Reduced Google matrix analysis and Art: Painters.

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Abstract—This study concentrates on extracting painting art history knowledge from the network structure of Wikipedia. Therefore, we construct theoretical networks of webpages representing the hyperlinked structure of articles of 6 Wikipedia language editions. These 6 networks are analyzed to extract the most influential painters in each edition using Google matrix theory. Importance of webpages of over 3000 painters are measured using PageRank. The 30 most influential painters are enlisted and their ties are studied with the reduced Google matrix analysis. Reduced Google Matrix is a powerful model that captures both direct and hidden interactions between our selection of 30 nodes. From this study, we show that it is possible to extract from the components of the reduced Google matrix meaningful information on the ties between these painters. For instance, our analysis groups together painters that belong to the same painting movement and shows meaningful ties between painters of different movements. We argue that this approach gives meaningful information about art and that it could be a part of largest work of network analysis on human knowledge and cultures.

I. INTRODUCTION

"The art is the expression or application of human creative skill and imagination, typically in a visual form such as painting or sculpture, producing works to be appreciated primarily for their beauty or emotional power" [1]. Artists use different approches and techniques to create emotions. Since the beginning of mankind, painters have offered masterpieces in the form of paintings and drawings to the world. Depending on historical periods, cultural context and available techniques, painters have followed different art movements. Art historians group painters into art movements to capture the fact that they have worked in the same school of thought. But a painter could be placed in several movements as his works evolve with time and its individual intellectual path development [2]–[8].

The major finding of this paper is to show that it is possible to automatically extract this common knowledge on art history by analyzing the hyperlinked network structure of the global and free online encyclopedia Wikipedia [9]. The analysis conducted in this work is solely based on a graph representation of the Wikipedia articles where vertices represent the articles and the edges the hyperlinks linking these articles together. The actual content of the articles is never processed in our developments.

Wikipedia has become the largest open source of knowledge being close to Encyclopedia Britanica [10] by the accuracy

TABLE I
LIST OF NAMES OF 30 SELECTED PAINTERS AND THEIR PAGERANK
ORDER FOR FRWIKI, ENWIKI AND DEWIKI, ORDERED BY CATEGORY.

Name	Category	Colour	FrWiki	EnWiki	DeWiki
Picasso	Cubism	Red	1	2	2
Braque	Cubism	Red	17	20	20
Léger	Cubism	Red	19	24	24
Mondrian	Cubism	Red	25	22	22
Gris	Cubism	Red	29	28	25
Delaunay	Cubism	Red	28	27	26
Matisse	Fauvism	Blue	6	11	12
Gauguin	Fauvism	Blue	13	15	18
Derain	Fauvism	Blue	22	25	27
Dufy	Fauvism	Blue	27	26	29
Rouault	Fauvism	Blue	30	30	28
Vlaminck	Fauvism	Blue	24	29	30
Monet	Impressionists	Green	4	9	11
Cézanne	Impressionists	Green	8	12	9
Manet	Impressionists	Green	12	13	16
Renoir	Impressionists	Green	15	14	17
Degas	Impressionists	Green	18	16	21
Pissarro	Impressionists	Green	23	19	23
da Vinci	Great masters	Orange	2	1	1
Michelangelo	Great masters	Orange	3	3	4
Raphael	Great masters	Orange	5	4	5
Rembrandt	Great masters	Orange	9	5	6
Rubens	Great masters	Orange	10	7	7
Durer	Great masters	Orange	14	8	3
Dali	Modern 20-21	Pink	7	10	13
Warhol	Modern 20-21	Pink	11	6	8
Kandinsky	Modern 20-21	Pink	20	17	10
Chagall	Modern 20-21	Pink	21	18	15
Miró	Modern 20-21	Pink	16	21	19
Munch	Modern 20-21	Pink	26	23	14

of its scientific entries [12] and overcoming the later by the enormous quantity of available information. A detailed analysis of strong and weak features of Wikipedia is given in [13], [14]. Unique to Wikipedia is that articles make citations to each other, providing a direct relationship between webpages and topics. As such, Wikipedia generates a larger directed network of article titles with a rather clear meaning. For these reasons, it is interesting to apply algorithms developed for search engines of World Wide Web (WWW), those like the PageRank algorithm [15](see also [16]), to analyze the ranking properties and relations between Wikipedia articles. For various language editions of Wikipedia it was shown that

the PageRank vector produces a reliable ranking of historical figures over 35 centuries of human history [18]–[21] and a solid Wikipedia ranking of world universities (WRWU) [18], [22]. It has been shown that the Wikipedia ranking of historical figures is in a good agreement with the well-known Hart ranking [23], while the WRWU is in a good agreement with the Shanghai Academic ranking of world universities [24].

