Quantum computation in presence of imperfections and decoherence

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Nodes and key persons:

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- London (RHUL), UK - R.Schack (node leader)

Future direction line for QIPC theory in FP6:

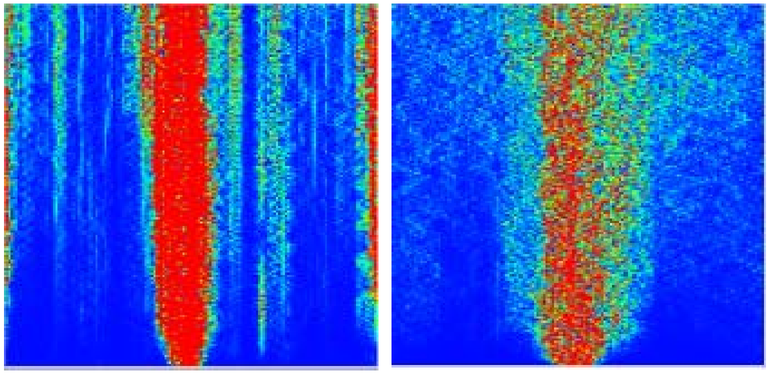
"... Theoretical work should be aimed at further developing quantum information theory and in particular to elucidating the physics of quantum information, for instance key concepts like decoherence and imperfections, quantum error-correcting codes, multi-particle entanglement and their relationship with classical computer science. This area includes the development of new quantum algorithms." EDIQIP suggestion

Effects of Decoherence and Imperfections for Quantum Information Processing (EDIQIP)

Coordinator: D.Shepelyansky (www.quantware.ups-tlse.fr)
Nodes: G.Alber (TUD), G.Benenti (INFM), R.Schack(RHUL)

Objective

- Effects of realistic imperfections on quantum computer operability and accuracy
- Decoherence and quantum chaos induced by inter-qubit couplings
- New efficient algorithms for simulation of quantum and classical physical systems
- Numerical codes with up to 30 qubits
- Development and test of error-correcting codes for quantum chaos and noisy gates



Anderson metal-insulator transition with 7 qubits

Objective Approach

- Analytical methods developed for many-body systems (nuclei, atoms, quantum dots)
- Random matrix theory and quantum chaos
- Large-scale numerical simulations of many qubits on modern supercomputers
- Stability of algorithms to quantum errors

Status

- Project started January 1, 2003
- New quantum algorithms and imperfection effects for Anderson transition and quantum wavelet transform; numerics with 7-28 qubits
- Universal law for fidelity decay induced by static imperfections (random matrix theory)
- Quantum trajectories and error-correction

Fidelity decay due to errors

Accuracy measure of quantum computation is fidelity: $f(t) = |<\psi(t)|\psi_\varepsilon(t)>|^2$.

Quantum algorithm: $|\psi(t)> = U^t\,|\psi(0)>$, $U = \underbrace{U_{N_g} \cdot \ldots \cdot U_1}_{\text{elementary gates}}$.

Errors: $U_j \;\rightarrow\; U_j\, e^{i\delta H}$, $\delta H \sim \varepsilon$.

(i) Decoherence due to residual couplings of quantum computer to external bath:

δH random and different at each j and t,

e.g.: random phase fluctuations: $\delta\phi \in [-\varepsilon, \varepsilon]$ in phase-shift gates.

