Dynamics and chaos in astronomy and physics

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Asteroid families inside mean motion resonances

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PLAN

1. **ESTIMATING FAMILY AGES**
2. **THE HILDA REGION: 3/2 RESONANCE**
3. **YARKOVSKY EFFECT IN ORDER 1 RESONANCES**
4. **THE ERODED HILDA FAMILY**
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6. **YARKOVSKY EFFECT IN 1/1 RESONANCE**
7. **THE FOSSIL FAMILIES OF TROJANS**
8. **2/1 AND 9/5 RESONANCE**

1.1 COLLISIONAL HISTORY OF EACH LARGE FAMILY

Our most recent classification is based on synthetic proper elements for $510\,746$ numbered/multi-opposition asteroids; it has 124 families with $121\,750$ members.

Proper elements are quantities changing very slowly, almost only by chaotic diffusion and non-gravitational perturbations. Synthetic proper elements are computed by an accurate numerical integration for $2 \div 10$ My, followed by a fit to a simplified quasiperiodic model.

What is the point of such a large classification? Medium/Large families (> 300 members) allow precise statistical studies. The shape of these families is not just for the eye, it has to be used for least squares fits, giving objective data with uncertainty. Anticipation of the conclusions: we want to compute, with a uniform method, the age of the largest possible set of collisional events in the asteroid main belt.

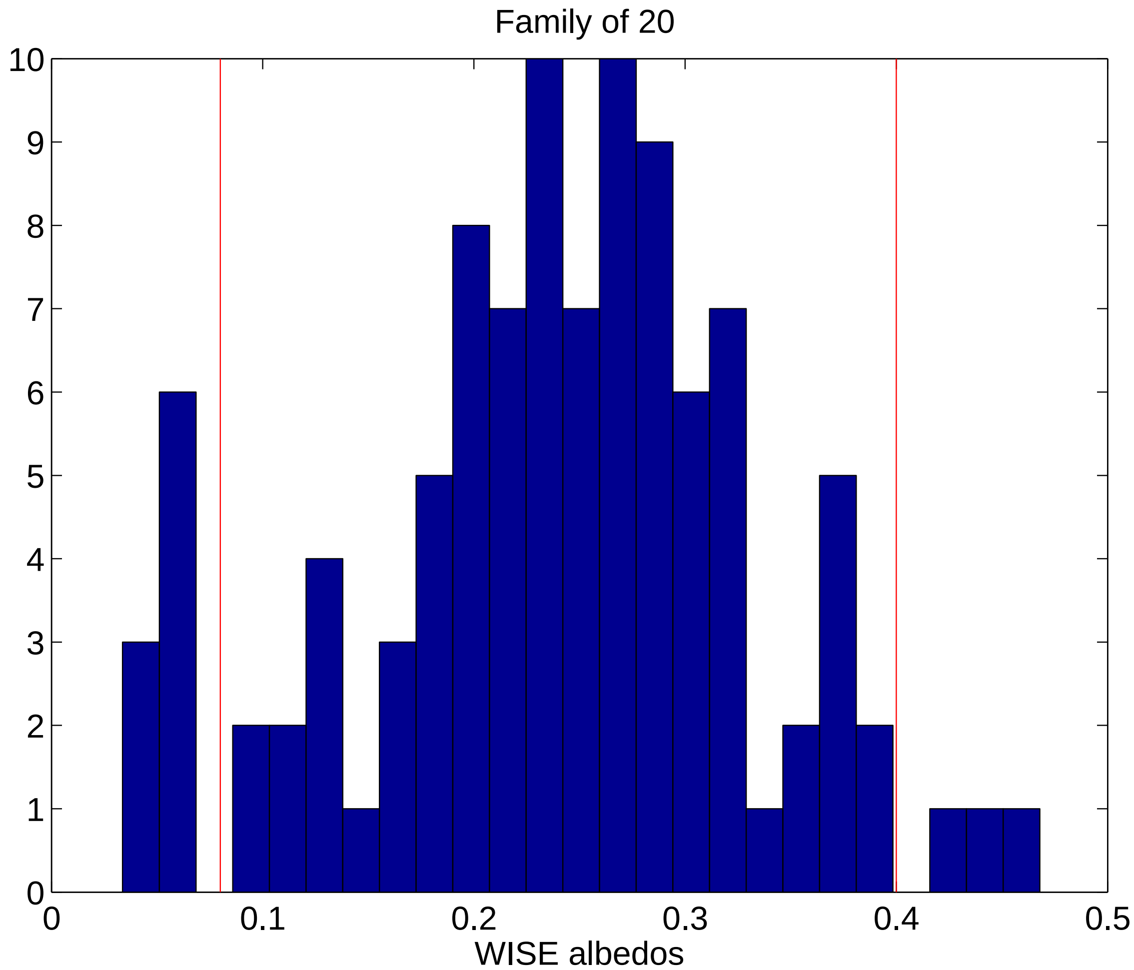
1.2 HOW TO ESTIMATE AGES: BOUNDARIES

With age, families spread in proper a in both directions, until they hit a resonance strong enough to transport family members far enough not to be recognized as members. These boundaries have to be used in family age determination.

number/ name	cause	min proper a	min D IN	cause	max proper a	min D OUT
4 Vesta	7/2	2.25	2.50	3/1	2.50	2.94
15 Eunomia	3/1?	2.52	5.00	8/3	2.71	5.00
20 Massalia	10/3	2.33	1.00	3/1	2.50	0.91
10 Hygiea	11/5	3.07	7.14	2/1	3.25	7.69
31 Euphrosyne	11/5	3.07	6.67	2/1	3.25	6.67
3 Juno	FB?	2.62	2.00	8/3	2.70	2.50
163 Erigone	10/3	2.33	2.50	2/1M	2.42	2.50

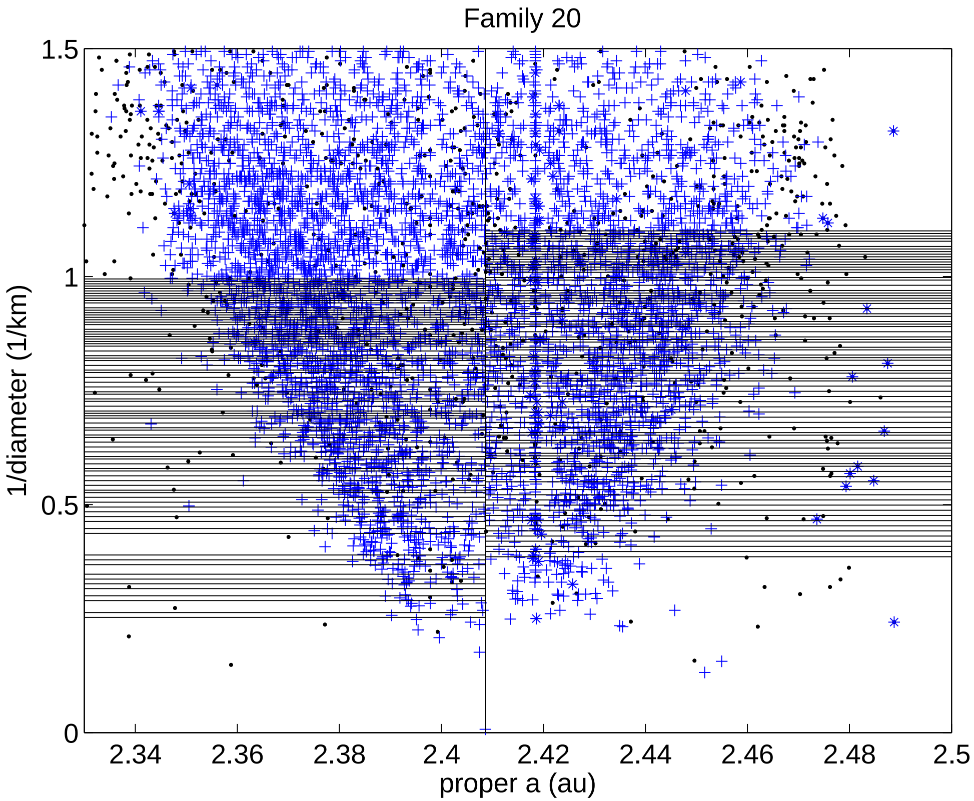
The table shows only the families of the cratering type (fragments $< 12\%$ in volume) for which we have computed ages. Almost all of the boundaries in a are set by resonances: they limit the range in D usable for the age estimate.

