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Analysis of world terror networks from the reduced Google matrix of Wikipedia

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Abstract. We apply the reduced Google matrix method to analyze interactions between 95 terrorist groups and determine their relationships and influence on 64 world countries. This is done on the basis of the Google matrix of the English Wikipedia (2017) composed of 5 416 537 articles which accumulate a great part of global human knowledge. The reduced Google matrix takes into account the direct and hidden links between a selection of 159 nodes (articles) appearing due to all paths of a random surfer moving over the whole network. As a result we obtain the network structure of terrorist groups and their relations with selected countries including hidden indirect links. Using the sensitivity of PageRank to a weight variation of specific links we determine the geopolitical sensitivity and influence of specific terrorist groups on world countries. The world maps of the sensitivity of various countries to influence of specific terrorist groups are obtained. We argue that this approach can find useful application for more extensive and detailed data bases analysis.

1 Introduction

"A new type of terrorism threatens the world, driven by networks of fanatics determined to inflict maximum civilian and economic damages on distant targets in pursuit of their extremist goals" [1]. The origins of this world wide phenomenon are under investigation in political, social and religious sciences (see e.g. [1–4] and references therein). At the same time the number of terrorist groups is growing in the world [5] reaching over 100 officially recognized groups acting in various countries of the world [6,7]. These numbers become quite large and the mathematical analysis of multiple interactions between these groups and their relationships to world countries is getting of great timeliness. The first steps in this direction are reported in a few publications (see e.g. [8,9]) showing that the network science methods (see e.g. [10]) should be well adapted to such type of investigations. However, it is difficult to obtain a clear network structure with all dependencies which are emerging from the surrounding world with all its complexity.

In this work we use the approach of the Google matrix G and PageRank algorithm developed by Brin and Page for large scale WWW network analysis [11]. The mathematical and statistical properties of this approach for various networks are described in [12,13]. The efficiency of these methods are demonstrated for Wikipedia and world trade networks in [14–16]. For the analysis of the terror networks we use the reduced Google matrix approach developed recently [17–19]. This approach selects from a global large scale network a subset of nodes of interest and constructs the reduced Google matrix G_{R} for this subset including all indirect links connecting the subset nodes via the global network. The analysis of political leaders and world countries subsets of Wikipedia networks in various language editions demonstrated the efficiency of this analysis [18,19]. Here, for the English Wikipedia network (collected in May 2017), we target a subset of $N_g = 95$ terrorist groups referenced in Wikipedia articles of groups enlisted as terrorist groups for at least two countries in [7] (see Tab. 1). The collection of 24 editions of Wikipedia networks dated by May 2017 is available at [20]. In addition we select the group of $N_c = 64$ related world countries given in Table 2. This gives us the size of G_{R} being $N_r = N_g + N_c = 159$ that is much smaller then the global Wikipedia network of $N = 5\,416\,537$ nodes (articles) and $N_\ell = 122\,232\,932$ links generated by quotation links from one article to another. The method of the reduced Google matrix and the obtained results for interactions between terrorist groups and countries are described in the next sections.

We note that the analysis of Wikipedia data and related networks is now in development by various groups (see e.g. [21–23]). Here we used the matrix methods for analysis of Wikipedia networks. These methods have their roots at the investigations of random matrix theory and quantum chaos [24].

Here we present results for English Wikipedia edition but the different cultural views of other language editions of Wikipedia attract growing interest of researchers (see

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Table 1. List of selected terrorist groups (from [7]) attributed to 6 categories marked by color, KG gives the local PageRank index of terrorist groups.

