Quantum state preparation

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The problem

Given

a probability distribution

$$\{x\} \mapsto p(x)$$

in the form of a classical algorithm that computes $p(x)$ from input x

(see also Grover & Rudolph arXiv:quant-ph/0208112)

Find

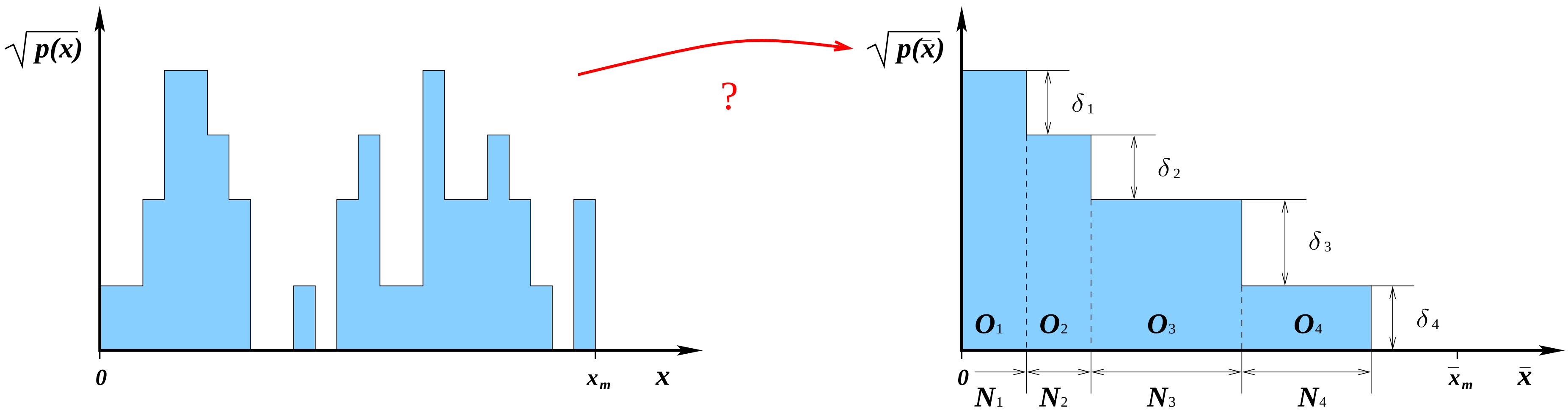
a quantum algorithm that takes the equal superposition state

$$|\psi_0\rangle \propto \sum_x |x\rangle$$

into the state $\quad \big\downarrow \; ?$

$$|\psi_{\rm fin}\rangle = \sum_x \sqrt{p(x)}|x\rangle$$

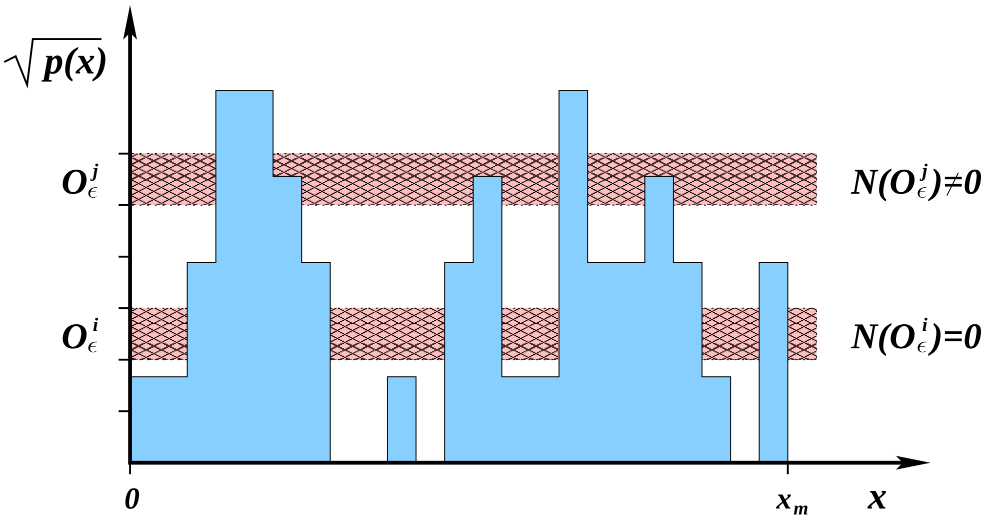
Stage 1: Quantum compilation of $p(x)$.



