Quantum comets

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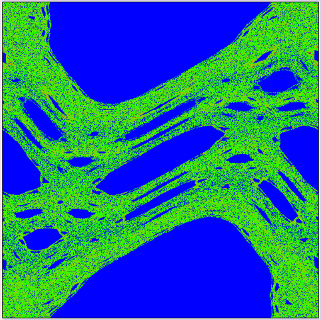


- (1969-79) Chirikov standard map:
  $\bar{p} = p + K \sin x$, $\bar{x} = x + \bar{p}$ (K=1.1)
- (1979) Quantum map (kicked rotator):
  $\bar{\psi} = e^{-i\hat{p}^2/2\hbar} e^{-iK/\hbar \cos \hat{x}} \psi$ (Chirikov group 1981-1987): Anderson or dynamical localization
- (1974) Microwave ionization of hydrogen/Rydberg atoms (Bayfield-Koch experiment, Yale), quantum localization of chaos: theory (1983-1990), experiment Koch, Bayfield, Walther (1988-91)
- (1986-90) Kepler map, Halley comet: Petrosky, Chirikov-Vecheslavov, DS, Shevchenko
- (2009-16) Dark matter capture: Khriplovich, DS, Lages, Rollin + Heggie (1975)

Microwave ionization of hydrogen/Rydberg atoms

Bayfield, Koch PRL (1974) - experiments at Yale:

Hydrogen principle quantum number $n_0 \approx 66$, microwave $\omega/2\pi = 9.9GHz$, field amplitude $\epsilon \approx 10V/cm$ being smaller than static ionization border $\epsilon_{st} \approx 30V/cm$; $N_I \approx 76$ photons are required for atom ionization

Hamiltonian (in atomic units):

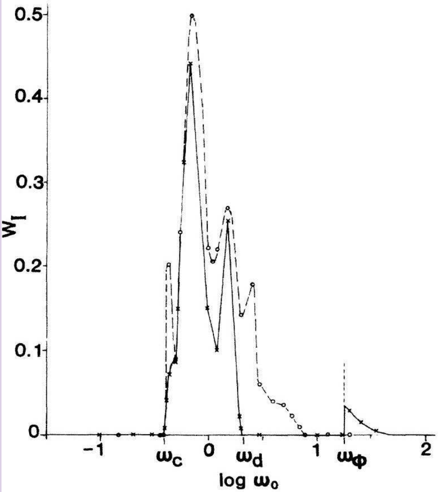
$H(p, r) = p^2/2 - 1/|r| - \epsilon r \cos \omega t$

Classical description/scaling :

$\omega_0 = \omega {n_0}^3 \approx 0.43$,

$\epsilon_0 = \epsilon {n_0}^4 \approx 0.03 < 0.13$

Right (1986): Ionization probability as a function of ω_0 (numerics: dashed - classical; full - quantum)



History of the problem: DS Scholarpedia (2012)

Kepler map

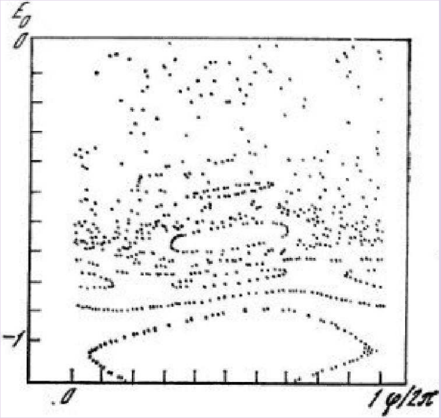
variation of energy and phase on one orbital period

Classical hydrogen atom in 1d (1983 - 1987)

$\bar{N} = N + k \sin \phi$
$\bar{\phi} = \phi + 2\pi\omega(-2\omega\bar{N})^{-3/2}$

