Google Matrix Analysis of DNA Sequences

Vivek Kandiah and Dima Shepelyansky

Laboratoire de Physique Théorique, IRSAMC, UMR 5152 du CNRS
Université Paul Sabatier, Toulouse

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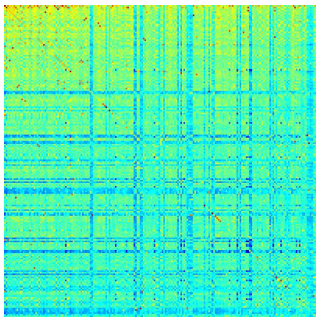


Overview

- Introduction : from DNA sequence to network.
- Statistics of Google matrix elements : similarities and differences with WWW.
- Spectrum and PageRank
- PageRank correlations : statistical similarity between species.
- Conclusion

Introduction : motivation

- Large and accurate genomic dataset available for several species[1].
- Interest in detection of specific/rare patterns in a given sequence.
- New viewpoint of directed network.

Google matrix : $G_{ij} = \alpha S_{ij} + (1 - \alpha)/N$

with $S_{i,j} = T_{i,j} / \sum_j T_{i,j}$ where T describes the transitions between nearby words.

[1] http://www.ensembl.org/

Introduction : from DNA sequence to network

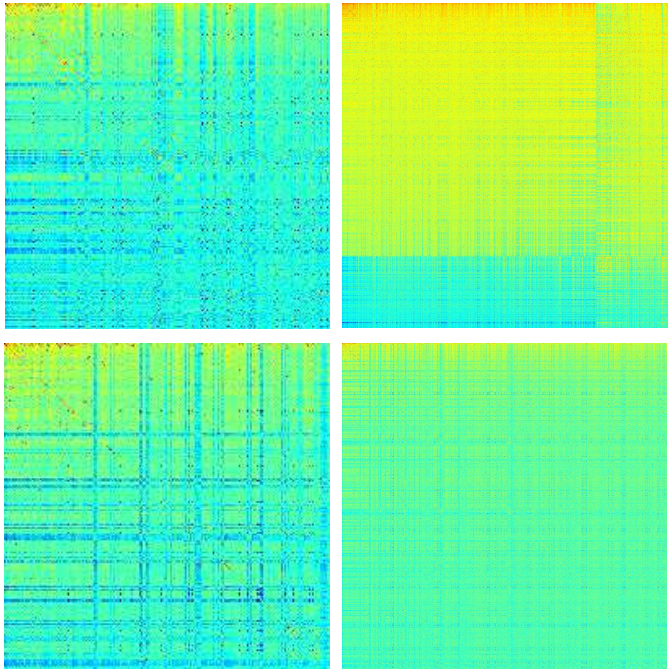
- Single string of DNA sequences of length L base pairs, read in the nat ural direction. **Dataset** 5 species : Bos Taurus (Bull,$L \approx 2.9 \cdot 10^9 bp$); Canis Familiaris (Dog,$L \approx 2.5 \cdot 10^9 bp$); Loxondonta Africana (Elephant,$L \approx 3.1 \cdot 10^9 bp$); Homo Sapiens (Human,$L \approx 1.5 \cdot 10^{10} bp$) and Danio Rerio (Zebrafish,$L \approx 1.4 \cdot 10^9 bp$).
- Only words with A,C,G and T are considered, words containing unknown nuc leotides are discarded.
- Analysis are performed with $m = 5$, $m = 6$ and $m = 7$ letters words $\rightarrow$ size of the space of states (matrix size) are $N = 4^m = 1024$, $N = 4096$ and $N = 16384$ at $\alpha = 1$.

$$...TCG \underbrace{ATAT}_{W_{k-1}} \underbrace{CTGG}_{W_k} \underbrace{TAAC}_{W_{k+1}} CTA...$$

$$\rightarrow W_{k-1} \rightarrow W_k \rightarrow W_{k+1} \rightarrow$$

- $T_{ij} \rightarrow T_{ij} + 1$ whenever word j points to word i. At the end, all empty columns elements are replaced by $1/N$.

