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(c1) Time scales for reliable quantum information processing in the presence of external decoherence

The implementation of quantum information protocols has to face the problem of the coupling of quantum processors with the environment: Knowing the sources and strength of quantum noise and controlling it is a fundamental goal to be achieved in quantum computation

Main tool: Quantum Trajectories allow storing only a stochastically evolving state vector, instead of a density matrix

This has an enormous advantage in memory requirements: if the Hilbert space has size N, we store only a state vector of size N instead of a density matrix of size $N \times N$

By averaging over many runs we get the same probabilities (within statistical errors) as the ones obtained by solving the density matrix directly

The theory of quantum trajectories is of widespread use in quantum optics and quantum foundations, but with the exception of a few examples [see Barenco, Brun, Schack, Spiller, Phys. Rev. A **56**, 1177 (1997)] it has not been explored as a tool in quantum computation and information

Quantum trajectories in the Markov approximation

If a system interacts with the environment, its state is described by a density operator ρ. Under the Markov assumption, the dynamics of the system is described by a (Lindblad) master equation:

$$\dot{\rho} = -\frac{i}{\hbar}[H_s, \rho] - \frac{1}{2}\sum_k \{L_k^\dagger L_k, \rho\} + \sum_k L_k \rho L_k^\dagger,$$

H_s is the system's Hamiltonian, $\{\ ,\ \}$ denotes the anticommutator and L_k are the Lindblad operators, with $k \in [1, \ldots, M]$ (the number M depending on the particular model of interaction with the environment)

- The first two terms of the above equation can be regarded as the evolution performed by an effective non-hermitian Hamiltonian, $H_{\mathrm{eff}} = H_s + iK$, with $K = -\hbar/2 \sum_k L_k^\dagger L_k$:

$$-\frac{i}{\hbar}[H_s, \rho] - \frac{1}{2}\sum_k \{L_k^\dagger L_k, \rho\} = -\frac{i}{\hbar}[H_{\mathrm{eff}}\rho - \rho H_{\mathrm{eff}}^\dagger].$$

- The last term is the one responsible for the so called quantum jumps

If the initial density matrix describes a pure state ($\rho(t_0) = |\phi(t_0)\rangle\langle\phi(t_0)|$), then, after an infinitesimal time dt, it evolves into the statistical mixture

$$\rho(t_0 + dt) = (1 - \sum_k dp_k)|\phi_0\rangle\langle\phi_0| + \sum_k dp_k|\phi_k\rangle\langle\phi_k|,$$

where $dp_k = dt\langle\phi(t_0)|L_k^\dagger L_k|\phi(t_0)\rangle$, and the new states are defined by

$$|\phi_0\rangle = \frac{(I - iH_{\text{eff}}dt/\hbar)|\phi(t_0)\rangle}{\sqrt{1 - \sum_k dp_k}}$$

and

$$|\phi_k\rangle = \frac{L_k|\phi(t_0)\rangle}{||L_k|\phi(t_0)\rangle||}.$$

Therefore with probability dp_k a jump occurs and the system is prepared in the state $|\phi_k\rangle$. With probability $1 - \sum_k dp_k$ there are no jumps and the system evolves according to the effective Hamiltonian H_{eff}. (normalization is included because the evolution is non-hermitian)

Numerical method

• Start the time evolution from a pure state $|\phi(t_0)\rangle$

• At intervals dt much smaller than the time scales relevant for the evolution of the system, choose a random number ϵ from a uniform distribution in the unit interval $[0, 1]$

• 1) If $\epsilon \le dp$, where $dp = \sum_k dp_k$, the state of the system jumps to one of the states $|\phi_k\rangle$ (to $|\phi_1\rangle$ if $0 \le \epsilon \le dp_1$, to $|\phi_2\rangle$ if $dp_1 < \epsilon \le dp_1 + dp_2$, and so on)
2) if $\epsilon > dp$ the evolution with the non-hermitian Hamiltonian H_{eff} takes place and we end up in the state $|\phi_0\rangle$

• Repeat this process as many times as $n_{\text{steps}} = \Delta t/dt$, where Δt is the total evolution time

This procedure describes a stochastically evolving wave vector, and we say that a single evolution is a quantum trajectory

• Average over different runs to recover, up to statistical errors, the probabilities obtained using the density operator. Given an operator A, we can write the mean value $\langle A\rangle_t = \mathrm{Tr}[A\rho(t)]$ as the average over $\mathcal{N}$ trajectories:

$$\langle A\rangle_t = \lim_{\mathcal{N}\to\infty} \frac{1}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \langle \phi_i(t)|A|\phi_i(t)\rangle$$

