Effects of Decoherence and Imperfections for Quantum Information Processing (EDIQIP)

coordination

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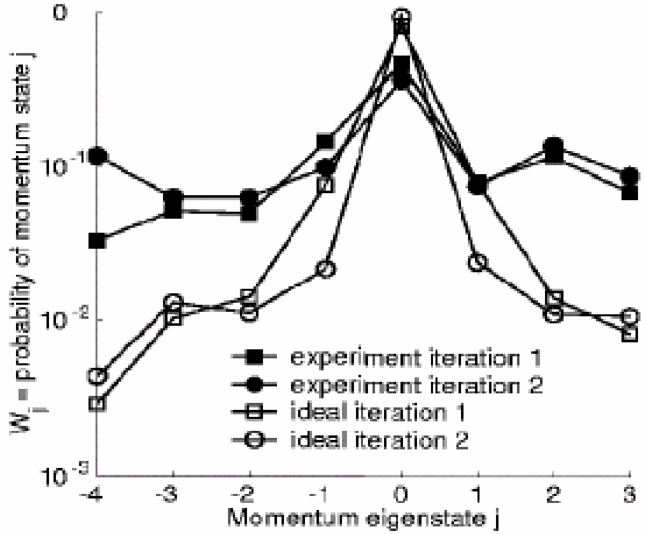
Coordinator:D.Shepelyansky(www.quantware.ups-tlse.fr)
Nodes:G.Alber(TUD),G.Benenti(INFM),R.Schack (RHUL)

Objective

- Effects of realistic imperfections on quantum computer operability and accuracy
- Decoherence and quantum chaos induced by inter-qubit couplings
- New efficient algorithms for simulation of quantum and classical physical systems
- Numerical codes with up to 30 qubits
- Development and test of error-correcting codes for quantum chaos and noisy gates



Objective Approach

- Analytical methods developed for many-body systems (nuclei, atoms, quantum dots)
- Random matrix theory and quantum chaos
- Large-scale numerical simulations of many qubits on modern supercomputers
- Stability of algorithms to quantum errors

Status

- Dissipative decoherence, quantum trajectories in sawtooth map, Grover; Ehrenfest explosion
- Quantum algorithms for small-world dynamics, state preparation
- YQC with magnetic field gradient
- PAREC method impovement
- MIT NMR experiment for sawtooth algorithm

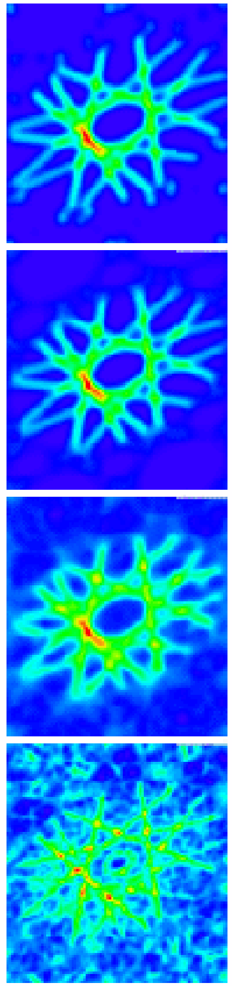
Effects of Decoherence and Imperfections for Quantum Information Processing (EDIQIP)

www.quantware.ups-tlse.fr/EDIQIP

(collaboration with ARO/NSA/ARDA QC program)

