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Dissipative quantum chaos: transition from wave packet collapse to explosion,
Phys. Rev. Lett. **95**, 164101 (2005)

(c3) Quantum Simulation of Dissipative Chaotic Systems

Quantum simulation is a special instance of quantum computation

In particular, optical lattices allowed the observation of the superfluid to Mott insulator and could be used for the study of a wide range of phenomena, from strongly correlated condensed matter physics to spin glasses

We investigate the possibilities opened by optical lattices for the quantum simulation of complex dissipative systems

G.G. Carlo, G. Benenti and D.L. Shepelyansky

Dissipative quantum chaos: transition from wave packet collapse to explosion,
Phys. Rev. Lett. **95**, 164101 (2005)

A test bed to study the effects of dissipative decoherence on nonlinear dynamics: cold atoms in optical lattices

The laser fields create an effective kicked potential implementing the kicked rotator model, which has been realized by the groups of M.Raizen (Texas), d'Arcy et al. (Oxford), Amman et al. (Auckland) and Ringot et al. (Lille)

These experiments allowed the investigation of several important physical phenomena, such as dynamical localization, decoherence, chaos assisted tunneling, (anti)Zeno effect, and ratchet effect

Dissipative quantum chaos: transition from wave packet collapse to explosion,
Phys. Rev. Lett. **95**, 164101 (2005)

Dissipative quantum chaos: transition from wave packet collapse to wave packet explosion

The instability of classical dynamics leads to exponentially fast spreading of the quantum wave packet on the logarithmically short Ehrenfest time scale

$$t_E \sim \frac{|\ln \hbar|}{\lambda}$$

λ Lyapunov exponent, $\hbar$ effective Planck constant

After the logarithmically short Ehrenfest time a description based on classical trajectories is meaningless for a closed quantum system

What is the interplay between wave packet explosion (delocalization) induced by chaotic dynamics and wave packet collapse (localization) caused by dissipation?

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Dissipative quantum chaos: transition from wave packet collapse to explosion,
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A model of dissipative chaotic dynamics

Markovian master equation $\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] - \frac{1}{2}\sum_{\mu}\{\hat{L}_{\mu}^{\dagger}\hat{L}_{\mu}, \hat{\rho}\} + \sum_{\mu}\hat{L}_{\mu}\hat{\rho}\hat{L}_{\mu}^{\dagger}$

Kicked rotator Hamiltonian $\hat{H} = \frac{\hat{I}^2}{2} + k\cos(\hat{x})\sum_{m=-\infty}^{+\infty}\delta(\tau - mT)$

Dissipation described by the Lindblad operators

$$\hat{L}_1 = g\sum_{I}\sqrt{I+1}\,|I\rangle\,\langle I+1|,\ \hat{L}_2 = g\sum_{I}\sqrt{I+1}\,|-I\rangle\,\langle -I-1|$$

At the classical limit, the evolution of the system in one period is described by the Zaslavsky map

$$\begin{cases} I_{t+1} = (1-\gamma)I_t + k\sin x_t, \\ x_{t+1} = x_t + TI_{t+1}, \end{cases}$$

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Quantum trajectories

The changing state of a single open quantum system is represented directly by a stochastically evolving quantum wave function, as for a single run of a laboratory experiment - a single evolution is termed a quantum trajectory

The stochasticity is due to the (indirect) quantum measurement process: As an example we can consider the atomic decay and the measurement of the emitted photon

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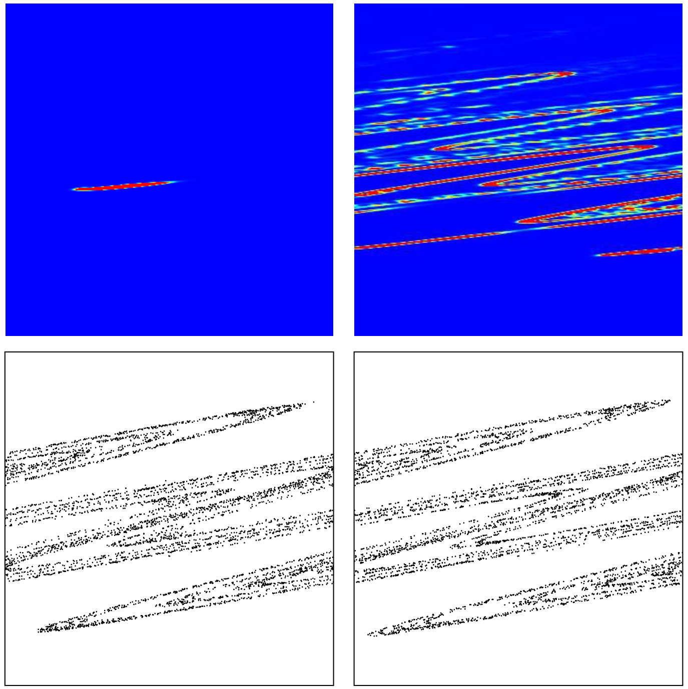
Collapse to explosion transition (going from strong to weak dissipation)