Name	KG	Color	Name	KG	Color
Islamic State of Iraq and the Levant	1	BL	Hezb-e Islami Gulbuddin	49	RD
Al-Qaeda	2	BL	Kach and Kahane Chai	50	BK
Taliban	3	RD	Palestine Liberation Front	51	OR
Provisional Irish Republican Army	4	BK	Harkat-ul-Mujahideen	52	RD
Hamas	5	OR	Kurdistan Free Life Party	53	BK
Hezbollah	6	OR	Indian Mujahideen	54	RD
Muslim Brotherhood	7	BL	Abu Nidal Organization	55	OR
Liberation Tigers of Tamil Eelam	8	RD	Hizbul Mujahideen	56	RD
Kurdistan Workers' Party	9	BK	Libyan Islamic Fighting Group	57	GN
Al-Shabaab (militant group)	10	GN	Islamic State of Iraq and the Levant in Libya	58	GN
ETA (separatist group)	11	BK	Revolutionary People's Liberation Party/Front	59	BK
FARC	12	BK	Al-Mourabitoun	60	GN
Houthis	13	PK	Revolutionary Organization 17 November	61	BK
Al-Nusra Front	14	PK	Holy Land Foundation for Relief and Development	62	OR
Boko Haram	15	GN	Ansar al-Sharia (Libya)	63	GN
Ulster Volunteer Force	16	BK	Al-Itihaad al-Islamiya	64	GN
Shining Path	17	BK	Al-Haramain Foundation	65	BL
Popular Front for the Liberation of Palestine	18	OR	Ansar Bait al-Maqdis	66	PK
Lashkar-e-Taiba	19	RD	Ansaru	67	GN
Hizb ut-Tahrir	20	BL	Babbar Khalsa	68	BL
Al-Qaeda in the Arabian Peninsula	21	PK	Jamaat-ul-Mujahideen Bangladesh	69	RD
Tehrik-i-Taliban Pakistan	22	RD	Force 17	70	OR
Islamic Jihad Mov. in Palestine	23	OR	Kata'ib Hezbollah	71	PK
Ulster Defence Association	24	BK	Kurdistan Freedom Hawks	72	BK
Abu Sayyaf	25	RD	Islamic Jihad Union	73	RD
Real Irish Republican Army	26	BK	Abdullah Azzam Brigades	74	PK
Ansar Dine	27	GN	Moroccan Islamic Comb. Group	75	GN
Jemaah Islamiyah	28	RD	Ansar al-Sharia (Tunisia)	76	GN
Al-Qaeda in the Islamic Maghreb	29	GN	Al-Qaeda, Indian Subcontinent	77	RD
Egyptian Islamic Jihad	30	PK	Jund al-Aqsa	78	PK
Al-Jama'a al-Islamiyya	31	PK	Hezbollah Al-Hejaz	79	PK
Jaish-e-Mohammed	32	RD	Jamaat-ul-Ahrar	80	RD
Aum Shinrikyo	33	RD	Jamaah Ansharut Tauhid	81	RD
United Self-Defense Forces of Colombia	34	BK	Islamic State of Iraq and the Levant ??? Algeria Province	82	GN
Armed Islamic Group of Algeria	35	GN	Osbat al-Ansar	83	PK
Continuity Irish Republican Army	36	BK	International Sikh Youth Federation	84	RD
Movement for Oneness and Jihad in West Africa	37	GN	East Turkestan Liberation Organization	85	RD
Quds Force	38	PK	Great Eastern Islamic Raiders' Front	86	BK
Al-Aqsa Martyrs' Brigades	39	OR	Aden-Abyan Islamic Army	87	PK
Com. Party of the Philippines	40	RD	Al-Aqsa Foundation	88	OR
Caucasus Emirate	41	RD	Khalistan Zindabad Force	89	RD
Haqqani network	42	RD	Mujahidin Indonesia Timur	90	RD
Turkistan Islamic Party	43	RD	Al-Badr	91	RD
Ansar al-Islam	44	PK	Soldiers of Egypt	92	PK
Izz ad-Din al-Qassam Brigades	45	OR	National Liberation Army	93	BK
Lashkar-e-Jhangvi	46	RD	Jundallah	94	RD
Harkat-ul-Jihad al-Islami	47	RD	Army of Islam	95	PK
Islamic Movement of Uzbekistan	48	RD			

Table 2. List of selected countries.

Rank	Name	abr	Rank	Name	abr
1	United States	US	33	Portugal	PT
2	France	FR	34	Ukraine	UA
3	Germany	DE	35	Czech Republic	CZ
4	United Kingdom	GB	36	Malaysia	MY
5	Iran	IR	37	Thailand	TH
6	India	IN	38	Vietnam	VN
7	Canada	CA	39	Nigeria	NG
8	Australia	AU	40	Afghanistan	AF
9	China	CN	41	Iraq	IQ
10	Italy	IT	42	Bangladesh	BD
11	Japan	JP	43	Syria	SY
12	Russia	RU	44	Morocco	MA
13	Spain	ES	45	Algeria	DZ
14	Netherlands	NL	46	Saudi Arabia	SA
15	Poland	PL	47	Lebanon	LB
16	Sweden	SE	48	Kazakhstan	KZ
17	Mexico	MX	49	Albania	AL
18	Turkey	TR	50	United Arab Emirates	AE
19	South Africa	ZA	51	Yemen	YE
20	Switzerland	CH	52	Tunisia	TN
21	Philippines	PH	53	Jordan	JO
22	Austria	AT	54	Libya	LY
23	Belgium	BE	55	Uzbekistan	UZ
24	Pakistan	PK	56	Kuwait	KW
25	Indonesia	ID	57	Qatar	QA
26	Greece	GR	58	Mali	ML
27	Denmark	DK	59	Kyrgyzstan	KG
28	South Korea	KR	60	Tajikistan	TJ
29	Israel	IL	61	Oman	OM
30	Hungary	HU	62	Turkmenistan	TM
31	Finland	FI	63	Chad	TD
32	Egypt	EG	64	South Sudan	SS

e.g. [14,25]) and we think that the extension of this research to other editions will be of significant interest.

2 Reduced Google matrix

It is convenient to describe the network of N Wikipedia articles by the Google matrix G constructed from the adjacency matrix A_{ij} with elements 1 if article (node) j points to article (node) i and zero otherwise. In this case, elements of the Google matrix take the standard form $G_{ij} = \alpha S_{ij} + (1-\alpha)/N$ [11–13], where S is the matrix of Markov transitions with elements $S_{ij} = A_{ij}/k_{out}(j)$, $k_{out}(j) = \sum_{i=1}^{N} A_{ij} \neq 0$ being the node j out-degree (number of outgoing links) and with $S_{ij} = 1/N$ if j has no outgoing links (dangling node). Here $0 < \alpha < 1$ is the damping factor which for a random surfer determines the probability $(1-\alpha)$ to jump to any node; below we use the standard value $\alpha = 0.85$. The right eigenvector of G with the unit eigenvalue gives the PageRank probabilities $P(j)$ to find a random surfer on a node j. We order all nodes P getting them ordered by the PageRank index $K = 1, 2, \ldots, N$ with a maximal probability at $K = 1$. From this global ranking we obtain the local ranking of groups and countries given in Tables 1 and 2.