- <u>**Final Report Results 2003 - 2005**</u>

- Quantum chaos border for quantum computer hardware
- **New Quantum algorithms (QA):** sawtooth map, tent map, Anderson transition, electrons in a magnetic field, small-world => **polynomial number of gates** but also only **polynomial speed up** for information extraction
- **QA for classical dynamics: possible exponential gain** for Poincaré recurrences in the Arnold cat map; correlations in strange attractor map
- QA for arbitrary pure state preparation with fidelity arbitrary close to unity
- **Numerical simulations of imperfections in new QA with up to 28 qubits**
- **Universal laws for fidelity decay** induced by static imperfections (using random matrix theory), unitary random errors, dissipative decoherence
- Static imperfections and dissipative decoherence in the Grover algorithm
- **Pauli random error correction for static imperfections:** generic method to elliminate coherent errors and increase fidelity (parametric improvement)
- Dissipative quantum dynamics of cold atom with quantum trajectories
- **First experimental implementations of QA developed by EDIQIP**
- **Quantum Numerical Recipes** at Quantware Library
- EVENTS: E.Fermi summer school, Varenna, Italy, 5-15 July 2005; Institut Henri Poincaré, Paris, 4 Jan – 7 April, 2006
- Publications: **67 papers (7 PRLs)**; 2 PhD QIPC theses at EDIQIP



Dissemination of results

All information is at **www.quantware.ups-tlse.fr/EDIQIP**

During third year 2005:

- 29 papers published and submitted including

$\longrightarrow$ 3 PRLs

$\longrightarrow$ lectures at E.Fermi Varenna School

- 20 talks and posters on international conferences in EU, Japan, Korea and Mexico

Organization of schools in QIPC:

- Enrico Fermi summer school on QIPC, Varenna, 5 - 15 July, 2005
(directors: G.Casati, D.Shepelyansky, P.Zoller) $\longrightarrow$ about 90 participants
- Institut Henri Poincare, QIPC trimestre, Paris, 4 Jan - 7 April 2006
(directors: Ph.Grangier, M.Santha, D.Shepelyansky), www.quantware.ups-tlse.fr/IHP2006/
$\longrightarrow$ about 140 participats

EDIQIP research group at Toulouse

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José Lages (postdoc EDIQIP, 2004 - 2005, now MC at Univ. at Besancon)

O.Giraud (postdoc CNRS, 2004 - 2005, now CR2 CNRS at LPT)

14 papers in 2005 (2 PRLs)

Fidelity decay due to errors

Accuracy measure of quantum computation is fidelity: $f(t) = |<\psi(t)|\psi_\varepsilon(t)>|^2$.

Quantum algorithm: $|\psi(t)> = U^t\,|\psi(0)> \quad , \quad U = \underbrace{U_{N_g} \cdot \ldots \cdot U_1}_{\text{elementary gates}}$.

Errors: $U_j \;\to\; U_j\, e^{i\delta H} \quad , \quad \delta H \sim \varepsilon$.

(i) Decoherence due to residual couplings of quantum computer to external bath:

δH random and different at each j and t,

e.g.: random phase fluctuations: $\delta\phi \in [-\varepsilon, \varepsilon]$ in phase-shift gates.

(ii) Static imperfections in the quantum computer itself:

δH (random but) constant at each j and t,

e.g.: $$\delta H = \sum_{j=0}^{n_q-1} \delta_j\, \sigma_j^{(z)} + 2 \sum_{j=0}^{n_q-2} J_j\, \sigma_j^{(x)}\, \sigma_{j+1}^{(x)} \quad , \quad J_j, \delta_j \in [-\varepsilon, \varepsilon] \;.$$

(iii) Non-unitary errors in quantum computation:

$e^{i\delta H}$ is non-unitary ($\delta H \neq \delta H^\dagger$, density matrix and quantum trajectories approach)

Dissipative decoherence in the quantum sawtooth algorithm

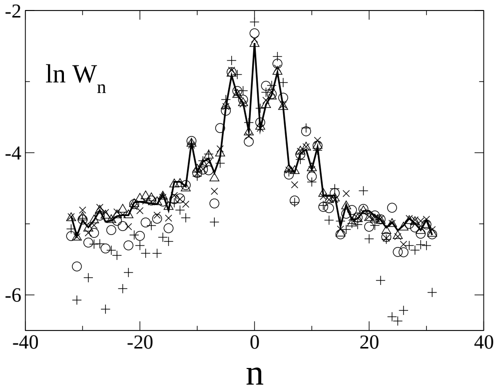
$$H(t) = \frac{T p^2}{2} + V(\theta) \sum_{n=-\infty}^{\infty} \delta(t-n)$$