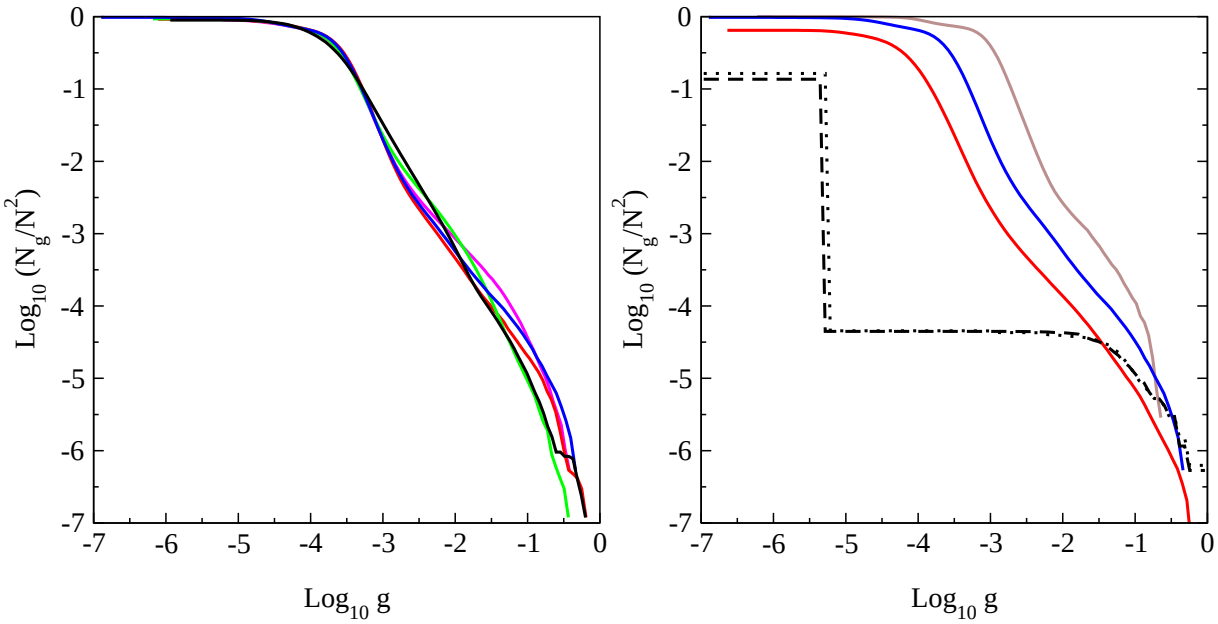
Statistics of Google matrix elements



DNA Google matrix of Homo sapiens (HS) constructed for words of 5-letters (top) and 6-letters (bottom) length. Matrix elements $G_{KK'}$ are shown in the basis of PageRank index K (and K'). Here, x and y axes show K and K' within the range $1 \leq K, K' \leq 200$ (left) and $1 \leq K, K' \leq 1000$ (right). The element G_{11} at $K = K' = 1$ is placed at top left corner. Color marks the amplitude of matrix elements changing from blue for minimum zero value to red at maximum value.

- Full matrix limit, $L/mN^2 \approx 10$ to 100 transitions per elements at $m = 6$.
- Webpages ≈ 10 links per node on average with $N \approx 2 \cdot 10^5$.

