Mesoscopic quantum mechanics: mesoscopic solid-state qubits and quantum measurements

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Outline

1. Mesoscopic condensed-matter qubits

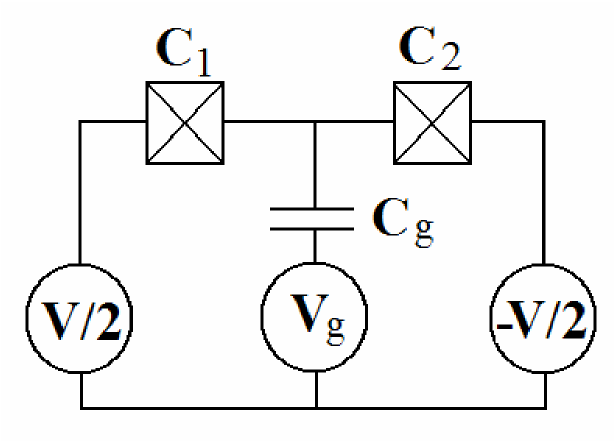
   ``Single-particle'' physics and qubits

   - Coulomb-blockade phenomena
   - Cooper-pair and atomic qubits
   - Electron (quantum dot) and FQHE quasiparticle (anti-dot) qubits

   Generic superconductor (flux, charge-flux, ``phase'') qubits

2. Decoherence in mesoscopic qubits
3. Quantum measurements and mesoscopic detectors

Cooper-pair (Bloch) transistor



$$U(n_1,n_2)=E_C(n_1-n_2-q)^2-eV(n_1+n_2),$$
$$q=C_gV_g/2e,\ E_C=2e^2/C_\Sigma,\ C_\Sigma=C_1+C_2+C_g,$$
$$H=U(n_1,n_2)-E_{J1}\cos\varphi_1-E_{J2}\cos\varphi_2.$$

$$\{n_1,n_2\}\Rightarrow\{n,N\}:\quad n=n_1-n_2,\quad N=(n_1+n_2)/2,$$
$$\chi=(\varphi_1-\varphi_2)/2,\quad \varphi=\varphi_1+\varphi_2,\quad [n,\chi]=[N,\varphi]=i.$$

$$E_{J1}=E_{J2},\quad H=E_C(n-q)^2-2eVN-2E_J\cos(\varphi/2)\cos\chi.$$

Thus, we have either

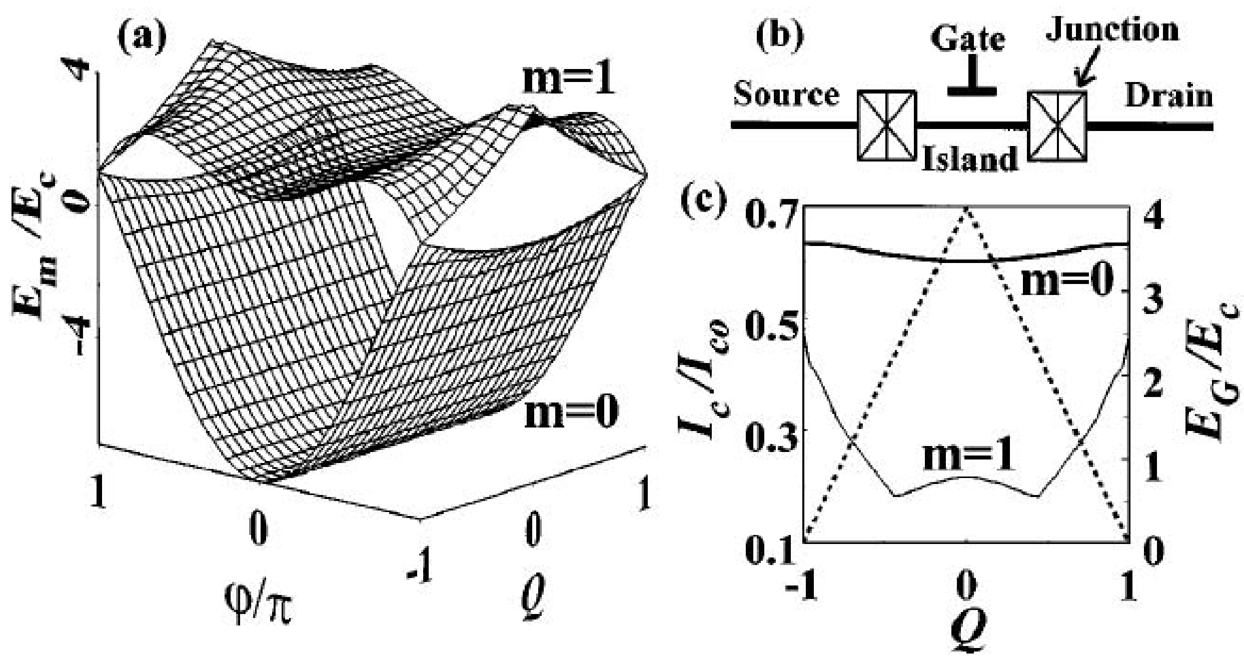
- Cooper-pair box with phase-controlled tunnel coupling; or
- Josephson junction with charge-controlled critical current.

$$V = 0, \quad \varphi = const.$$

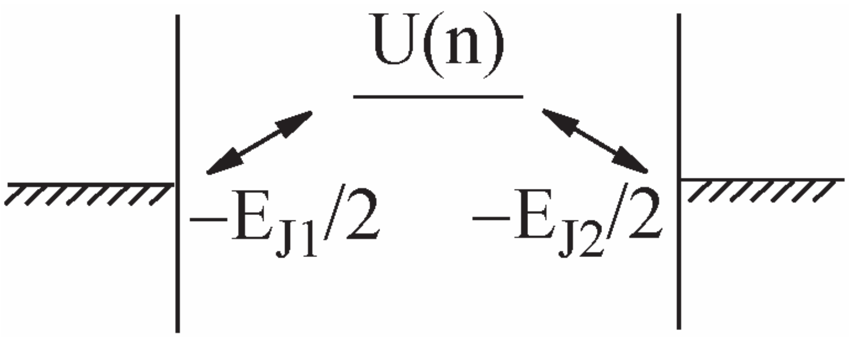
$$H = E_C (n-q)^2 - 2E_J \cos(\varphi/2)\cos\chi \quad =$$

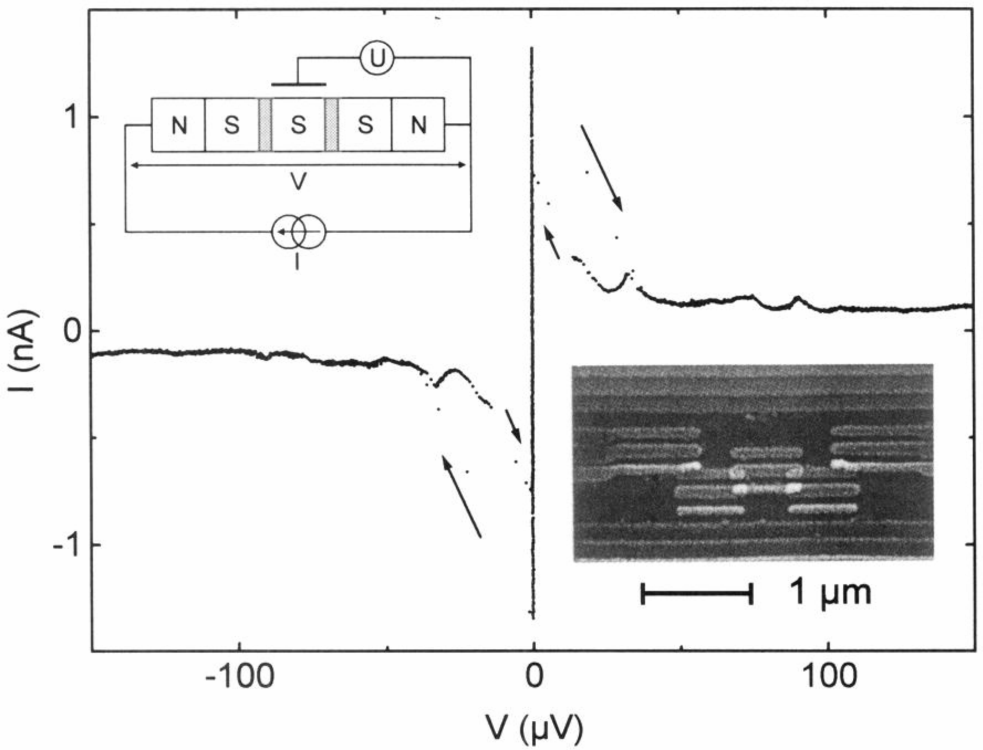
$$= E_C (n-q)^2 - 2E_J \cos(\varphi/2)(|n\rangle\langle n\pm 1| + |n\pm 1\rangle\langle n|) \Rightarrow \quad E_m(q,\varphi).$$

$$I_C^{(m)} = \max_\varphi [I_S(\varphi)], \quad I_S(\varphi) = (2e/\hbar)\partial E_m(q,\varphi)/\partial\varphi.$$