Classical map : $V(\theta) = (\theta - \pi)^2/2$

$$p_{n+1} = p_n - k(\theta_n - \pi)$$
$$\theta_{n+1} = \theta_n + T\, p_{n+1}$$

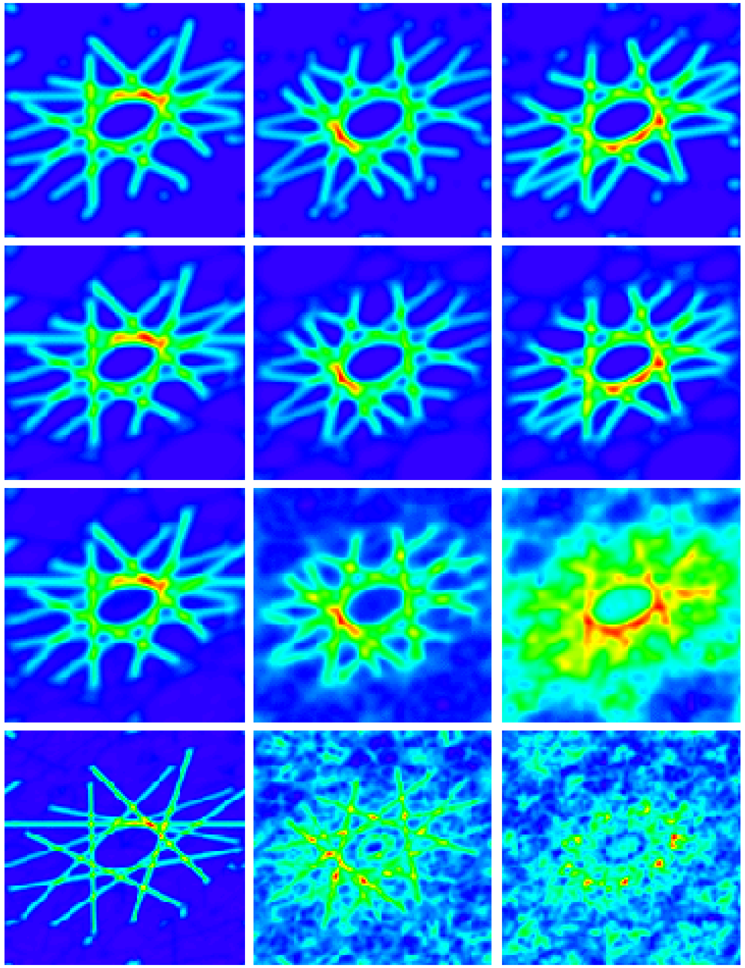
Quantum map : $p = -i\partial/\partial\theta$

$$|\psi(t+1)> = U\,|\psi(t)>$$
$$U = e^{-iTp^2/2}\, e^{-ikV(\theta)}$$



Quantum register with $N = 2^{n_q}$ states is used to store ψ; $n_g = 3n_q^2 + n_q$ elementary quantum gates for one map iteration

One-qubit per gate decay rate Γ: $K = kT = \sqrt{3}, k = \sqrt{2}, t = 30, \Gamma = 0.001, n_q = 8$; the Lindblad equation (full curve), quantum trajectories $M = 20$ to 10^3 (symbols).



