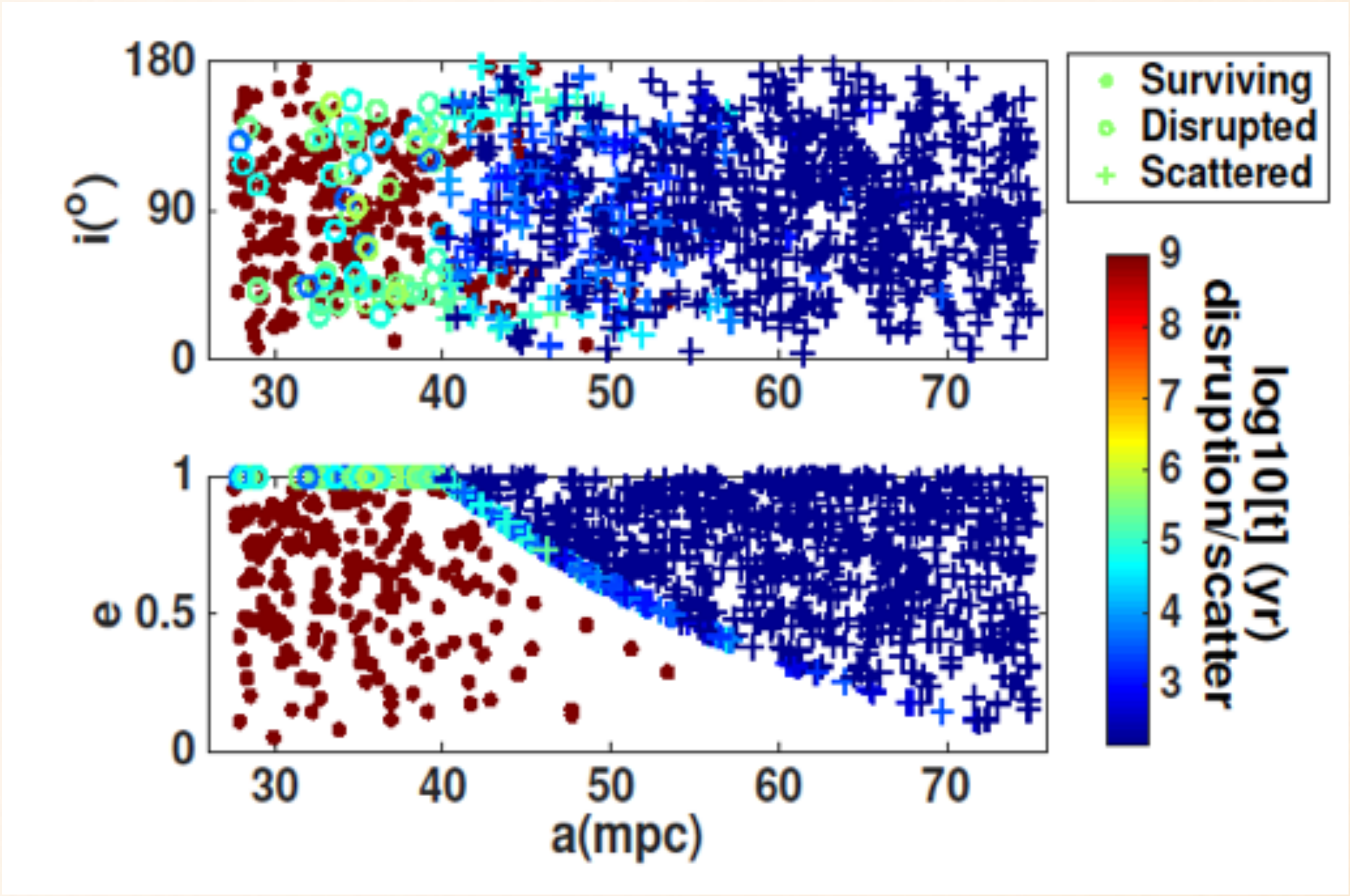
(Li et al. 2015)

EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB

- 57/1000 disrupted; 726/1000 scattered.

=> Scattered stars may change stellar density profile of the BHs.

=> Disruption rate can reach ~10^{-3}/yr.

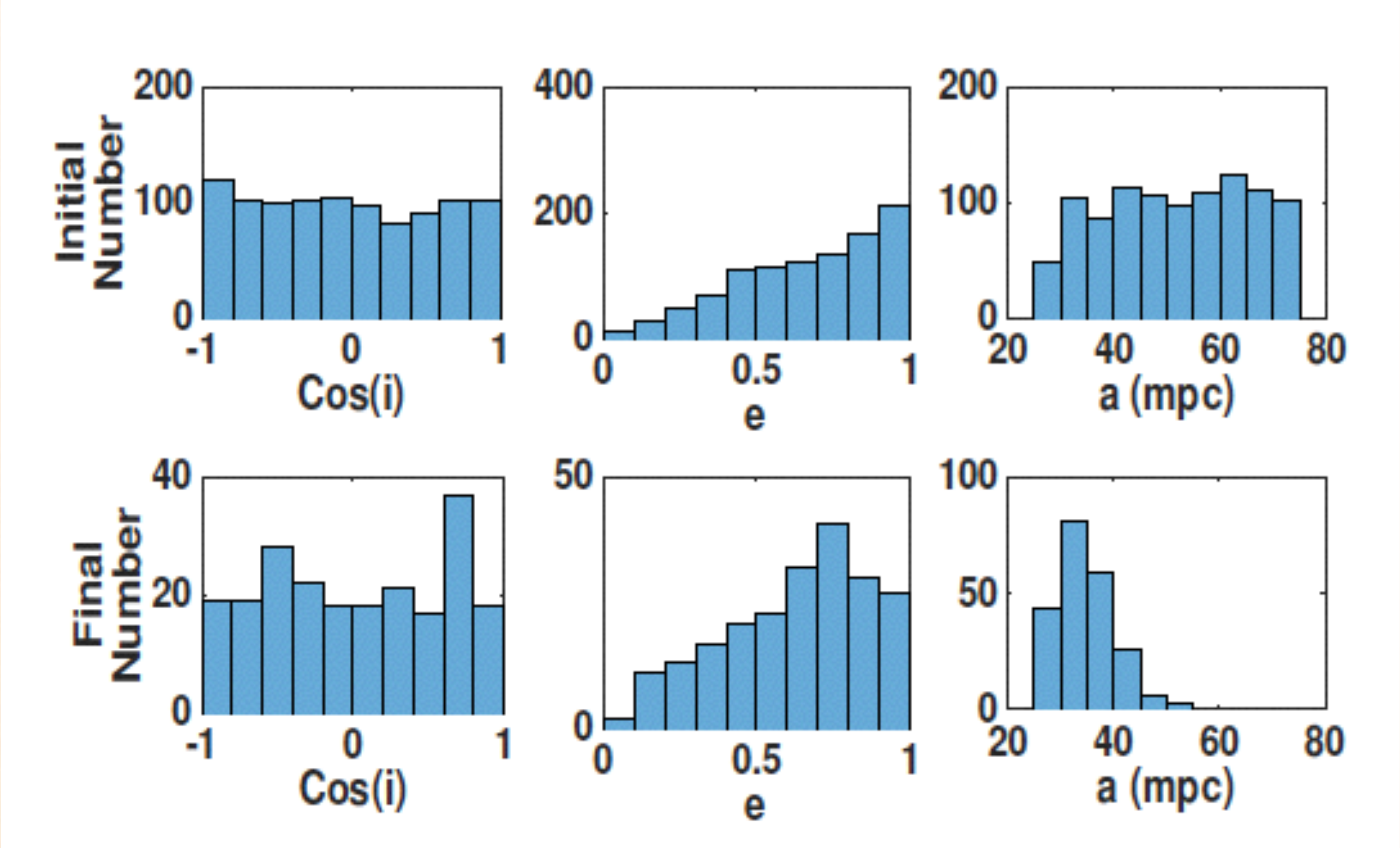


- Example: $m_1 = 10^7 M_\odot$, $m_2 = 10^8 M_\odot$, $a_2 = 0.5$pc, $e_2 = 0.5$, Run time: 1Gyr.

(Li et al. 2015)

EFFECTS OF EKM ON STARS SURROUNDING BBH

- Example: $m_1 = 10^7 M_\odot$, $m_2 = 10^8 M_\odot$, $a_2 = 0.5$pc, $e_2 = 0.5$, $\alpha = 1.75$ (Run time: 1Gyr)



(Li et al. 2015)

TAKE HOME MESSAGES

- Perturbation of the outer object can produce retrograde inner orbit and excite inner orbit eccentricity
- Under tidal dissipation, the perturbation of a farther companion can produce misaligned hot Jupiters
- Perturbation of a SMBH in a SMBHB can enhance the tidal disruption rate of stars to $10^{-2} \sim {}^{-3}$/yr.

THANK YOU!

MORE EXAMPLES OF HIERARCHICAL 3-BODY DYNAMICS

- For stellar systems:

Short Period Binaries

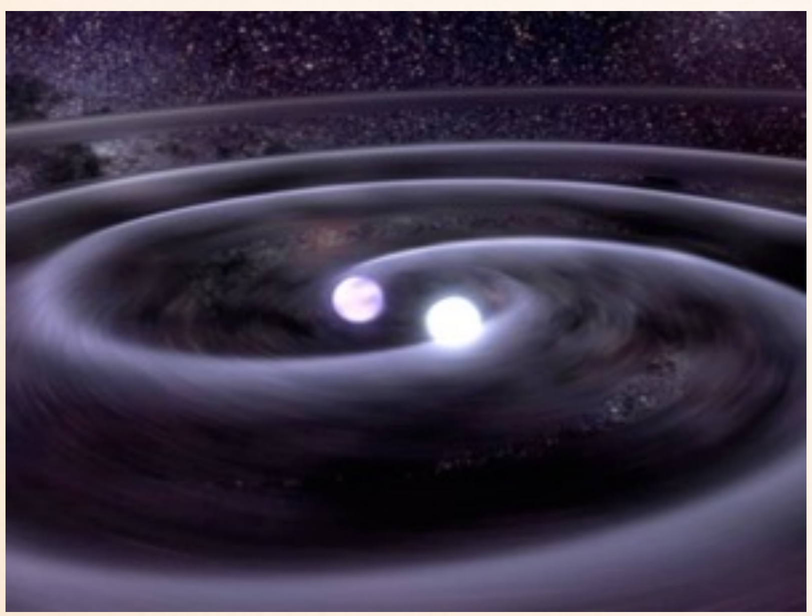


Image credit: NASA/Tod Strohmayer/Dana Berry

e.g., Harrington 1969; Mazeh & Shaham 1979; Ford et al. 2000; Eggleton & Kiseleva-Eggleton 2001; Fabrycky & Tremaine 2007; Shappee & Thompson 2013

Blue Stragglers

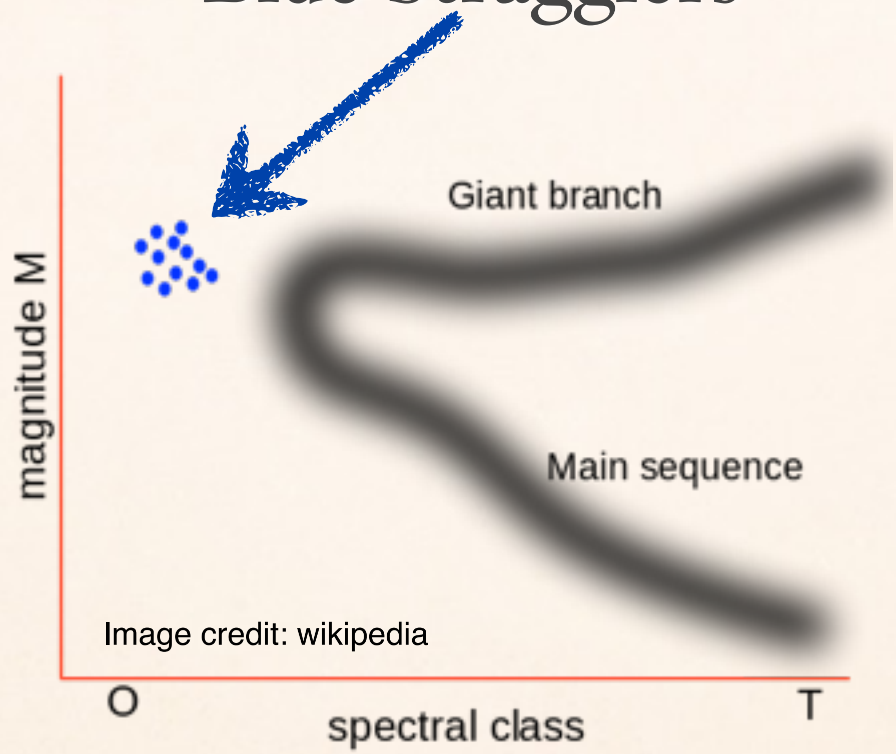
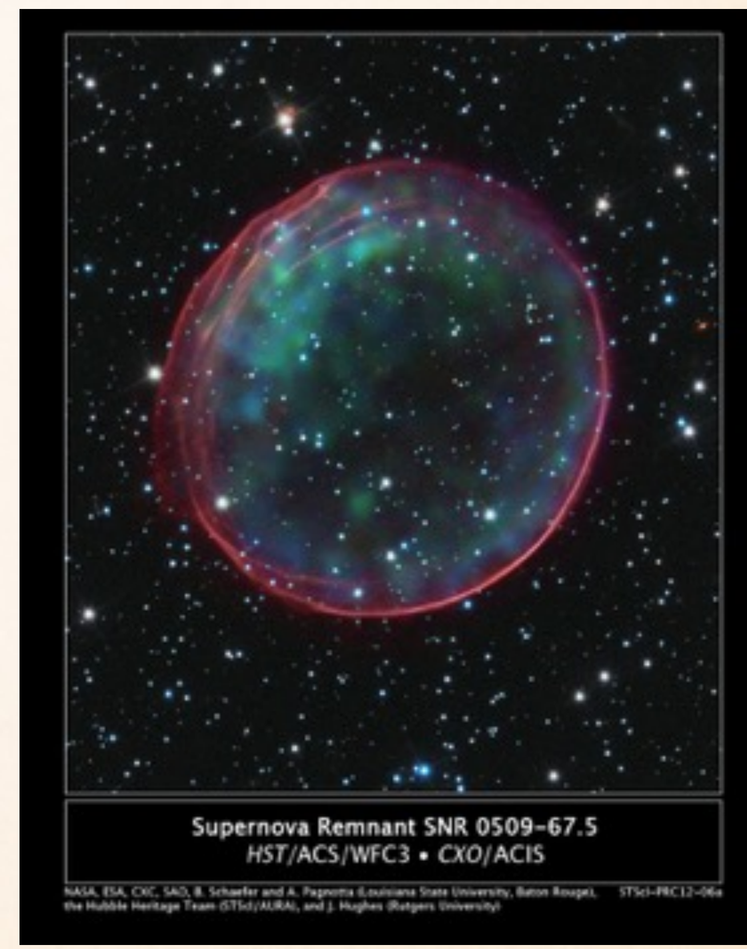


Image credit: wikipedia

e.g., Perets & Fabrycky 2009; Naoz & Fabrycky 2014

Type Ia Supernova



e.g., Katz & Dong 2012; Kushnir et al. 2013

MORE EXAMPLES OF HIERARCHICAL 3-BODY DYNAMICS

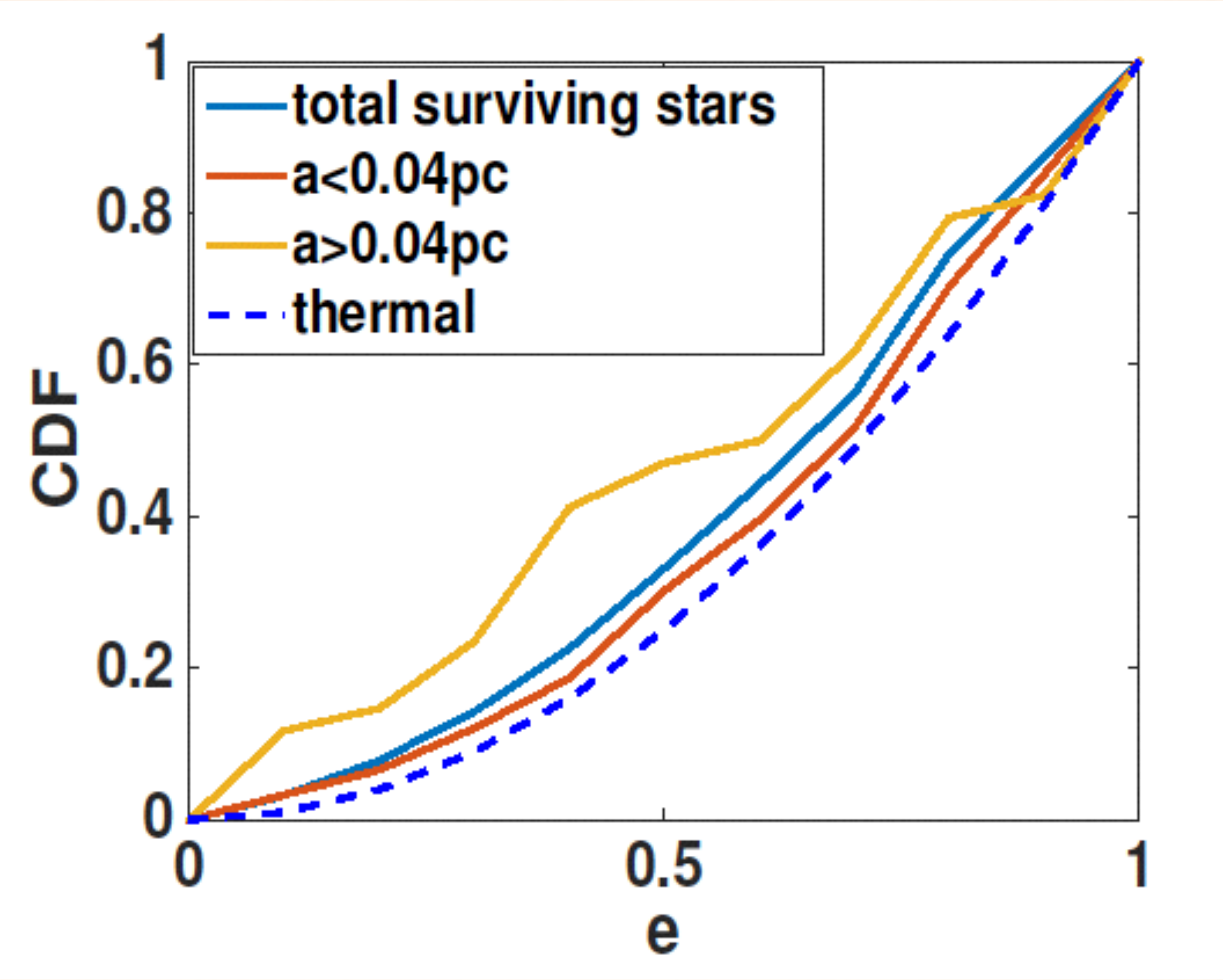
- Black hole systems:

  Merger of short period black hole binaries

e.g., Blaes et al. 2002; Miller & Hamilton 2002; Wen 2003; Bode & Wegg 2014;

EFFECTS OF EKM ON STARS SURROUNDING BBH

- Example: $m_1 = 10^7 M_\odot$, $m_2 = 10^8 M_\odot$, $a_2 = 0.5$pc, $e_2 = 0.5$, $\alpha = 1.75$. Run time: 1Gyr.



(Li et al. 2015)

Systematic Study of the Parameter Space

- Identify the resonances and the chaotic region.

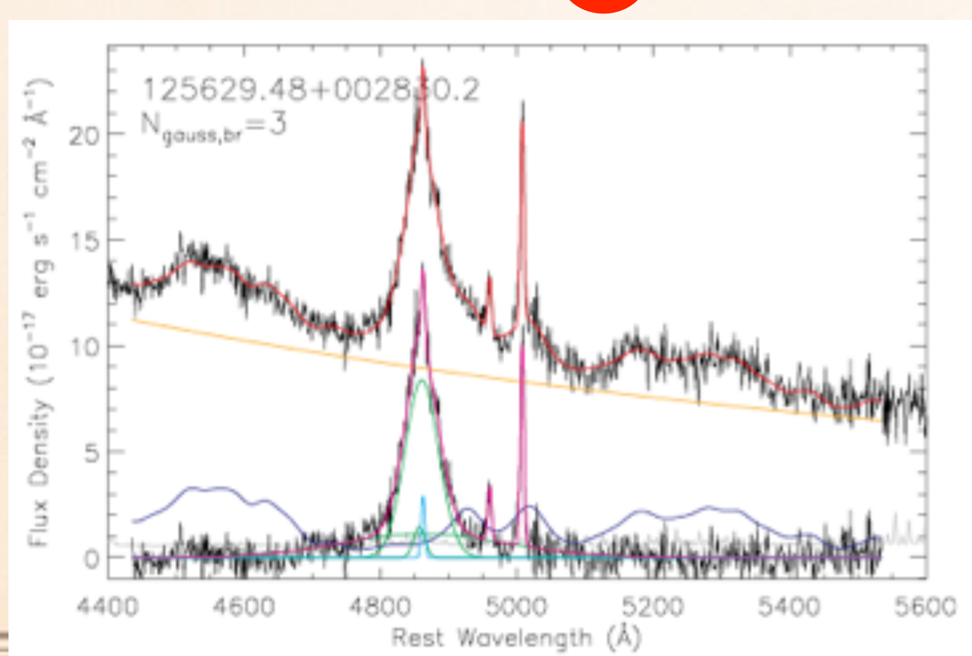
- Characterize the parameter space that give rise to the interesting behaviors --- eccentricity excitation and orbital flips.

STARS SURROUNDING SMBHB

- At ~1pc separation it is more difficult to identify SMBHBs. SMBHBs can be observed with photometric or spectral features.

  (e.g., Shen et al. 2013, Boroson & Lauer 2009, Valtonen et al. 2008, Loeb 2007)

Example of multi-epoch spectroscopy (Shen et al. 2013):

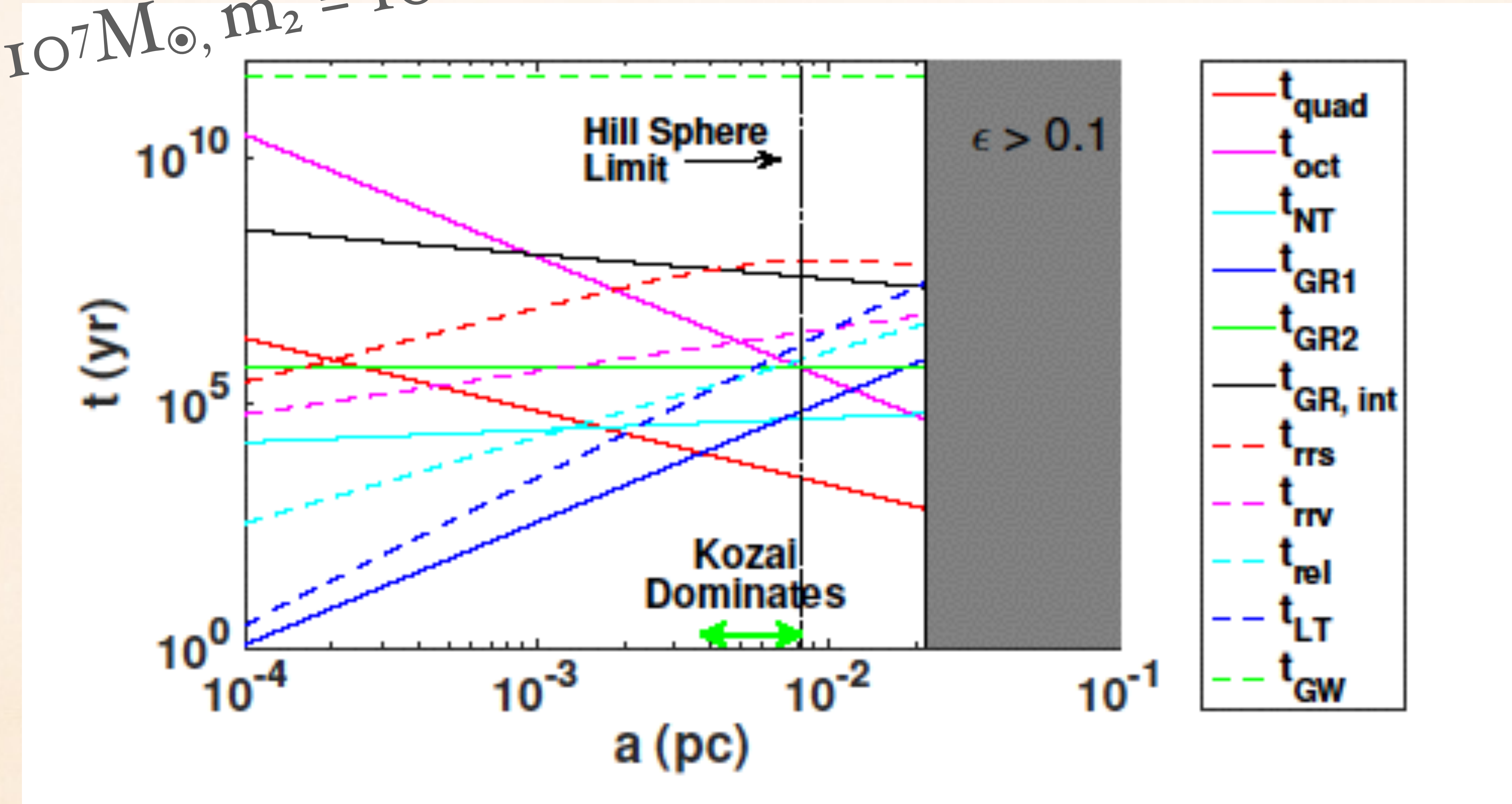


active BH dominates the BL features, multi-epoch BL features => binary orbital parameters

SUPPRESSION OF EKL

- Eccentricity excitation suppressed when precession timescale < Kozai timescale.