1.3 HOW TO ESTIMATE AGES: ALBEDO



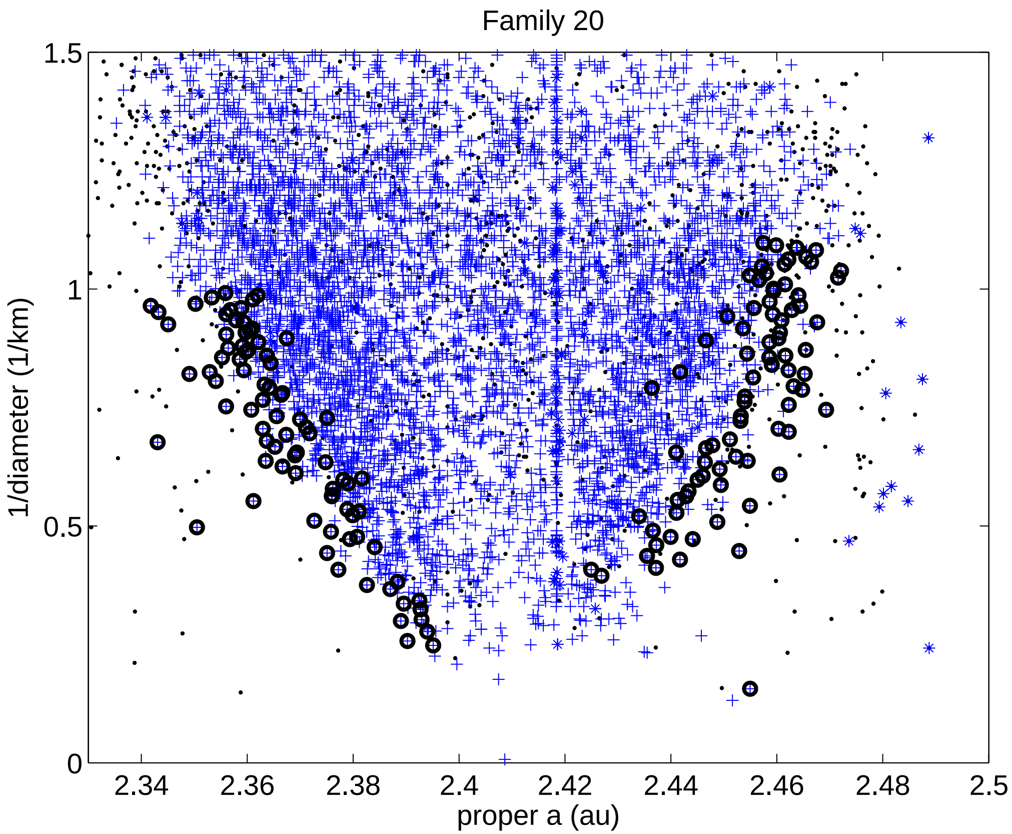
The albedos of the family members for the family of (20) Massalia for which there are WISE albedo data with $S/N > 3$: tail rejection and averaging gives an estimated albedo of 0.25 ± 0.07. The STD measures inhomogeneity in albedo, some of it could be due to measurement error.

1.4 HOW TO ESTIMATE AGES: BINS



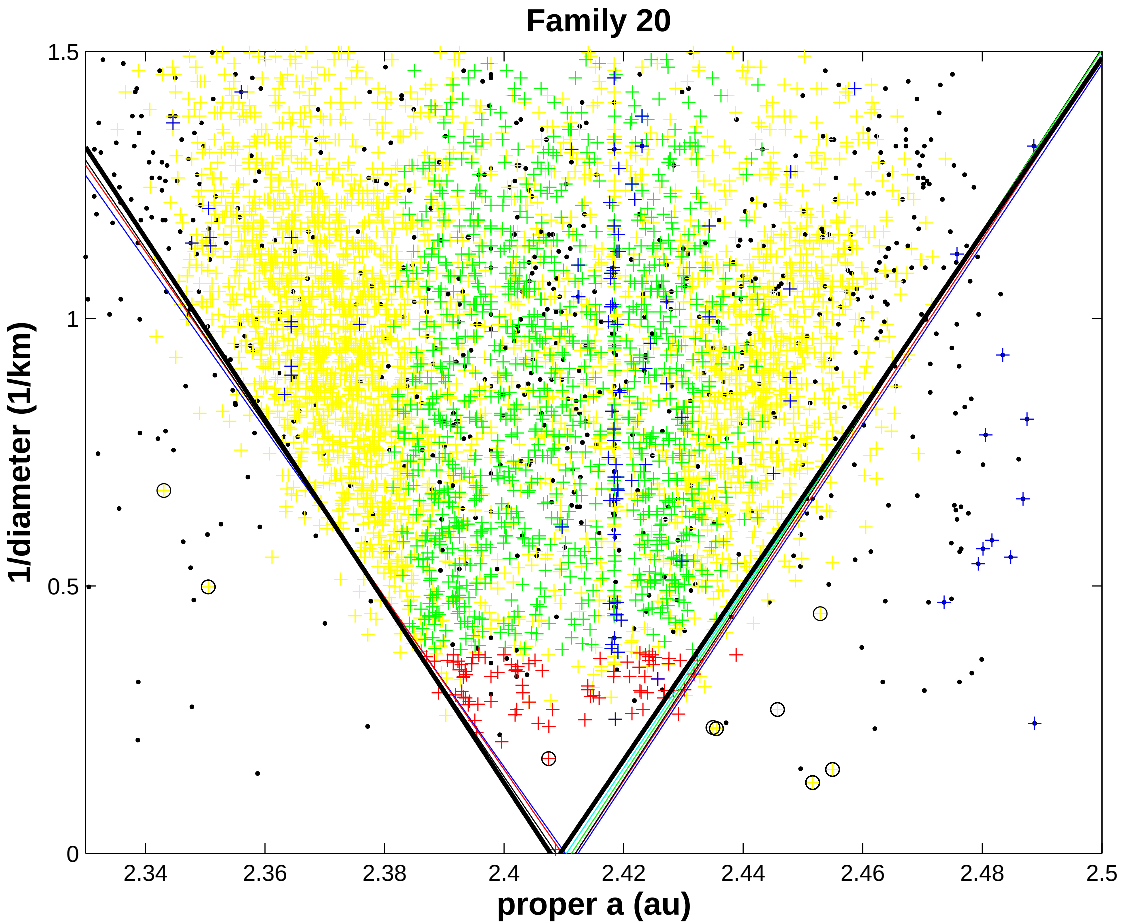
The $1/D$ values are split by bins (separately on IN and OUT sides) in such a way that they contain approximately the same number of family members.