Periodic oscillations of the critical current (1)





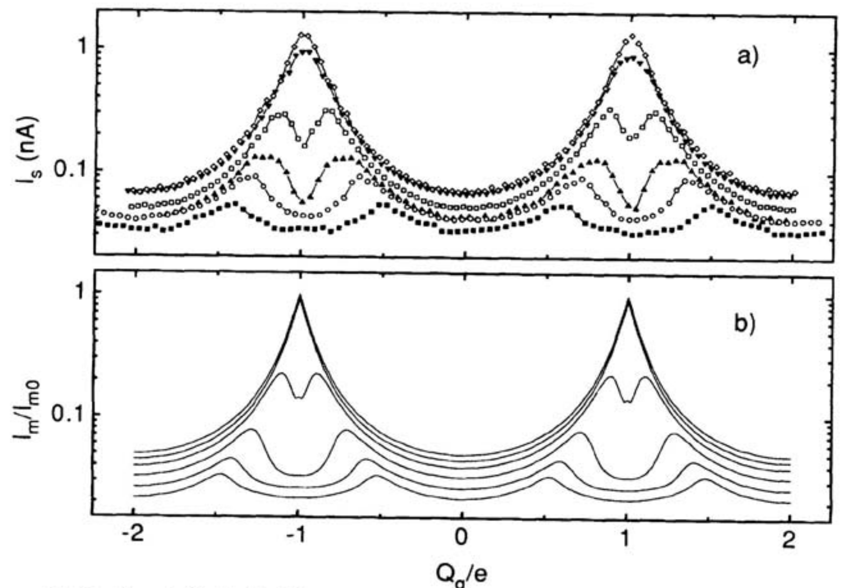


FIG. 2. (a) Switching current as a function of gate charge, for several values of the magnetic field H, at $T=65$ mK. Top to bottom: $H=0$, 0.07, 0.11, 0.14, 0.16, 0.17 T. The dip at odd integer values of Q_g/e corresponds to the poisoning of Josephson tunneling by the entrance of one quasiparticle in the island. (b) Theoretical runaway current as a function of gate charge, for the same field values as in (a).

P. Joyez *et al.*, Phys. Rev. Lett. **72**, 2458 (1994).

Periodic oscillations of the critical current (2)

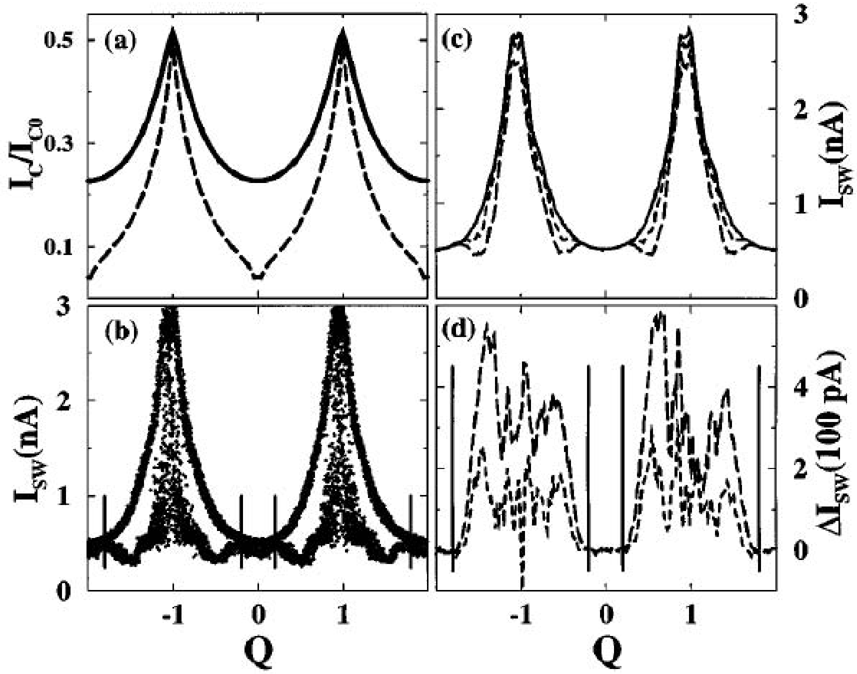


FIG. 3. (a) Calculated critical currents for the lowest energy bands of BT2. (b)–(c) $I_{sw}(Q)$ of BT2 in the presence and absence of 34 GHz radiation ($T \sim 30$ mK). (b) At high power levels the characteristic shape of the current modulation in the excited band is evident but I_c is suppressed even in the ground band (near $Q \sim Q_{\text{even}}$). The solid vertical lines mark the band gap threshold determined from an independent measurement of C_Σ. (c) Averaged $I_{sw}(Q)$ data in the low power limit. The solid, short-dashed, and long-dashed lines represent increasing power levels from $P = 0$ (solid line). (d) $\Delta I_{sw} = I_{sw}|_{P=0} - I_{sw}|_{P>0}$ derived from (c) show a nearly power independent band gap threshold. The vertical lines are the same as in (b).

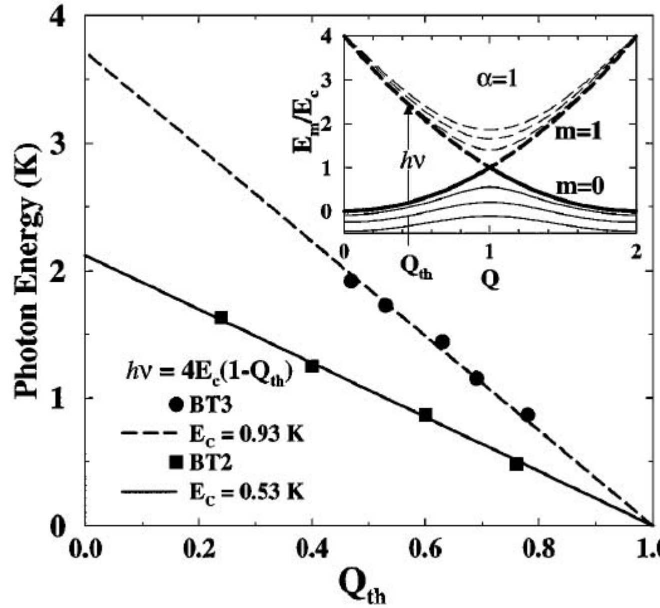


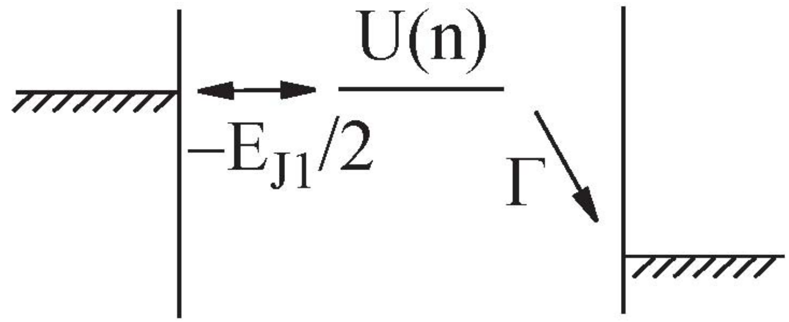
FIG. 4. Measured frequency dependence of Q_{th}. The solid and dashed lines are fits to Eq. (2) using E_c as a fitting parameter. The inset shows energy bands vs Q (for $\varphi = 0, \pi/2, 3\pi/4, \pi$) in a BT with $\alpha = 1$.

D.J. Flees *et al.*, Phys. Rev. Lett. **78**, 4817 (1997).

