Treatment of Sound on Quantum Computers

Quantware MIPS Center, Laboratoire de Physique Théorique
UMR 5152 du CNRS, Université Paul Sabatier
Toulouse, France www.quantware.ups-tlse.fr

and Ecole Normale Supérieure, rue d'Ulm, Paris, France

Jae-Weon Lee (EC IST-FET project EDIQIP, CNRS Toulouse)

Alexei Chepelianskii (ENS, Paris)

Dima Shepelyansky (CNRS, Toulouse)

SUPPORT: ARO/NSA/ARDA program and EU IST-FET EDIQIP

Abstract

We study numerically how a sound signal stored in a quantum computer can be recognized and restored with a minimal number of measurements in presence of random quantum gate errors. A method developed uses elements of MP3 sound compression and allows to recover human speech and sound of complex quantum wavefunctions.

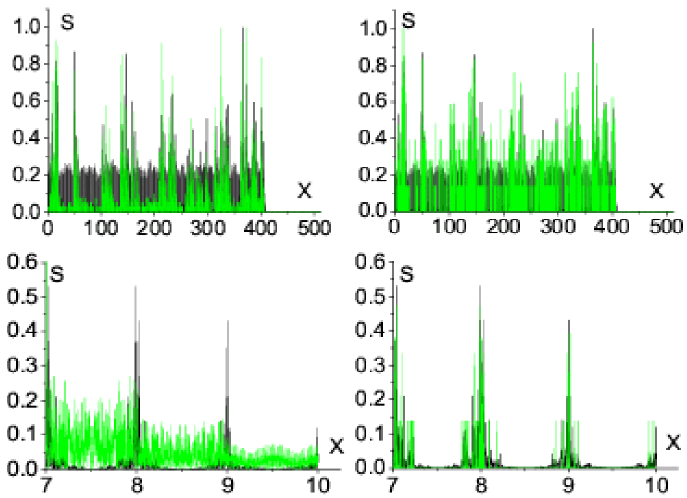
Note: quantum sounds are available at www.quantware.ups-tlse.fr/qaudio/

"Good afternoon, gentlemen. I am a HAL 9000 computer. I became operational at the H.A.L. lab in Urbana, Illinois on the 12th of January ..."

For the standard audio sampling rate of $44kHz$ a quantum computer with 20 qubits (two-level quantum systems) can store a mono audio signal of 23 seconds. A quantum computer with 50 qubits may store an amount of information exceeding all modern supercomputer capacities (1000 years of sound). Thus the development of readout methods, which in presence of imperfections can recognize and restore the sound signal via a minimal number of quantum measurements, becomes of primary importance.

To study this problem we choose the following soundtrack pronounced by HAL in the Kubrick movie "2001: a space odyssey": *"Good afternoon, gentlemen. I am a HAL 9000 computer. I became operational at the H.A.L. lab in Urbana, Illinois on the 12th of January"*. The duration of this recording is 26 seconds and at a sampling rate $f = 8kHz$ it can be encoded in the wavefunction of a quantum computer with $n_q = 18$ qubits (the HAL speech is thus zero padded to last 32 seconds). This rate gives good sound quality and is more appropriate for our numerical studies. Digital audio signals can be represented by a sequence of samples with values s_n in the interval $(-1, 1)$ so that the n-th sample gives the sound at time $t = n/f$. This signal can be encoded on a quantum computer by the following wavefunction $\psi = A \sum_n s_n |n>$, where A is normalization constant. The state $|n>$ represents the multiqubit eigenstate $|a_1...a_i...a_{n_q}>$ where a_i is 0 or 1 corresponding to the lower or upper qubit state, the sequence of a_i gives the binary representation of n.

