Asteroids in three-body mean motion resonances with planets

E.A. Smirnov
Pulkovo Observatory, St.-Petersburg, Russia

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- An essential role in the asteroidal dynamics is played by the mean motion resonances.
- Two-body planet-asteroid resonances are widely known, due to the Kirkwood gaps.
- Besides, there are so-called *three-body resonances*.
- In the latter case the resonance represents a commensurability between the mean frequencies of the orbital motions of an asteroid and two planets (e.g. Jupiter and Saturn):

Three-body resonances are the linear combination of the mean frequences of the orbital motion of two planets and an asteroid:

$$m_{\mathrm{P1}}\dot{\lambda_{\mathrm{P1}}} + m_{\mathrm{P2}}\dot{\lambda_{\mathrm{P2}}} + m\dot{\lambda} \approx 0,$$

where $\dot{\lambda_{\mathrm{P1}}}$, $\dot{\lambda_{\mathrm{P2}}}$, $\dot{\lambda}$ — derivatives of mean longitudes first and second planet and an asteroid respectively and m_{P1}, m_{P2}, m — are integers.

The three-body mean motion resonances seem to be the main actors structuring the dynamics in the main asteroid belt.

Nesvorny, Morbidelli, ApJ 116 (1998)

Resonant argument

To identify the resonance a special parameter called "resonant argument" is introduced. It is linear combination of the mean longitudes and the longitudes of periapsis. In planar problem it is defined by the following formula:

$$\sigma = m_{\mathrm{P1}}\lambda_{\mathrm{P1}} + m_{\mathrm{P2}}\lambda_{\mathrm{P2}} + m\lambda + p_{\mathrm{P1}}\varpi_{\mathrm{P1}} + p_{\mathrm{P2}}\varpi_{\mathrm{P2}} + p\varpi,$$

where λ_{P1}, λ_{P2}, λ, ϖ_{P1}, ϖ_{P2}, ϖ — mean longitudes and longitudes of periapsis of two planets and an asteroid respectively and m_{P1}, m_{P2}, m, p_{P1}, p_{P2}, p — are integers followed by D'Alembert rule (see Morbidelli, 2002):

$$m_{\mathrm{P1}} + m_{\mathrm{P1}} + m + p_{\mathrm{P1}} + p_{\mathrm{P2}} + p = 0.$$

Order of the resonance

- One more important variable is *the resonant order*.
- Width of the resonance, corresponding number of sub-resonances depend on this parameter (Nesvorny, Morbidelli, ApJ, 1998).
- The resonant order is given by formula:

$$q = \left| m_{\mathrm{P1}} + m_{\mathrm{P2}} + m \right|.$$

Stage 1

Identification matrix

Identification matrix

- It consists of two primary columns.
- The first one contains designations of the resonance.
- The second one contains the corresponding resonant values of the semimajor axis.

Table: An extract from the identification matrix

Resonance	a_{res} (AU)
$5\mathrm{J}-2\mathrm{S}-2$	3.1746
$4\mathrm{J}-6\mathrm{U}-1$	2.4189
$4\mathrm{E}-7\mathrm{M}-1$	2.3641
$3\mathrm{V}-6\mathrm{E}-1$	2.3850
$1\mathrm{M}-3\mathrm{J}-1$	2.3464

Stage 2

Dynamical identification

Dynamical identification

- First of all, each asteroid's orbit from the adopted set of ≈ 460000 objects (thanks to AstDyS) is computed for 10^5 yr (thanks to mercury6 and orbit9). The perturbations from all planets (from Mercury to Neptune) and Pluto are taken into account.
- For each asteroid we find possible set of three-body resonances and calculated related resonant arguments.
- Each argument is then analyzed automatically on the presence of libration/circulation, using the computed trajectory of the object.
- We distinguish two types of resonant libration: pure and transient.

Pure resonant asteroid 463 Lola

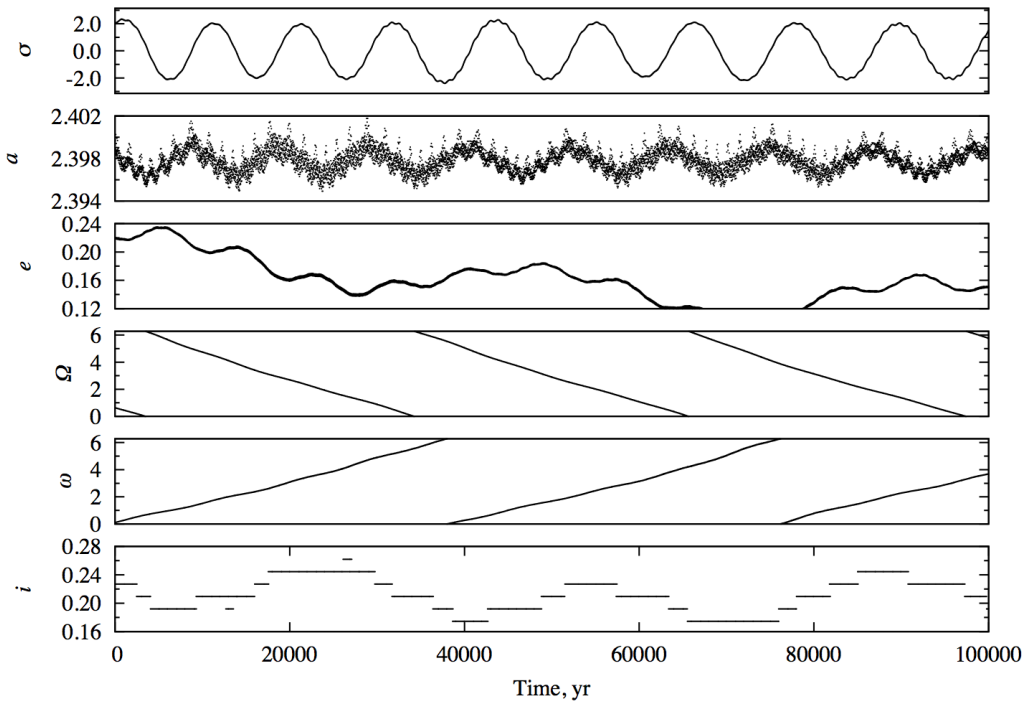


Figure: Resonant argument and orbital elements of pure resonant asteroid 463 Lola, resonance 4J-2S-1.