$m_0 = 10^7 M_\odot$, $m_2 = 10^9 M_\odot$

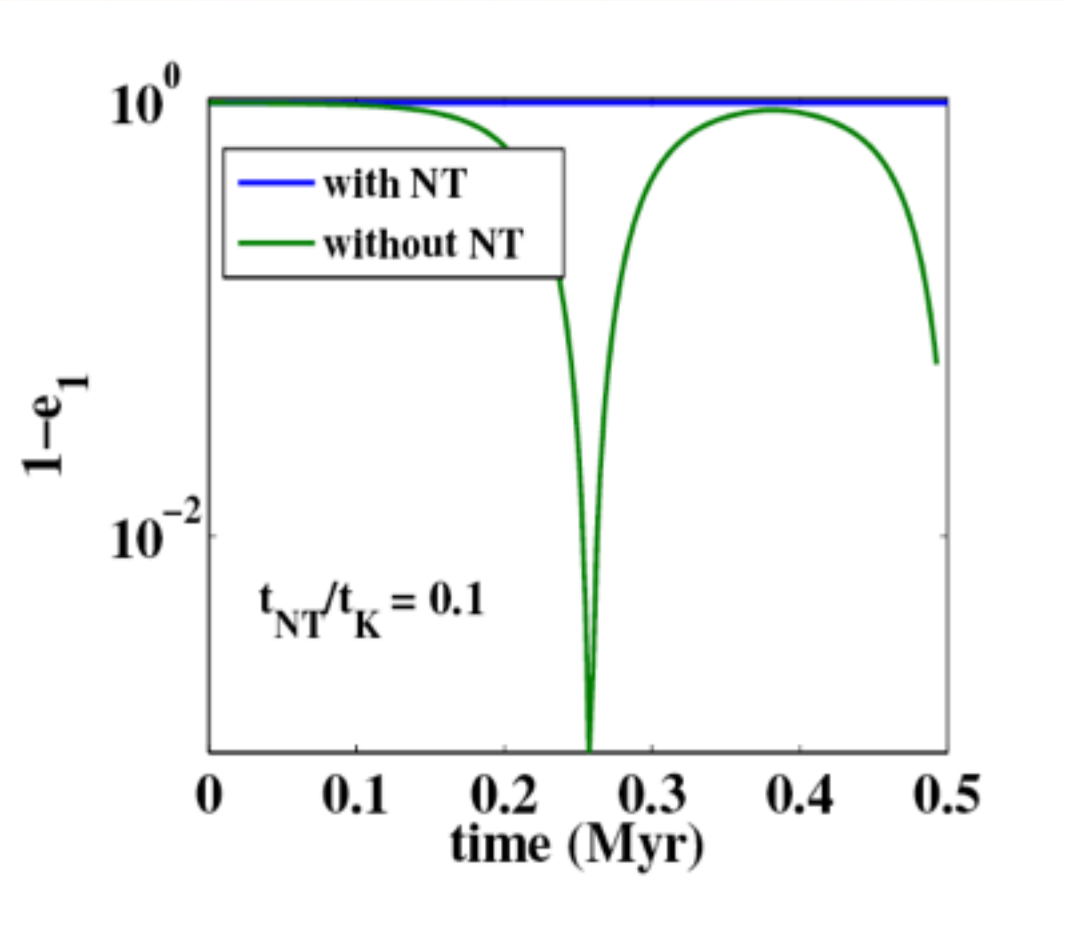


$e_1 = 2/3$, $a_2 = 0.3$ pc, $m_1 = 1 M_\odot$, $e_2 = 0.7$.

(Li et al. 2015)

SUPPRESSION OF EKL

- Eccentricity excitation suppressed when precession timescale < Kozai timescale.



$m_0 = 10^7 M_\odot$, $m_2 = 10^9 M_\odot$, $e_1 = 2/3$, $a_2 = 0.3$ pc, $m_1 = 1 M_\odot$, $e_2 = 0.7$. (Li et al. 2015)

EFFECTS ON STARS SURROUNDING AN IMBH IN GC

- Example: $m_1 = 10^4$ M$_\odot$, $m_2 = 4\times10^6$M$_\odot$, $a_2 = 0.1$pc, $e_2 = 0.7$ (Run time: 100 Myr)

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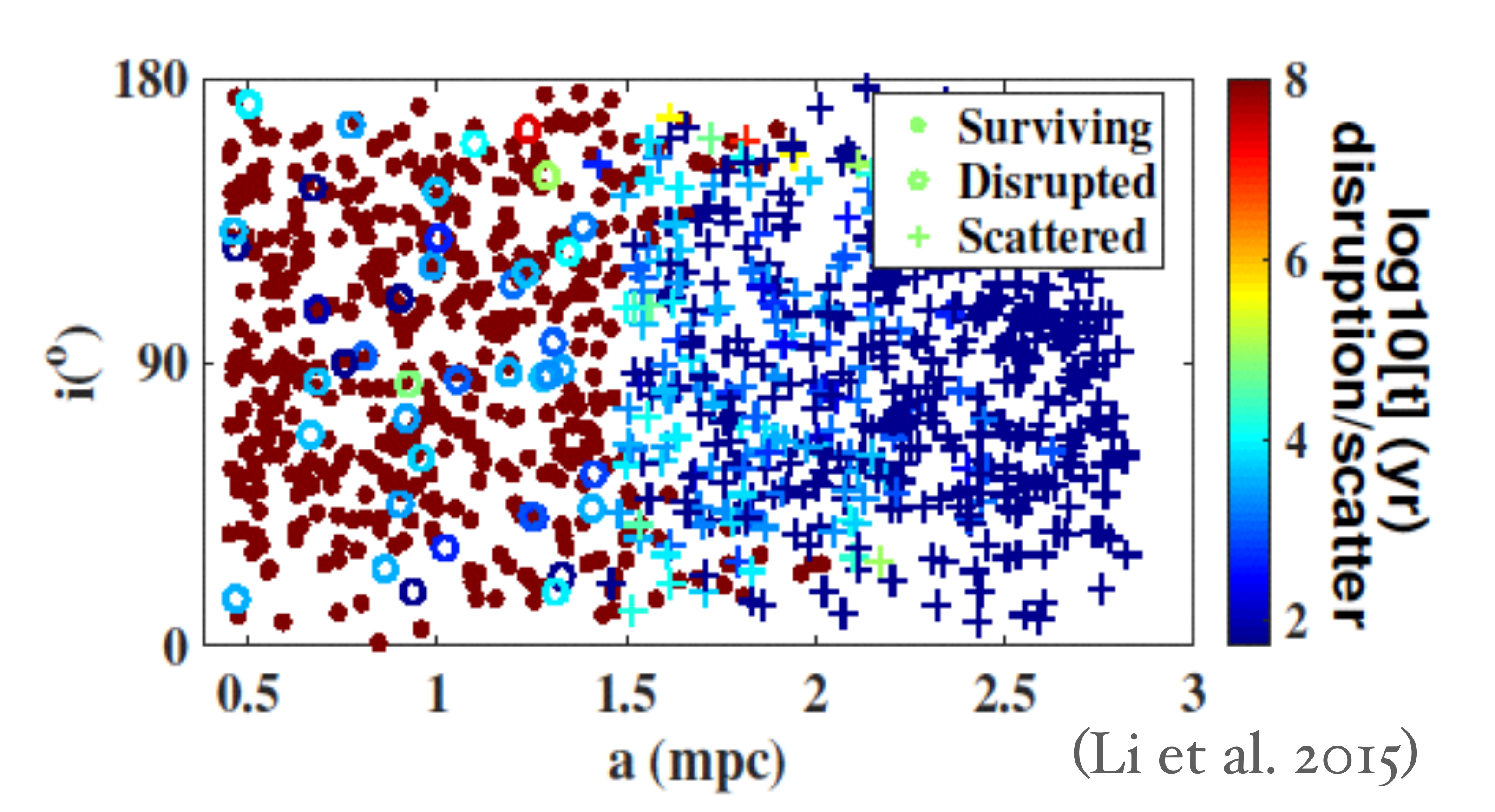


(Li et al. 2015)

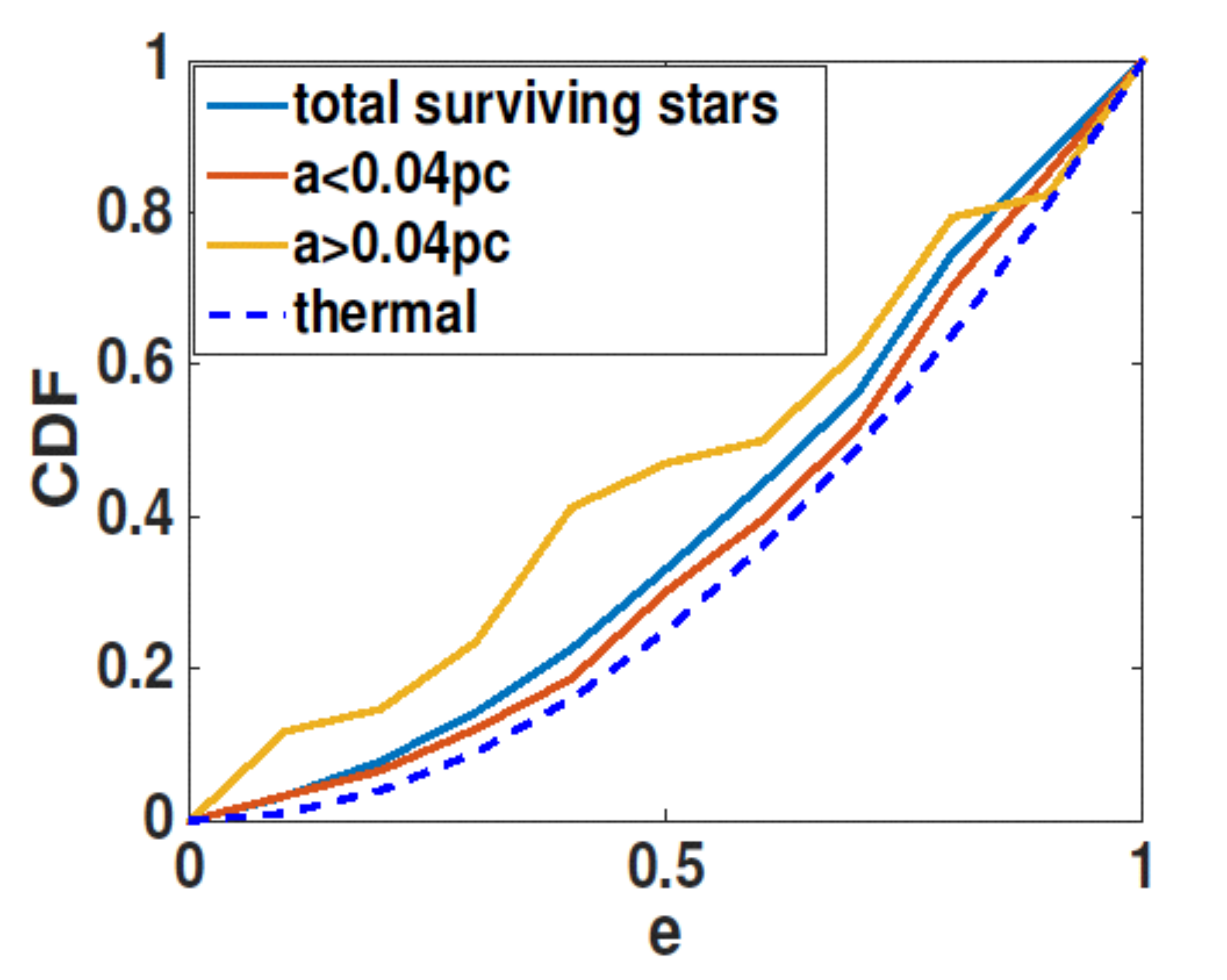
- 40/1000 disrupted; 500/1000 scattered.

=> ~50% stars survived.

=> Disruption rate can reach ~10^{-4}/yr.

EFFECTS OF EKM ON STARS SURROUNDING BBH

- Example: $m_1 = 10^7 M_\odot$, $m_2 = 10^8 M_\odot$, $a_2 = 0.5$pc, $e_2 = 0.5$, $\alpha = 1.75$. Run time: 1Gyr.



(Li et al. 2015)

EFFECTS ON STARS SURROUNDING AN IMBH IN GC

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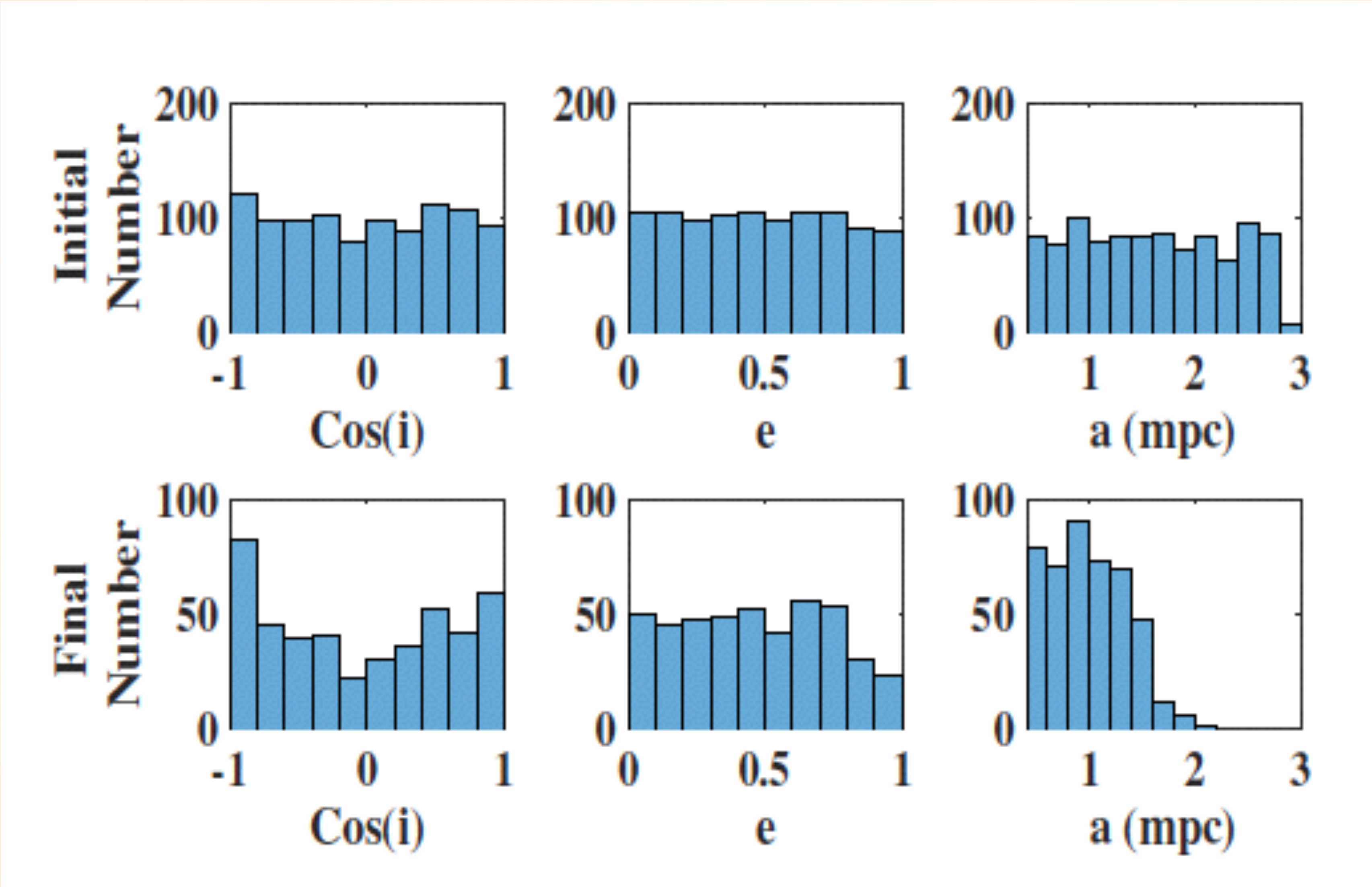
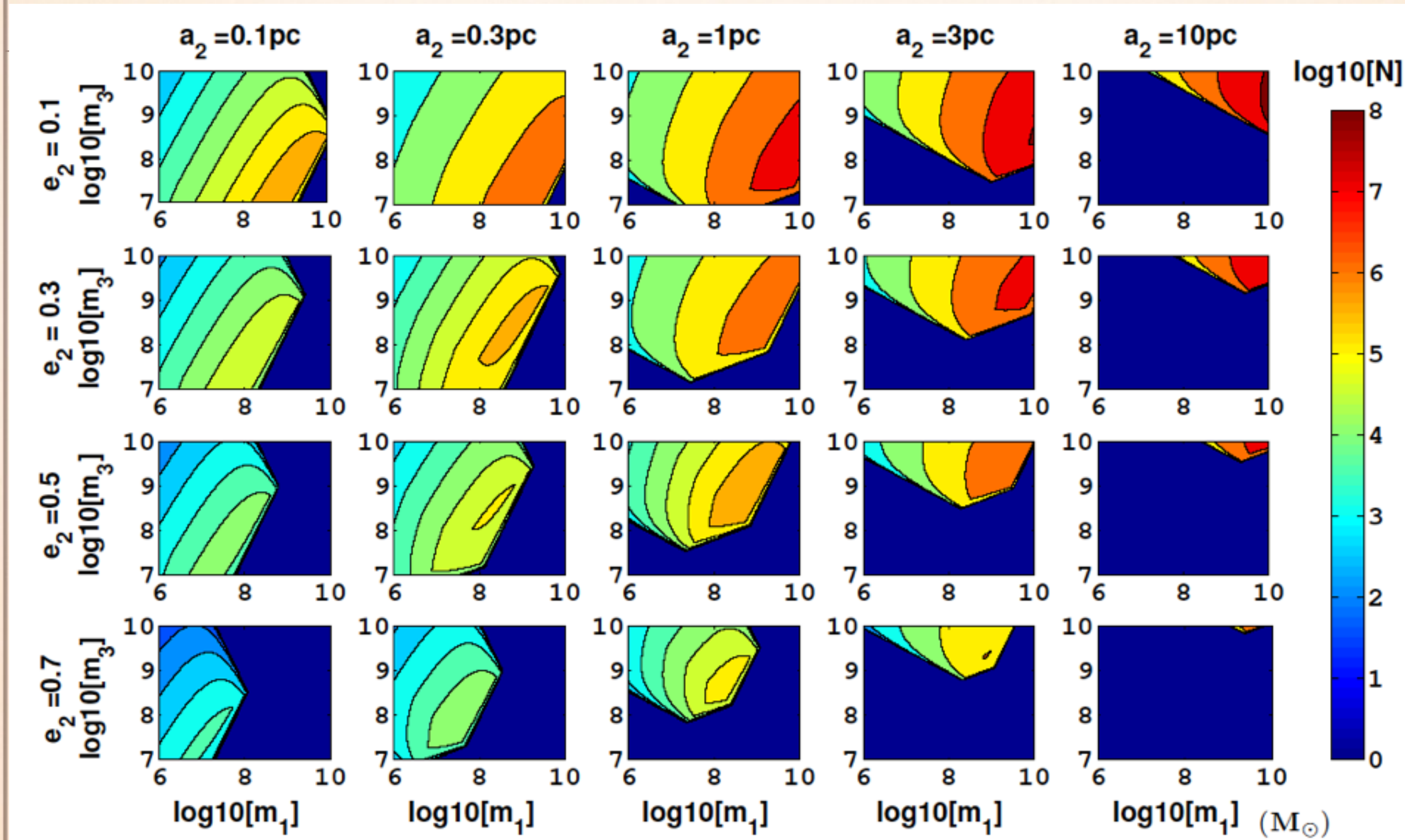


(Li et al. 2015)