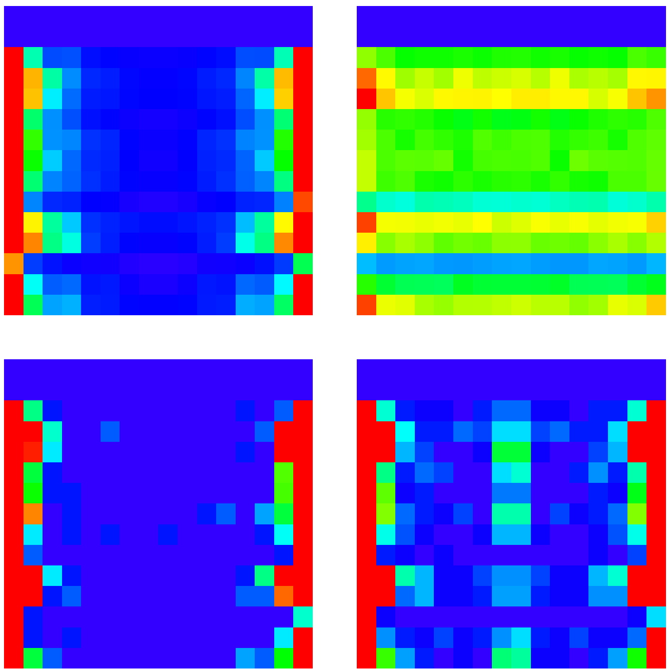
Using numerical simulations we test various approaches to the readout problem of the above signal encoded in the wavefunction of a quantum computer. Direct measurements of the wavefunction do not allow to keep track of the sign of s_n and many measurements are required to determine the amplitude $|s_n|$ with good accuracy. Another strategy is to use the analogy with the MP3 coding. With this aim we divide the sound into consecutive frames of fixed size $\Delta n = 2^{n_f}$ where n_f can be viewed as the number of qubits required to store one frame. We choose these qubits to be the n_f least significant qubits in the binary representation of $n = (a_1 ... a_{n_q - n_f + 1} ... a_{n_q})$. Then we perform QFT on these n_f qubits that corresponds to applying FFT to all the $2^{n_q - n_f}$ frames of the signal in parallel. This requires $n_f(n_f + 1)/2$ quantum gates contrary to $O(n_f 2^{n_f})$ classical operations for FFT. After this transformation the wavefunction represents the instantaneous spectrum of the sound signal evolving in time from one frame to another. This way the most significant $n_q - n_f$ qubits store the frame number k while the least significant n_f qubits give the frequency harmonic number j. Hence, after QFT the wave function has the form $\psi = \sum_{k,j} S_{k,j} |k, j>$ where $S_{k,j}$ is the complex amplitude of the j-th harmonic in the k-th frame. The measurements in this representation gives the amplitudes $|S_{k,j}|$ while phase information is lost. However for sound the main information is stored in the spectrum amplitudes and the ear can recover the original speech even if the phases are all set to zero. Thus the recovered signal is obtained by the inverse classical FFT and is given by $s'_n = \sum_j |S_{k,j}| e^{2\pi i j m / \Delta n}$ with $n = k\,\Delta n + m$ (to listen sound we use $Re(s'_n)$). For time domain measurements of s_n the recovered signal is $\tilde{s}_n = |s_n|$.



Sound signal spectrum as a function of frame number $x = n/\Delta n$. On all panels the black curves show the spectrum of the original signal s_n. On the left top panel the green/gray curve represents the spectrum of $\tilde{s}_n$ obtained with $M = 5$ measurements per frame in time domain. On the right top panel it shows the spectrum of s'_n for the same number of measurements M performed after QFT. Bottom panels show these spectra on a smaller scale.

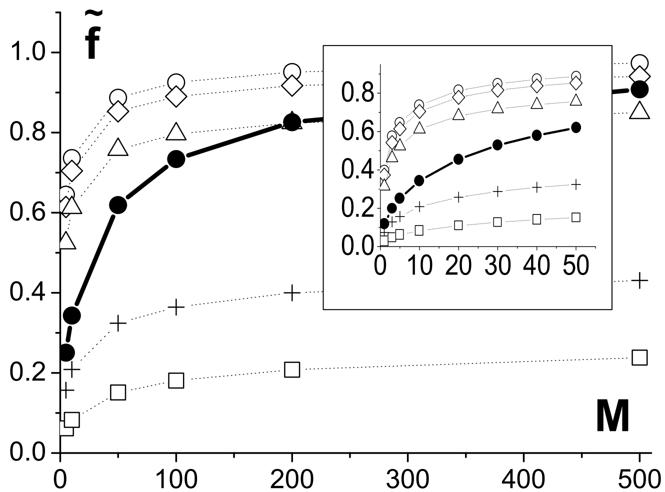
What are effects of error in quantum gates on sound reconstruction?