Pure resonant asteroid 2096 Väinö

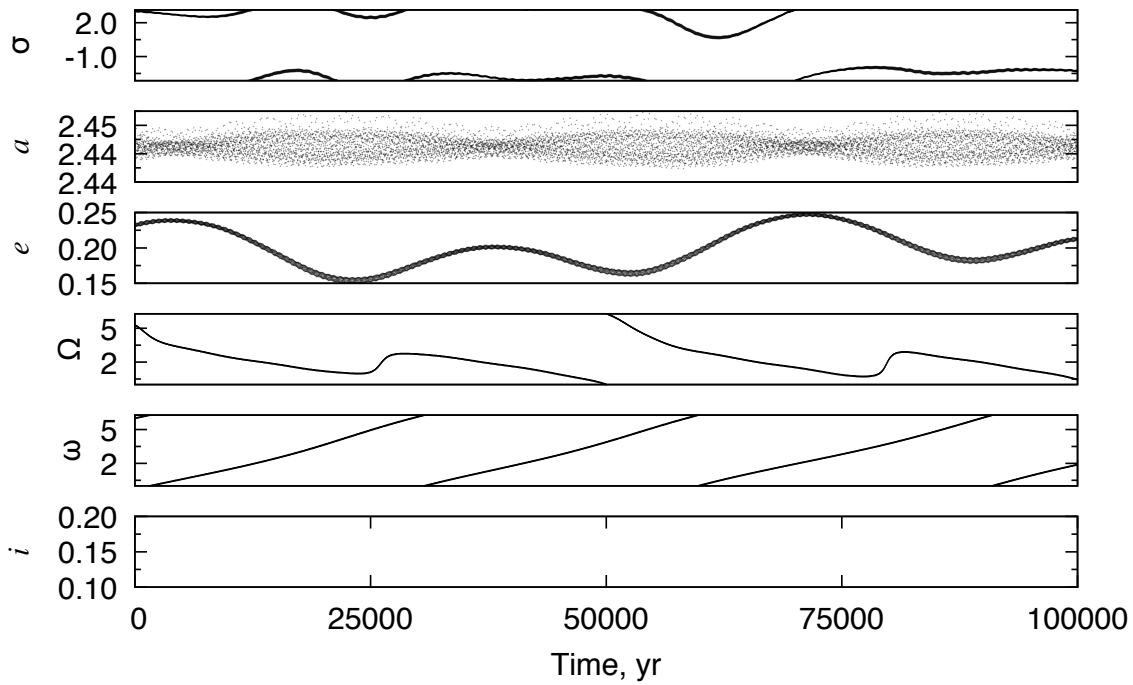


Figure: Resonant argument and orbital elements of pure resonant asteroid 2096 Väinö, resonance 1M+3J-3.

Transient resonant asteroid 490 Veritas

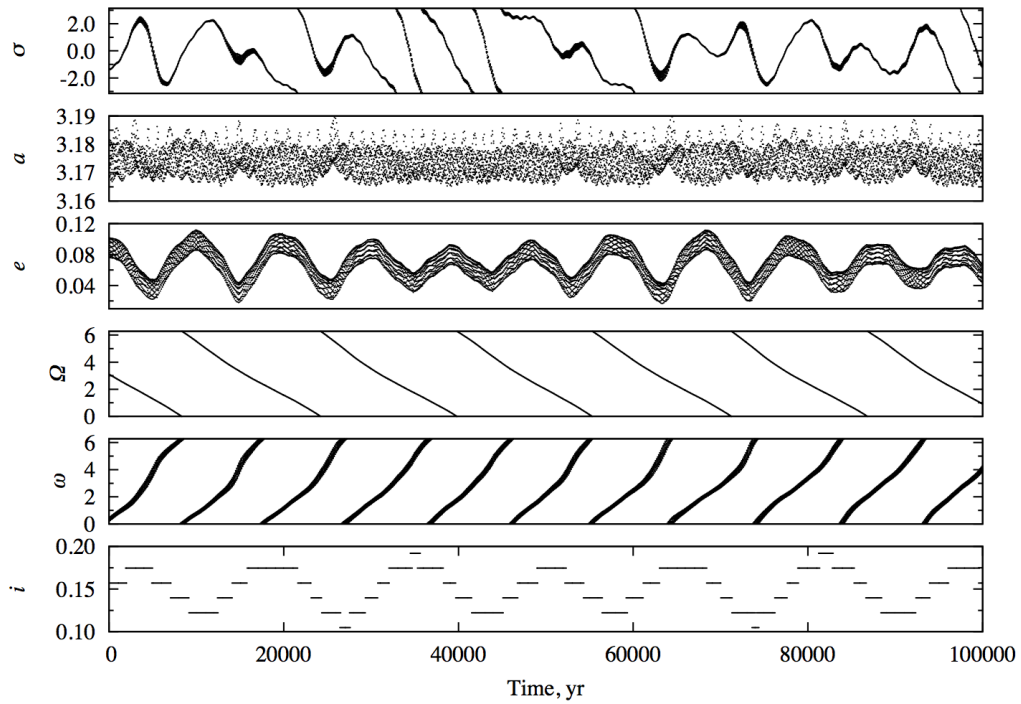


Figure: Resonant argument and orbital elements of pure resonant asteroid 490 Veritas, resonance 5J-2S-2.