$N = -1/2\omega n^2 = E/\hbar\omega$ is photon number, $\phi = \omega t$ at perihelion;

valid for distance at perihelion $q = l^2/2 < (1/\omega)^{2/3}$
linearization of equation for phase near resonant values $\bar{\phi} - \phi = 2\pi m$ gives $\bar{\phi} = \phi + T\bar{N}$; $T = 6\pi\omega^2 {n_0}^5$
Chirikov standard map with $K = kT = \epsilon_0/\epsilon_c$; chaotic, diffisive ionization for $\epsilon_0 > \epsilon_c = 1/(49{\omega_0}^{1/3})$; diffusion rate $D = k^2/2$



"Kepler map" term coined in Phys. Rev. A **36**, 3501 (1987)

Quantum Kepler map and photonic localization

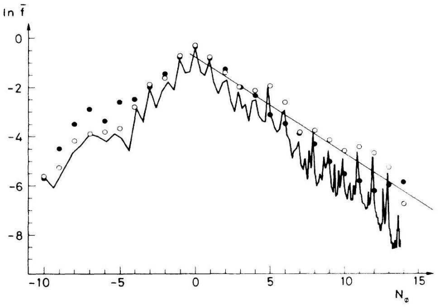
Classical hydrogen atom in 1d (1983 - 1987)

Operator commutator $[\hat{N}, \hat{\phi}] = -i$ in
$\bar{N} = N + k \sin \phi$,
$\bar{\phi} = \phi + 2\pi\omega(-2\omega\bar{N})^{-3/2}$
or $\bar{\psi} = e^{-i\hat{H}_0} \hat{P} e^{-ik \cos \hat{\phi}} \psi$
$\hat{H}_0 = 2\pi[-2\omega(N_0 + \hat{N}_\phi)]^{-1/2}$,
$N_0 = -1/(2\omega {n_0}^2) = -N_I$,
$\hat{N}_\phi = -i\partial/\partial\phi$.

quantum localization of diffusion (like Anderson localization (1958) in disordered solids)

$\ell_\phi = D = k^2/2 = 3.33\epsilon^2/\omega^{10/3}$
$f_N \propto \exp(-2|N - N_0|/\ell_\phi)$

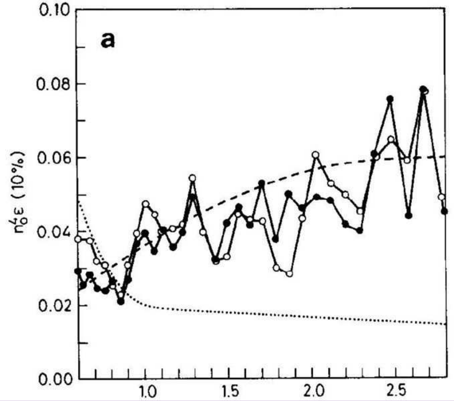
Right: $n_0 = 100$, $\epsilon_0 = 0.04$, $\omega_0 = 3$ (open circles - 1d Schrodinger eq., black circles - the quantum Kepler map, straight line - theory)



Delocalization transition

$\ell_\phi > N_I = 1/(2\omega n_0{}^2) = n_0/2\omega_0$
or
$\epsilon_0 > \epsilon_q = \omega_0^{7/6}/(6.6 n_0)^{1/2} = 0.4\omega^{1/6}\omega_0$

Right: ionization threshold ϵ_0 vs ω_0 for Koch (1988) experiment at 36GHz (open circles), $45 \leq n_0 \leq 80$, $n_I = 90$; quantum Kepler map (full circles); dashed/dotted curve - quantum/classical Kepler map theory; interaction time 100 microwave periods (no fit parameters).



Physica A **163**, 205 (1990)
1d Kepler map gives a good description of real ionization of 3d atom

Kepler map for comets

Petrosky Phys. Lett. A (1986)

a planet on a 2d circular orbit (radius $r_p = 1$, planet velocity $v_p = 1$) around a star at mass ratio $\mu = m_p/M$, comet perihelion distance $q \gg r_p$