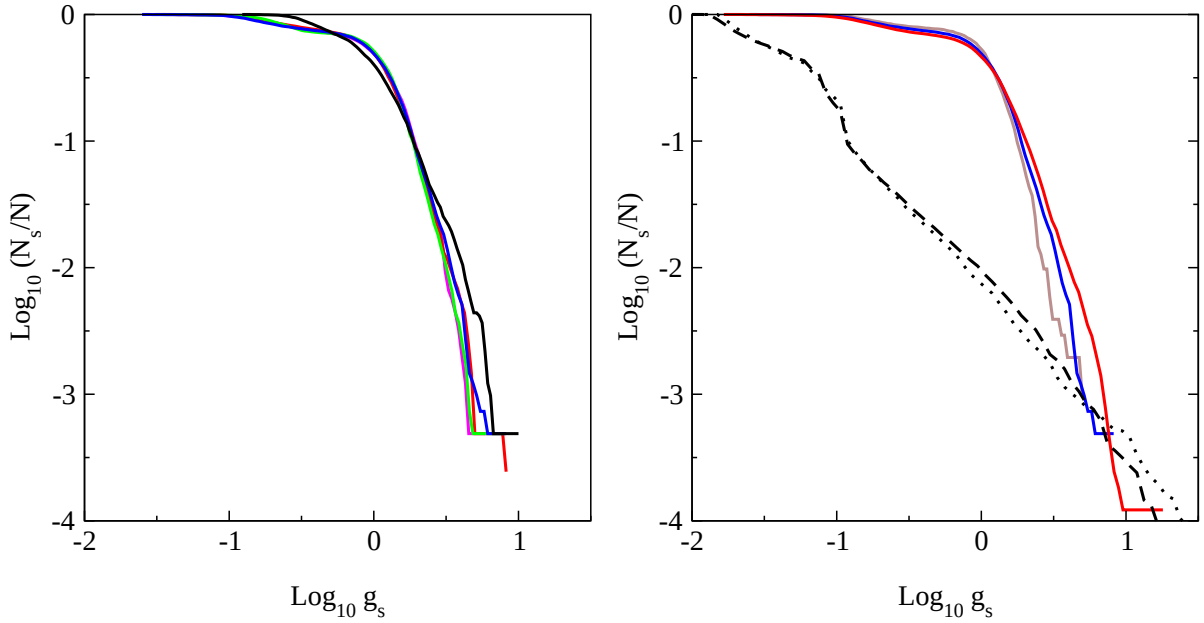
Statistics of Google matrix elements



Integrated fraction N_g/N^2 of Google matrix elements with $G_{ij} > g$ as a function of g. *Left panel :* Various species with 6-letters word length: bull BT (magenta), dog CF (red), elephant LA (green), Homo sapiens HS (blue) and zebrafish DR(black). *Right panel :* Data for HS sequence with words of length $m = 5$ (brown), 6 (blue), 7 (red). For comparison black dashed and dotted curves show the same distribution for the WWW networks of Universities of Cambridge and Oxford in 2006 respectively.

- Long range algebraic decay as $N_g \propto 1/g^{\nu-1}$. Fit in the range $-5.5 < \log_{10} g < -0.5$ gives : $\nu = 2.46 \pm 0.025$ (BT), 2.57 ± 0.025 (CF), 2.67 ± 0.022 (LA), 2.48 ± 0.024 (HS), 2.22 ± 0.04 (DR). For HS : $\nu = 2.68 \pm 0.038$ at $m = 5$ and $\nu = 2.43 \pm 0.02$ at $m = 7$.
- Oscillations but universal decay law with $\nu \approx 2.5$.
- Distribution of outgoing links in WWW networks decay with $\tilde{\nu} \approx 2.7$.

Statistics of Google matrix elements



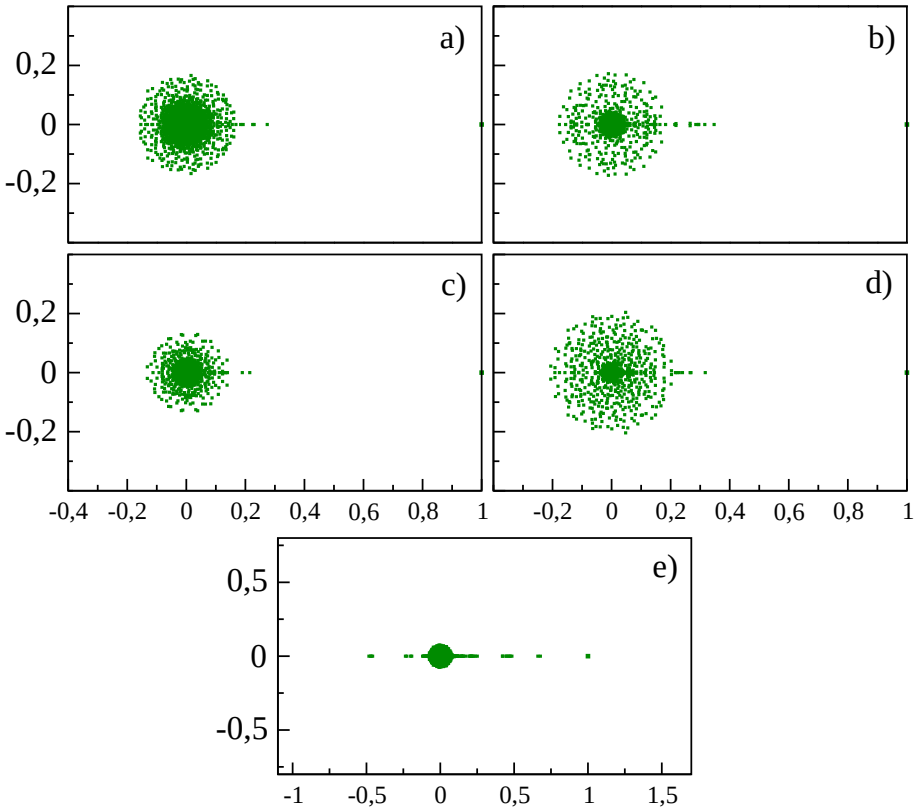
Integrated fraction N_S/N of sum of ingoing matrix elements with $\sum_{j=1}^{N} G_{i,j} \geq g_S$. Left and right panels show the same cases as above in same colors. The dashed and dotted curves are shifted in x-axis by one unit left to fit the figure scale.

