EDIQIP Final Report

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Years 1 and 2

- (i) State preparation
- (ii) Simulation of nonunitary dynamics: Bayesian hypothesis elimination
- (iii) Quantum symbolic dynamics
- (iv) Decoherence functional for iterated quantum maps

Year 3

- (i) State preparation without need of auxiliary qubits
- (ii) Simulation of nonunitary dynamics: generalisation
- (iii) Quantum symbolic dynamics: hypersensitivity to perturbation
- (iv) Decoherence for coarse-grainings of the quantum baker's map

State preparation

- Our algorithms prepare a $\log_2 N$ qubit register in an arbitrary pure state

$$|\Psi\rangle = \sum_{x=0}^{N-1} \sqrt{p(x)}\, e^{2\pi i \phi(x)} |x\rangle \ ,$$

with arbitrary fidelity.

- Objection: A typical state of a quantum register cannot be efficiently described, let alone prepared!

- Assumption: We are given classical algorithms to compute $p(x)$ and $\phi(x)$.

Efficient for which $p(x)$?

Consider a sequence of probability functions $p_N : \{0, \ldots, N-1\} \rightarrow [0, 1]$, $N = 1, 2, \ldots$. For any N, the algorithm prepares the quantum register in a state $|\tilde{\Psi}\rangle$ such that, with probability greater than $1 - \nu$, the fidelity obeys the bound

$$|\langle\tilde{\Psi}|\Psi\rangle| > 1 - \lambda \; .$$

If there exists $\eta < 1$ such that

$$p_N(x) \leq \frac{1}{\eta N} \quad \text{for all} \quad N \quad \text{and} \quad x \; ,$$

the resources needed by our state preparation algorithm are polynomial in the number of qubits, $\log_2 N$, and the inverse parameters η^{-1}, λ^{-1} and ν^{-1}.

Performance

Given: • classical algorithm for computing p: $p(x) \leq 1/(\eta N)$.
• target state $|\Psi_p\rangle = \sum_{x=0}^{N-1} \sqrt{p(x)}\,|x\rangle$.

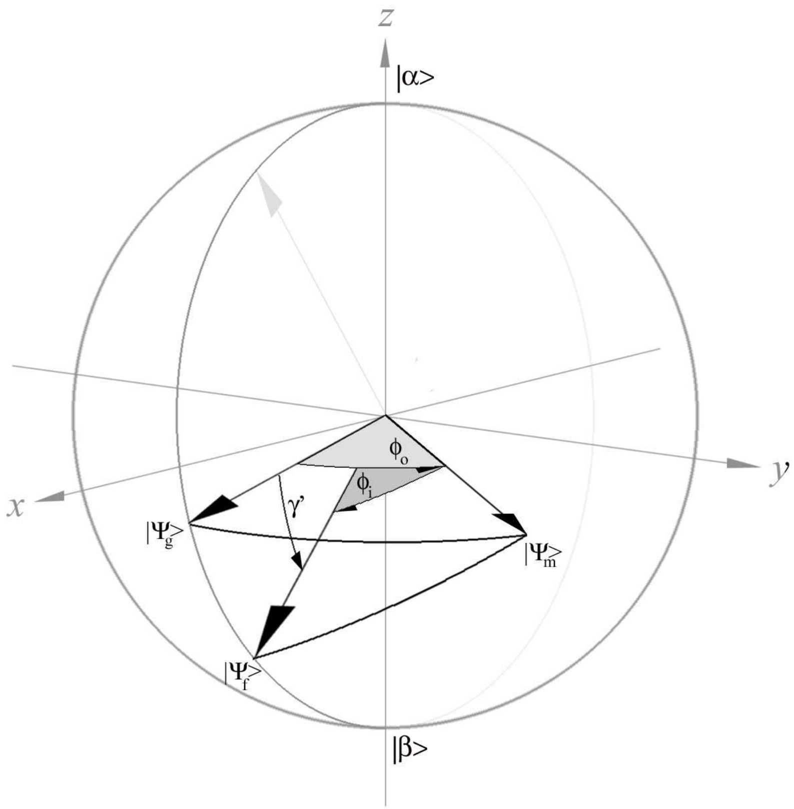
We prepare: $|\Psi_{\tilde{p}}\rangle = \sum_{x=0}^{N-1} \sqrt{\tilde{p}(x)}\,|x\rangle$, such that with probability $1-\nu$, $|\langle\Psi_{\tilde{p}}|\Psi_p\rangle| > 1-\lambda$ and choose $\epsilon < \lambda\eta/3$.

	oracle calls	auxiliary qubits
counting	$\frac{27(1+4\nu)}{\nu\epsilon^6}$	$\log_2 \frac{27(1+4\nu)}{\nu\epsilon^5}$
preparing $\vert\Psi_{\tilde{p}}\rangle$	$\frac{3\pi}{\epsilon^3\sqrt{\epsilon}}$	$3 + 3\log_2 \frac{1}{\epsilon}$

