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Quantum error correction and quantum algorithms

Reliable performance of quantum algorithms is limited by

- uncontrolled couplings to reservoirs $\longrightarrow$ decoherence and dissipation
- uncontrolled Hamiltonian couplings between qubits $\longrightarrow$ coherent errors

We have developed

- embedded jump codes for spontaneous decay processes (2003)
- a new quantum error correcting method for coherent errors (2004)
  PAREC ($\equiv$ Pauli-random-error-correction) $\longrightarrow$ no redundancy
  O. Kern, G. Alber, D. Shepelyansky, Eur. Phys. J. D **32**, 153 (2005)

O. Kern, G. Alber, D. Shepelyansky: *Quantum error correction of coherent errors by randomization*, Eur. Phys. J. D **32**, 153 (2005)

PAREC - a new error correcting method for coherent errors

The problem: ideal dynamics of a quantum computer are modified by unknown Heisenberg-type interactions $\longrightarrow$ coherent errors

$$\delta\hat{H} = \sum_{j=0}^{n_q-1} \delta_j\, \hat{\boldsymbol{\sigma}}_j^{(z)} + \sum_{j=0}^{n_q-2} J_j \hat{\boldsymbol{\sigma}}_j \cdot \hat{\boldsymbol{\sigma}}_{j+1} \longrightarrow \hat{U} = e^{-i\delta\hat{H}}$$

- random imperfections unknown parameters (δ_j, J_j) vary randomly during computation

$\longrightarrow$ (linear) exponential decay of fidelity

$$\longrightarrow\ f(t) = |\ {}_t\langle\psi^{(\mathrm{ideal})}|\psi\rangle_t\ |^2 = e^{-t/t_c}$$

with the characteristic decay time $t_c = 1/(\epsilon^2 n_q n_g^2)$ and with $\delta_j, J_j \in [-\sqrt{3}\epsilon, \sqrt{3}\epsilon]$

O. Kern, G. Alber, D. Shepelyansky: *Quantum error correction of coherent errors by randomization*, Eur. Phys. J. D **32**, 153 (2005)

- static imperfections unknown parameters (δ_j, J_j) stay constant during computation

$\longrightarrow$ (quadratic) exponential decay of fidelity due to quantum interference

$$\longrightarrow \quad f(t) = e^{-t/t_c - t^2/(t_c t_h)}$$

with the Heisenberg-time $t_h = 0.5 \times 2^{nq}$

$\longrightarrow$ severely restricts computation time of a realistic quantum computer!

K. M. Frahm, R. Fleckinger, D.L. Shepelyansky, Eur. Phys. J. D. **29**, 139 (2004)

The idea: randomization of the coherent errors $\longrightarrow$ PAREC

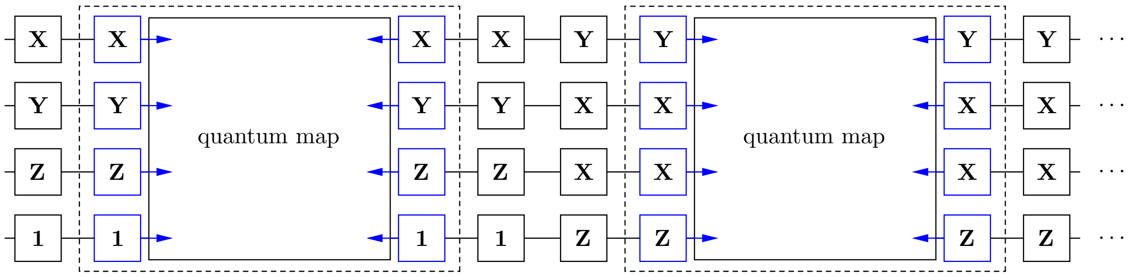
$\longrightarrow$ change of the exponential decay from quadratic to linear

$\longrightarrow$ significant increase of reliable computation times!