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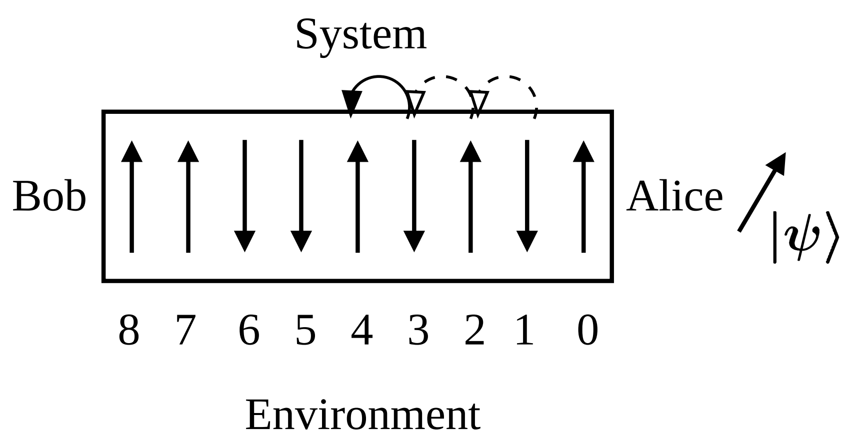
Teleportation in a noisy environment

We assume that the delivery of one of the qubits of the EPR pair required in the teleportation protocol is done by means of SWAP gates along a noisy chain of qubits



- Assume that the initial state of the chain is given by

$$\sum_{i_{n-1},\ldots,i_2} c_{i_{n-1},\ldots,i_2} |i_{n-1}\ldots i_2\rangle \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),$$

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where $i_k = 0, 1$ denotes the down or up state of qubit k

- In order to deliver one of the qubits of the EPR pair to Bob, we implement a protocol consisting of $n-2$ SWAP gates, each one exchanging the states of a pair of qubits:

$$\sum_{i_{n-1},\dots,i_2} \frac{c_{i_{n-1},\dots,i_2}}{\sqrt{2}}(|i_{n-1}\dots i_2 00\rangle + |i_{n-1}\dots i_2 11\rangle)$$

$$\rightarrow \sum_{i_{n-1},\dots,i_2} \frac{c_{i_{n-1},\dots,i_2}}{\sqrt{2}}(|i_{n-1}\dots 0 i_2 0\rangle + |i_{n-1}\dots 1 i_2 1\rangle) \rightarrow$$

$$\dots \rightarrow \sum_{i_{n-1},\dots,i_2} \frac{c_{i_{n-1},\dots,i_2}}{\sqrt{2}}(|0 i_{n-1}\dots i_2 0\rangle + |1 i_{n-1}\dots i_2 1\rangle).$$

- To model the transmission of the qubit through a chaotic quantum chain we take random coefficients $c_{i_{n-1},\dots,i_2}$, that is they have amplitudes of the order of $1/\sqrt{2^{n-2}}$ (to assure wave function normalization) and random phases

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Amplitude damping channels

In the single-qubit case the dissipative environment is modeled by means of the following Kraus operators:

$$M_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \quad M_1 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}$$

Different possible generalizations to the many-qubit case:

1) Collective interaction: a single damping probability describes the action of the environment, irrespective of the internal many-body state of the system

Example: start from the four-qubit pure state $\rho(t_0) = |1011\rangle\langle 1011|$; after a time dt we obtain the statistical mixture

$$\rho(t_0+dt) = \left(1 - \frac{\Gamma dt}{\hbar}\right) |1011\rangle\langle 1011| + \frac{\Gamma dt}{3\hbar}(|0011\rangle\langle 0011| + |1001\rangle\langle 1001| + |1010\rangle\langle 1010|)$$

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2) Single-qubit interactions: each qubit has its own interaction with the environment, independently of the other qubits

$$M_\mu = I \otimes \cdots \otimes I \otimes M_1 \otimes I \otimes \cdots \otimes I, \quad (\mu = 1, \ldots, n)$$

In the previous example,

$$\rho(t_0+dt) = \left(1 - \frac{3\Gamma dt}{\hbar}\right) |1011\rangle\langle 1011| + \frac{\Gamma dt}{\hbar}(|0011\rangle\langle 0011| + |1001\rangle\langle 1001| + |1010\rangle\langle 1010|$$