- Power law decay as $N_S \propto 1/g^{\mu-1}$. Fit gives $\mu = 5.59 \pm 0.15$ (BT), 4.90 ± 0.08 (CF), 5.37 ± 0.07 (LA), 5.11 ± 0.12 (HS), 4.04 ± 0.06 (DR). For HS at $m = 5, 7$ we have $\mu = 5.86 \pm 0.14$ and 4.48 ± 0.08.
- Distribution of ingoing links in WWW networks decay with $\tilde{\mu} \approx 2.1$.
- Visible differences between species but close to universal decay curve.

Statistics of Google matrix elements

- WWW outgoing links decay with $\tilde{\nu} \approx 2.7 \rightarrow$ **DNA matrix elements** distribution decay with $\nu \approx 2.5 \rightarrow$ similar to **WWW outgoing links** distribution.
- Sum of Ingoing matrix elements distribution similar to ingoing links distribution : **Webpages** decay with $\tilde{\mu} = 2.1$ and **DNA** decay with $\mu \approx 5$.

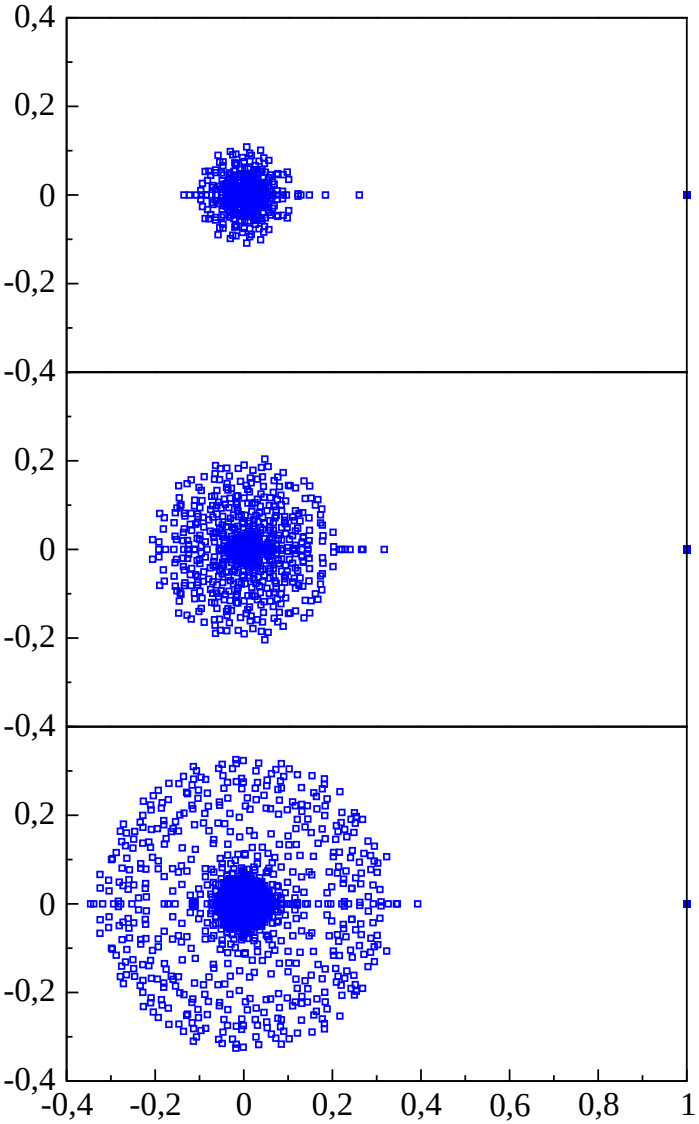
Spectrum and PageRank



Eigenvalue spectrum at $m = 6$ of a) Bos Taurus, b) Canis Familiaris, c) Loxodonta Africana, d) Homo Sapiens and e) Danio Rerio.