At present, directed networks of real systems can be very large (about 4.2 million articles for the English Wikipedia edition in 2013 [17] or 3.5 billion web pages for a publicly accessible web crawl that was gathered by the Common Crawl Foundation in 2012 [45]). For some studies, one might be interested only in the particular interactions between a very small subset of nodes compared to the full network size. For instance, in this paper, we are interested in capturing the interactions of the 30 painters represented in Tab. I using the networks extracted from six Wikipedia language editions covering a few millions of articles each. However, the interactions between these 30 nodes should be correctly determined taking into account that there are many indirect links between the 30 webpages via all other nodes of the network. In previous works, a solution to this general problem has been proposed in [26]–[28] by defining the reduced Google matrix theory. Main elements of Reduced Google matrix G_{R} will be presented in Section II.

In this paper, we extract from G_{R} and its decomposition into direct and indirect matrices a high-level *reduced network of 30 painters*. This high-level network can be computed for either direct or hidden (i.e. indirect) interactions. More specifically, we deduce a fine-grained classification of painters that captures what we call the *hidden friends* of a given painter. The structure of these graphs provides relevant information that offers new information compared to the direct networks of relationships.

The aforementioned networks of direct and hidden interactions can be calculated for different Wikipedia language editions. In this paper, reduced Google matrix analysis is applied to the same set of 30 painters on networks originating from six different Wikipedia langage editions (English, French, German, Spanish, Russian and Italian). We will refer to these editions using EnWiki, FrWiki, DeWiki, EsWiki, RuWiki and ItWiki in the remainder of this paper.

This paper introduces first the main elements of reduced Google matrix theory in Section II. Next, Section III presents the reduced Google matrices calculated for 30 painters and from six different langage editions. Specific emphasis is given to the very different English, French and German editions. Then, networks of friendship from direct and hidden interaction matrices are created and discussed in Section IV. We show that the networks of friends completely capture the well-established history of painting by i) interconnecting densely painters of the same movement and ii) showing reasonable links between painters of different movements. Finally, Section V concludes this paper.

II. REDUCED GOOGLE MATRIX THEORY

It is convenient to describe the network of N Wikipedia articles by the Google matrix G constructed from the adjacency matrix A_{ij} with elements 1 if article (node) j points to article (node) i and zero otherwise. Elements of the Google matrix take the standard form $G_{ij} = \alpha S_{ij} + (1 - \alpha)/N$ [15]–[17], where S is the matrix of Markov transitions with elements $S_{ij} = A_{ij}/k_{out}(j)$, $k_{out}(j) = \sum_{i=1}^{N} A_{ij} \neq 0$ being the node j out-degree (number of outgoing links) and with $S_{ij} = 1/N$ if j has no outgoing links (dangling node). The damping factor $0 < \alpha < 1$ is which for a random surfer determines the probability $(1 - \alpha)$ to jump to any node; below we use the standard value $\alpha = 0.85$. The right eigenvector of G with the unit eigenvalue gives the PageRank probabilities $P(j)$ to find a random surfer on a node j. We order nodes by decreasing probability P getting them ordered by the PageRank index $K = 1, 2, ...N$ with a maximal probability at $K = 1$. From this global ranking we capture the top 50 painters mentioned in Tab. II.

Reduced Google matrix is constructed for a selected subset of nodes (articles) following the method described in [26]–[28] and based on concepts of scattering theory used in different fields of mesoscopic and nuclear physics or quantum chaos. It captures in a N_r-by-N_r Perron-Frobenius matrix the full contribution of direct and indirect interactions happening in the full Google matrix between the N_r nodes of interest. In addition the PageRank probabilities of selected N_r nodes are the same as for the global network with N nodes, up to a constant multiplicative factor taking into account that the sum of PageRank probabilities over N_r nodes is unity. Elements of reduced matrix $G_{\mathrm{R}}(i, j)$ can be interpreted as the probability for a random surfer starting at web-page j to arrive in web-page i using direct and indirect interactions. Indirect interactions refer to paths composed in part of web-pages different from the N_r ones of interest. Even more interesting and unique to reduced Google matrix theory, we show here that intermediate computation steps of G_{R} offer a decomposition of G_{R} into matrices that clearly distinguish direct from indirect interactions: $G_{\mathrm{R}} = G_{rr} + G_{\mathrm{pr}} + G_{\mathrm{qr}}$ [27]. Where G_{rr} is given by the direct links between selected N_r nodes in the global G matrix with N nodes, G_{pr} is rather close to the matrix in which each column is given by the PageRank vector P_r, ensuring that PageRank probabilities of G_{R} are the same as for G (up to a constant multiplier). Therefore G_{pr} doesn't provide much information about direct and indirect links between selected nodes. The one playing an interesting role is G_{qr}, which takes into account all indirect links between selected nodes appearing due to multiple paths via the global network nodes N (see [26]–[28]). The matrix $G_{\mathrm{qr}} = G_{\mathrm{qrd}} + G_{\mathrm{qrnd}}$ has diagonal (G_{qrd}) and non-diagonal (G_{qrnd}) parts. G_{qrnd} describes indirect interactions between nodes. The explicit formulas as well as the mathematical and numerical computation methods of all three components of G_{R} are given in [26]–[28].

TABLE II
LIST OF 50 TOP PAINTERS FROM FRWIKI, ENWIKI, DEWIKI, ITWIKI, ESWIKI AND RUWIKI BY INCREASING PAGERANK.