classical

$\Gamma = 0$
$n_q = 8$

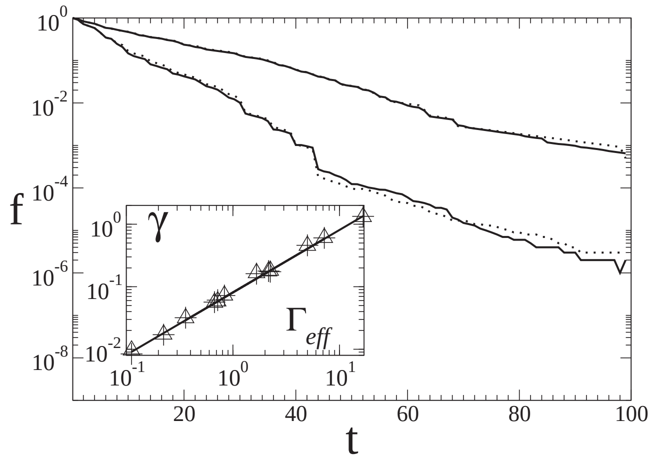
$\Gamma = 0.0005$
$n_q = 8$

$\Gamma = 0.0005$
$n_q = 10$

$t = 9$ $t = 40$ $t = 90$

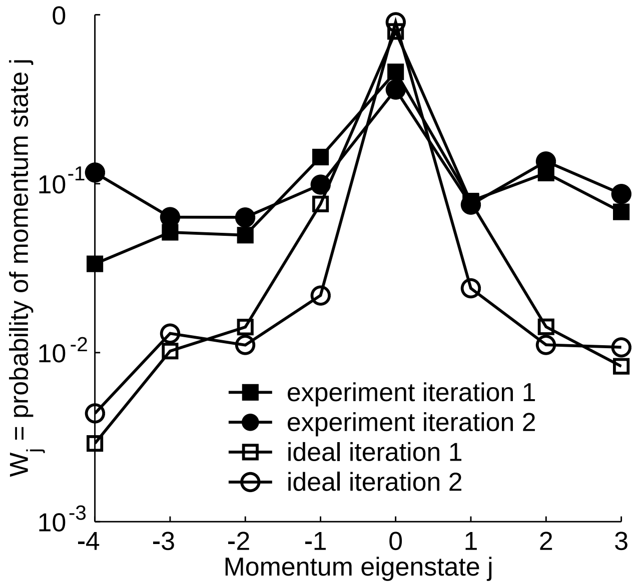
qubit spontaneous decay rate Γ
$K = -0.5, T = 2\pi/2^{nq}$,
$|p = 0.1 \times 2^{nq}\rangle$

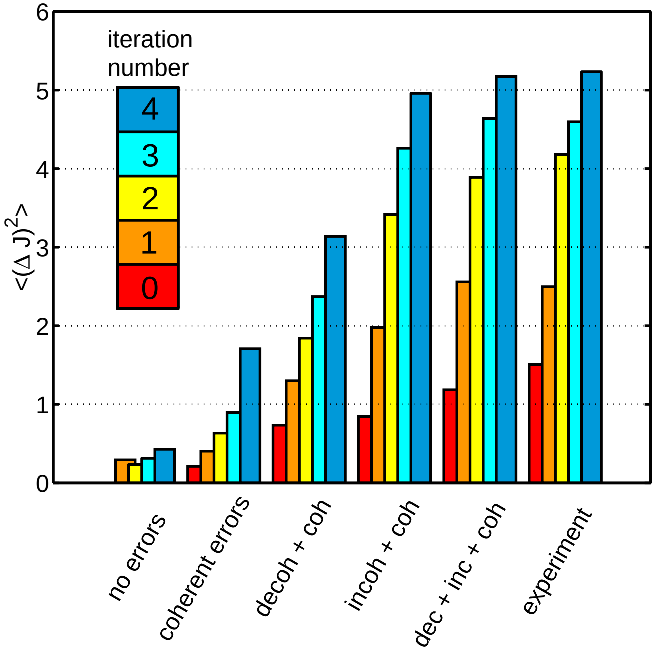
linear exponential fidelity decay
$f(t) = |_t \langle \psi^{(\text{ideal})} | \psi \rangle_t |^2$
with effective rate
$\gamma \approx \Gamma_{\text{eff}}/10 = \Gamma n_q n_g/10$ (*)



(*) $\longrightarrow$ for the Grover QA we find $\gamma \approx 0.4\Gamma_{\text{eff}}$ (see Ref.[12])