Spectral diagram of sound signal



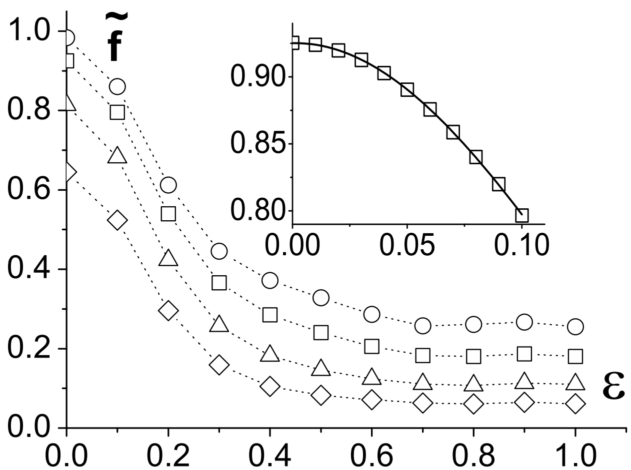
Coarse grained diagram for the signal of Fig. 1 obtained by measurements of qubits 1 to 4 and 10 to 13 (see text). Top left: original signal; bottom left: distribution obtained after QFT with the same number of measurements as in Fig. 1; bottom right: same as bottom left with the noise in the quantum gates ($\epsilon = 0.05$); top right: diagram obtained from measurements in time domain (with the same number of measurements as in bottom right). The color represents amplitude of the spectrum: blue for zero and red for maximal values. The horizontal/vertical axis corresponds to coarse grained frequency/time.

Fidelity properties



Fidelity $\tilde{f}$ for sound signals s'_n (open circles) and $\tilde{s}_n$ (full circles) as a function of number of measurements M. For s'_n fidelity is shown for various amplitudes of noise in the QFT gates with $\epsilon = 0$ (o), 0.05 (diamonds), 0.1 (triangles), 0.3 (+), 1 (squares). Inset shows data at small M scale.

The global quality of the recovered signal $\tilde{s}_n$ (or s'_n) obtained via a finite number of measurements is convenient to characterize by the fidelity defined as $\tilde{f} = |\sum_n \tilde{s}_n^{(')*} \tilde{s}_n^{(')}(M,\epsilon)|/R$ with $R = (\sum_n |\tilde{s}_n^{(')}|^2 \sum_n |\tilde{s}_n^{(')}(M,\epsilon)|^2)^{1/2}$. Here, $\tilde{s}_n^{(')}(M,\epsilon)$ is the signal obtained in a way described above with M measurements per frame in time domain ($\tilde{s}_n(M)$) or in frequency domain after QFT with noisy gates ($s'_n(M,\epsilon)$).

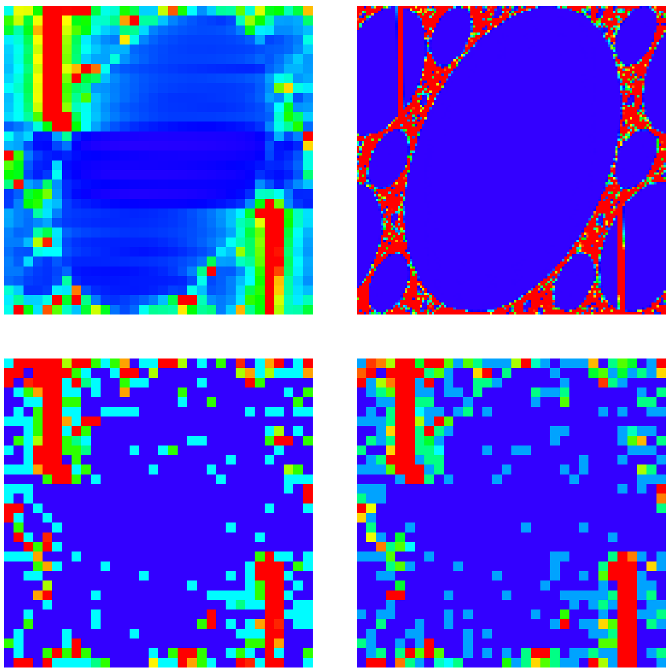


Dependence of fidelity $\tilde{f}$ for the sound signal s'_n on the noise amplitude ϵ for the number of measurements $M = 1000$ (o), 100 (squares), 20 (triangles), 5 (diamonds). Inset shows a data fit $1 - \tilde{f} = 0.36\epsilon^2 n_f^2$ (full curve) at small ϵ, $M = 100$.