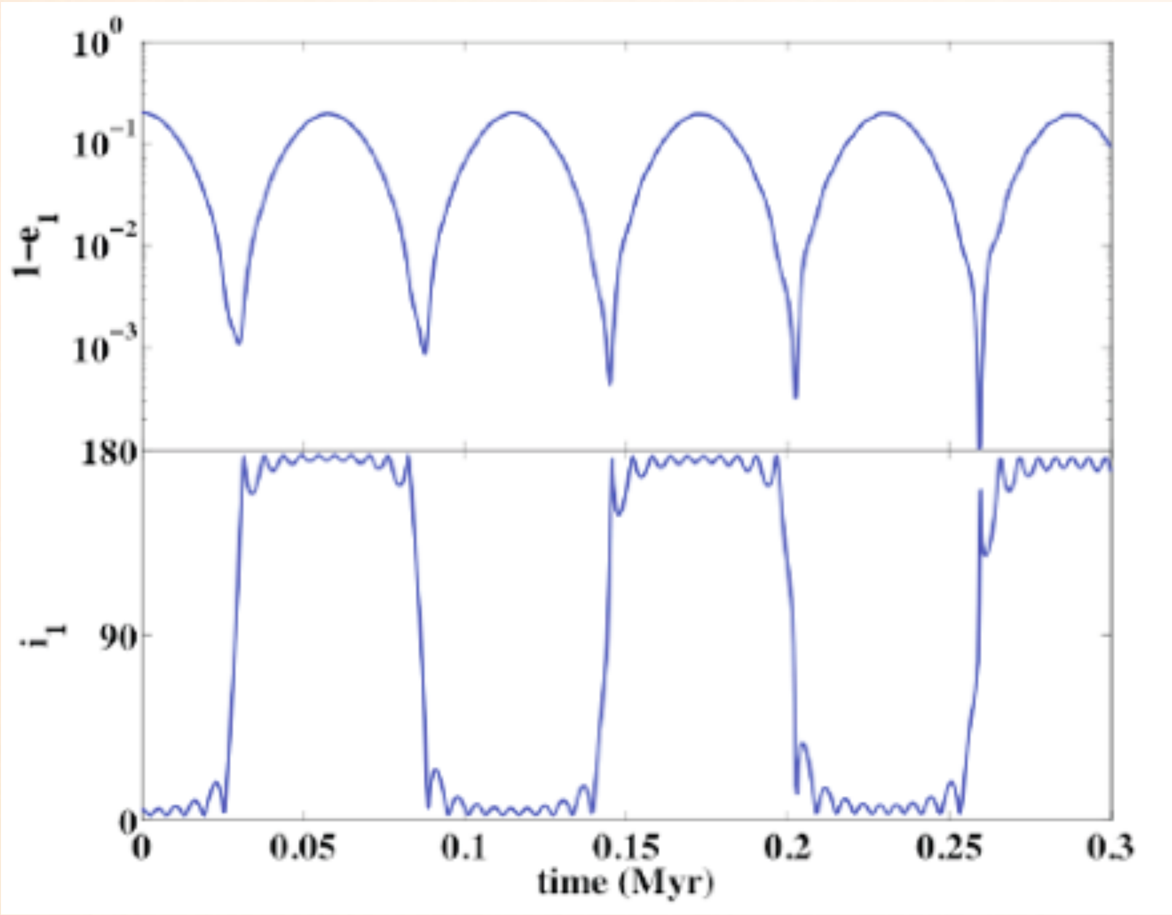
SUPPRESSION OF EKL



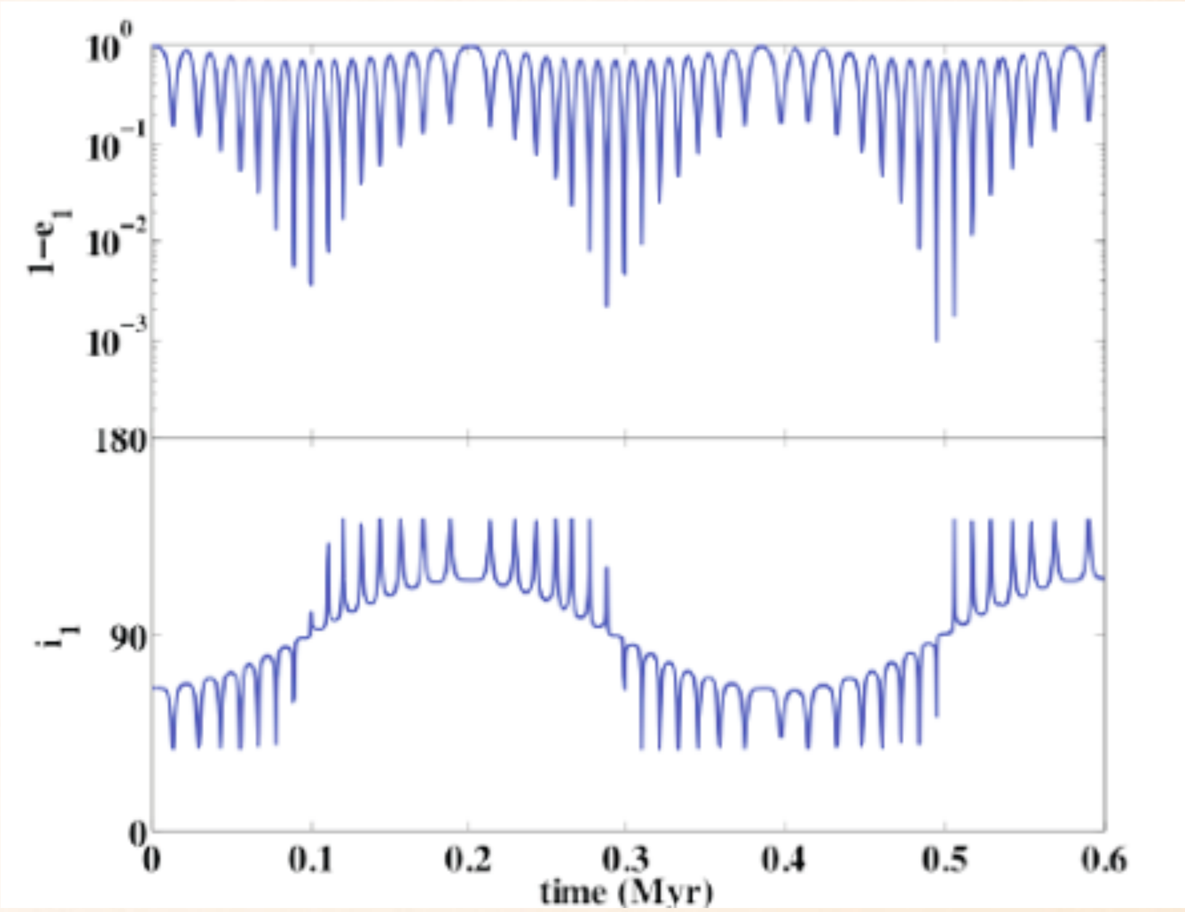
(Li et al. 2015)

DIFFERENCES BETWEEN HIGH/LOW I FLIP

Low inclination flip



High inclination flip



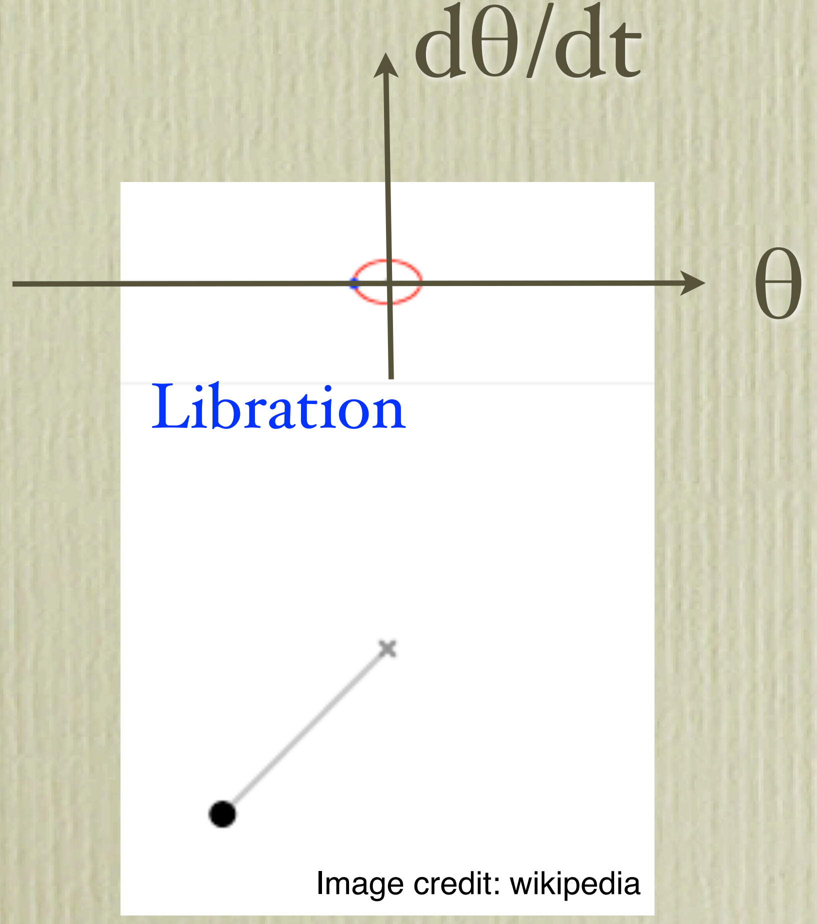
Low inclination flips:

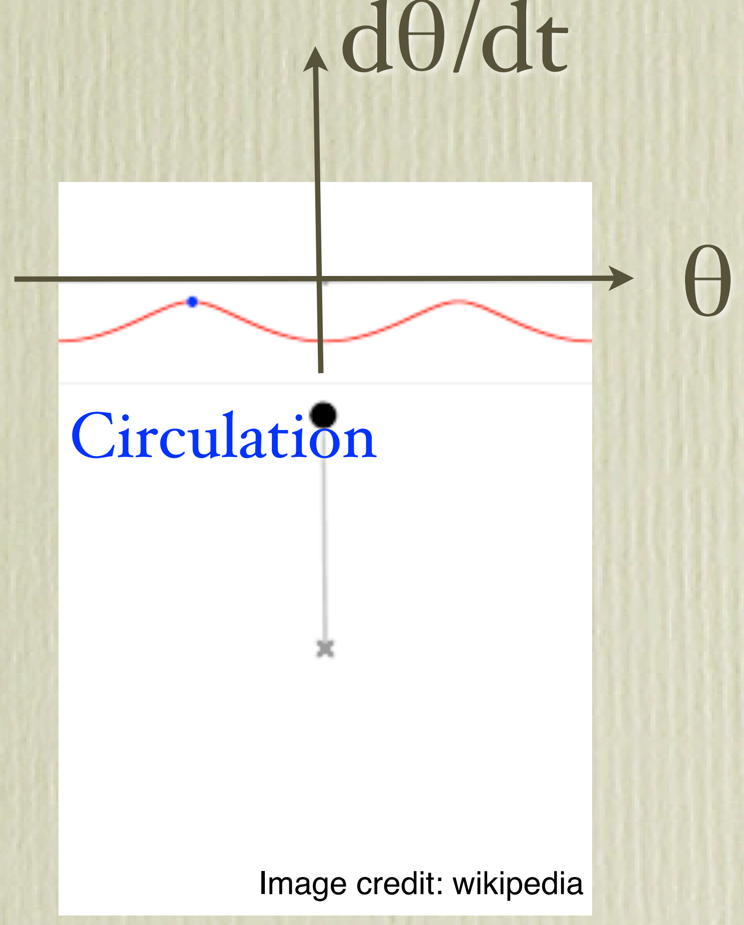
- e_1 ↑ monotonically, inclination stays low before flip.
- Flip occurs faster.

(Li et al. 2014a)

Resonances and Chaotic Regions

- The Hamiltonian H_{res} takes form of a pendulum.
- Two dynamical regions: libration region and circulation region.

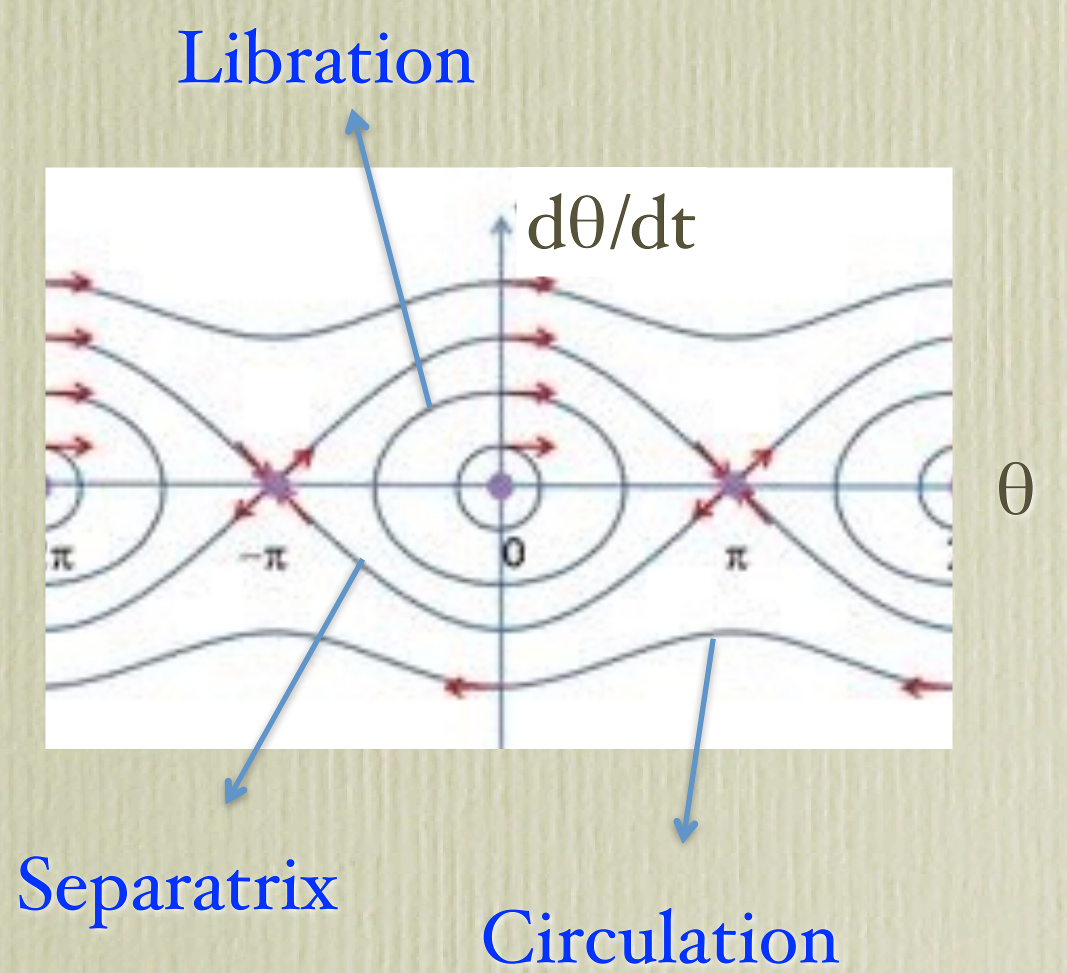




Resonances and Chaotic Regions

- The Hamiltonian H_{res} takes form of a pendulum.
- Two dynamical regions: libration region and circulation region, separated by separatrix.

Phase Diagram:



Resonances and Chaotic Regions

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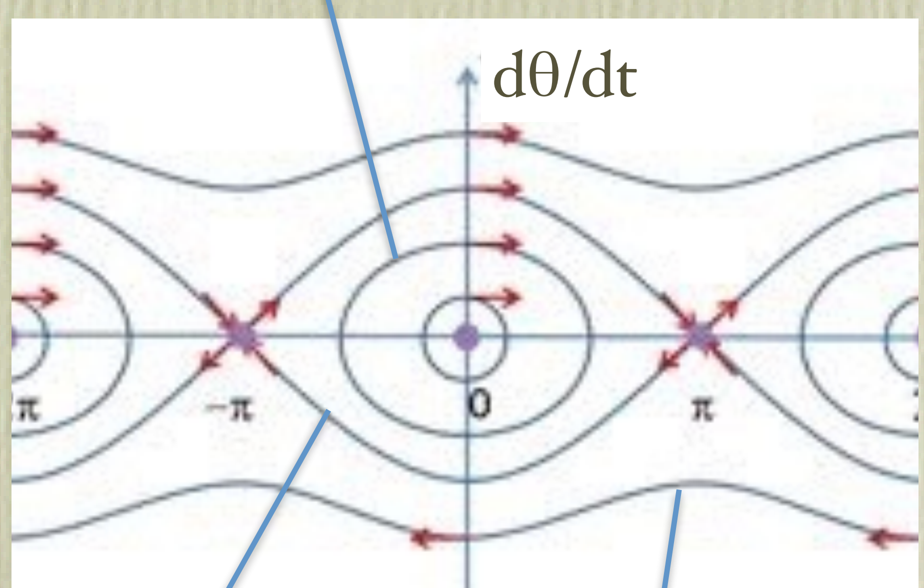
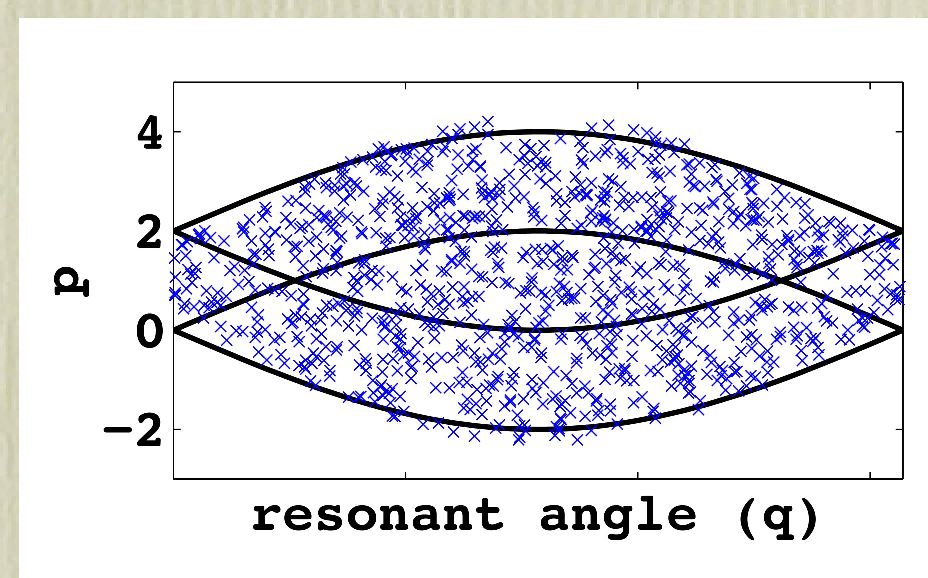
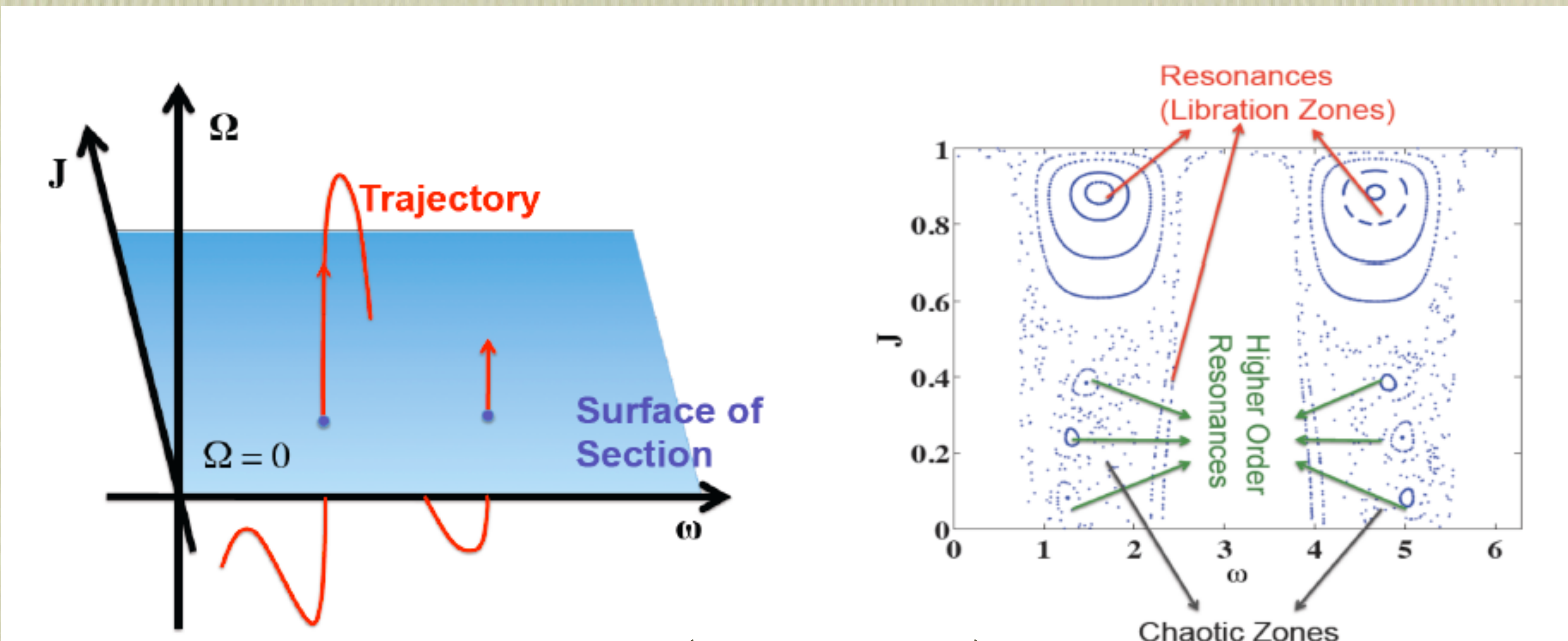


Overlap of resonances can cause chaos



Surface of Section

Example of a 2-degree freedom H (J, ω, Jz, Ω)



(Li et al. 2014b)

- Resonant zones: points fill 1-D lines.
  trajectories are quasi-periodic.
- Chaotic zones: points fill a higher dimension.

Surface of Section

- Surface of section of hierarchical three-body problem in the test particle limit in the J − ω Plane.
- $J = \sqrt{1 - e_1^2}$ (specific angular momentum);

ω: argument of periapsis

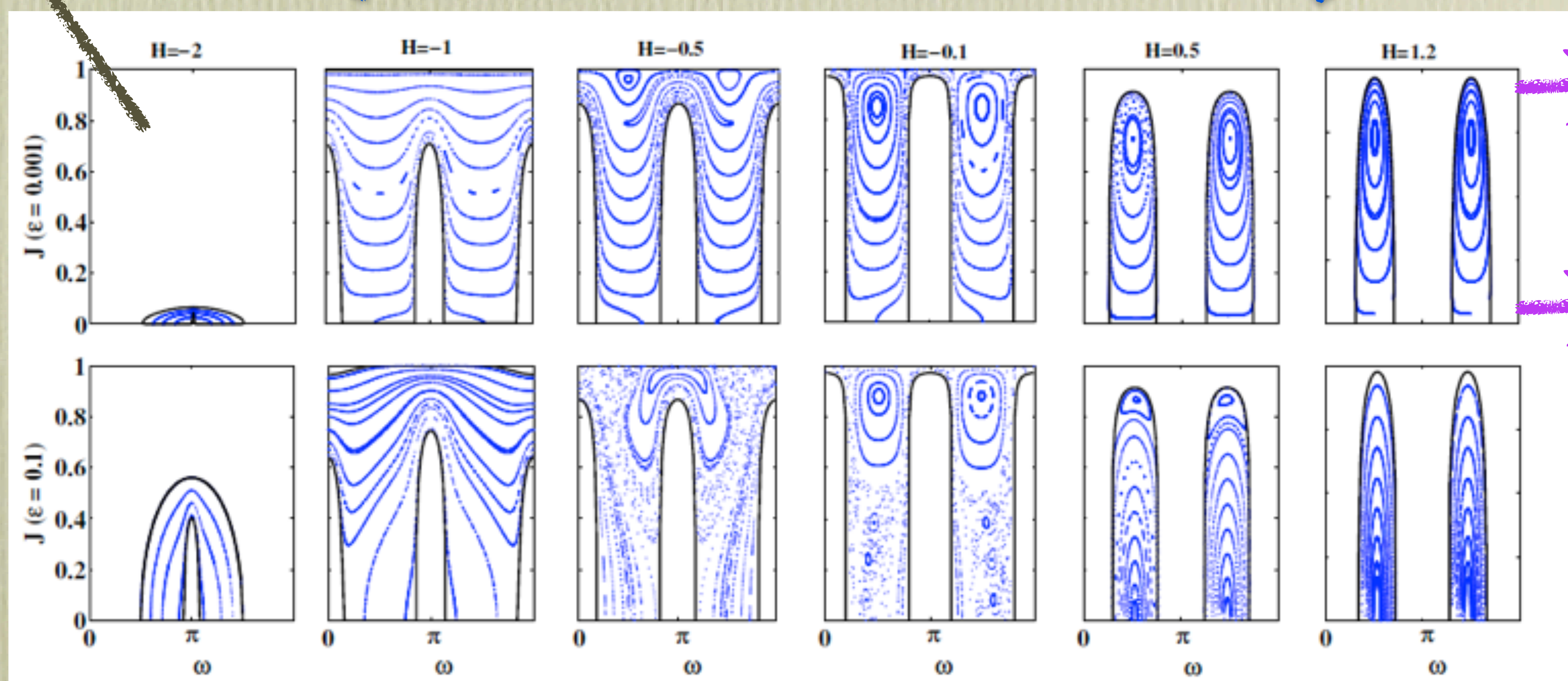
Low i ⟷ High i (40-60°) ⟷ i~90°

Low H ⟷ High H

No physical solution

Quadrupole order dominates

Octupole order stronger



low e

high e

Li et al. 2014b

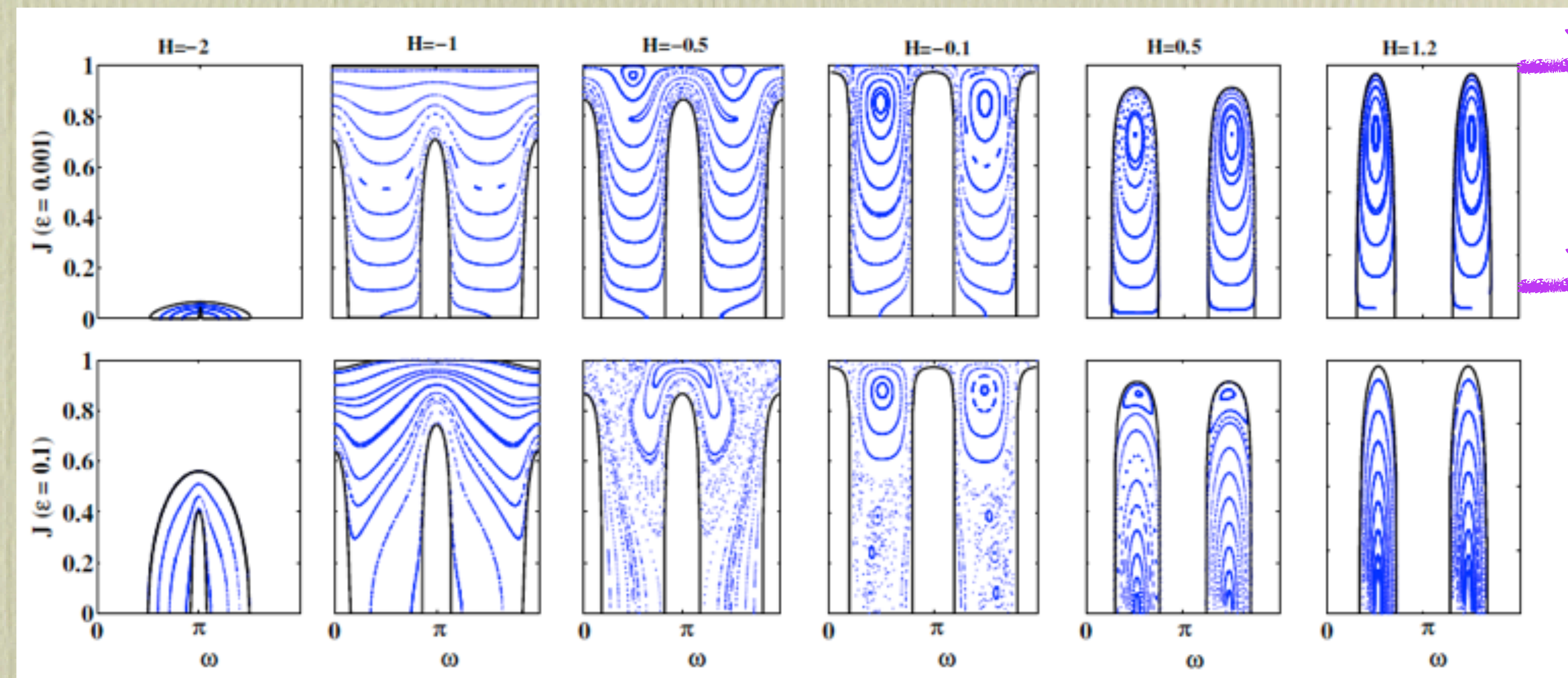
Surface of Section

Resonances exist for all surfaces:

Low i ⟷ High i (40-60°) ⟷ i~90°

Quadrupole order dominates

Octupole order stronger



low e

high e

Quadrupole resonances:

centers at low e_1, $\omega=\pi/2$ and $3\pi/2$ (*e.g. Kozai 1962*)

Octupole resonances:

centers at high e_1, $\omega=\pi$ or $\pi/2$ and $3\pi/2$

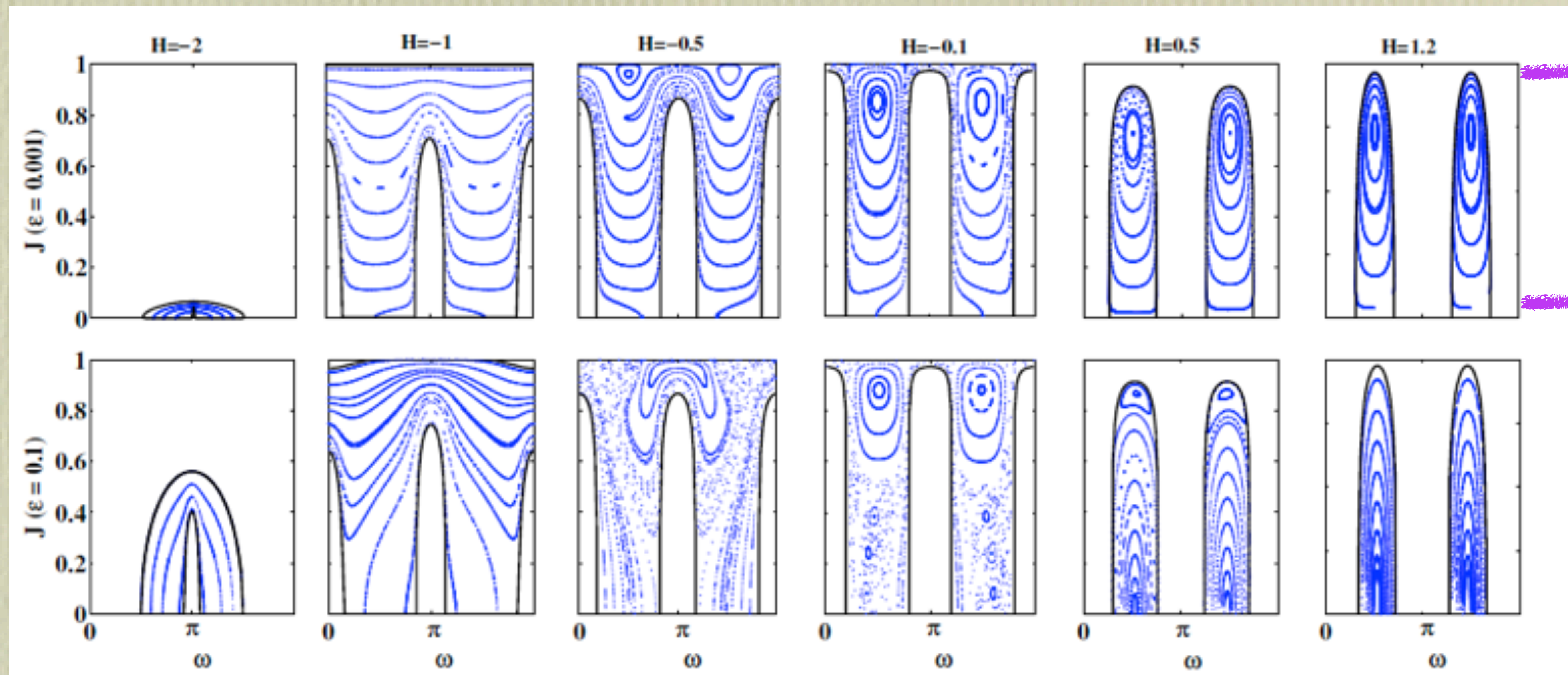
Li et al. 2014b

Surface of Section

Low i ⟷ High i (40-60°) ⟷ i~90°

Quadrupole order dominates

Octupole order stronger



low e

high e

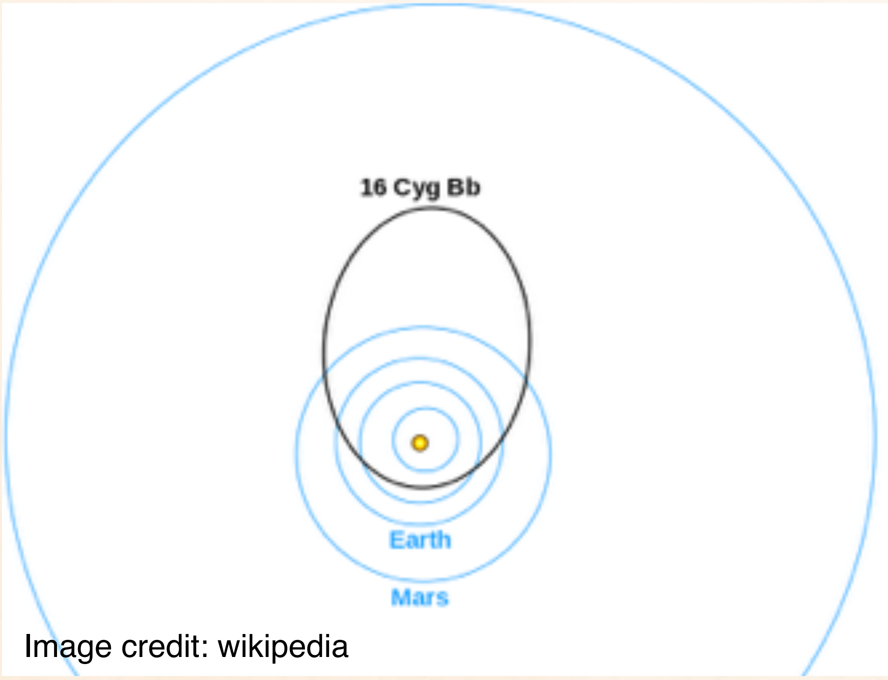
- e_1 excitation (J→0) are caused by octupole resonances.
- Near coplanar flip due to octupole resonances alone.
- High inclination flip due to both quadrupole and octupole order resonances.

Li et al. 2014b

EXAMPLES OF HIERARCHICAL 3-BODY DYNAMICS

- Exoplanetary systems:

Eccentric Orbits



e.g., Holman et al. 1997; Ford et al. 2000; Wu & Murray 2003;

Exoplanets with large spin-orbit misalignment

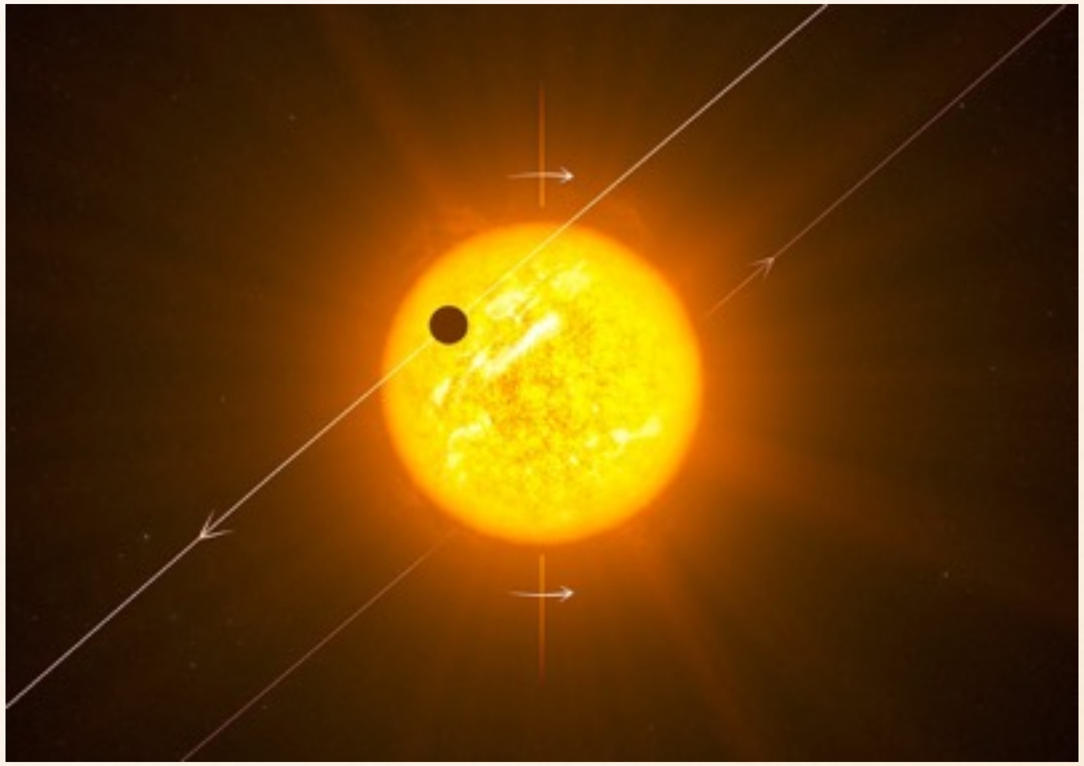


Image credit: ESO/A. C. Cameron

e.g., Fabrycky & Tremaine 2007; Naoz et al. 2011, 2012; Petrovich 2014; Storch et al. 2014; Anderson et al. 2016

Summary

- Hierarchical Three Body Dynamics:
  - Starting with near coplanar configuration, the inner orbit of a hierarchical 3-body system can flip by ~180°, and $e_1 \rightarrow 1$.
  - This mechanism is regular, and the flip criterion and timescale can be expressed analytically.
  - This mechanism can produce counter orbiting hot exoplanets, and can enhance collision/tidal disruption rate.

- Underlying resonances:
  - Flips and e_1 excitations are caused by octupole resonances.
  - High inclination flips are chaotic, with Lyapunov timescale ~ $6t_K$.

Summary

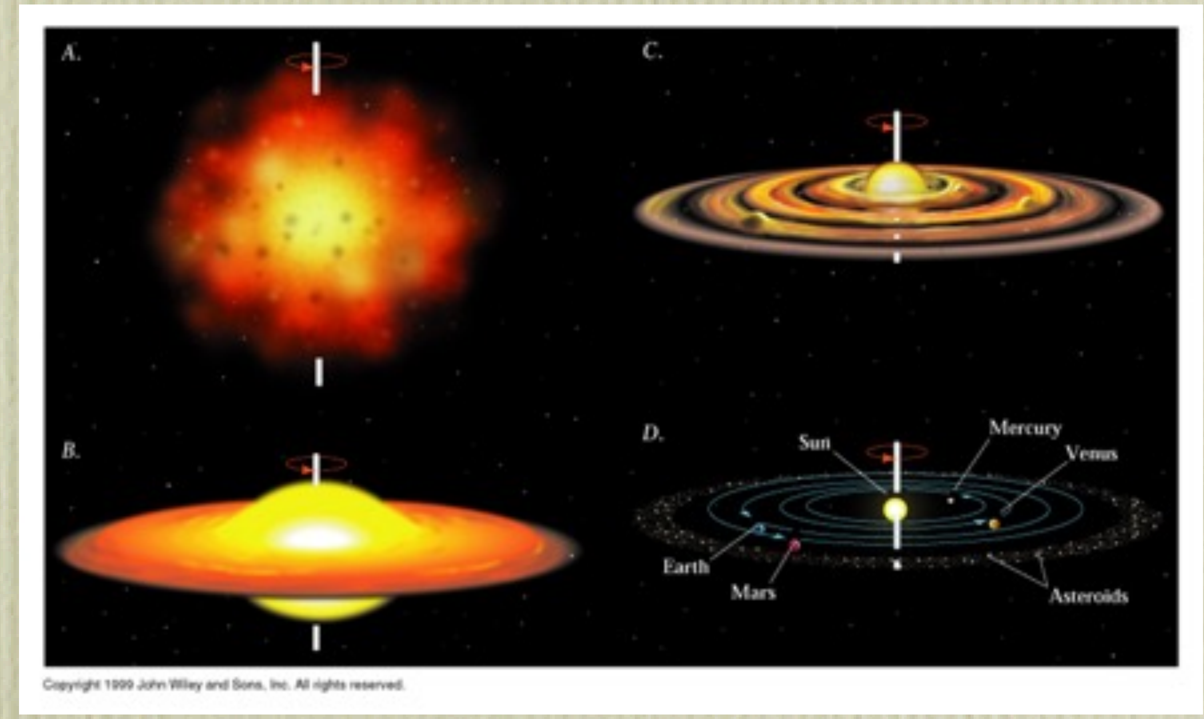
- Coplanar flip:
  - Starting with near coplanar configuration, the inner orbit of a hierarchical 3-body system can flip by ~180°, and $e_1 \rightarrow 1$.
  - This mechanism is regular, and the flip criterion and timescale can be expressed analytically.
  - This mechanism can produce counter orbiting hot exoplanets, and can enhance collision/tidal disruption rate.
- Characterization of parameter space:
  - Near coplanar flip and e_1 excitations are caused by octupole resonances.
  - High inclination flips are chaotic, with Lyapunov timescale ~ $6t_K$.

Potential Applications

- Captured stars in BBH systems may affect stellar distribution around the BHs (e.g., Ann-Marie Madigan, Smadar Naoz, Ryan O'Leary).
- Tidal disruption and collision events for planetary systems (e.g., Eugene Chiang, Bekki Dawson, Smadar Naoz).
- Production of supernova (e.g., Rodrigo Fernandez, Boaz Katz, Todd Thompson).
- Other aspects:
  - Involving more bodies (e.g., Smadar Naoz, Todd Thompson).
  - Obliquity variation of planets.

COHJ Contradict with popular Planets' Formation Theory

- Formation Theory:



- Planet systems form from cloud contraction.
- Spin of the star ends up aligned with the orbit of the planets

Analytical Overview --- Test Particle Limit

- Hamiltonian has two degrees of freedom:

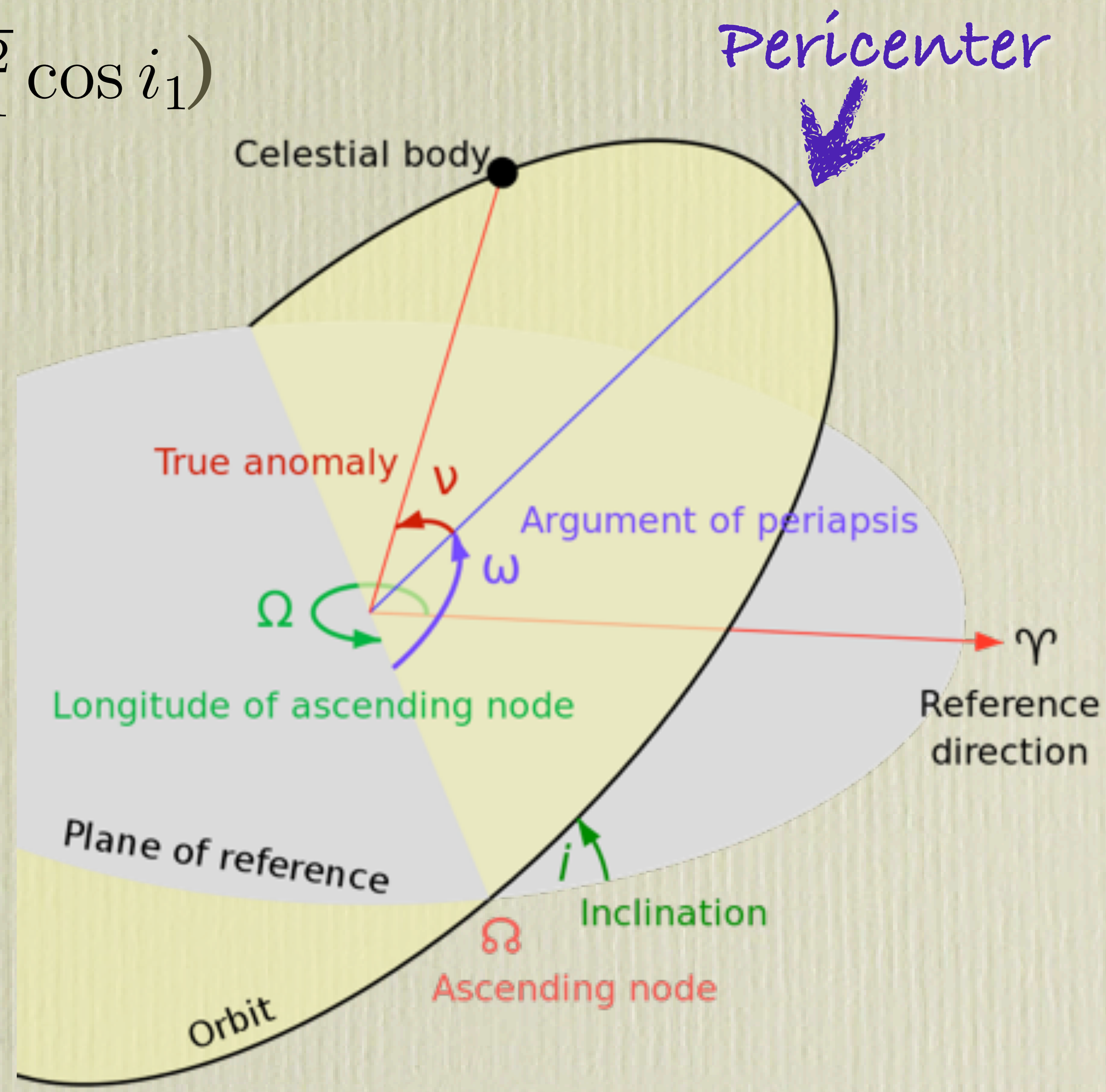
isolated 3-body: 6 dof $\xrightarrow{\text{secular}}$ 4 dof $\xrightarrow{\text{test-particle}}$ 2 dof

2 conjugate pairs: J & ω, Jz & Ω

$(J = \sqrt{1 - e_1^2},\ \ Jz = \sqrt{1 - e_1^2} \cos i_1)$

ω: orientation in orbital plane.

Ω: orientation in reference plane.



ANALYTICAL OVERVIEW

- Hamiltonian has two degrees of freedom in test particle limit:

$$(J = \sqrt{1-e_1^2},\ Jz = \sqrt{1-e_1^2}\cos i_1,\ \omega,\ \Omega\ )$$

2 conjugate pairs: J & ω, Jz & Ω

- The Hamiltonian up to the Octupole order:

$$H = F_{quad}(J, Jz, \omega) + \epsilon F_{oct}(J, Jz, \omega, \Omega)$$

Quadrupole order: Independent of Ω => Jz constant

ϵ : hierarchical parameter: $\epsilon = \frac{a_1}{a_2}\frac{e_2}{1-e_2^2}$

Octupole order: Depend on both Ω & ω => J and Jz not constant

Analytical Overview

- Hamiltonian (Harrington 1968, 1969; Ford et al., 2000):
  - In the octupole order: $H = -F_{quad} - \varepsilon F_{oct}$, $\varepsilon = (a_1/a_2)e_2/(1-e_2^2)$

$$F_{quad} = -(e_1^2/2) + \theta^2 + 3/2 e_1^2 \theta^2 + 5/2 e_1^2 (1-\theta^2)\cos(2\omega_1),$$

$$F_{oct} = \frac{5}{16}(e_1 + (3e_1^3)/4) \times ((1 - 11\theta - 5\theta^2 + 15\theta^3)\cos(\omega_1 - \Omega_1) + (1 + 11\theta - 5\theta^2 - 15\theta^3)\cos(\omega_1 + \Omega_1)) - \frac{175}{64} e_1^3 ((1 - \theta - \theta^2 + \theta^3)\cos(3\omega_1 - \Omega_1) + (1 + \theta - \theta^2 - \theta^3)\cos(3\omega_1 + \Omega_1)),$$

- Independent of Ω_1, J_z const.
- Depend on both ω_1 and Ω_1 ➔ both J and J_z are not const.

$$t_K = \frac{8}{3} P_{in} \frac{m_1}{m_2} \left(\frac{a_2}{a_1}\right)^3 (1 - e_2^2)^{3/2}$$

Analytical Derivation for Flip Criterion and Timescale

- Hamiltonian (at O(i)):
  - Evolution of e_1 only due to octupole terms:

    => e_1 does not oscillate before flip.
  - Depend on only J_1 and $\varpi_1 = \omega_1 + \Omega_1$

    => System is integrable.

