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3 papers in 2003

*Detected-jump-error-correcting quantum codes, quantum error designs,
and quantum computation*, Phys. Rev. A **68**, 012316-1-10 (2003)

Quantum Error Correcting Codes for Spontaneous Decay Processes

We have developed

embedded quantum codes (jump codes) for correcting
spontaneous decay processes

- multiple-error correcting codes
- bounds on redundancies

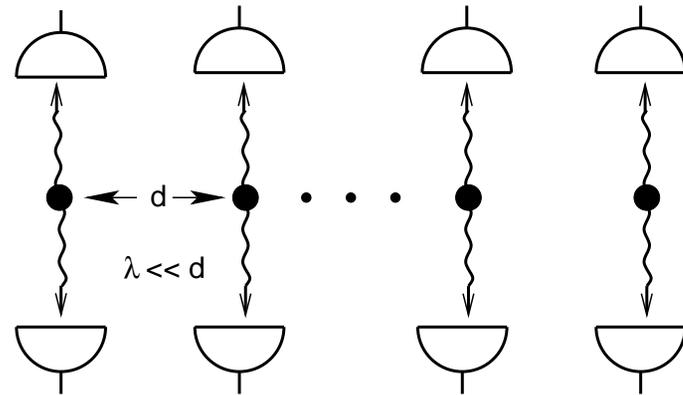
collaboration with Karlsruhe University: Th. Beth, Ch. Charnes, M. Grassl

Detected-jump-error-correction quantum codes, quantum error designs, and quantum computation, Phys. Rev. A **68**, 012316-1-10 (2003)

Jump codes and spontaneous decay:

The problem:

spontaneous decay into
statistically independent reservoirs



error correction by continuous observation of a qubit array

Decoherence arises due to:

- modified dynamics between successive spontaneous emission events
- spontaneous emissions events

Detected-jump-error-correction quantum codes, quantum error designs, and quantum computation, Phys. Rev. A **68**, 012316-1-10 (2003)

Main idea of jump codes

- correct modified dynamics between successive jumps **passively**
- correct spontaneous decays **actively**

Theoretical description of decoherence and error correction by
quantum trajectory method

Detected-jump-error-correction quantum codes, quantum error designs, and quantum computation, Phys. Rev. A **68**, 012316-1-10 (2003)

Jump codes and error designs

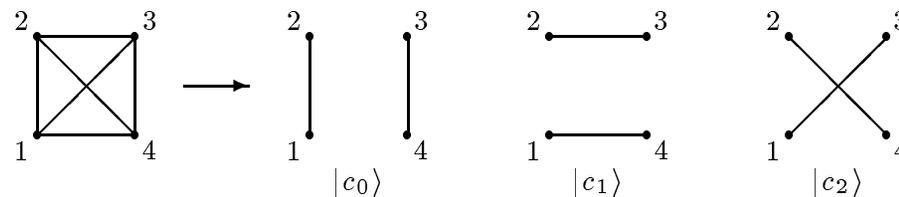
The simplest example of a 1-JC(4,2,3)

$$|c_0\rangle = \frac{1}{\sqrt{2}}[|1100\rangle + e^{i\varphi}|0011\rangle]$$

$$|c_1\rangle = \frac{1}{\sqrt{2}}[|1001\rangle + e^{i\varphi}|0110\rangle]$$

$$|c_2\rangle = \frac{1}{\sqrt{2}}[|0101\rangle + e^{i\varphi}|1010\rangle]$$

equal weights guarantee preservation of quantum coherence



jump code is associated with possible pairs of parallels of an affine plane

Multiple-Error Correcting Jump Codes

Error designs → two- and three- error correcting jump codes

A three- error correcting 3-JC(8,4,3)

$$\begin{aligned} |c_1\rangle &= |00110011\rangle + |00111100\rangle + |01010101\rangle + |01011010\rangle \\ &\quad + |01100110\rangle + |01101001\rangle + |10010110\rangle + |10011001\rangle \\ &\quad + |10100101\rangle + |10101010\rangle + |11000011\rangle + |11001100\rangle \\ |c_2\rangle &= |00110110\rangle + |00111001\rangle + |01010011\rangle + |01011100\rangle \\ &\quad + |01100101\rangle + |01101010\rangle + |10010101\rangle + |10011010\rangle \\ &\quad + |10100011\rangle + |10101100\rangle + |11000110\rangle + |11001001\rangle \\ |c_3\rangle &= |00110101\rangle + |00111010\rangle + |01010110\rangle + |01011001\rangle \\ &\quad + |01100011\rangle + |01101100\rangle + |10010011\rangle + |10011100\rangle \\ &\quad + |10100110\rangle + |10101001\rangle + |11000101\rangle + |11001010\rangle \end{aligned}$$

*Detected-jump-error-correction quantum codes, quantum error designs,
and quantum computation*, Phys. Rev. A **68**, 012316-1-10 (2003)