The reduced Google matrix G_{R} is constructed for a selected subset of nodes (articles) following the method described in [17,18] and based on concepts of scattering theory used in different fields of mesoscopic and nuclear physics or quantum chaos [24]. This matrix has N_r nodes and belongs to the class of Google matrices. In addition the PageRank probabilities of selected N_r nodes are the same as for the global network with N nodes, up to a constant multiplicative factor taking into account that the sum of PageRank probabilities over N_r nodes is unity. The matrix G_{R} is represented as a sum of three matrices (components) $G_{\mathrm{R}} = G_{rr} + G_{\mathrm{pr}} + G_{\mathrm{qr}}$ [18]. The first term G_{rr} is given by the direct links between selected N_r nodes in the global G matrix with N nodes, the second term G_{pr} is rather close to the matrix in which each column is given by the PageRank vector P_r, ensuring that PageRank probabilities of G_{R} are the same as for G (up to a constant multiplier). Therefore G_{pr} does not provide much information about direct and indirect links between selected nodes. The most interesting is the third matrix G_{qr} which takes into account all indirect links between selected nodes appearing due to multiple links via the global network nodes N [17,18]. The matrix $G_{\mathrm{qr}} = G_{\mathrm{qrd}} + G_{\mathrm{qrnd}}$ has diagonal (G_{qrd}) and nondiagonal (G_{qrnd}) parts. The part G_{qrnd} represents the main interest since it describes indirect interactions between nodes. The explicit formulas as well as the mathematical and numerical computation methods of all three components of G_{R} are given in [17–19].

The selected groups and countries are given in Tables 1 and 2 in order of their PageRank probabilities (given by KG rank column for groups and Rank column for countries, respectively). All countries have PageRank probabilities being larger than those of terrorist groups so that they are well separated.

3 Results

In this work we extract from G_{R} a network of 64 countries and 95 groups. This network reflects direct and indirect interactions between countries and groups, which motivates us to study the relative influence of group alliances on the other ones and on the countries. The matrix G_{R} and its three components G_{rr}, G_{pr} and G_{qr} are computed for $N_r = 159$ Wikipedia network nodes formed by $N_c = 64$ country nodes and $N_g = 95$ group nodes. The weights of these three G_{R} components are $W_{rr} = 0.0644$, $W_{\mathrm{pr}} = 0.8769$ and $W_{\mathrm{qr}} = 0.0587$ (the weight is given by the sum of all matrix elements divided by N_r, thus $W_{rr} + W_{\mathrm{qr}} + W_{\mathrm{qr}} = 1$). The dominant component is G_{pr} but as stated above it is approximately given by columns of the PageRank vector so that the most interesting information is provided by G_{rr} and especially the component G_{qr} given by indirect links [18,19].

The matrix elements of $G_{\mathrm{R}}, G_{rr}, G_{\mathrm{qr}}$ corresponding to the part of 95 terrorist groups are shown in the color maps of Figure 1 (indices are ordered by increasing values of KG as given in Tab. 1, thus element with KG1 = KG1 is located at the top left corner). The largest matrix elements of G_{R} are the ones of top PageRank groups of Table 1. Such large values are enforced by G_{pr} component which is dominated by PageRank vector. The elements of G_{rr} and G_{qr} are smaller but they determine direct and indirect interactions between groups.

According to Figure 1 the strong interactions between groups can be found by analyzing G_{qr} looking at new links appearing in G_{qr} and being absent from G_{rr}. As an example we list:

- Tehrik-i-Taliban Pakistan (KG22) and Jundallah (KG94);
- Hamas (KG5) and Izz ad-Din al-Qassam Brigades (KG45);
- Taliban (KG3) and Al-Qaeda in the Arabian Peninsula (KG21);
- Kurdistan Freedom Hawks (KG72) and Kurdistan Workers' Party (KG9).

3.1 Network structure of groups

To analyze the network structure of groups we attribute them to 6 different categories marked by 6 colors in Table 1:

- C1 for the International category of groups operating worldwide (color BL – blue, top group is KG1 ISIS);
- C2 for the groups targeting Asian countries (color RD – red, top group is KG3 Taliban);
- C3 for the groups related with the Israel-Arab conflict (color OR – orange, top group is KG5 Hamas);
- C4 for the groups targeting African countries (color GN – green, top group is KG10 Al-Shabaab);
- C5 for the groups related to Arab countries at Middle East and the Arabian Gulf (color PK – pink, top group is KG13 Houthis);
- C6 for all remaining groups (color BK – black, top group is KG4 IRA).

These 6 categories of groups are related to their activity and their geographical location. Only the category C1 has global international activity, other categories have more local geographical activity. We will see that the network analysis captures these categories.

We order the terror groups by their local PageRank index KG in Table 1 (highest probability of PageRank vector for groups is at $KG = 1$, G is for group). The selected countries are ordered by their local PageRank index K in Table 2 (highest probability of PageRank vector for countries is at $K = 1$).

We analyze the network structure of groups by selecting the top group node of each category in Table 1 and then, their top 4 friends in $G_{rr} + G_{\mathrm{qrnd}}$ (i.e. the nodes with the 4 largest matrix elements of $G_{rr} + G_{\mathrm{qrnd}}$ in the column representing the group of interest. It corresponds to the 4 largest outgoing link weights). From the set of top group nodes and their top 4 friends, we continue to extract the top 4 friends of friends until no new node is added to this network of friends. The obtained network structure of groups is shown in Figure 2. This network structure clearly highlights the clustering of nodes corresponding to selected categories. It shows the leading role of top PageRank nodes for each category appearing as highly central nodes with large in-degree. We note that we speak about networks of friends and followers using the terminology of social networks. Of course, this has only associative meaning (we do not mean that some country is a friend of terrorist group).

