Resonances and chaos in the dynamics of exoplanets

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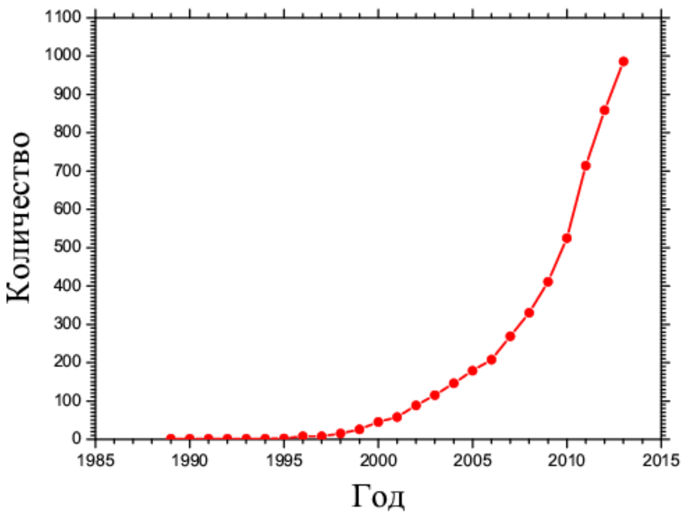
Introduction

The dynamics of exoplanetary systems (of both single and binary stars) is considered. An especial attention is paid to resonances and manifestations of dynamical chaos.

- A basis for moden theoretical ideas on the structure, dynamics, and formation of planetary systems, as well as the place of the Solar system among them, is provided by the observational data on exoplanets.

Statistics of exoplanet discovery



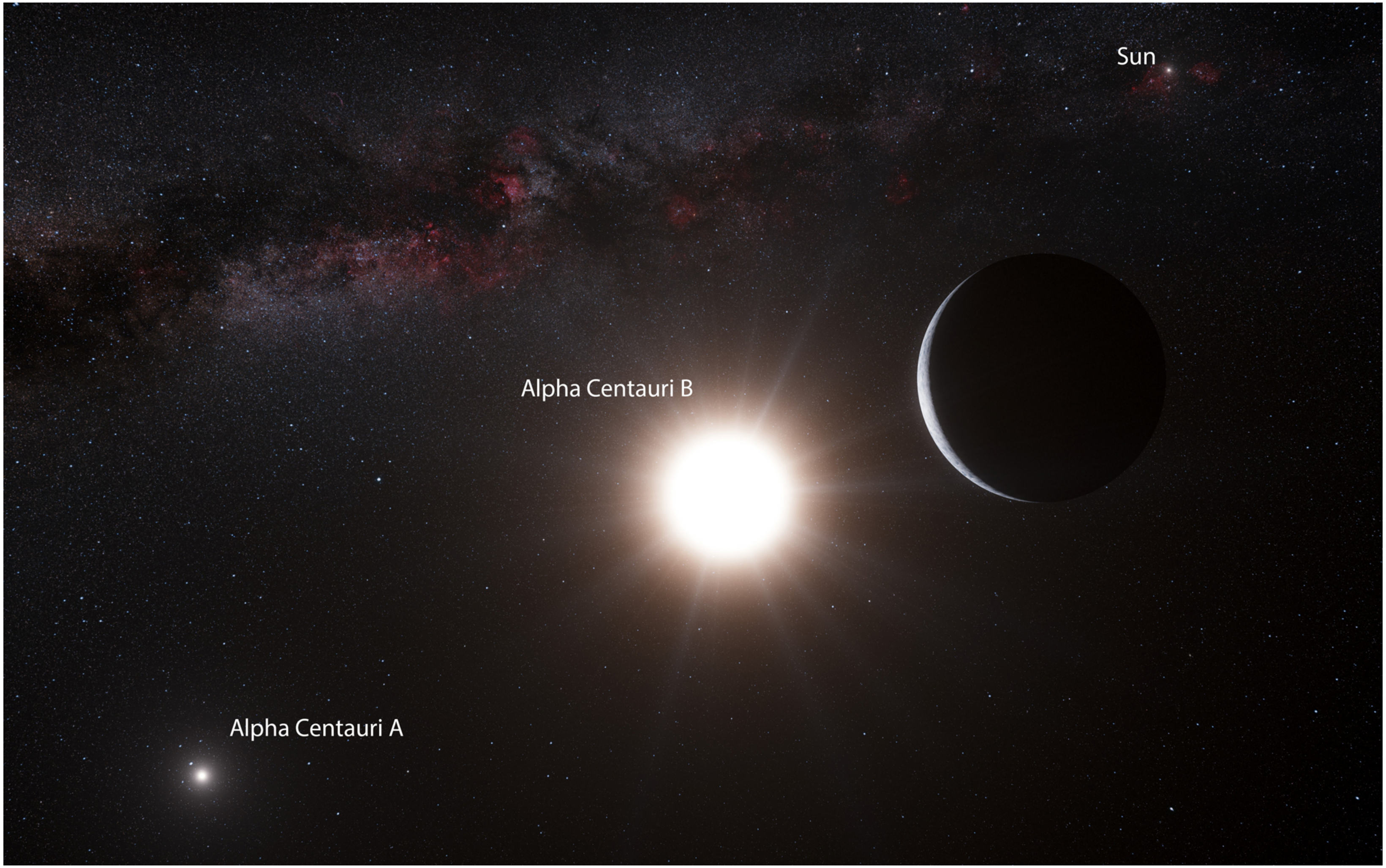
~3400 exoplanets have been discovered up to now.

They belong to ~2600 exoplanetary systems, among which ~580 are multiplanet (i.e., have two or more planets),

~130 are planetary systems of multiple (mostly binary) stars;

21 are circumbinary systems.

Observational biases (selection effects): large planets close to their parent stars are discovered first of all.



ESO image (artistic representation)

Methods of discovery

Indirect methods: astrometry, Doppler spectroscopy (measurement of periodic variations of radial velocity of a star), measurement of variations in time of the radio signals from pulsars, observations of microlensing events, observations of planetary transits (passages of planets across stellar disks), TTV method (transit timing variations).

Direct methods: differential spectrophotometry during transits, coronography, polarimetry.

The effects of observational selection: primarily large planets in close-to-star orbits are discovered.

The most successful method: analysis of transits

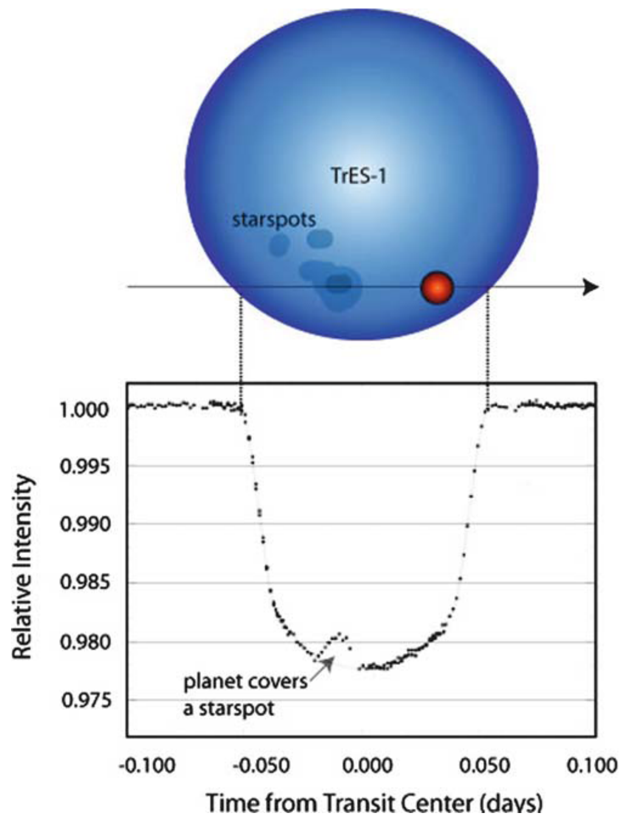


Image by G.Laughlin, http://oklo.org

A lightcurve of TrES-1 (D.Charbonneau et al., 2007), HST. A small peak during eclipse: a transit over a stellar spot.