Transient resonant asteroid 192 Nausikaa

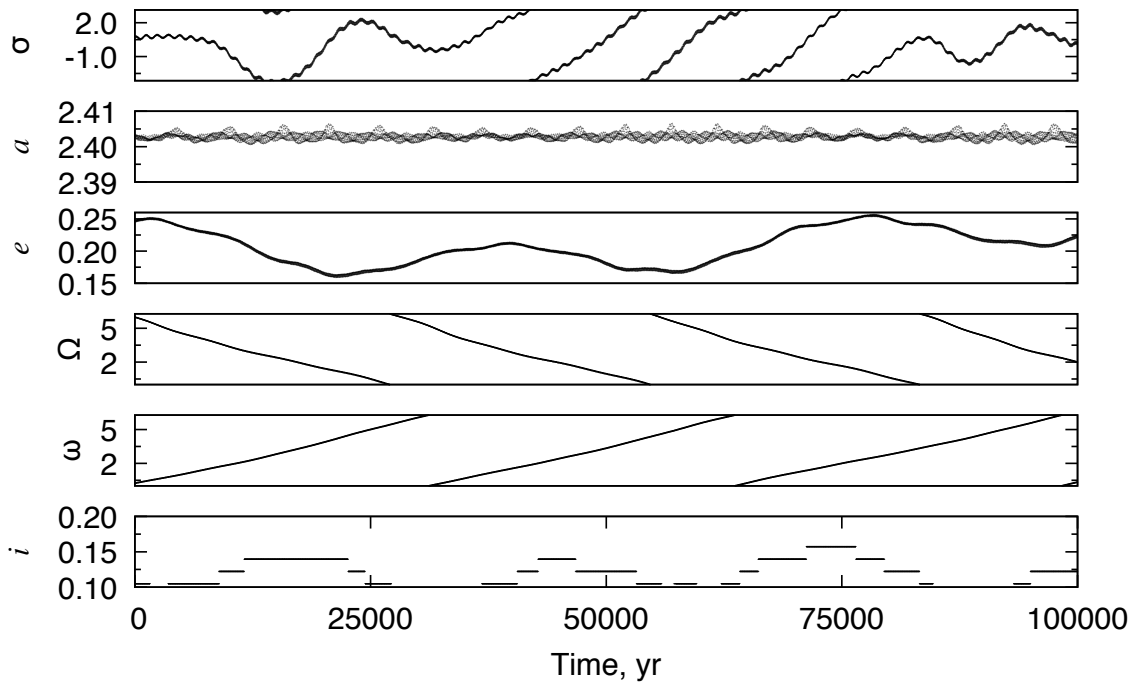


Figure: Resonant argument and orbital elements of pure resonant asteroid 192 Nausikaa, resonance 3M-3J-5.

Top 10 most populated three-body resonances

Resonance	a_{res}	Total	Pure
4J −6U −1	2.4189	2163	11
5J −2S −2	3.1747	1400	307
3J −2S −1	3.0801	1269	251
4J −2S −1	2.3981	1234	1051
3J −1S −1	2.7530	1061	397
4J −3S −1	2.6235	1008	196
2J +2S −1	2.6151	946	46
2J +4U −1	2.7765	732	96
2J +5U −1	2.6791	732	54
3J −6U −1	3.1193	718	121

The same, without J-S and J-U

Resonance	a_{res}	Total	Pure
1V +5S −6	2.4237	670	42
4E −7M−1	2.3641	483	123
3V −6E −1	2.3850	450	81
1M−3J −1	2.3464	437	99
1M−2S −2	2.6575	269	31
2M−4J −3	2.5812	235	30
2M−3J −3	2.3965	214	30
1M+6S −3	2.5592	207	25
1M−1S −2	2.5331	196	30
2M−3S −4	2.5934	191	37

Asteroids in three-body resonances, statistics I

Planet 1	Planet 2	T+P	% of total	Pure	% of pure
Venus	Earth	1307	0.28	35	2.68
Venus	Mars	349	0.08	14	4.01
Venus	Jupiter	342	0.07	8	2.34
Venus	Saturn	856	0.19	5	0.58
Venus	Uranus	1027	0.22	25	2.43
Earth	Mars	2988	0.65	165	5.52
Earth	Jupiter	570	0.12	33	5.79
Earth	Saturn	239	0.05	3	1.26
Earth	Uranus	1819	0.39	55	3.02
Mars	Jupiter	4714	1.02	288	6.11
Mars	Saturn	3751	0.81	197	5.25
Mars	Uranus	5644	1.22	280	4.96

Asteroids in three-body resonances, statistics II

Planet 1	Planet 2	T+P	% of total	Pure	% of pure
Jupiter	Saturn	22102	4.78	1759	**7.76**
Jupiter	Uranus	21706	4.69	857	3.95
Jupiter	**Neptune**	11376	2.46%	494	4.34
Saturn	Uranus	2136	0.46	57	2.67
Uranus	Neptune	119	2.41	21	**17.65**
	Total	**81045**	**17.53%**	**4295**	**5.30%**

Number of asteroids vs planet

Table: Number of transient plus pure (T+P) three-body resonant asteroids and pure (P) resonant asteroids vs planet involved in the resonance.

Planet	T+P	% of total	Pure	% of pure
Venus	3881	0.84	87	2.24
Earth	6923	1.50	291	4.20
Mars	13446	2.91	944	7.02
Jupiter	49434	10.69	2945	5.96
Saturn	29084	6.29	2021	6.95
Uranus	32332	6.99	1274	3.94