    => $e_1(t)$ can be solved.
  - Flip at $e_{1,\,max} \sim 1$

    => The flip timescale can be derived.
  - Flip when $\varpi_1 = 180^o$

    => The flip criterion can be derived.

$$\varepsilon > \frac{8}{5} \frac{1 - e_1^2}{7 - e_1\left(4 + 3e_1^2\right)\cos\left(\omega_1 + \Omega_1\right)}$$

Li et al., 2013

Analytical Overview

- Hamiltonian has two degrees of freedom:

$$(J = \sqrt{1-e_1^2},\ Jz = \sqrt{1-e_1^2}\cos i_1,\ \omega,\ \Omega)$$

2 conjugate pairs: J & ω, Jz & Ω

- Hamiltonian (*Harrington 1968, 1969; Ford et al. 2000*):

In the octupole order:

Interaction Energy (H) of two orbital wires:

$$H = F_{quad}(J, Jz, \omega) + \epsilon F_{oct}(J, Jz, \omega, \Omega)$$

Quadrupole order: Independent of Ω => Jz constant

ϵ: hierarchical parameter: $\epsilon = \frac{a_1}{a_2}\frac{e_2}{1-e_2^2}$

Octupole order: Depend on both Ω & ω => J and Jz not constant

Analytical Der… Flip Criterion
an… le

put equation in hidden slides

- Hamiltonian (at O(i)) depend on only e_1 and $\varpi_1 = \omega_1 + \Omega_1$:
- Evolution of e_1 only due to octupole terms:

$$\dot{e}_1 = \frac{5}{8} J_1\left(3J_1^2 - 7\right)\varepsilon \sin(\varpi_1) \qquad \dot{\varpi}_1 = J_1\left(2 + \frac{5\left(9J_1^2 - 13\right)\varepsilon\cos(\varpi_1)}{\sqrt{1-J_1^2}}\right)$$

- $e_1(t)$ can be solved =>

The flip criterion and the flip timescale can be derived:

$$\varepsilon > \frac{8}{5}\frac{1-e_1^2}{7 - e_1\left(4 + 3e_1^2\right)\cos(\omega_1 + \Omega_1)}$$

Li, et al., 2013

DYNAMICS OF HIERARCHICAL THREE-BODY SYSTEMS

Quadrupole resonances:

$i > 40^o$: e, i oscillations (*e.g.*, Kozai 1962)

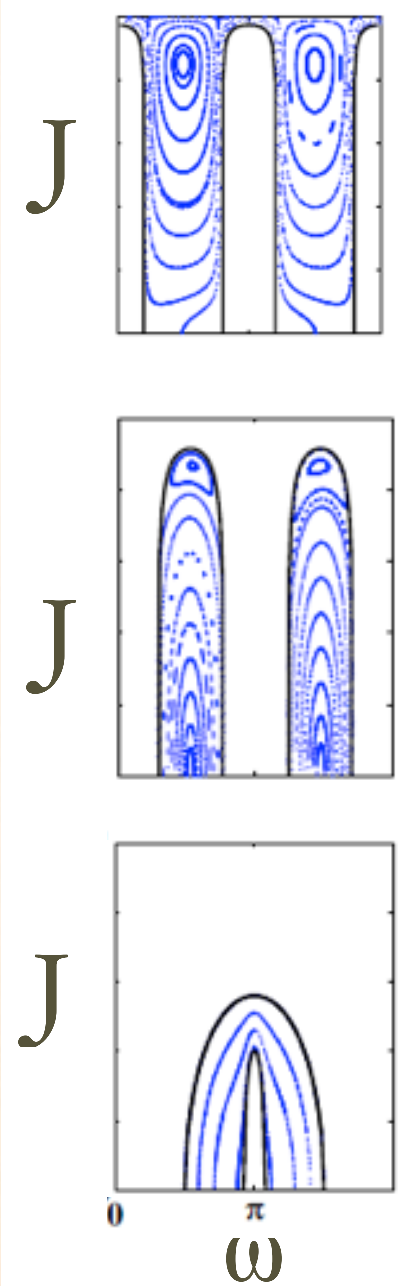
Octupole resonances:

$i > 40^o$: $e \rightarrow 1$, orbit flips (Naoz et al. 2011), flip criterion at $j_z \sim 0$ ($i \sim 90^o$)

can be obtained (Katz et al. 2011)

$i \sim 0^o$: $e \rightarrow 1$, orbit flips over 180^o,

dynamics regular, flip criterion and flip timescale can be obtained (Li et al. 2014a)



Li et al. 2014b

FLIP CRITERION

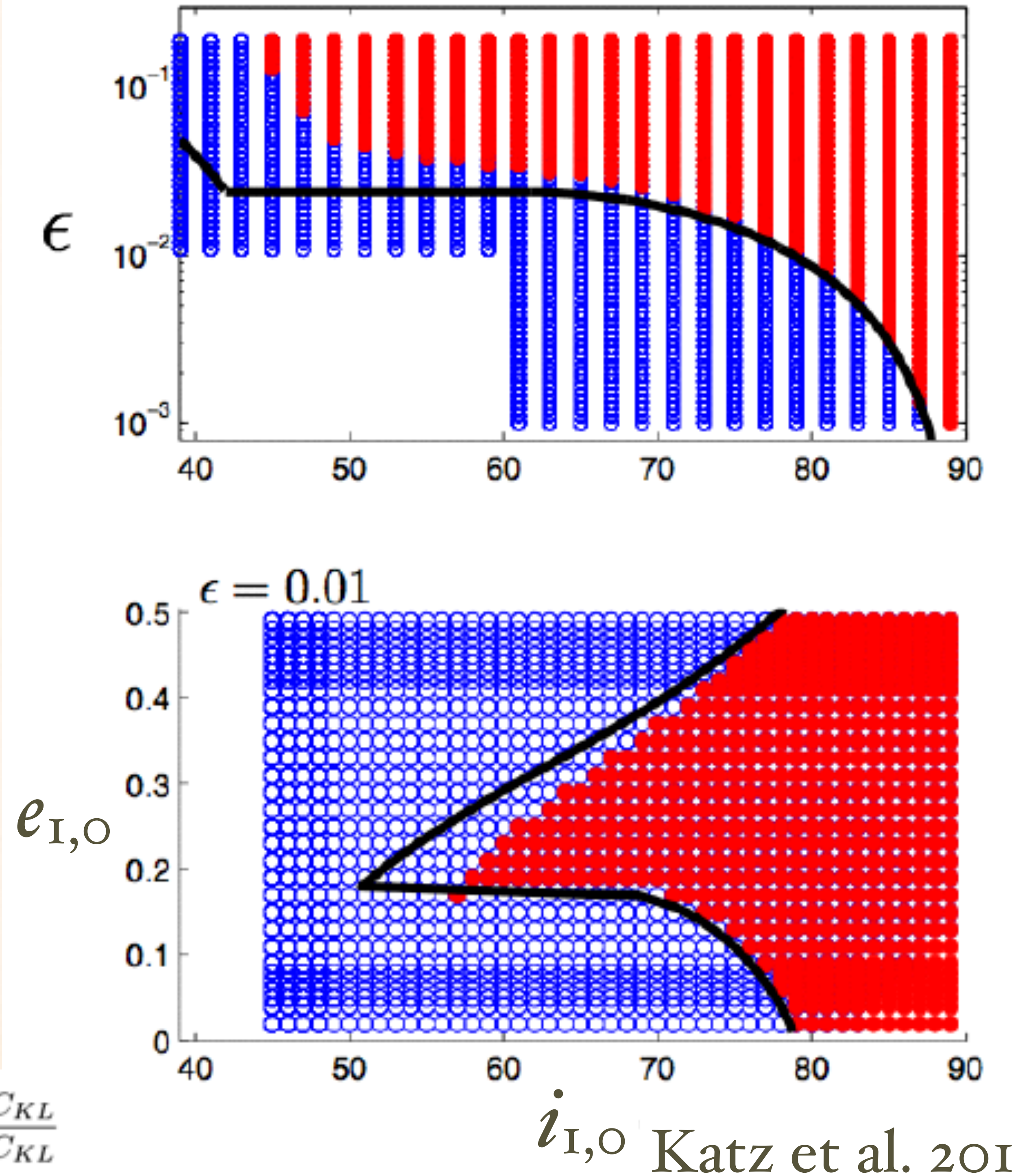
- Averaging the quadrupole oscillations in limit $j_z \sim 0$, Katz et al. 2011 obtain the constant:

$$f(C_{KL}) + \epsilon \frac{\cos i_{tot} \sin \Omega_1 \sin \omega_1 - \cos \omega_1 \cos \Omega_1}{\sqrt{1 - \sin^2 i_{tot} \sin^2 \omega_1}}$$

- Requiring $j_z = 0$, during the flip:

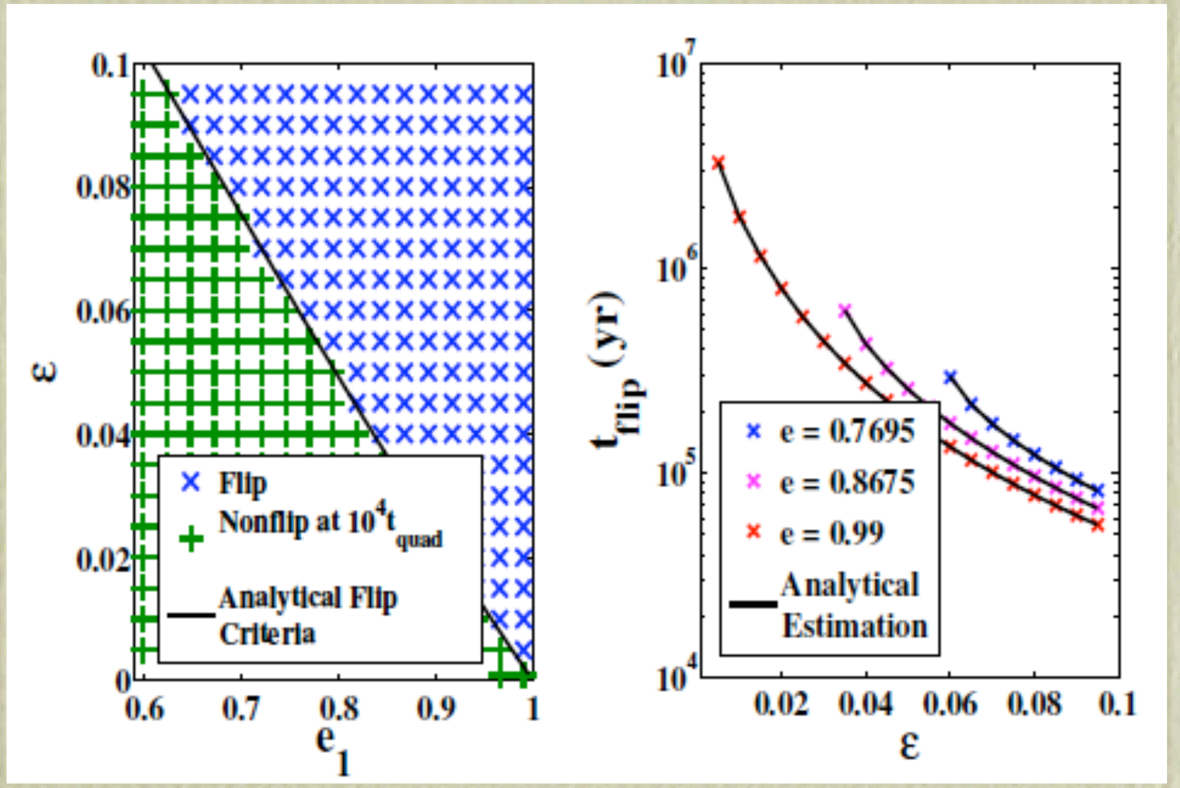
$$\epsilon_c = \frac{1}{2} f\left(\frac{1}{2} \cos^2 i_{tot,0}\right)$$

$$f(C_{KL}) = \frac{32\sqrt{3}}{\pi} \int_{x_{min}}^{1} \frac{K(x) - 2E(x)}{(41x - 21)\sqrt{2x+3}} dx \quad \text{and} \quad x_{min} = \frac{3 - 3C_{KL}}{3 + 2C_{KL}}$$



Katz et al. 2011

Analytical Results v.s. Numerical Results



Why do analytical results with low inclination approximation work?

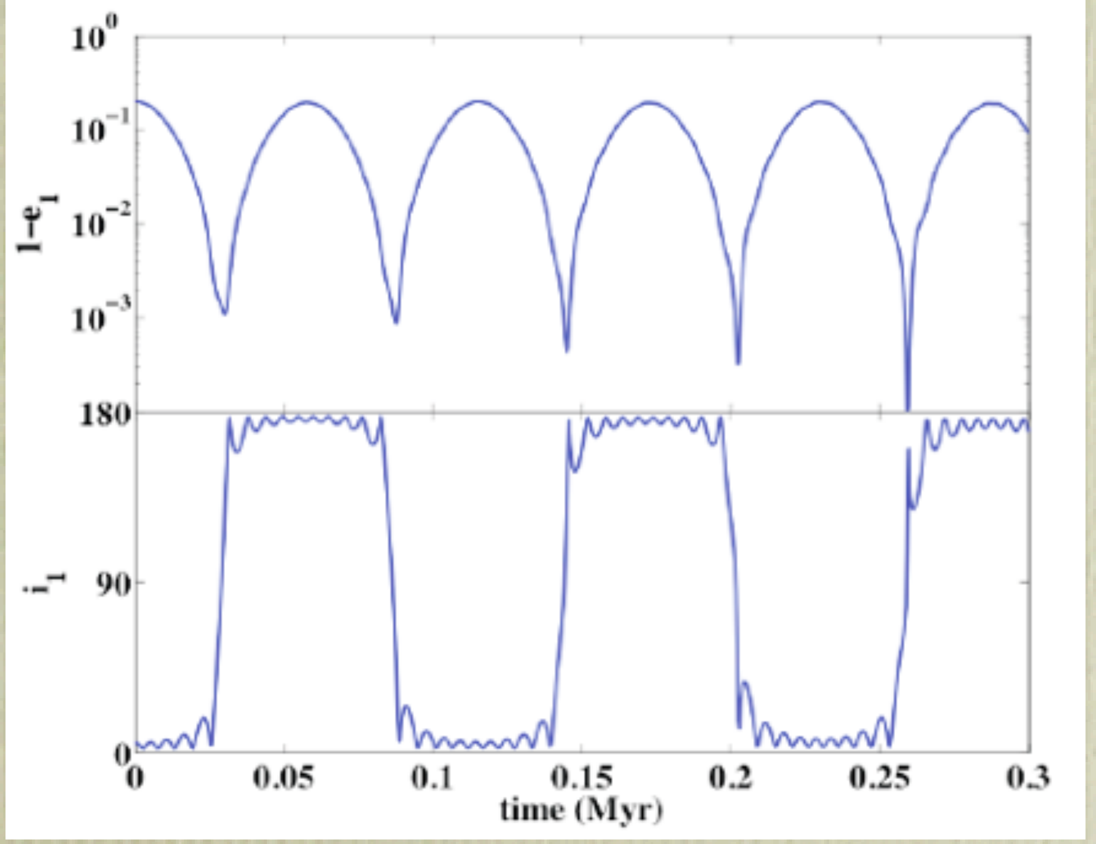
IC: $m_1 = 1M_\odot$, $m_2 = 0.1M_\odot$, $a_1 = 1AU$, $a_2 = 45.7AU$, $\omega_1 = 0^o$, $\Omega_1 = 180^o$, $i_1 = 5^o$.

Li, et al., 2013

Analytical Results v.s. Numerical Results

Why do analytical results with low inclination approximation work?

Small inclination assumption holds for most of the evolution.



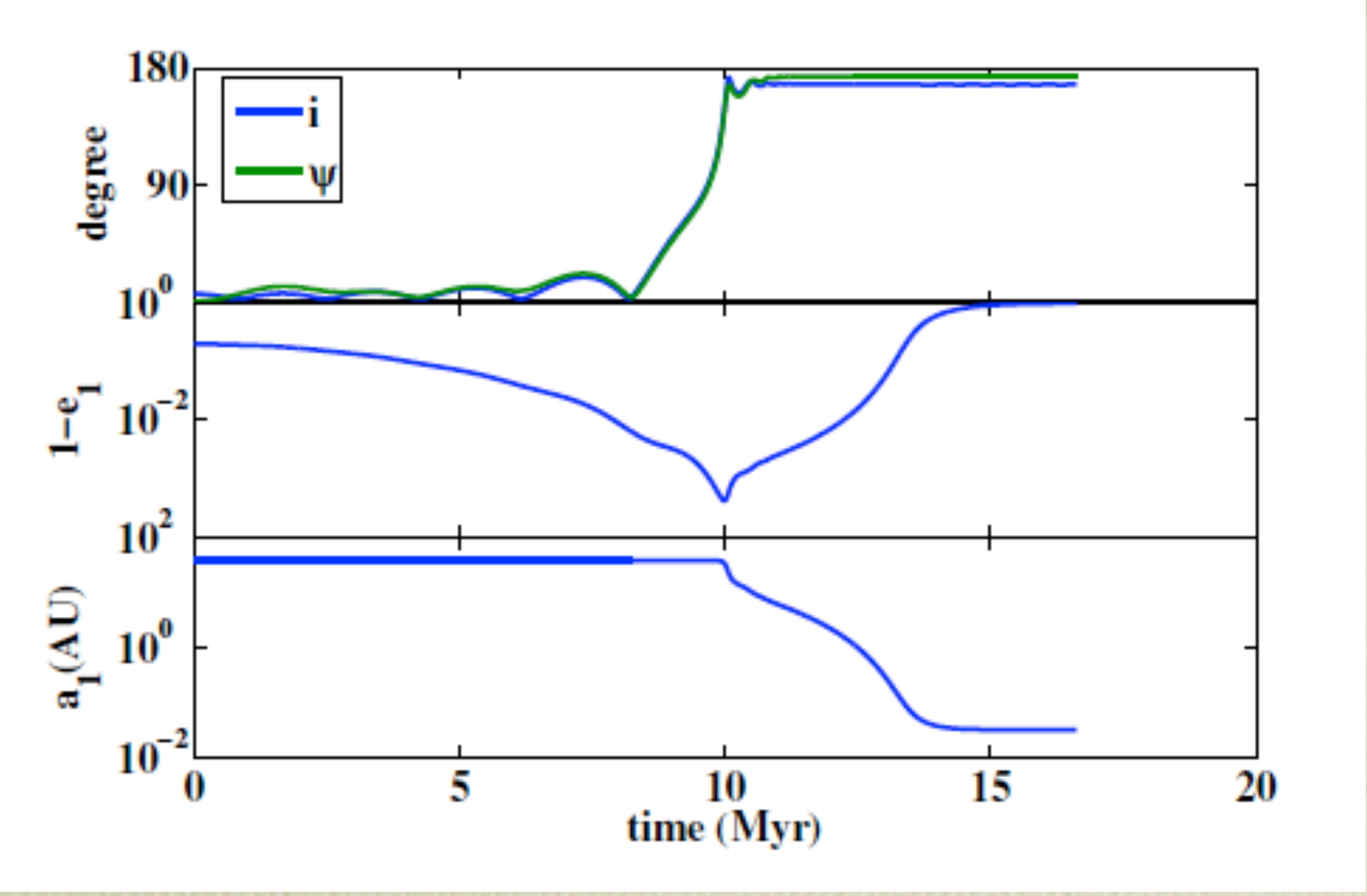
IC: $m_1 = 1\ M_\odot$, $m_J=1M_J$, $m_2 = 0.3\ M_\odot$, $\omega_1 = 0^o$, $\Omega_1 = 180^o$, $e_2=0.6$, $a_1 = 4$ AU, $a_2 = 50$ AU, $e_1 = 0.8$, $i = 5^o$

Li, et al., 2013

Examples --- 1. Produce Counter Orbiting Hot Jupiters (+ tide)

Question: Does this mechanism produce a peak at ψ≈180°?

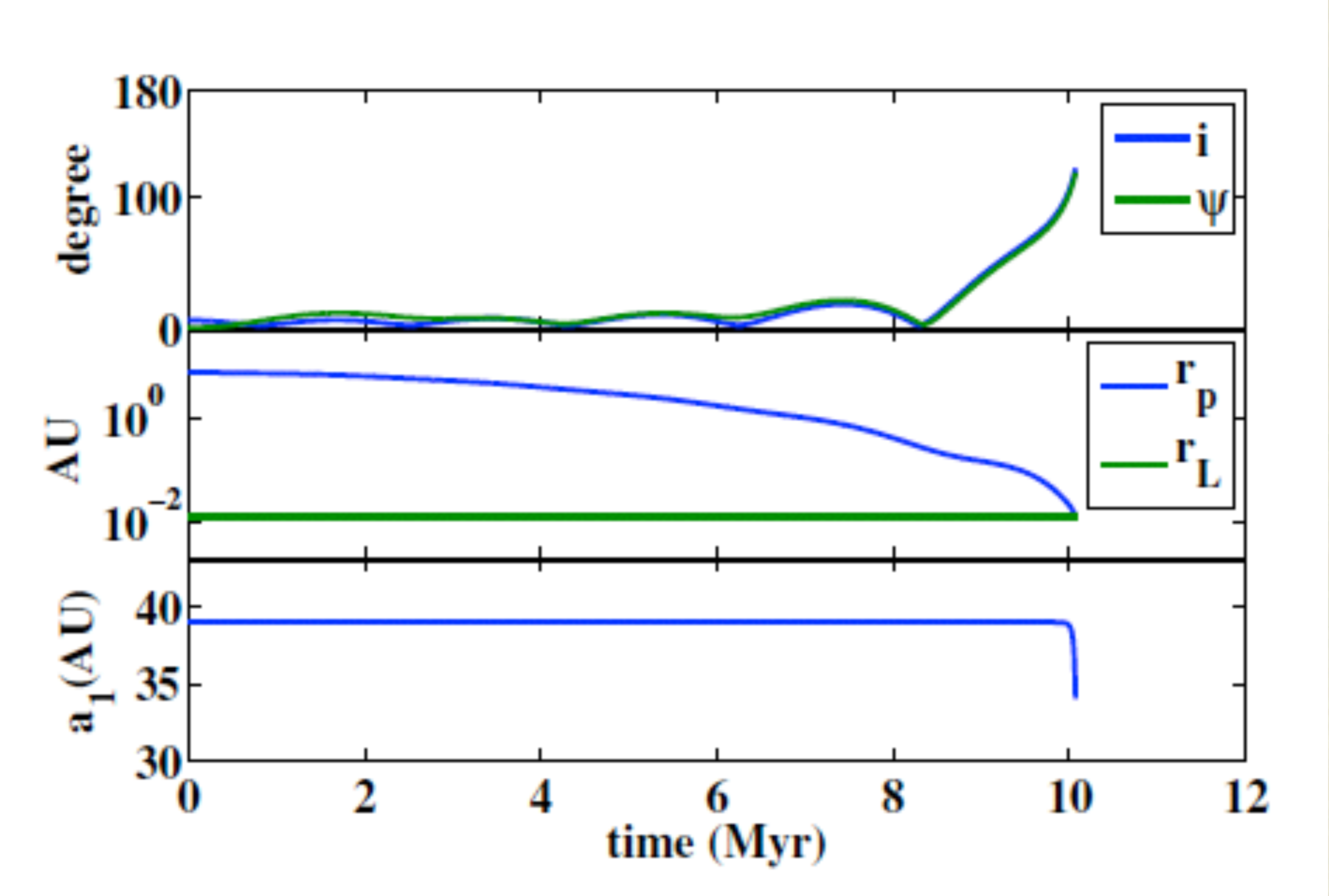
No.



Li et al., 2014a

Examples --- 1. Produce Counter Orbiting Hot Jupiters (+ tide)

Question:
Will planet be tidally disrupted?



Yes!

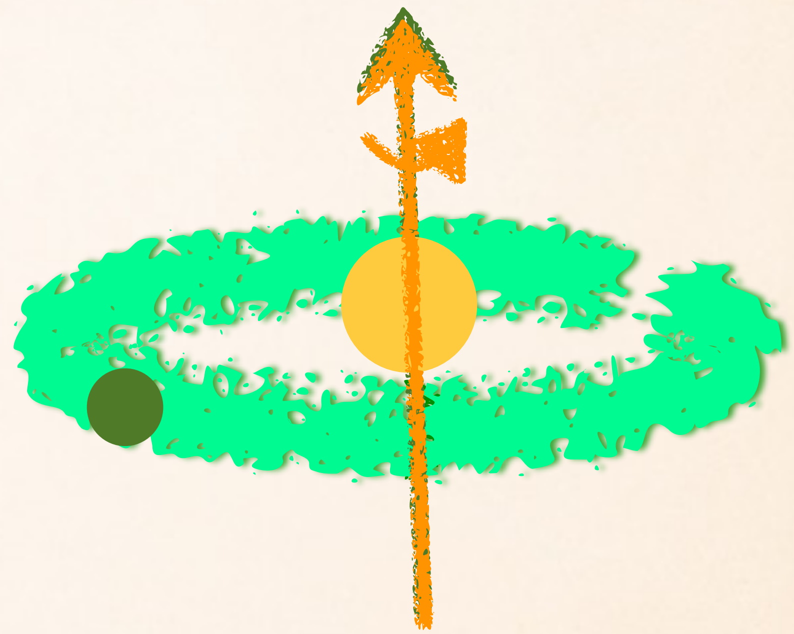
Li et al., 2014a

ORIGIN OF SPIN-ORBIT MISALIGNMENT

- **Smooth Migration:** planets move close due to interaction with proto-planetary disk.

Star tilts through magnetic interaction

(Lai et al. 2011)

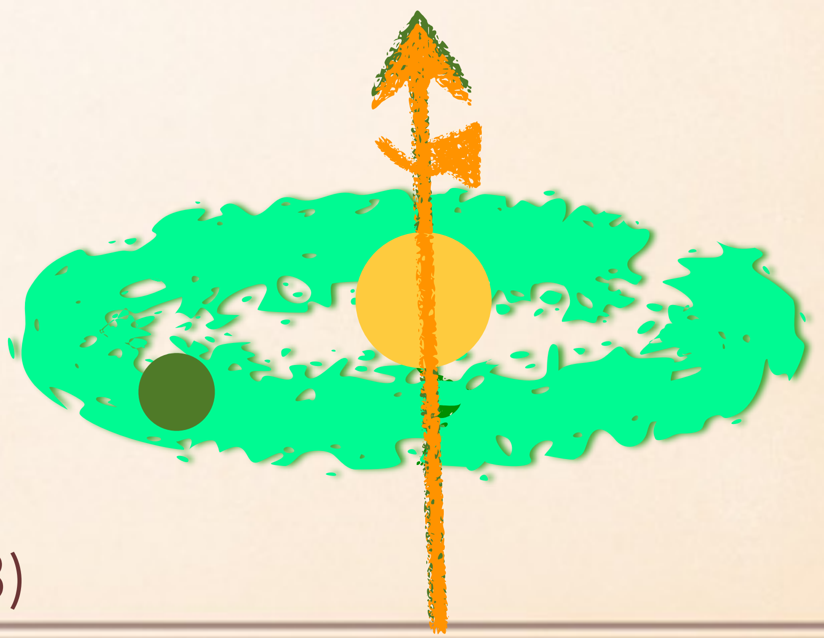


or stellar oscillation effects

(Rogers et al. 2012, 2013)

Disk tilts through inhomogeneous collapse of the molecular cloud

(Bate et al. 2010; Thies et al. 2011; Fielding et al. 2015)



or the torque from nearby stars.