1.5 HOW TO ESTIMATE AGES: MAX AND MIN A



The black marks are the members of the family with minimum proper a in each bin on the left, and maximum proper a in each bin in the right. The family does not have an exact V-shape, not even a smooth boundary, but this is mostly due to the error in the estimate of $1/D$. Thus we assign a STD to each value of $1/D$, from an error model including STD error H and STD variation of albedo (from WISE data).

1.6 HOW TO ESTIMATE AGES: FIT WITH OUTLIER REJECTION



We do not use a lower envelope V-shape, but a V-shape fit to the data, thus some members are below the fit line. Black circles are outliers rejected from the fit. Note the 3 steps in the family assemblage: Red core family Green attached Yellow satellite families; this is essential to get enough range in $1/D$.

1.8 RESULTS: SLOPES

Slopes of the V-shape, computed for cratering families: 7 cases, 5 peculiar results.

number/ name	no. members	side	S	$1/S$	STD $1/S$	ratio	STD ratio
4 Vesta	8620	IN	-2.983	-0.335	0.040		
		OUT	1.504	0.665	0.187	1.98	0.61
15 Eunomia	7476	IN	-1.398	-0.715	0.057		
		OUT	2.464	0.406	0.020	0.57	0.05
20 Massalia	5510	IN	-15.062	-0.066	0.003		
		OUT	14.162	0.071	0.006	1.06	0.10
10 Hygiea	2615	IN	-1.327	-0.754	0.079		
		OUT	1.329	0.752	0.101	1.00	0.17
31 Euphrosyne	1137	IN	-1.338	-0.747	0.096		
		OUT	1.507	0.663	0.081	0.89	0.16
3 Juno	960	IN	-5.261	-0.190	0.038		
		OUT	7.931	0.126	0.049	0.66	0.29
163 Erigone &	429	IN	-7.045	-0.142	0.035		
5026 Martes	380	OUT	6.553	0.153	0.013	1.08	0.28

Vesta: 2 collisional families (Rhea Silvia and Veneneia?), as suspected from shape.

Eunomia: 2 coll.families. Juno: possibly 2 coll. families.

1.9 THE YARKOVSKY CALIBRATION

The V-shape can be interpreted as (essentially) the effect of the Yarkovsky perturbation over the age of the family. Since an inverse slope $1/S$ is expressed in units of Δa for unit $1/D$, then if we can calibrate the Yarkovsky secular perturbation da/dt for a $D = 1$ km asteroid of the same composition as the family (with obliquity either 0° or 180°) we can compute the age since the formation of the family:

$$\Delta t = \frac{\Delta a}{da/dt|_{D=1}}$$

The uncertainty of the slope, computed from the fit covariance matrix, can be easily propagated to the estimated age.

The Yarkovsky calibration is obtained by scaling from the best measured such effect, the one on (101955) Bennu, from which a porosity decreasing the density with respect to the one of (704) Interamnia can be derived. If the same porosity of Bennu is assumed for $D = 1$ km asteroids with different composition, a density can be estimated from the one of a similar large asteroid, e.g., Vesta for V, Hygiea for C, Eunomia for S.

Thus beside the uncertainty of $1/S$ in the estimated age there is another error term, due to the uncertainty of the calibration. These have been estimated to have a relative STD between 0.2 and 0.3, depending upon the amount of information available on the taxonomic type of the family members.

1.10 AGE ESTIMATES

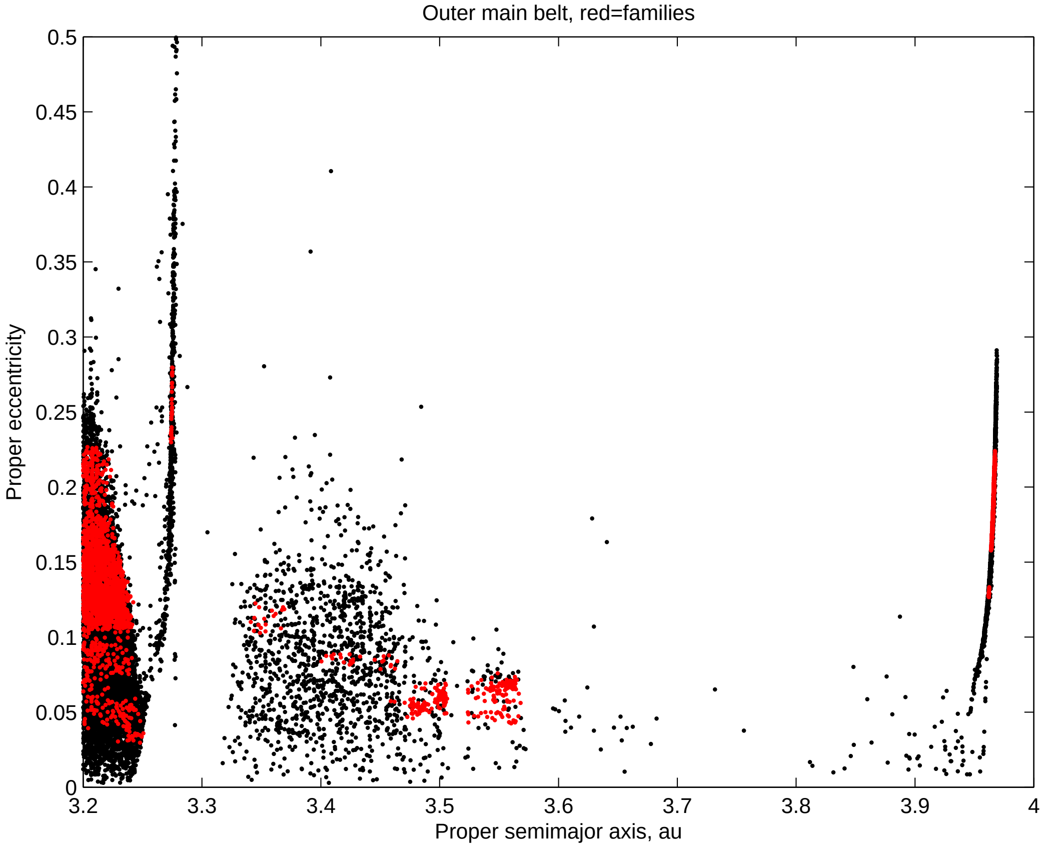
Ages for cratering families: 8 dynamical families have $9 \div 10$ ages.

number/ name	side IN/OUT	da/dt $10^{-10} au/y$	Age My	STD(fit) My	STD(cal) My	STD(age) My
4 Vesta	IN	-3.60	930	112	186	217
	OUT	3.49	1906	537	381	659
15 Eunomia	IN	-3.66	1955	155	391	421
	OUT	3.55	1144	57	229	236
20 Massalia	IN	-3.81	174	7	35	35
	OUT	3.73	189	16	38	41
10 Hygiea	IN	-5.67	1330	139	266	300
	OUT	5.50	1368	183	274	329
31 Euphrosyne	IN	-5.71	1309	169	262	312
	OUT	5.72	1160	142	232	272
3 Juno	IN	-3.46	550	110	110	156
	OUT	3.41	370	143	74	161
163 Erigone &	IN	-6.68	212	53	42	68
5026 Martes	OUT	6.64	230	46	19	50