The appearance of links due to indirect relationships between groups is confirmed by well-known facts. For instance, it can be seen that Al-Qaeda in the Arabian Peninsula (KG21) is linking Al-Shabaab (KG10) and Houthis (KG13). Al-Qaeda in the Arabian Peninsula is primarily active in Saudi Arabia. It is well known that Saudi Arabia is an important financial support of Al-Shabaab [26] and that Houthis is confronting Saudi Arabia. As such, it makes sense that Al-Qaeda in the Arabian Peninsula links both groups as it is tied to Saudi Arabia.

Another meaningful example is the one of Hezbollah (KG6) and Houthis that share the same ideology, since they are both Shiite and are strongly linked to Iran. From Figure 2, it can be seen that Hezbollah is a direct friend of Houthis. The case of Hamas (KG5) and Hezbollah, that share the same ideology in facing Israel, is highlighted as well in our results. Moreover, Figure 2 shows as well that Hezbollah is the linking group between Hamas and Houthis. Finally, the network of Figure 2 clearly shows

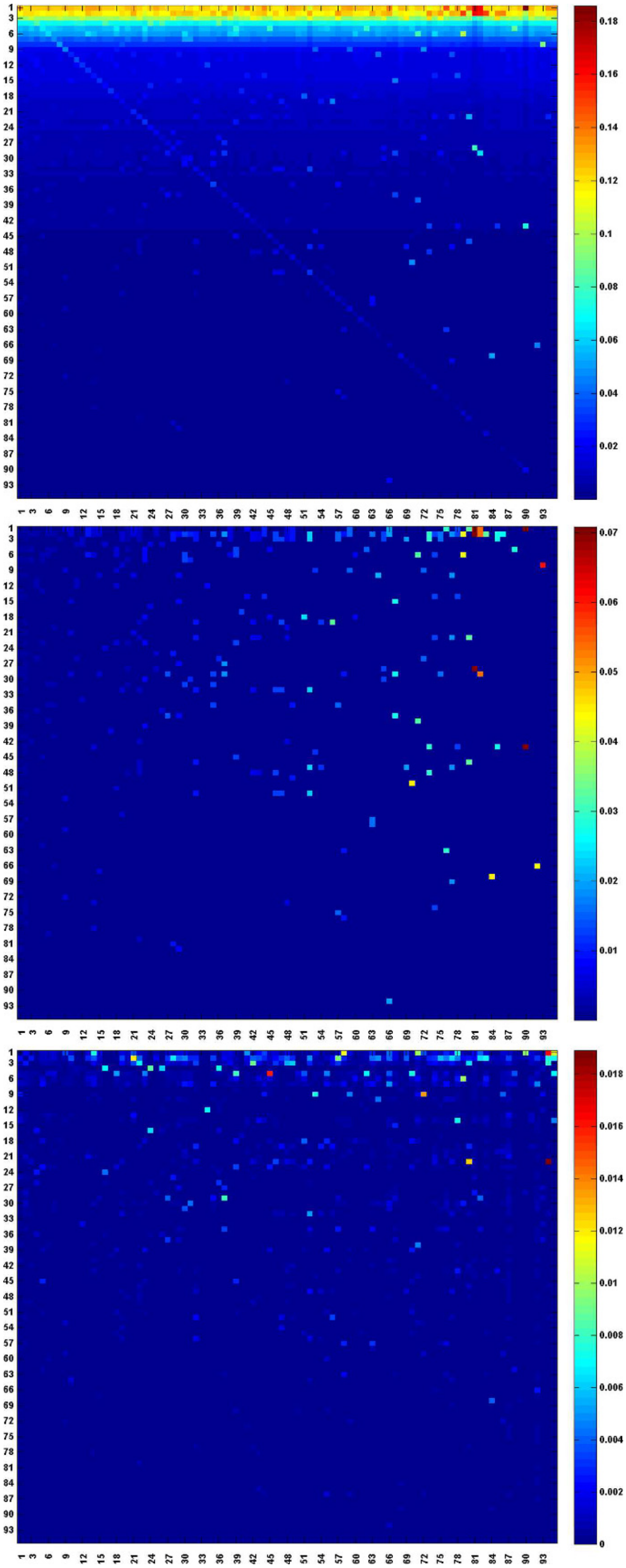

Fig. 1. Density plots of matrices G_{R}, G_{rr} and G_{qrnd} (top, middle and bottom; color changes from red at maximum to blue at zero); only 95 terrorist nodes of Table 1 are shown.

that the groups that are listed as International (blue color) are clearly playing that role by having lots of ingoing links from the other categories.

3.2 Relationships between groups and countries

The interactions between groups and countries are characterized by the network structure shown in Figures 3 and 4. For clarity, we first show in Figure 3 the top 4 country friends of the 6 terrorist groups identified as leading each category. In Figure 4 we show for the same 6 leading terrorist groups the top 2 country friends and top 2 terrorist groups friends. This latter representation shows altogether major ties between groups and countries and in-between groups. Very interesting and realistic relations between groups and countries can be extracted from this network. For instance, Taliban (KG3) is an active group in Afghanistan and Pakistan that represents an Islamist militant organization that was one of the prominent factions in the Afghan Civil War [5,27,28]. As shown in Figures 3 and 4 Afghanistan and Pakistan are the countries the most influenced by Taliban.