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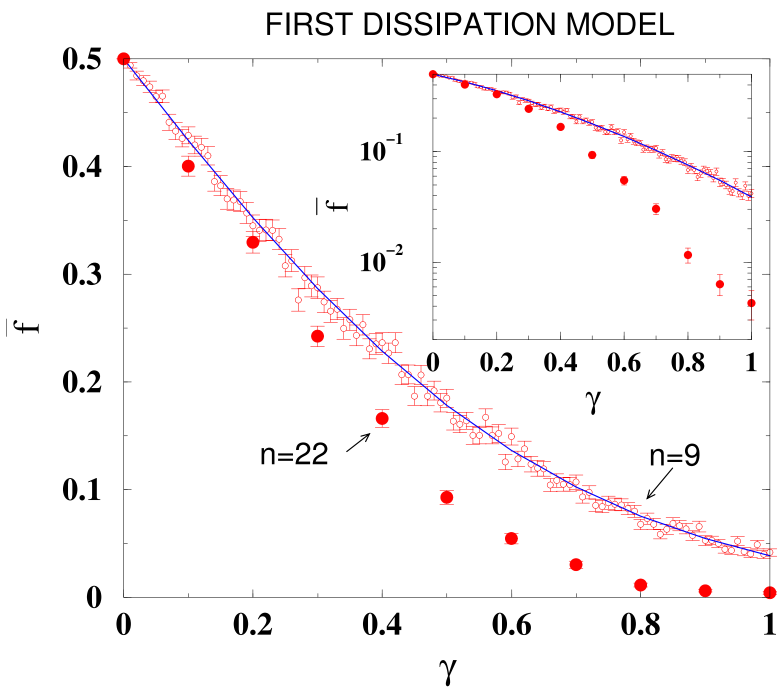
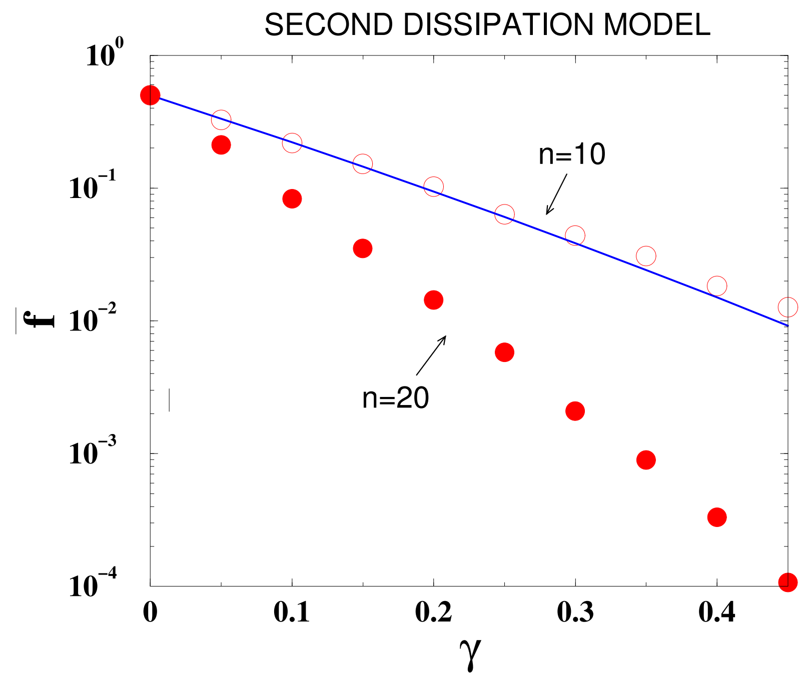
Numerical results with up to 24 qubits

We have computed the fidelity $\bar{f} = f - f_\infty$ of teleportation in the presence of a dissipative environment, as a function of the dimensionless damping rate γ and for up to $n_q = 24$ qubits





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Fidelity time scale for quantum computation in a noisy environment

To be concrete, let us consider the quantum baker's map, a prototypical map for theoretical studies of quantum chaos, which can be simulated efficiently on a quantum computer and has already been implemented on a 3-qubit NMR-based quantum processor [Weinstein, Lloyd, Emerson, Cory, PRL **89**, 157902 (2002)]

The classical baker's transformation maps the unit square $0 \leq q, p < 1$ onto itself according to

$$(q,p) \to (q',p') = \begin{cases} (2q, \frac{1}{2}p), & \text{if } 0 \leq q \leq \frac{1}{2}, \\ (2q-1, \frac{1}{2}p + \frac{1}{2}), & \text{if } \frac{1}{2} < q < 1. \end{cases}$$

This corresponds to compressing the unit square in the p direction and stretching it in the q direction, then cutting it along the p direction, and finally stacking one part on top of the other