TTV techniques

TTV (transit timing variations) techniques: if a planetary system has more than one planet, or a parent star system is multiple, then the transiting planet passes between the observer and the star at irregular time intervals. Due to perturbations of the orbital elements, the transit time varies in relation to a strictly periodic signal. Theoretical studies (Agol et al. 2005; Holman, Murray, 2005) showed that TTV-simulations allowed one to obtain a virtually complete information about the orbital parameters of planets in the observed system.

TTV were first discovered and modeled in systems with several transiting planets (Lissauer et al., 2011). Applying this method, Nesvorny et al. (2012) discovered a non-transiting planet, analyzing its TTV signal. Thus, “TTV analysis brings Celestial Mechanics back to the glorious time when Le Verrier predicted the existence and the position of Neptune from the analysis of the anomalies of the motion of Uranus. The “miracle” of Le Verrier now repeats routinely.” (A.Morbidelli.)

Definition of a "planet" in the Solar system

By resolutions of the IAU 26th General Assembly (Prague 2006), Pluto is no longer considered a planet in the strict sense of the term. It is referred to a new class of objects, that of dwarf planets.

«The IAU therefore resolves that planets and other bodies in our Solar System, except satellites, be defined into three distinct categories in the following way:

(1) **A "planet" is a celestial body that (a) is in orbit around the Sun, (b) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape, and (c) has cleared the neighbourhood around its orbit.**

(2) A "dwarf planet" is a celestial body that (a) is in orbit around the Sun, (b) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape [2], (c) has not cleared the neighbourhood around its orbit, and (d) is not a satellite.

(3) All other objects, except satellites, orbiting the Sun shall be referred to collectively as "Small Solar-System Bodies".» (Resolution 5A of the 26th IAU GA.)

Planets and brown dwarfs

The planet is considered to be a body not massive enough ($M < 0.013\ M_{sun}$) to maintain the reaction of fusion of deuterium nuclei.

Brown dwarfs are objects intermediate between planets and stars ($0.013\ M_{Sun} < M < 0.075\ M_{Sun}$): they are not massive enough to maintain the reaction of fusion of ordinary hydrogen, but may maintain the reaction of fusion of deuterium; the temperature at the center of a body is lower than $6 \cdot 10^6$ K ($M < 0.08\ M_{Sun}$).

Hot Jupiters and Super-Earths

Hot Jupiters: the giant exoplanets with Jovian masses in close-to-star orbits (observed orbital periods are about several days).

The problem with hot Jupiters is that the giant planets are formed far from their parent stars, therefore a mechanism of migration is required, delivering the planet to its observed location. The discovery of the giant planets orbiting in close-to-star orbits initiated studies of the role of tidal effects in their dynamics (Batygin et al., 2009; Lovis et al., 2011; Van Laerhoven and Greenberg, 2012; Correia et al., 2013).

Super-Earths: the planets with masses from 1 to 13 M_{Earth} with non-dominant atmospheres (the height of the atmosphere is much less than the radius of a planet). The surface can be either rocky or oceanic.

Rogue (free-flloating) planets

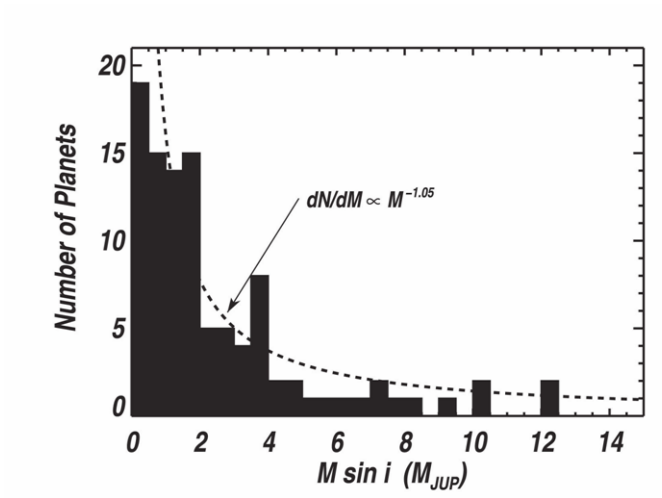
Planets in interstellar space: ***free-floating planets***, rogue planets, orphan planets. They are the planets that do not belong to planetary systems of stars.

Mercury can be ejected from the Solar system on the timescale of the order of a billion yr (J.Laskar, 1994, AA, 287, L9).

A large fraction of the planets formed in systems of double stars may escape (H.Zinnecker, 2001, in: ASP Conf. Series, 239, 223).

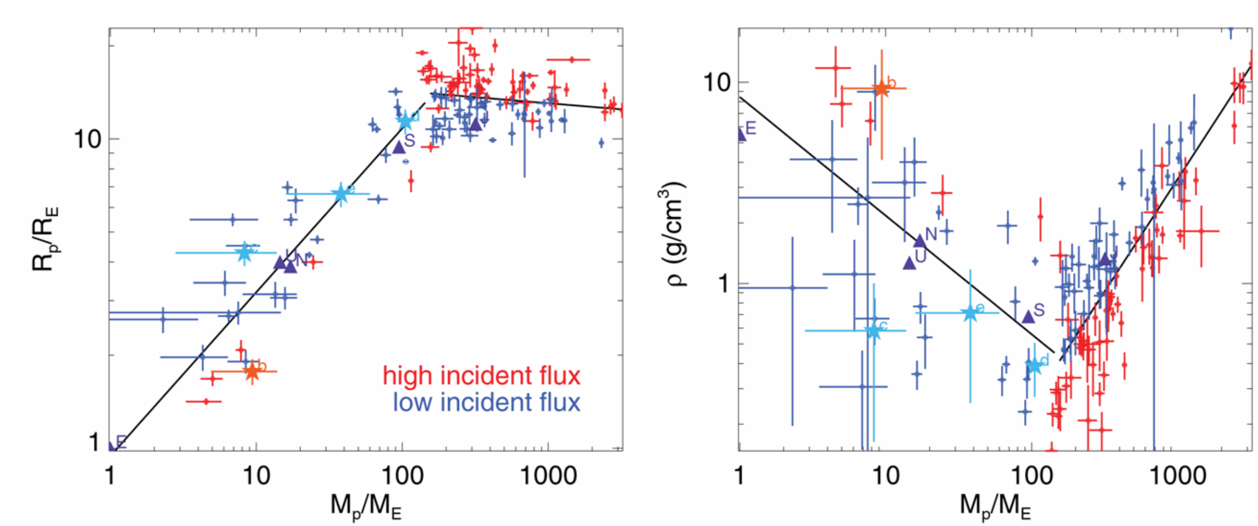
Such objects are discovered in a star cluster in the Orion nebula (M.R.Zapatero Osorio et al., 2000, Science, 290, 103).

Mass distribution and mass-radius relation



- A sharp decline of the number of planets at large masses.
- The maximum at jovian masses («hot Jupiters»).
- However, the number of discovered planets with Neptunian and smaller masses (down to super-Earth's) permanently grows.

Marcy, G., et al., 2005, Progr. Theor. Phys. Suppl. 158, 24.



L.M.Weiss et al., 2013, ApJ, 768, 14.

Stability and resonant structure of planetary systems

Resonances, interaction of resonances**,** and the ***chaotic behaviour***, caused by this interaction, play an essential role in the dynamics of planetary systems at various stages of their evolution, in many respects determining the observed architecture of planetary systems.

The orbital resonances are subdivided to ***mean motion resonances*** and ***secular resonances***. In the first case, the commensurabilities between mean orbital frequencies (of two or even a greater number of orbiting bodies) are implied; in the second case, those between orbital precession frequencies.