At small ϵ the drop of $\tilde{f}$ is quadratic in ϵ $(1 - \tilde{f} \sim \epsilon^2 n_f^2)$ since each gate transfers about of ϵ^2 amount of probability from ideal computational state to all other states.

The obtained results show that the MP3-like strategy adapted to the quantum signals allows to recover human speech with a significantly smaller number of measurements with a reduction factor of 10-20.

Sound of quantum wavefunctions



Left: coarse grained diagram of $S^{(g)}$ for the sound signal s'_n obtained from the quantum computation of the wavefunction in quantum saw-tooth map after $t = 100$ map iterations, top panel shows exact distribution $S^{(g)}$ and bottom panel is for a number of measurements as in Figs. 1,2. Bottom right: same as bottom left but with noise amplitude $\epsilon = 0.05$ in the quantum gates. Right top: the exact Husimi distribution $h(l, \theta)$. The initial state is a momentum eigenstate at $l = 100 - N/2$ (for $h(l, \theta)$ x and y axes corresponds to l and θ respectively; we use sampling rate $f = 1$ kHz for this case with $n_q = 14$, $K = -0.5$).

For certain quantum objects such an encoding can be done efficiently. As an example we consider the wavefunction evolution described by the quantum sawtooth map

$$\overline{\psi} = \hat{U}\psi = e^{-iT\hat{l}^2/2} e^{ik\hat{\theta}^2/2}\psi, \quad (1)$$

where $\hat{l} = -i\partial/\partial\theta$, $\hbar = 1$, k, T are dimensionless map parameter and $\overline{\psi}$ is the value of ψ after one map iteration (we set $\hbar = 1$). In the semiclassical limit $k \gg 1$, $T \ll 1$ the chaos parameter of the model is $K = kT = const$. The efficient quantum algorithm for the simulation of this complex dynamics was already developed and tested (PRL **86**, 2890 (2001)). The computation is done for the wavefunction ψ on a discrete grid with $N = 2^{nq}$ points with $\theta_n = 2\pi n/N$, $n = 1, ..., N$ in θ-representation and $l + N/2 = 1, .., N$ in momentum representation. Here, as before n_q is the number of qubits in a quantum computer and in θ-representation $\psi = \sum_n \psi(\theta_n)|n>$ is encoded in the register $|n> = |a_1...a_i...a_{n_q}>$. The transition between n and θ representations is done by QFT and one map iteration is computed in $O(n_q^2)$ quantum gates for an exponentially large vector of size 2^{nq}. To study the sound of quantum wavefunctions of map (1) we choose here a case with $K = -0.5$, $T = 2\pi/N$ and $n_q = 14$ corresponding to a complex phase space structure.

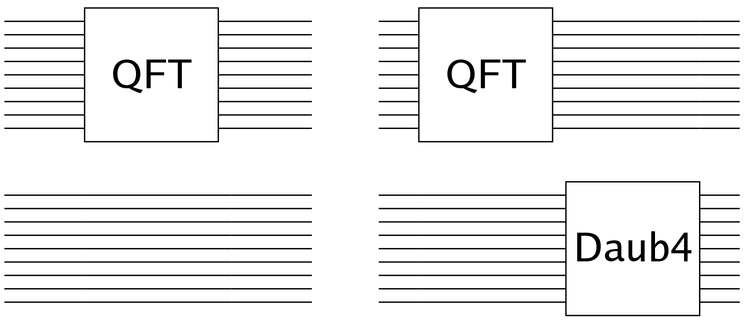
The coarse graining of $S_{k,j}$ is done by measuring 5 most significant and 5 less significant qubits corresponding to $a_1 a_2 \ldots a_5$ and $a_{10} a_{11} \ldots a_{14}$ that gives coarse grained distribution $S^{(g)}$ in 32×32 cells. The diagrams of $S^{(g)}$ obtained for infinite and finite number of measurements are displayed in Fig. 5 (left top and bottom respectively). The exact diagram shows an interesting structure which is recovered with a finite number of measurements. This structure remains robust against noise in the quantum gates used for computation of 100 map iterations and final QFT