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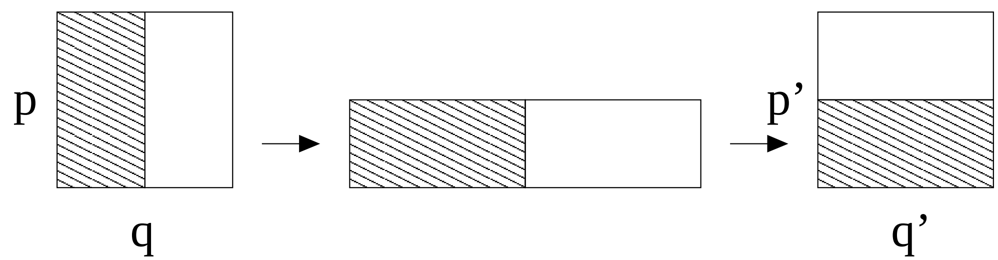
(similarly to the way a baker kneads dough).



The baker's map is a paradigmatic model of classical chaos. It exhibits sensitive dependence on initial conditions, which is the distinctive feature of classical chaos: any small error in determining the initial conditions amplifies exponentially in time. In other words, two nearby trajectories separate exponentially, with a rate given by the maximum Lyapunov exponent $\lambda = \ln 2$

The baker's map can be quantized (following Balazs, Voros and Saraceno): we introduce the position (q) and momentum (p) operators, and denote the eigenstates of these operators by $|q_j\rangle$ and $|p_k\rangle$, respectively. The corresponding eigenvalues are given by $q_j = j/N$ and $p_k = k/N$, with $j, k = 0, ..., N-1$, N being the dimension of the Hilbert space. Note that, to fit N levels onto the unit square, we must set $2\pi\hbar = 1/N$.

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It can be shown that the quantized baker's map can be defined in the position basis by the transformation

$$|\psi\rangle \rightarrow |\psi'\rangle = B\,|\psi\rangle = F_n^{-1} \begin{pmatrix} F_{n-1} & 0 \\ 0 & F_{n-1} \end{pmatrix} |\psi\rangle,$$

where F is the discrete Fourier transform and n is the number of qubits $(N = 2^n)$. Therefore, the quantum baker's map can be implemented efficiently on a quantum computer [Schack, PRA **57**, 1634 (1998); Brun and Schack, PRA **59**, 2649 (1999)]

The environment is modeled as a phase shift channel for each qubit, corresponding to the Kraus operators

$$M_0 = \sqrt{1-p}\,I, \quad M_1 = \sqrt{p}\,\sigma_z$$

We perform t steps of the noisy evolution of the baker's map considering a random initial state and measure the fidelity

$$f(t) = \langle \psi_{\text{exact}}(t) | \rho_{\text{noise}}(t) | \psi_{\text{exact}}(t) \rangle$$

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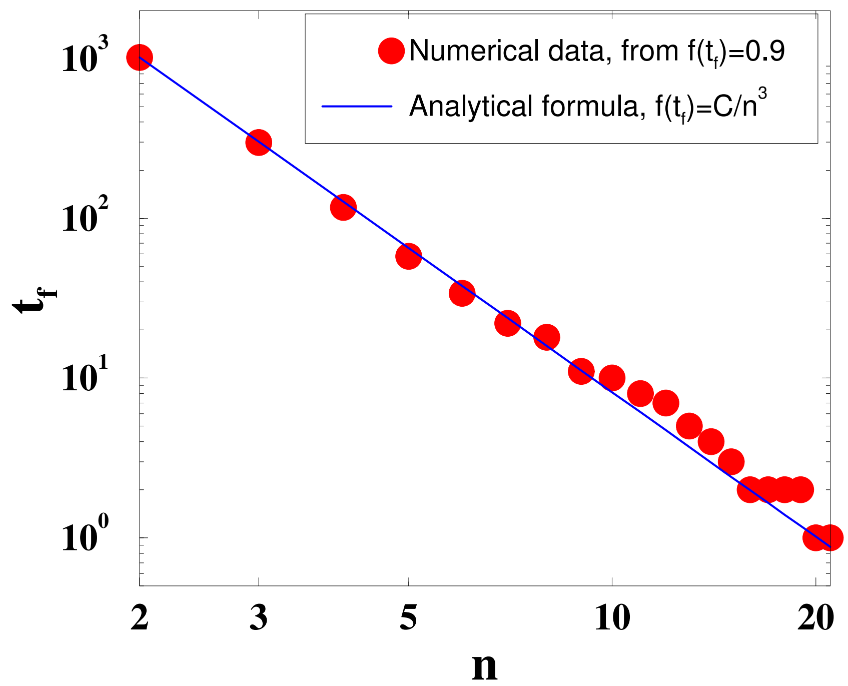
Theoretical expectations:

$$f(t) = \exp(-n\gamma N_g) = \exp(-2\gamma n^3 t),$$

t number of map steps
γ dimensionless decay rate
N_g total number of quantum gates

The number of gates that can be reliably implemented without quantum error correction drops only polynomially with the number of qubits: $(N_g)_f \propto 1/n$



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Effect of a dissipative environment on cold atoms transport in laser fields

Can we find suitable conditions under which noise, combined with the system's dynamics, may drive a directed transport of quantum information?