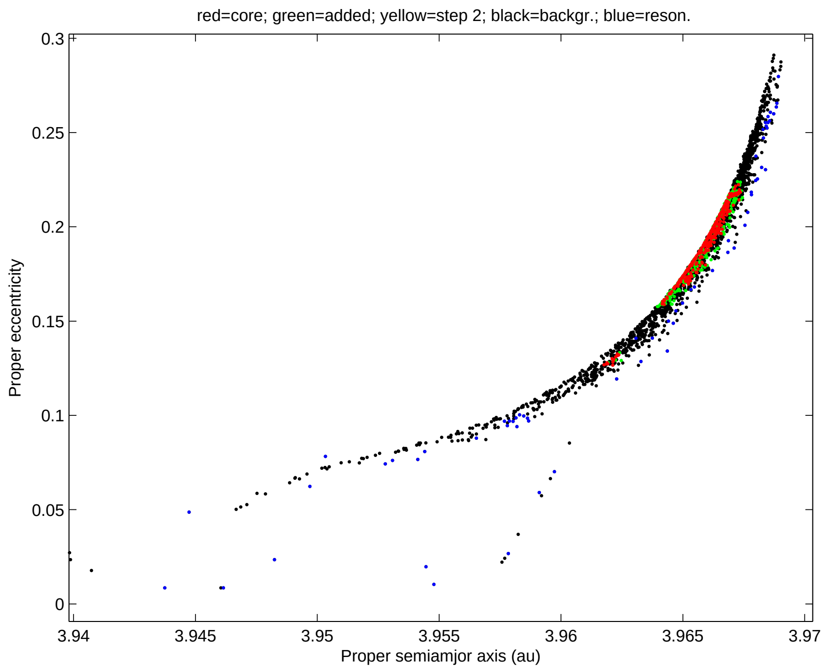
Ages are estimated for families in the range $150 < \Delta t < 2000$ million years.

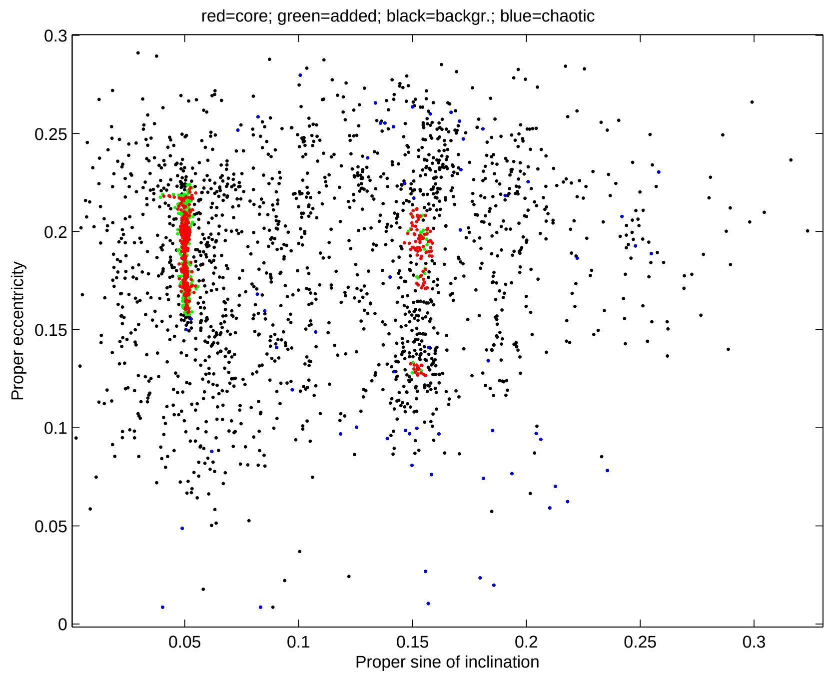
2.1 OUTER MAIN BELT: DEPLETION AND STABLE ISLANDS

Here we focus on the Outermost Main Belt, with $3.26 <$ proper a < 4 au (family members in red), mostly depleted in population, with family of (87) Sylvia split by $9/5$. Two gaps opened by the $2/1$ and the $3/2$ mean motion resonance with Jupiter. The stable regions are named after (1362) Griqua and (153) Hilda.



2.2 THE HILDA GROUP: 3/2 RESONANCE

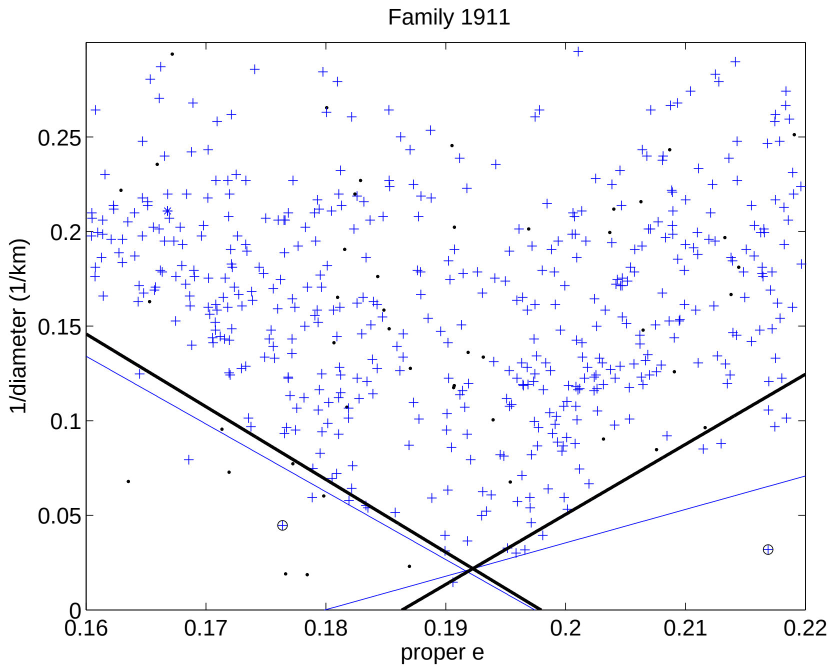


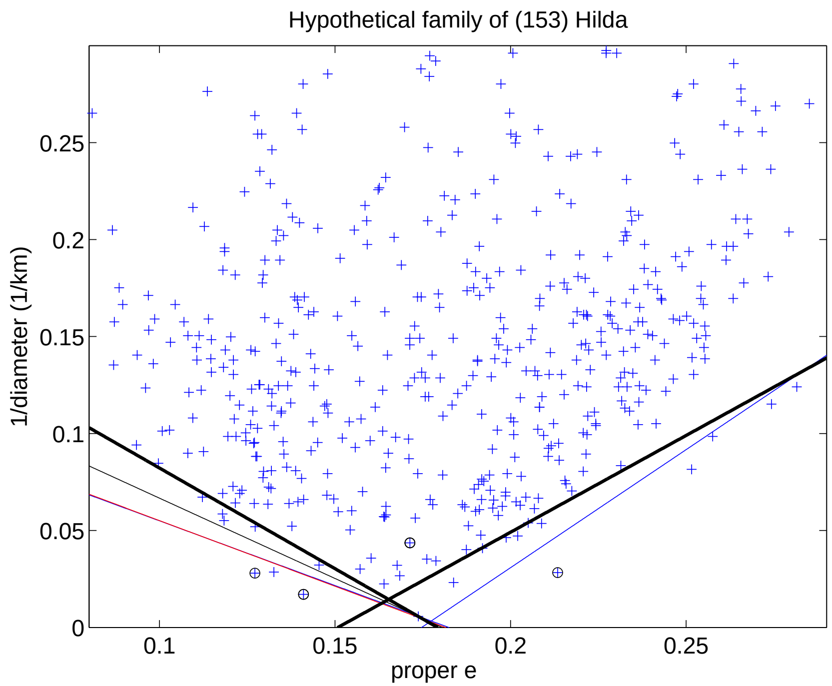


The Hilda group is an island stable for Gy, inside the resonance 3/2 with Jupiter The semi-proper a is obtained by digital filtering and averaging, not accounting for the resonance, and has a narrow range of values (left). This does not affect the family classification: the maximum amplitude of libration $\Delta a = 0.063$ au gives a contribution to distance 35 times smaller than the spread in proper e and $\sin I$.

