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Quantum error correction and quantum algorithms

reliable performance of quantum algorithms is limited by

- uncontrolled couplings to reservoirs $\rightarrow$ decoherence and dissipation
- uncontrolled Hamiltonian couplings between qubits $\rightarrow$ coherent errors

main results

- jump codes for correcting spontaneous decay processes (2003-2005)

  O. Kern, G. A., Eur.Phys. J.D **36**, 241 (2005)
- dynamical decoupling methods for suppressing coherent errors (2004-2005)

  $\longrightarrow$ no redundancy

  PAREC ($\equiv$ Pauli-random-error-correction) - a new stochastic decoupling method

  O. Kern, G. A., D. L. Shepelyansky, Eur. Phys. J. D **32**, 153 (2005)

  a new embedded dynamical decoupling method - more efficient error suppression

  Kern, G. A., Phys.. Rev. Lett. **95**, 250501 (2005)

Deterministic Decoupling Methods

consider quantum memory perturbed by Heisenberg-type interactions

$$\hat{H}_0/\hbar = \sum_{k=0}^{nq-1} \delta_k \ \hat{Z}_k + \sum_{k<l=0}^{nq-1} J_{kl}(\hat{X}_k\hat{X}_l + \hat{Y}_k\hat{Y}_l + \hat{Z}_k\hat{Z}_l)$$

main idea: suppression of perturbing dynamics by a

periodic deterministic sequence of unitary operations

e.g. bang-bang methods

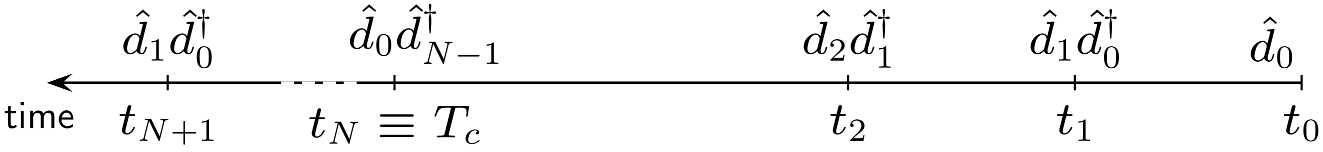
accumulated error after time $T = nT_c$: T_c ... periodicity

$$f(T) \quad \equiv \quad |\langle\psi|\hat{U}(T = nT_c)|\psi\rangle|^2 \geq 1 - \left(\frac{\|\hat{H}_0\|^2 T_c}{2\hbar}\right)^2 (T/\hbar)^2 \qquad (1)$$

coherent accumulation of errors $\longrightarrow$ decay quadratic in time

Bang-Bang Methods

use sudden deterministic unitary operations $\{\hat{d}_j; j = 0, 1, ..., N-1\}$



$$\hat{U}(T_c) = \hat{d}_{N-1}^\dagger e^{-i\hat{H}_0(t_N - t_{N-1})/\hbar}\hat{d}_{N-1}\hat{d}_{N-2}^\dagger \cdots \hat{d}_1\hat{d}_0^\dagger e^{-i\hat{H}_0(t_1-t_0)/\hbar}\hat{d}_0 \equiv \mathcal{T}\prod_{j=0}^{N-1} e^{-i\hat{\tilde{H}}_j(t_{j+1}-t_j)/\hbar}$$

$$\equiv e^{-i\hat{\overline{H}}T_c/\hbar}$$

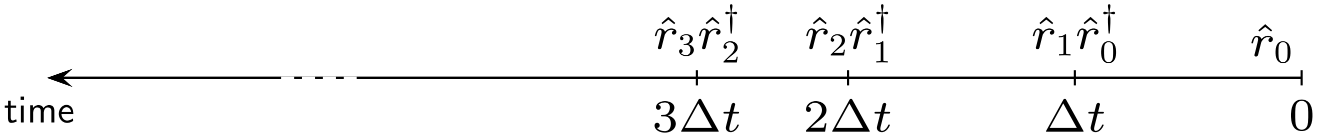
$\hat{\tilde{H}}_j = \hat{d}_j^\dagger \hat{H}_0 \hat{d}_j \quad \cdots$ interaction picture Hamiltonian

basic properties:

- $\hat{H}_0 \equiv 0 \quad \longrightarrow \quad \hat{U}(T_c) = \mathbf{1}$
- $\sum_{j=0}^{N-1} \hat{\tilde{H}}_j(t_{j+1} - t_j) = 0 \quad \longrightarrow$ cancellation to lowest order perturbation theory for $T = nT_c$

Stochastic Decoupling Methods

statistically independent unitary operations, e.g. PAREC (Pauli operations)



$$\hat{U}(T) = \hat{r}^\dagger_{N-1} e^{-i\hat{H}_0\Delta t/\hbar}\hat{r}_{N-1}\hat{r}^\dagger_{N-2}\cdots\hat{r}_1\hat{r}_0^\dagger e^{-i\hat{H}_0\Delta t/\hbar}\hat{r}_0 \equiv \mathcal{T}\prod_{j=0}^{N-1} e^{-i\hat{\hat{H}}_j\Delta t/\hbar}$$

$$\hat{\hat{H}}_j = \hat{r}_j^\dagger \hat{H}_0 \hat{r}_j \quad \cdots \quad \text{interaction picture Hamiltonian}$$

accumulated error after time $T = n\Delta t$:

$$\mathbb{E}f(T) = 1 - \Gamma T + \mathcal{O}\left((\Gamma T)^2\right) \geq 1 - \frac{\|\hat{H}_0\|^2 T\Delta t}{\hbar^2}$$

Fermi's Golden rule with $\Gamma = (2\pi/\hbar)[\Delta t/(2\pi\hbar)]\Delta H_0^2$

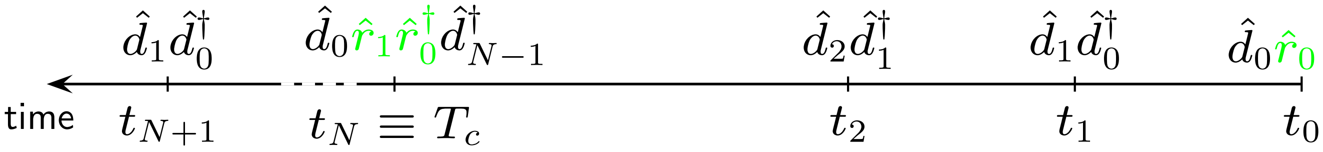
$$(\Delta H_0)^2 = \langle\psi|(\hat{H}_0 - \langle\psi|\hat{H}_0|\psi\rangle)^2|\psi\rangle$$

incoherent accumulation of errors $\longrightarrow$ decay linear in time

Embedded Decoupling Methods

main idea: embed a deterministic decoupling scheme into a stochastic one

$\longrightarrow$ strong error suppression also for long interaction times



accumulated error after time $T = nT_c$:

- deterministic method: $f(T) \equiv |\langle\psi|\hat{U}(T = nT_c)|\psi\rangle|^2 \geq 1 - \underbrace{\left(\|\hat{H}_0\|^2 T_c/(2\hbar)\right)^2}_{\equiv \hat{\overline{H}}^2} (T/\hbar)^2$

- stochastic method: $\mathbb{E}f(T) \geq 1 - \|\hat{H}_0\|^2 \left(T\Delta t/\hbar^2\right)$

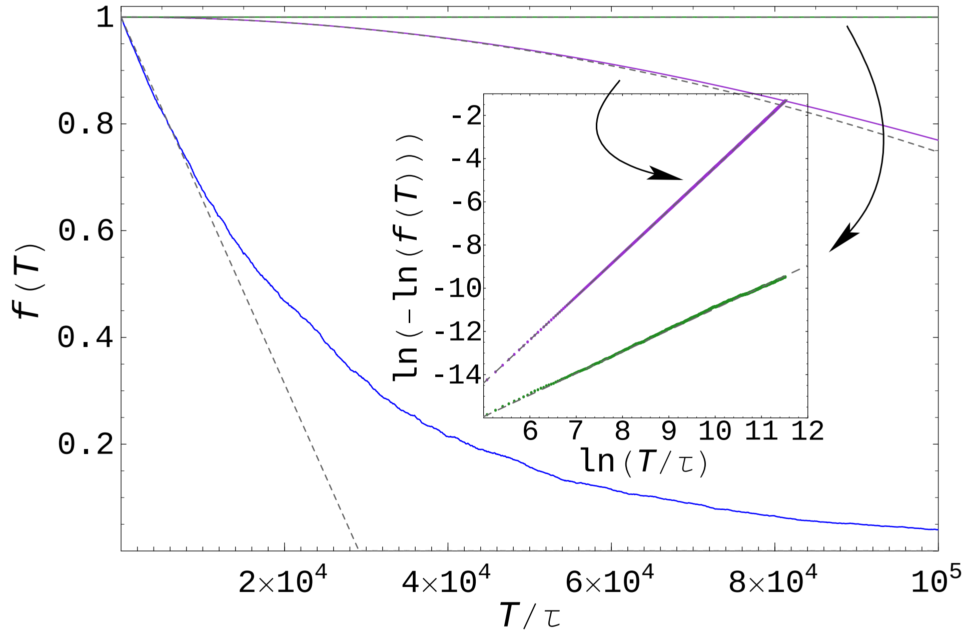
- embedded method: $\Delta t \;\rightarrow\; T_c,\;\; \hat{H}_0 \;\rightarrow\; \hat{\overline{H}}$

