Google matrix of the world network of economic activities

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Introduction

- Aim : Study relationships among multiple industrial sectors between countries, using Google matrix framework.
- Methodology : Google Personalized Vector Method (GPVM) [1] L.Ermann and D. Shepelyansky (2015) (COMTRADE)
- Datasets : Input/Output matrices generated by joint initiative (TiVA) of OECD and WTO
- Key results : 2D reduced rankings
 - Sensitivity to price variations

Introduction : GPVM

		М		Country 1			Country 2		
	cc'ss'		S1	S2	S3	S1	S2	S3	
in it for	<u></u> у1	S1	0.7	1.2	1.0	2.1	8.0	3.9	
	unti	S2	3.7	5.0	5.5	6.7	1.9	4.4	
A ANTIC	С О	S3	0.3	0.0	1.0	1.0	2.7	4.2	
	y 2	S1	8.3	7.9	3.5	3.2	4.6	4.8	
	untr	S2	0.9	7.1	1.3	3.5	5.5	3.8	
	Col	S3	0.0	3.3	2.4	2.7	4.0	0.0	

Matrix size $N = N_c$ countries $x N_s$ sectors

Nodes : pairs of elements composing the network, e.g here (country, sector)
Links : directed links e.g (country1, sector3) → (country2, sector2)
M_{cc'ss'}: money transfer matrix from country c' sector s' to sector s in country c

Introduction : GPVM

$\mathbf{M}_{\mathbf{cc'ss'}}$		Country 1			Country 2			
		S1	S2	S3	S1	S2	S3-	
Country 1	S1	0.7	1.2	1.0	2.1	8.0	3.9	
	S2	3.7	5.0	5.5	6.7	1.9	4.4	
	S3	0.3	0.0	1.0	1.0	2.7	4.2	
Country 2	S1	8.3	7.9	3.5	3.2	4.6	4.8	
	S2	0.9	7.1	1.3	3.5	5.5	3.8	
	S3	0.0	3.3	2.4	2.7	4.0	0.0	

 $V_{c's'} = \sum_{cs} M_{cc',ss'}$

► Total import
$$V_{cs} = \sum_{c',s'} M_{cc',ss'}$$

For comparison purpose, importRank (based on money volume) : $\hat{P}_i = V_{cs}/V$ with $i = s + (c - 1)N_s$ and total volume $V = \sum_{c,c',s,s'} M_{cc',ss'}$

Google matrix : $G_{ij} = \alpha S_{ij} + (1 - \alpha)v_i e_j$

$$S_{i,i'} = \begin{cases} M_{cc',ss'}/V_{c's'} & \text{if } V_{c's'} \neq 0\\ 1/N & \text{if } V_{c's'} = 0 \end{cases}$$

 v_i personalization vector normalized to 1

Introduction : GPVM

 $v_i = \frac{V_{cs}}{N_c \sum_{s'} V_{cs'}}$ to keep the relative trade volumes in sectors

1st iteration : Compute PageRank as Gp=p with α =0.5 **2nd** iteration : Compute PageRank as Gp=p with α =0.5 and $v'(i) = \frac{P_s}{N_c}$

Why this choice of vector ?

- → One of Google matrix's strength is the « democratic » treatment of nodes.
- → Here due to the multiple links type (sectors) the nodes are pairs of country and sectors.
- \rightarrow There is no meaning in trying to treat equaly the nodes in our case.

GPVM keeps the equal treatment in countries and breaks democracy in sectors

Reduced PageRank

The previous steps can be derived for the inverted network to get the ExportRank and the CheiRank, the following reductions also applies to CheiRank.

Tracing over sectors : $P_c = \sum_s P_{cs} = \sum_s P(s + (c-1)N_s)$ Tracing over countries : $P_s = \sum_c P(s + (c-1)N_s) = \sum_c P_{cs}$

K : index defined by ordering PageRank probabilities decreasingly, P(K) > P(K+1).

For CheiRank, the index is denoted by **K***.

Similar indexes can be defined for countries K_c, K_c^* for sectors K_s, K_s^* and for comparison purposes we define similar indexes for Import and Export Ranks.

Datasets : I/O matrices

2 major aspects of international economy

Commodity trade

- exchange of products
- large datasets, relatively easy to produce
- COMTRADE database

[1] L.Ermann and D. Shepelyansky (2015)

Activity trade

- exchange of economic activities and services
- difficult to produce, scarce data
- Input/Output matrices

[2] V.K, H.E and D. S (2015)

Our datasets : I/O matrices for years 1995 and 2008 Nc = 58 (57 countries and rest of the world) Ns = 37 sectors $N = Nc \times Ns = 2146$ Excluding internal exchanges !