$K = 7$, $\hbar = 0.012$, $\gamma = 0.5$ and $\gamma = 0.01$

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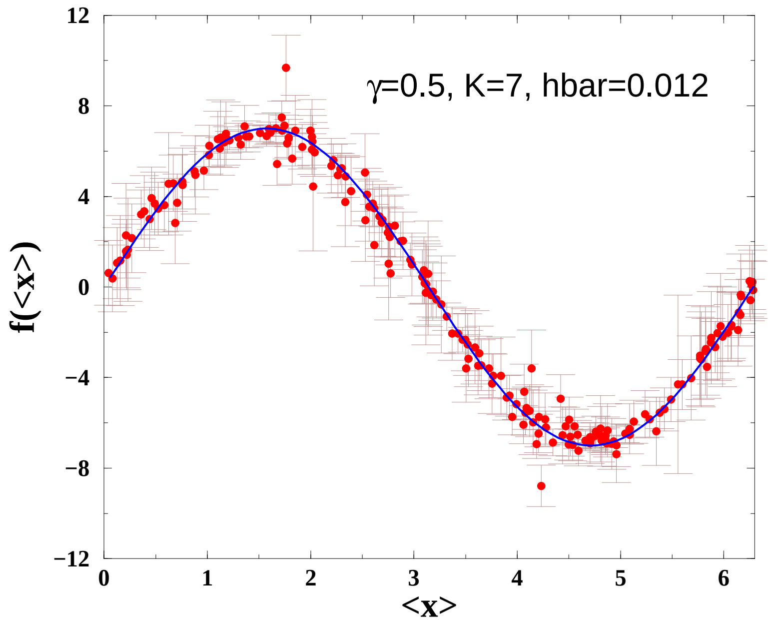
Classical-like evolution of quantum trajectories



$$f \equiv \langle p \rangle_{t+1} - (1-\gamma)\langle p \rangle_t, \quad \langle p \rangle_t = \langle x \rangle_t - \langle x \rangle_{t-1}$$

From classical dynamics we expect $f(x) = K \sin x$ - Quantum fluctuations $\propto \sqrt{\hbar}$

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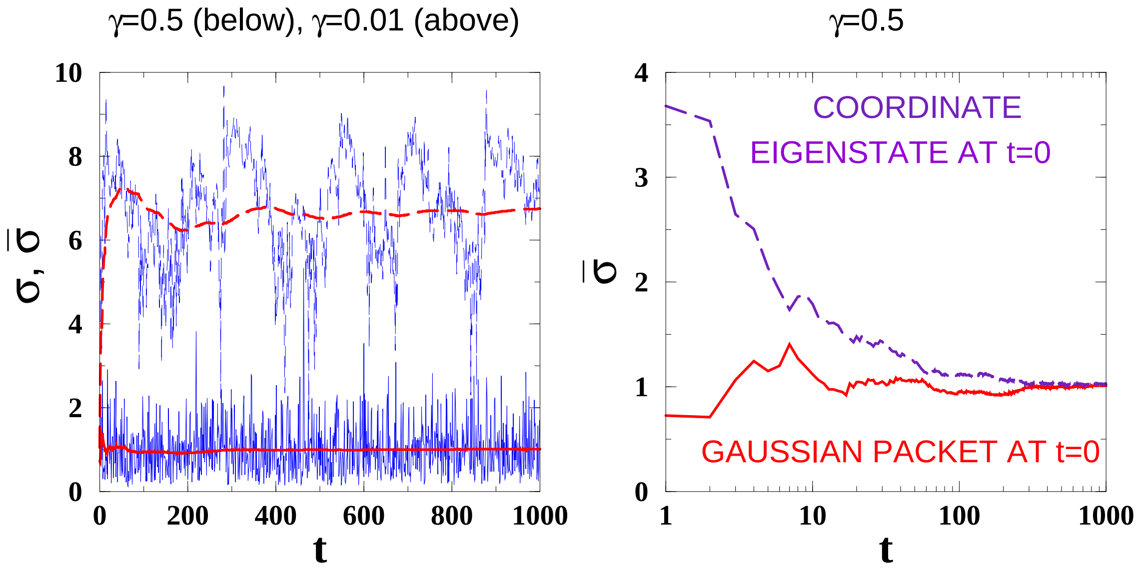
Wave packet dispersion

$$\sigma_t = \sqrt{(\Delta x)_t^2 + (\Delta p)_t^2}, \quad \text{cumulative average } \overline{\sigma}_t \equiv \frac{1}{t}\sum_{j=1}^{t}\sigma_j$$