The concentrations in the $(e, \sin i)$ plot (right) are the family of (1911) Schubart (508 members) on the left and the eroded family of (153) Hilda near the center. The different density is due to a different collisional and Yarkovsky history.

3.1 THE SHAPE OF RESONANT FAMILIES



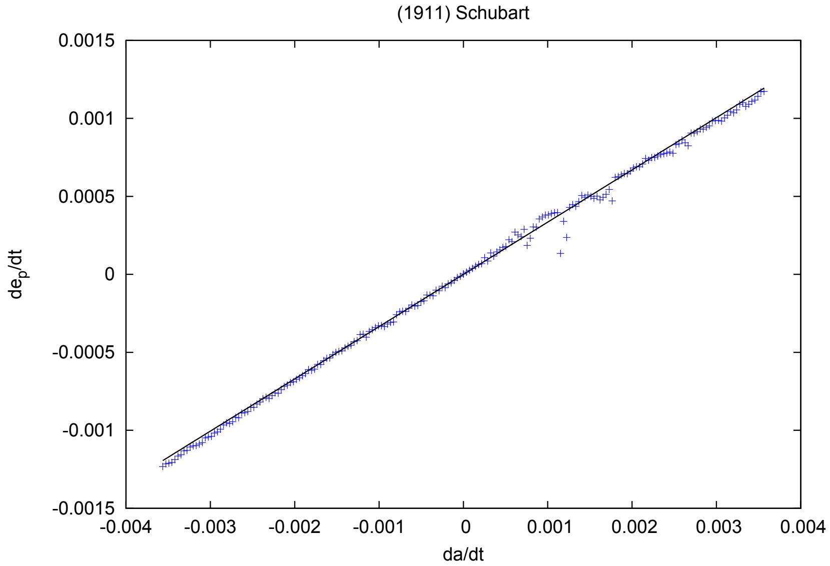


In the plane $(1/D, e)$ (inverse diameter, proper eccentricity) the families in order 1 resonances have a characteristic V-shape. Left: family Schubart; right: enlarged family Hilda, 484 asteroids in $0.075 < e < 0.29$; $0.14 < \sin I < 0.17$.

The cause is mostly non-gravitational: $1/D$ is proportional to Area/Mass. If the 3/2 resonance converts a secular $da/dt(NR)$ due to Yarkovsky into a secular de/dt, then the inverse slope of the V-shape two sides measure the age of the family.

The inverse slopes are $\Delta e(D=1)$ (1/km), for Schubart -0.26 ± 0.03 low e side, 0.27 ± 0.02 high e, for Hilda -0.96 ± 0.08 low e, 1.00 ± 0.15 high e.

3.2 YARKOVSKY AND RESONANCE, NUMERIC METHOD



Numerical method: assign a transverse force such that the secular change in proper a is $da/dt(NR)$, for 200 values in a realistic range of Yarkovsky effects

Propagate for 10 My, then compute synthetic proper elements for 9 intervals of 2 My each, fit a line to the points $(da/dt(NR), de/dt)$ of the 200 test points.

Above: for Schubart the ratio is $de/dt = 0.335 \cdot da/dt(NR)$, using calibration values $de/dt(D=1) = -1.68 \times 10^{-4}$ low e, 1.72×10^{-4} high e in 1/My, ages estimated are 1550 ± 490, 1570 ± 480 in My.

3.3 YARKOVSKY AND RESONANCE, ANALYTIC

Analytical argument: the main term of the 3/2 resonance contains the critical angle $\phi = 2\lambda - 3\lambda_J + \varpi$, appearing in a term $O(e)\ \cos\phi$ where the coefficient is also a function of a, giving a fundamental model of resonance in the (ϕ, e) plane.

As a is forced to change slowly, the resonant orbits evolve according to adiabatic invariant theory (Henrard and Lemaitre), with a changing guiding trajectory, in particular with libration center moving along a curve in the (a, e) plane, where a is semi-proper. Thus a secular drift de/dt is predicted by a well known theory.

Anyone volunteering for attempting to use adiabatic invariant theory to recompute the Yarkovsky calibration? This could give a strong confirmation. The numeric (or synthetic) method is probably more accurate, but could not be invented without the analytic understanding.

4.1 THE PRIMORDIAL FAMILY of (153) HILDA

For Hilda the calibration of the effects in proper e is $de/dt = 0.370 \cdot da/dt(NR)$, then combining with the calibration of $da/dt(NR)$ we get $de/dt(D=1) = -1.83 \times 10^{-4}$, 1.99×10^{-4} for low e, high e, respectively.

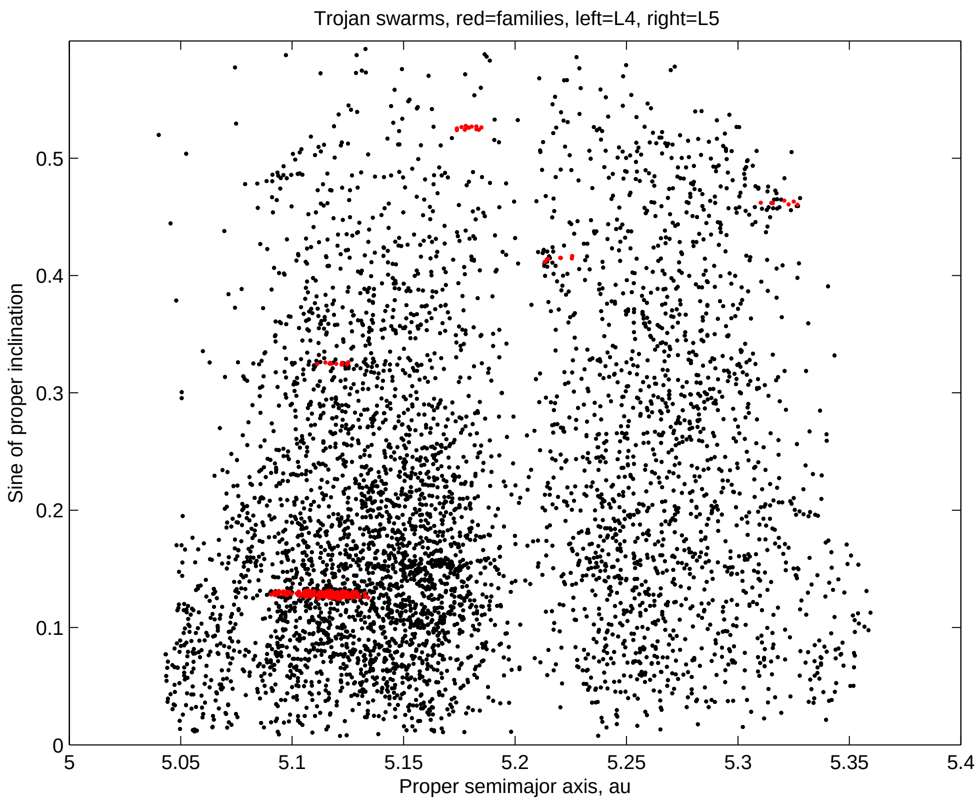
By using the slopes given before we get the estimates ages 5270 ± 1650 for the low e side, 5040 ± 1680 for the high e side, both in My. Thus the age is > 3620 My, from the low e result.