Number of asteroids vs planet

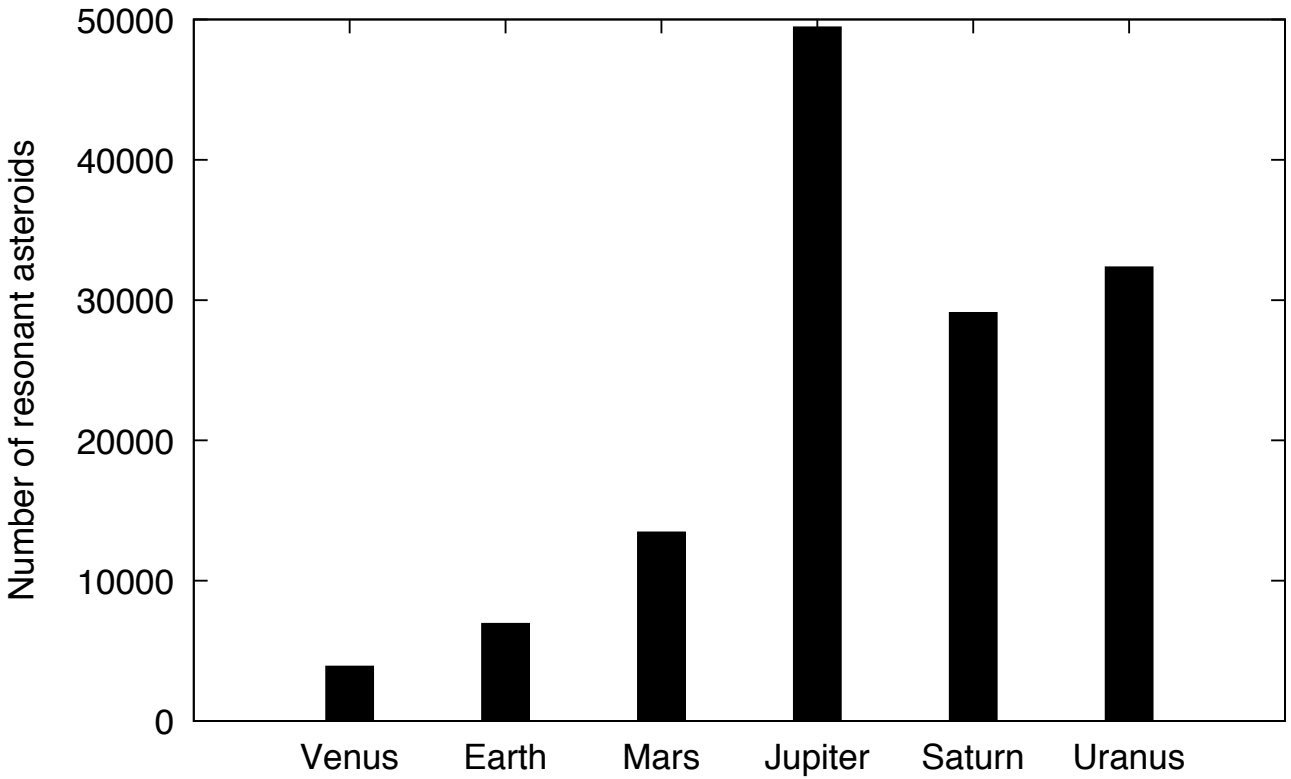


Figure: Number of resonant asteroids for the planets.