- Presence of large gap.
- HS $\sim$ CF and strong differences between mammalian and non mammalian sequences.
- Spectrum of G and G^* are identical.

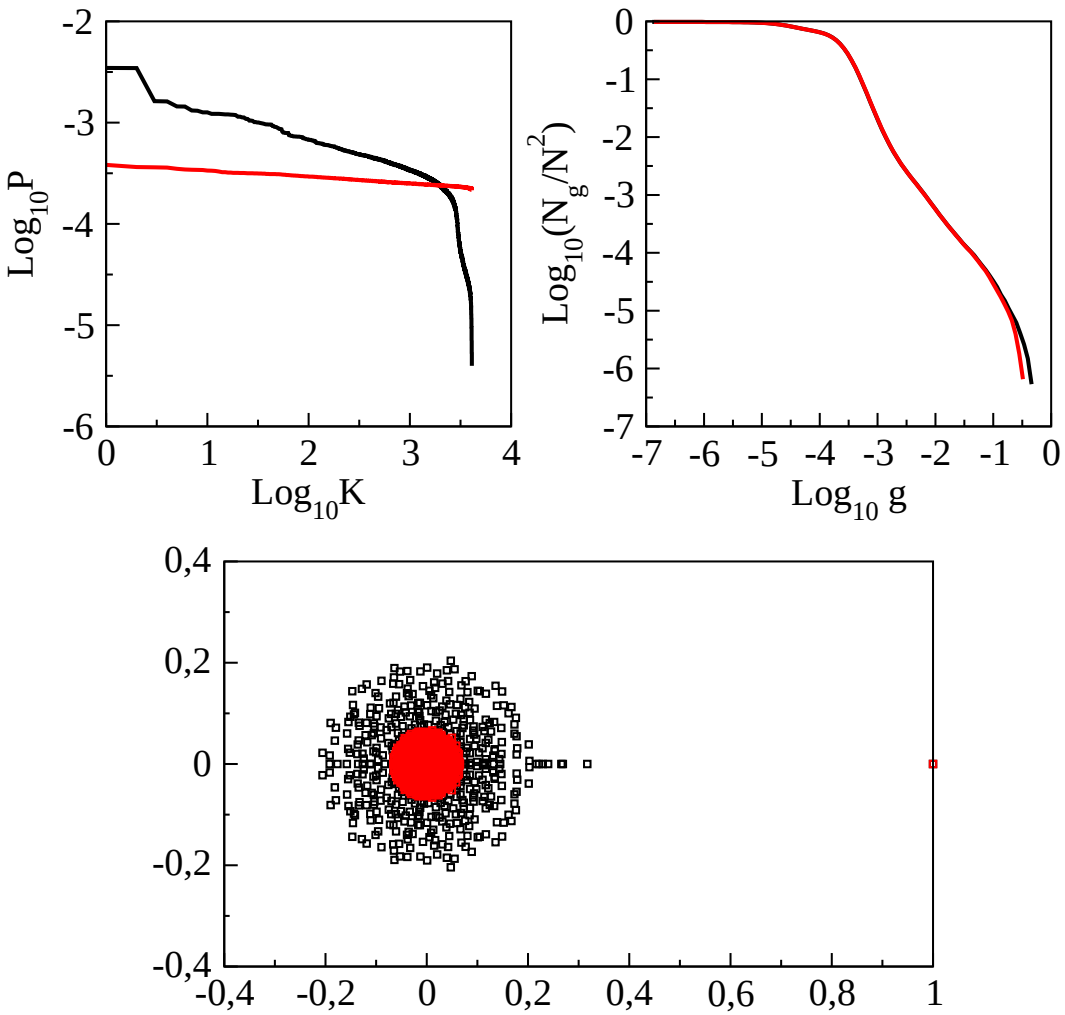
Spectrum and PageRank



Eigenvalue spectrum at $m = 5$, $m = 6$ and $m = 7$ of Homo Sapiens.