FrWiki	EnWiki	DeWiki	ItWiki	EsWiki	RuWiki
Pablo Picasso	Leonardo da Vinci	Leonardo da Vinci	Leonardo da Vinci	Leonardo da Vinci	Leonardo da Vinci
Leonardo da Vinci	Pablo Picasso	Pablo Picasso	Michelangelo	Francisco Goya	Pablo Picasso
Michelangelo	Michelangelo	Albrecht Durer	Raphael	Pablo Picasso	Michelangelo
Claude Monet	Raphael	Michelangelo	Pablo Picasso	Michelangelo	Rembrandt Van Rijn
Vincent Van Gogh	Rembrandt Van Rijn	Raphael	Giorgio Vasari	Raphael	Vincent Van Gogh
Jacques-Louis David	Vincent Van Gogh	Rembrandt Van Rijn	Titian	Diego Velázquez	Albrecht Durer
Eugène Delacroix	Francis Bacon	Peter Paul Rubens	Peter Paul Rubens	Salvador Dali	Ilya Repin
Raphael	Andy Warhol	Vincent Van Gogh	Caravaggio	Peter Paul Rubens	Nicholas Roerich
Henri Matisse	Peter Paul Rubens	Titian	Vincent Van Gogh	Titian	Titian
Salvador Dali	Albrecht Durer	Francis Bacon	Giotto Di Bondone	Francis Bacon	Henri Matisse
Paul Cézanne	William Blake	Andy Warhol	Rembrandt Van Rijn	Albrecht Durer	Salvador Dali
Rembrandt Van Rijn	Titian	Paul Klee	Sandro Botticelli	Rembrandt Van Rijn	Paul Cézanne
Peter Paul Rubens	Claude Monet	Paul Cézanne	Albrecht Durer	Vincent Van Gogh	Viktor Vasnetsov
Andy Warhol	Salvador Dali	Lucas Cranach the Elder	Francisco Goya	Sandro Botticelli	Ivan Aivazovsky
Marcel Duchamp	Henri Matisse	Wassily Kandinsky	Giuseppe Arcimboldo	Caravaggio	Diego Velázquez
Édouard Manet	Giorgio Vasari	Claude Monet	Piero Della Francesca	Henri Matisse	Marc Chagall
Giorgio Vasari	Paul Cézanne	Henri Matisse	Edvard Munch	Eugène Delacroix	Claude Monet
Paul Gauguin	Francisco Goya	Salvador Dali	Andrea Mantegna	Paul Cézanne	Valentin Serov
Albrecht Durer	Joseph Mallord William Turner	Giorgio Vasari	Masaccio	Andy Warhol	Paul Gauguin
Pierre Auguste Renoir	Eugène Delacroix	Edvard Munch	Claude Monet	Claude Monet	Hieronymus Bosch
Joan Miró	Caravaggio	Giotto Di Bondone	Jacques-Louis David	Giorgio Vasari	Henri de Toulouse-Lautrec
Jean-Auguste-Dominique Ingres	Jackson Pollock	Marc Chagall	Samuel Morse	Paul Gauguin	Karl Bryullov
Georges Braque	Édouard Manet	Caspar David Friedrich	Wassily Kandinsky	Diego Rivera	Eugène Delacroix
Edgar Degas	Anthony van Dyck	Édouard Manet	Diego Velázquez	Giotto Di Bondone	Wassily Kandinsky
Francisco Goya	Pierre Auguste Renoir	Otto Dix	Pieter Bruegel The Elder	Jacques-Louis David	Édouard Manet
Gustave Courbet	Jacques-Louis David	Caravaggio	Fra Angelico	Édouard Manet	Francisco Goya
Fernand Léger	Diego Velázquez	Francisco Goya	Salvador Dali	Tintoretto	Kazimir Malevich
Titian	William Hogarth	Pierre Auguste Renoir	Pierre Auguste Renoir	Bartolomé Esteban Murillo	Andrei Rublev
Caravaggio	Paul Gauguin	Paul Gauguin	Andy Warhol	Anthony van Dyck	Giorgio Vasari
Jackson Pollock	Hans Holbein The Younger	Max Ernst	Giovanni Battista Tiepolo	Georges Braque	Jacques-Louis David
Wassily Kandinsky	Edgar Degas	Gustav Klimt	Paul Cézanne	Edgar Degas	Igor Grabar
Nicolas Poussin	Johannes Vermeer	Eugène Delacroix	Giovanni Bellini	Joan Miró	Pierre Auguste Renoir
Marc Chagall	Marcel Duchamp	Joan Miró	Domenico Ghirlandaio	Wassily Kandinsky	Samuel Morse
Honoré Daumier	Sandro Botticelli	Jan Van Eyck	Pietro Perugino	Piero Della Francesca	Caravaggio
Max Ernst	Giotto Di Bondone	Pieter Bruegel The Elder	Jan Van Eyck	Hieronymus Bosch	Edgar Degas
Diego Velázquez	Willem De Kooning	Max Liebermann	Paolo Veronese	Andrea Mantegna	Mikhail Vrubel
Gustave Doré	Nicolas Poussin	Diego Velázquez	Giorgione	Jackson Pollock	Nicolas Poussin
Sandro Botticelli	Pieter Bruegel The Elder	Sandro Botticelli	Nicolas Poussin	Henri de Toulouse-Lautrec	Anthony van Dyck
Giotto Di Bondone	John Constable	Marcel Duchamp	Tintoretto	Johannes Vermeer	Joseph Mallord William Turner
Jean-Baptiste Camille Corot	Wassily Kandinsky	Gerhard Richter	Paul Gauguin	Francisco De Zurbaran	Jean-Auguste-Dominique Ingres
Henri de Toulouse-Lautrec	Marc Chagall	Max Beckmann	Antonio da Correggio	William Blake	Alexandre Benois
William Bouguereau	El Greco	Hans Holbein The Younger	Edgar Degas	Marcel Duchamp	Giotto Di Bondone
Pieter Bruegel The Elder	Lucas Cranach the Elder	El Greco	Édouard Manet	Pierre Auguste Renoir	Konstantin Korovin
Antoine Watteau	Benjamin West	Jacques-Louis David	Lucas Cranach the Elder	Hans Holbein The Younger	Isaac Levitan
Georges Seurat	Gustave Doré	Georges Braque	Eugène Delacroix	Pieter Bruegel The Elder	Gustave Courbet
Rene Magritte	Henri de Toulouse-Lautrec	Johannes Vermeer	Gustave Doré	Nicolas Poussin	William Blake
André Derain	Georgia O'keefe	Henry van de Velde	Marc Chagall	Jan Van Eyck	Tove Jansson
Paul Klee	James Abbot Mac Neil Whistler	Edgar Degas	Guido Reni	William Bouguereau	Ivan Kramskoi
François Boucher	Jan Van Eyck	Lovis Corinth	William Blake	Gustave Courbet	
Camille Pissarro	Thomas Gainsborough	FRANZ MARC			