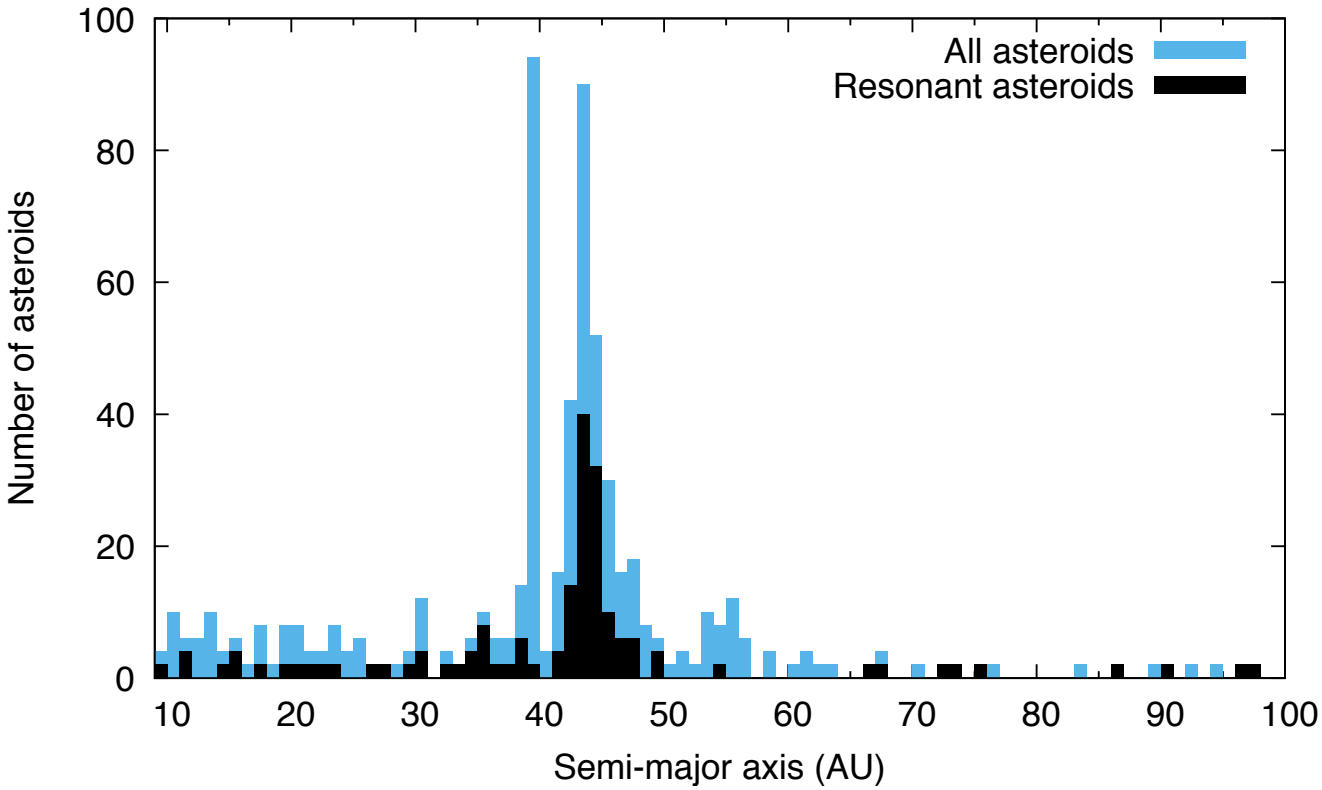
TNO resonant asteroids

Table: Top five most "populated" resonance with Uranus and Neptune

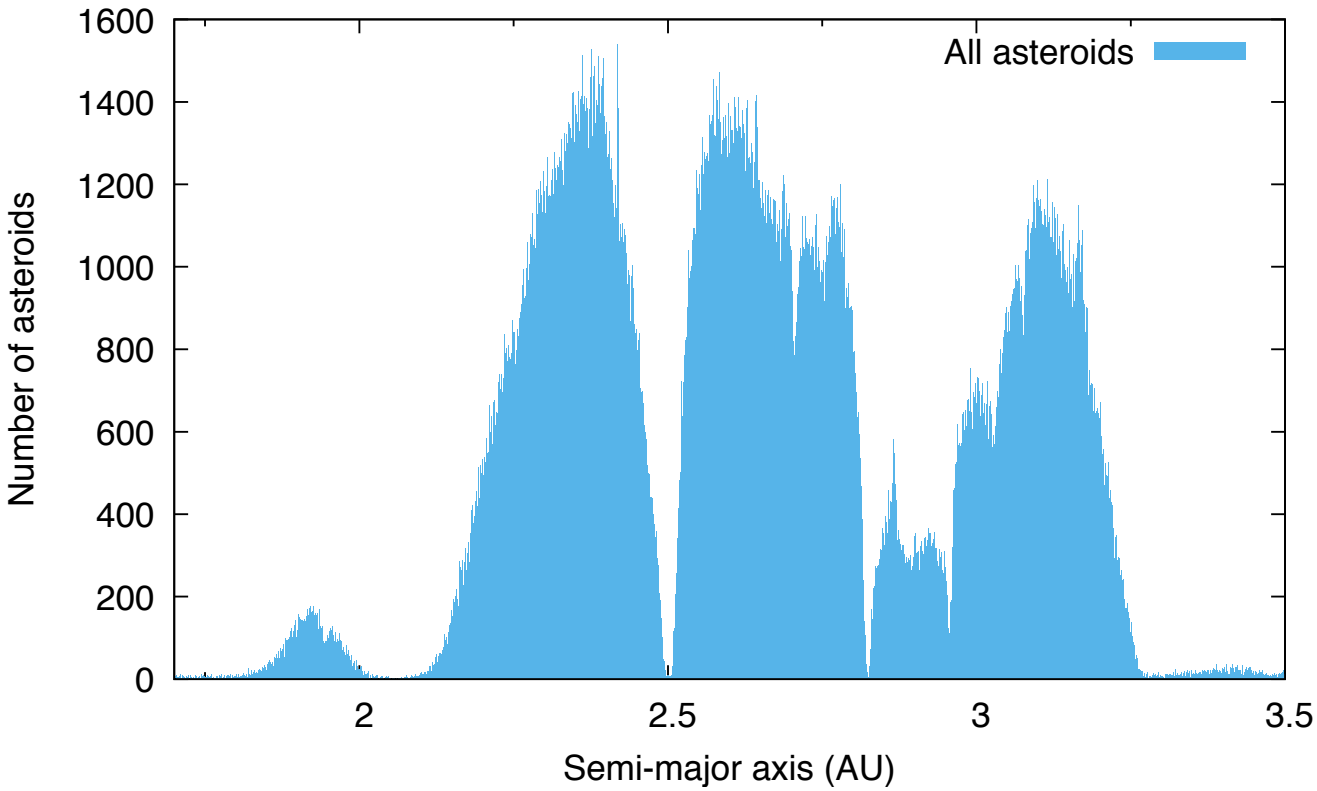
Resonance			a_{res}	T+P	P
3U	-7N	2	45.1133	12	2
4U	-5N	-5	44.4904	8	0
1U	-6N	7	44.0420	7	0
3U	-3N	-5	44.0558	7	1
2U	-1N	-5	43.6328	6	0

- Number of asteroids with semimajor axis in $[39.0, 55.0]$ is equal to **201**, number of resonant — **75** (37.31%).