The captures of planetary systems into orbital resonances are believed to be a natural outcome of a primordial ***migration*** of the bodies within the protoplanetary disk. The presence of mean motion resonances and their interaction implies the possibility for ***dynamical chaos*** in the orbital dynamics, as e.g. in the case of the Kepler-36 biplanet system (K.Deck et al., 2012, ApJ, 755, L21).

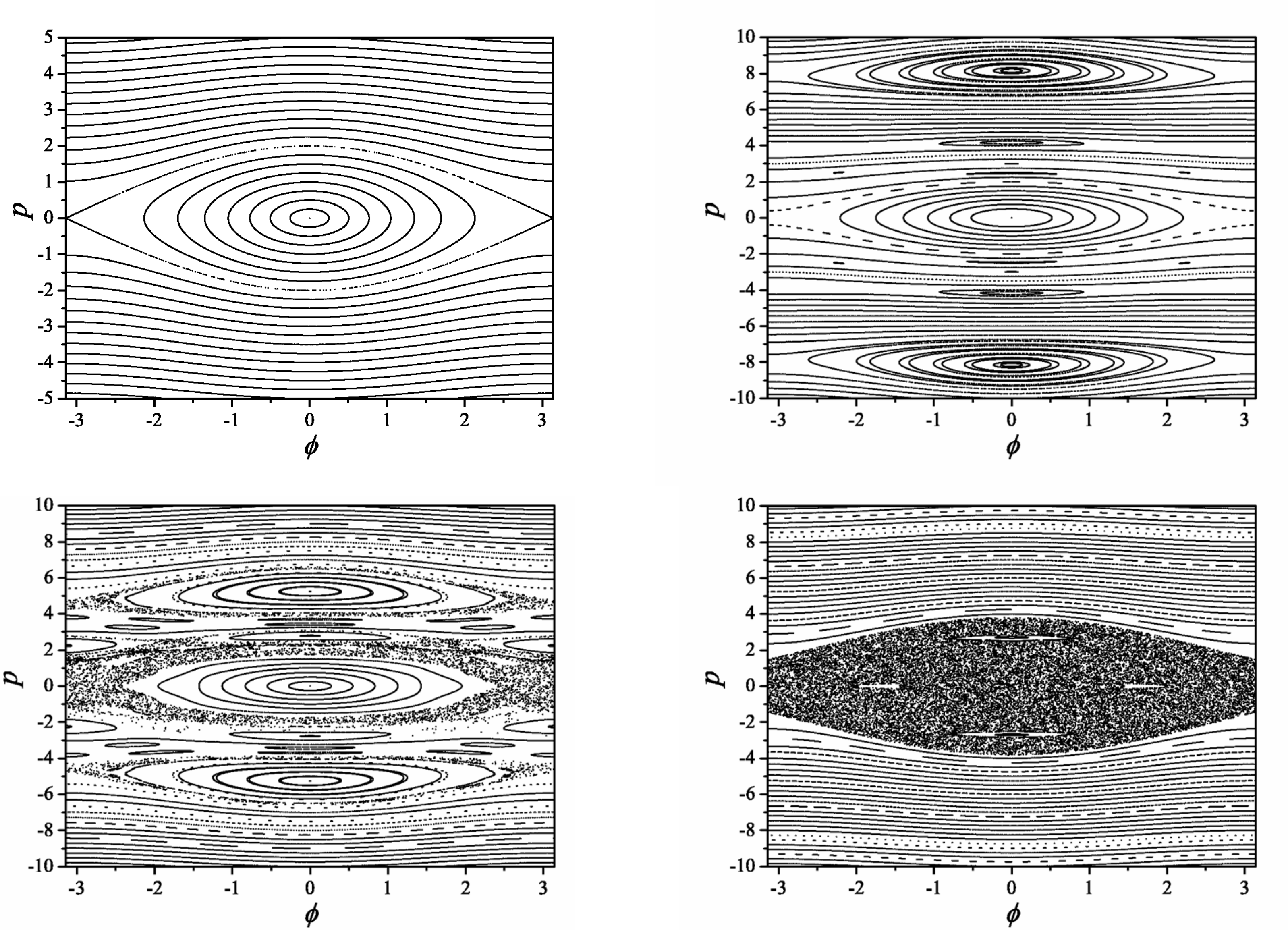
Stability and resonant structure of planetary systems

In the Solar system, some ***approximate commensurabilities*** of the orbital periods of planets are well-known: Jupiter-Saturn (the ratio of orbital frequencies ≈ **5/2**), Saturn-Uranus (≈ **3/1**), Uranus-Neptune (≈ **2/1**); not to mention the Neptune-Pluto resonance (**3/2**).

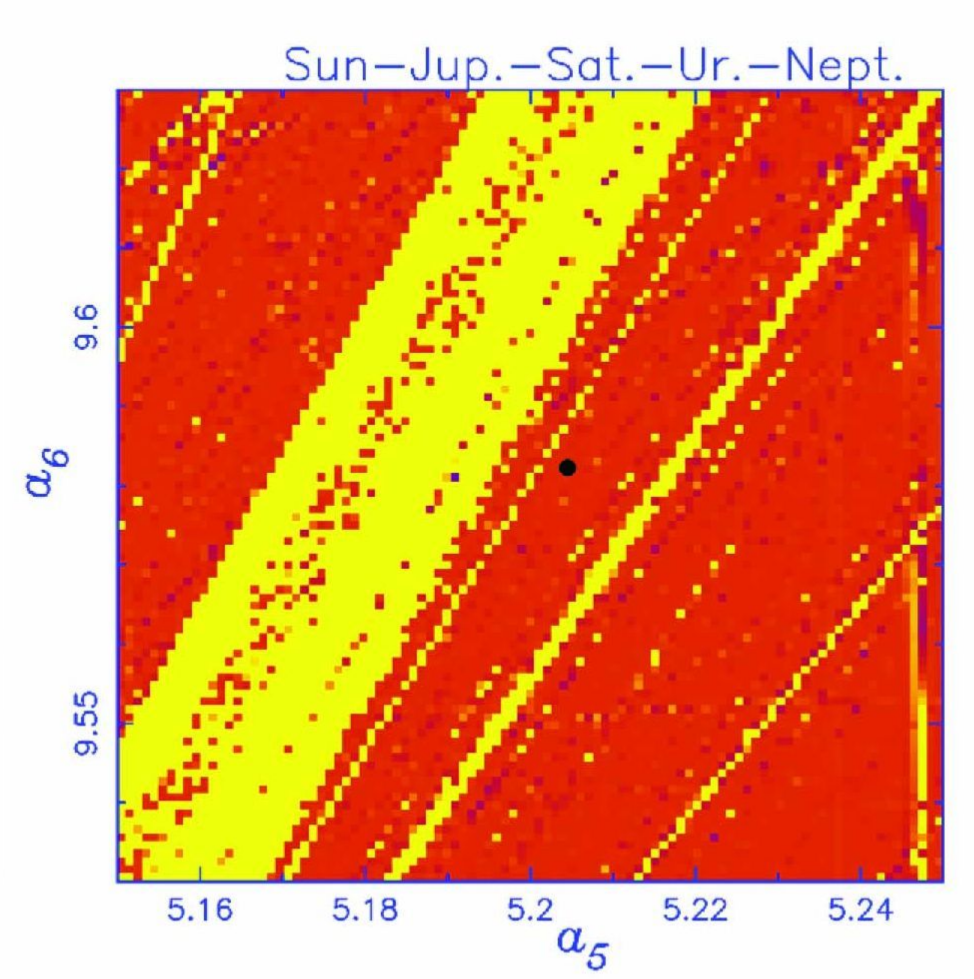
At the end of eighties, Sussman and Wisdom (1988, Science 241, 433; 1992, Science 257, 56) and Laskar (1989, Nature 338, 237) made first estimates of the ***Lyapunov time*** of the Solar planetary system in numerical integrations. It turned out to be not at all infinite, i.e., the motion of the Solar system is not regular; moreover, its Lyapunov time is by three orders of magnitude less than its age.