The fact that Saudi Arabia links Houthis, Taliban and Al Shabaab can be explained by the fact that Saudi Arabia is in war with Houthis [29,30]. Also, the main funding sources for groups active in Afghanistan and Pakistan originate from Saudi Arabia [31]. Moreover, Al-Shabaab advocates for the Saudi-inspired Wahhabi version of Islam [32]. Referring to [33], ISIS (KG1) was born in 2006 in Iraq as Islamic State of Iraq (ISI). Its main activities are in Syria and Iraq. As shown in Figures 3 and 4 a strong relationship exists among the two countries and ISIS. Hamas and Hezbollah are the leading groups in MEA facing Israel. As shown in Figures 3 and 4, with the knowledge of the relationship between Hezbollah and Houthis, we can explain why Israel is a linking node between Houthis and Hamas. Finally, we find that Iran links Houthis with ISIS. This could be explained by the fact that both groups are in conflict with Saudi Arabia.

3.3 Sensitivity analysis

To analyze more specifically the influence of given terrorist groups on the selected 64 world countries we introduce the sensitivity F determined by the logarithmic derivatives of PageRank probability P obtained from G_{R}. At first we define δ_{ij} as the relative fraction to be added to the relationship from node j to node i in G_{R}. Knowing δ_{ij}, a new modified matrix $\tilde{G}_{\mathrm{R}}$ is calculated in two steps. First, element $\tilde{G}_{\mathrm{R}}(i,j)$ is set to $(1+\delta_{ij}) \cdot G_{\mathrm{R}}(i,j)$. Second, all elements of column j of $\tilde{G}_{\mathrm{R}}$ are normalized to 1 (including element i) to preserve the column-normalized property of this matrix from the class of Google matrices. After that $\tilde{G}_{\mathrm{R}}$ reflects an increased probability for going from node j to node i.

It is now possible to calculate the modified PageRank eigenvector $\tilde{P}$ from $\tilde{G}_{\mathrm{R}}$ using the standard $\tilde{G}_{\mathrm{R}}\tilde{P} = \tilde{P}$ relation and compare it to the original PageRank probabilities P calculated with G_{R} using $G_{\mathrm{R}}P = P$. Due to the relative change of the transition probability between nodes i and

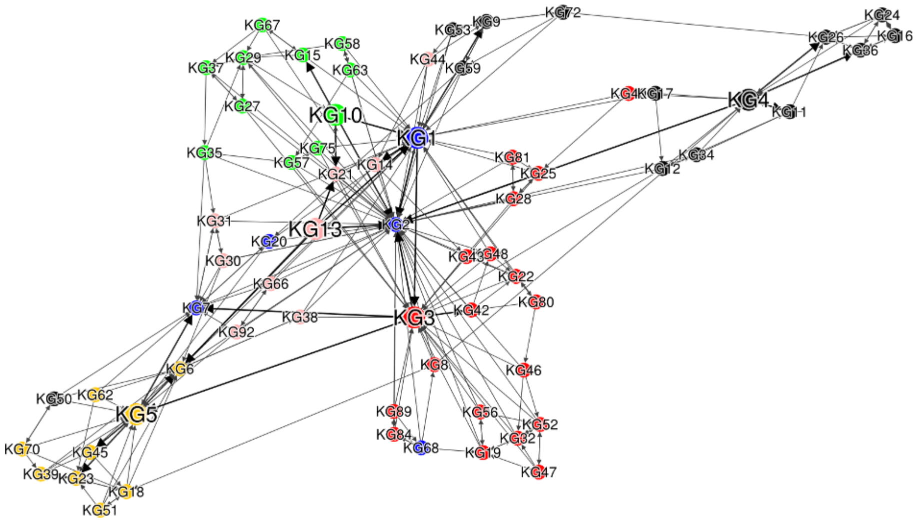

Fig. 2. Friendship network structure between terrorist groups obtained from $G_{\rm qr}+G_{rr}$; colors mark categories of nodes and top nodes are given in text and Table 1; circle size is proportional to PageRank probability of nodes; bold black arrows point to top 4 friends, gray tiny arrows show friends of friends interactions computed until no new edges are added to the graph (drawn with [38,39].

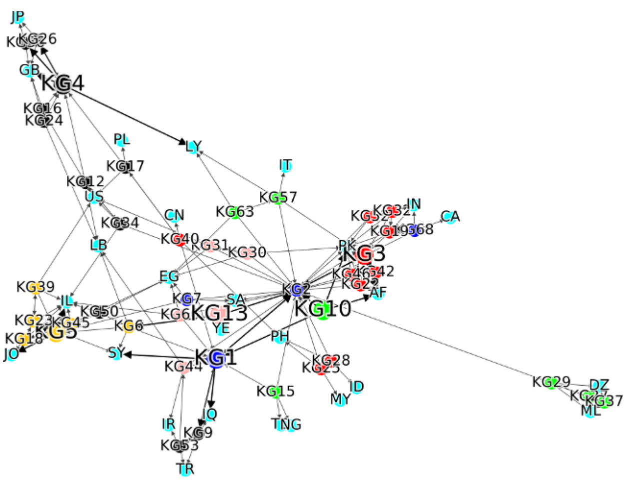

Fig. 3. Friendship network structure extracted from $G_{\rm qr} + G_{rr}$ with the top terrorist groups (marked by their respective colors) and countries (marked by cyan color). The network structure is shown in case of 2 friends for top terrorist groups of each category and top friend 2 countries for each group. Networks are drawn with [38,39].