Resonant tunneling of Cooper pairs

Dynamics of the density matrix

$$\dot{\rho} = -i[H, \rho] + \Gamma\{\rho\},$$
$$H = [\varepsilon\sigma_z - E_J\sigma_x]/2.$$



Relaxation terms can be described in the Markov approximation, since $eV \sim E_C \gg E_J$

$$\dot{\rho}_{11} = -\dot{\rho}_{00} = E_J \operatorname{Im}(\rho_{10}) - \Gamma\rho_{11},$$
$$\dot{\rho}_{10} = -i\varepsilon\rho_{10} - i(E_J/2)(\rho_{11} - \rho_{00}) - \Gamma\rho_{10}/2.$$

Stationary current

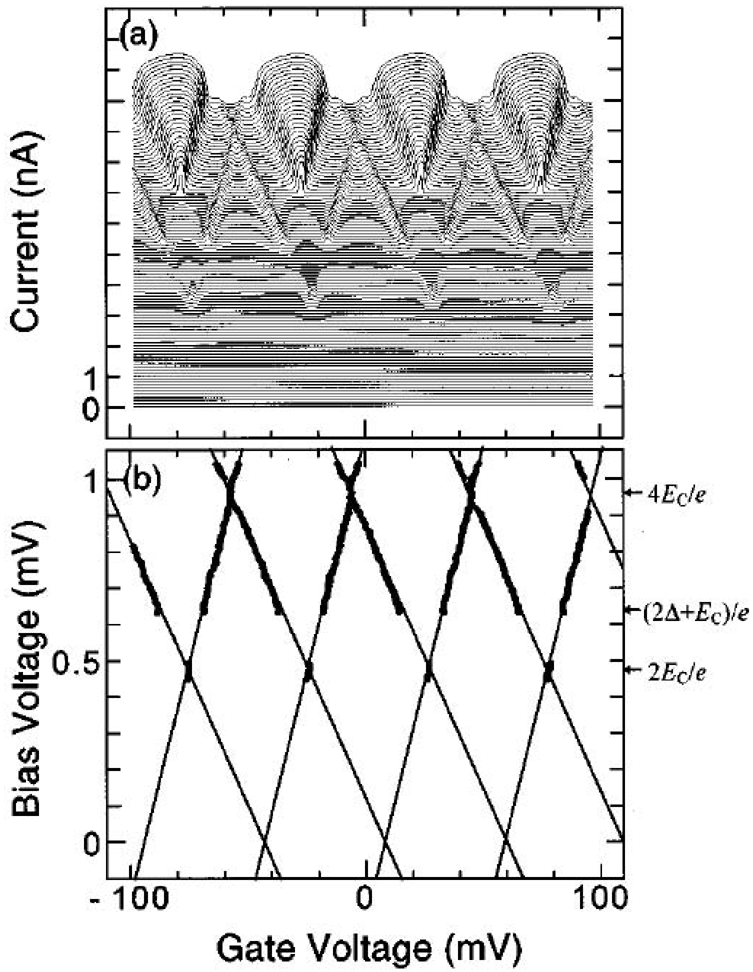
$$\langle I \rangle = \Gamma\rho_{11}^{(st)} = \frac{\Gamma E_J^2}{\Gamma^2 + 4\varepsilon^2 + 2E_J^2}.$$

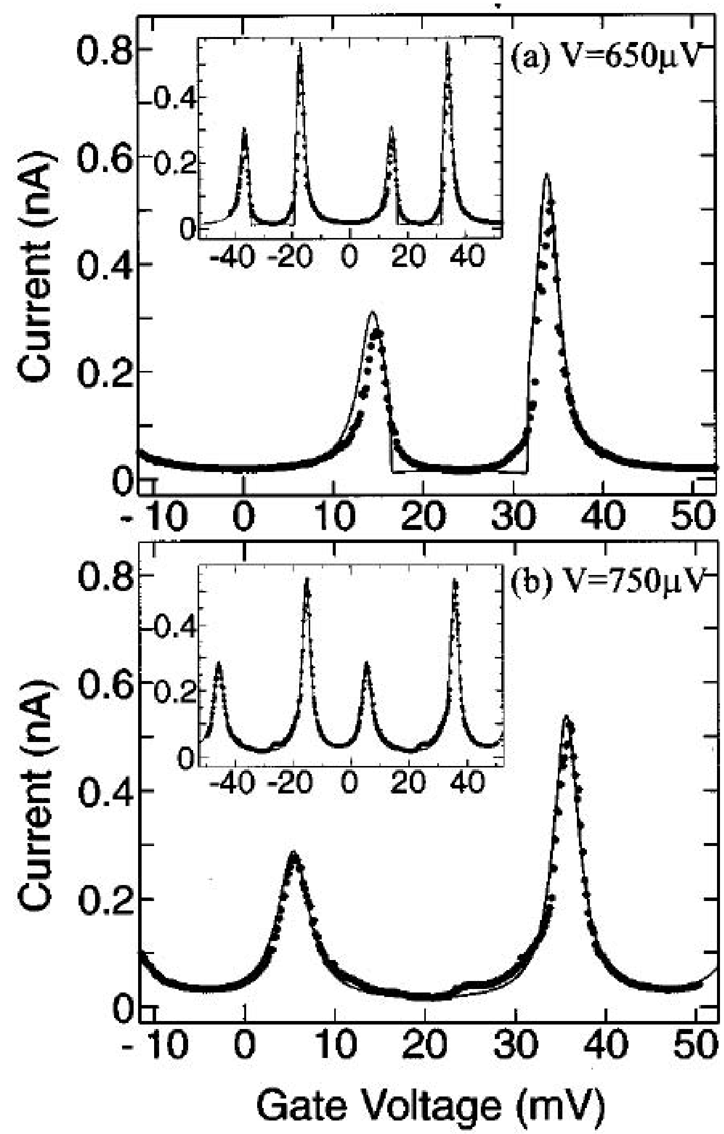
D.V.A. and V.Ya. Aleskin, JETP Lett. **50**, 367 (1989); T.A. Fulton *et al.*, Phys. Rev. Lett. **63**, 1307 (1989).