Comet dynamics is described by the Kepler map

$\bar{w} = w + F \sin x$, $\bar{x} = x + w^{-3/2}$

$w = v^2$ is comet rescaled energy; x is planet phase divided by 2π

$F \approx 2\mu q^{-1/4} \exp(-0.94 q^{3/2})$

Petrosky (1986); Chirikov-Vecheslavov (BINP 1986) - (A&A 1989)

kick function from 46 times at perehelion for Halley comet

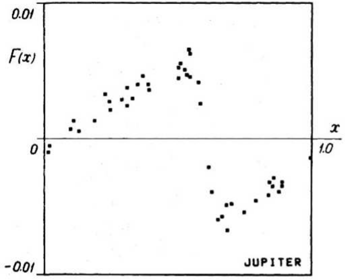


Fig. 1. The full perturbation of comet Halley vs. Jupiter's phase

F-kick function for Halley comet from Chirikov-Vecheslavov: diffusive ionization in time $t_I \sim T_J(2/F^2) \sim 10^7$ *years*

Chaotic Halley comet

Chirikov-Vecheslavov (1986-1989)

Comet dynamics is described by the Halley (modified Kepler) map

$\bar{w} = w + F(x)$, $\bar{x} = x + w^{-3/2}$

Main contribution from Jupiter, Saturn
Chaotic diffusion, average ionization time is approximately 10^7 years

More about kick function: Rollin, Haag, Lages Phys. Let. A **379**, 1017 (2015)

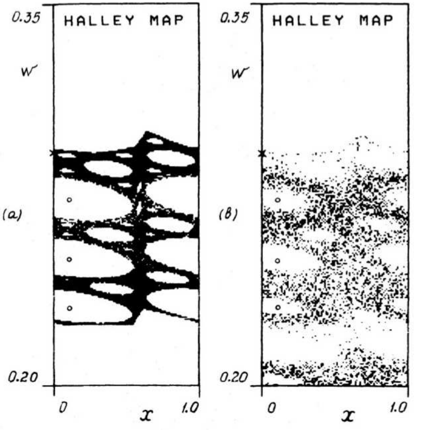


Fig. 3a and b. Phase trajectory of map (3) in the STA (6). Initial conditions (crosses) $w_1 = 0.29164$; $x_1 = 0$ (in 1986, see Table 1): **a** Jupiter's perturbation only, $N = 1.5\ 10^5$ iterations; **b** perturbation by both Jupiter and Saturn, $N = 4000$

Chaotic autoionization of molecular Rydberg states

Rydberg electron interaction with charged rotation core
rotating dipole + Coulomb interaction (atomic units)

$$H = ({p_x}^2 + {p_y}^2)/2 - 1/r + d(x\cos\omega t + y\sin\omega t)/r^3$$

that is approximately

$$H = ({p_x}^2 + {p_y}^2)/2 - [(x + d\cos\omega t)^2 + (y + d\sin\omega t)^2]^{-1/2}$$

Exact Kramers-Henneberger transformation gives Hamiltonian of excited hydrogen atom in a circular polarized microwave field with effective $\epsilon = d\omega^2$

$$H = ({p_x}^2 + {p_y}^2)/2 - 1/r - \omega m + d\omega^2 r\cos\psi$$

where ψ conjugated to momentum m is the polar angle between direction to electorn and field direction in the rotating frame.

Conditions of applicability:

$d < a_{core} < q = r_{min} = l^2/2 < r_\omega = 1/\omega^{2/3}$;

$r_\omega >> a_{core}$ (core size) for $\omega \ll 1/a_{core}^{3/2}$

Phys. Rev. Lett. **72**, 1818 (1994)

Kepler map for rotating dipole

$\bar{N} = N + k \sin \phi$,
$\bar{\phi} = \phi + 2\pi\omega(-2\omega\bar{N})^{-3/2}$

$k \approx 2.6 d\omega^{1/3}[1 + l^2/2n^2 + 1.09 l\omega^{1/3}]$
Chaotic diffusion, average ionization time is approximately
$t_I \approx N_I^2/D \approx 2/[(2n_0\omega^2)k^2]$
$D = k^2/2$