Murray and Holman (1999) conjectured that the revealed chaos is due to interaction of ***subresonances*** in a ***multiplet*** corresponding to a particular ***three-body resonance*** Jupiter-Saturn-Uranus.

Interaction of resonances

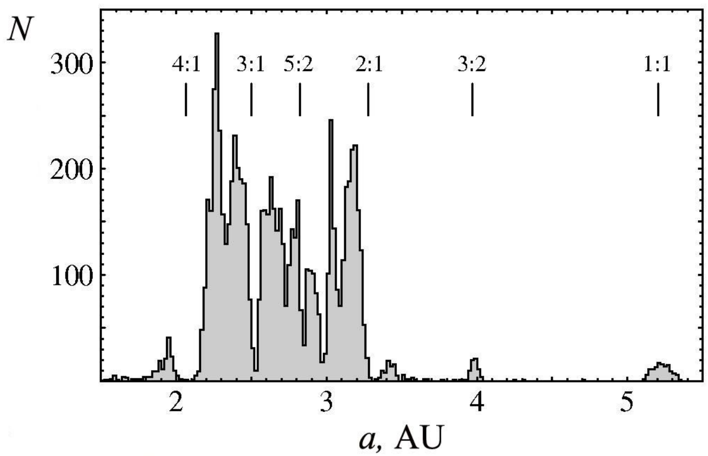


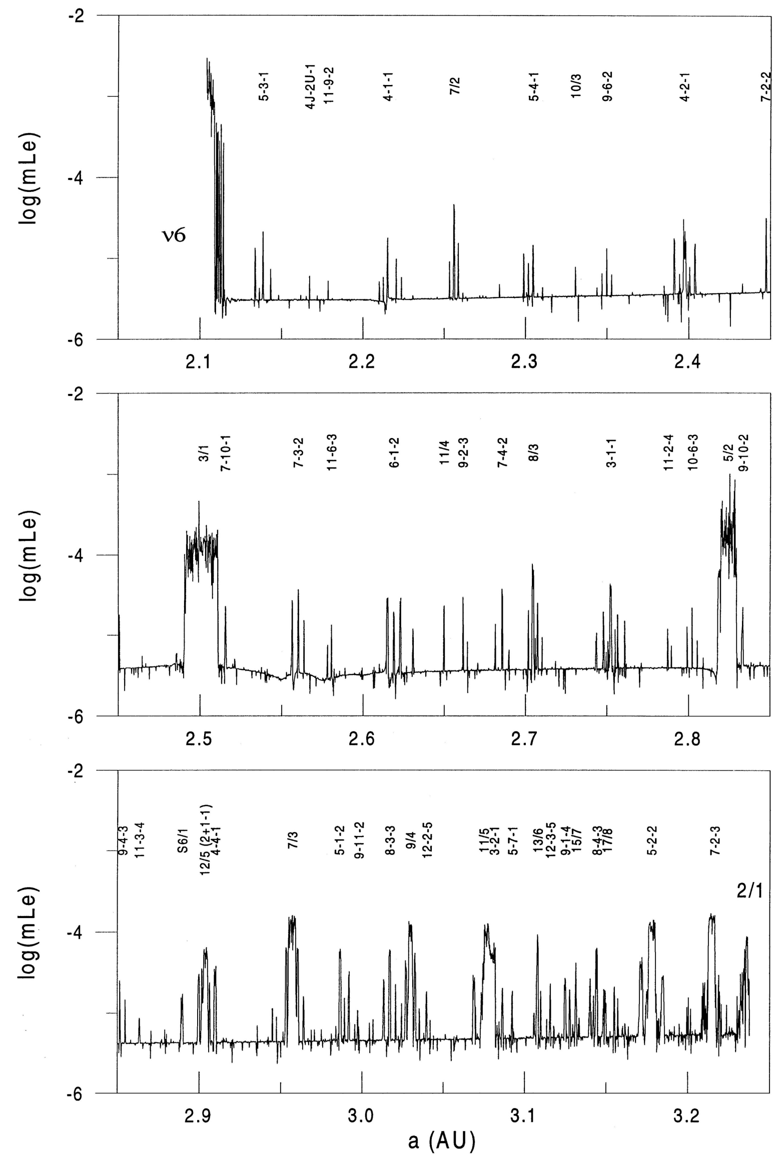
Dynamical chaos in the Solar system



Guzzo M., 2005, *Icarus* 174, 273.

Resonances in asteroidal dynamics

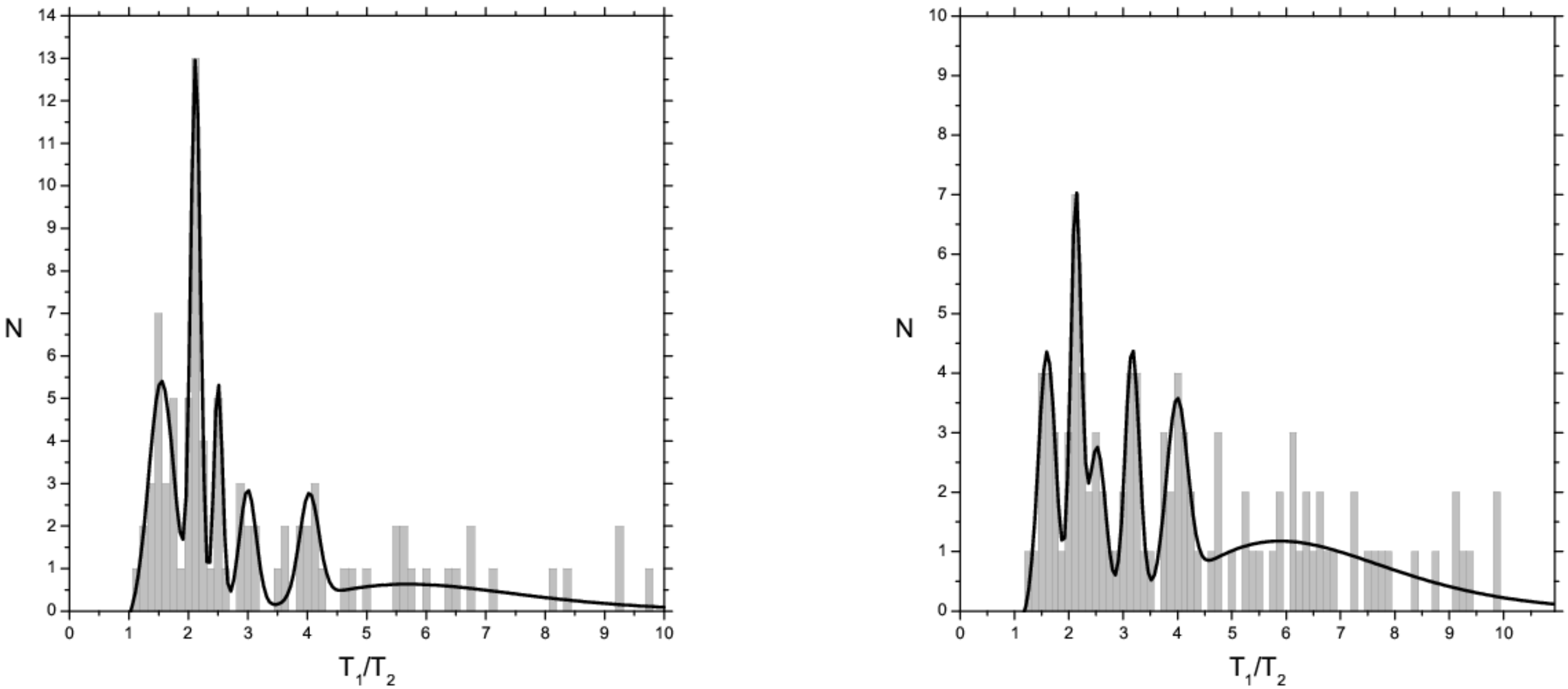




Murray C.D., Dermott S.F.
Solar System Dynamics (1999).