Datasets : I/O matrices

Examples of sectors and their correspondance to the International Standard Industrial Classification

	OECD ICIO Category	ISIC Rev. 3 correspondence
		01 - Agriculture, hunting and related service activities
		02 - Forestry, logging and related service activities
1	C01T05 AGR	05 - Fishing, operation of fish hatcheries and fish farms; service activities incidental to fishing
		10 - Mining of coal and lignite; extraction of peat
		11 - Extraction of crude petroleum and natural gas; service activities incidental to oil and gas extraction
		12 - Mining of uranium and thorium ores
		13 - Mining of metal ores
2	C10T14 MIN	14 - Other mining and quarrying
		15 - Manufacture of food products and beverages
3	C15T16 FOD	16 - Manufacture of tobacco products
		17 - Manufacture of textiles
		18 - Manufacture of wearing apparel; dressing and dyeing of fur
4	C17T19 TEX	19 - Tanning and dressing of leather; manufacture of luggage, handbags, saddlery, harness and footwear
		20 - Manufacture of wood and of products of wood and cork, except furniture;
5	C20 WOD	Manufacture of articles of straw and plaiting materials
		21 - Manufacture of paper and paper products
6	C21T22 PAP	22 - Publishing, printing and reproduction of recorded media
7	C23 PET	23 - Manufacture of coke, refined petroleum products and nuclear fuel
8	C24 CHM	24 - Manufacture of chemicals and chemical products
0	CON DDD	

Spectrum and PageRank

PageRank and reduced PageRank probability decays



2D Rankings - (K,K^{*})

GPVM(PageRank/CheiRank) (2008) Volume (Import/Export) (2008)



Position of USA improved while China weakened :

► Possibly due to broader network of USA in important activity sectors

2D Rankings - (K,K)



- Countries are distributed around diagonal
- e.g Russia and Saudi Arabia much more important (imrpoved ranking)

2D Rankings - (K,K)



• Progression of emerging countries are visible : India, China and Russa improved, but Brazil weakened a bit.



- Some sectors are oriented in export (e.g s=2) others in import (e.g s=23)
- GPVM highlights node connectivity : e.g s=23 is a leading export sector while in monetary volume it would be s=7

35 C85 HTH 36 C90T93 OTS

37 C95 PVH



The change of sector importance in time is well visible for the higest ranking nodes



Sensitivity to price variation

- For a given sector s, increase the money transfert in $\mathbf{M}_{\mathbf{cc'ss'}}$ from 1 to $\mathbf{1+\delta}_{\mathbf{s}}$
- Recompute G and G* and get new P and new P*
- Determine the derivatives $\mathbf{D} = \mathbf{dP}/\mathbf{d\delta}_{s}$ and $\mathbf{D}^{*} = \mathbf{dP}^{*}/\mathbf{d\delta}_{s}$
- To highlight relative changes, compute $\mathbf{D}_1 = d \ln(\mathbf{P})/d\delta_s$ and $\mathbf{D}_1^* = d \ln(\mathbf{P}^*)/d\delta_s$



Example of derivatives computed for price increase in sector 7 with data from 2008 ($s{=}2$ MIN / $s{=}7$ PET /s=8 CHM / s=18 MTR ~/ s=25 TRN)

Sensitivity to price variation

For comparison, the derivatives computed using ImportRank and ExportRank (Volume) for the same dataset are shown below :



PageRank/CheiRank give a better picture of price sensitivity than volume ranks because the network structure is taken into account

Trade Balance

Balance :

$$B_{c} = \sum_{s} (P_{cs}^{*} - P_{cs}) / \sum_{s} (P_{cs}^{*} + P_{cs})$$
$$= (P_{c}^{*} - P_{c}) / (P_{c}^{*} + P_{c})$$

Top panel shows balance computed with PageRank for year 2008, bottom panel shows the same datasets with volume ranks.





Trade Balance Sensitivity

- Compute new P and P* after price increase in a given sector
- Determine new balance value
- Compute derivative dB/dδ_s

Top panel shows the balance sensitivity to cost increase in sector 7 (manufacture of coke, refined petroleum and nuclear fuels) For comparison, the same is shown with volume ranks in bottom panel





Trade Balance Sensitivity

Examples of balance derivatives profile For two different countries (black Curves).

China has significant positive values for sectors s=4, 14, 16. (manufacture of textile ; office machinery ; radio etc) and negative values for s=7, 25, 27 (petroleum ; land transport ; financial intermediation).

USA has significant positive values for sectors s=23, 29, 32. (sales of motor vehicles ; renting machinery ; other business) and negative values for s=11(manufacture of basic metals).



B_c Sensitivity to labor cost

What about the response of the world to a price increase of all industries in a given country ?

- Small increase 1 to $1 + \sigma_{a}$
- Compute new P,P* and B
- Determine **dB/do**

Here the effects of labor cost increase are shown for 2008, the increase happens in China (top panel) and USA (bottom).



Conclusion

- Better understanding of international trade network could provide insight and help to policy makers
- Google matrix approach is quite simple and powerful to take the network structure into account and giving a more correct picture than pure monetary rankings
- COMTRADE and I/O matrices provide two complementary datasets to apply network approach an gain insight of international economy
- Price shocks are a very complicated phenomenon with highly non trivial effects, but the GPVM gives a first decent approach to tackle this problem

References

[1] L.Ermann and D.L. Shepelyansky, Google matrix analysis of the multiproduct world trade network, Eur. Phys. J. B 88, 84 (2015)

[2] V. Kandiah, H. Escaith and D.L. Shepelyansky, Google matrix of the world network of economic activities, Eur. Phys. J. B 88, 84 (2015)