Effects of Decoherence and Imperfections for Quantum Information Processing (EDIQIP)

coordination

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nodes and key persons

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Main aims

- new quantum algorithms

  first generation of quantum information processors $(10 - 30$ qubits)

  simulation of non-trivial complex quantum dynamics

- characteristic aspects of imperfections and decoherence

  Are there universal decay laws? $\longrightarrow$ time scales?

- new efficient methods for suppressing imperfections and decoherence

  quantum error correction and dynamical decoupling methods

New quantum algorithms

Algorithms:

- quantum baker map $\longrightarrow$ experiment: Y.S.Weinstein et al., PRL **89**, 157902 (2002)
- quantum sawtooth map $\longrightarrow$ MIT-experiment: M.K. Henry et al. (2005) - quant/ph-0512204
- Arnold cat map $\longrightarrow$ exponential gain for period finding of Poincaré recurrences
- strange-attractor map $\longrightarrow$ realistic numerical simulations with 28 qubits (world record)
- quantum tent map
- Anderson metal-insulator transition
- electron on a lattice in a magnetic field (Harper model)
- information propagation in small-world networks

General features: exponential or polynomial gain

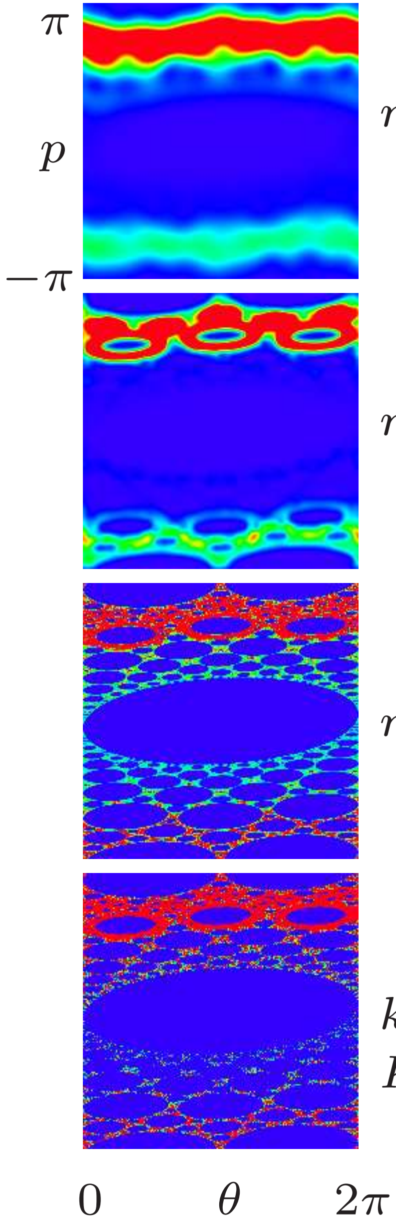
- quantum parallelism (linear superpositions of 2^{n_q} basis states)
- sometimes also measurements with polynomial complexity possible

Complex quantum phenomena:

dynamical localization, Anderson transitons, anomalous diffusion, $\cdots$

anomalous diffusion

The quantum sawtooth map and Husimi functions



$n_q = 6$

$n_q = 9$

$n_q = 16$

$k \to \infty$, $T \to 0$

$K = kT = \text{const}$

$n_g = 3n_q^3 + n_q$ quantum gates for one iteration

$$|\psi^{(1)}\rangle = e^{-iT\hat{p}^2/2} e^{ikV(\hat{\theta})/2} |\psi^{(0)}\rangle$$

with the sawtooth potential $V(\theta) = (\theta - \pi)^2 \ (\theta \in [0, 2\pi))$