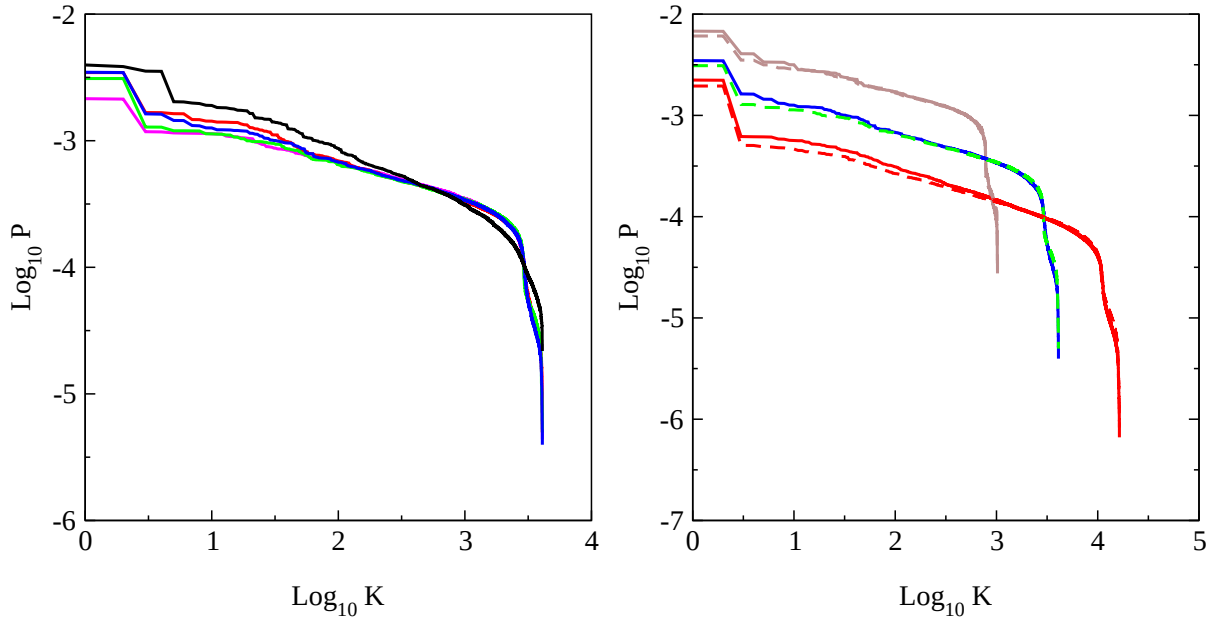
- Increase in word length leads to an increase of eigenvalue cloud radius, $\lambda_c \approx 0.1$, $\lambda_c \approx 0.2$ and $\lambda_c \approx 0.35$ for $m = 5$, $m = 6$ and $m = 7$.
- The spectrum is not reproducible with simple RMT model.

Spectrum and PageRank



Random matrix model with distribution of elements corresponding to HS at $m = 6$.

Spectrum and PageRank



PageRank probability decay of several species at $m = 6$ (left) and Homo Sapiens at $m = 5$, $m = 6$ and $m = 7$ (right).