How can this randomization be realized in practice?

consider 4 qubits and Heisenberg-tpye coherent perturbations of the form

$$\delta \hat{H} \quad = \quad \sum_{j=0}^{n_q-1} \delta_j \, \hat{\boldsymbol{\sigma}}_j^{(z)} + \sum_{j=0}^{n_q-2} J_j \hat{\boldsymbol{\sigma}}_j \cdot \hat{\boldsymbol{\sigma}}_{j+1}$$



repeated random changes of the computational basis

- select random sequences of Pauli operations between quantum gates, e.g. $(\hat{X}, \hat{Y}, \hat{Z}, \hat{\mathbf{1}})$
- transform elementary quantum gates appropriately, e.g. $\hat{X}\hat{Z}\hat{X} = -\hat{Z}$

$\rightarrow$ repeated random changes of computational basis randomize coherent errors $\delta \hat{H}$

PAREC $\equiv$ Pauli-random-error-correction

Appropriate choice of universal quantum gates

How can the unitary transformations of the quantum gates be performed efficiently?
Implementation of unitary transformations might be costly and might introduce new errors

An appropriate set of universal quantum gates

$$\begin{aligned} \hat{S}_{\pm X_j}(\Delta\phi) &= e^{\mp i \hat{X}_j \Delta\phi}, \hat{S}_{\pm Z_j}(\Delta\phi) = e^{\mp i \hat{Z}_j \Delta\phi}, \\ \hat{S}_{\pm X_k X_j}(\Delta\phi) &= e^{\mp i \hat{X}_k \hat{X}_j \Delta\phi} \end{aligned}$$

$\rightarrow$ required gate transformations reduce to simple permutations, such as

$$\hat{R}_j \hat{S}_{\pm X_j}(\Delta\phi) \hat{R}_j = \begin{cases} \hat{S}_{\pm X_j}(\Delta\phi) & \text{if } \hat{R}_j \in \{\mathbf{1}_j, \hat{X}_j\} \\ \hat{S}_{\mp X_j}(\Delta\phi) & \text{if } \hat{R}_j \in \{\hat{Y}_j, \hat{Z}_j\}. \end{cases}$$

no elaborate unitary quantum transformations are required

Stabilization of coherent errors of the tent map

dynamics of the tent map in the presence of uncontrolled Heisenberg-type interactions

tent map

$$\mid \psi\rangle' \quad = \quad e^{-iT\hat{p}^2/2} e^{-iV(\hat{\Theta})} \mid \psi\rangle \ \text{ with } \ V'(\Theta) = \begin{cases} k(\frac{\pi}{2} - \Theta), & 0 \leq \Theta < \pi \\ k(-\frac{3\pi}{2} + \Theta), & \pi \leq \Theta < 2\pi \end{cases}$$

computational basis

$$\begin{aligned} \hat{p} \mid 0000000000\rangle_L &= -512 \mid 0000000000\rangle_L \\ \cdots & \quad \cdots \\ \hat{p} \mid 1000000000\rangle_L &= 0 \\ \cdots & \quad \cdots \\ \hat{p} \mid 1111111111\rangle_L &= 511 \mid 1111111111\rangle_L \end{aligned}$$