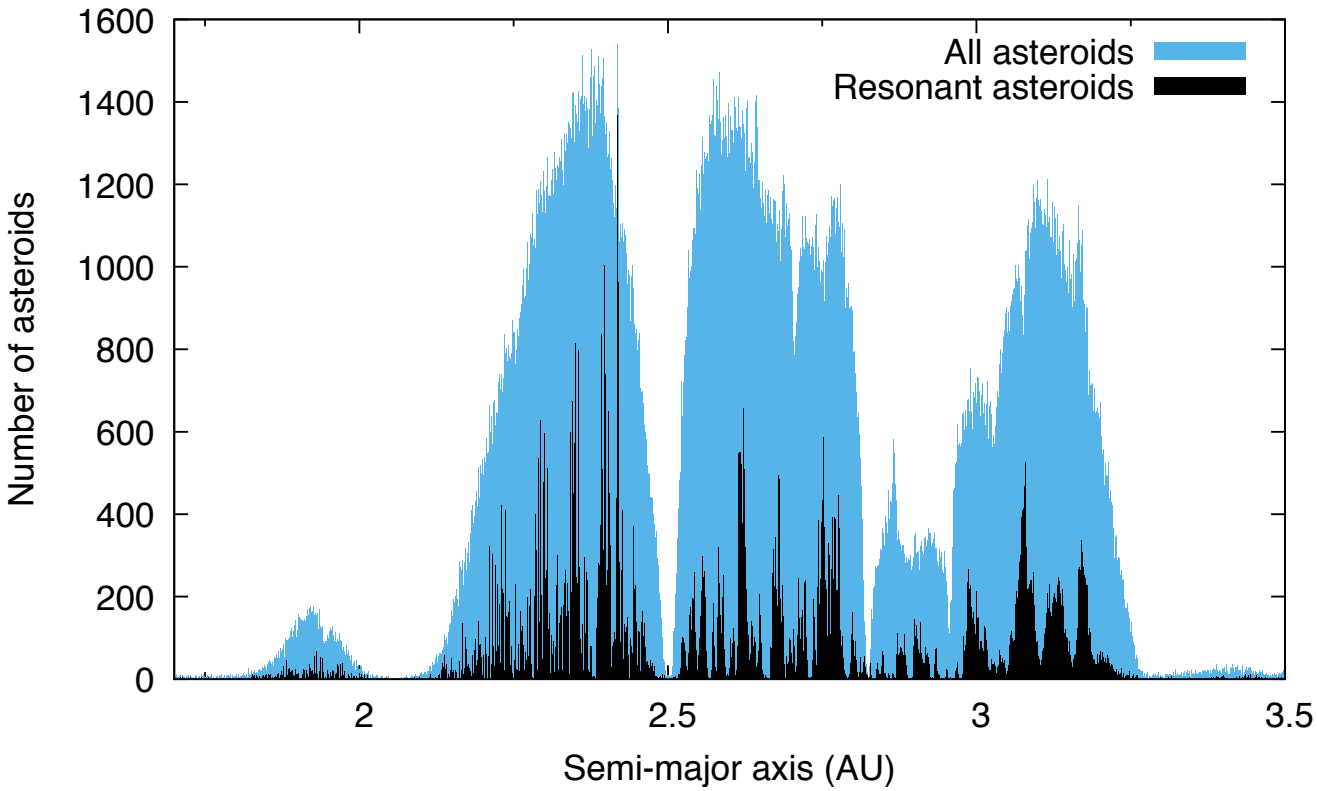
Distribution of resonant TNO



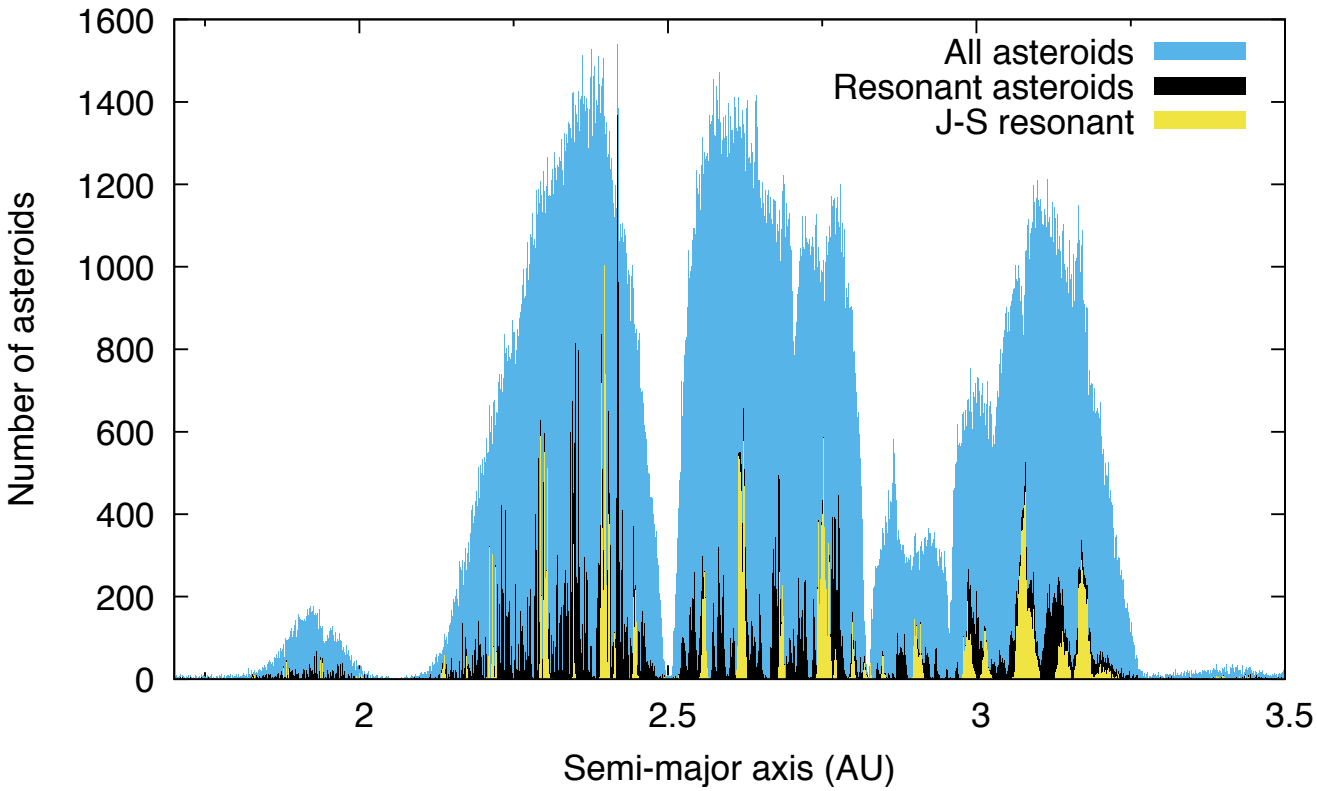
Distribution of asteroids in main belt