Bias conditions

$$\Delta U_j = U(n_j+1) - U(n_j) = 0:$$

$$eV + (-1)^j 2eC_g V_g / C_\Sigma = E_C.$$

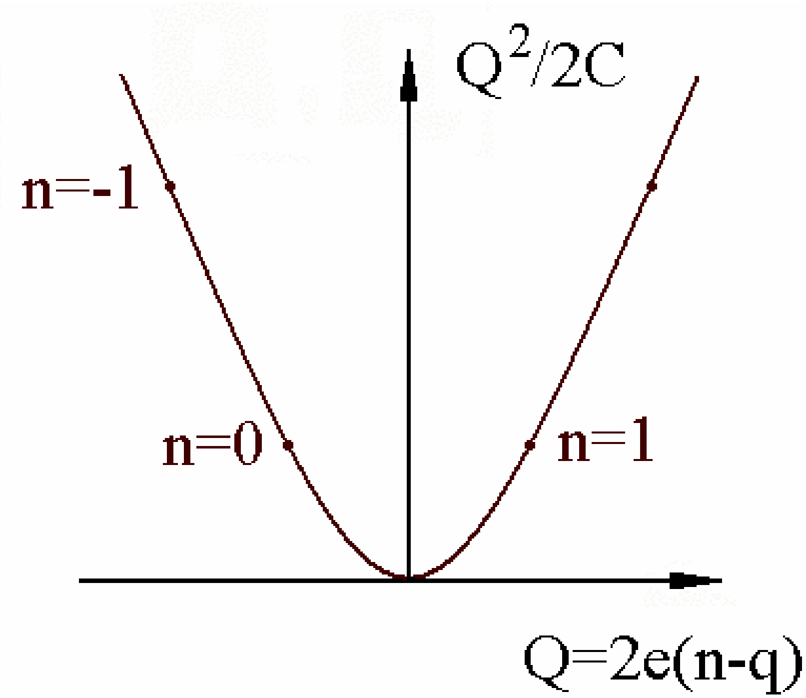


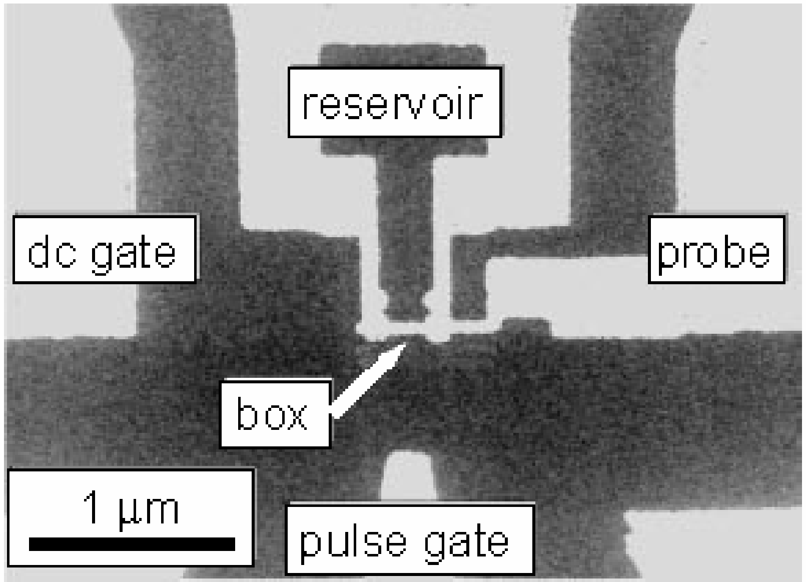


Y. Nakamura *et al.,* Phys. Rev. B **53**, 8234 (1996).

Cooper-pair box as the charge qubit

$$E_J << E_C, \quad q \approx 1/2$$

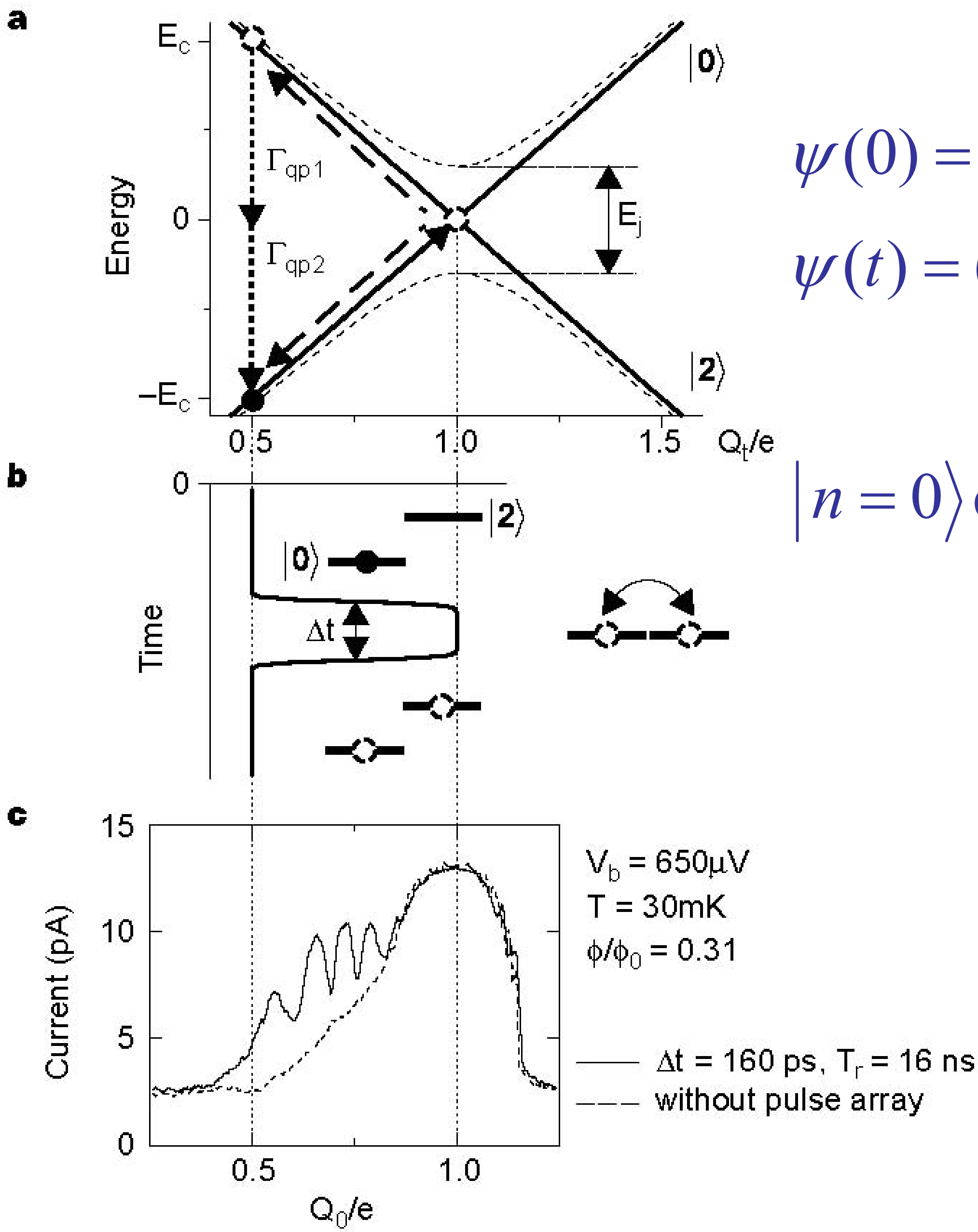




$$H = -E_C(q - 1/2)\sigma_z - (E_J/2)\sigma_x$$

Y. Nakamura *et al.*,
Nature **398**, 786 (1999).

Charge qubit (1)

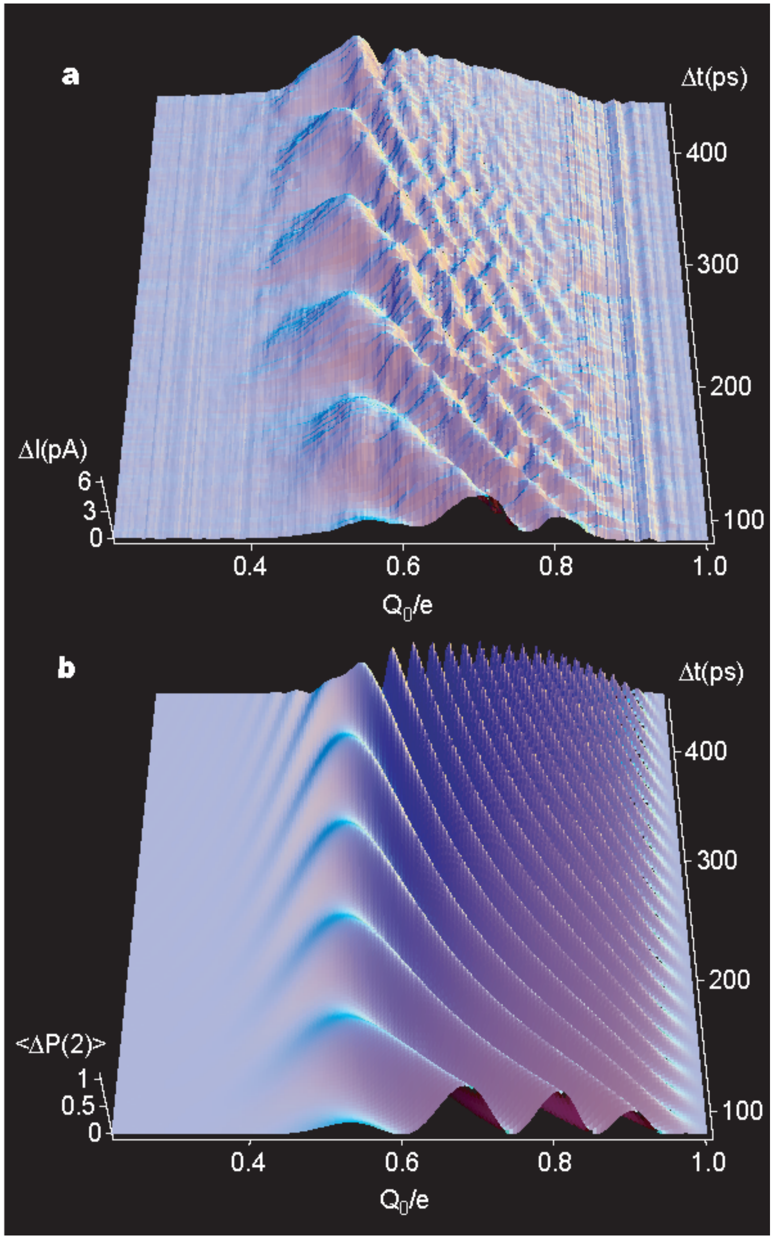


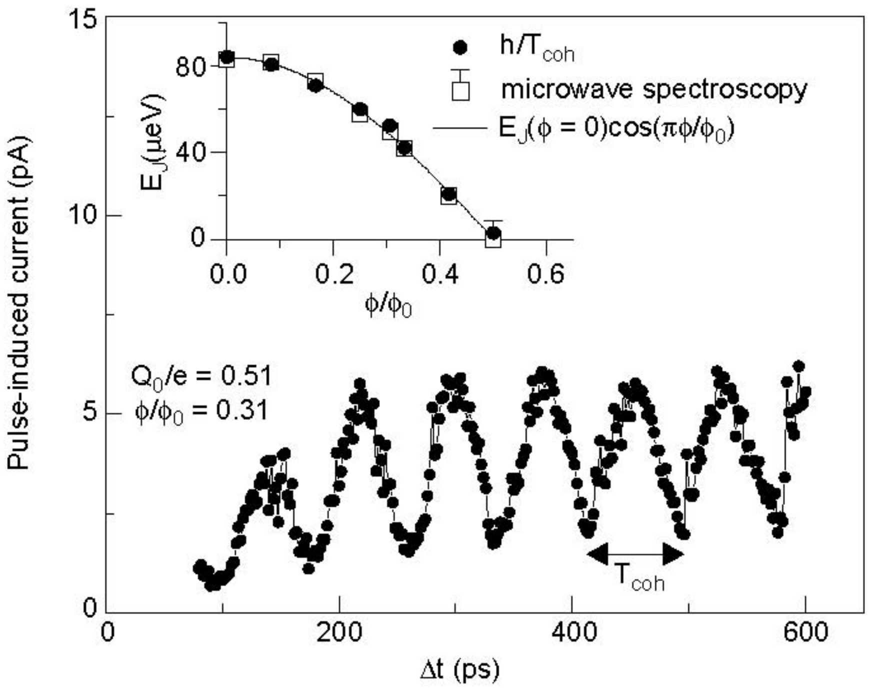
$$\psi(0) = |n=0\rangle = (|+\rangle + |-\rangle)/\sqrt{2},$$