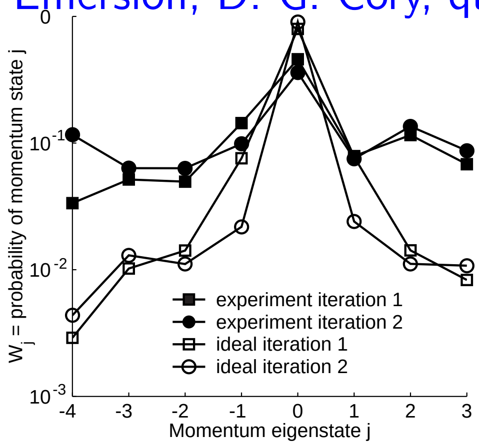
initial momentum eigenstate with $p = 0.38 \times 2^{nq}$

$$T = 2\pi/2^{nq},\ K = kT = -0.1$$

G. Benenti et al., PRA **67**, 052312 (2003)

Experimental observation of localization on a QIP

M. K. Henry, J. Emersion, D. G. Cory, quant-ph/05122204



Quantum computation under realistic conditions

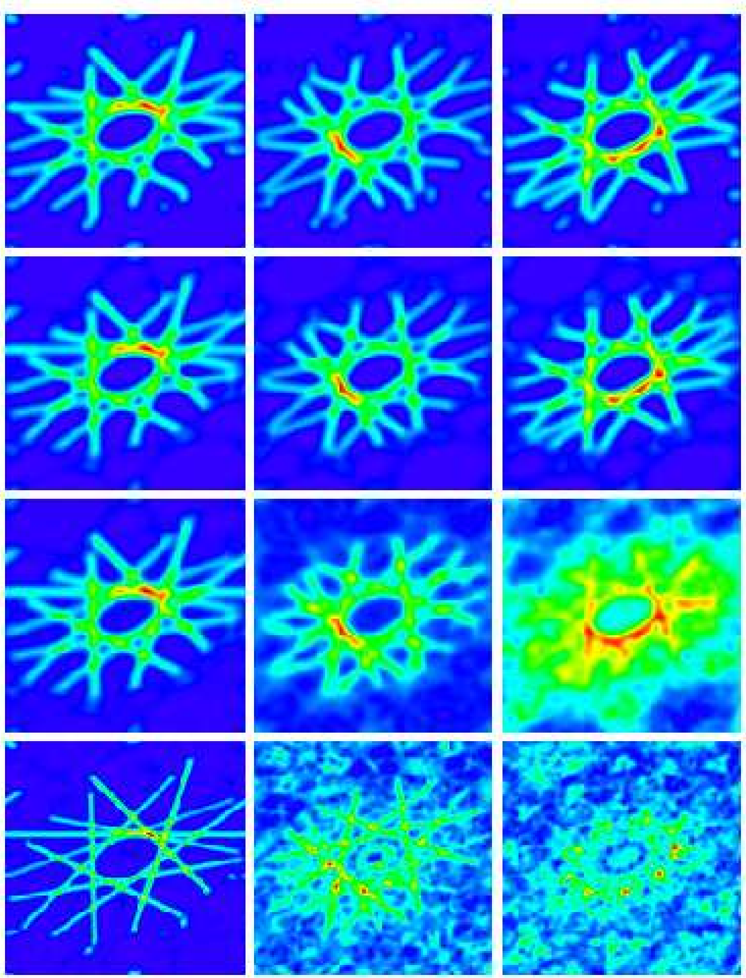
- uncontrolled couplings to the outside world
  decoherence and dissipation
  e.g. spontaneous decay

- unkown coherent interqubit couplings inside a quantum computer
  constant during an algorithm $\longrightarrow$ static imperfections $\longrightarrow$ quantum chaos
  time-dependent during an algorithm $\longrightarrow$ random imperfections

major challenges

- characteristic dynamical properties - universal decay laws?
- methods for efficient stabilization?