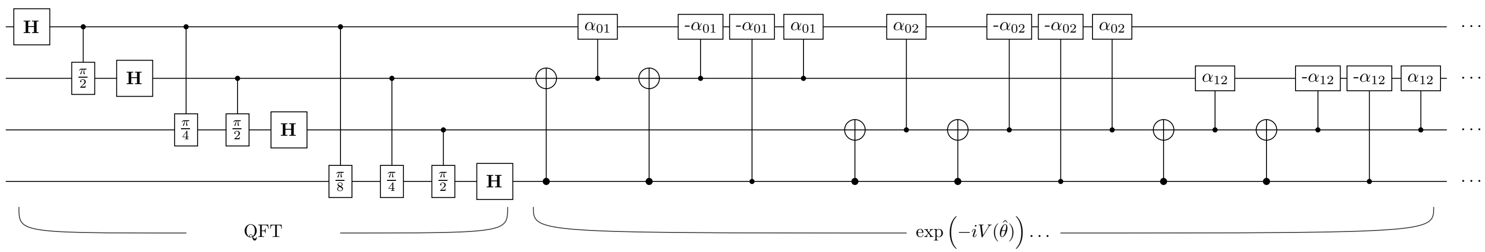
10 logical qubits

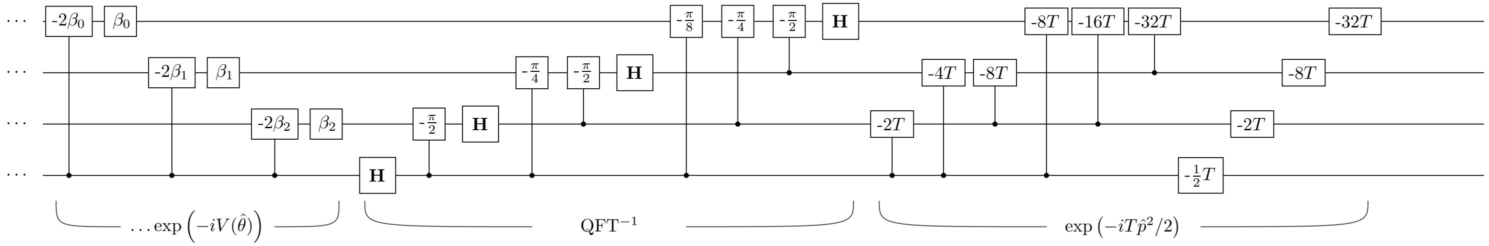
Ideal dynamics and gate sequences

$\hat{p}$-basis and discrete quantum Fourier transform

1 iteration with $n_q = 4$ qubits $\longrightarrow$ 54 quantum gates (Hadamard gates, controlled phase gates)

$$(n_g = 9n_q^2/2 - 11n_q/2 + 4)$$





Dynamical evolution and coherent errors

$$t = 20000, \ \epsilon = 5 \times 10^{-7}$$
$$n_q = 10, \ T = 2\pi/2^{n_q}, \ k = 1.7/T$$

ideal dynamics

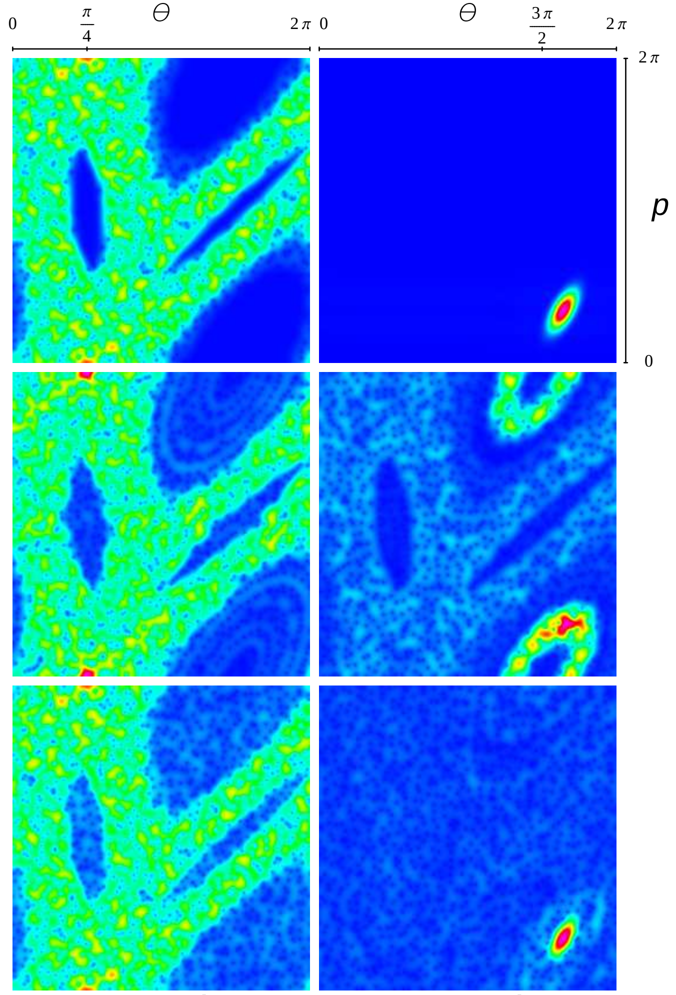
coherent initial states $(p = 0)$
unstable fixed point (left:$\Theta = \pi/2$)
stable fixed point (right:$\Theta = 3\pi/2$)