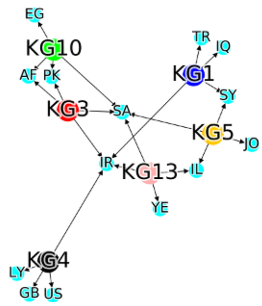

Fig. 4. Friendship network structure extracted from $G_{\rm qr} + G_{rr}$ with the top terrorist groups (marked by their respective colors) and countries (marked by cyan color). The network structure is shown with the top terrorist groups of each category and their top 4 friend countries. Networks are drawn with [38,39].

j, the steady state PageRank probabilities are modified. This reflects a structural modification of the network and entails a change of importance of nodes in the network. These changes are measured by a logarithmic derivative of the PageRank probabilities:

$$D_{(j\to i)}(k) = ({\rm d}P_k/{\rm d}\delta_{ij})/P_k = (\tilde{P}_k - P_k)/(\delta_{ij}P_k), \qquad (1)$$

so that the derivative $D_{(j\to i)}(k)$ gives for node k its sensitivity to the change of link j to i. We note that this approach is similar to the sensitivity analysis of the world trade network to the price of specific products (e.g. gas or petroleum) as studied in [15].

Figures 5 and 6 show maps of the sensitivity influence D of the top groups of the 6 categories on all 64 countries. Here we see that Taliban (KG3) has important influence on Afghanistan, Pakistan, and Saudi Arabia and less influence on other countries. In contrast ISIS (KG1) has a strong worldwide influence with the main effects on Canada, Libya, USA, Saudi Arabia. The world maps show that the groups of Figure 5 (Taliban, Hamas, Houthis) produce mainly local influence in the world. In contrast, the groups of Figure 6 (ISIS, Al Shabaab, IRA) spread their influence worldwide. Even if IRA mainly affects UK it still spreads its influence on other Anglo-Saxon countries. The presented results determine the geopolitical influence of each terrorist group.

Figure 7 shows the influence of a relation between one selected country c and one selected terrorist group i on the other countries j. The results are shown for two countries

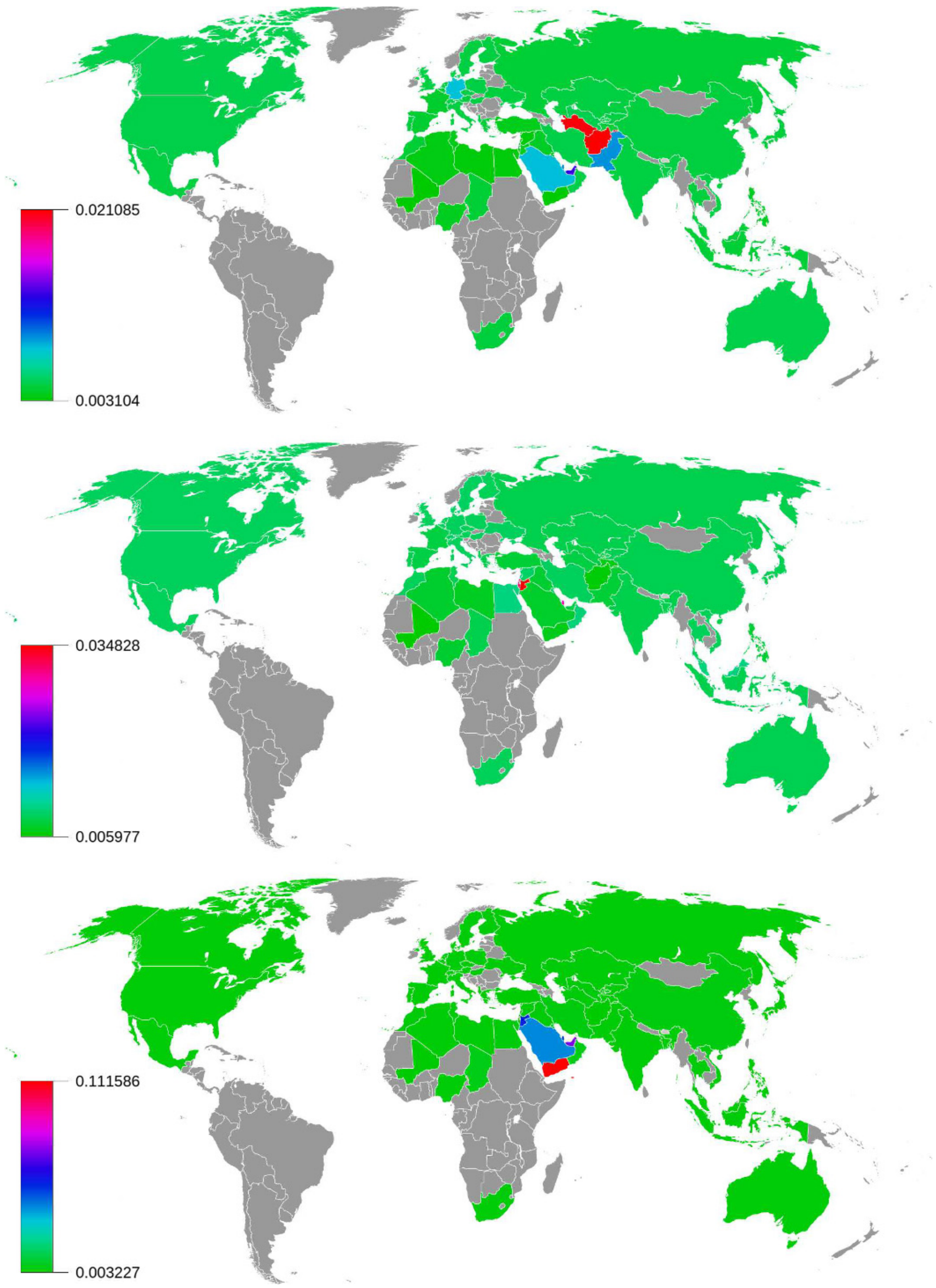

Fig. 5. World map of the influence of terrorist groups on countries expressed by sensitivity $D_{(j \to i)}(j)$ where j is the country index and i the group index, see text). The influence of Taliban KG3, Hamas KG5, Houthis KG13 is shown in panels (top to bottom). Color bar marks $D_{(j \to i)}(j)$ values with red for maximum and green for minimum influence; gray color marks countries not considered is this work.

being US (top panel – $c = 1$) and Saudi Arabia (bottom panel – $c = 46$). Each element (i, j) of the given matrices is expressed by $D_{(c \to i)}(j)$). Results show the enormous influence of Saudi Arabia on terrorist groups and other countries (almost all panel is in red). The influence of USA is more selective.