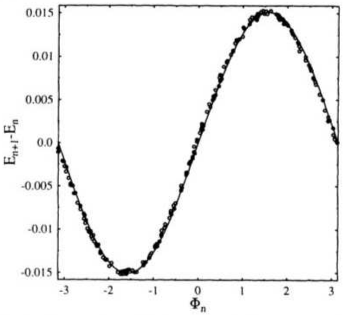
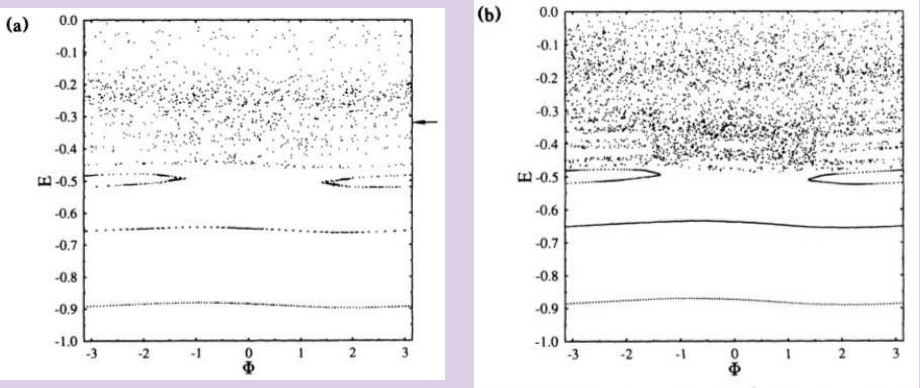


FIG. 1. Comparison of the numerically computed values $\Delta N = \bar{N} - N$ (dots) obtained by solving the system (3) for $dn_s^{-2} = 0.000625$, $\omega n_s^3 = 4$, $l/n_s = 0.3$, $n_0/n_s = 1.25$ and the theoretical curve $k \sin\Phi$ (full curve), with the value of k taken from (6). The value n_s fixes the classical scale.

The map is approximate since the orbital momentum is only approximately concerved (e.g. Dvorak, Kribbel A&A **227**, 264 (1990))

Kepler map for rotating dipole



The phase space (En_0^2, ϕ) for the rotating dipole $d/n_0^2 = 0.000625$, $\omega n_0^3 = 4$, $l/n_0 = 0.3$, (a) - continuous equations, (b) - the Kepler map, initial energy is marked by arrow

Kepler map for rotating quadrupole (planet/asteroid)

$$H = ({p_x}^2 + {p_y}^2)/2 - 0.5[(x - d\sin\omega t)^2 + (y - d\cos\omega t)]^{-1/2}$$
$$-0.5[(x + d\sin\omega t)^2 + (y + d\cos\omega t)]^{-1/2}$$

$\bar{w} = w + A\sin 2\phi$,
$\bar{\phi} = \phi + 2\pi\omega\bar{w}^{-3/2}$

$A \sim d^2\omega^2 \sim \Delta Q\omega^2$
($\Delta Q \sim a^2_{core} \sim d^2$ being quadrupole moment)
Chaos border
$\Delta Q/R^2 > 1/(50{\omega_0}^3)$
where ΔQ is rotating part of the quadrupole of rigid body, ω_0 is the ration between the quadrupole rotaion frequency and the satellite frequency.

$q < r_\omega = 1/\omega^{2/3}$

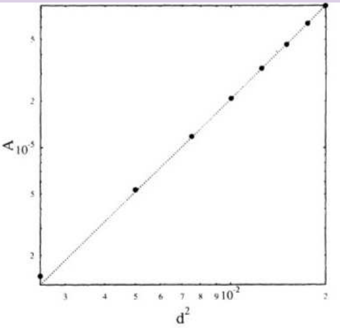


FIG. 3. The dependence of the energy change A on d for the quadrupole case with $\omega n_0^3 = 5$, $l/n_0 = 0.6$.