(ii) Static imperfections in the quantum computer itself:

δH (random but) constant at each j and t,

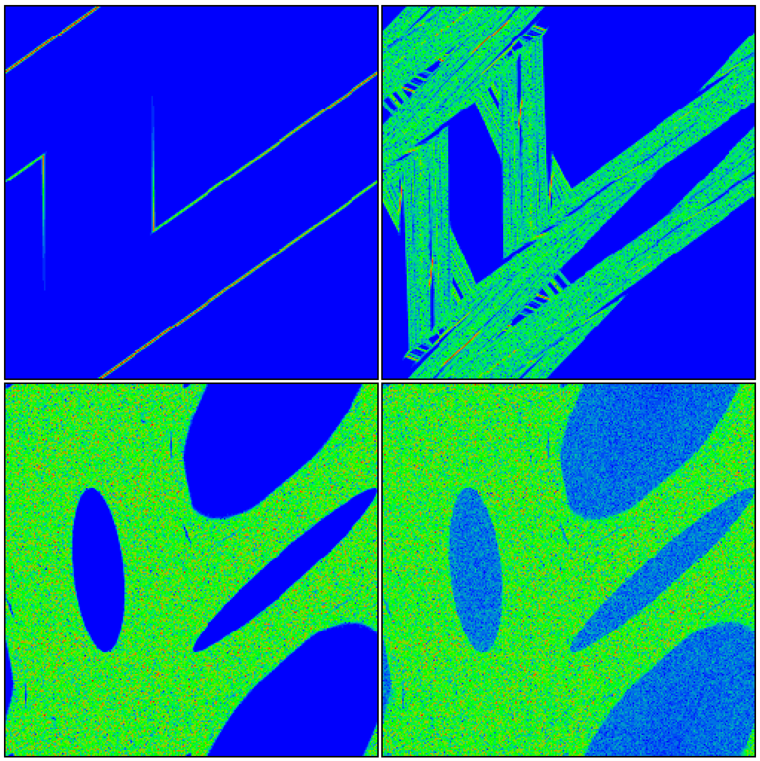
e.g.: $$\delta H = \sum_{j=0}^{n_q-1} \delta_j\, \sigma_j^{(z)} + 2 \sum_{j=0}^{n_q-2} J_j\, \sigma_j^{(x)}\, \sigma_{j+1}^{(x)} \quad , \quad J_j, \delta_j \in [-\varepsilon, \varepsilon] \; .$$

(iii) Non-unitary errors in quantum computation:

$e^{i\delta H}$ is non-unitary ($\delta H \neq \delta H^\dagger$, density matrix and quantum trajectories approach)

Quantum tent map: Husimi function

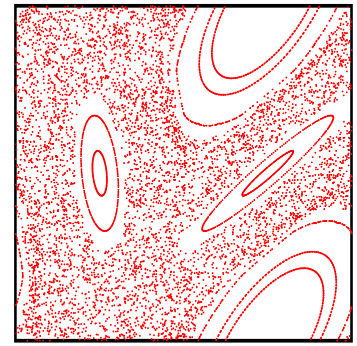
$t = 5$ 16 qubits $t = 15$



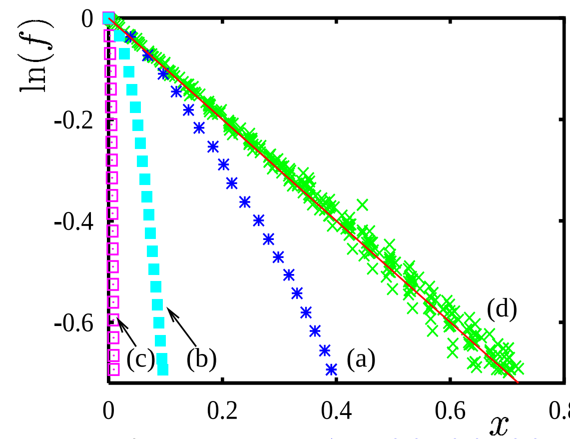
$t = 5625, \varepsilon = 0$ $\varepsilon = 7 \cdot 10^{-7}$

$\hbar_{\text{eff}} = T = 2\pi/N$, $N = 2^{nq}$, $n_g \approx 9n_q^2/2$ (gates #)

Poincaré section ($K = kT = 1.7$)