A. Top Painters

In order to get the top painters in each Wikipedia edition of our selection, we create a Matlab code to get all the painters' names from the "List of painters by name" created by Wikipedia that includes painters from all ages and part of the world [11]. We have collected *3334* names, then we check the existence of each node (name) in our 6 selected editions. Next, we get their nodes' number from our database of Wikipedia. However, we note that some names are not necessarily known for their paintings (e.g. Hitler), so we did a second check to keep only names known for their art painting production. The regular Google matrix of each Wikipedia edition is constructed following the standard rules described in Sec. II. From this Google matrix, PageRank K of all N nodes present in all 6 wikipedia editions is determined. From this vector of N values, we extract rank of identified painters and we reorder them by decreasing PageRank value. Tab. II shows the list of top 50 painters from the 6 selected Wikipedia editions. Not surprisingly, the order of top painters changes with respect to the editions due to cultural bias but main trends are there, e.g.:

- Leonardo da Vinci ranks first place in 5 out of 6 editions,
- Michelangelo and Picasso belong to the top 4 in all editions,
- Russian painters, like Viktor Vasnetsov and Ivan Aivazovsky, are in the top 20 of RuWiki but don't appear before rank 50 in other editions.

Using the PageRank of all 3334 painters computed for the 6 langage edition, we have extracted 223 painters by creating the union set of the top 100 painters of each langage edition. Fig. 2 represents on a world map the number of painters born in each country. There is a clear predominance of European painters in this selection with an strong part of Russian artists as well.

B. Selected painters

Our selected painting categories are: Cubism, Impressionists, Fauvism, Great masters and Modern art (20th century). Following an average ranking score [21] among the selected Wikipedia editions, we have selected the top 5 painters of
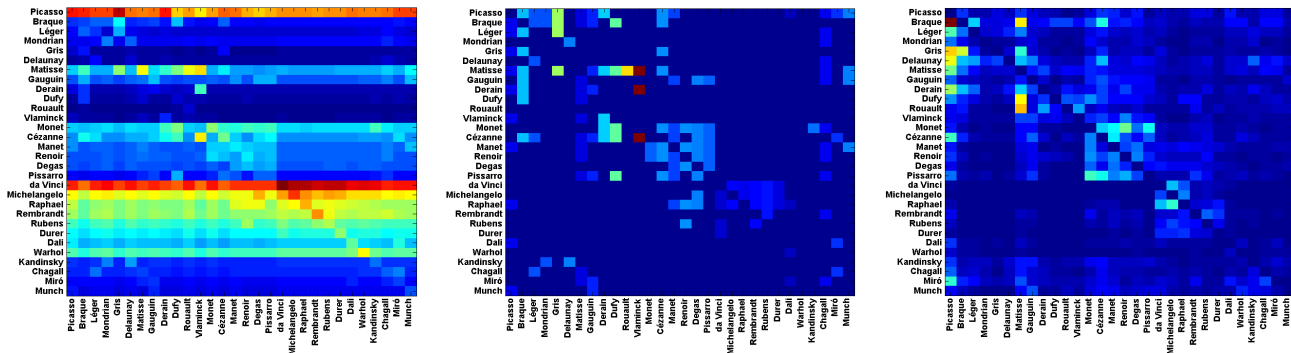
Fig. 1. Density plots of matrices G_R (left), G_{rr} (middle) and G_{qrnd} (right) for the reduced network of 30 painters in the EnWiki network. Color scale represents maximum values in red, intermediate in green and minimum in blue.