$$(K = 7,\, \hbar = 0.012)$$

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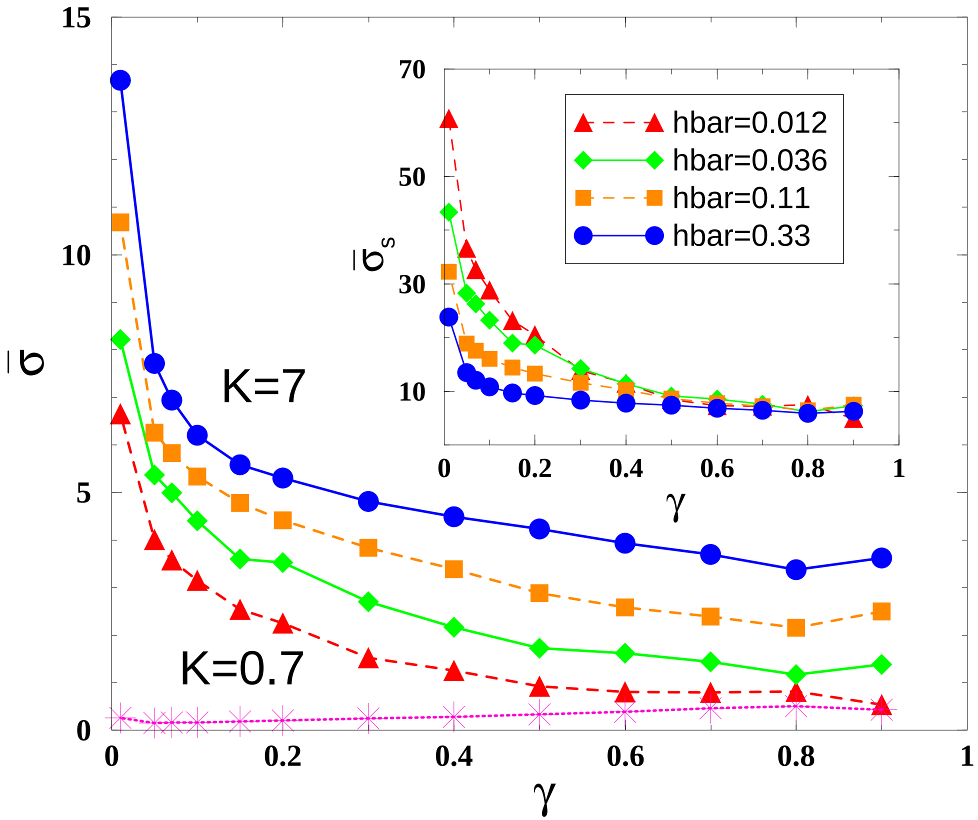
Localization - delocalization crossover



$\overline{\sigma}_s \equiv \overline{\sigma}/\sqrt{\hbar}$ scaled dispersion

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Ehrenfest explosion

Due to the exponential instability of chaotic dynamics the wave packet spreads exponentially and for times shorter than the Ehrenfest time we have $\sigma_t \sim \sqrt{\hbar}\exp(\lambda t)$

The dissipation localizes the wave packet on a time scale of the order of $1/\gamma$

Therefore, for $1/\gamma \ll t_E \sim |\ln \hbar|/\lambda$, we obtain $\overline{\sigma} \sim \sqrt{\hbar}\exp(\lambda/\gamma) \ll 1$

In contrast, for $1/\gamma > t_E$ the chaotic wave packet explosion dominates over dissipation and we have complete delocalization over the angle variable

In this case, the wave packet spreads algebraically due to diffusion for $t > t_E$: for $t \gg t_E$ we have $\sigma_t \sim \sqrt{D(K)t}$, $D(K) \approx K^2/2$ being the diffusion coefficient; this regime continues up to the dissipation time $1/\gamma$, so that $\overline{\sigma} \sim \sqrt{D(K)/\gamma}$

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Dissipative quantum chaos: transition from wave packet collapse to explosion,
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Tthe transition from collapse to explosion (Ehrenfest explosion) takes place at

$$t_E \sim \frac{|\ln \hbar|}{\lambda} \sim \frac{1}{\gamma}$$

Therefore, even for infinitesimal dissipation strengths the quantum wave packet is eventually localized when $\hbar \to 0$: we have $\lim_{\hbar \to 0} \overline{\sigma} = 0$; in contrast, in the Hamiltonian case $(\gamma = 0)$ $\lim_{\hbar \to 0} \overline{\sigma} = \infty$

Only for open quantum systems the classical concept of trajectory is meaningful for arbitrarily long times; on the contrary, for Hamiltonian systems a description based on wave packet trajectories is possible only up to the Ehrenfest time scale

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(d1) Model of a deterministic detector and dynamical decoherence

The quantum measurement problem has recently gained a renewed interest due to its relevance for quantum information processing