Morbidelli A., Nesvorný D., 1999,
Icarus 139, 295.

Resonances in dynamics of exoplanets



«Inner» (a) and «outer» (b) resonances.

E.A.Popova, I.I.Shevchenko, 2014, J. Phys. Conf. Series., 572, 012006.

The Solar system among other planetary systems

In contrast to the Solar system planets, exoplanets usually have large orbital eccentricities; besides, in many discovered exosystems, giant planets move in close-to-star orbits (hot Jupiters and Neptunes). There are no super-Earths in the Solar system.

However, there do exist multiplanet systems quite similar to the Solar one, e.g., Gliese 581, 47 UMa, μ Arae (HD 160691).

Stability criteria

It is not infrequent that observational errors of orbital parameters of planets are much greater than the intervals of these parameters' values on which the long-term stability of the system is provided. That is why the stability analysis allows one to impose valuablel restrictions on the orbital parameters.

Up to now, analytical, as well as numerical-experimental, ***criteria of stability*** of planetary systems have been developed.

In the first case, they are based on adaptation of the ***Hill criterion*** (Gladman, 1993) and ***Chirikov's resonance overlap criterion*** (Duncan et al., 1989; Mudryk, Wu, 2006).

In the second case, they are based on computations of ***MEGNO*** (Cincotta et al., 2003; Goździewski, 2003; Goździewski et al., 2013), ***Lyapunov exponents*** (Popova, Shevchenko, 2013), fundamental frequencies of motion (the ***frequency analysis***, Correia et al., 2009; Laskar, Correia, 2009), as well as on numerical assessment of escape/encounter conditions (Holman, Wiegert; 1999; Pilat-Lohinger et al., 2003; Kholshevnikov, Kuznetsov, 2011).

Stability criteria

The Hill criterion

The radius of the stability inner zone is directly proportional to the radius of the Hill sphere calculated at the secondary's pericenter:

$$r_{\rm H} \approx (\mu/3)^{1/3} a(1-e),$$

where $\mu = M_{\rm sec}/M_{\rm prim}$ is the secondary-primary mass ratio. This formula renders the so-called "Hill sphere at pericenter scaling". The Hill sphere ordinary radius is $r_{\rm H} = (\mu/3)^{1/3} a$.

Hamilton, D.P., & Burns, J.A. 1992, Icarus 96, 43

Stability criteria

The Wisdom criterion

In the planar circular restricted three-body problem, the radial half-width of the instability neighborhood of the pertuber's orbit is given by

$$\Delta a_{\text{overlap}} \approx 1.3\mu^{2/7}a',$$

where $\mu = m_2/(m_1 + m_2)$ is the mass parameter, a' is the semimajor axis of the perturber; $e \lesssim 0.15$. The particles with a within the interval $a' \pm \Delta a_{\text{overlap}}$ move chaotically.

The value of p, critical for the overlap of the first order resonances $(p+1) : p$, is

$$p_{\text{overlap}} \approx 0.51\mu^{-2/7}.$$

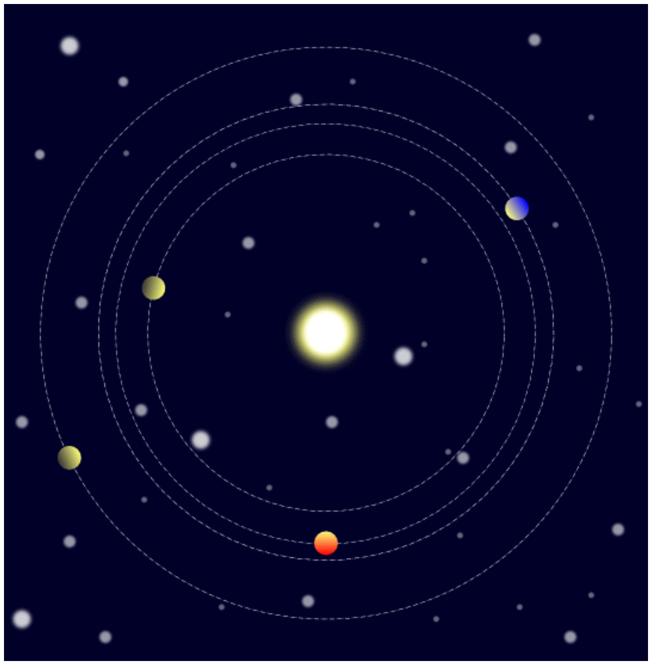
Wisdom, J. 1980, Astron. J., 85, 1122

Duncan, M., Quinn, T., & Tremaine, S. 1989, Icarus, 82, 402

Multiplanet systems

- About one third of the discovered exoplanets reside in ***multiplanet systems***, i.e., the systems with two or more planets (H.Rein, 2012, MNRAS, 427, L21). About ~600 multiplanet systems are known.
- Orbital resonances are ubiquitous in planetary systems, as confirmed in computations of the behaviour of resonant arguments. The occurrence of low-order resonances (such as **2/1** and **3/2**) is statistically significant (Wright et al., 2011; Fabrycky et al., 2012).
- Well-known systems with planets in the ***2/1 resonance*** are Gliese 876 and HD 82943, in the ***3/1 resonance*** is the 55 Cnc system. Gliese 876 is an example of a system where three planets are involved in the ***Laplace resonance 4:2:1*** (Martí et al., 2013) like the innermost three Galilean satellites of Jupiter.
- A closely packed multi-planet resonant system, Kepler-223, exhibiting a *8:6:4:3 mean motion resonance*, was reported (Lissauer et al., 2011).

Kepler-223 (KOI-730)



Kepler-223 is a yellow dwarf (G5V) with 4 planets, discovered by the transit method.
This planetary systemis the most remarkable example of a ***closely packed resonant system***.

- Radii of planets: b, c, d, e: 1.8, 2.1, 2.8, 2.4 R_{earth}.
- Orbital periods: 7.4, 9.8, 14.8, 19.7 d.
- The orbital frequencies satisfy the ratios **8:6:4:3**.

"This resonant chain is potentially the missing link that explains how planets that are subject to migration in a gas or planetesimal disk can avoid close encounters with each other, being brought to a very closely packed, yet stable, configuration."
(J.J.Lissauer et al., 2011, ApJ Suppl. Ser., 197, 8).

Image Wikimedia Commons

Planetary systems of binary stars

Credit: IAU/L. Calçada.

Upsilon And

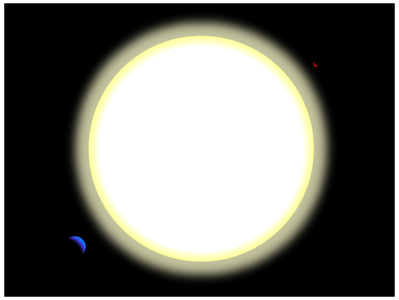


Image Wikimedia Commons

Upsilon And is a double star consisting of a yellow (F8) dwarf and a red dwarf; the size of the binary is 750 AU. Three giant planets orbit around the first one (R.P.Butler et al.,1999, ApJ 526, 916).

Masses of the planets b, c, d: >0.687, >1.97, >3.93 M_J.

Orbital periods: 4.6, 241, 1290 d.

Planets c and d are close to ***resonance 11/2***. The system as a whole is stable (T.A.Michtchenko, R.Malhotra, 2004, Icarus, 168, 237).

Planetary systems of binary stars

More than half of all observed main sequence stars are in binaries and multiples (A.Duquennoy, M.Mayor, 1991, Astron. Astrophys. 248, 485; R.Mathieu et al., 2000, in: Protostars and Planets IV, Univ. Arizona Press, p.703). Planets are known to belong to ~130 multiple stars (all in all ~2600 exoplanet systems are known).