The origin of this structure becomes clear after its comparison with the coarse grained Wigner function called the Husimi distribution $h(\theta, l)$ which is shown in Fig. 5 (right top) which is very close to $S^{(g)}$ (left top). Indeed, $h(\theta, l)$ is defined in the phase space (l, θ) by

$$h(l,\theta) = \sum_{l'=l-N/2}^{l+N/2} G(l'-l)\psi(l')e^{il'\theta} \tag{2}$$

where the gaussian smoothing function is $G(l'-l) = (T/\pi)^{1/4} e^{-T(l'-l)^2/2}/\sqrt{N}$. The Husimi distribution is always positive and gives a direct comparison between the classical phase space Liouville density distribution and a quantum wavefunction.

Further Wavelet compression

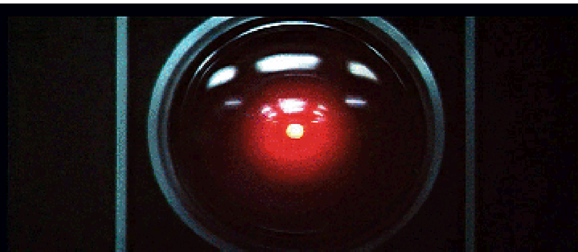


	Hal	QFT	Daub4
entropy	10.9	8.41	8.25
participation ratio	29003	1405	207

We tried to apply a Daubechies 4 wavelet transform on the high order (most significant) qubits in order to take advantage of the similarities between identical Fourier components at different times. The participation ratio is reduced by a factor of 7 but the reconstructed audio and the signal obtained from measurements in frequency domain are of similar quality.

References

1. http://www.mpeg.org/MPEG/audio.html
2. M.A. Nielsen and I.L. Chuang *Quantum Computation and Quantum Information*, Cambridge Univ. Press, Cambridge (2000).
3. Sound is available at http://www.palantir.net/cgi-bin/file.cgi?file=wav/hal9000.wav
4. B. Georgeot and D. L. Shepelyansky, Phys. Rev. Lett. **86**, 2890 (2001).
5. G. Benenti, G. Casati, S. Montangero and D. L. Shepelyansky, Phys. Rev. Lett. **87**, 227901 (2001).
6. E. Wigner Phys. Rev. **40**, 749 (1932); M. V. Berry, Phil. Trans. Royal Soc. **287**, 237 (1977).
7. S.-J. Chang and K.-J. Shi, Phys. Rev. A **34**, 7 (1986).
8. K.M.Frahm, R.Fleckinger and D.L.Shepelyansky, Eur. Phys. J. **D 29** (2004), 139 [highlight paper of the issue] (quant-ph/0312120)



The HAL Quantum Speech

- original HAL speech

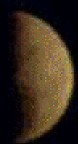
measurements in frequency domain
- 10 measurements per frame

measurements in frequency domain with quantum errors (amplitude of gate errors 0.05)
- 10 measurements per frame

measurements in time domain
- 10 measurements per frame

quantum sound of coarse grained Wigner function
- 10 measurements per frame

- download all sound files (2 Mb)



"Good afternoon, gentlemen. I am a HAL 9000 computer. I became operational at the H. A. L. lab in Urbana, Illinois on the 12th of January"

This sound is enconded on an 18 qubit quantum computer and it is recovered with a finite number of quantum measurments performed in time or frequency domain (MP3 like) in presence of gate imperfections.

We can also listen the sound of a complex quantum wavefunction generated by an efficient quantum algorithm (14 qubits).

Quantum computations are done
on Quantium I simulated numerically on Pentium IV

The paper is at http://xxx...

The original HAL speech and *2001 A Space odyssey* pictures are taken from http://www.palantir.net/2001/