The readout problem has been solved in ion-trap quantum computation using quantum jump detection

In solid-state implementations, the single-qubit measurement is very challenging and has been widely discussed and various readout schemes have been experimentally realized, for instance by the groups of D. Esteve (Saclay), J.E. Mooij (Delft) and Nakamura (NEC, Japan)

A detector can be seen as a complex quasi-classical object coupled to a quantum system: investigate the dynamical evolution of concrete system-detector models

J.W. Lee, D.V. Averin, G. Benenti and D.L. Shepelyansky

Model of a deterministic detector and dynamical decoherence,
Phys. Rev. A **72**, 012310 (2005)

The model

Let us consider a two-level system coupled to a chaotic dynamical system in the semiclassical regime (it could be implemented by means of optical lattices)

$$\begin{aligned}
\hat{H} &= \hat{H}_s + \hat{H}_d + \hat{H}_{int}, \\
\hat{H}_s &= \delta\hat{\sigma}_x, \\
\hat{H}_d &= \frac{\hat{p}^2}{2} + K\cos(\hat{\theta})\sum_m \delta(t-m), \\
\hat{H}_{int} &= \epsilon_c\hat{\sigma}_z\cos(\hat{\theta})\sum_m \delta(t-m).
\end{aligned}$$

We have $[\hat{p},\hat{\theta}] = -i\hbar$, where $\hbar$ is the effective dimensionless Planck constant

- The classical limit for the detector corresponds to $\hbar \to 0$, while keeping K constant
- The classical motion is chaotic when $K \gg 1$

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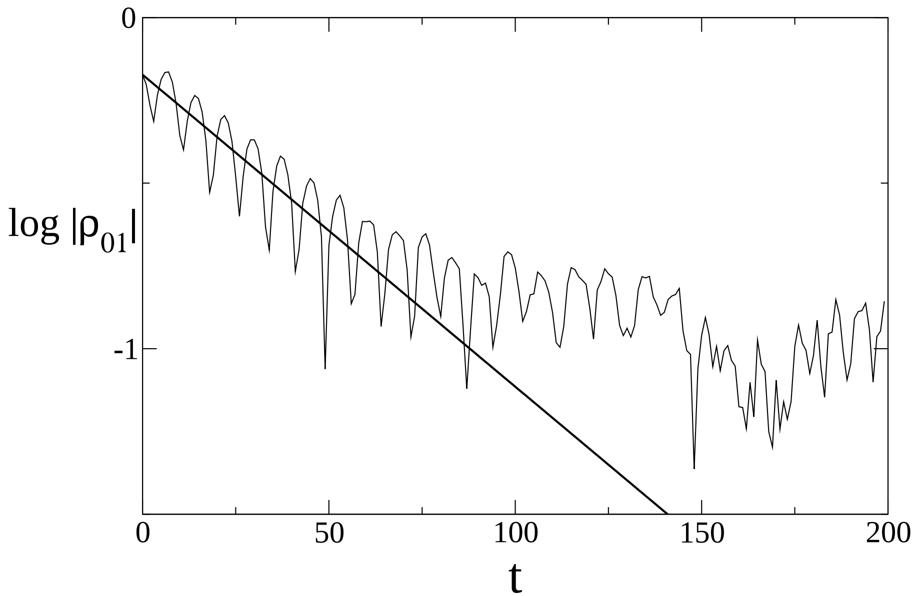
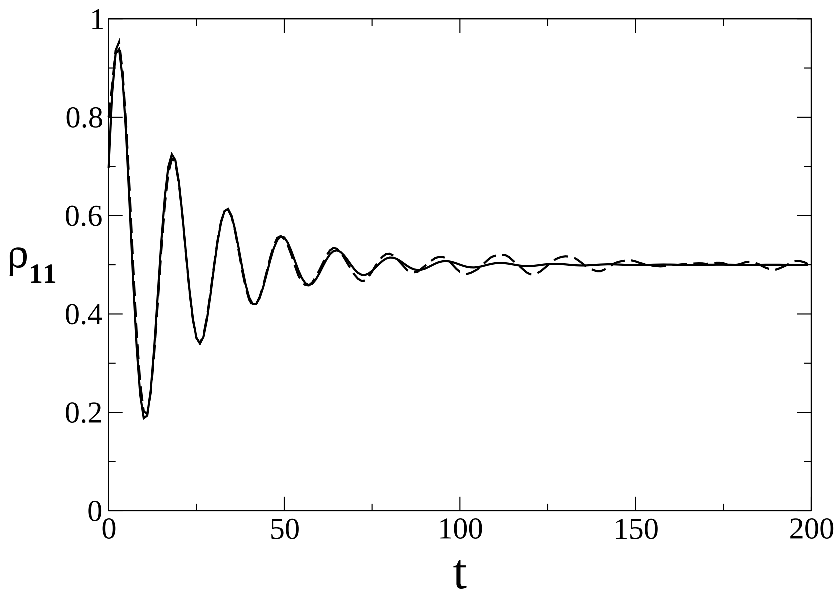
Phys. Rev. A **72**, 012310 (2005)