The slope of the size distribution of the family members is $N(D) \sim 1/D^{-1.75}$, that is, extremely shallow. From the V-shape the family has been eroded by loosing most members with $D < 10$ km on the low e side, with $D < 7$ km on the high e side.

Anyway the important result is that the super-Hilda family, if it exists, is Primordial (formed before Late Heavvy Bombardement). Most families that old cannot be detected in proper elements data, or maybe appear as “shadows” which cannot be confirmed by statistical significance, because they have been eroded by loosing most small members, thus the density contrast with the background has become not significant.

5.1 THE TROJAN REGION: 1/1 RESONANCE

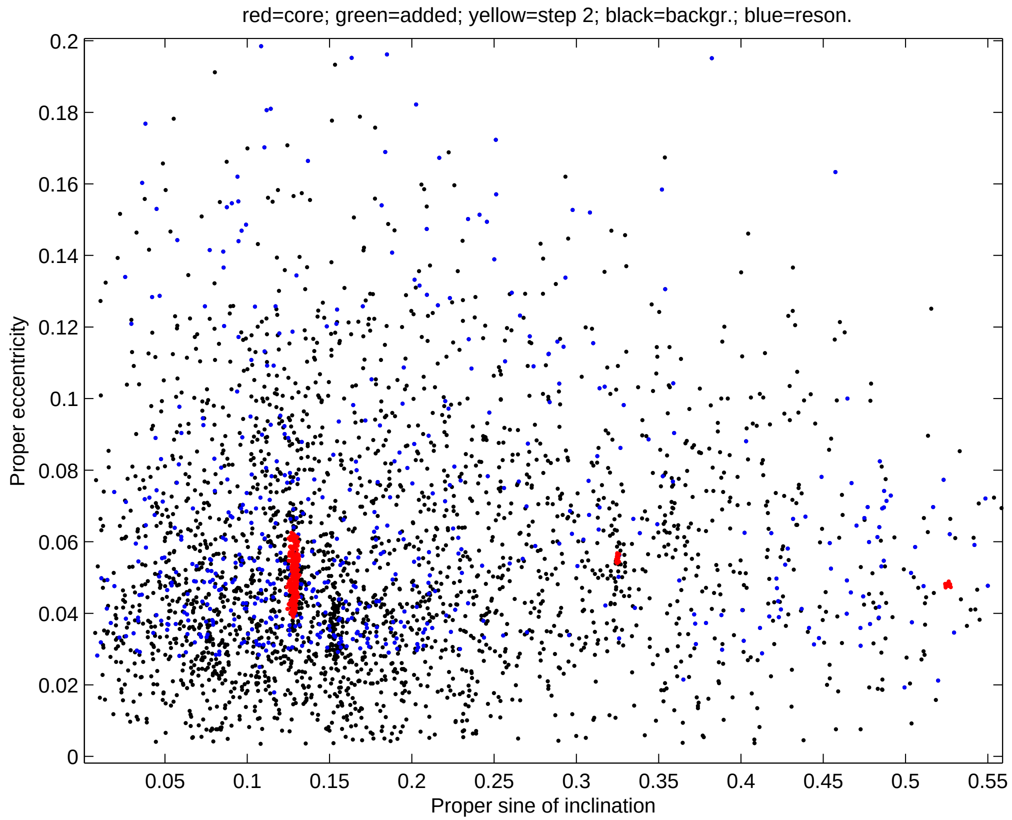
Inside the resonance $1/1$ with Jupiter (argument $\phi = \lambda - \lambda_J$) there are two large islands of stability, with ϕ librating around $+60^\circ$ (L4) and around -60° (L5). This libration affects also the semi-major axis, thus we can compute an amplitude of libration (both in ϕ and is $a - a_j$). Proper elements according to the synthetic theory of Milani (1993) are now available for $6,020$ Trojans ($3,357$ in L4, $1,663$ in L5).



L4 on the left, L5 on the right. Max $\Delta a = 0.163$, larger contrib. than for Hildas.

5.2 THE FAMILIES IN L4

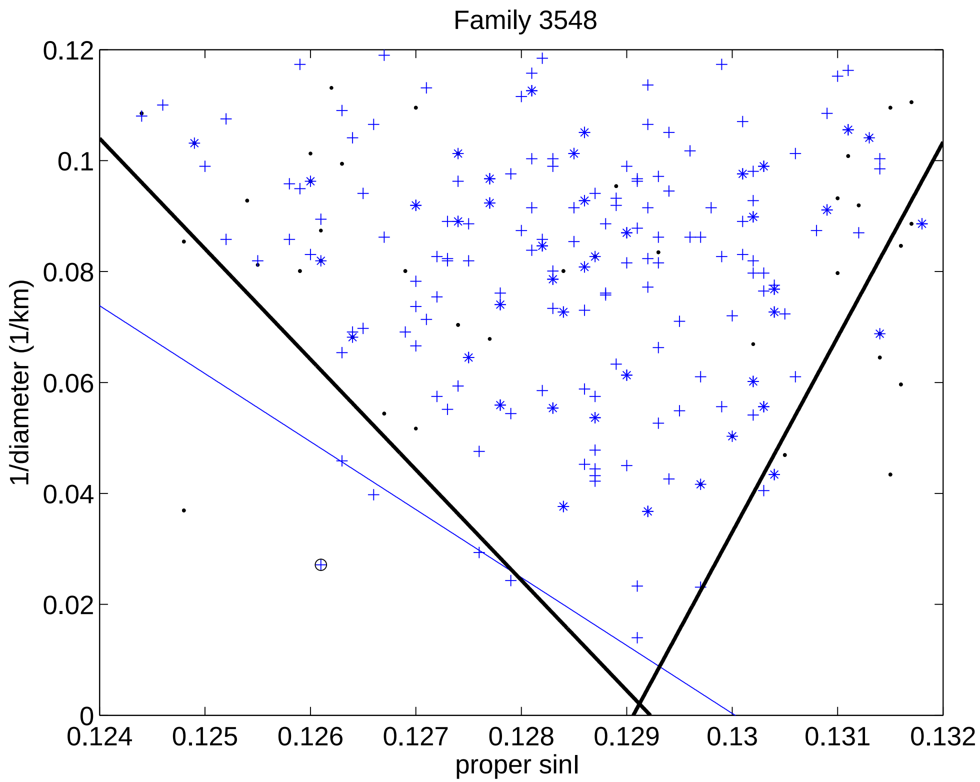
Then we can compute the same distance by using the amplitude Δa in place of a; L4 and L5 have to be handled separately. With a standard HCM procedure, in L4 we find 5 families (red). The largest one (172 members) has as lowest numbered (3548) Eurybates; two small are satellites of 3548.



The two others, of (624) Hektor and (9799) 1996 RJ have large inclination and are very compact.