First (NMR-based) experimental implementation





(M.K. Henry, J. Emerson, D.G. Cory, quant-ph/0512204)

Localization is a purely quantum effect, quite fragile in the presence of noise: a significant degree of quantum control has been achieved

Small-world networks

Classical small-world networks

⇒ Small-world networks describe social and biological networks, Internet connections, airline flights, and many others

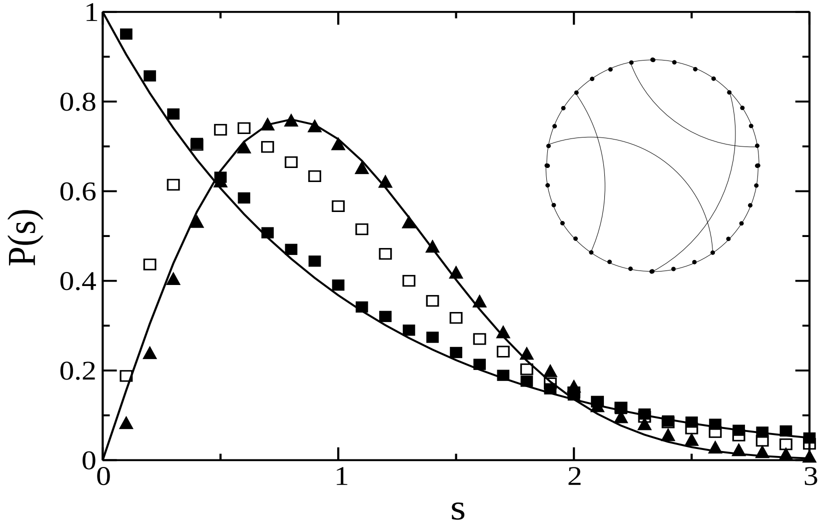
⇒ Possibility to go from a given point to any other through only a small number of shortcut links.

Quantum small-world networks with disorder (Anderson model + shortcuts)

pN random shortcut links are added to a disordered circular chain of N sites

$p = 0 \longrightarrow$ localization $l \approx 100(V/W)^2$

At $p > p_c \approx 1/(4l)$ a delocalization transition takes place (change in the spectral statistics)



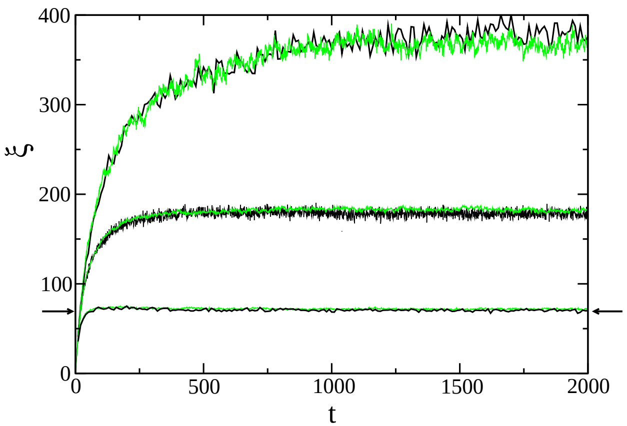
Level spacing statistics for 2^{14} vertices, and $p = 1/32$, for three values of the disorder: $W/V = 0.5$ (triangles), 1.3 (empty squares) and 3 (full squares). The transition from Poisson to Wigner distribution reflects a delocalization transition

Quantum algorithm for quantum small-world network

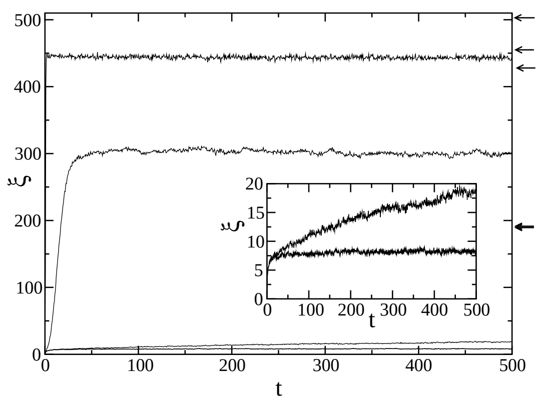
⇒ Approximate algorithm, needs $O(n_r^2)$ gates and $3n_r + 3$ qubits for $N = 2^{n_r}$ vertices.