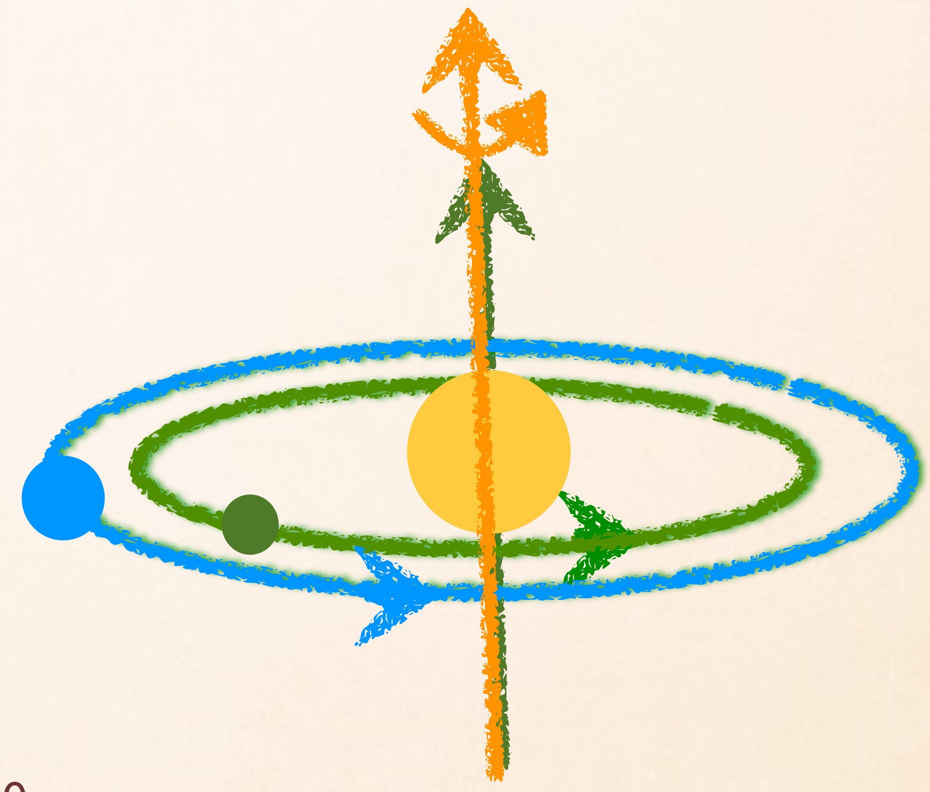
(Tremaine 1989; Batygin 2012; Xiang-Gruess & Papaloizou 2013)

ORIGIN OF SPIN-ORBIT MISALIGNMENT

- **Violent Migration (Dynamical Origin):** planets move close due to interactions with companion stars/planets.

Planetary orbit tilts under planet-planet scattering

(e.g., Chatterjee et al. 2008, Petrovich 2014)

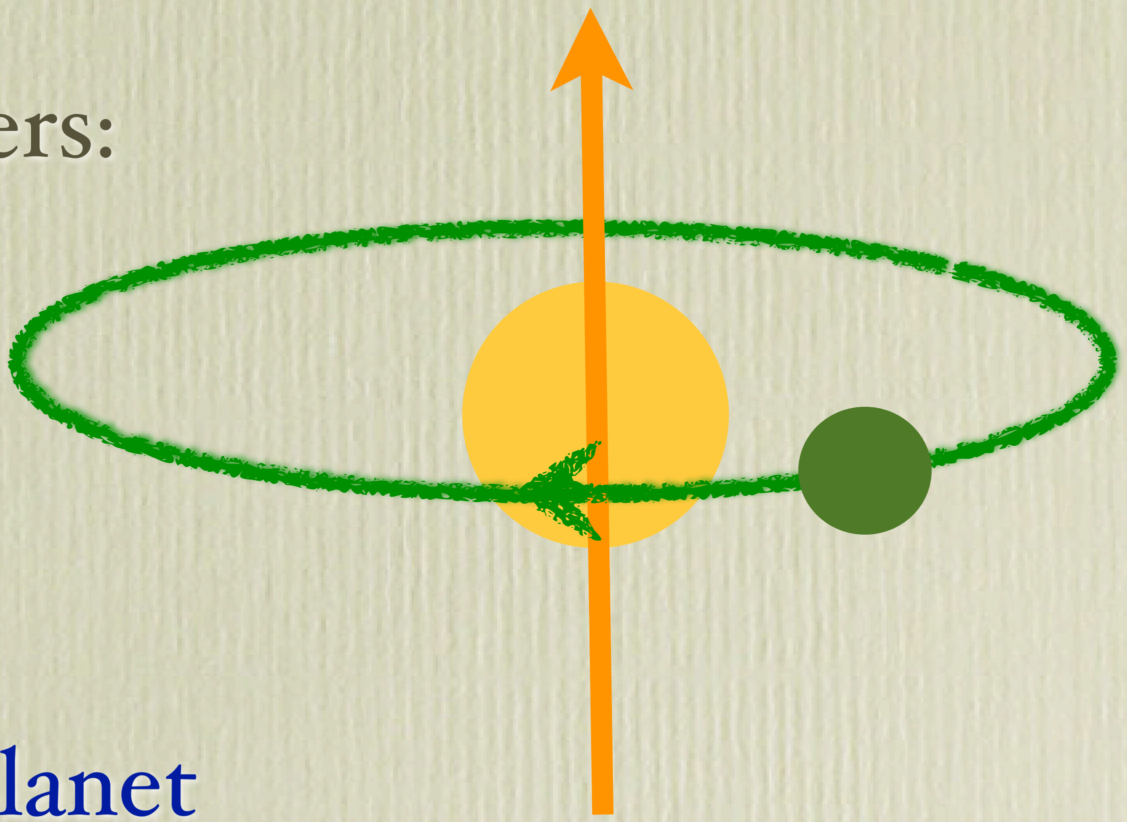


or long-term secular dynamical effects between planets or stellar companion.

(e.g., Fabrycky and Tremaine 2007; Nagasawa et al. 2008; Naoz et al. 2011, 2012; Wu and Lithwick 2011; Li et al. 2014; Valsecchi and Rasio 2014)

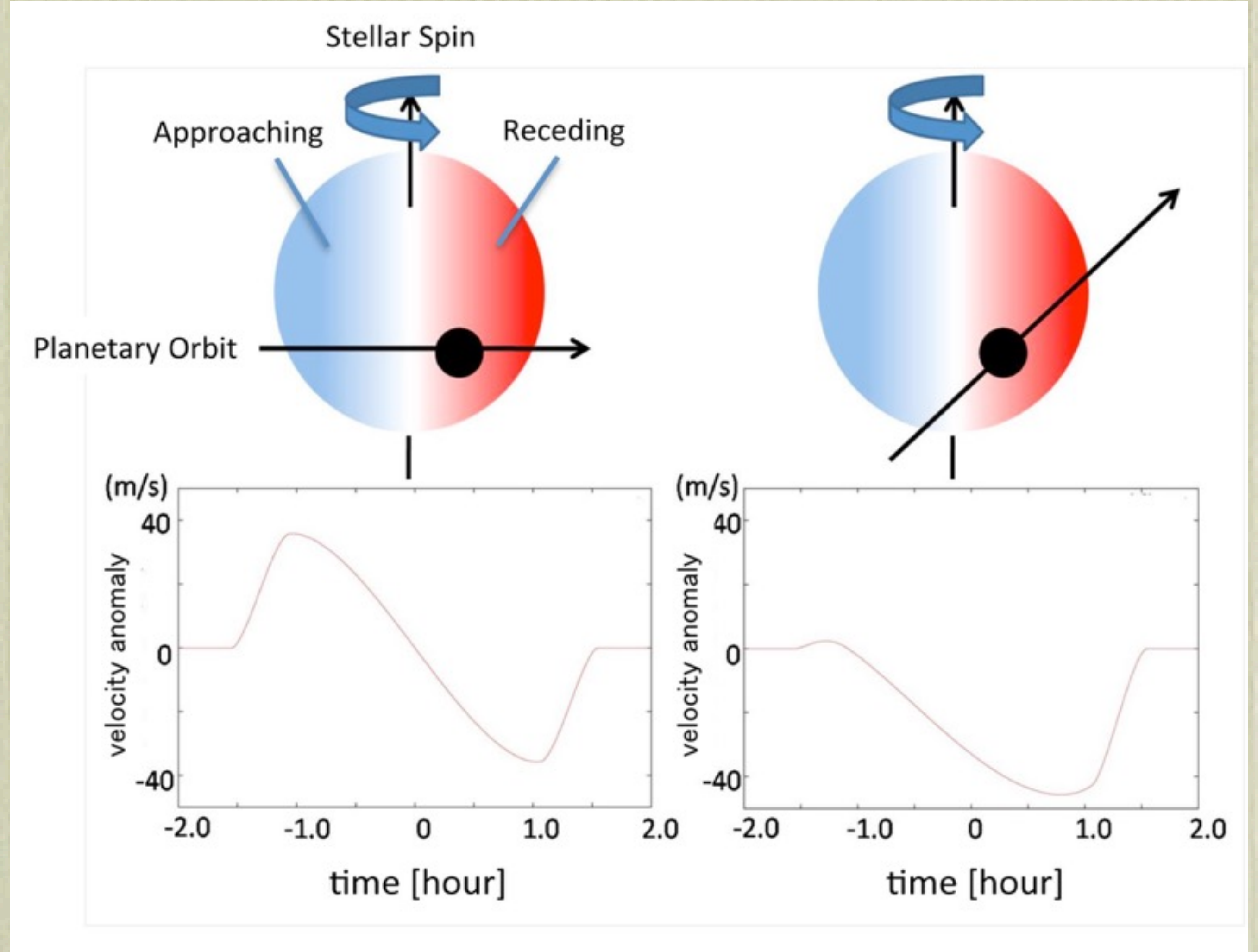
Applications --- 1. Produce Counter Orbiting Hot Jupiters (+ tide)

- Hot Jupiters:
  - massive exoplanets ($m \geq m_J$) with close-in orbits (period: 1-4 day).
- Counter Orbiting Hot Jupiters:
  - Hot Jupiters that orbit in exactly the opposite direction to the spin of their host star.



- Disagree with the classical planet formation theory:
the orbit aligns with the stellar spin.

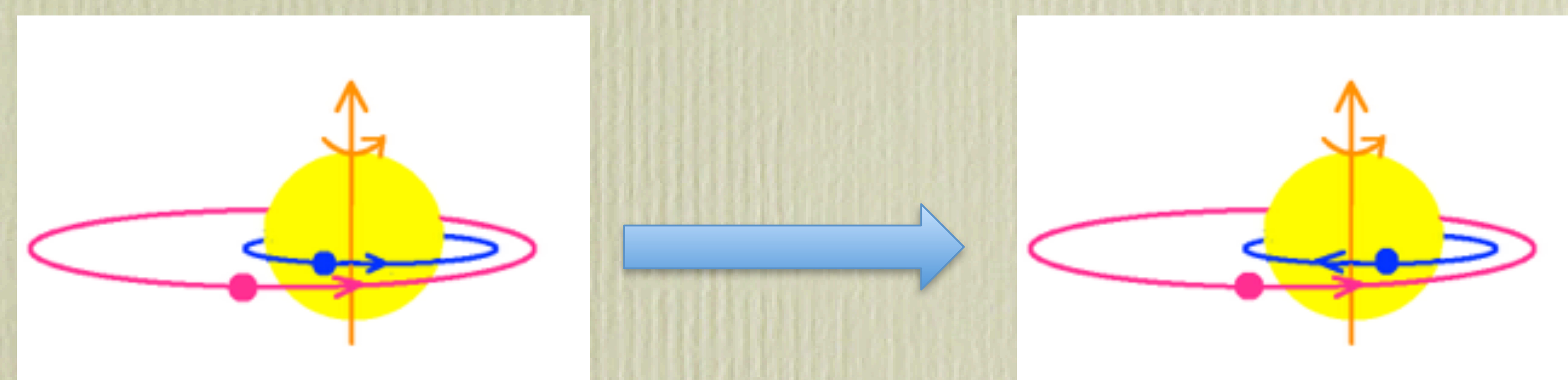
Rossiter-McLaughlin Method



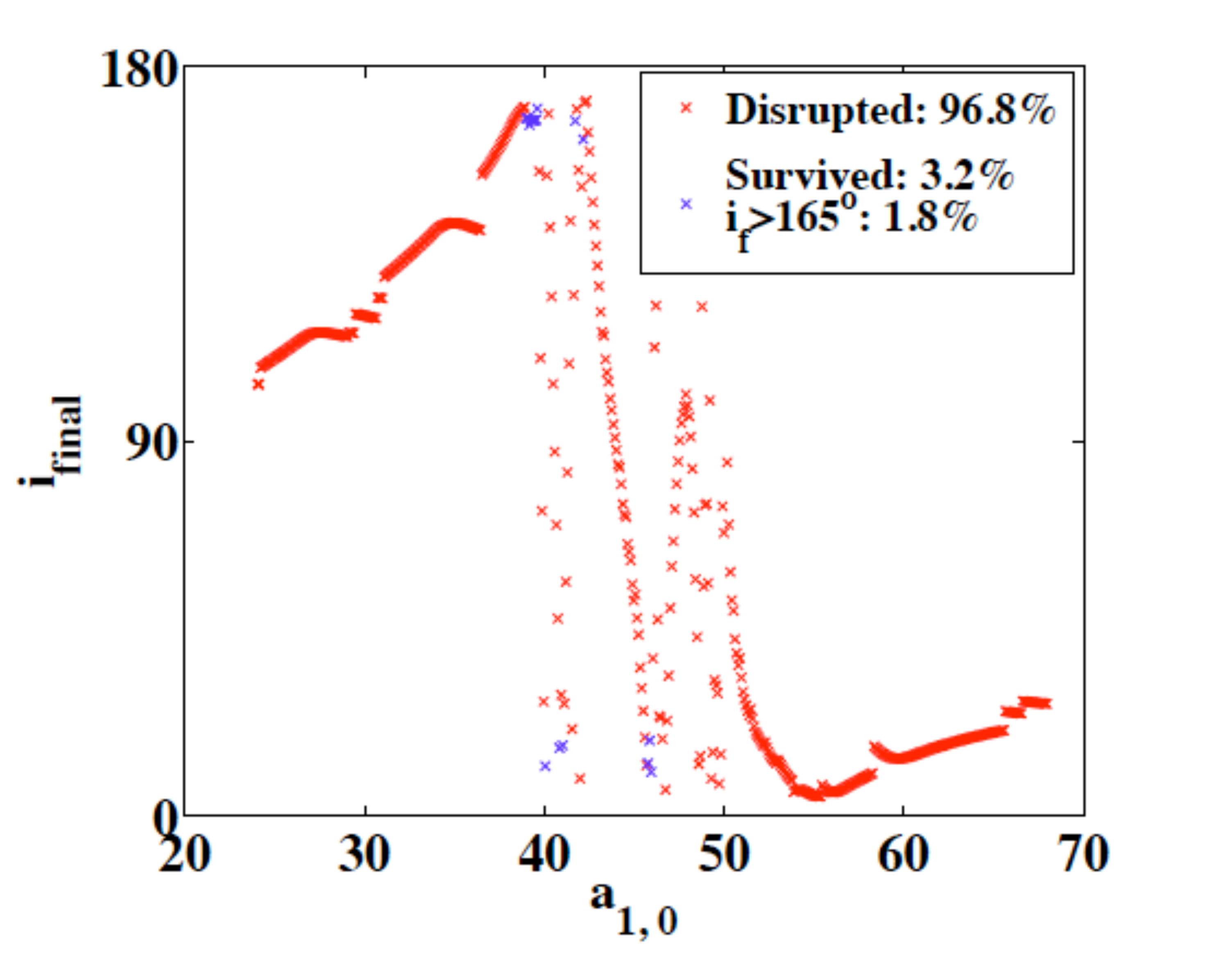
http://www.subarutelescope.org/

Take Home Message

- Eccentric Coplanar Kozai Mechanism can flip an eccentric coplanar inner orbit to produce counter orbiting exoplanets



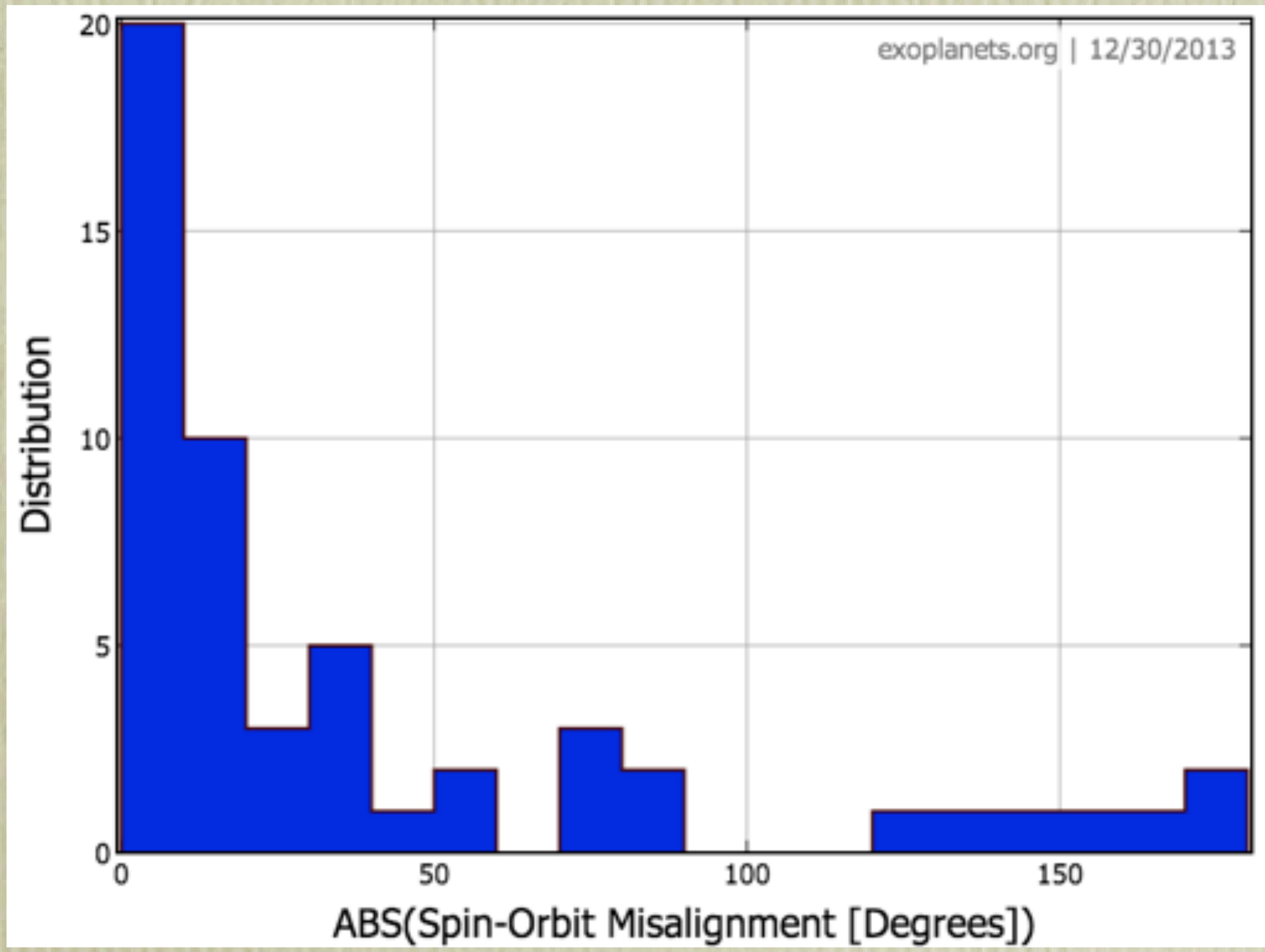
Eccentric inner orbit flips due to eccentric coplanar outer companion

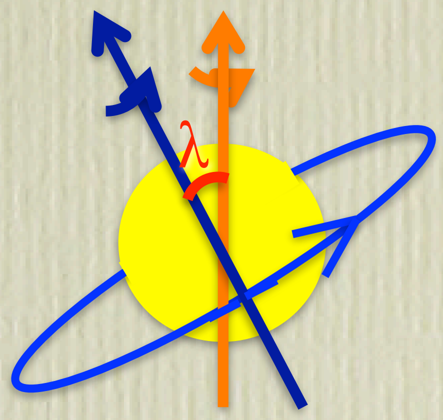


180
90
0
20
30
40
50
60
70
i_{final}
$a_{1,0}$
Disrupted: 96.8%
Survived: 3.2%
i_f>165°: 1.8%

Observational Links to Counter Orbiting Hot Jupiters

- Distribution of sky projected spin-orbit angle (λ) of Hot Jupiters





There are retrograde hot jupiters (λ>90°)

It is possible to have counter orbiting planets.