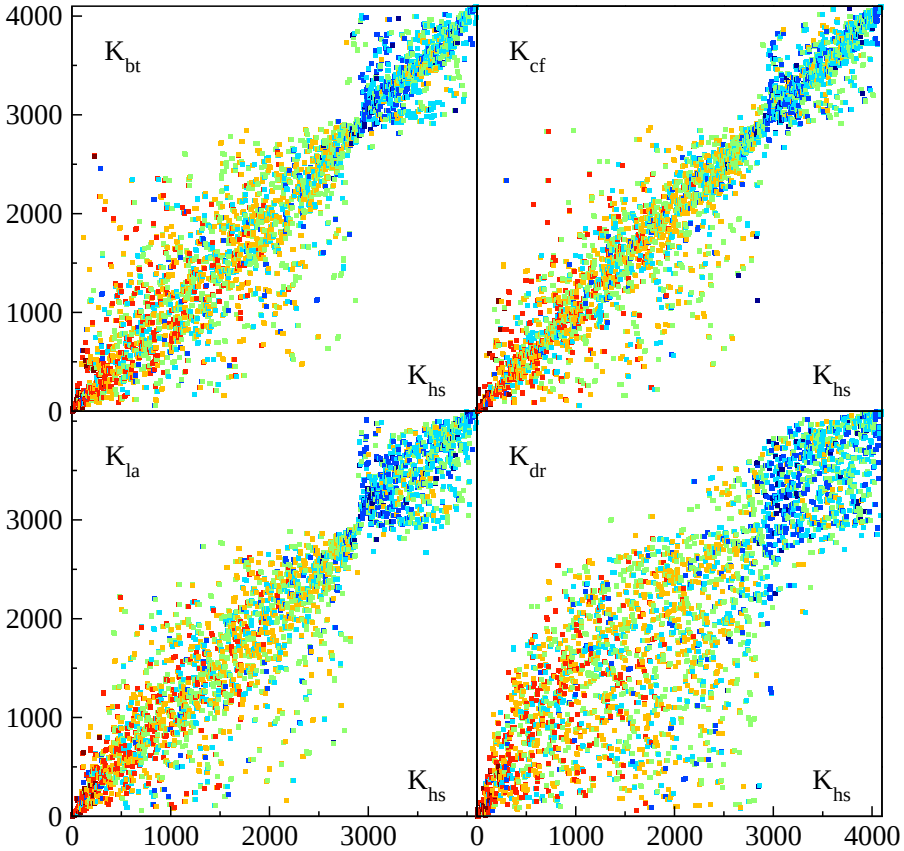
- PageRank $\sim$ frequency of words.
- $P(K) \sim 1/K^{\beta}$ with $\beta = 1/(\mu - 1)$.
- At $m = 6$: $\beta = 0.273 \pm 0.005$ (BT), 0.340 ± 0.005 (CF), 0.281 ± 0.005 (LA), 0.308 ± 0.005 (HS), 0.426 ± 0.008 (DR) in the range $1 \leq \log_{10} K \leq 3.3$. Small variation between mammalian species, stable with word length.

Top five (top) and last five (bottom) PageRank entries of DNA sequences.

BT	CF	LA	HS	DR
TTTTTT	TTTTTT	AAAAAA	TTTTTT	ATATAT
AAAAAA	AAAAAA	TTTTTT	AAAAAA	TATATA
ATTTTT	AATAAA	ATTTTT	ATTTTT	AAAAAA
AAAAAT	TTTATT	AAAAAT	AAAAAT	TTTTTT
TTCTTT	AAATAA	AGAAAA	TATTTT	AATAAA

BT	CF	LA	HS	DR
CGCGTA	TACGCG	CGCGTA	TACGCG	CCGACG
TACGCG	CGCGTA	TACGCG	CGCGTA	CGTCGG
CGTACG	TCGCGA	ATCGCG	CGTACG	CGTCGA
CGATCG	CGTACG	TCGCGA	TCGACG	TCGACG
ATCGCG	CGATCG	CGCGAT	CGTCGA	TCGTCG