$$\psi(t) = (|+\rangle \exp\{-iE_J t/2\} + |-\rangle \exp\{iE_J t/2\})/\sqrt{2} = |n=0\rangle \cos(E_J t/2) + |n=1\rangle \sin(E_J t/2)$$

$$p_{n=1}(t) = [1 - \cos(E_J t)]/2$$

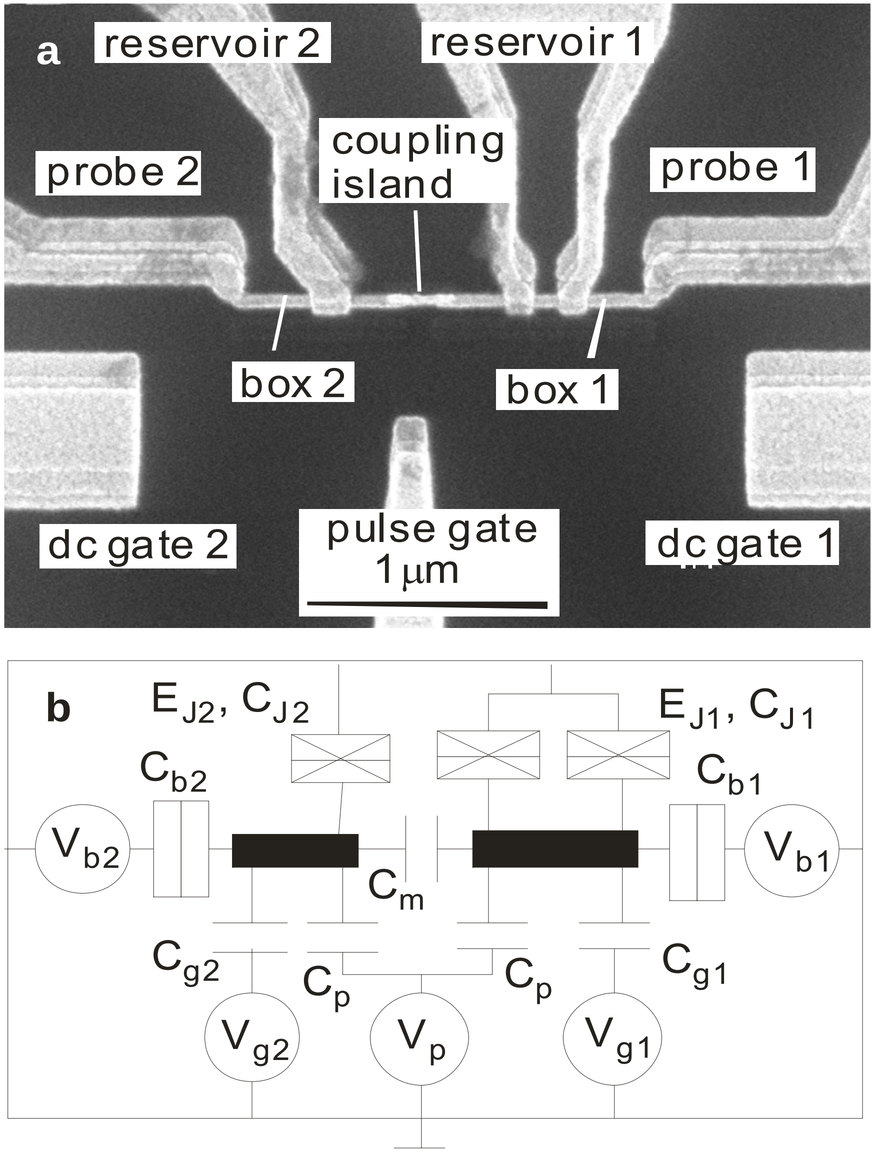
Charge qubit (2)





Y. Nakamura *et al.*,
Nature **398**, 786 (1999).

Two coupled charge qubits (1)



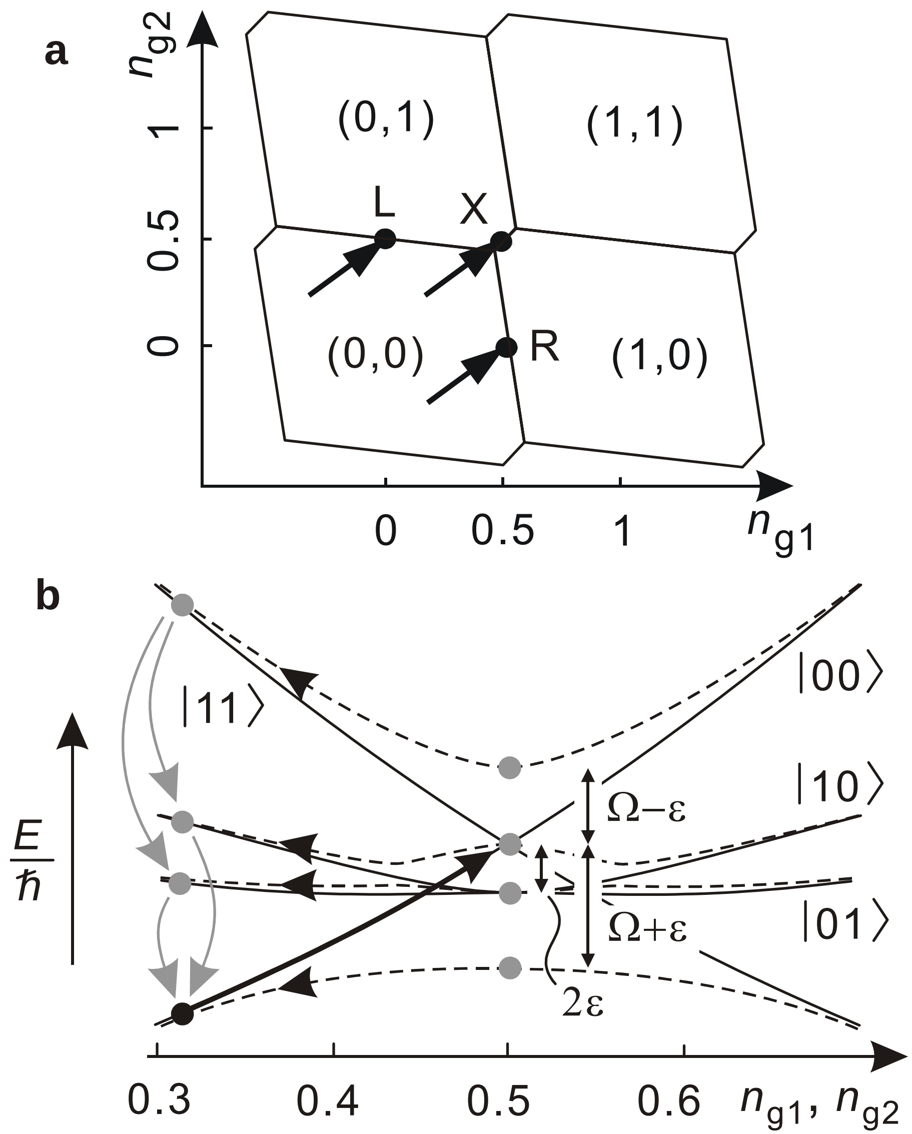
$$H = \begin{bmatrix} E_{00} & -\frac{1}{2}E_{J1} & -\frac{1}{2}E_{J2} & 0 \\ -\frac{1}{2}E_{J1} & E_{10} & 0 & -\frac{1}{2}E_{J2} \\ -\frac{1}{2}E_{J2} & 0 & E_{01} & -\frac{1}{2}E_{J1} \\ 0 & -\frac{1}{2}E_{J2} & -\frac{1}{2}E_{J1} & E_{11} \end{bmatrix}$$