⇒ Gain depends on p, and varies from better than quadratic to possibly exponential.

⇒ Numerical results show that the quantum algorithm approximates well the dynamics, even in presence of moderately large static imperfections.



Exact (black) and approximate (green) algorithm. IPR vs time, for $W/V = 0.5$, $p = 1/32$ and (from bottom to top) $n_r = 8, 10, 12$.



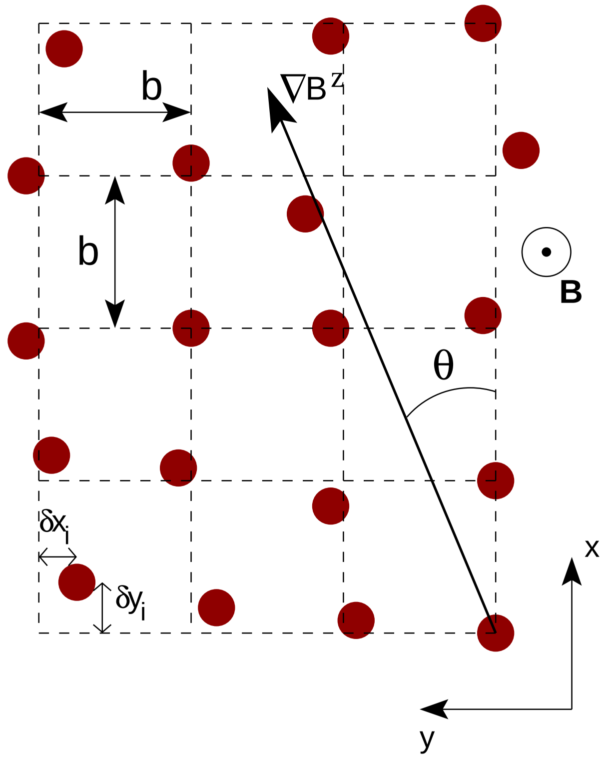
Effect of static errors, $n_r = 10$, $W/V = 3$, $p = 1/32$ and from top to bottom $\epsilon = 10^{-5}$, 10^{-4}, 10^{-6}, 10^{-7} and 0

Yamamoto quantum computer with magnetic field gradient

YQC $\Rightarrow$ all-silicon QC proposed by Yamamoto *et al.* PRL **89**, 017901 (2002)
$\Rightarrow$ Magnetic field gradient + dipole-dipole interactions between nuclei spins

	$\lvert\gamma/2\pi\rvert$	b	d	$\omega_g = 10d$	$g \sim 10d/b$
^{31}P	17.2 MHz/T	4 Å	154 Hz	1.54 kHz	0.224 T/μm
^{29}Si	8.47 MHz/T	1.9 Å	346 Hz	3.46 kHz	2.15 T/μm
^{29}Si	8.47 MHz/T	1 Å	2374 Hz	23.74 kHz	28.0 T/μm