All data for the matrices discussed above, figures and sensitivity are available at [7].

We note that above we analyzed the world terror networks. However, at present the statistical data for human crime activity become available [34,35] and the extension of the described methods to this area would be of interest.

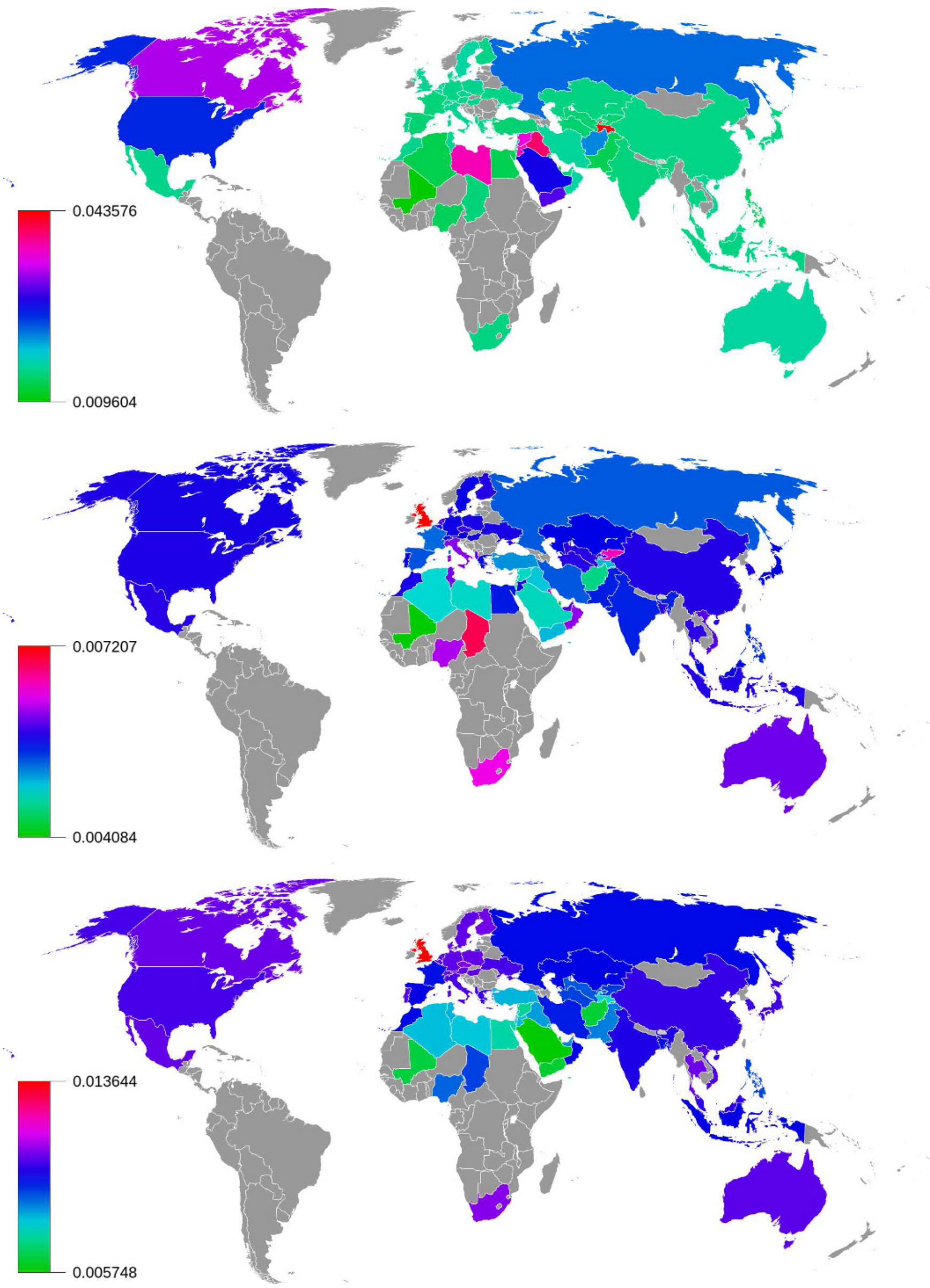

Fig. 6. World map of the influence of terrorist groups on countries expressed by sensitivity $D_{(j\to i)}(j)$ where j is the country index and i the group index, see text). The influence of ISIS KG1, Al Shabaab KG10, IRA KG4 is shown in panels (top to bottom). Color bar marks $D_{(j\to i)}(j)$ values with red for maximum and green for minimum influence; gray color marks countries not considered is this work.