PRL **72**, 1818 (1994))

Capture of dark matter in the Solar system

Flow of dark matter particles (DMP): $f(v)\,dv = \sqrt{\frac{54}{\pi}}\,\frac{v^2 dv}{u^3}\,\exp\left(-\frac{3}{2}\frac{v^2}{u^2}\right)$;
$\rho_g \approx 4 \cdot 10^{-25} g/cm^3$, $u \approx 220 km/s$

Dimension argument:

$\Delta m_p = \rho_g T_d < \sigma v >; \; < \sigma v > \sim \sqrt{54\pi}\;\; \frac{G^2\, m_p\, M}{u^3};\; \Delta m_p \sim \rho_g T_d \sqrt{54\pi}\;\; \frac{G^2\, m_p\, M}{u^3}$

For $T_d \approx 4.5 \cdot 10^9 years$ one gets $\Delta m_p \sim 10^{21} g$ for Jupiter, density $6 \cdot 10^{-22} g/cm^3$ assuming r_p volume. But in reality $T_d \sim 10^7 years$ is given by diffusion escape time as for Halley comet.

From the Kepler map only DMP with $|w| < F \approx 5 m_p {v_p}^2/M$ are captured with $q < r_p$. On infinity $q = (v r_d)^2/2GM$ and $q \sim r_p$ gives cross-section:

$\sigma \sim \pi r_d^2 \sim 2\pi G M r_p/v^2 \sim 2\pi r_p^2 (v_p/v)^2 \sim 2\pi r_p^2 M/(5 m_p) \gg \pi r_p^2$
(also Heggie MNRAS (1975))

Typical capture/escape velocity $v^2 \sim 5 m_p {v_p}^2/M$; for Sun-Jupiter $v \sim 1 km/s$ in agreement with numerics of A.Peter PRD (2009)

Khriplovich, DS Int. J. Mod. Phys. D (2009)

Captured mass of dark matter in the Solar system

Capture process continues during time $T_d \approx 10^7 years$
for Sun-Jupiter (Chirikov-Vecheslavov):

$$\Delta m_p \sim \rho_g T_d \sqrt{54\pi} \quad \frac{G^2 m_p M}{u^3}$$

$$T_d \sim 1/D \sim (M/m_p)^2$$

$$\Delta m_p \sim \rho_g G^2 M^3/m_p u^3 \sim 10^{-14} M$$

DMP density in vicinity of Earth-Jupiter:

$\rho_{EJ} \sim 5 \cdot 10^{-29} g/cm^3 \ll \rho_g \approx 4 \cdot 10^{-25} g/cm^3$

BUT

$\rho_{EJ} \gg \rho_{gH} \approx 1.4 \cdot 10^{-32} g/cm^3$
(4000 times enhancement at $u/v_p = 17$
for galactic density in one kick range $0 < |w| < w_H = F$)

Global density enhancement is also possible at $u/v_p < 1$.

=> SEE TALK of José Lages

Lages, DS MNRAS Lett (2013)

Quantum effects for dark matter in binaries?

DMP energy change in number of photons

$\bar{w} = w + F(x)$, $\bar{x} = x + \bar{w}^{-3/2}$

$\Delta E = m_d F v_p^2$, $\Delta N_\phi = m_d F v_p^2 T_p / 2\pi\hbar = k$

diffusion per period, localization:

$l_\phi \approx D \approx k^2/2 < N_l = m_d v_p^2 T_p / 4\pi$

with $v_p = r_p T/2\pi$, $v_p^2 = 2MG/r_p$

This gives

$m_d < \hbar (M/m_p)^2 / [6c\sqrt{r_S r_p}]$,

$r_S = 2MG/c^2$ Schwarzschild radius

This gives for Sun-Jupiter

$m_d < 2 \cdot 10^{-16} m_e$

This mass is too small and

thus quantum effects are not important for DMP

ALL THIS

FROM 46 appearences of Halley comet

Table 1. Comet Halley's dynamics: perihelion passage times (after Yeomans and Kiang, 1981)