each category which represents the order of appearance in Tab. I. Tab. I also lists local PageRank index for painters in the French, English and German Wikipedia editions. Painters that belong to the same movement or having a common piece of history may probably exhibit stronger interactions in Wikipedia. As such, we have created a color code that groups together painters that either belong to the same movement (e.g. Fauvism, Cubism, impressionist) or share a big part of history (e.g. Great Masters, Modern). Color code is as follows: Red, Blue, Green, Orange and Pink represents Cubism, Fauvism, Impressionists, Great masters and Modern (20-21st century) respectively (see Tab. I).
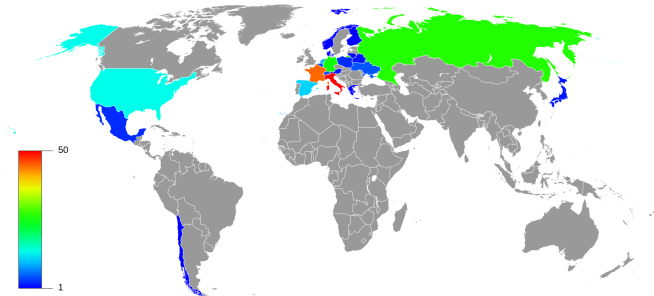
Fig. 2. Geographic birthplace distribution of the 223 painters that appear at least one time in the top 100 of the 6 langage editions analyzed.

C. Density plots of G_R, G_{rr} and G_{qrnd}

To illustrate the matrices derived by the reduced Google matrix analysis, we plot G_{rr}, G_R and G_{qrnd} in Fig. 1 for the EnWiki edition. Columns and lines are ordered with the list of painters given in Tab I. G_R is per-column normalized and dominated by the projector G_{pr} contribution, which is proportional to the global PageRank probabilities (more details in [26], [27]). As such, we clearly see that the density of each line of G_R is proportional to the importance of the painter in the full network. The matrices are interpreted in the following way : painter of column j is linked with the probability of element (i, j) to the painter of line i.

G_{rr} provides information only on direct links between painters. In other words, it represents the probability for a random surfer to reach the painter of line i from the article of the painter of column j using a hyperlink linking article j to article i in Wikipedia. On the contrary, G_{qrnd} offers a much more unified view of painters interactions as it captures more general indirect (or hidden) interactions via the $N-2$ other nodes of the full Wikipedia network. In other words, it represents the probability linking the painter of column j to the painter of line i related to all indirect paths linking article j to article j in the full network. An indirect path starts with a hyperlink linking the article of painter j to an article k that doesn't belong to the N_r painter nodes and ends with a hyperlink ending on the article of painter i.

Reading Fig. 1, we can extract strong and meaningful interactions between painters. New links appearing in G_{qrnd} and being absent from G_{rr} exist. As an example we list the links between Picasso and Braque, Pissaro and Monet, Rouault and Matisse. These relationships are very well known in art history, but looking at the pure structure of the network (i.e. reading G_{rr} matrix), they are absent. They appear clearly in the higher order mathematical analysis of the network using G_{qrnd}. For instance, it is common knowledge that since his visit to Picasso's studio, Braque became impressed by Picasso's paintings. They even became friends [35], which confirms our result. Pissaro and Monet are both impressionists. Monet succeeded in reaching England after entrusting a number of his works to Pissaro [36]. Rouault and Matisse were both students of Gustave Moreau [37] and were deeply influenced by him throughout their life [38]. Their relationship began in 1906 and lasted all their life. All these interactions can be extracted from the network of Wikipedia webpages using G_{qrnd} matrix.

III. RESULTS

Our study focuses on the networks extracted from 6 different Wikipedia editions[1] out of a set of 24 analyzed in [21]. EnWiki, FrWiki, RuWiki, DeWiki, ItWiki and EsWiki contain 4.212, 1.353 , 0.966, 1.533, 1.017 and 0.974 millions of article respectively.

[1] Data collected mid February 2013

Top Painter	all 6 editions	5 out of 6 editions	4 out of 6 editions
Picasso	Braque - Gris		
Matisse	Rouault		Braque - Dufy
Monet	Renoir	Pissarro	
da Vinci	Michelangelo - Raphael	Durer	Degas
Dali	Miró		

A. Friendships

In order to better capture the interactions provided by G_{qrnd}, we have listed for the selected Wikipedia editions the top 4 friends of a set of 5 leading painters. We can say that larger matrix elements in a column j of a given painter can be considered as top friends (i.e. high probability to end in node i from node j). One leading painter per group has been selected: Pablo Picasso for Cubism, Henri Matisse for Fauvism [34], Claude Monet for Impressionists [30], [31], Leonardo da Vinci for Great Masters and Dali for Modern. To pick them inside each group, we have chosen the painters whose average ranking score over all 6 selected Wikipedia editions is the highest. For each leading painter, we extract from matrix G_{qrnd} the 4 top friends. Tab. III shows a summary of cross-editions friends from G_{qrnd} for the top painter of each category. G_{qrnd} seems to emphasize more fine-grained regional interactions and by looking at the interactions (and in addition to relationships explained in Sec. II-C), we can see the strong relationship between da Vinci, Michelangelo and Raphael which can be explained by the fact that they were the nucleus of fifteenth-century Florentine art [32]. Another strong relation could be snapped between Mirò and Dali, as both are inspired by Picasso [33].