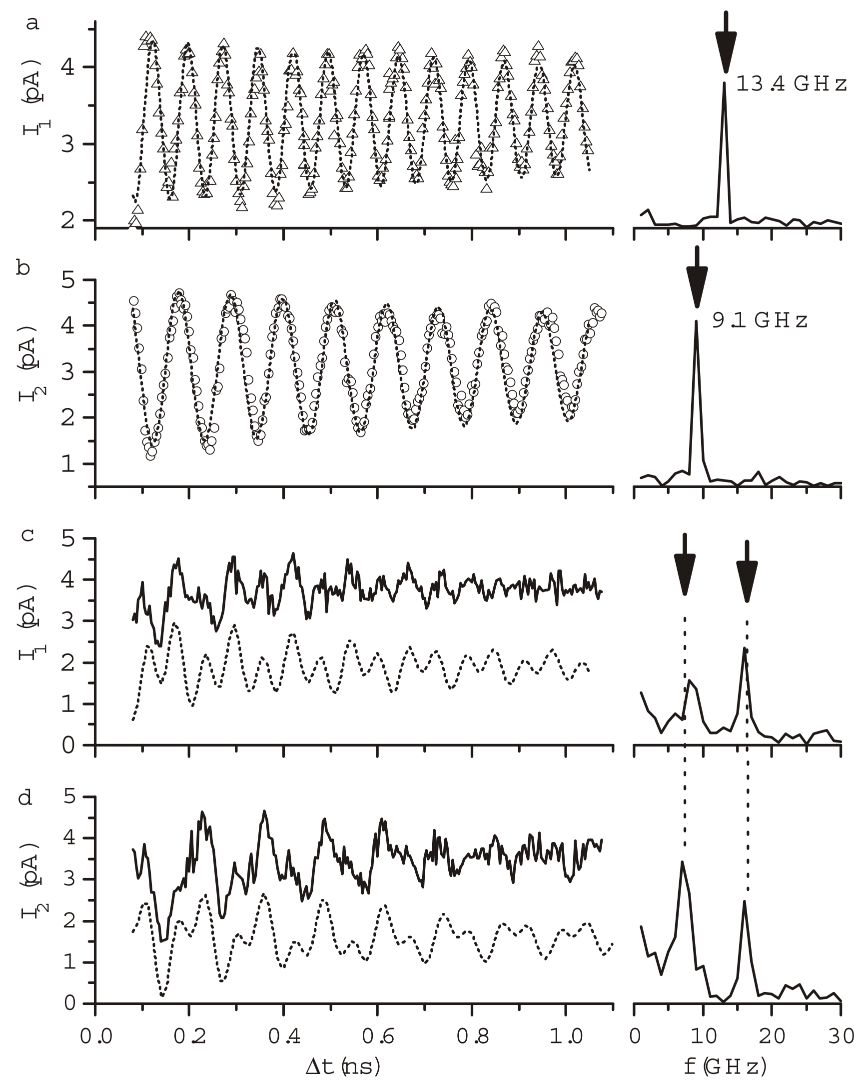
$$E_{n1n2} = E_{C1}(n_{g1}-n_1)^2 + E_{C2}(n_{g2}-n_2)^2 + E_m(n_{g1}-n_1)(n_{g2}-n_2),$$

$$E_m = 4e^2C_m/(C_{\Sigma 1}C_{\Sigma 2} - C_m{}^2)$$

Yu. A. Pashkin *et al.*,
Nature **421**, 823 (2003).

Two coupled charge qubits (2)





Two coupled charge qubits (3)

\`\`Co-resonance'' point: $$H = -(1/2)\sum_{j=1,2} E_{Jj}\sigma_x^{(j)} + E_m \sigma_z^{(1)}\sigma_z^{(2)}.$$

In the new basis,

$$|\pm\rangle = (|0\rangle \pm |1\rangle)\sqrt{2}\ :$$

$$\Delta \equiv \left(E_{J1} + E_{J2}\right)/2, \quad \delta \equiv \left(E_{J2} - E_{J1}\right)/2,$$

$$\Omega = \left(\Delta^2 + \nu^2\right)^{1/2}, \quad \varepsilon = \left(\delta^2 + \nu^2\right)^{1/2}, \quad \nu \equiv E_m / 4.$$

$$\begin{array}{cc} & \begin{array}{cccc} ++ & -- & -+ & +- \end{array} \\ H = & \begin{bmatrix} -\Delta & \nu & 0 & 0 \\ \nu & \Delta & 0 & 0 \\ 0 & 0 & -\delta & \nu \\ 0 & 0 & \nu & \delta \end{bmatrix} \end{array}$$

Probability p_j to have a Cooper pair on the *j*th box:

$$p_{1,2}(1) = \frac{1}{2}\left[1 - \cos(\Omega t)\cos(\varepsilon t) - \frac{E_{J1,2}^2 - E_{J2,1}^2 + E_m^2}{4\Omega\varepsilon}\sin(\Omega t)\sin(\varepsilon t)\right].$$

Entanglement between the two coupled qubits

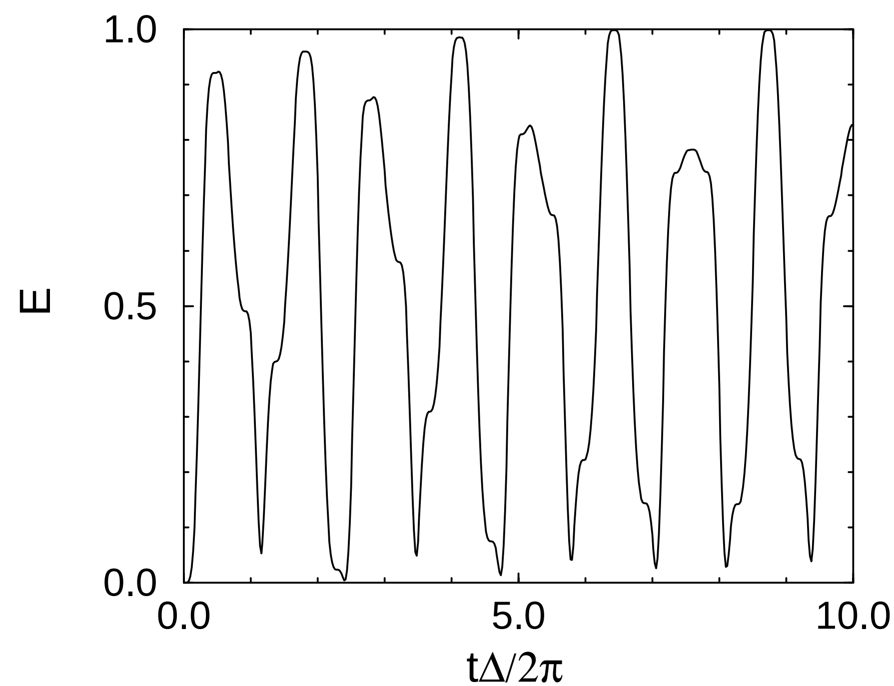
$$E = 1 - \frac{1}{2}\sum_{\pm} p_{\pm} \log_2 p_{\pm}, \quad p_{\pm} = 1 \pm \sqrt{1-r}.$$

Entanglement in the oscillation process

$$r = \frac{1}{2}\Big[1 - \frac{E_m^2}{\Omega^2}\sin^2(\Omega t) - \frac{E_m^2}{\varepsilon^2}\sin^2(\varepsilon t) + \frac{2E_m^4}{\Omega^2\varepsilon^2}\sin^2(\Omega t)\sin^2(\varepsilon t) +$$

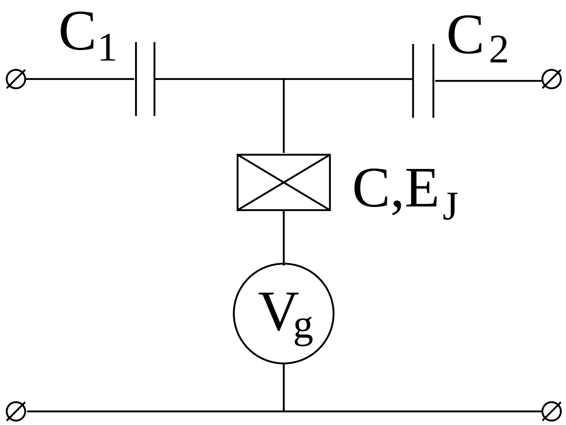
$$+ \frac{\Delta^2 E_m^2}{\Omega^4}\sin^4(\Omega t) + \frac{\delta^2 E_m^2}{\varepsilon^4}\sin^4(\varepsilon t) +$$

$$+ \frac{E_m^2}{2\Omega\varepsilon}\sin(2\Omega t)\sin(2\varepsilon t)\Big]^{1/2}.$$