A test bed to study the effects of dissipative decoherence on nonlinear dynamics: cold atoms in optical lattices

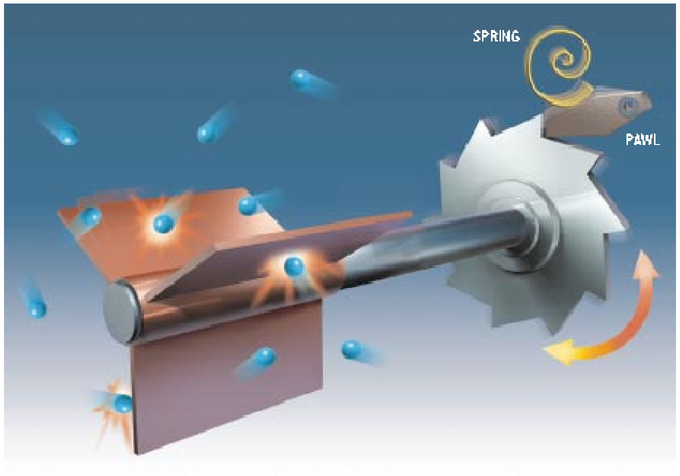
The laser fields create an effective kicked potential implementing the kicked rotator model, which has been realized by the groups of M.Raizen (Texas), d'Arcy et al. (Oxford), Amman et al. (Auckland) and Ringot et al. (Lille)

These experiments allowed the investigation of several important physical phenomena, such as dynamical localization, decoherence, chaos assisted tunneling, and (anti)Zeno effect

Cold atoms in optical lattices also allowed the demonstration of the ratchet phenomenon, which is characterized by directed transport in the absence of any net force

The Feynman ratchet

Can useful work be extracted out of unbiased microscopic random fluctuations if all acting forces and temperatures gradients average out to zero?



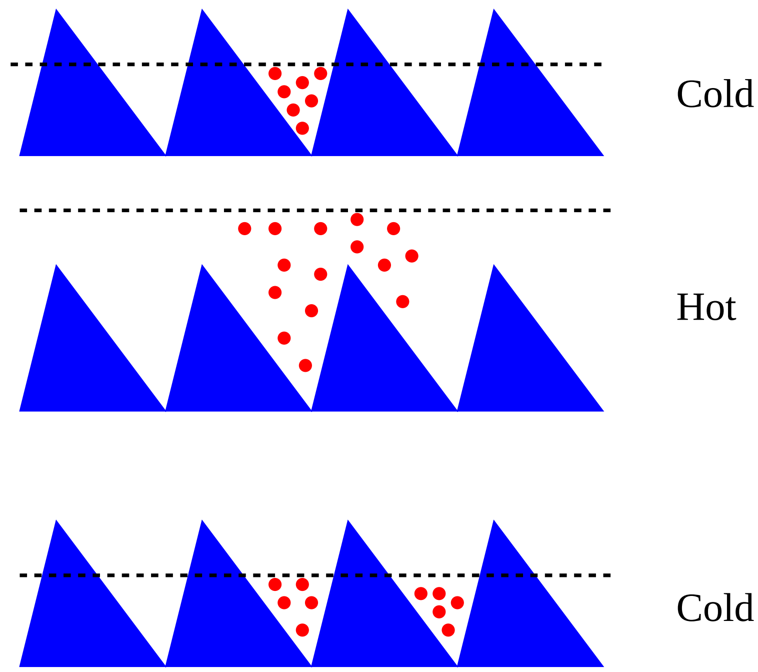
(taken from D.Astumian, Scientific American, July 2001)

Thermal equilibrium: the gas surrounding the paddles and the ratchet (plus the pawl) are at the same temperature

In spite of the built asymmetry no preferential direction of motion is possible. Otherwise, we could implement a perpetuum mobile, in contradiction to the second law of thermodynamics