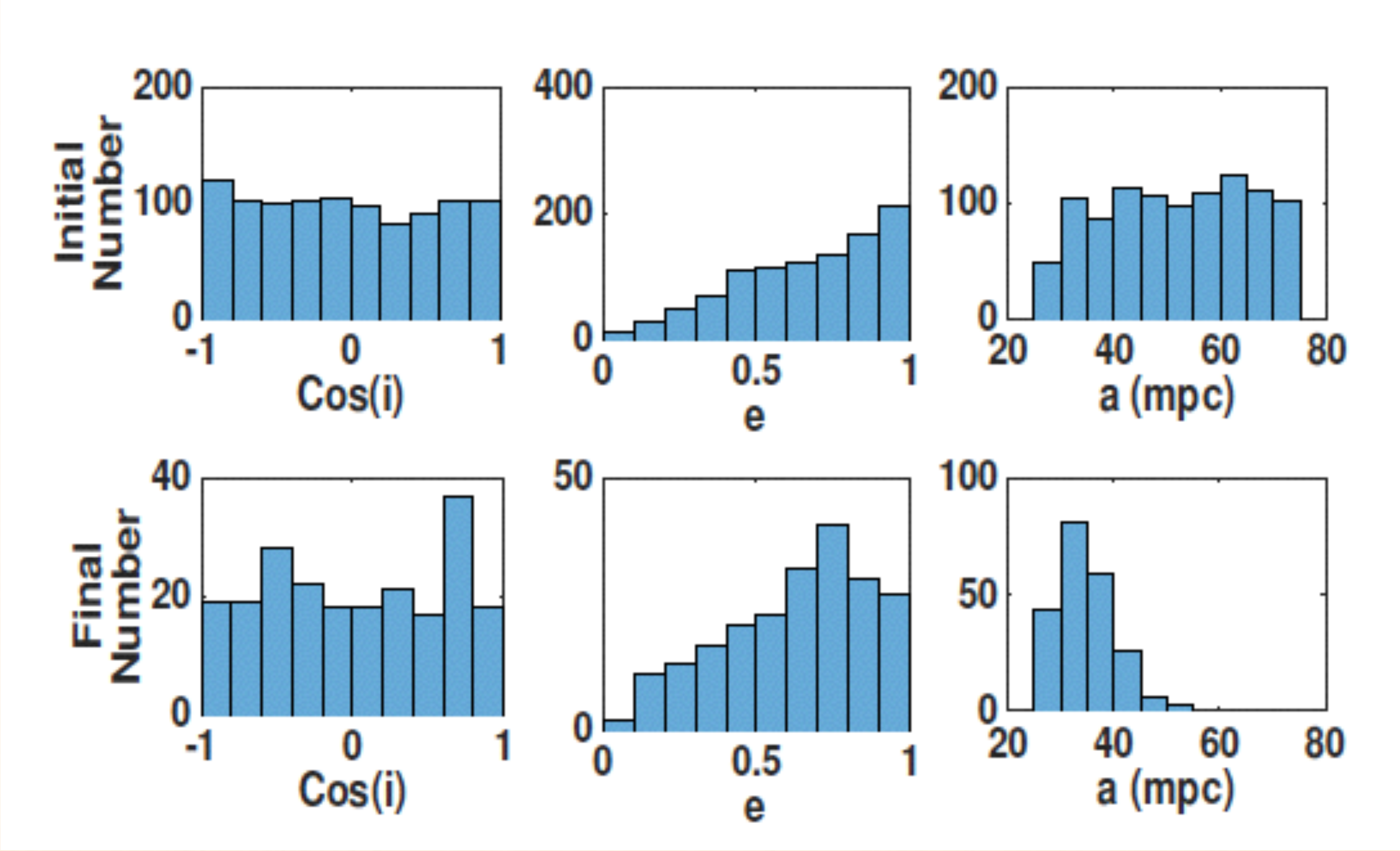
Applications --- 2. Effects of EKM of Stars Surrounding BBH

- Tidal disruption rate is highly uncertain:
  - It is observed to be 10-5~-4/galaxy/yr from a very small sample by Gezari et al. 2008.
  - It roughly agrees with theoretical estimates. (e.g. Wang & Merritt 2004)

- The disruption rate may be greatly enhanced:
  - due to non-axial symmetric stellar potential. (Merritt & Poon 2004)
  - due to SMBHB (Ivanov et al. 2005, Wegg & Bode 2011, Chen et al. 2011)
  - due to recoiled SMBHB (Stone & Loeb 2011)

Examples --- 3. Effects of EKM of Stars Surrounding BBH

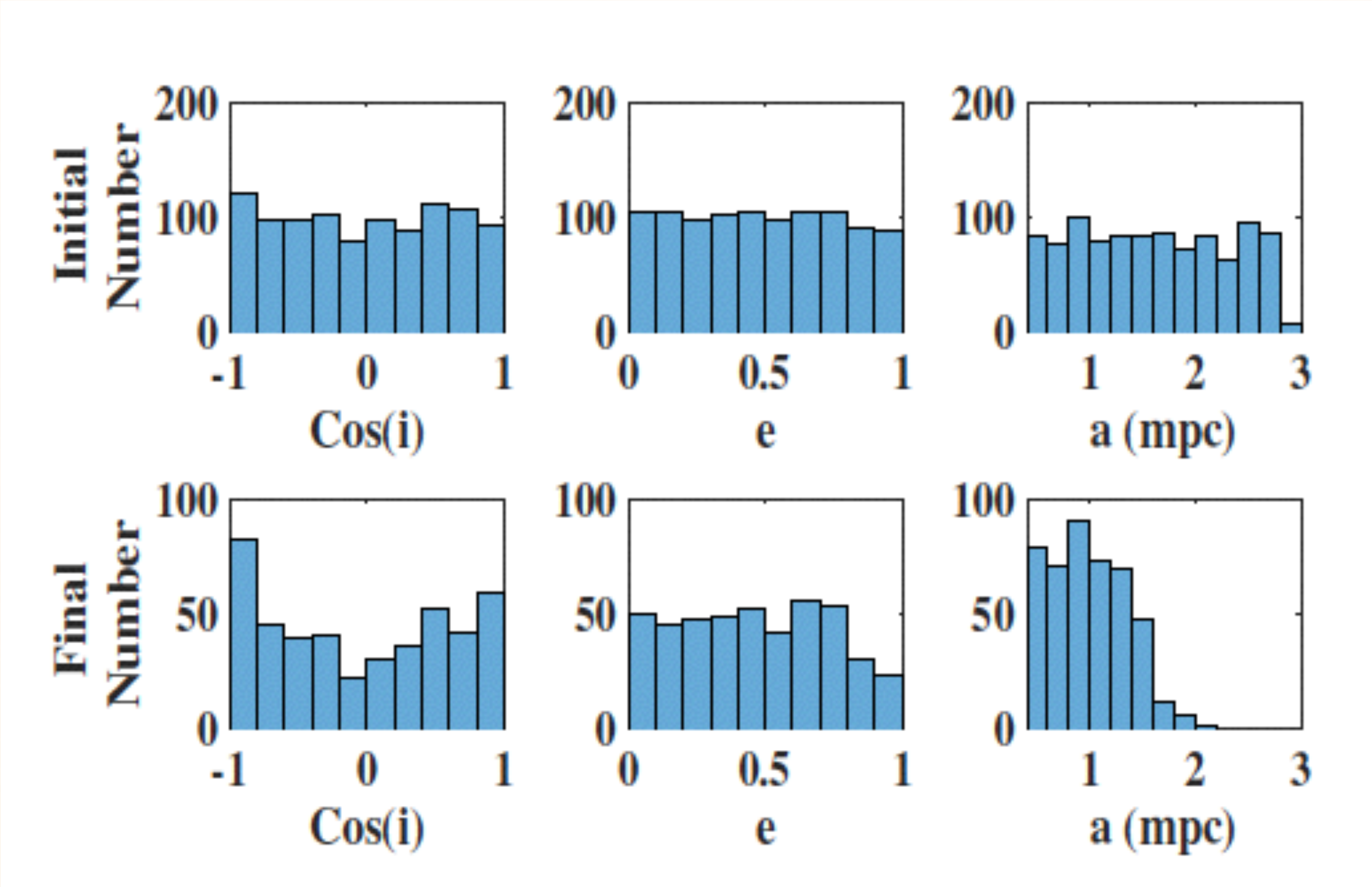
- Example: $m_1 = 10^7 M_\odot$, $m_2 = 10^8 M_\odot$, $a_2 = 0.5$pc, $e_2 = 0.5$, $\alpha = 1.75$ (stellar distribution), normalized by M-σ relation. Run time: 1Gyr.



(Li, et al. submitted 2015)

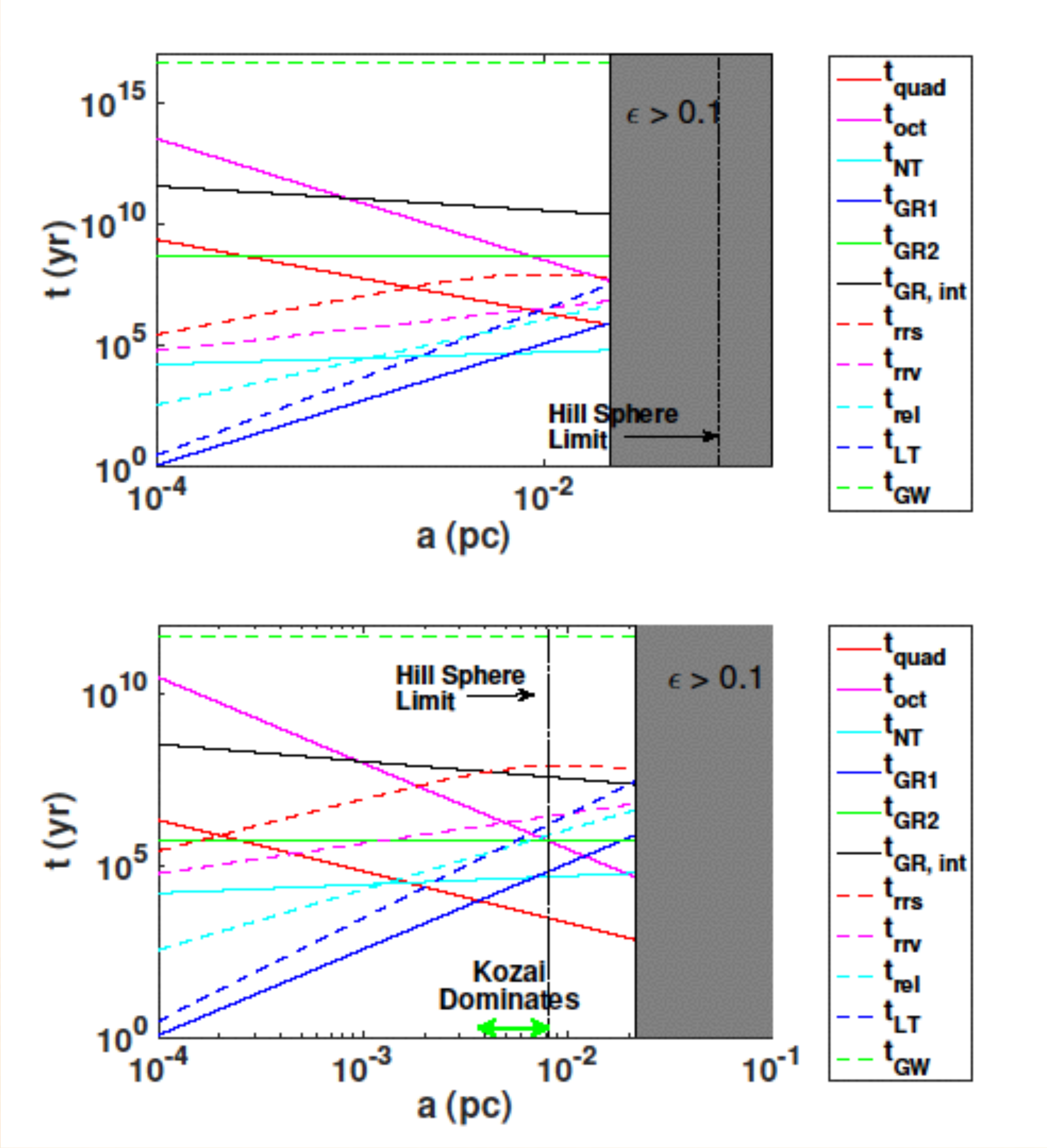
Examples --- 3. Effects of EKM of Stars Surrounding BBH

- Example: $m_1 = 10^4 M_\odot$, $m_2 = 4\times10^6 M_\odot$, $a_2 = 0.1$pc, $e_2 = 0.7$, $\alpha = 1.75$ (stellar distribution), normalized by M-σ relation. Run time: 100Myr.

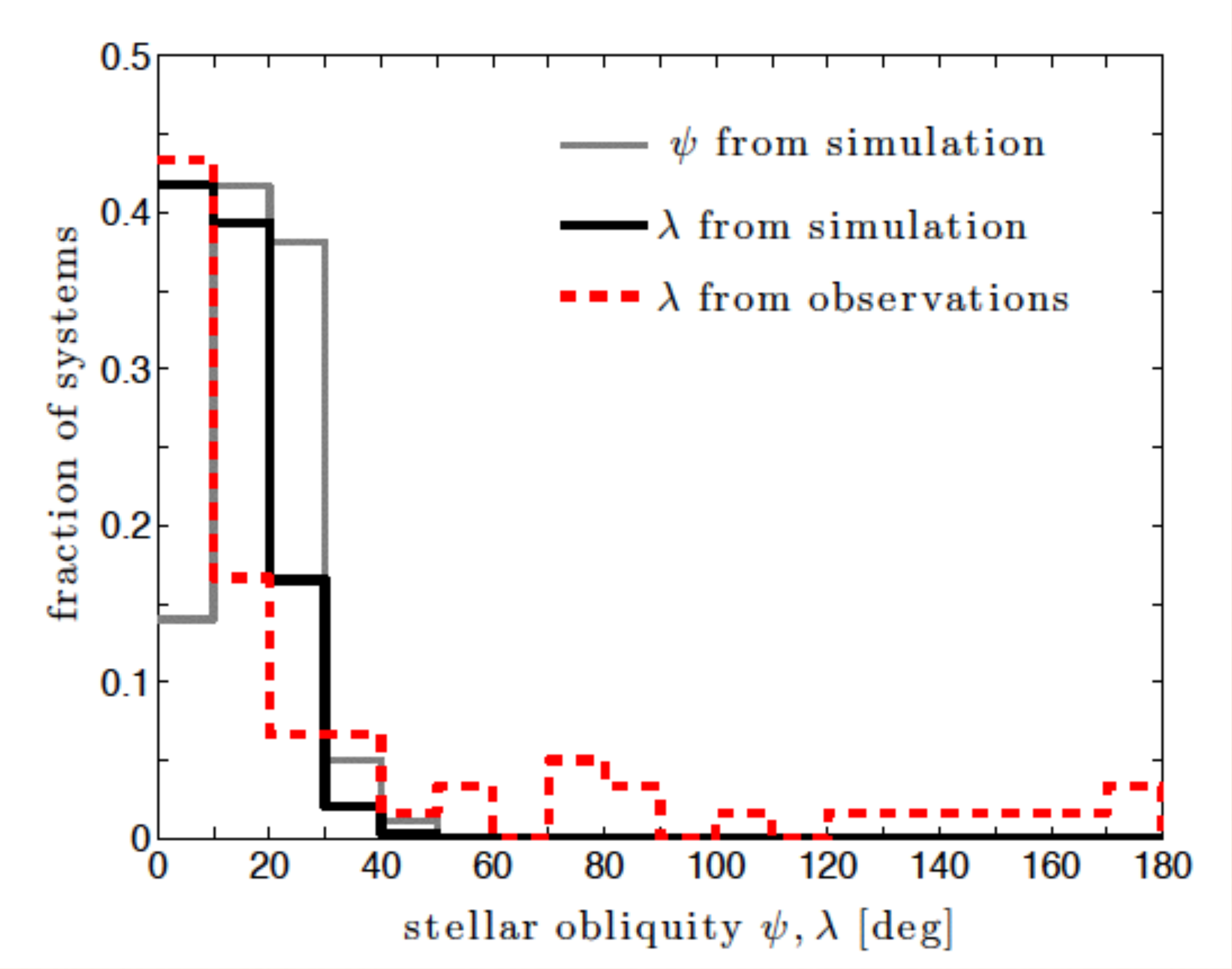


(Li, et al. submitted 2015)

COMPARISON OF TIMESCALES



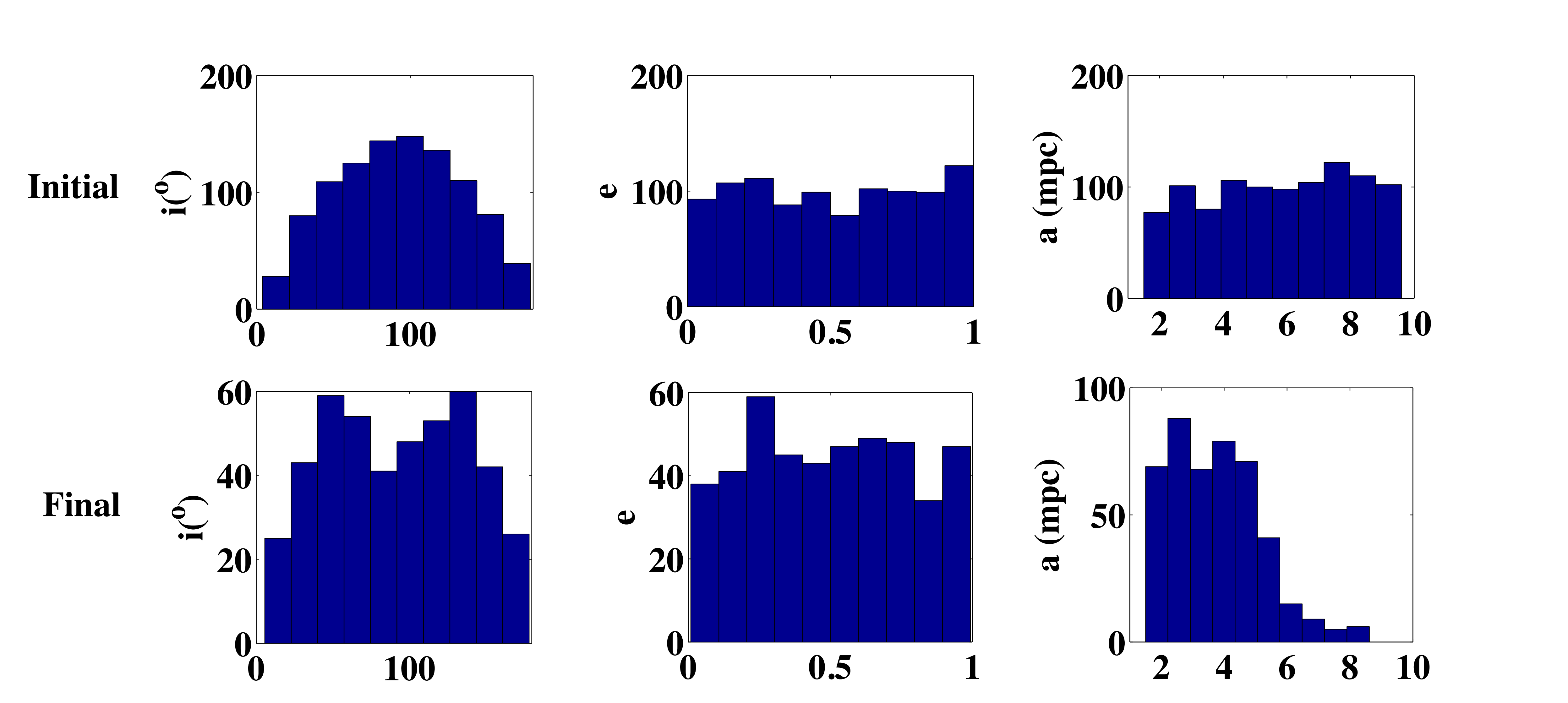
COPLANAR HIGH ECCENTRICITY MIGRATION



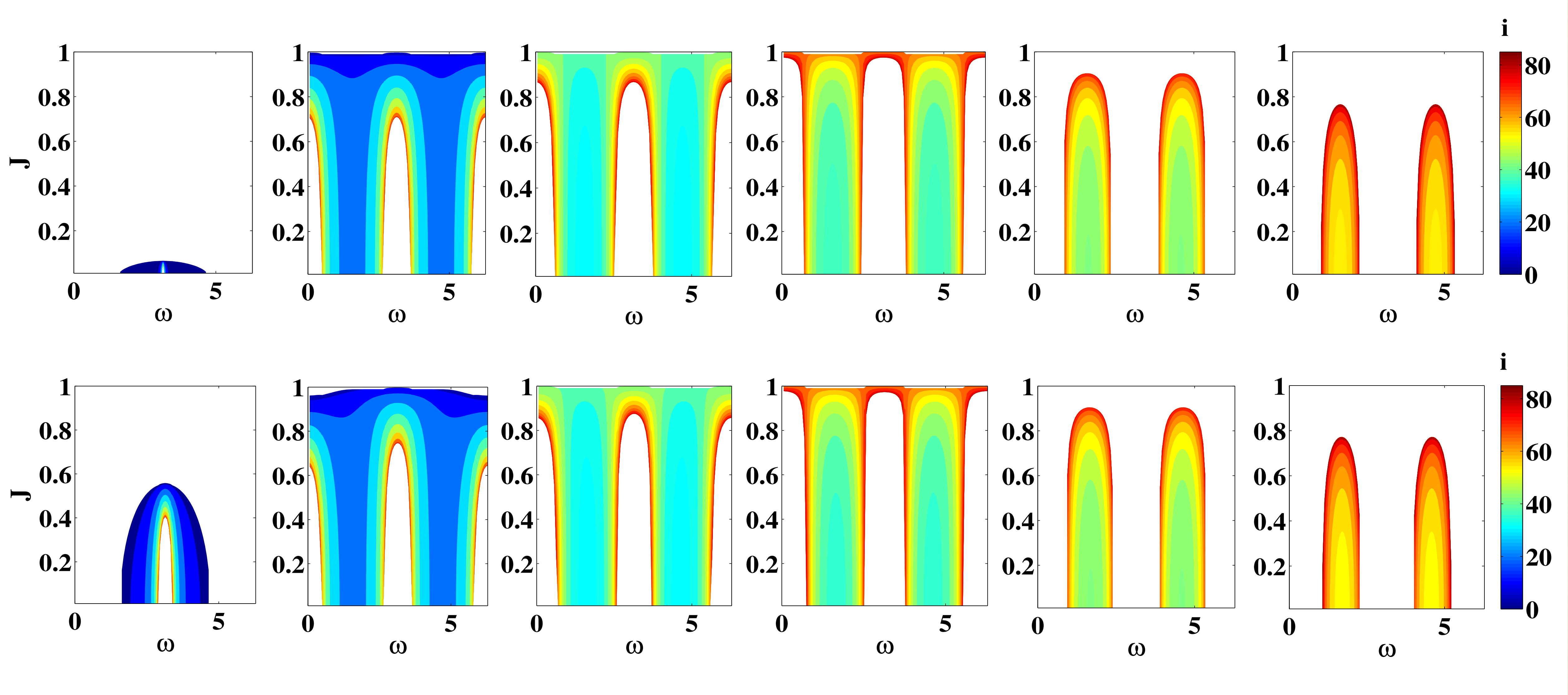
Population synthesis study.
tv=0.1yr

Initial v.s. Final Distribution

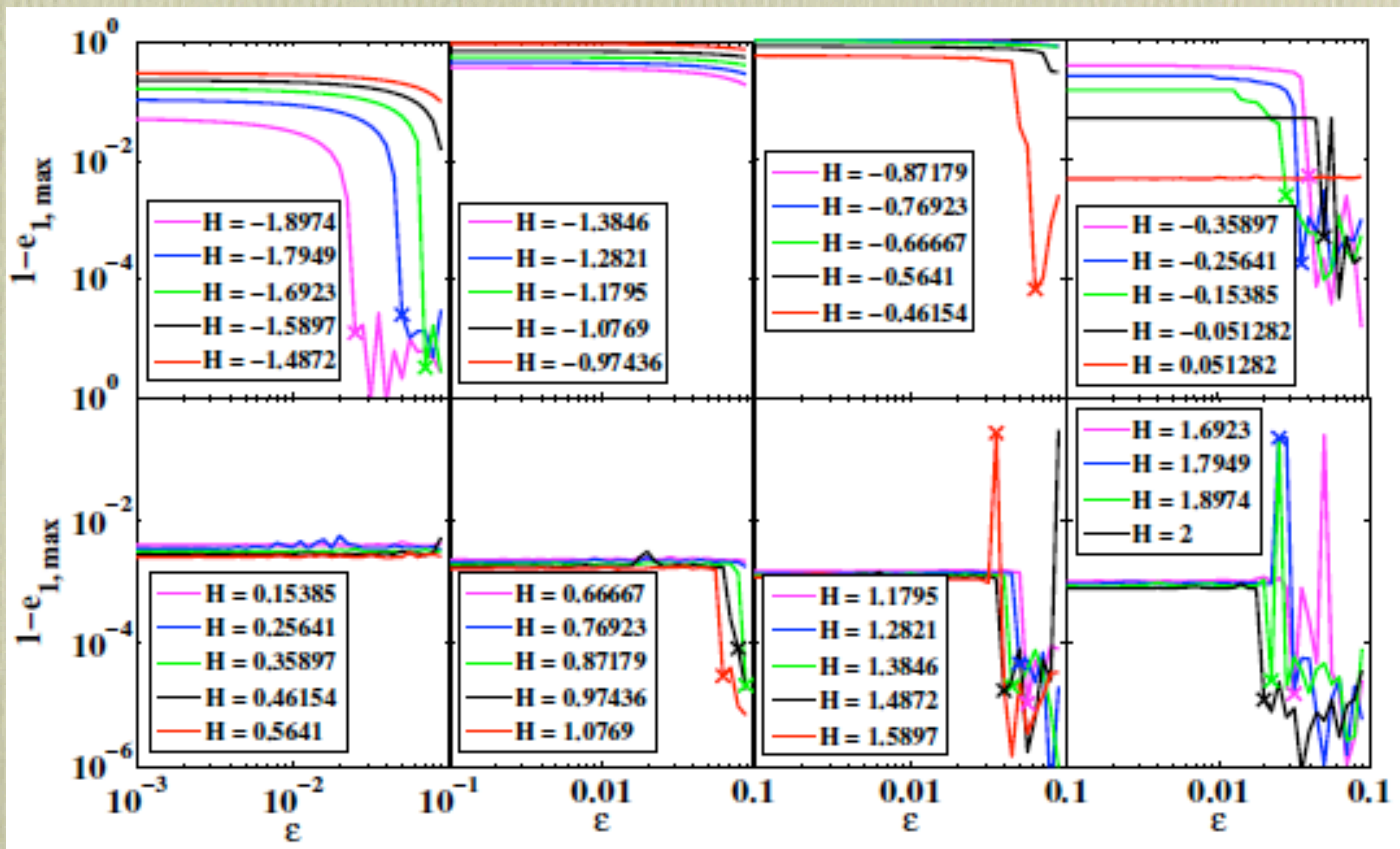
- Example: $m_1 = 10^6 M_\odot$, $m_2 = 10^{10} M_\odot$, $a_2 = 1$pc, $e_2 = 0.7$, $\alpha = 1.75$ (stellar distribution), normalized by M-σ relation. Run time: 1Gyr.