Decoherence - the quantum sawtooth map



classical

$\Gamma = 0$
$n_q = 8$

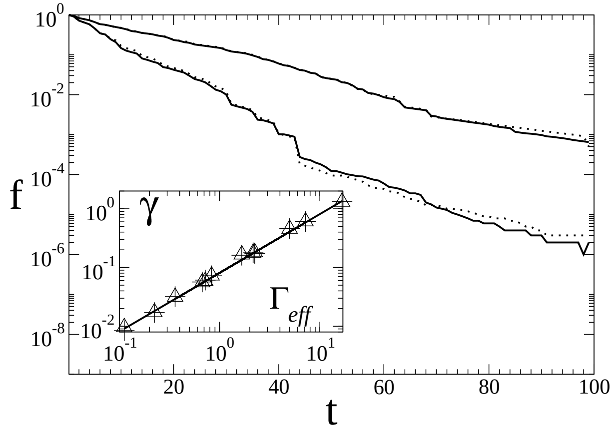
$\Gamma = 0.0005$
$n_q = 8$

$\Gamma = 0.0005$
$n_q = 10$

$t = 9$ $t = 40$ $t = 90$

consider spontaneous decay of qubits
with spontaneous decay rate Γ
$K = -0.5, T = 2\pi/2^{n_q},$
$|p = 0.1 \times 2^{n_q}\rangle$

linear exponential fidelity decay
$f(t) = |_t \langle \psi^{(\mathrm{ideal})} | \psi \rangle_t|^2$
with effective rate $\Gamma_{\mathrm{eff}} = \Gamma n_q n_g$



J. W. Lee, D. L. Shepelyansky, PRE **71**, 056202 (2005)

Imperfections by inter-qubit couplings universal decay laws

The problem: ideal dynamics of a quantum computer are modified by unknown Heisenberg-type interactions $\longrightarrow$ unitary (coherent) errors

$$\delta \hat{H} = \sum_{j=0}^{n_q-1} \delta_j \, \hat{\boldsymbol{\sigma}}_j^{(z)} + \sum_{j=0}^{n_q-2} J_j \hat{\boldsymbol{\sigma}}_j \cdot \hat{\boldsymbol{\sigma}}_{j+1} \quad \longrightarrow \quad \hat{U} = e^{-i\delta \hat{H}}$$

- random imperfections

phases of quantum gates vary randomly during computation in the interval $\pm\epsilon$

$\longrightarrow$ (linear) exponential decay of fidelity

$$\longrightarrow \quad f(t) = | \;_t\langle \psi^{(\mathrm{ideal})} | \psi \rangle_t \; |^2 = e^{-t/t_c}$$

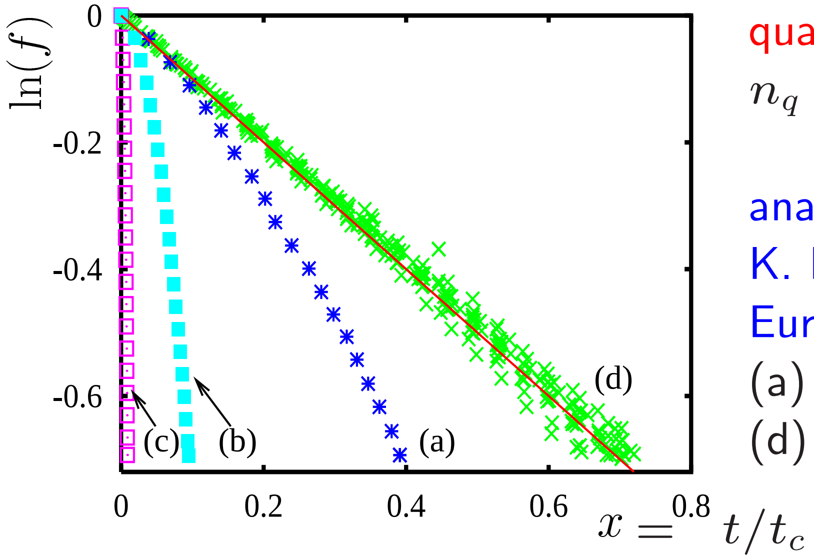
with the characteristic decay time (averaged over initial states) $t_c^{-1} = \epsilon^2 n_g / 47$

• static imperfections unknown parameters (δ_j, J_j) stay constant during computation