Most of the planets discovered in binary systems follow ***inner orbits*** (***S-type***, around one of the components of a binary), while others ***outer (circumbinary) orbits*** (***P-type***, around both of the components).

Analysis of transits observed from the *Kepler* space telescope revealed several ***circumbinary planetary systems*** of main sequence stars: Kepler-16, 34, 35, 38, 47 (L.Doyle et al., 2011, Scence 333, 1602; W.Welsh et al., 2012, Nature 481, 475; J.Orosz et al., 2012, ApJ, 758, 87; 2012, Science 337, 1511). Moreover, the Kepler-47 system is ***multiplanet*** (contains two planets). Scenarios of planetary formation and dynamics of circumbinary systems (often at the brink of stability) present a challenge to theorists.

Stability criteria

The Holman–Wiegert criterion

In the planar problem, the radius $a_{\rm cr}$ of the instability inner zone for the initially circular prograde outer particle orbits is given by the numerical-experimental fitting relation

$$a_{\rm cr}/a_{\rm b} = 1.60 + 5.10 e_{\rm b} - 2.22 e_{\rm b}^2 + 4.12\mu - 4.27 e_{\rm b}\mu - 5.09\mu^2 + 4.61 e_{\rm b}^2\mu^2,$$

where $\mu = m_2/(m_1 + m_2)$ is the binary's mass parameter, $a_{\rm b}$ and $e_{\rm b}$ are the binary's semimajor axis and eccentricity.

Holman, M.J., & Wiegert, P.A. 1999, Astron. J., 117, 621

Stability criteria

The "Kepler map" criterion

The energy width of a one-sided chaotic band in the vicinity of the perturbed parabolic orbit scales as the power 2/5 of the system mass parameter (Petrosky, 1986):

$$\Delta E_{\mathrm{cr}} \propto \mu^{2/5},$$

where $\mu = m_2/(m_1 + m_2)$ is the mass parameter. The particles with E within the interval $-\Delta E_{\mathrm{cr}} < E < 0$ move chaotically.

Using the formulas for the parameters λ and W of the Kepler map (Shevchenko, 2011), one has

$$\Delta E_{\mathrm{cr}} \simeq A \mu^{2/5} q^{-1/10} \exp\left(-B q^{3/2}\right),$$

where $A = 3^{2/5}\pi^{3/5}2^{-1/2}K_G^{-2/5} = 2.2061\ldots$, $B = 2^{5/2}/15 = 0.3771\ldots$, $K_G = 0.9716\ldots$.

The critical eccentricity is given by

$$e_{\mathrm{cr}} = 1 - 2q\Delta E_{\mathrm{cr}}.$$

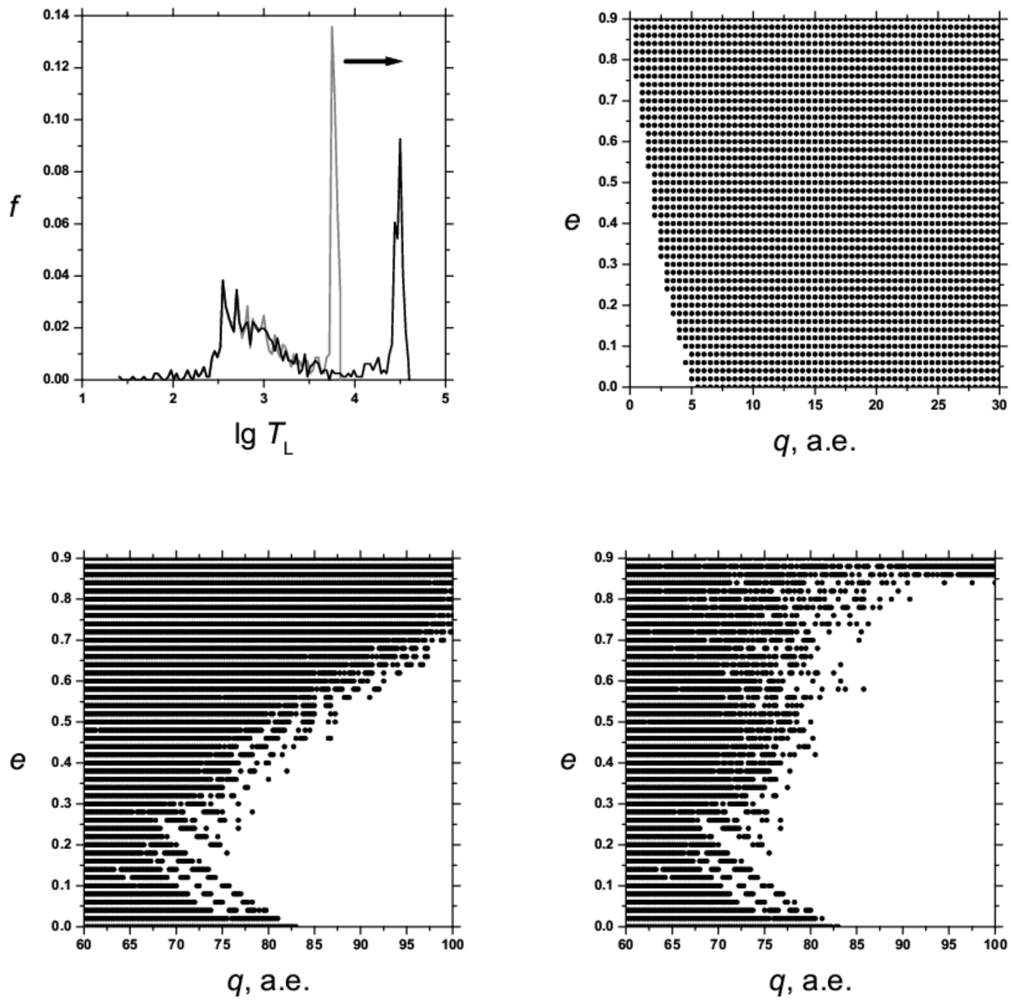
The orbits with $e > e_{\mathrm{cr}}$ for a given value of q are chaotic.

Planetary systems of binary stars

System	m_1 (m_S)	m_2 (m_S)	m_p (m_J)	a_b (AU)	e_b	a_p (AU)	e_p
α Centauri	1.11	0.93	−	23.4	0.52	−	−
Kepler-16	0.69	0.20	0.33	0.22	0.16	0.71	0.007
Kepler-34	1.05	1.02	0.22	0.23	0.52	1.09	0.18
Kepler-35	0.89	0.81	0.13	0.18	0.14	0.60	0.042

D.Pourbaix et al., 1999, AA, 344, 172;
L.Doyle et al., 2011, Science, 333, 1602;
W.F.Welsh et al., 2012, Nature, 481, 475.

Planetary dynamics in the α Centauri system



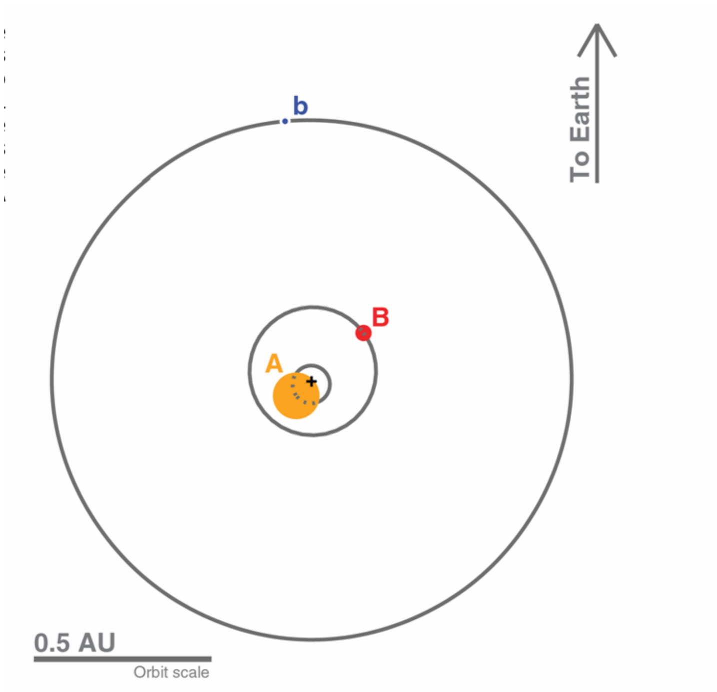
The outer border of ***chaotic zone*** on the stability diagrams corresponds to the semimajor axis of planet's orbit ~80 AU (if the initial orbit is circular), and the inner border is strongly dependent on the choice of initial conditions.