Initial Condition in i



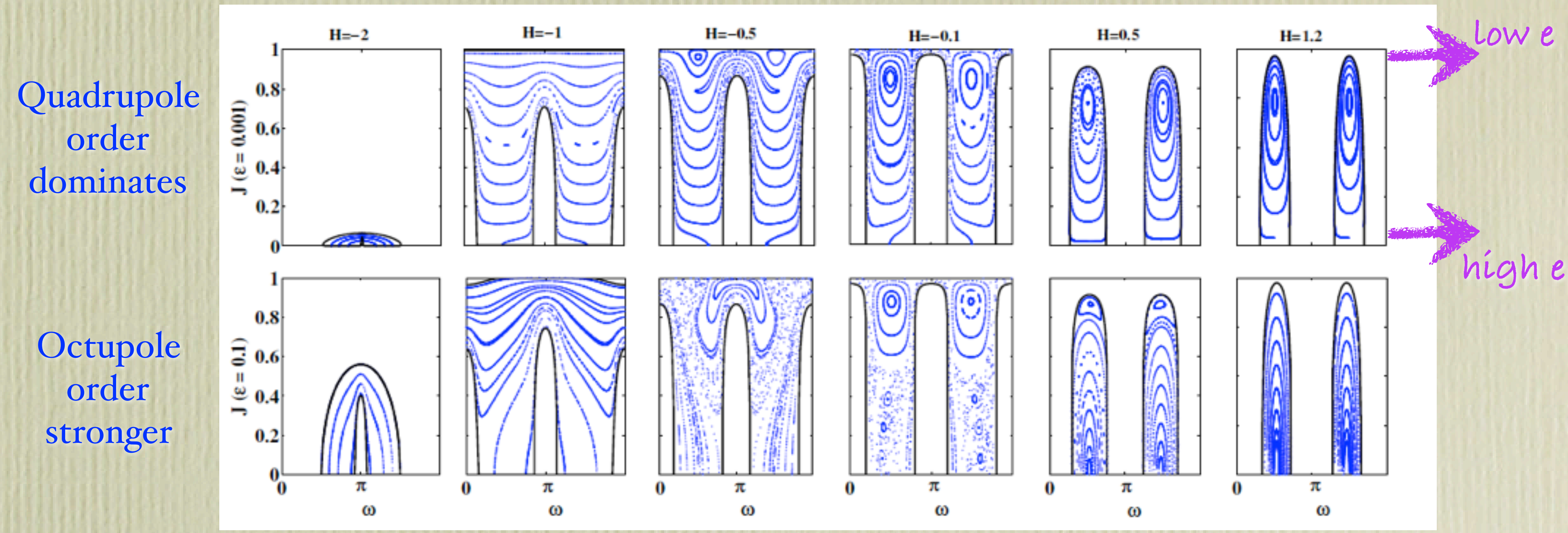
Maximum e_I for different H and ϵ



Maximum e_I for low i, high e_I case, and high i cases

Surface of Section

Low i ⟷ High i (40-60°) ⟷ i~90°



- Trajectories chaotic only for H=-0.5, -0.1 at high ϵ .
- High inclination flips are chaotic.
- Overall evolution of the trajectories: evolution sensitive on the initial angles.

Li et al. 2014b

Surface of Section

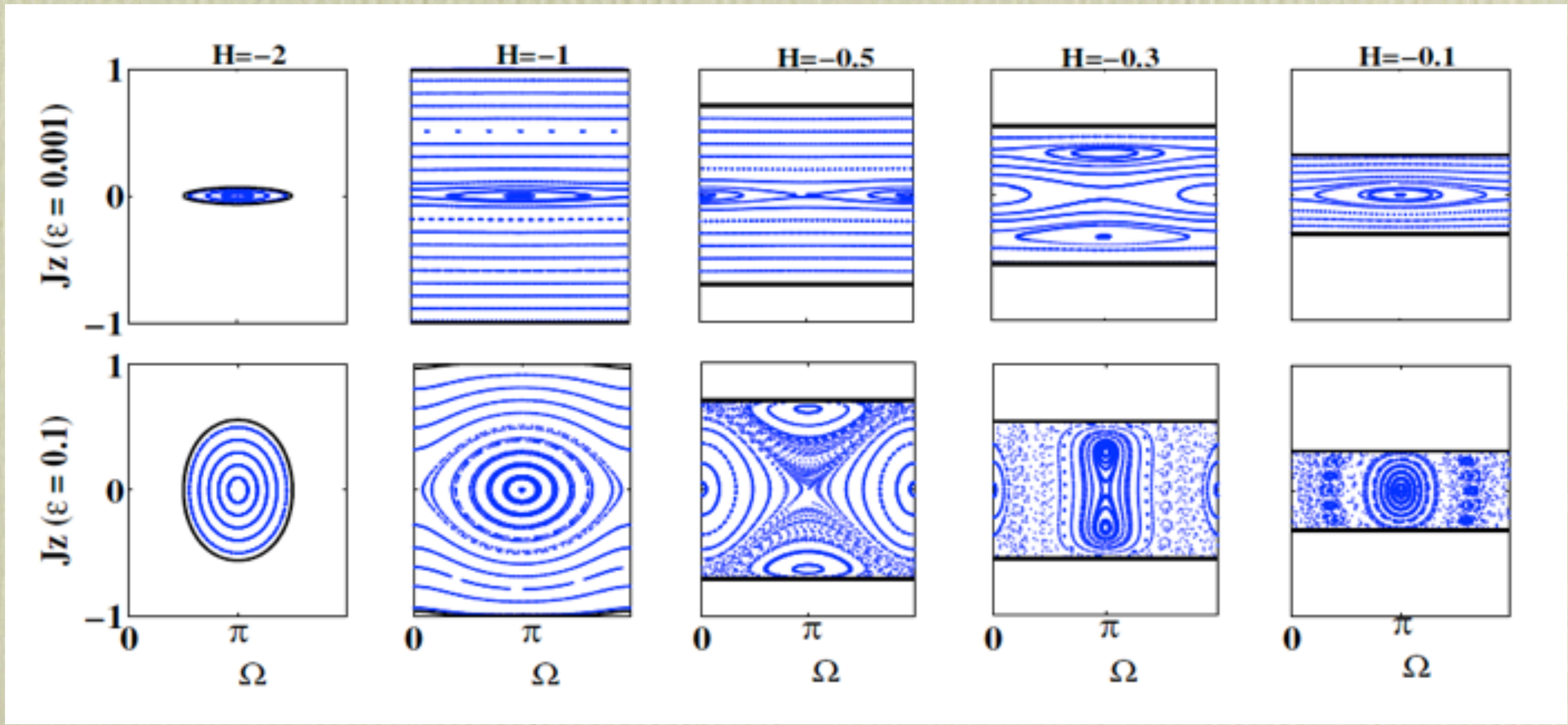
- Surface of section in the Jz − Ω plane

$Jz = \sqrt{1 - e_1^2} \cos i_1$ Ω: longitude of node

Low i, high e_1 → High i, low e_1

Quadrupole order dominates

Octupole order dominates



- All features are due to octupole effects.
- Trajectories are chaotic only possible when H=-0.5, -0.3, -0.1, for high ϵ.

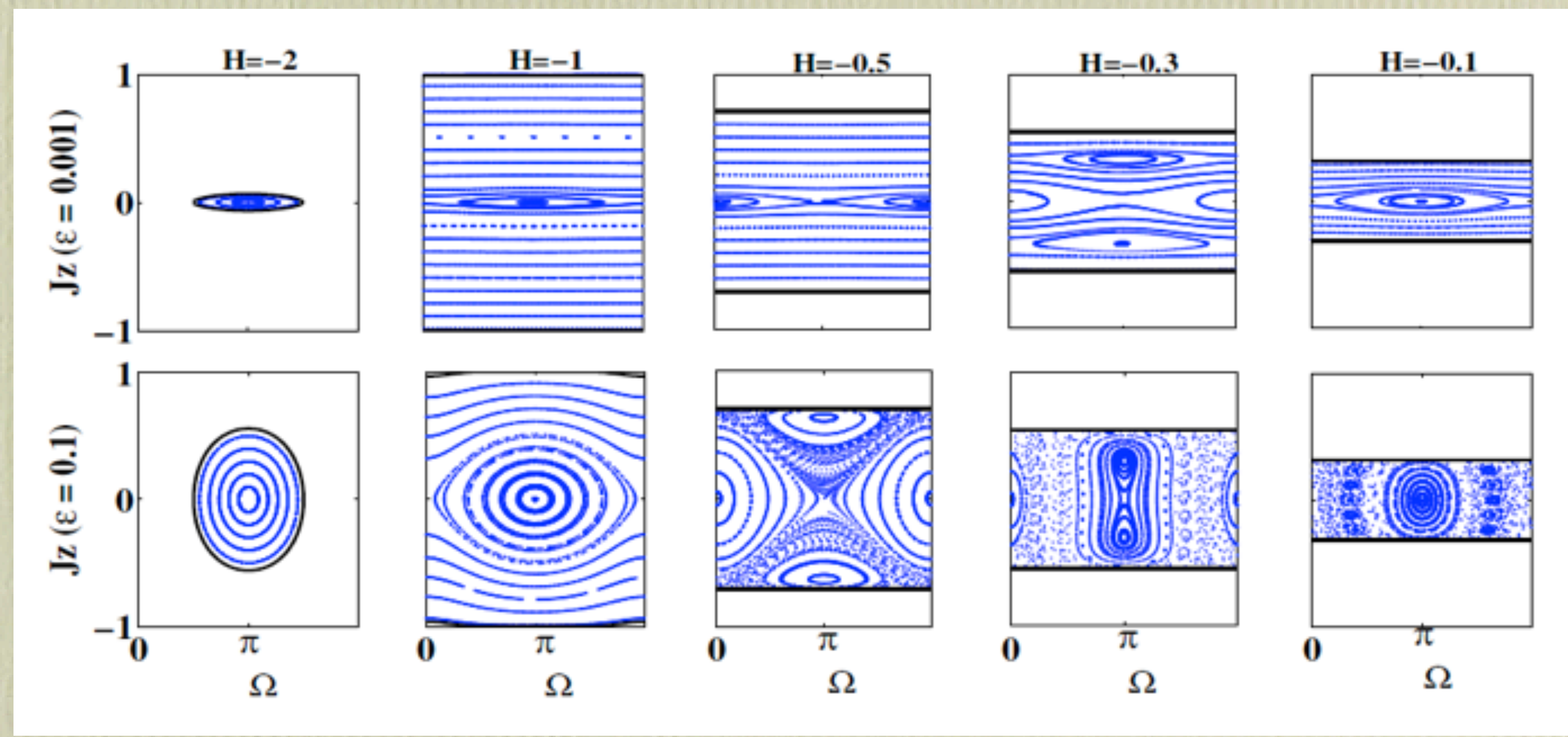
Li et al. 2014b

Surface of Section

Low i, high e_1 → High i, low e_1

Quadrupole order dominates

Octupole order dominates



- All features are due to octupole effects.
- Trajectories are chaotic only when H≤0.
- Flips are due to octupole resonances.

(Li, et al., 2014 in prep)

Applications --- 2. Tidal Disruption of Stars Surrounding BBH

- SMBHBs originate from mergers between galaxies. Following the merger, the distance of the SMBHB decreases.

  (Complete numerical simulations: e.g. Khan et al. 2012)

- SMBHBs with ~kpc separation have been observed with direct imagine.

  (e.g. Fabbiano et al. 2011, Green et al. 2010, Civano et al. 2010, Komossa et al. 2003, Hutchings & Neff 1989)

- At ~1pc separation it is more difficult to identify SMBHBs. SMBHBs have been observed with optical spectra, light variability and radio lines.

  (e.g. Boroson & Lauer 2009, Valtonen et al. 2008, Rodriguez et al. 2006)

- Motivation of tidal disruption of stars by ~1pc SMBHB:

  Identify SMBHB at ~1 pc separation with tidal disruption rate

Effects on Stars Surrounding BBH

- Dynamics of stars around BH or BBH:
  - Secular dynamics introduce instability in eccentric stellar disks around a single BH (e.g. *Madigan, Levin & Hopman 2009*)
  - Tidal disruption event rate can be enhanced due to BBH and the recoil of BBH (*Ivanov et al. 2005, Wegg & Bode 2011, Chen et al. 2011, Stone & Loeb 2011*)
  - Relic stellar clusters of recoiled BH may uncover MW formation history (e.g. *O'Leary & Loeb 2009*).

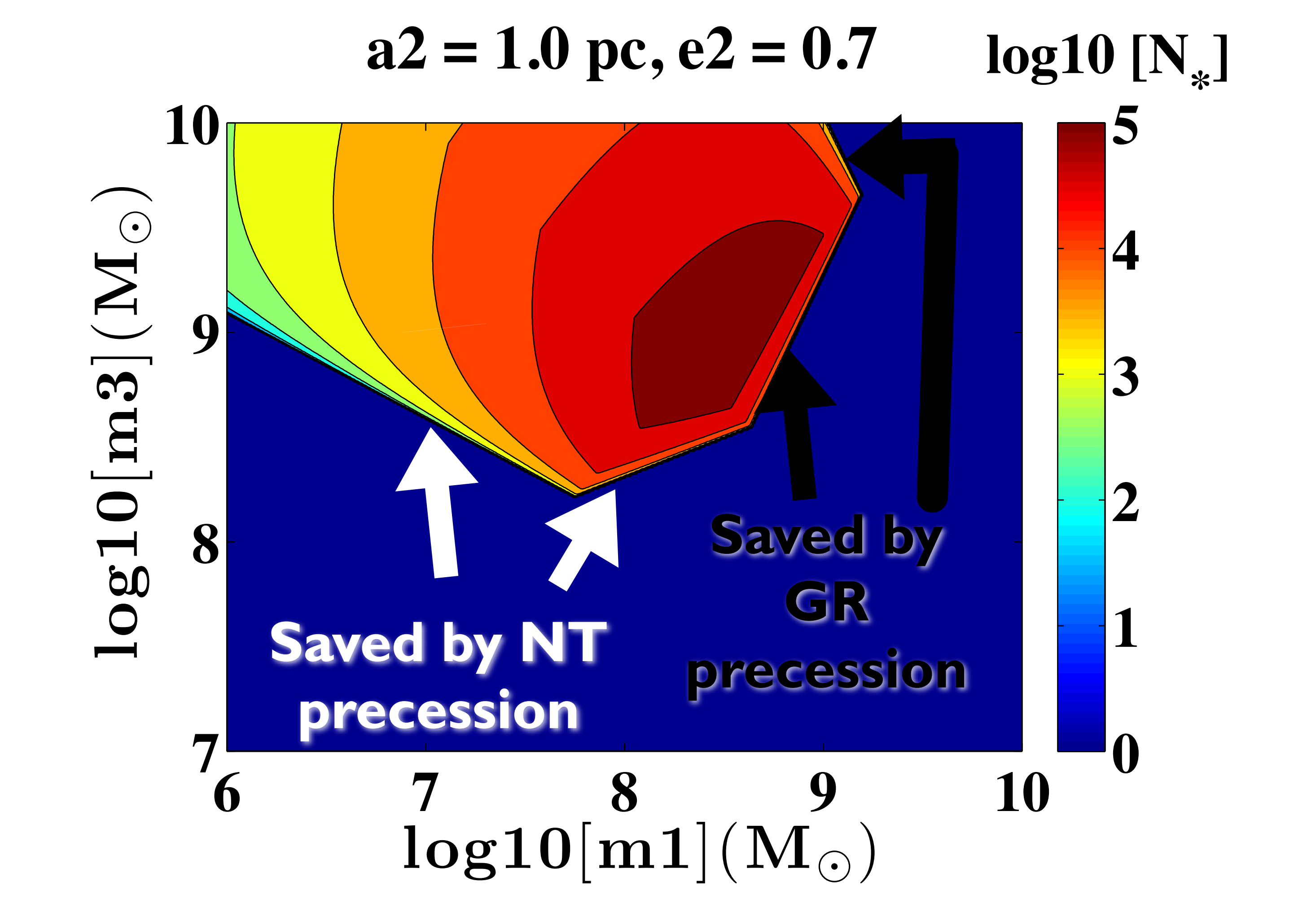
- Here we study the effect of EKM to stars surrounding BBH

Effects of EKM on Stars Surrounding BBH

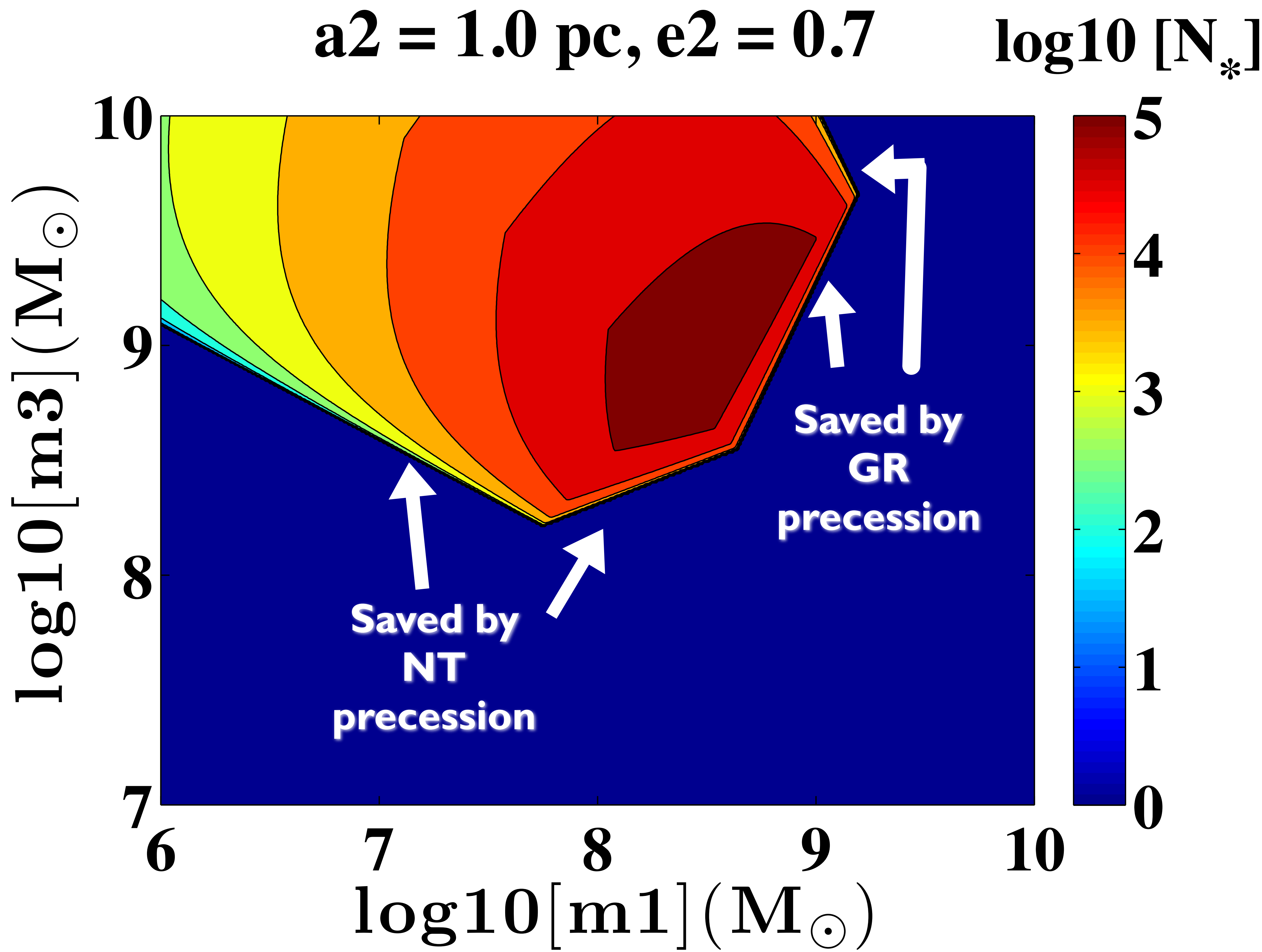
- Study the role of **eccentric ($e_2 \neq 0$) Kozai mechanism** in the presence of **general relativistic (GR) precession** and **Newtonian (NT) precession** for stars surrounding SMBHB.

- Set the separation of the BBH at $a_2=1pc$, $e_2=0.7$ and assuming $\rho_* \propto a^{-1.75}$, normalized by M-σ relation.

- N_* is the number of stars affected by the eccentric Kozai Mechanism. (Requirement: $t_{GR} < t_{Kozai}$, $t_{NT} < t_{Kozai}$, $\varepsilon < 0.1$, $a_1 < r_{RL}$).



(Li, et al., in prep)



a2 = 1.0 pc, e2 = 0.7
log10 [N*]
Saved by GR precession
Saved by NT precession
log10[m3](M⊙)
log10[m1](M⊙)

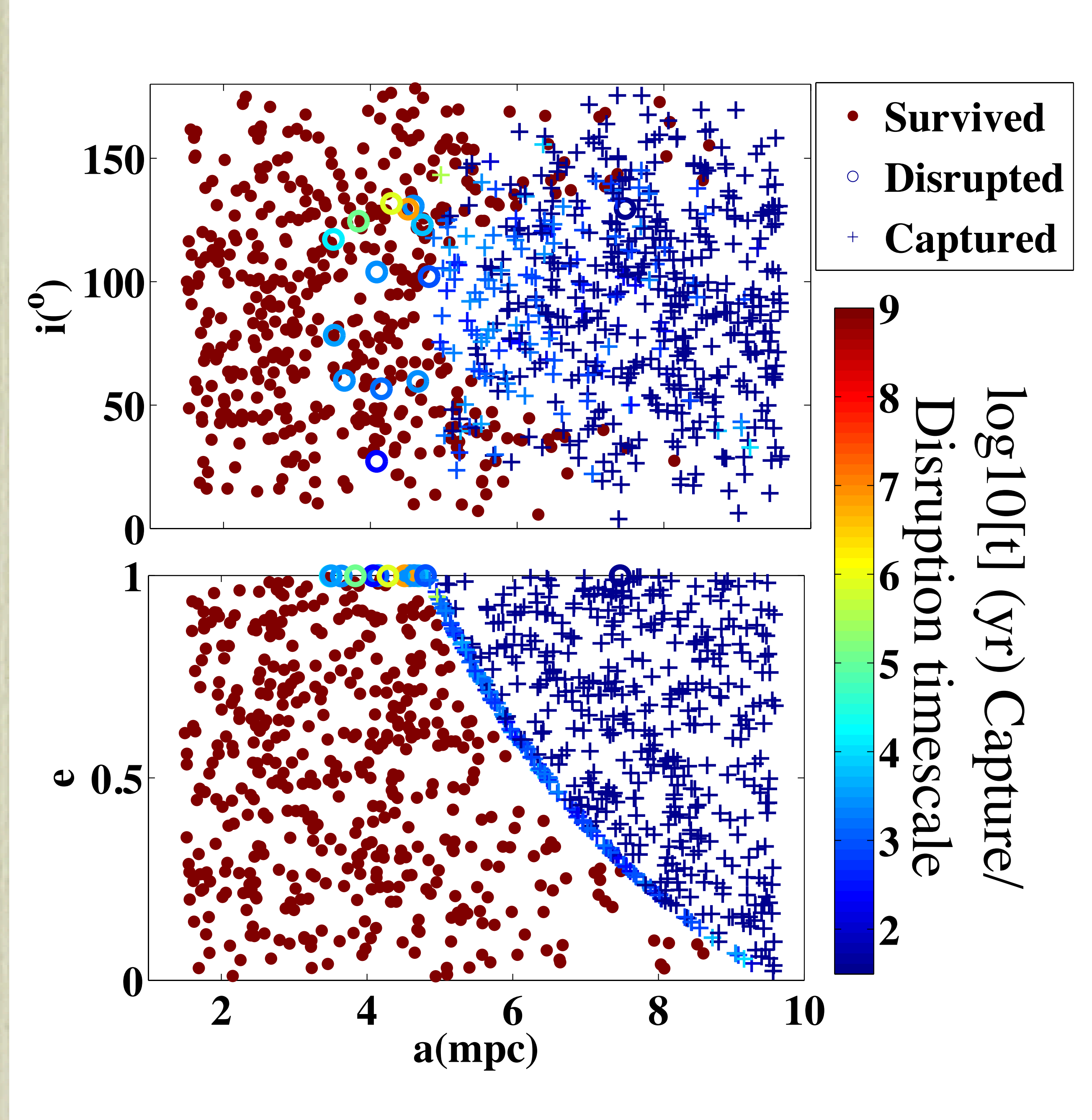
Effects of EKM on Stars Surrounding BBH

- Example: $m_1 = 10^6 M_\odot$, $m_2 = 10^{10} M_\odot$, $a_2 = 1$pc, $e_2 = 0.7$, Run time: 1Gyr.

- 14/1000 disrupted; 535/1000 captured. Disruption/capture timescales are short.

=> Captured stars may change stellar density profile of the other BH

=> With rapid diffusion, disruption rate ~10^{-3}/yr.



(Li, et al., in prep)