$\rightarrow$ in addition quadratic exponential decay due to quantum interference

$$\rightarrow \quad f(t) = e^{-t/t_c - t^2/(t_c t_h)}$$

with the Heisenberg-time $t_h \approx 2^{n_q}/3$ and $t_c{}^{-1} \approx \epsilon^2 n_q n_g^2$



quantum tent map with $K = 1.7$

$n_q = 10$, $n_g = (9/2)n_q^2 - (11/2)n_q + 4$

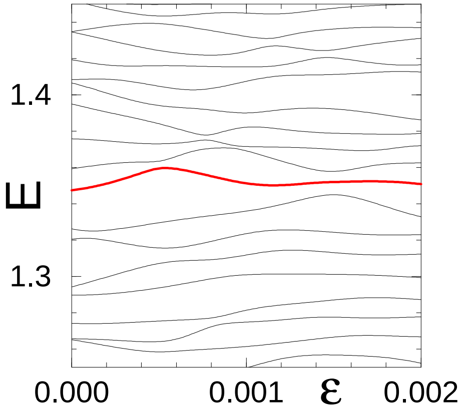
analytical approach by random matrix theory:
K. M. Frahm, R. Fleckinger, D.L. Shepelyansky,
Eur. Phys. J. D. **29**, 139 (2004)

(a) $\epsilon = 0.00003$, (b) $\epsilon = 6 \times 10^{-6}$, (c) $\epsilon = 5 \times 10^{-7}$
(d) random imperfections

$\rightarrow$ restricts computation time of a realistic quantum computer

Energy eigenstates of an operating quantum computer

hypersensitivity of energy eigenstates to static imperfections



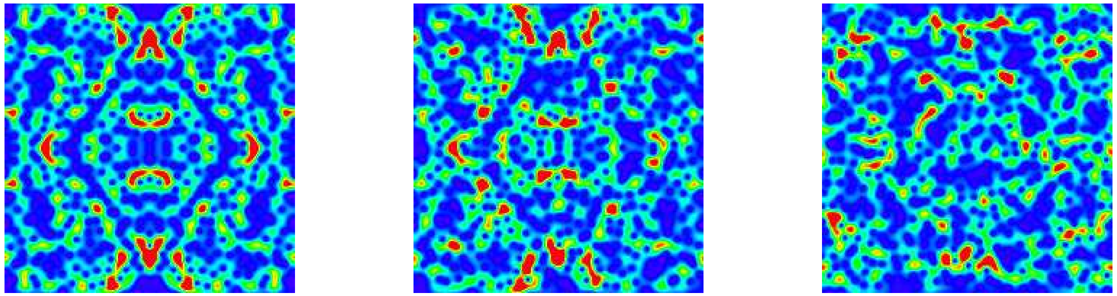
quasi-energies of the quantum sawtooth map

$$(K = kT = \sqrt{2}, T = 2\pi/2^{n_q}, n_q = 9, J = 0)$$

$$e^{-iT\hat{n}^2/4} e^{ik(\hat{\theta}-\pi)^2/2} e^{-iT\hat{n}^2/4} |\psi\rangle \quad = \quad e^{-iE}|\psi\rangle$$

for various static imperfections ϵ

Husimi plots of energy eigenstate



$\epsilon = 0$ $\quad$ $\epsilon = 0.0004$ $\quad$ $\epsilon = 0.001$

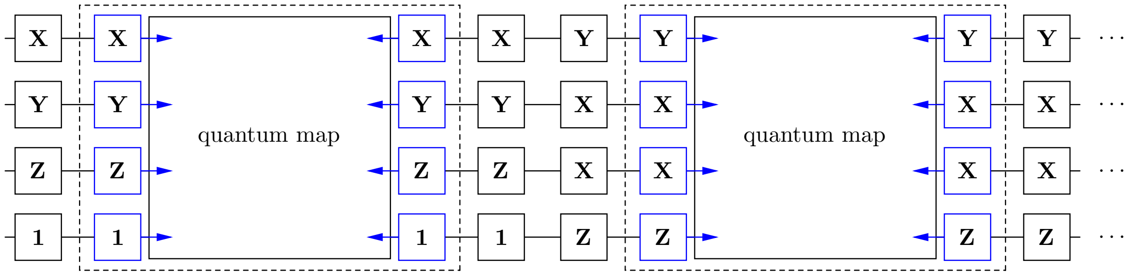
Random dynamical decoupling of inter-qubit couplings

The basic idea: randomization of coherent errors

$\longrightarrow$ change of the exponential decay from quadratic to linear

$\longrightarrow$ significant increase of reliable computation times, no redundancy required!