Redundancy of Multiple-Error Correcting Jump Codes

A general upper bound

$$D \leq \binom{N-t}{\lfloor N/2 \rfloor - t}$$

D ... dimension of the code space

N ... number of physical qubits

t ... number of errors which can be corrected

Detected-jump-error-correction quantum codes, quantum error designs, and quantum computation, Phys. Rev. A **68**, 012316-1-10 (2003)

Achievable bounds

$n \setminus t$	1	2	3	4	5	6
4	3_2					
5	3_3					
6	10_3	$2 - 3_3$				
7	$10_3 - 15$	$3_3 - 5$				
8	35_4	$4_5 - 15$	$3_4 - 5$			
9	$35_4 - 56$	$4_5 - 21$	$3_4 - 6$			
10	126_5	$6_5 - 56$	$3_4 - 21$	$2_5 - 6$		
11	$126_5 - 210$	$10_5 - 84$	$4_5 - 28$	$3_5 - 7$		
12	462_6	$10_5 - 210$	$6_6 - 84$	$3_5 - 28$	$3_6 - 7$	
13	$462_6 - 792$	55_4	$6_6 - 120$	$3_5 - 36$	$3_6 - 8$	
14	1716_7	$55_6 - 792$	$6_6 - 330$	$5_5 - 120$	$3_6 - 36$	$2_7 - 8$

Stabilization of the Saw-Tooth Map by One-Error Correcting Jump Codes

Dynamics of the quantum saw-tooth map in the presence of spontaneous decay

The saw-tooth map $|\psi\rangle' = e^{-iT\hat{n}^2/2} e^{ik(\hat{\Theta}-\pi)^2/2} |\psi\rangle$

Computational basis

$$\begin{aligned} \hat{n} |000000\rangle_L &= -32 |000000\rangle_L \\ &\dots \dots \\ \hat{n} |100000\rangle_L &= 0 \\ &\dots \dots \\ \hat{n} |111111\rangle_L &= 31 |111111\rangle_L \end{aligned}$$

6 logical qubits

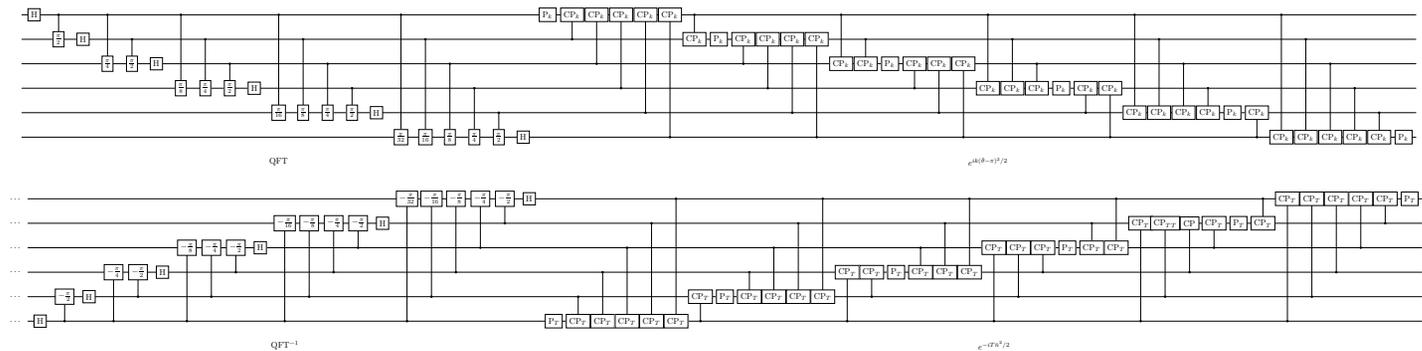
Encoding in error correcting code space of a 1 – JC(10, 5, 126)

$$\frac{1}{\sqrt{2}}(| 0000011111 \rangle + | 1111100000 \rangle) = | 000000 \rangle_L$$

$$\frac{1}{\sqrt{2}}(| 0000101111 \rangle + | 1111010000 \rangle) = | 000001 \rangle_L$$