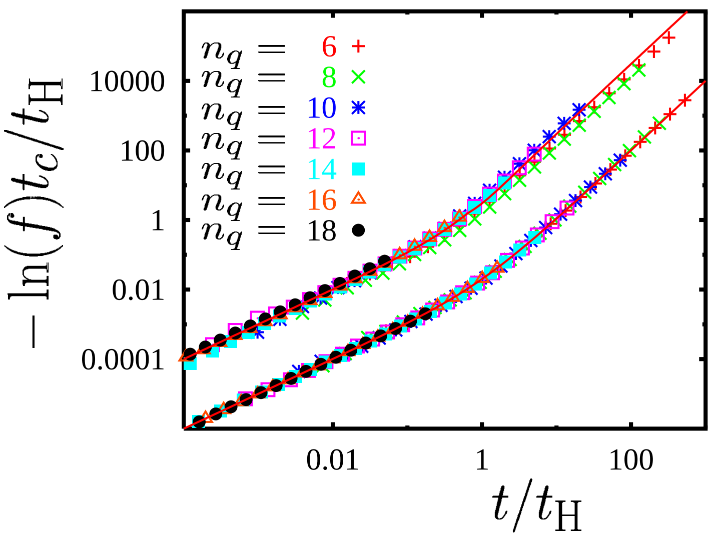


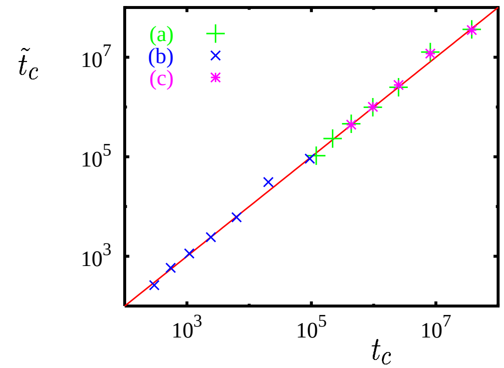
Fidelity decay with errors

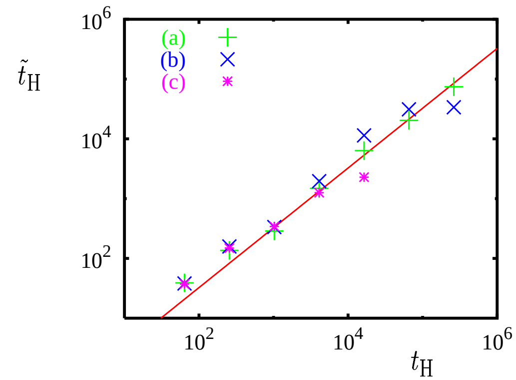


(static: $x = t/t_c$, (a), (b), (c),
random: $x = t/t_r$, (d))

Random matrix theory for fidelity decay







Upper curve: with theoretical values:

$t_{\rm H} = 2^{n_q}$ and $t_c = 1/(\varepsilon^2\, n_q\, n_g^2)$

Lower curve: with fit values $\tilde{t}_c$ and $\tilde{t}_{\rm H}$ from: $-\ln(f(t)) = \dfrac{t}{\tilde{t}_c} + \dfrac{t^2}{\tilde{t}_c \tilde{t}_{\rm H}}$ $\quad (\tilde{t}_{\rm H} \approx t_{\rm H}/3)$

Time scale of reliable quantum computations (b1-b3)

Time scale t_f with $f(t_f) = 0.9$:

Theory from RMT-approach:

If $\varepsilon \gg (2^{n_q} n_g^2 n_q)^{-1/2}$:

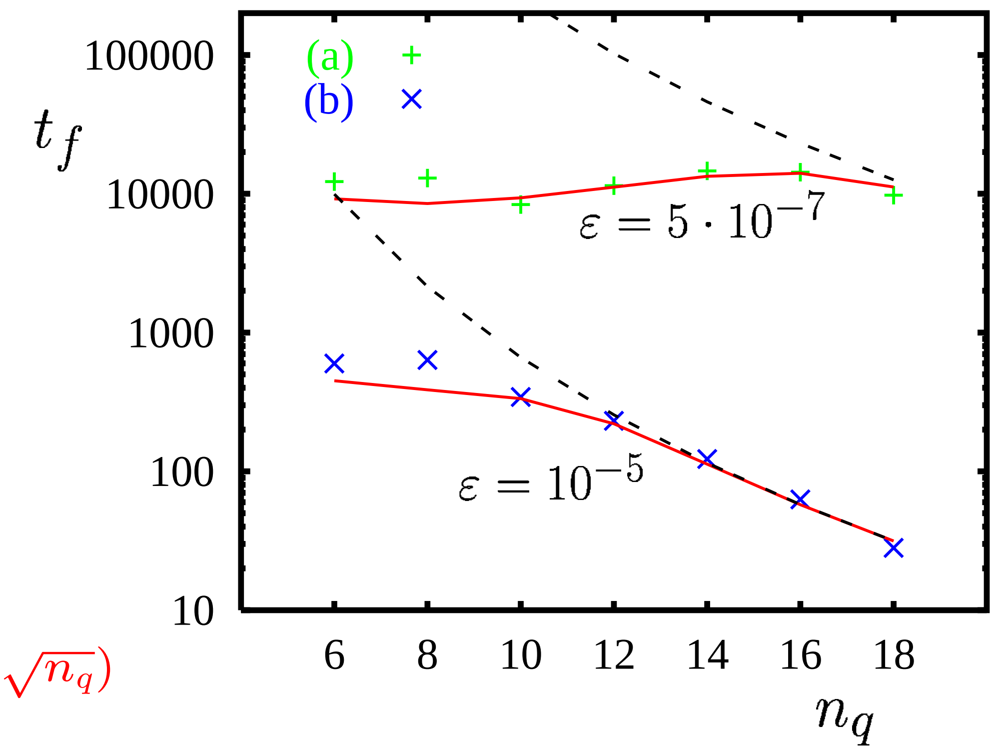
$$t_f \approx 0.1\, t_c \approx 1/(10\varepsilon^2\, n_q\, n_g^2)$$

$$N_g = t_f\, n_g \approx 1/(10\varepsilon^2\, n_q\, n_g)$$

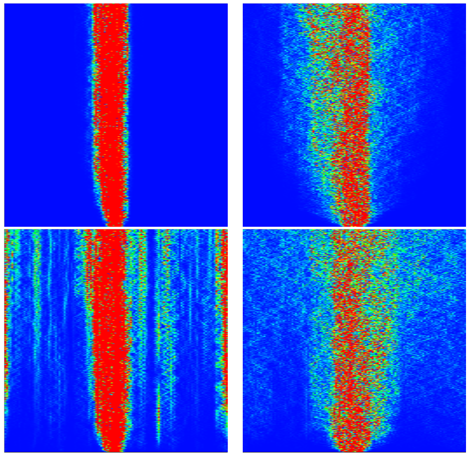
If $\varepsilon \ll (2^{n_q} n_g^2 n_q)^{-1/2}$:

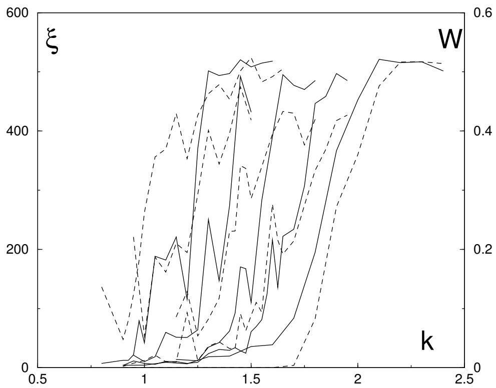
$$t_f \approx 0.2\sqrt{t_c t_{\rm H}} \approx 2^{n_q/2}/(5\varepsilon\, n_g \sqrt{n_q})$$