How can this randomization be realized in practice?



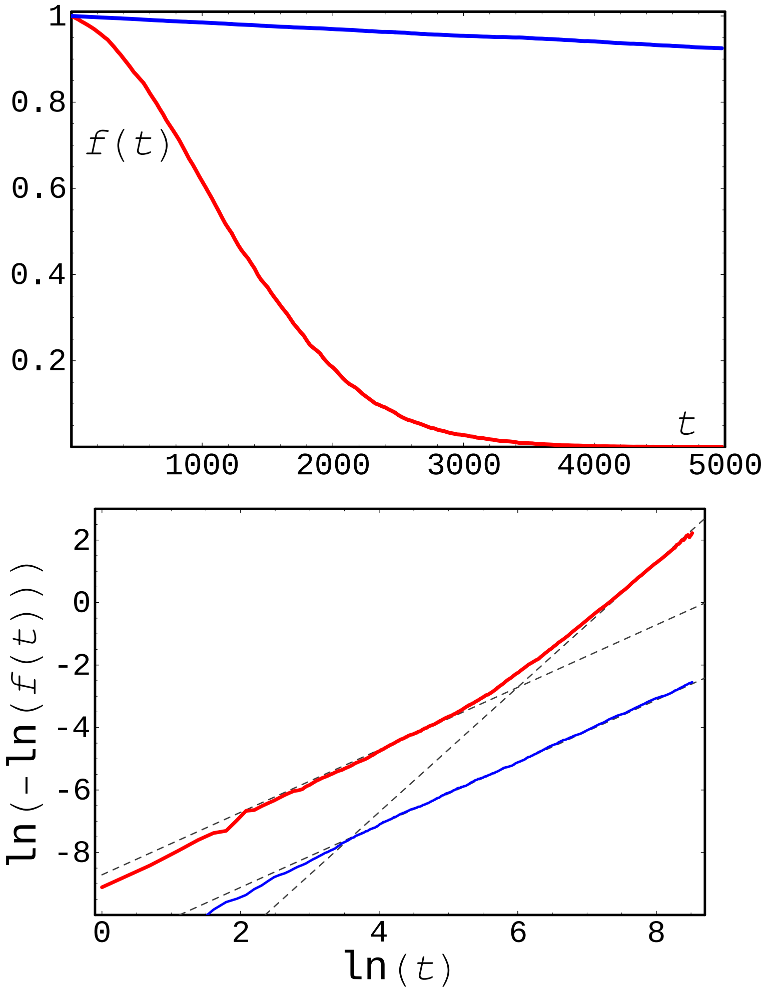
$$\delta\hat{H} = \sum_{j=0}^{nq-1} \delta_j \, \hat{\boldsymbol{\sigma}}_j^{(z)} + \sum_{j=0}^{nq-2} J_j \hat{\boldsymbol{\sigma}}_j \cdot \hat{\boldsymbol{\sigma}}_{j+1}$$

repeated random changes of computational basis $\longrightarrow$ PAREC $\equiv$ Pauli-random-error-correction

- select random sequences of Pauli operations between quantum gates, e.g. $(\hat{X},\ hatY, \hat{Z}, \hat{\mathbf{1}})$
- transform elementary quantum gates appropriately, e.g. $\hat{X}\hat{Z}\hat{X} = -\hat{Z}$

  achievable by appropriate permutations of elementary quantum gates

Time evolution of fidelity $f(t) = |\ {}_t\langle\psi^{(\text{ideal})}|\psi\rangle_t\ |^2$ $n_q = 8$



with PAREC (Pauli-random-error-correction)

without error suppression

without error suppression

$\rightarrow\ \ln f(t) = -t/t_c - t^2/(t_c t_h)$

with PAREC

$\rightarrow\ \ln f(t) = -t/t_c$

Embedded dynamical decoupling of inter-qubit couplings

combine stabilizing properties of PAREC at long times with
strong error suppression of deterministic decoupling methods at shorter times