Dynamical decoherence





Dephasing and population relaxation for the qubit at $K = 8$, $\hbar = 7.67 \times 10^{-4}$, $\delta = 0.2$, $\epsilon = \epsilon_c/\hbar = 0.2$

J.W. Lee, D.V. Averin, G. Benenti and D.L. Shepelyansky

Model of a deterministic detector and dynamical decoherence,
Phys. Rev. A **72**, 012310 (2005)

Simulating decoherence

Our fully deterministic dynamical model can reproduce the main features (relaxation times T_1 and T_2,...) of the single-qubit decoherence due to a heat bath

The evolution of the reduced density matrix is the same as in the Caldeira-Leggett model with purely dephasing qubit-environment coupling

In this model, the interaction with the environment induces phase kicks:

$$\rho \to \frac{1}{2\pi}\int_0^{2\pi} d\theta R(\theta)\rho R^\dagger(\theta), \quad R(\theta) = \begin{bmatrix} e^{-i\epsilon\cos\theta} & 0 \\ 0 & e^{i\epsilon\cos\theta} \end{bmatrix}.$$

That is, in the qubit-detector interaction Hamiltonian the angle θ is drawn from a random uniform distribution in $[0, 2\pi]$ (random phase approximations)

This paves the way to very convenient numerical simulations of more complicated environments not easy to treat analytically (such as non-Markovian environments, an issue of interest, for instance, in solid state quantum computation)

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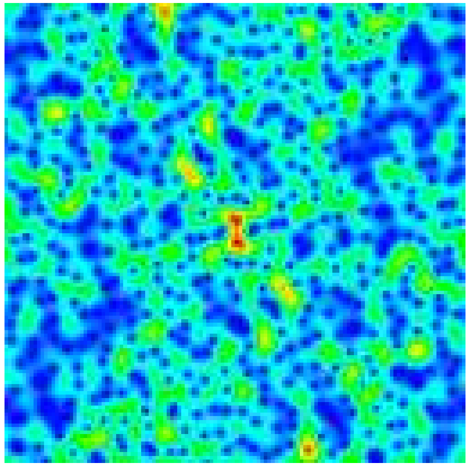
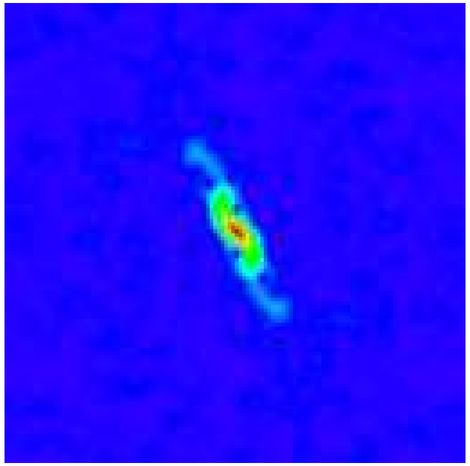
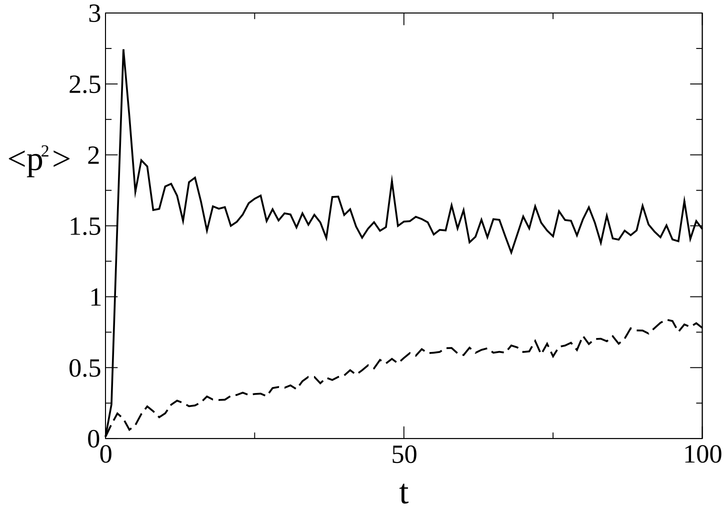
Model of a deterministic detector and dynamical decoherence,
Phys. Rev. A **72**, 012310 (2005)

Detection in the strong coupling model

In the strong coupling limit we can distinguish the spin un/down states







Husimi functions at $t = 10$ and evolution of $\langle p^2 \rangle$ for the detector for spin up (left Husimi and solid curve) and spin down (right Husimi and dashed curve) at $K = 4.5$, $\hbar = 1.23 \times 10^{-2}$, $\epsilon_c = 0.8$. $\delta = 0.1$