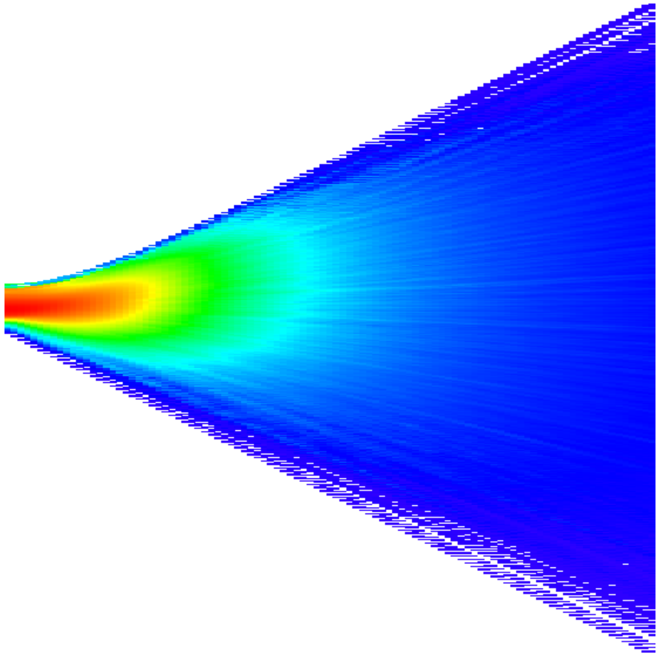
TABLE I: Typical physical parameters for different nuclei corresponding to $\omega_g/d = 10$ in our simulations.



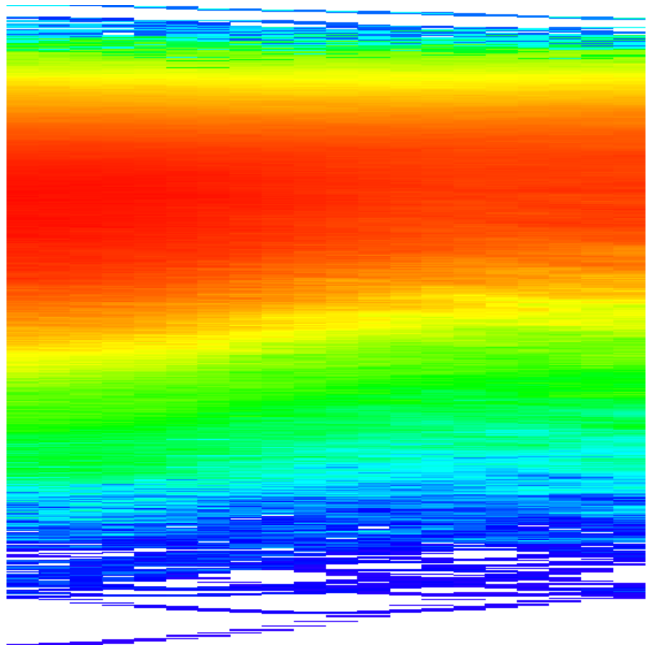
Dipole interaction $d = \mu_0\gamma^2\hbar^2/8\pi b^3$; $\omega_g = max(\delta\omega_i) = \gamma b g/2\pi$; $\nabla B = g$; δ grid shift

Quantum chaos in YQC hardware

⇒ Quantum chaos border (central band): $\omega_g^c \approx d\sqrt{n_q}$; $(\theta \sim 1)$



Entropy S_n as a function of ω_g/d (0 to 20); S_n=0 to 13; $n_q = 4 \times 4$, $\theta = 0.3$, $\delta = 0.1$



Entropy for $0 \leq \theta \leq \pi/4$ at $\omega_g/d = 4$

Future prospects

⇒ application of EQIP results to concrete physical realisations of quantum computers

⇒ realistic pulse shapes and concrete gates implementations

⇒ realisation of one and two qubit gates in YQC, ion traps, superconducting qubits

⇒ from ideal gates to real gates implementations

⇒ further developments of numerical codes and Quantware Library

⇒ application of developed numerical methods to cold atoms and Josephson junctions

⇒ implications of PAREC method for the fault-tolerance threshold

⇒ development of new quantum algorithms

Publications of EDIQIP in 2005

Scientific deliverables are marked by D4,D7,D8,D11,D12.

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[29] Quantware Library: *Quantum numerical recipes* (numerical codes for realistic quantum simulations

created in the frame of EDIQIP project), publicly available at the EDIQIP web site http://www.quantware.ups-tlse.fr/QWLIB/ (D12)