Deterministic decoupling method:

apply periodic external Hamiltonian $\hat{H}(t)\ \ (0 \leq t \leq T_c) \ \rightarrow\ \hat{U}(t)$

$$\frac{1}{T_c}\int_0^{T_c} dt\ \underbrace{\hat{U}^\dagger(t)\delta\hat{H}(t)\hat{U}(t)}_{\text{toggled Hamiltonian}} = 0$$

residual perturbation coherent $\longrightarrow$ quadratic exponential decay

Idea of embedding procedure:

- remove large parts of perturbation $\delta\hat{H}(t)$ by deterministic method
- remove parts of residual perturbation by PAREC by

embedding the deterministic cycle between two subsequent PAREC operations

$\longrightarrow$ significant improvement of stabilizing properties

O. Kern, G. Alber, Phys. Rev. Lett. **95**, 250501 (2005)

Summary - Outloook

- development of new quantum algorithms
  poynomial and exponential gain
  numerical tests with up to 28 qubits

- universal decay laws for imperfections
  static imperfections $\longrightarrow$ quadratic-in-time fidelity decay
  analytical treatment by random matrix theory

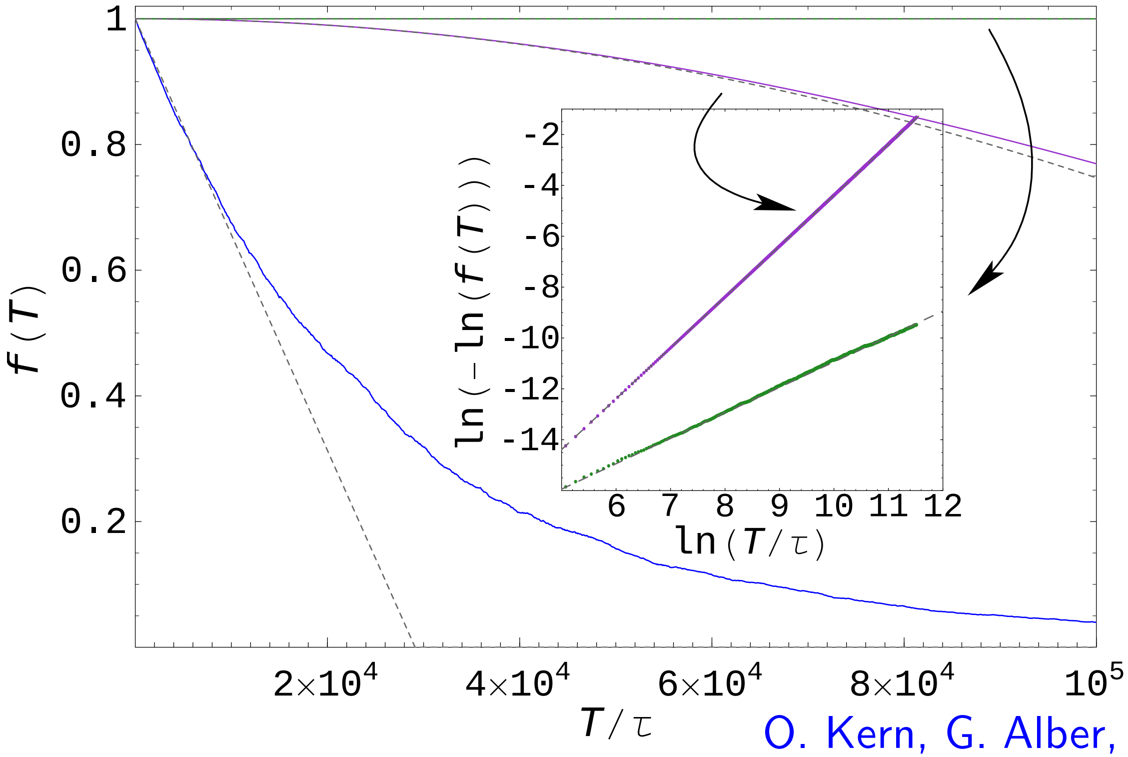
- new methods for error suppression
  quantum error correction and dynamical decoupling methods
  embedding procedures $\longrightarrow$ useful for efficient error suppression