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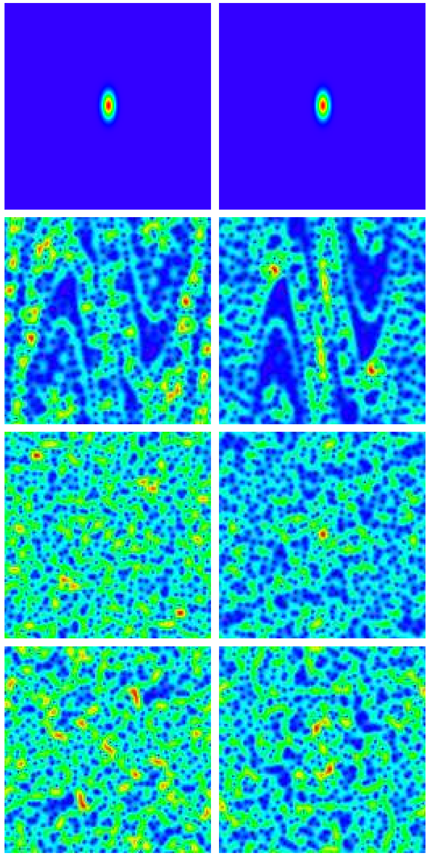
Model of a deterministic detector and dynamical decoherence,
Phys. Rev. A **72**, 012310 (2005)

Detection in the weak coupling limit

Due to chaos, the dynamical evolution of the detector is strongly sensitive to the state of the qubit. However, it is unclear how to extract a signal from a measurement coarse-grained on a scale much larger than $\hbar$

Husimi functions for $K = 8$, $\hbar = 1.23 \times 10^{-2}$, $\delta = 0.2$, $\epsilon_c = \epsilon\hbar = 0.4\hbar \ll 1$, $t = 0, 4, 8, 12$ (from top to bottom), spin up (left) and spin down (right)



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Quantum dephasing and classical chaos

The relation between classical dynamical chaos and quantum dephasing is still an open important problem, potentially relevant also for the prospects of quantum computation

In order to elucidate this problem, we consider the quantum Loschmidt echo or fidelity $\mathcal{F}$ for a mixed initial state $\rho_0 = \sum_k p_k |\psi_k\rangle\langle\psi_k|$:

$$\mathcal{F}(t) = \left| \sum_k p_k f_k(t) \right|^2, \quad f_k(t) = \langle\psi_k|U_0(t)^\dagger U_\epsilon(t)|\psi_k\rangle$$

$\mathcal{F}(t)$ is just the quantity which is measured in the Ramsey type experiments performed on cold atoms in optical lattices and in atom optics billiards

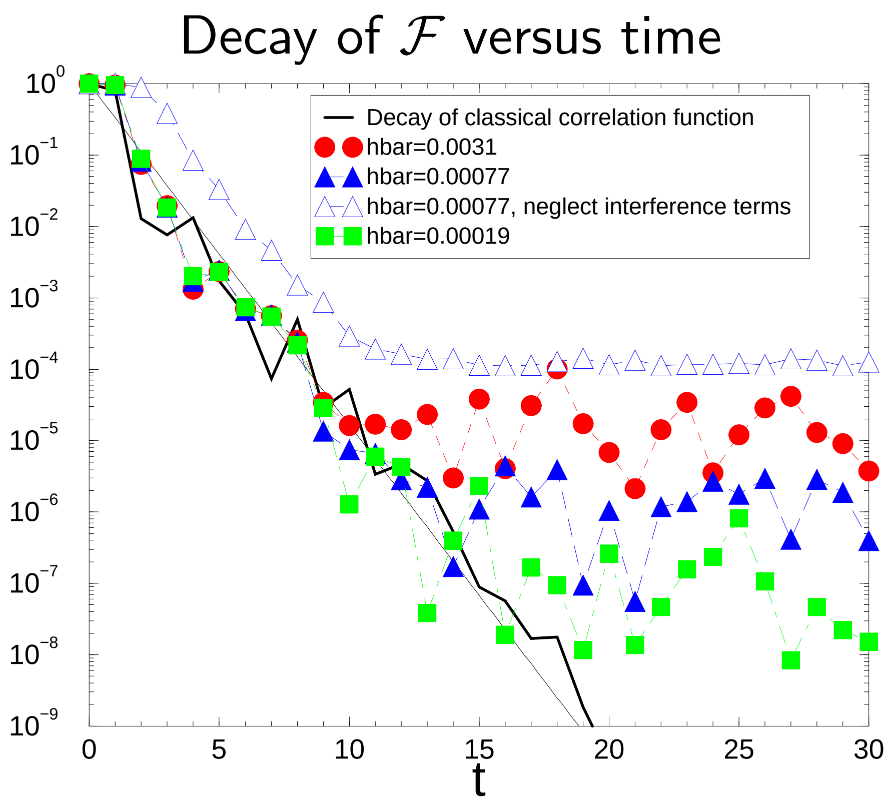
The function $\mathcal{F}$ accounts for quantum interference and is expected to retain quantal features even in the deep semiclassical region