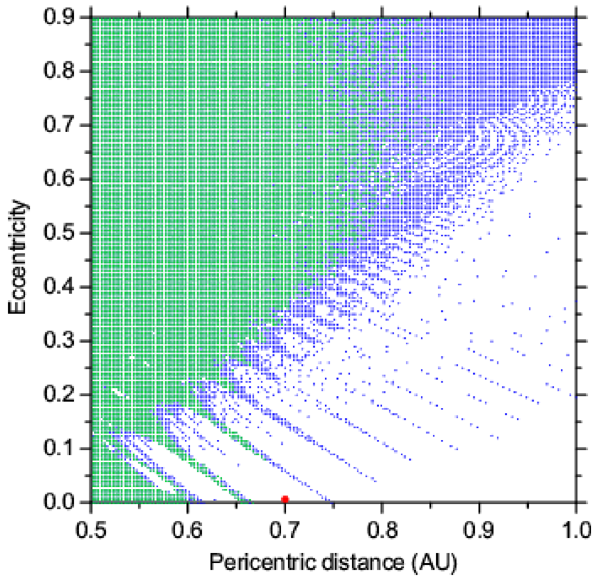
The most likely ***Lyapunov times*** of the chaotic motion of planets in zones of instability are ~500 years for outer orbits and ~60 for inner orbits.

Fractal structure of chaos borders in the diagrams is due to the presence of orbital resonances.

E.A.Popova, I.I.Shevchenko, 2012, Astron. Lett., 38, 581.

Circumbinary dynamics in the Kepler-16 system

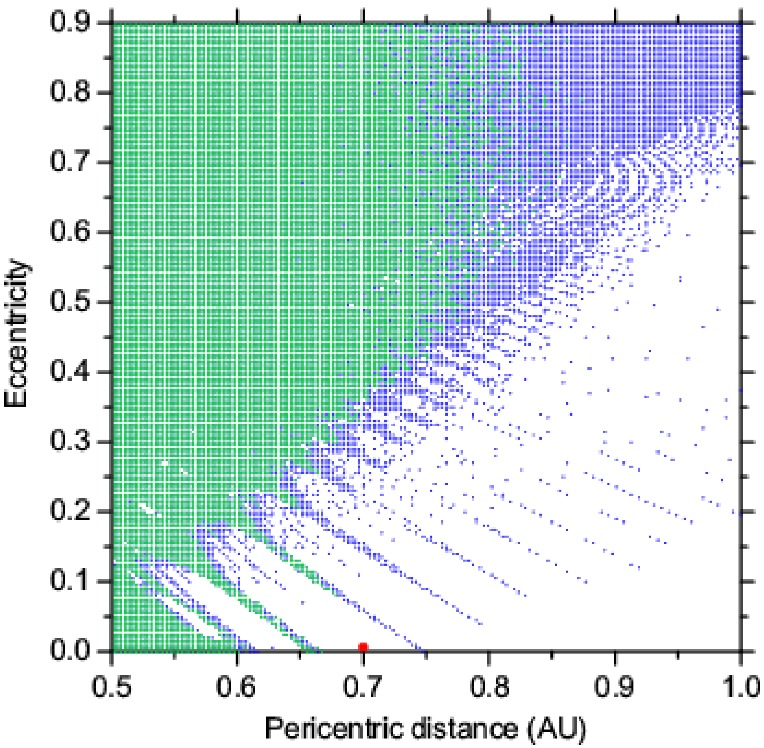


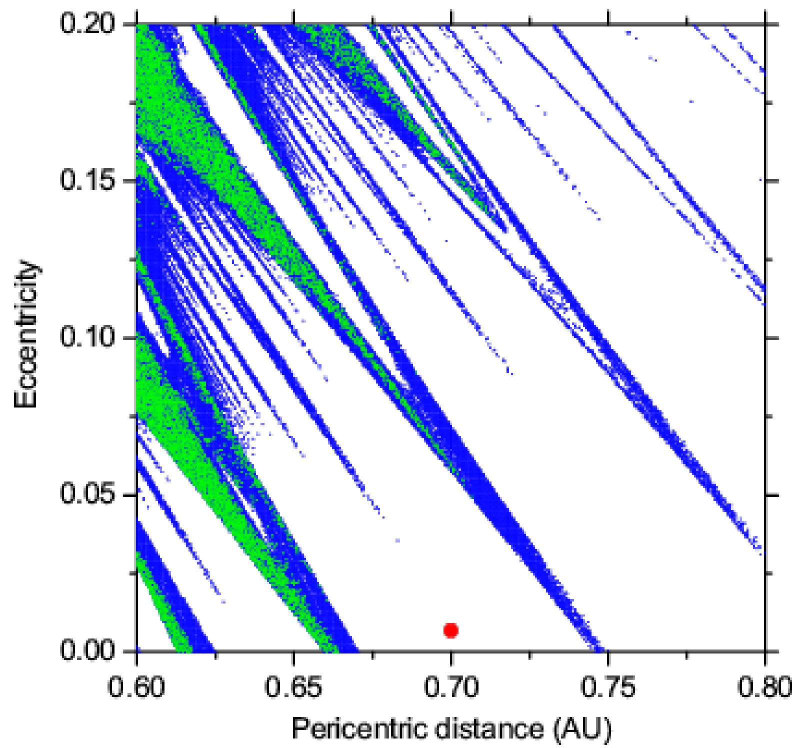


L.Doyle et al., 2011, Science, 333, 1602.

E.A.Popova, I.I.Shevchenko, 2013, ApJ, 769.

Circumbinary dynamics in the Kepler-16 system





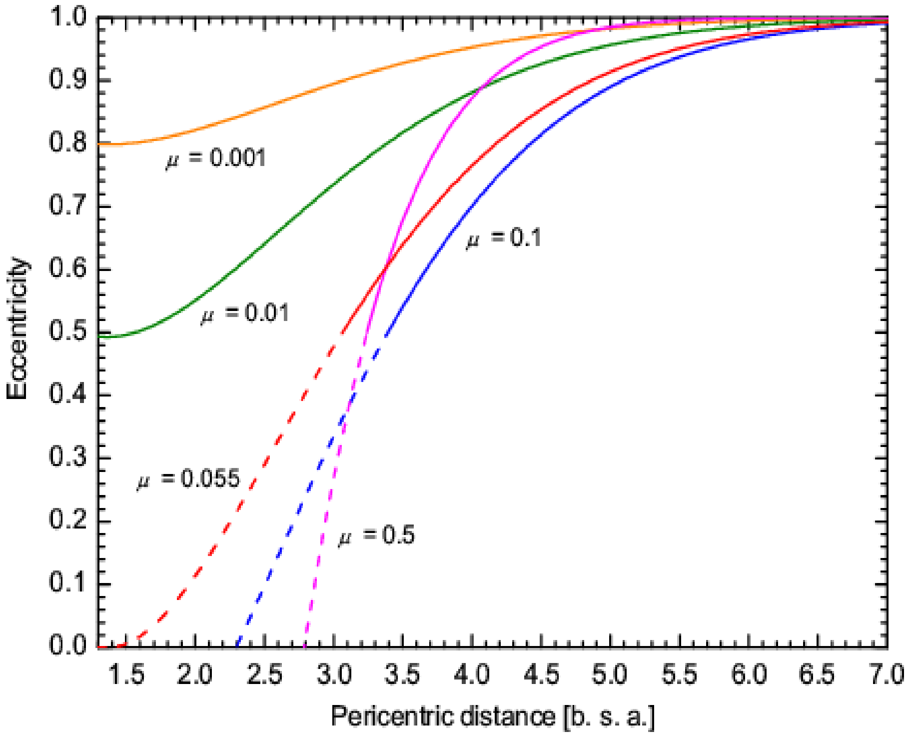
E.A.Popova, I.I.Shevchenko, 2013, ApJ, 769, 152.