n	Year	Perihelion passage, t_n (JD)	Jupiter's phase X_n	Saturn's phase Y_n
1	1986	2446470.9518*	0.	0.
2	1910	2418781.6777	6.39083584	2.57350511
3	1835	2391598.9387	12.6647606	5.09993167
4	1759	2363592.5608	19.1287858	7.70290915
5	1682	2335655.7807	25.5767473	10.2994180
6	1607	2308304.0406	31.8896785	12.8415519
7	1531	2280492.7385	38.3086791	15.4263986
8	1456	2253022.1326	44.6490451	17.9795802
9	1378	2224686.1872	51.1891362	20.6131884
10	1301	2196546.0819	57.6840264	23.2285948
11	1222	2167664.3229	64.3500942	25.9129322
12	1145	2139377.0609	70.8789490	28.5420157
13	1066	2110493.4340	77.5454480	31.2265267
14	989	2082538.1876	83.9976717	33.8247519
15	912	2054365.1743	90.5001572	36.4432169
16	837	2026830.7700	96.8552482	39.0023280
17	760	1998788.1713	103.327633	41.6086720
18	684	1971164.2668	109.703382	44.1761014
19	607	1942837.9758	116.241244	46.8088124
20	530	1914909.6300	122.687259	49.4045374
21	451	1885963.7491	129.368127	52.0948344
22	374	1857707.8424	135.889745	54.7210039
23	295	1828915.8984	142.535083	57.3969935
24	218	1800819.2235	149.019949	60.0083634
25	141	1772638.9340	155.524114	62.6275046
26	66	1745189.4601	161.859602	65.1787221
27	−11	1717323.3485	168.291253	67.7686629
28	−86	1689863.9617	174.629030	70.3208017
29	−163	1661838.0660	181.097560	72.9255932
30	−239	1633907.6180	187.544060	75.5215136
31	−314	1606620.0237	193.842186	78.0576857
32	−390	1578866.8690	200.247766	80.6371280
33	−465	1551414.7388	206.583867	83.1885924
34	−539	1524318.3270	212.837867	85.7069955
35	−615	1496638.0035	219.226637	88.2796687
36	−689	1469421.7792	225.508291	90.8092075
37	−762	1442954.0301	231.617192	93.2691812
38	−835	1416202.8066	237.791521	95.7555018
39	−910	1388819.7203	244.111687	98.3005491
40	−985	1361622.0640	250.389054	100.828362
41	−1058	1334960.1638	256.542767	103.306381
42	−1128	1309149.3447	262.500045	105.705298
43	−1197	1283983.7325	268.308406	108.044248
44	−1265	1259263.8959	274.013879	110.341767
45	−1333	1234416.0059	279.748908	112.651187
46	−1403	1208900.1811	285.638100	115.022687

* After Kalyuka et al., 1985.

Effective periods for Jupiter 4332.653; for Saturn 10759.362 (days).

Chaotic notes on resonant nonlinear interactions of asteroids

Chirikov, DS Sov. J. Nucl. Phys. (1982)

3d ocsillator Hamiltonian
$H = (p_x^2 + p_y^2 + p_z^2)/2 + (x^2 + y^2 + z^2)/2 + (x^2y^2 + x^2z^2 + y^2z^2)/2$

Kolmogorov-Sinai entropy (max Lyapunov exponent, $H \to 0$)
$h/H = h_R = const$

measure of chaos at $H \to 0$ about 50%

+ Mulansky, Ahnert, Pikovsky, DS J. Stat. Phys. **145**, 1256 (2011)
chaos measure $\mu \sim \epsilon$, $\lambda \sim \epsilon^{1/2}$

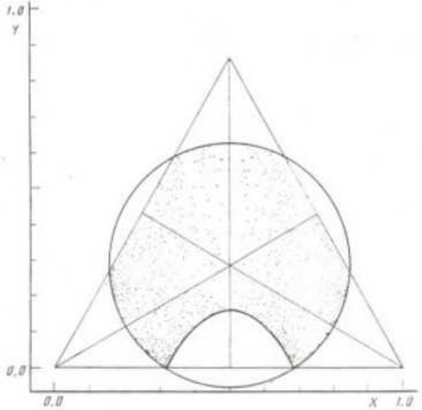


FIG. 3. Same as in Fig. 2; $H_R = 0.324$, $h_R = 0.14$.