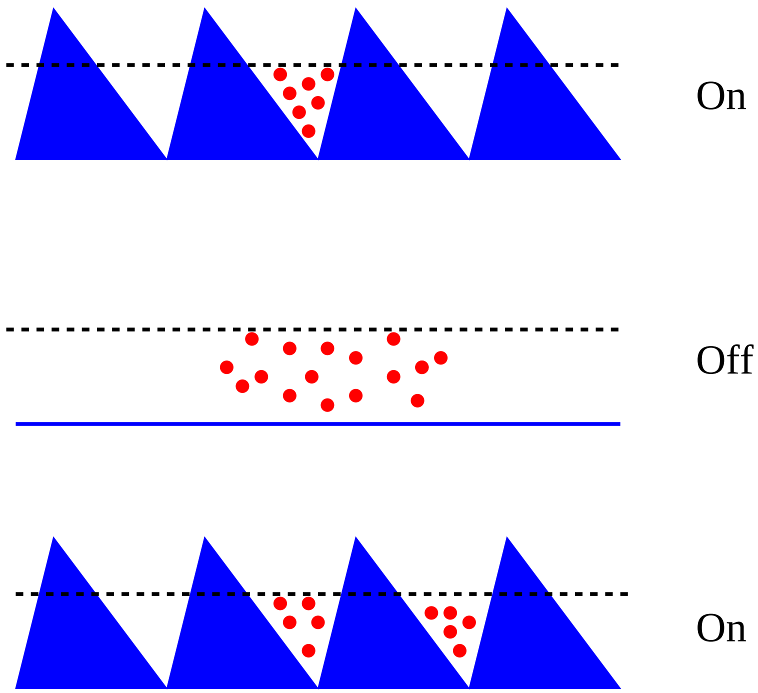
Brownian motors

To build a Brownian motor drive the system out of equilibrium



Working principle of a Brownian motor driven by temperature oscillation

Another model of Brownian motor: a pulsating (flashing) ratchet



Quantum ratchets in dissipative chaotic systems,
preprint cond-mat/0407702

A deterministic model of quantum chaotic dissipative ratchet

Particle moving in a kicked periodic asymmetric potential

$$V(x,\tau) = k\left[\cos(x) + \frac{a}{2}\cos(2x+\phi)\right]\sum_{m=-\infty}^{+\infty}\delta(\tau - mT),$$

Classical evolution in one period described by the map

$$\begin{cases} \overline{n} = \gamma n + k(\sin(x) + a\sin(2x+\phi)), \\ \overline{x} = x + T\overline{n}, \end{cases}$$

$0 < \gamma < 1$ dissipation parameter (velocity proportional damping):
$\gamma = 0$ overdamping – $\gamma = 1$ Hamiltonian evolution

Introducing the rescaled momentum variable $p = Tn$, one can see that classical dynamics depends on the parameter $K = kT$ (not on k and T separately)

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Study of the quantized model

Quantization rules: $x \to \hat{x}$, $n \to \hat{n} = -i(d/dx)$ (we set $\hbar = 1$)

Since $[\hat{x}, \hat{p}] = iT$, the effective Planck constant is $\hbar_{\rm eff} = T$

In order to simulate a dissipative environment in the quantum model we consider a master equation in the Lindblad form for the density operator $\hat{\rho}$ of the system:

$$\dot{\hat{\rho}} = -i[\hat{H}_s, \hat{\rho}] - \frac{1}{2} \sum_{\mu=1}^{2} \{\hat{L}_\mu^\dagger \hat{L}_\mu, \hat{\rho}\} + \sum_{\mu=1}^{2} \hat{L}_\mu \hat{\rho} \hat{L}_\mu^\dagger$$

$\hat{H}_s = \hat{n}^2/2 + V(\hat{x}, \tau)$ system Hamiltonian
$\hat{L}_\mu$ Lindblad operators
$\{\ ,\ \}$ denotes the anticommutator

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The dissipation model

We assume that dissipation is described by the following lowering operators:

$$\hat{L}_1 = g \sum_n \sqrt{n+1}\, |n\rangle \langle n+1|,$$
$$\hat{L}_2 = g \sum_n \sqrt{n+1}\, |-n\rangle \langle -n-1|, \quad n = 0, 1, \ldots$$

These Lindblad operators can be obtained by considering the interaction between the system and a bosonic bath. The master equation is then derived, at zero temperature, in the usual weak coupling and Markov approximations

Requiring that at short times $\langle p \rangle$ evolves like in the classical case, as it should be according to the Ehrenfest theorem, we obtain $g = \sqrt{-\ln \gamma}$

Simulation of quantum dissipation with quantum trajectories: this approach has enormous advantages in memory requirements, as it allows us to store only a stochastically evolving state vector, instead of the full density matrix