IV. Networks of 30 painters

From G_{qrnd} we extract the top 4 friends of leading painters to plot the graphs of Fig. 3. Note that Fig. 3 essentially highlights hidden links. The black thick arrows identify the top 4 friends interactions. Red arrows represent the friends of friends interactions that are computed recursively until no new edge is added to the graph. The graphs are plotted using the Yifan Hu layout of Gephi [43] that groups together nodes more densely interconnected.

Impressionists, Fauvism, Cubism and Great masters create, in all editions, a cluster of nodes densely interconnected. The group of Modern painters plays a role by connecting the other categories: 1) Dali seems to be the common interconnection node between Fauvism and Cubism categories in EnWiki. 2) Kandinsky connects Fauvism and Cubism in FrWiki. 3) Munch connects Impressionists and Fauvism in DeWiki. The networks of G_{qrnd} end up almost spanning the full set of 30 painters. These links show that the interactions between the painters groups are coherent. These graphs picture the essence of painting history by grouping together painters that belong to the same movement and by interconnecting them in a reasonable and close-to reality way.

For instance, our graphs are consistent with the history of modern art which starts with the Impressionists movement (1870-1890) that searched for the exact analysis of the effects of color and light in nature. The painters we have selected are among the most important ones of the movement and they create a clear cluster of nodes in Fig. 3 (see green nodes) as they exhibit a tight relationship (friends) in G_{R}. The Fauvism movement emerged after impressionist (1899-1908) [39]–[41]. Fauvist painters were concerned with the impression created with colors. This movement was inspired by different artists such as Matisse. The *Fauves* members were a loosely shaped group of artists with shared interests. Henri Matisse became later the leader of the group of artists [34]. He introduced unnaturalistic and intense color into their paintings to describe light and space. The fauvism movement is the precursor of the Cubism movement [42]. Our result shows deep relationships between Fauvism and Cubism, noting that Braque is always the core of this interconnection. Cubism movement (1907- 1922) is pretty distinct from Impressionism, which is underlined as well in our graphs with only a few red links connecting these two clusters of nodes.