Due to dephasing induced by the underlying chaotic classical dynamics, the decay of $\mathcal{F}$ can be directly connected to the decay of an appropriate classical correlation function (of classical phases)

Contrary to decoherence produced by an external noise, in our case dephasing is of purely dynamical nature

Quantum dephasing and decay of classical correlation functions in chaotic systems,
preprint quant-ph/0504141

Results derived analytically for the example of a nonlinear driven oscillator and numerically confirmed for the kicked rotator model



V.V. Sokolov, G. Benenti and G. Casati

Effects of single-qubit quantum noise on entanglement purification,
preprint quant-ph/0505177, to be published in Eur. Phys. J. D.

Effects of single-qubit quantum noise on entanglement purification

A central problem of quantum communication is how to reliably transmit quantum information through a noisy quantum channel

If a member of a maximally entangled EPR pair is transmitted from Alice to Bob, then noise in the channel can degrade the amount of entanglement of the pair

This problem is of primary importance for entanglement-based quantum cryptography, in protocols such as E91 (Ekert, 1991)

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Entanglement purification

Entanglement purification techniques exist and have been applied, in particular, to quantum cryptography

A quantum privacy amplification iterative protocol was proposed by Deutsch *et al.* (1996), that eliminates entanglement with an eavesdropper by creating a small number of nearly perfect (pure) EPR states out of a large number of partially entangled states

This protocol is based on the so-called LOCC, that is on local quantum operations (quantum gates and measurements performed by Alice and Bob on their own qubits), supplemented by classical communication

Effects of single-qubit quantum noise on entanglement purification,
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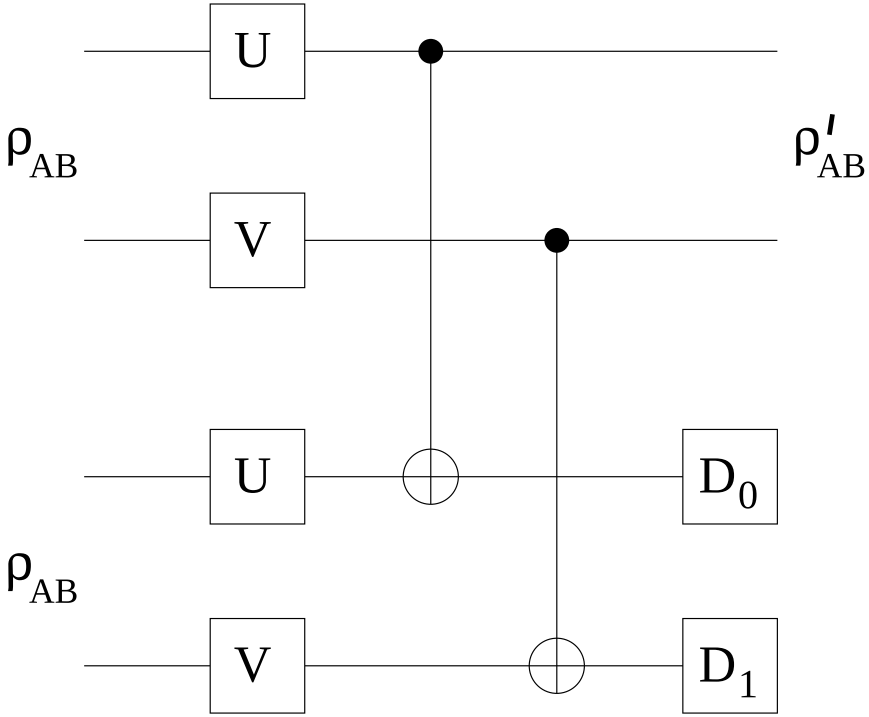

Accept ρ'_{AB} only if D_0 and D_1 give the same outcome

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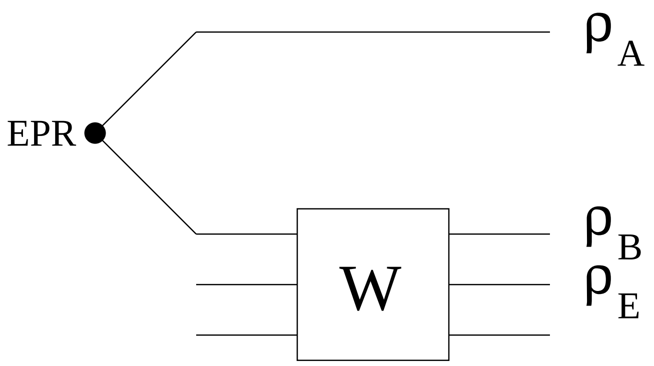
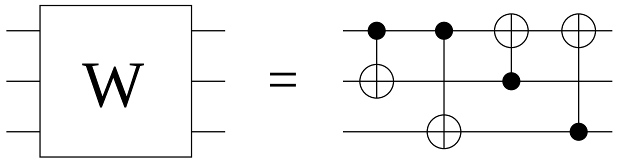
Effects of single-qubit quantum noise on entanglement purification,
preprint quant-ph/0505177, to be published in Eur. Phys. J. D.