$$\longrightarrow \quad \mathbb{E}f(T) \geq 1 - \left(\|\hat{H}_0\|^2 T_c/(2\hbar)\right)^2 \left(TT_c/\hbar^2\right)$$

$\longrightarrow$ decay linear in time and $\sim \|\hat{H}_0\|^4$

Embedded Decoupling, Deterministic Decoupling, and PAREC - a comparison

time evolution of the fidelity $\hat{H}_0/\hbar = \sum_{k=0}^{n_q-1} \delta_k \hat{Z}_k + \sum_{kl=0}^{n_q-1} J_{kl}(\hat{X}_k\hat{X}_l + \hat{Y}_k\hat{Y}_l + \hat{Z}_k\hat{Z}_l)$



$n_q = 9$

τ ... time between subsequently applied unitary operations,

$J_{kl} \in [-\sqrt{3} \times 10^{-3}, \sqrt{3} \times 10^{-3}]$

$\hat{d}_j = \hat{\sigma}_{l_1} \otimes \ldots \otimes \hat{\sigma}_{l_{n_q}}$

with $(l_1, \ldots, l_{n_q})$ defined by OA(32,9,4,2)

$l_j \in \{0, 1, 2, 3\}$,

$\hat{\sigma}_0 = \mathbf{1},\ \hat{\sigma}_1 = \hat{X},\ \hat{\sigma}_2 = \hat{Y},\ \hat{\sigma}_3 = \hat{Z}$

lowest curve: PAREC method
middle curve: bang-bang method
uppermost curve: embedded decoupling method

O.Kern, G. A., Phys. Rev. Lett. **95**, 250501 (2005)

Summary - Outlook

dynamical decoupling methods - powerful tool for suppressing errors

no redundancy

embedded decoupling methods

combine advantages of deterministic and stochastic methods

strong error suppression also for long interaction times

applications to quantum algorithms possible by recoupling methods

Orthogonal Arrays and Deterministic Decoupling Schemes

definition: $OA(\lambda s^t, k, s, t)$ is a $k \times \lambda s^t$ matrix
with elements from an s-letter alphabet so that in each
$t \times \lambda s^t$ submatrix each possible (vertical) t-tupel appears exactly λ times

$OA(32, 9, 4, 2)$ as an example:

$s = 4$: $l_j \in \{0, 1, 2, 3\}$ with $\hat{\sigma}_0 = \mathbf{1}, \hat{\sigma}_1 = \hat{X}, \hat{\sigma}_2 \hat{Y}, \hat{\sigma}_3 = \hat{Z}$
$k = 9$: $n_q = 9$
$t = 2$: $\rightarrow$ $s^t = 16$ $\rightarrow$ $\lambda = 2$

0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3
0	1	2	3	0	1	2	3	1	0	3	2	1	0	3	2	2	3	0	1	2	3	0	1	3	2	1	0	3	2	1	0
0	2	0	2	3	1	3	1	1	3	1	3	2	0	2	0	2	0	2	0	1	3	1	3	3	1	3	1	0	2	0	2
0	3	2	1	3	0	1	2	1	2	3	0	2	1	0	3	2	1	0	3	1	2	3	0	3	0	1	2	0	3	2	1
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0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3