V. Conclusion

This work offers a new perspective for future art studies. It is possible to extract from multi-cultural Wikipedia networks a global understanding of the interactions between the painters. We have applied the Google matrix (Tab. II) and the reduced Google matrix analysis (Fig. 1) to the network of articles of 6 Wikipedia editions to get the top painters in each edition and also to analyse the network structure of 30 painters and the interconnection between painting categories. This approach takes into account all human knowledge accumulated in Wikipedia, leveraging all indirect interactions existing between the 30 selected articles and the huge information contained by more than 10 millions articles of Wikipedia. The network structure obtained for the painters (Fig. 3) clearly show the presence of 5 categories of painters. The main painters in each category are determined from their PageRank. We show that the indirect or hidden links between painters play an important role and are, in many cases, predominant over direct links. The obtained results, tested on the publicly available data of Wikipedia, show the efficiency of the analysis. We argue that the reduced Google matrix approach can find further important applications for terror networks analysis using more advanced and detailed databases.
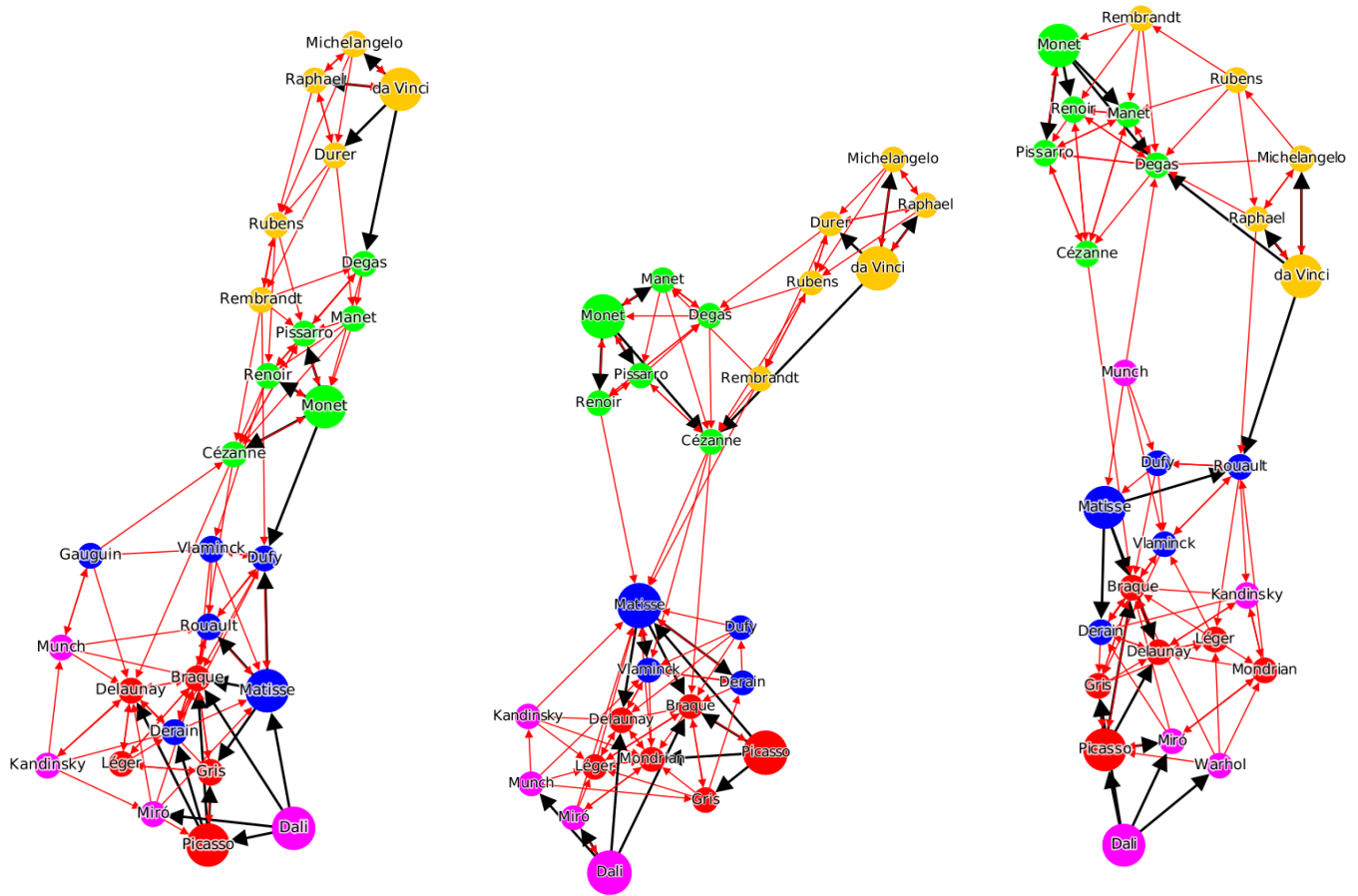
Fig. 3. Network structure of friends induced by the 5 top painters of each group in G_{qrnd}. Results are plotted for EnWiki (left), FrWiki (middle) and DeWiki (right). Red, Blue, Green, Orange and Pink nodes represents Cubism, Fauvism, Impressionists, Great masters and Modern(20-21) respectively. The top painter node points with a bold black arrow to its top 4 friends. Red arrows represent the friends of friends interactions computed until no new edges are added to the graph. All graphs are automatically plotted using *Gephi* [43].

REFERENCES

[1] Oxford Dictionaries. https://en.oxforddictionaries.com/definition/art. Accessed Oct 2017

[2] Melvin L. Alexenberg. *The Future of Art in a Digital Age: From Hellenistic to Hebraic Consciousness*, Intellect Ltd (2008)

[3] J. Fandel. *Pablo Picasso*, Mankato, MN : Creative Education (2016)

[4] J. Richardson. *A Life of Picasso. Vol. 1*, Random House (1991)

[5] Herschel B. Chipp. *Picasso's Guernica: History, Tranformations, Meanings*, University of California Press (1988)

[6] J. Richardson. *A Life of Picasso, Volume II: 1907-1917 - The Painter of Modern Life*, Random House ebooks.

[7] N. Wolf. *Expressionism*, Taschen (2004).

[8] Timothy O. Benson, L. Easton, C. Grammont and F. Josenhans. *Expressionism in Germany and France: From Van Gogh to Kandinsky*, LACMA (2014).

[9] http://www.wikipedia.org (accessed August 2016)

[10] Encyclopaedia Brittanica http://www.britannica.com/ (accessed August 2016).

[11] https://en.wikipedia.org/wiki/List_of_painters_by_name.