static imperfections

detunings and Heisenberg-type interactions

static imperfections and error correction

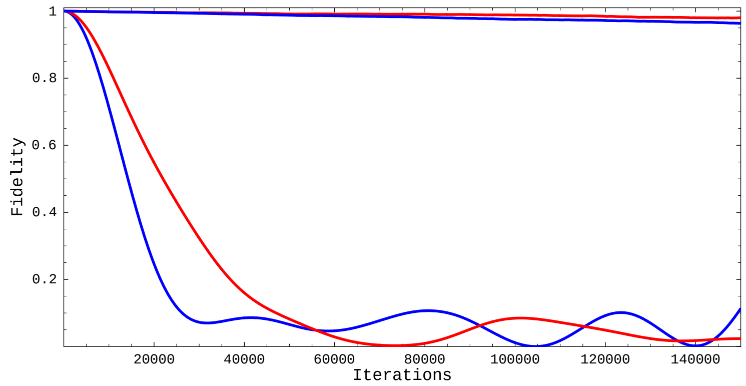
randomization of errors, i.e.
change signs of δ_j, J_j randomly



unstable stable

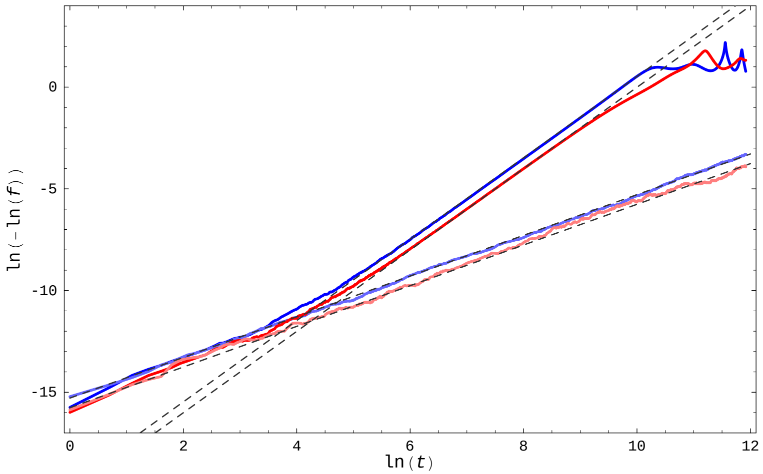
Time evolution of fidelity

$f(t) = |\ {}_t\langle\psi^{(\text{ideal})}|\psi\rangle_t\ |^2 \quad n_q = 8$



with error correction

without error correction



without error correction

$\longrightarrow\ \ln f(t) = -t/t_c - t^2/(t_c t_h)$

with error correction

$\longrightarrow\ \ln f(t) = -t/t_c$

stable fixed point

unstable fixed point

Outlook

so far developed

- embedded jump codes against spontaneous decay (redundancy, perfect)
- PAREC against static imperfections (no redundancy, not perfect)

Open problems:

- ultimate limit of PAREC?
- error correction of incoherent *and* coherent errors?
- compatibility of PAREC and jump codes?
- universal sets of quantum gates?