Fidelity decay of a quantum memory

Heiseberg-type imperfections

$$\delta\hat{H}(t) = \hbar\sum_{k=0}^{n_q-1}\delta_k\hat{Z}_k + \hbar\sum_{k<l=0}^{n_q-1}J_{kl}(\hat{X}_k\hat{X}_l + \hat{Y}_k\hat{Y}_l + \hat{Z}_k\hat{Z}_l)$$

$$\delta_k, J_{kl} \in [-\sqrt{3}\times 10^{-3}, \sqrt{3}\times 10^{-3}]$$



embedded decoupling method $\Delta t = T_c$

deterministic (bang-bang) method OA(32,9,4,2) with $T_c = 32\tau$

PAREC method $\Delta t = \tau$

O. Kern, G. Alber, Phys. Rev. Lett. **95**, 250501 (2005)

Correction of static imperfections by PAREC

$$t = 20000,\ \epsilon = 5 \times 10^{-7}$$
$$n_q = 10,\ T = 2\pi/2^{n_q},\ k = 1.7/T$$

ideal dynamics of the tent map

coherent initial states ($p = 0$)

unstable fixed point (left:$\Theta = \pi/2$)

stable fixed point (right:$\Theta = 3\pi/2$)

static imperfections

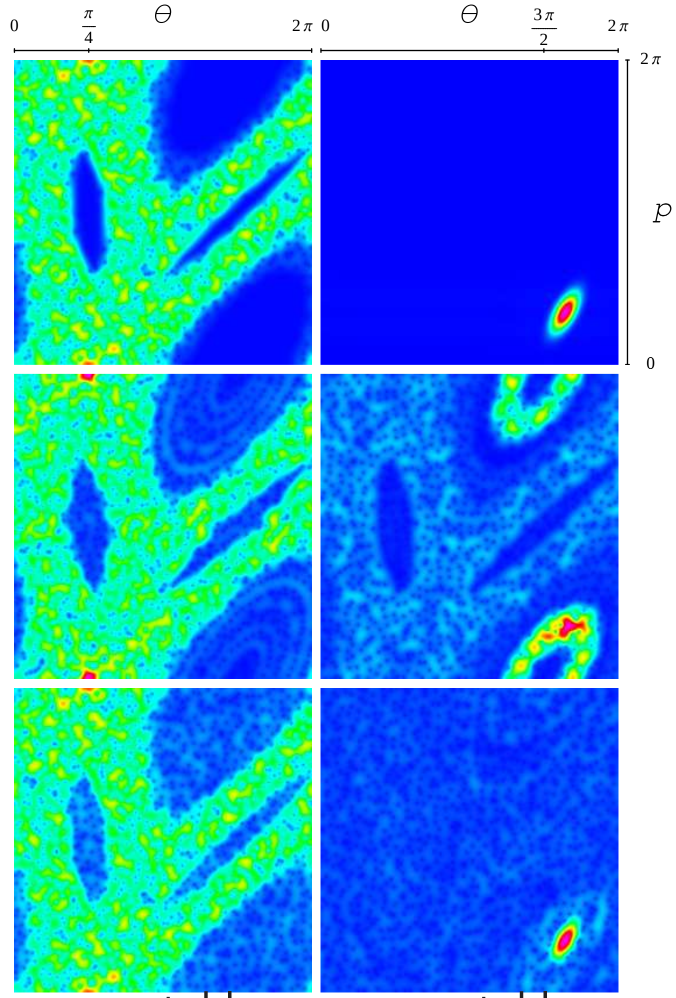
detunings and Heisenberg-type interactions

static imperfections and PAREC

randomization of errors, i.e.

change signs of δ_j, J_j randomly

O. Kern, G. Alber, D. L. Shepelyansky,
Eur. Phys. J. D. **32**, 153 (2004)



unstable stable