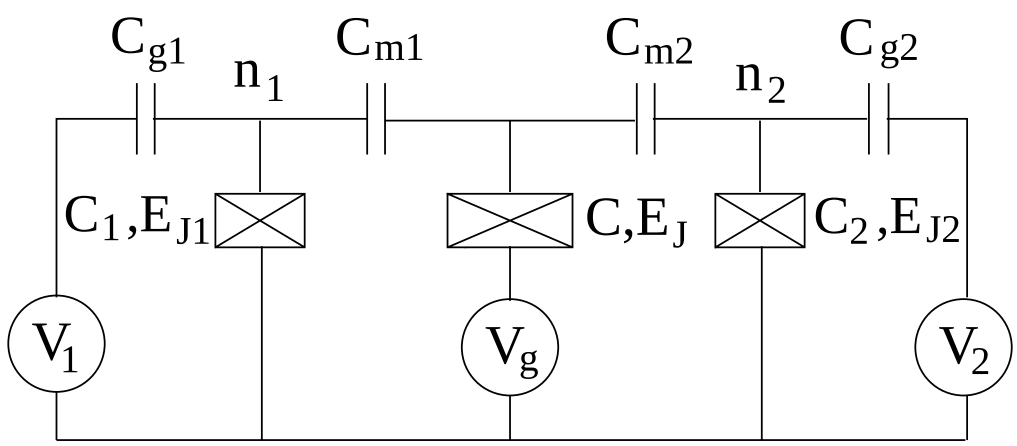


Variable electrostatic transformer: controlled coupling of charge qubits

Equivalent circuit of the transformer



Gate-controlled qubit coupling



coupling capacitance:

$$C \equiv \partial V_{out} / \partial q = \partial^2 E_0(q_g + q) / \partial q^2$$

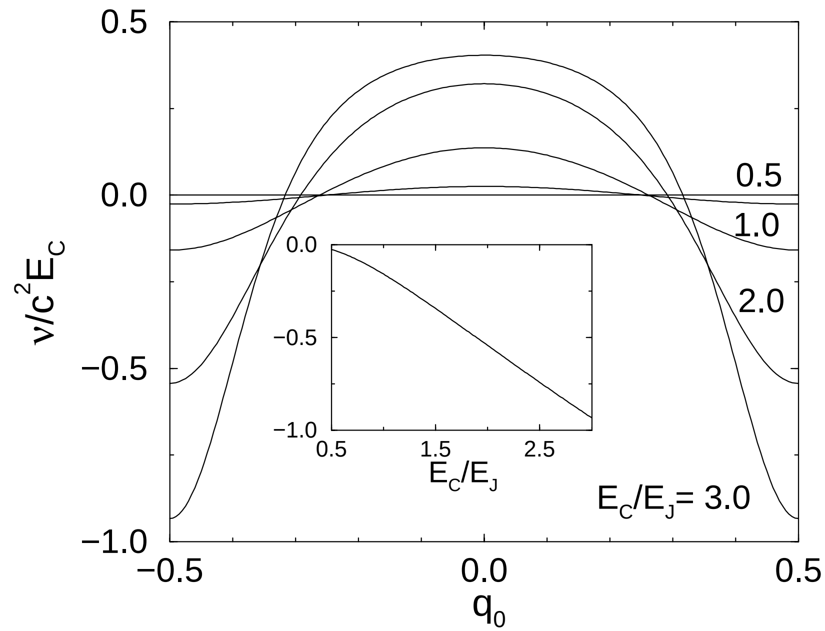
D.V.A. and C. Bruder, Phys. Rev. Lett. **91**, 057003 (2003).

Coupling strength

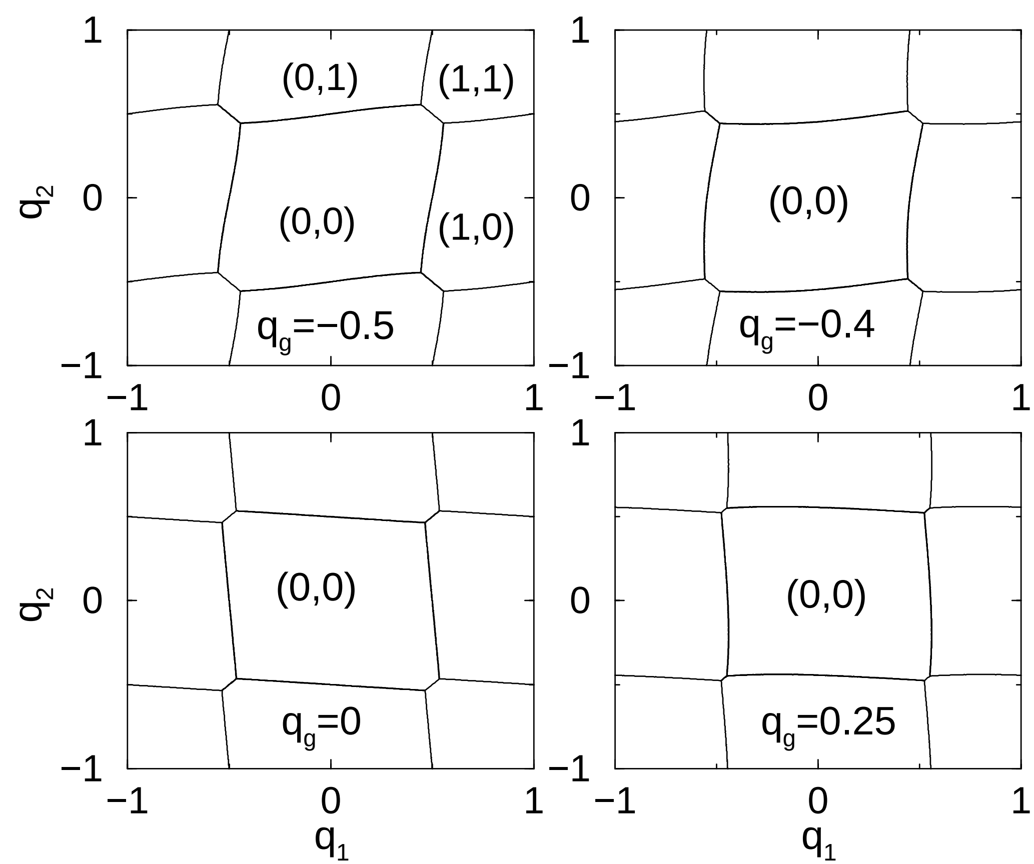
$$H = \nu\sigma_z^{(1)}\sigma_z^{(2)}, \quad c = C_m / C_\Sigma\,,$$

$$\nu = [\varepsilon_0(q_0 + c) + \varepsilon_0(q_0 - c) - \\ - 2\varepsilon_0(q_0 + c)]/4,$$

$$q_0 = q_g + c\sum_{i=1,2}(q_i - 1/2).$$



Charging diagram (transition from positive to negative coupling):



Variable ``quantum'' capacitance

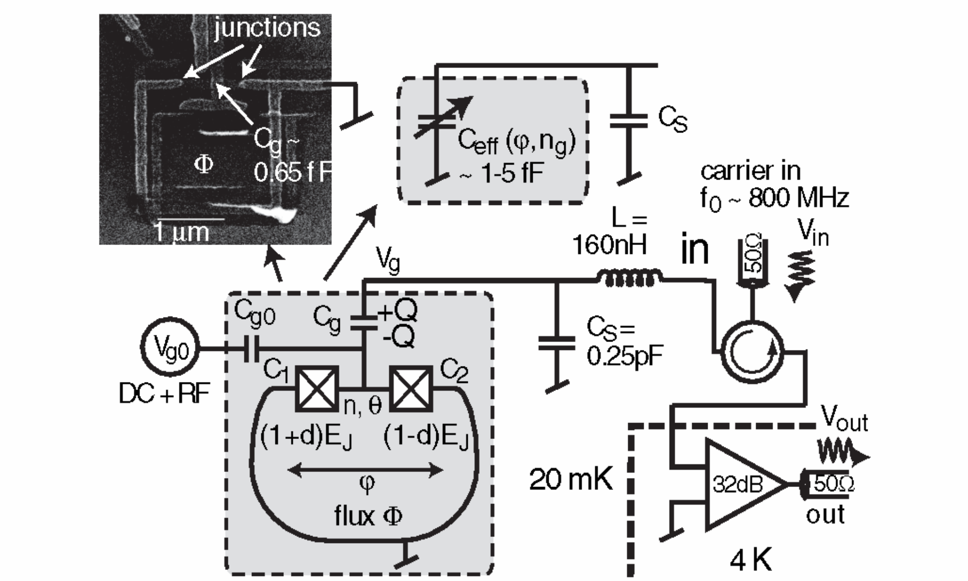
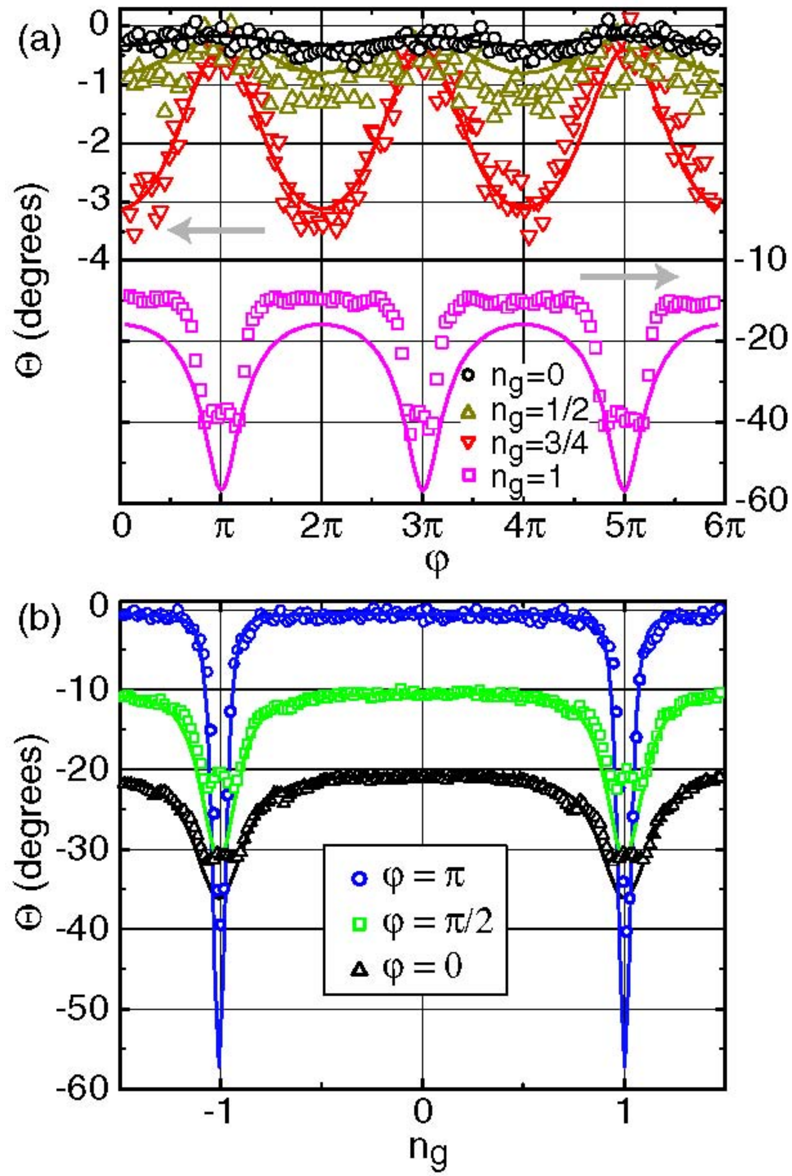