6.1 V-SHAPES AND YARKOVSKY FOR TROJANS

What about Yarkovsky effect on Trojans? Is the shape of the Trojan familes evidence for Yarkovsky-driven dispersion? The effect of Yarkovsy on Δa is very small, to the point that the secular $d\Delta a/dt$ is smaller by a factor 10^{-5} than $da/dt(NR)$ (within the numerical error from 0). This small "resonance transmission factor" is qualitatively like the one in the 3/2, but $> 1,000$ times smaller.



Thus we try drawing the family in the planes $(e, 1/D)$ and $(\sin I, 1/D)$: the most interesting is the latter, showing a bilateral V-shape (the other appears one-sided).

6.2 YARKOVSKY IS TOO SLOW FOR TROJANS

Given the slopes of the two sides of the V-shape, we need the calibration, that is to compute the ratio between the secular Yarkovsky effect on a (without the resonance) and the one on e inside the resonance: with the same procedure used for Hildas we find

$$\frac{de}{dt} = 3 \times 10^{-3} \frac{da}{dt}(NR) \;\; ; \;\; \frac{d\sin I}{dt} = 4.6 \times 10^{-3} \frac{da}{dt}(NR)$$

E.g., in $(\sin I, 1/D)$, the inverse slope $1/S = -0.028 \pm 0.03$ for high I, the Yarkovsky calibration out of resonance $da/dt(NR) = 4.3 \times 10^{-4}$ in 1/My we get an age

$$\Delta t = \frac{0.28}{4.6 \times 10^{-3} \times 4.3 \times 10^{-4}} = 14,200 \, My$$

more than the current estimate of the age of the universe. Values by using either the low I slope of the e slope are even larger.

This is the computation, but what is the astrophysical interpretation of such an absurd result?

6.3 WHY YARKOVSKY IS INEFFECTIVE FOR TROJANS?

Analytical argument: the main term of the 1/1 resonance contains the critical angle $\phi = \lambda - \lambda_J$, appearing in a term $c(a,e,I)\,\cos\phi$ where the coefficient, by D'Alembert third rule, has a lowest order part containing neither e nor I. The lowest order terms containing eccentricity and inclination are either $O(e^2)$ or $O(\sin I\,\sin I_S)$.

Thus the secular effect of Yarkovsky is decreased by a factor $e^2 = 0.0437^2 \simeq 2 \times 10^{-3}$ for (3548), while for (1911) the factor is $e = 0.191$: the effect among Trojans is about $1/100$ of the one for Hildas. This is only an order of magnitude estimate, but the point is that there is no surprise if the Yarkovsky calibrations are smaller by two order of magnitudes with respect to the ones of the Hildas.

What is the meaning of the V-shape in $(\sin I, 1/D)$ of (3548)? It cannot be explained by Yarkovsky, thus it must be originary, due mostly to the dispersion of V_∞ (after escaping from the gravitation of the parent body) of the fragments ejected. Thus the families found among Trojans are fossil, essentially frozen at the original shape and size in proper elements space.

7.1 THE FOSSIL FAMILIES OF TROJANS: EURYBATES

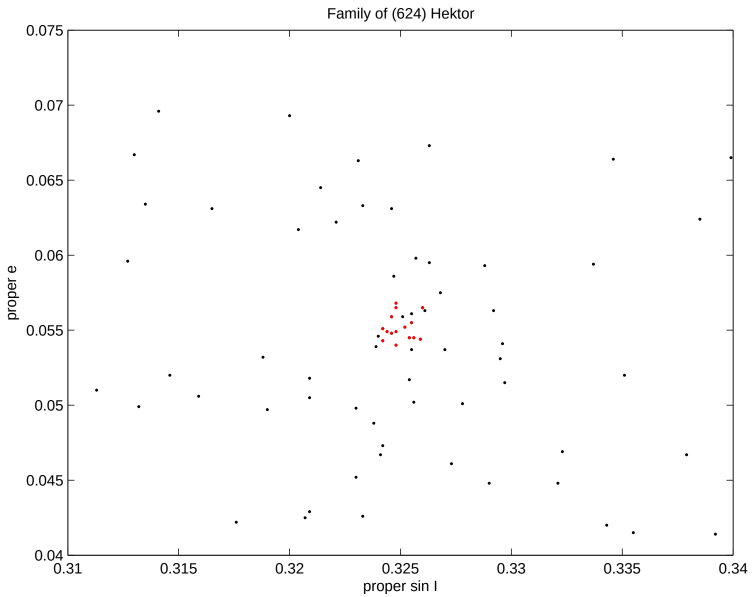
The standard deviations of the proper elements of the family of (3548) Eurybates are $\sigma(e) = 0.0048$, $\sigma(\sin I) = 0.0015$ and $\sigma(\Delta a) = 0.0107$ au, corresponding to relative velocities V_∞ of 62, 20 and 13 m/s, respectively.

The parent body can be estimated from the family volume to have had $D = 93$ km, the escape velocity can be roughly estimated (with very poor density data) at about 100 m/s. Thus all the dispersion of the proper elements could be the result of the original dispersion; of course also chaotic diffusion can have contributed. In this way what we get from the dispersion of the proper elements is not an age but an upper limit for the orginal field of velocity V_∞.

In the main belt, we have (almost always) interpreted the $1/S$ of the V-shapes as due only to the Yarkovsky effect accumulated over the family age. The original velocity ditribution can contribute some slope, because smaller fragments are ejected at higher speed, but for old families (age > 100 My) this effect is much smaller than Yarkovsky. Only young families (with age < 100 My) can show a mixture of the two effects, and indeed for the family of (1547) Nele, the age estimated from the V-shape (14 ± 5 My) can be significantly contaminated by the original velocities.

7.2 THE FOSSIL FAMILIES OF TROJANS: HIGH INCLINATION FAMILIES

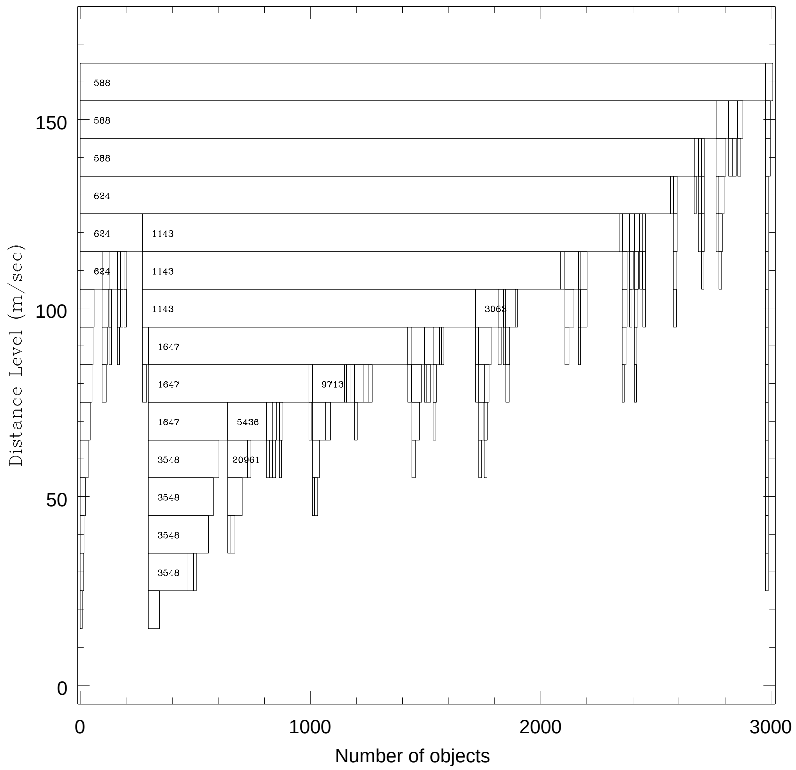
The families of (624) Hektor and of (9799) 1996 RJ, with 15 objects and 13, respectively, correspond to two extremely compact groupings in L4, at high inclination ($\sin(I) \simeq 0.33$ for 624, $\sin(I) \simeq 0.53$ for 9799).