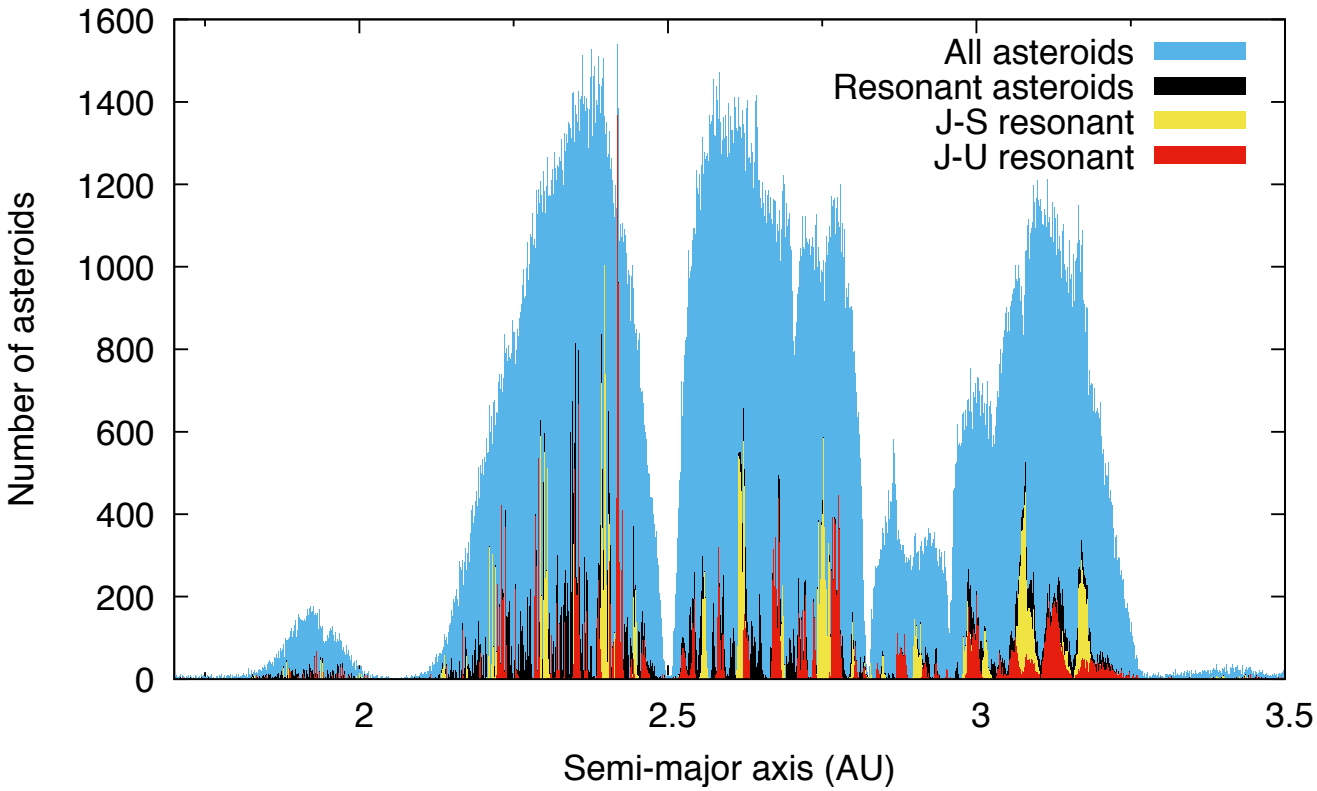
Distribution of resonant asteroids in main belt



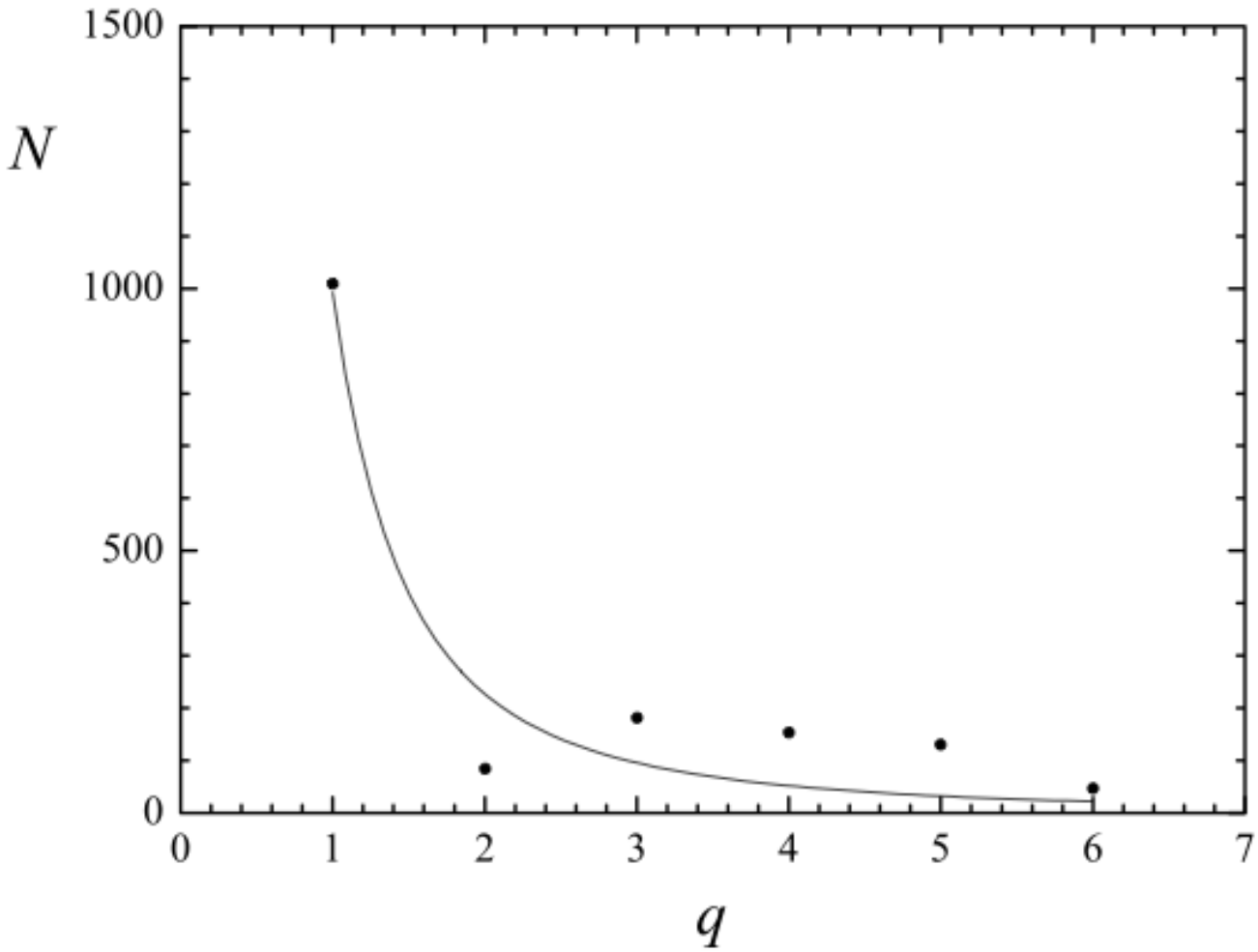
Distribution of resonant asteroids in main belt