Given the algorithm for $p(x)$ find a set of triples $\{(O_k, N_k, \delta_k)\}$, where each O_k is an oracle that marks all N_k values of x with the amplitude $\sqrt{p(x)} = \sum_{s=k}^{s_{\max}} \delta_s$.

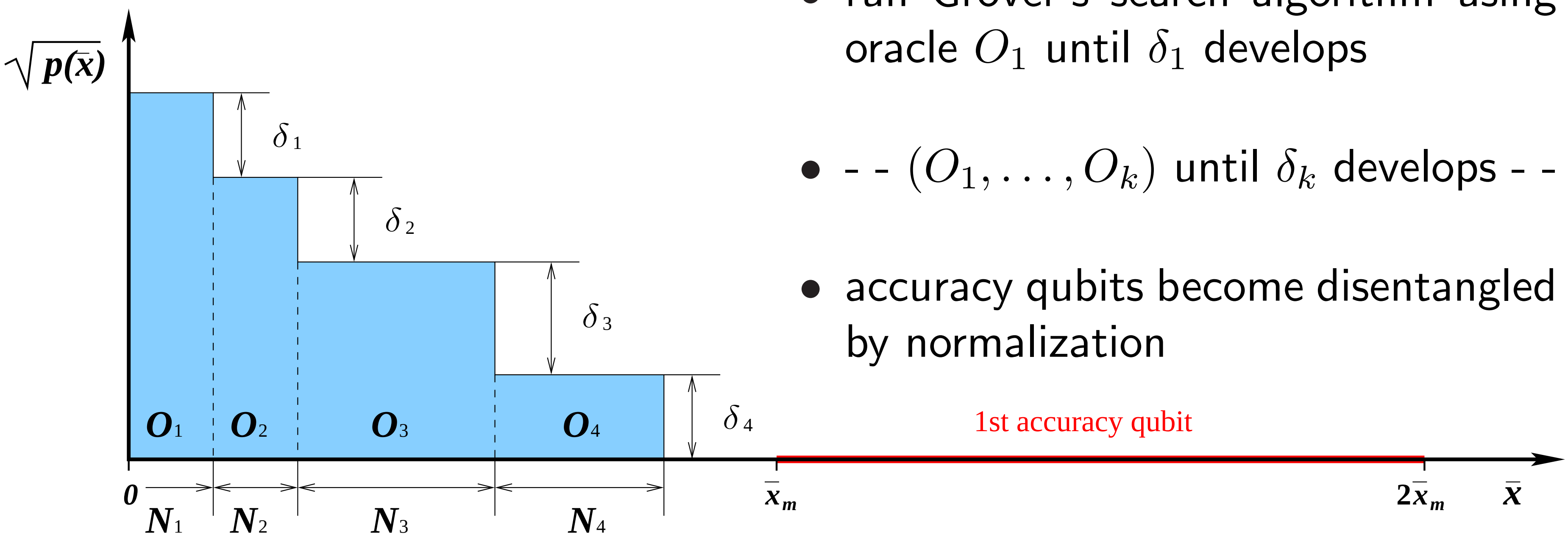
The set $\{(O_k, N_k, \delta_k)\}$ completely specifies the function $p(x)$.

Quantum compilation: details



- Fix acceptable counting accuracy $\eta = \Delta N/N$.
- divide the range of $\sqrt{p(x)}$ into regions of size ϵ.
- for every region count the number of x-values.
- create the list $\{(O_k, N_k, \delta_k)\}$

Stage 2: state preparation



- run Grover's search algorithm using oracle O_1 until δ_1 develops
- - - $(O_1, \ldots, O_k)$ until δ_k develops - -
- accuracy qubits become disentangled by normalization

Perfomance

- Counting to within accuracy η demands at most $-\log\sqrt{2\eta}$ auxiliary qubits and $1/\sqrt{2\eta}$ calls of the algorithm for $p(x)$.

- modeling small steps $\delta < \epsilon$ requires at most $-2\log\frac{\epsilon}{2\sqrt{2}}$ auxiliary qubits.

- worst case running time $O(\epsilon^{-1}\eta^{-1/2})$

Related work

- L. Grover and T. Rudolph, quant-ph/0208112 (alternative algorithm).

- E. Biham, O. Biham, D. Biron, M. Grassl and D. Lidar, quant-ph/9807027 (some mathematical methods).