Noise effects

We study the the impact of all possible single-qubit quantum noise channels on the quantum privacy amplification protocol for the purification of entanglement in quantum cryptography

We assume that the E91 protocol is used by two communicating parties (Alice and Bob) and that the eavesdropper Eve uses the isotropic Bužek-Hillery quantum copying machine to extract information





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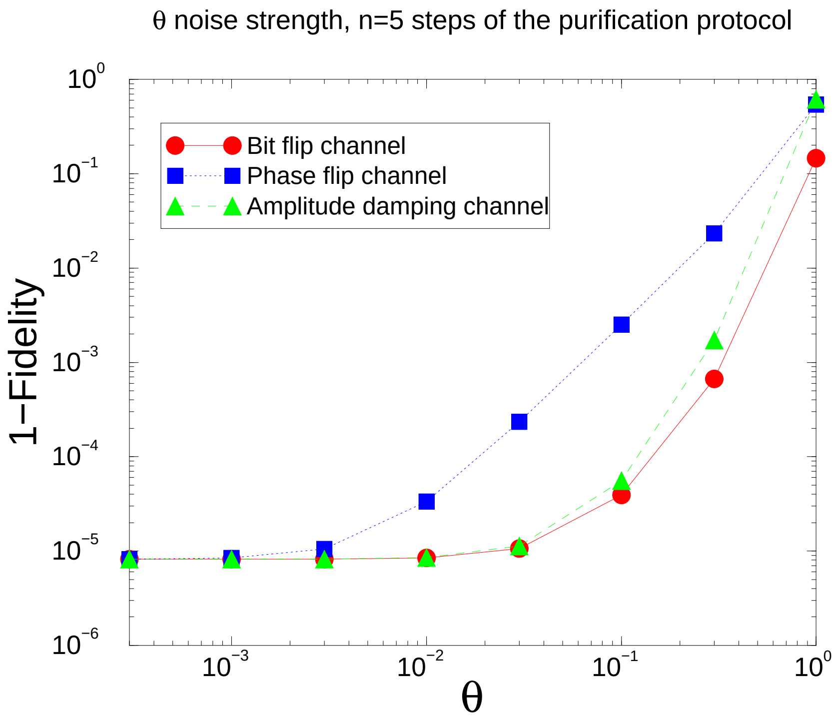
Effects of single-qubit quantum noise on entanglement purification,
preprint quant-ph/0505177, to be published in Eur. Phys. J. D.

We find that both the qualitative behavior of the fidelity of the purified state as a function of the number of purification steps and the maximum level of noise that can be tolerated by the protocol strongly depend on the specific noise channel



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Papers from the Como group in 2005

[1] S. Montangero, A. Romito, G. Benenti and R. Fazio, *Chaotic dynamics in superconducting nanocircuits*, Europhys. Lett. **71**, 893 (2005) [cond-mat/0407274].

[2] G.G. Carlo, G. Benenti, G. Casati and D.L. Shepelyansky, *Quantum ratchets in dissipative chaotic systems*, Phys. Rev. Lett. **94**, 164101 (2005) [cond-mat/0407702].

[3] C. Mejía-Monasterio, G. Benenti, G.G. Carlo and G. Casati, *Entanglement across a transition to quantum chaos*, Phys. Rev. A **71**, 062324 (2005) [quant-ph/0410246].

[4] J.W. Lee, D.V. Averin, G. Benenti and D.L. Shepelyansky, *Model of a deterministic detector and dynamical decoherence*, Phys. Rev. A **72**, 012310 (2005) [quant-ph/0501153].

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[7] G. Benenti, S. Felloni and G. Strini, *Effects of single-qubit quantum noise on entanglement purification*, preprint quant-ph/0505177, to be published in Eur. Phys. J. D.

[8] G. Benenti and G. Casati, *Quantum computers: Where do we stand?*, Europhysics News **36/1**, 16 (2005).

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[10] G. Casati and G. Benenti, *Quantum chaos, decoherence and quantum computation*, to be published in the Proceedings of the "Enrico Fermi" School on *Quantum Computers, Algorithms and Chaos*, Varenna, Italy, 5-15 July, 2005.

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