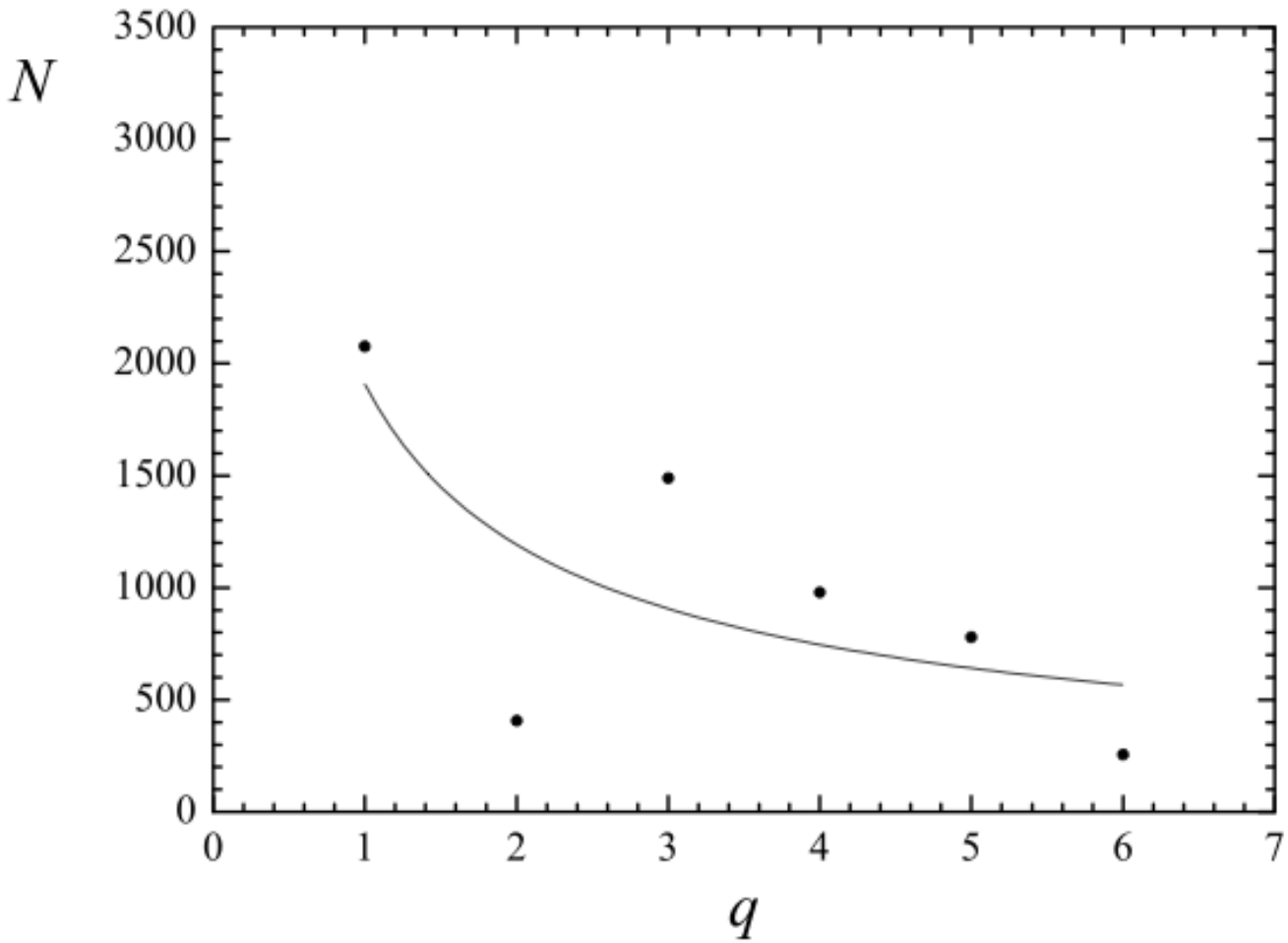
Distribution of resonant asteroids in main belt



Abundances for pure resonant asteroids



Abundances



Number of asteroids vs order vs planets

Planets	0	1	2	3	4	5	6
V–E	1	82	10	225	580	153	256
V–J	50	71	64	44	14	87	12
V–U	174	346	176	136	81	48	66
E–M	260	316	234	251	892	492	543
E–J	30	145	108	68	108	6	105
E–U	141	319	322	324	299	136	278
M–J	483	929	711	1164	533	668	226
M–S	379	670	727	752	497	462	264
M–U	512	1360	813	894	628	873	564
J–S	2845	5576	2741	4620	2854	1890	1583
J–U	1436	3620	3336	5150	2856	2769	2539
S–U	123	183	161	365	425	496	383
U–N	11	17	30	14	16	13	18

A few words about automation

- Install software in one command (it doesn't matter what technologies were used);
- distribute software without additional complications;
- deploy application to the remote/local computers and build natively clusters.
- maintain and update application on every node in the cluster.
- safely pass commands to nodes over network.

https://www.docker.com

They were right!

The three-body mean motion resonances seem to be the main actors structuring the dynamics in the main asteroid belt.

Nesvorny, Morbidelli, ApJ 116 (1998)

Conclusions

- The fraction of asteroids in three-body resonances (transient plus pure) turns out to be $\approx 17.53\%$ (5.30% of them are pure) of the total studied set of ≈ 460000 asteroids.
- There are number of asteroids in each possible planet configuration.
- Three-body resonances are "more populated" for TNO ($\approx 37\%$ instead of $\approx 18\%$).