Statistical proximity

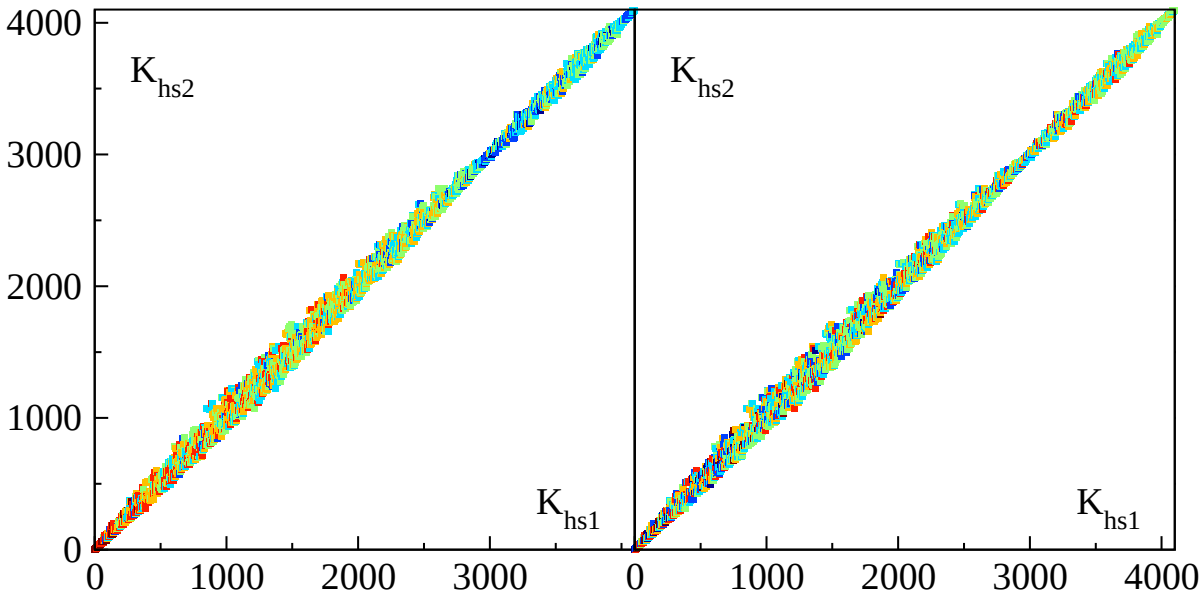


PageRank proximity K — K plane diagrams for different species in comparison with Homo Sapiens.

$$\zeta(s_1, s_2) = \frac{\sqrt{\sum_{i=1}^{N}(K_{s_1}(i) - K_{s_2}(i))^2)/N}}{\sigma_{rnd}}.$$

$$\begin{aligned}
\zeta(HS, CF) &= 0.206, \quad \zeta(HS, LA) = 0.238, \\
\zeta(HS, BT) &= 0.246, \quad \zeta(LA, CF) = 0.303, \\
\zeta(CF, BT) &= 0.308, \quad \zeta(LA, BT) = 0.324, \\
\zeta(DR, HS) &= 0.375, \quad \zeta(DR, CF) = 0.414, \\
\zeta(DR, LA) &= 0.422, \quad \zeta(DR, BT) = 0.425
\end{aligned}$$

Statistical proximity



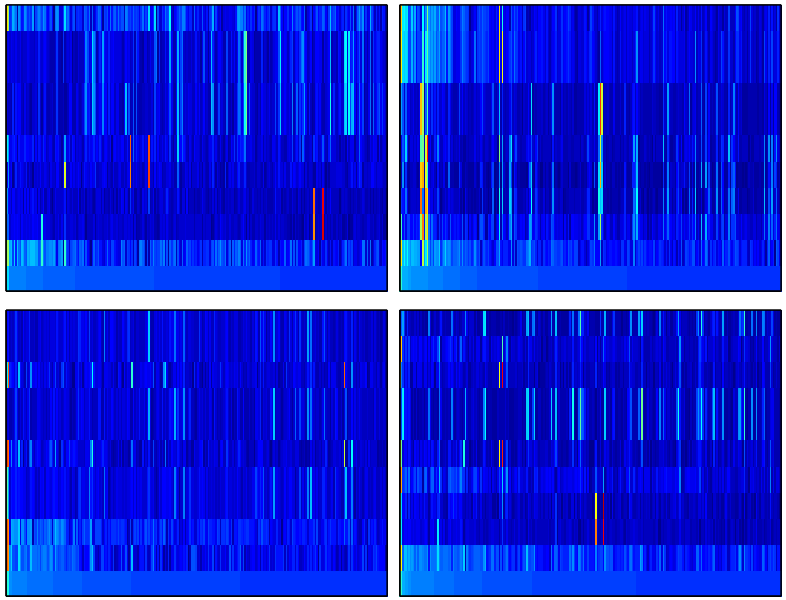
PageRank proximity K — K plane diagrams between two Homo Sapiens individuals.
$\zeta = 0.031$

Conclusion and Perspectives

- Complex and large gaped spectrum of Google matrix.
- DNA sequence $\mu \approx 5 \rightarrow$ slow PageRank decay $\beta \approx 0.25$ (For WWW $\beta \approx 0.9$).
- PageRank correlations show the statistical similarity between species from a Markov chain point of view.

Conclusion and Perspectives

- Structural differences and similarities of DNA with WWW through G_{ij}.
- PageRank useful to describe differences between species.
- Other eigenmodes might be highlight a relatively long living relaxation mode and might localize themselves in a paricular set of words.



Eigenstates corresponding to 10 largest eigenvalue are shown for the first 250 components in PageRank basis.

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