[12] J.Giles, *Internet encyclopaedias go head to head*, Nature **438**, 900 (2005)

[13] J.M. Reagle Jr. *Good Faith Collaboration: The Culture of Wikipedia*, MIT Press, Cambridge MA (2010)

[14] F.A. Nielsen, *Wikipedia research and tools: review and comments*, (2012), available at SSRN: dx.doi.org/10.2139/ssrn.2129874

[15] S. Brin and L. Page, *The anatomy of a large-scale hypertextual Web search engine*, Computer Networks and ISDN Systems **30**, 107 (1998)

[16] A.M. Langville and C.D. Meyer, *Google's PageRank and beyond: the science of search engine rankings*, Princeton University Press, Princeton (2006)

[17] L. Ermann, K.M. Frahm and D.L. Shepelyansky, *Google matrix analysis of directed networks*, Rev. Mod. Phys. **87**, 1261 (2015)

[18] A.O. Zhirov, O.V. Zhirov and D.L. Shepelyansky, *Two-dimensional ranking of Wikipedia articles*, Eur. Phys. J. B **77**, 523 (2010)

[19] Y.-H. Eom, K.M. Frahm, A. Benczur and D.L. Shepelyansky, *Time evolution of Wikipedia network ranking*, Eur. Phys. J. B **86**, 492 (2013)

[20] Y.-H. Eom and D.L. Shepelyansky, *Highlighting entanglement of cultures via ranking of multilingual Wikipedia articles*, PLoS ONE **8(10)**, e74554 (2013)

[21] Y.-H. Eom, P.Aragon, D.Laniado, A.Kaltenbrunner, S.Vigna and D.L. Shepelyansky, *Interactions of cultures and top people of Wikipedia from ranking of 24 language editions*, PLoS ONE **10(3)**, e0114825 (2015)

[22] J.Lages, A.Patt and D.L.Shepelyansky, *Wikipedia ranking of world universities*, Eur. Phys. J. B **89**, 69 (2016)

[23] M.H. Hart, *The 100: ranking of the most influential persons in history*, Citadel Press, N.Y. (1992)

[24] Academic Ranking of World Universities, http://www.shanghairanking.com/ (accessed Aug. 2016)

[25] A.D. Chepelianskii, *Towards physical laws for software architecture*, arXiv:1003.5455 [cs.SE] (2010)

[26] K.M. Frahm and D.L. Shepelyansky, *Reduced Google matrix*, arXiv:1602.02394[physics.soc] (2016)

[27] K.M. Frahm, K. Jaffrès-Runser and D.L. Shepelyansky, *Wikipedia mining of hidden links between political leaders*, Eur. Phys. J. B **89**, 269 (2016)

[28] K.M. Frahm, S. El Zant, K. Jaffrès-Runser, D.L. Shepelyansky, *Multi-cultural Wikipedia mining of geopolitics interactions leveraging reduced Google matrix analysis.* Phys. Lett. A **381**, 2677 (2017)

[29] J. Berger. *The Success and Failure of Picasso*, Vintage (1993).

[30] D. Wildenstein. *Monet or The Triumph of Impressionism*, Taschen (2014).

[31] P. Montebello, J.N. Wood, D. Wildenstein and C.S. Moffett *Monet's Years at Giverny: Beyond Impressionism*, Harry N.Abrams, INC, Publisher New York (1995).

[32] *Michelangelo and Leonardo da Vinci.* https://www.michelangelo.org/michelangelo-and-da-vinci.jsp. Accessed Oct 2017.

[33] Independent. *Picasso, Miró, Dalí: The Birth of Modernity.* http://www.independent.co.uk/arts-entertainment/art/reviews/picasso-mir-dal-the-birth-of-modernity-palazzo-strozzi-florence-picasso-in-paris-van-gogh-museum-2254050.html. Accessed Oct 2017.

[34] P. Schneider *Matisse*, Flammarion (2002).

[35] *Georges Braque and Pablo Picasso.* https://www.masterworksfineart.com/blog/georges-braque-and-pablo-picasso/. Accessed Oct 2017

[36] *Monet and Pissarro in London.* http://www.visual-arts-cork.com/history-of-art/claude-monet-camille-pissarro-in-london.htm. Accessed Oct 2017

[37] *Rouault / Matisse, correspondances au musée d'Art moderne de la Ville de Paris.* http://lucileee.blog.lemonde.fr/2007/01/08/rouault-matisse-correspondances-au-musee-dart-moderne-de-la-ville-de-paris/. Accessed Oct 2017

[38] *Matisse-Rouault : correspondance 1906-1953, « une vive sympathie d'art »* https://critiquedart.revues.org/13275. Accessed Oct 2017

[39] J.L. Ferrier *The Fauves: The Reign of Colour*, Terrail (1995).

[40] N. Brodskaia *The Fauves : The Masters Who Shook the World of Art (Schools & Movements Series)*, Parkstone Press (1996).

[41] S. Whitfield *Fauvism (World of Art)*, Thames & Hudson (1996)

[42] M. Antliff *Cubism and Culture*, Thames & Hudson (2001)

[43] M. Bastian, S. Heymann, M. Jacomy. *Gephi: An Open Source Software for Exploring and Manipulating Networks*. Proc. of International AAAI Conference on Weblogs and Social Media, 2009.

[44] S. Dorogovtsev, *Lectures on complex networks*, Oxford University Press, Oxford (2010).

[45] R. Meusel, S. Vigna, O. Lehmberg and C. Bizer, *The graph structure in the web analyzed on different aggregation levels*, J. Web Sci. **1**, 33 (2015)

[46] K.M. Frahm, B. Georgeot and D.L. Shepelyansky, *Universal emergence of PageRank*, J. Phys. A: Math. Theor. **44**, 465101 (2011)