4 Discussion

We have applied the reduced Google matrix analysis (Fig. 1) to the network of articles of English Wikipedia to analyze the network structure of 95 terrorist groups and their influence over 64 world countries (159 selected articles). This approach takes into account all human knowledge accumulated in Wikipedia, leveraging all indirect interactions existing between the 159 selected articles and the huge information contained by 5 416 537 articles of Wikipedia and its 122 232 932 links. The network structure obtained for the terrorist groups (Figs. 2 and 3) clearly show the presence of 6 types (categories) of groups. The main groups in each category are determined from their

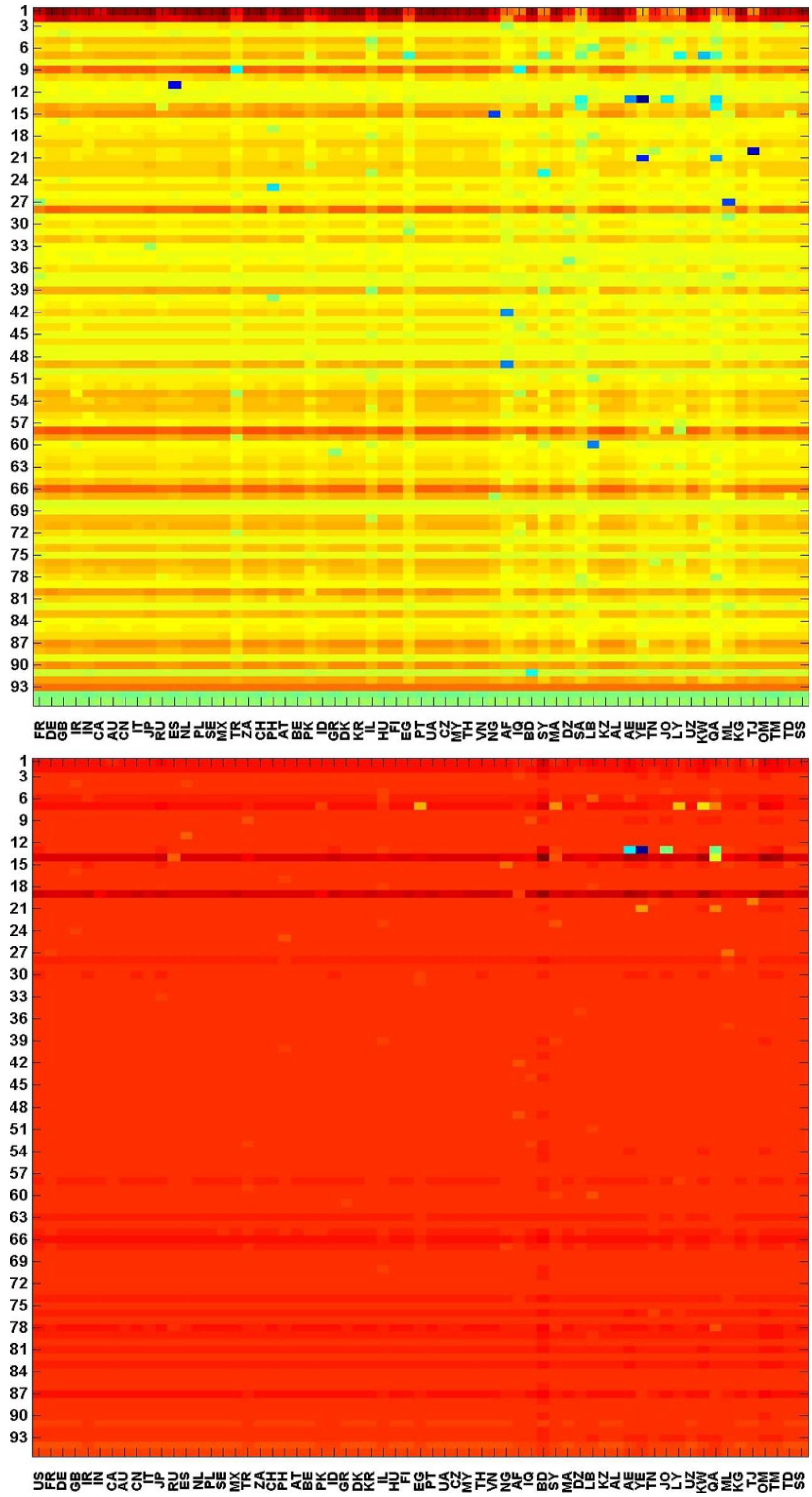

Fig. 7. Sensitivity influence $D_{(c\to i)}(j)$ for the relation between a selected country c and a terrorist group i (represented by group index i from Tab. 1 in vertical axis) on a world country j (represented by country index j from Tab. 2 in horizontal axis, $j = c$ is excluded) for two c values: USA (top), Saudi Arabia (bottom). Color shows $D_{(c\to i)}(j)$ value is changing in the range $(-2.8 \times 10^{-4}, 2.1 \times 10^{-4})$ for USA and $(-4.8 \times 10^{-3}, 10^{-3})$ for SA; minimum/maximum values correspond to blue/red.

PageRank. We show that the indirect or hidden links between terrorist groups and countries play an important role and are, in many cases, predominant over direct links. The geopolitical influence of specific terrorist groups on world countries is determined via the sensitivity of PageRank variation in respect to specific links between groups and countries (Fig. 4). We see the presence of terrorist groups with localized geographical influence (e.g. Taliban) and others with worldwide influence (ISIS). The influence of selected countries on terrorist groups and other countries is also determined by the developed approach (Fig. 6). The obtained results, tested on the publicly available data of Wikipedia, show the efficiency of the analysis. We argue that the reduced Google matrix approach can find further important applications for terror networks analysis using more advanced and detailed databases.

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Author contribution statement

All authors equally contributed to all aspects of this work.

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