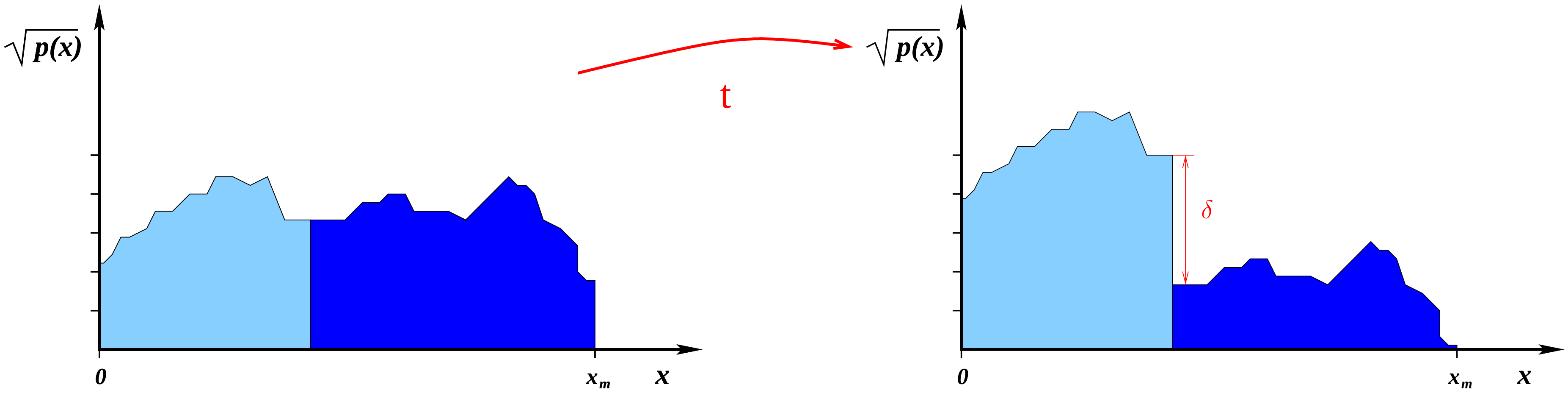
Eliminating the need for auxiliary qubits

- Reducing the number of qubits needed: important for experiments on early quantum computers.
- Method: In Grover iteration, replace oracles by phase oracles, and inversion about mean by partial inversion about mean.

Eliminating the need for auxiliary qubits



Difficulty



Grover iterations increase the average amplitude of "good" states while decreasing the average amplitude of the "bad" states. The rest of the amplitude profile remains intact. Generalised Grover iterations do not have this property.

Solution: Reach final state by a combination of three rotations.

Quantum Bayesian updating

Bayes rule:

$$p(h|d) = \frac{p(d|h)p(h)}{\sum_h p(d|h)p(h)} \; .$$

Quantum Bayesian updating:

$$\sum_{h=0}^{N-1} \sqrt{p(h)} \, |h\rangle \longmapsto \sum_{h=0}^{N-1} \sqrt{p(h|d)} \, |h\rangle \; .$$

In general, this evolution is nonunitary.

Prior given as a single copy

$$M_d|\Psi_{\mathrm{prior}}\rangle = |\Psi_{\mathrm{posterior}}\rangle$$

M_d must be trace-decreasing. Bound on success probability:

$$p_{\mathrm{success}} \leq \frac{p(d)}{\max_h p(d|h)}$$

Probabilistic algorithm (1)

- Prepare $|\Psi_{\rm prior}\rangle|0\rangle$.

- Construct a quantum circuit U_d that performs a conditional rotation of an auxiliary qubit so that

$$U_d|\Psi_{\rm prior}\rangle|0\rangle = \sum_h \sqrt{P(h)}|h\rangle\Big(A_1(h)|0\rangle + B_1(h)|1\rangle\Big)\,,$$

where

$$A_1(h) = c_1\sqrt{P(d|h)}, \quad B_1^2 = 1 - A_1^2 = 1 - c_1^2 P(d|h)\,,$$

and c_1 is a constant.

Probabilistic algorithm (2)

- Measure the auxiliary qubit to get the desired state

$$|\Psi_{\mathrm{posterior}}\rangle|0\rangle$$

with probability

$$p_1 = c_1^2 \sum_h P(h)P(d|h) = c_1^2 P(d)$$

Setting $c_1^2 = 1/\max_h P(d|h)$ achieves the theoretical bound on the success probability.

Deterministic updating

- Here we assume that the prior is given in the form of a unitary quantum circuit, U, such that $U|0\rangle = |\Psi_{\mathrm{prior}}\rangle$.

- The general updating algorithm can then be assembled using hypothesis elimination as a building block.

Quantum symbolic dynamics

- There is a one-parameter family of possible implementations of the quantum baker's map on a quantum register, corresponding to a family of different quantisations.
- In a 1999 proposal by Todd Brun in collaboration with the RHUL group, a realistic analysis was carried out of a suggested 3-qubit NMR experiment measuring hypersensitivity to perturbation in the quantum baker's map.
- We present numerical results that demonstrate how the rate of decoherence as well as the degree of hypersensitivity to perturbation depends on the specific way in which the baker's map is implemented on the quantum register.

Decoherence in the quantum baker's map

- Decoherence and entropy increase for different coarse-grainings of the quantum baker's map:

- We identify, in the framework of the decoherent histories formalism, those coarse-grainings that lead to the smallest entropy increase, i.e., those coarse-grainings that lead to the most predictable classical evolution.

- These are exactly those coarse-grainings that correspond to the natural representation of the map on a quantum register.