Random errors: $N_g \approx 5/\varepsilon^2$



Static imperfections in QA for Anderson transition





P.W.Anderson (1958) - Nobel prize 1977

The time evolution of the probability distribution $|\psi_n|^2$ in the localized (left column, $k = 1.2$) and delocalized (right column, $k = 2.4$) phases for $n_q = 7$ qubits ($N = 2^{n_q}$), with $0 \leq t \leq 400$ (vertical axis) and $-N/2 < n \leq N/2$ (horizontal axis); $k_c = 1.8$. The strength of static imperfections is $\epsilon = \mu = 0; 10^{-4}$ for top and bottom row.

Dependence of the IPR ξ and the excitation probability: $W = \sum_{n=(N/4,3N/4)} |\psi_n|^2$ (full and dashed curves for left and right scales respectively) on the kick strength k for $n_q = 10$ and $t \geq 10^5$, $\epsilon = 0$; 10^{-5}; 2×10^{-5}; 4×10^{-5}; 8×10^{-5} (curves from right to left); $\mu = 0$.

The complexity of quantum algorithm

The quantum states $n = 0, \dots, N-1$ are represented by one quantum register with n_q qubits so that $N = 2^{n_q}$. The initial state with all probability at $n_0 = 0$ corresponds to the state $|00\dots0\rangle$ (momentum n changes on a circle with N levels). The random phase multiplication $U_T = \exp(-iH_0(n))$ in the momentum basis is performed as a random sequence of one-qubit phase shifts and controlled-NOT gates. Then the kick operator $U_k = \exp(-ik(t)\cos\theta)$ is performed. One complete iteration of the algorithm requires n_g elementary gates where $n_g \approx 10k(n_q+2) + n_q^2$.

However, in the vicinity of critical point in real d-dimensions the number of states grows with time as $n^d \sim t$. Hence, up to time t the classical computation may use only N levels in each direction so that the total number of levels is $N^d \sim t$. Other levels are only very weakly populated on this time scale and therefore they can be eliminated with a good accuracy. Thus, the number of classical operations for t kicks can be estimated as $n_{gcl} \sim tN^d \log^d N \sim t^2 \log^d t$. At the same time the quantum algorithm will need $n_g \sim dn_q^2 t \sim t \log^2 t$ gates assuming d quantum registers with $N^d = 2^{dn_q} \sim t$ states. The coarse-grained characteristics of the probability distribution can be determined from few measurements of most significant qubits, *e.g.* W. Thus, even if each iteration step is efficient, the speedup is only quadratic near the critical point.

Effects of Decoherence and Imperfections for Quantum Information Processing (EDIQIP)

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(collaboration with ARO/NSA/ARDA QC program)

- First year progress

- New algorithms for classical dynamics: strange attractor (a1), Poincaré recurrences and periodic orbits (a2) => exponential gain
- Quantum chaos algorithms: dynamical localization in saw-tooth map, tent map, Wigner function (a3) => quadratic speed up
- Quantum algorithm (QA) for Anderson metal-insulator transition with quadratic speed-up; static imperfection effects on critical point; numerical tests with 7-12 qubits (future experiments ?) (a4,b2)
- Decoherence law for random errors in quantum gates: strange attractor, quantum chaos maps, wavelets (up to 28 qubits) (b1)
- Universal law for fidelity decay induced by static imperfections: random matrix theory works for 10 orders of magnitude variation, numerics for saw-tooth, tent maps, wavelets (b2,b3)
- Dynamics of entanglement in quantum computer hardware (c1) and quantum chaos algorithm (c2). Additivity implies strong super-additivity of entanglement of formation (Shor – Pomeransky) (c3)
- Quantum teleportation fidelity: quantum trajectories, 24 qubits (c4)
- Quantum sound treatment (d1); effects of measurements (d2)
- Quantum error-correction using knowledge of error location: numerical tests for Grover and saw-tooth algorithms (e1)