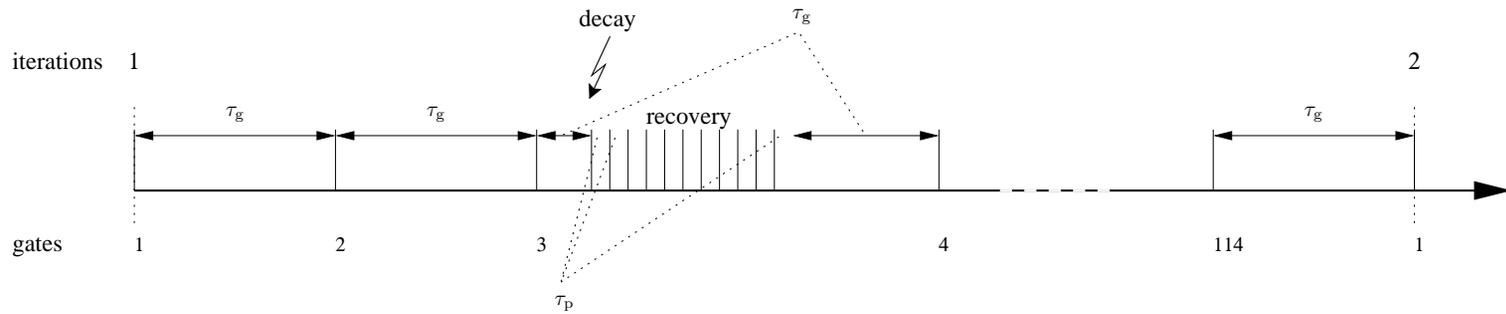
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Dynamics and gate sequences \hat{n} -basis and discrete quantum Fourier transform
 → 1 iteration → 114 quantum gates (Hadamard gates, controlled phase gates)



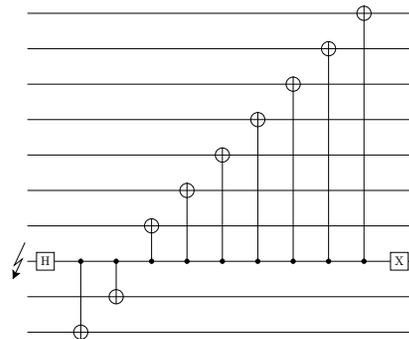
Error model of spontaneous decay

gates act instantaneously



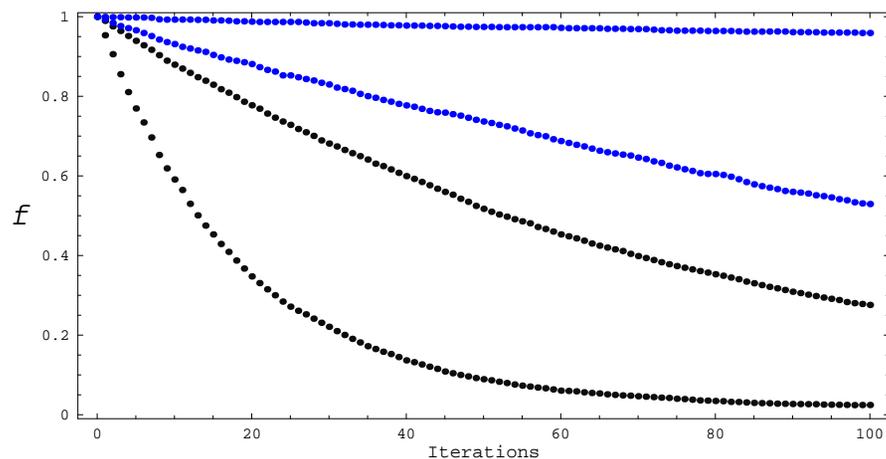
τ_g time between successive quantum gates \rightarrow **spontaneous decay**

$\tau_p = \tau_g/11$. . . time between successive quantum gates of the **recovery operation** (11 gates)



Stabilization of the saw-tooth map by error correction

Fidelity f with and without error correction

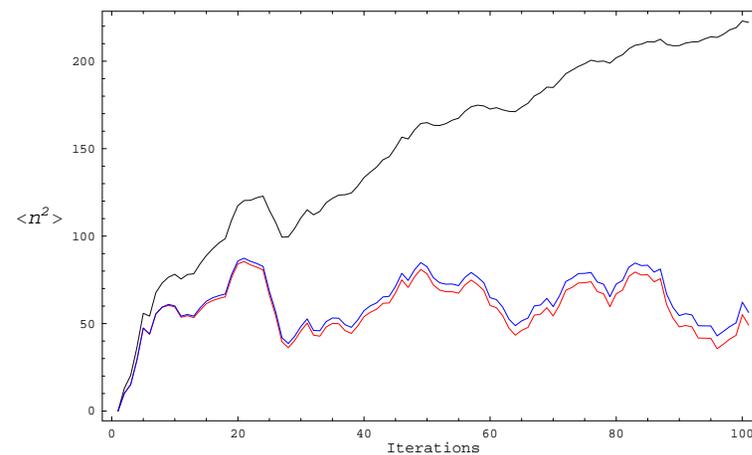


$$\kappa\tau_g = 5 \times 10^{-4} \text{ (upper curve)}$$

$$\kappa\tau_g = 2 \times 10^{-3} \text{ (lower curve)}$$

- . . . with error correction
- . . . without error correction
- . . . ideal case

$\langle n^2 \rangle$



Quantum Error Correction and Scalable Computer Architecture

Requirements for stabilizing quantum algorithms

- Quantum gates must not leave error correcting code space at any time
→ (Ising- and Heisenberg-type) Hamiltonian implementation of quantum gates
- Quantum gates should be independent of size of quantum memory
→ **scalable computer architecture**