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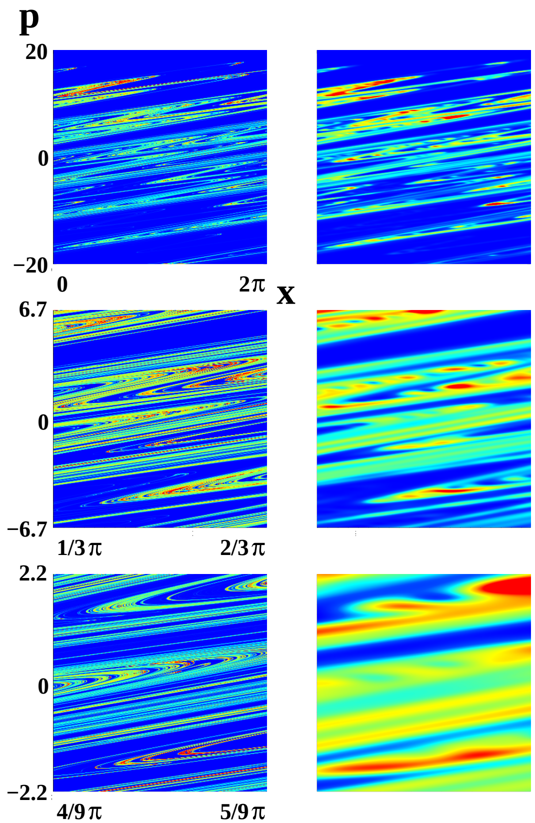
Asymmetric quantum strange attractor



Phase space pictures for $K = 7$, $\gamma = 0.7$, $\phi = \pi/2$, $a = 0.7$, after 100 kicks: classical Poincaré sections (left) and quantum Husimi functions at $\hbar_{\rm eff} = 0.012$ (right)

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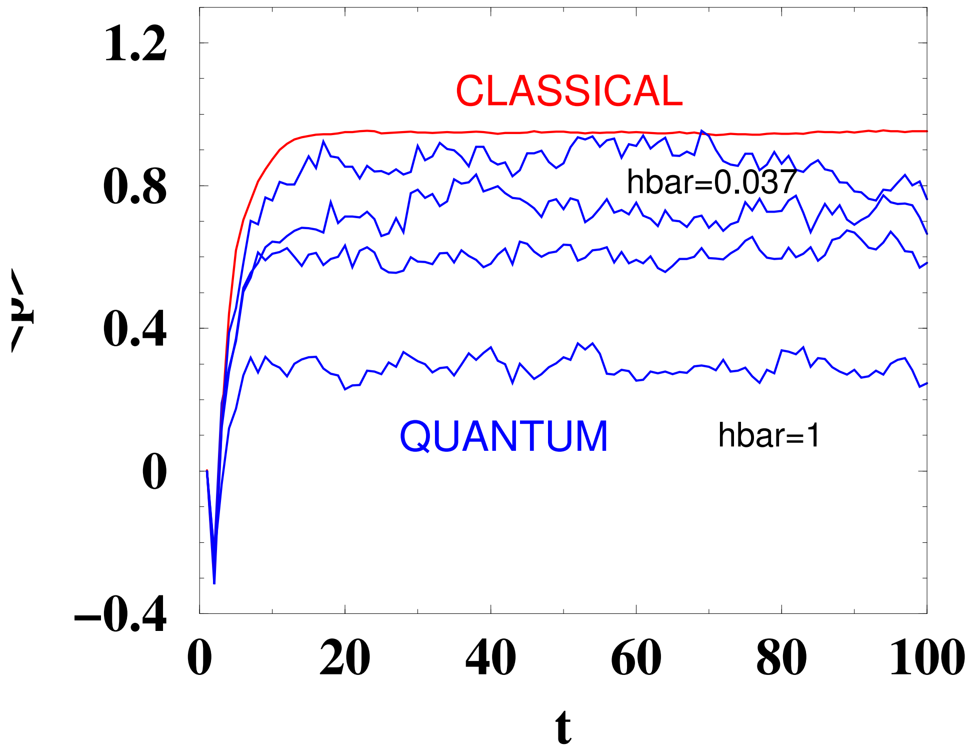
Ratchet effect



Average momentum $\langle p \rangle$ as a function of time t (measured in number of kicks)