For Hektor, the RMS of proper elements differences, with respect to those of the parent body (624), are $\sigma(\Delta a) = 0.0042$ au, $\sigma(e) = 0.0009$, $\sigma(\sin I) = 0.0006$, corresponding to V_∞ of $5, 11$ and 8 m/s respectively. The escape velocity from (624) Hektor is $\simeq 130$ m/s (size $370 \times 195 \times 195$ km).

7.3 THE FOSSIL FAMILY OF (624) HEKTOR

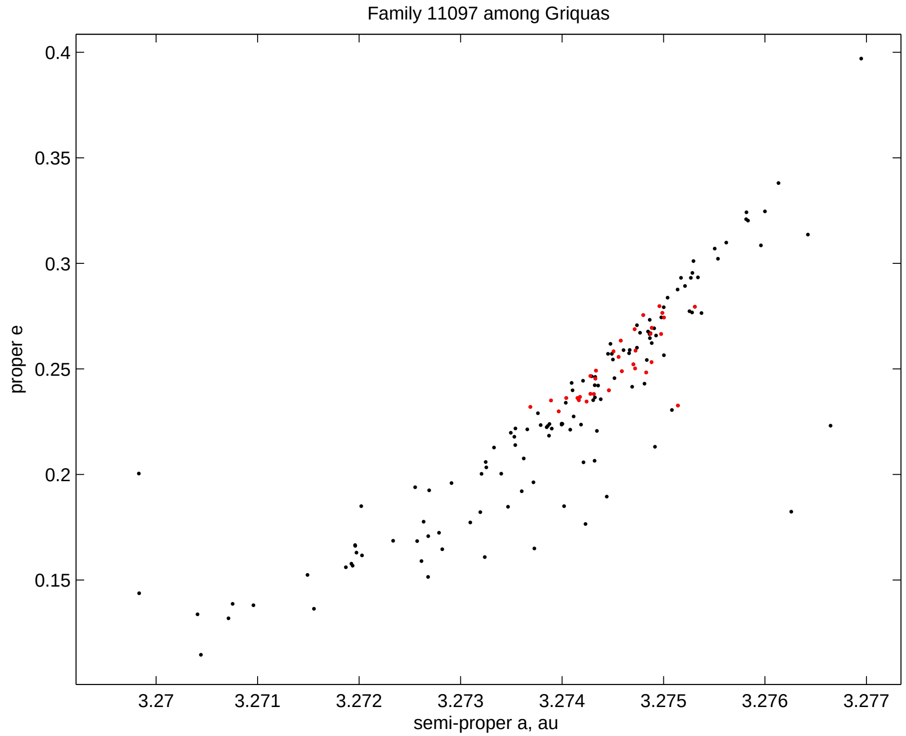
(624) has a satellite (in an orbit with high I and e) and is a contact binary. 624 is a cratering family with launch velocities just above the escape velocity.



The two high inclination families in L4 are deep and well isolated stalactite branches evident at the left (624) and rigth (9799) ends of the stalactite diagram above; families are identified at a distance $d_{max} = 30$ m/s. The stalactite of 624 is separate up to $d_{max} = 110$ m/s, at which level the family would contain 95 members.

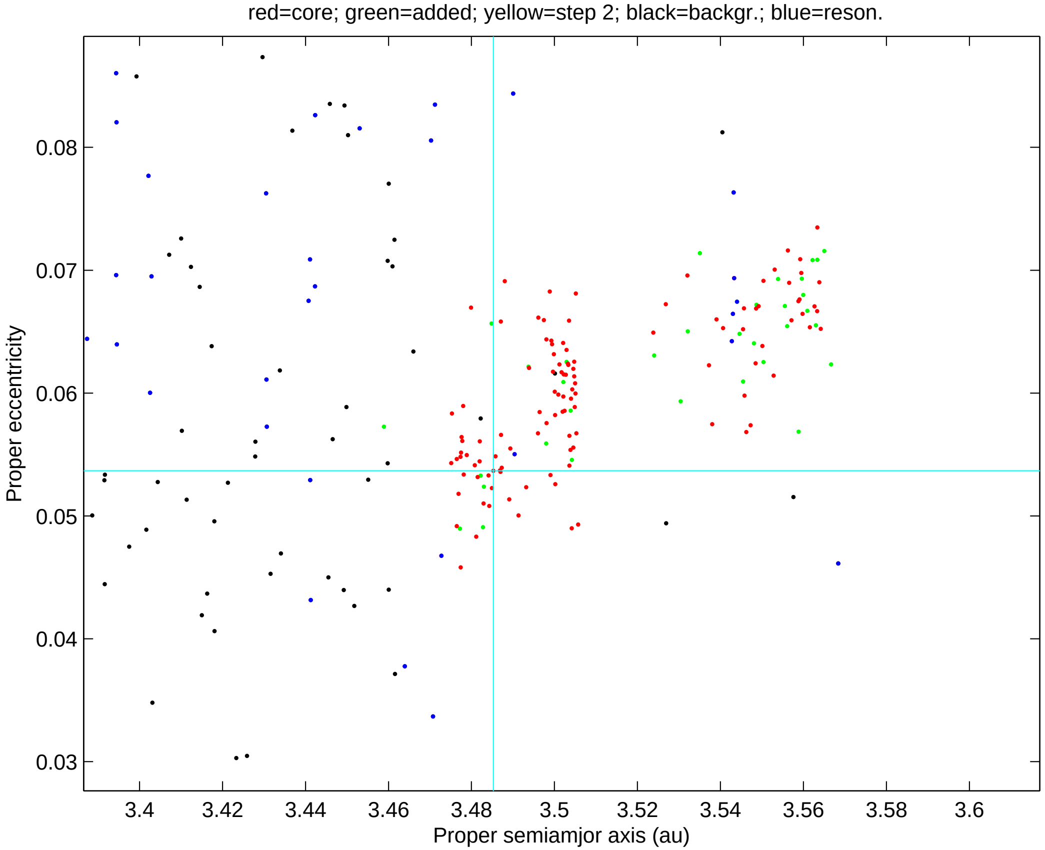
8.1 THE 2/1 RESONANCE: A SMALL FAMILY

Just to be complete: the same argument about the possibility to use a semi-proper a (filtered, averaged) applies inside the $2/1$ J resonance as well as in the $3/2$ resonance. There are only 649 asteroids with $3.26 <$ semi-proper $a < 3.28$, By using an HCM adapted to this low density region we find only one family, of (11097) 1994 UD1, with 33 members.



The clustering found by HCM does not fill the ostensibly stable region, populated by Griquas. Thus it can be a real family (as for (24) Phocea, see Knežević's talk).

8.2 THE FAMILY OF (87) SYLVIA



The family of (87) Sylvia appears complex in shape, but it can be interpreted as a single jet cut by a resonance. Indeed, at proper $a = 3.516$ au there is the $9/5$ mean motion resonance with Jupiter. No family asteroids in $9/5$, but few others.

This family (with 179 members) is not yet large enough for the age to be accurately estimated by our V-shape method.