Circumbinary dynamics in the Kepler-16 system

The stability diagrams in the plane of initial conditions "pericentric distance — eccentricity" show that Kepler-16b is in dangerous proximity to chaos domain. It is located between the "teeth" of instability in the space of orbital parameters. Kepler-16b survives because its orbit is close to the half-integer ***11/2 orbital resonance*** with the central binary.

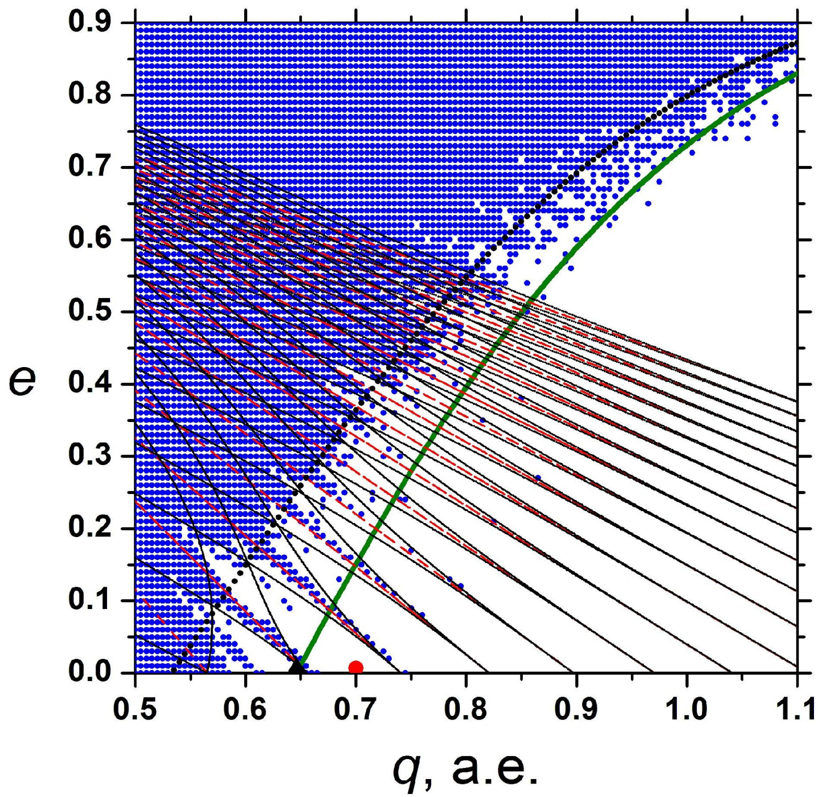
In the Solar system, this phenomenon is similar to the survival of Pluto and plutino located in the half-integer 3/2 orbital resonance with Neptune. The order of the occupied resonance increases with increasing the mass parameter of the central binary, because this shifts the boundary of stability outside; in the case of the Solar system, the corresponding "binary" is formed by the Sun and Neptune.

Central chaotic zones



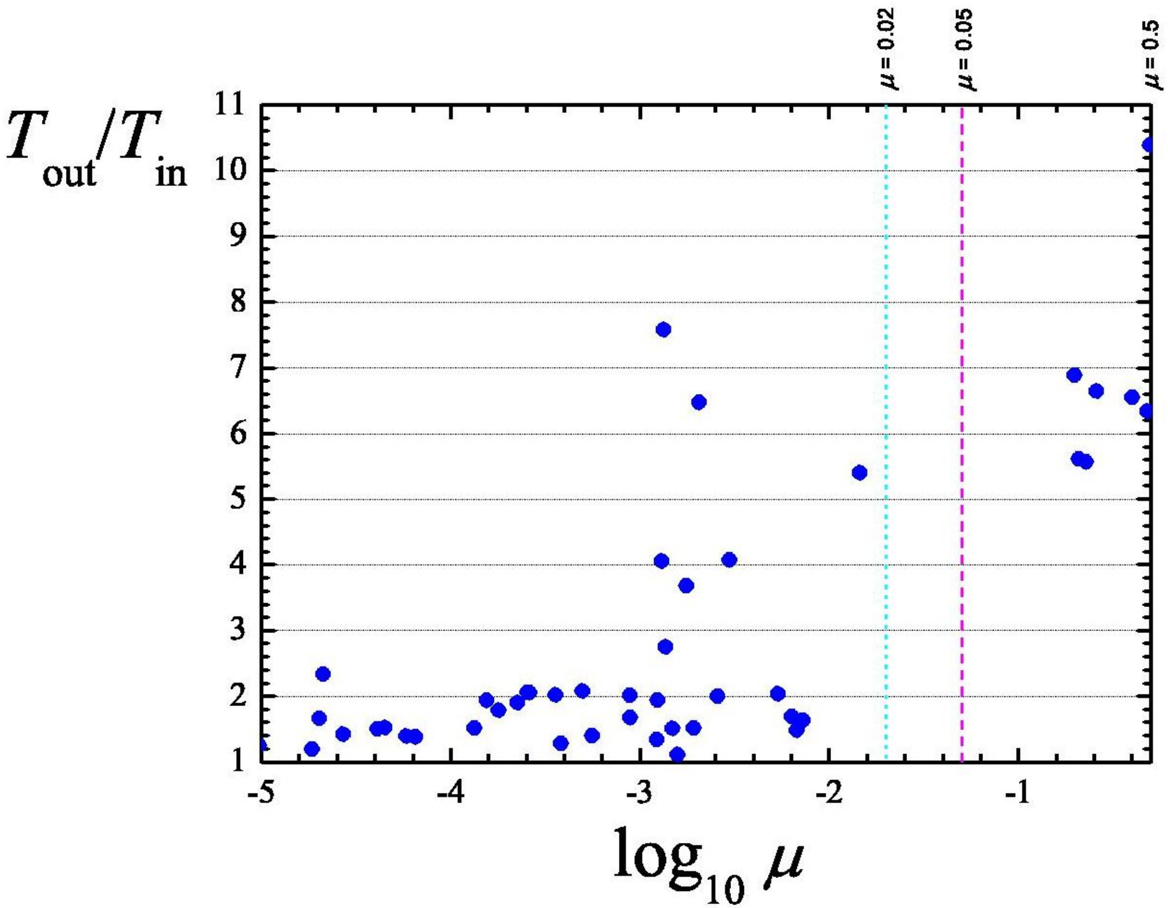
I.I.Shevchenko, 2015, Astrophys. J., 799, 8.

Analytical description of chaos borders



E.A.Popova, I.I.Shevchenko, 2016, Astron, Lett,, 42, 474.
Theory:
R.Mardling, 2008, Lect. Notes Phys., 760, 59;
I.I.Shevchenko, 2015, Astrophys. J., 799, 8.

Central chaotic zones



I.I.Shevchenko, 2015, Astrophys. J., 799, 8.

- Stability criteria development.
- Prevalence of particular resonances in exoplanetary systems.
- Resonant dynamics of closely packed exosystems.
- Resonant and near-resonant architectures of planetary systems of binary stars.
- Circumbinary planets at the “edge of chaos”.