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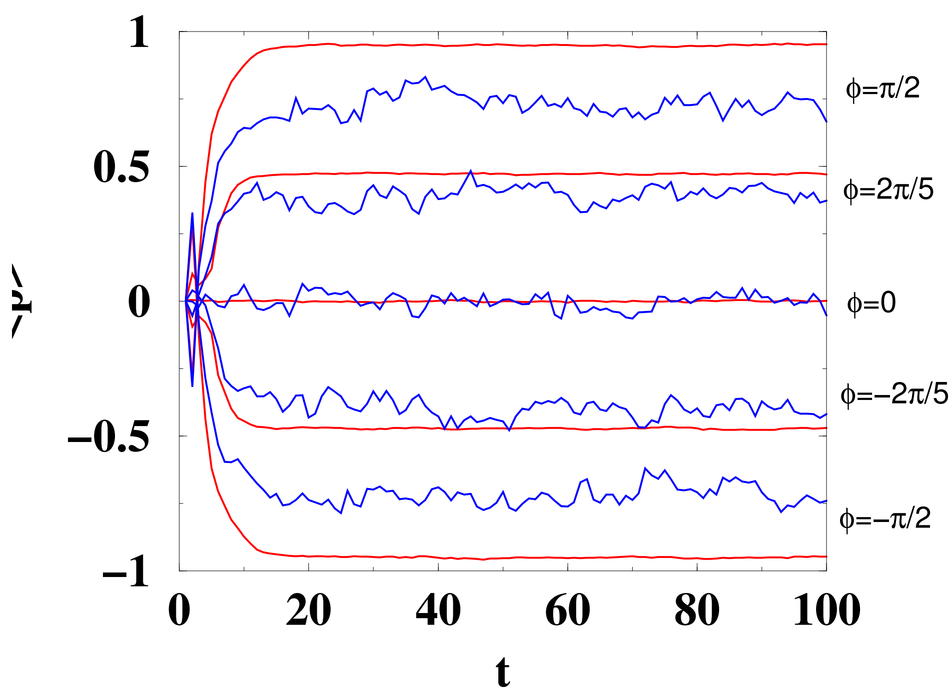
Control the direction of transport



Zero net current for $\phi = n\pi$, due to the space symmetry $V(x, \tau) = V(-x, \tau)$

In general $\langle p \rangle_{-\phi} = -\langle p \rangle_{\phi}$, due to the symmetry $V_{\phi}(x, \tau) = V_{-\phi}(-x, \tau)$

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Possible experimental implementation

Possible experimental implementations with cold atoms in a periodic standing wave of light

Values $K = 7$, $\hbar_{\text{eff}} \sim 1$ were used in the experimental implementations of the kicked rotor model

Friction force implemented by means of Doppler cooling techniques

State reconstruction techniques could in principle allow the experimental observation of a quantum strange ratchet attractor

The ratchet effect is robust when noise is added; due to the presence of a strange attractor, the stationary current is independent of the initial condition

Papers from the Como group in 2004

[1] D. Rossini, G. Benenti and G. Casati, *Entanglement Echoes in Quantum Computation* Phys. Rev. A **69**, 052317 (2004) [quant-ph/0309146].

[2] W.-G. Wang, G. Casati and B. Li, *Stability of Quantum Motion: Beyond Fermi-golden-rule and Lyapunov decay*, Phys. Rev. E **69**, 025201 (2004) [quant-ph/0309154].

[3] G. Casati and S.Montangero, *Measurement and Information Extraction in Complex Dynamics Quantum Computation* in *Decoherence and Entropy in complex Systems*, H.-T. Elze Ed., Lectures Notes in Physics Vol. 633, Springer-Verlag, Berlin 2004 [quant-ph/0307165].

[4] G.G. Carlo, G. Benenti, G. Casati and C. Mejía-Monasterio, *Simulating noisy quantum protocols with quantum trajectories*, Phys. Rev. A **69**, 062317 (2004) [quant-ph/0402102].

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[6] D. Rossini, G. Benenti and G. Casati, *Classical versus quantum errors in quantum computation of dynamical systems*, Phys. Rev. E **70**, 056216 (2004) [quant-ph/0405189].

[7] W.-G. Wang, G.Casati, B. Li and T. Prosen, *Uniform semiclassical approach to fidelity decay*, submitted to Phys. Rev. Lett. [quant-ph/0407040].

[8] G.G. Carlo, G. Benenti, G. Casati and D.L. Shepelyansky, *Quantum ratchets in dissipative chaotic systems*, submitted to Phys. Rev. Lett. [cond-mat/0407702].

[9] C. Mejía-Monasterio, G. Benenti, G.G. Carlo and G. Casati, *Entanglement across a Transition to Quantum Chaos*, submitted to Phys. Rev. A [quant-ph/0410246].

[10] S. Montangero, A. Romito, G. Benenti and R. Fazio, *Chaotic dynamics in superconducting nanocircuits*, submitted to Phys. Rev. Lett. [preprint cond-mat/0407274].

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"Enrico Fermi" School on Quantum Computers, Algorithms and Chaos

Villa Monastero, Varenna, Como Lake, Italy, July 5th-15th, 2005

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