FIG. 1. Schematic view of the experiment. The resonant frequency of the *LC* circuit (made using lumped elements) is tuned by the effective capacitance C_{eff} of the Cooper-pair box shown in the SEM image. For details, see text.



M.A. Sillanpää *et al.*, Phys. Rev. Lett. **95**, 206806 (2005).

Flux quantization in superconductors

Kinetic energy of Cooper pairs:

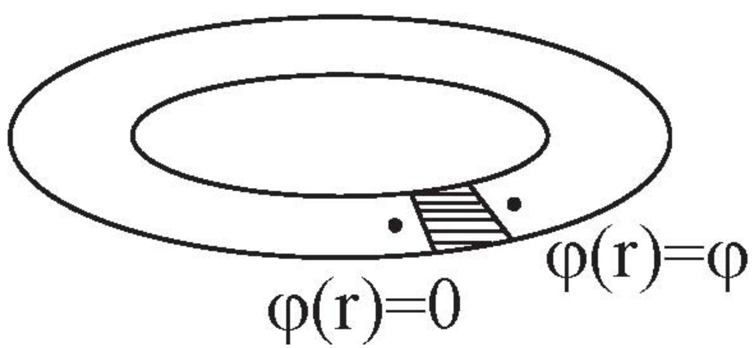
$$E = -\frac{\hbar^2}{2m}\int dr\psi^*(r)[\overline{\nabla} - i(2e/\hbar)\overline{A}]^2\psi(r), \quad \psi(r) = \sqrt{n}e^{i\varphi(r)}.$$

Inside a superconductor, velocity vanishes

$$\overline{\nabla}\varphi(r) - (2e/\hbar)\overline{A} = 0 \quad \Rightarrow \quad \oint dr\overline{\nabla}\varphi(r) = (2e/\hbar)\oint dr\overline{A} = (2e/\hbar)\Phi,$$

Thus, depending on geometry, either $\Phi = n\Phi_0, \quad \Phi_0 \equiv \pi\hbar/e,$

or



$$\varphi = 2\pi\Phi/\Phi_0.$$

Generic superconducting qubits

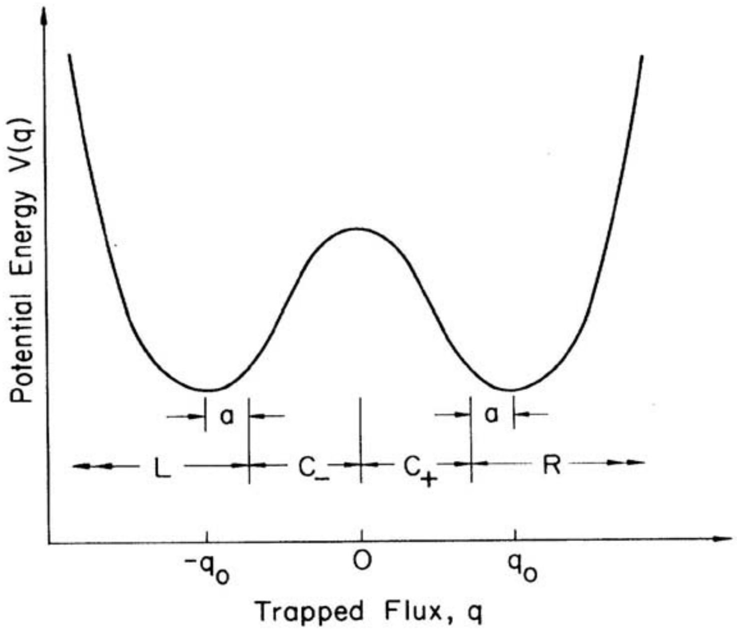
$$H = Q^2/2C - E_J \cos\varphi + (\Phi - \Phi_x)^2/2L,$$

$$\varphi = 2\pi\Phi/\Phi_0 = (2e/\hbar)\Phi, \quad [\Phi, Q] = i\hbar.$$

Qubits: flux, charge-flux, ``phase'' (=energy) .... .

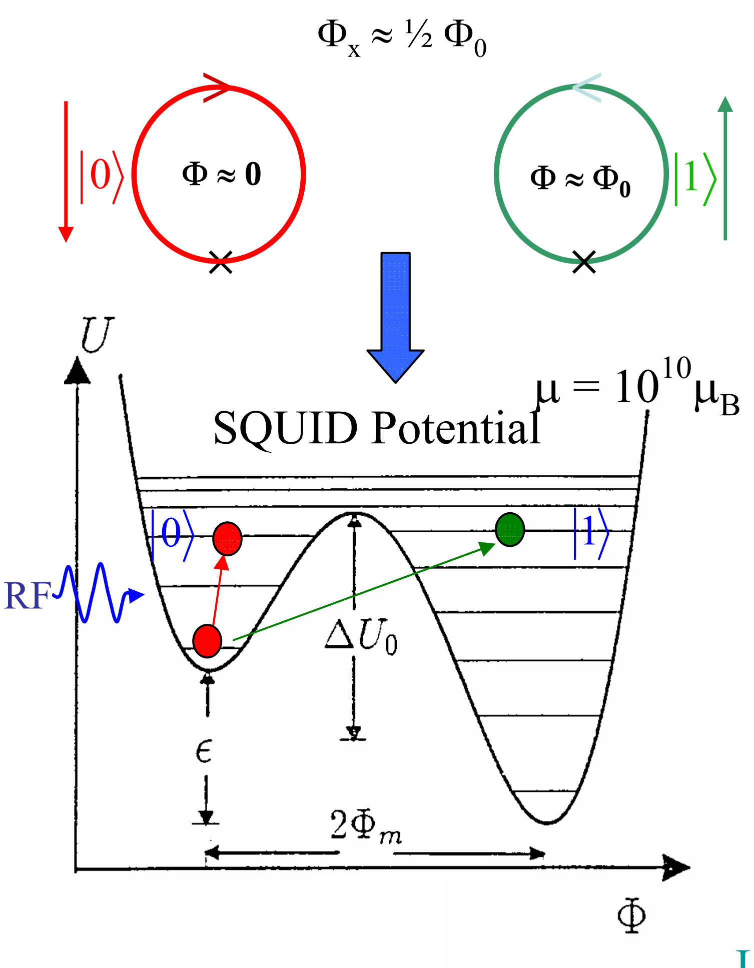
Flux [e.g., A.J. Leggett and A. Garg, PRL **54**, 857 (1985)]

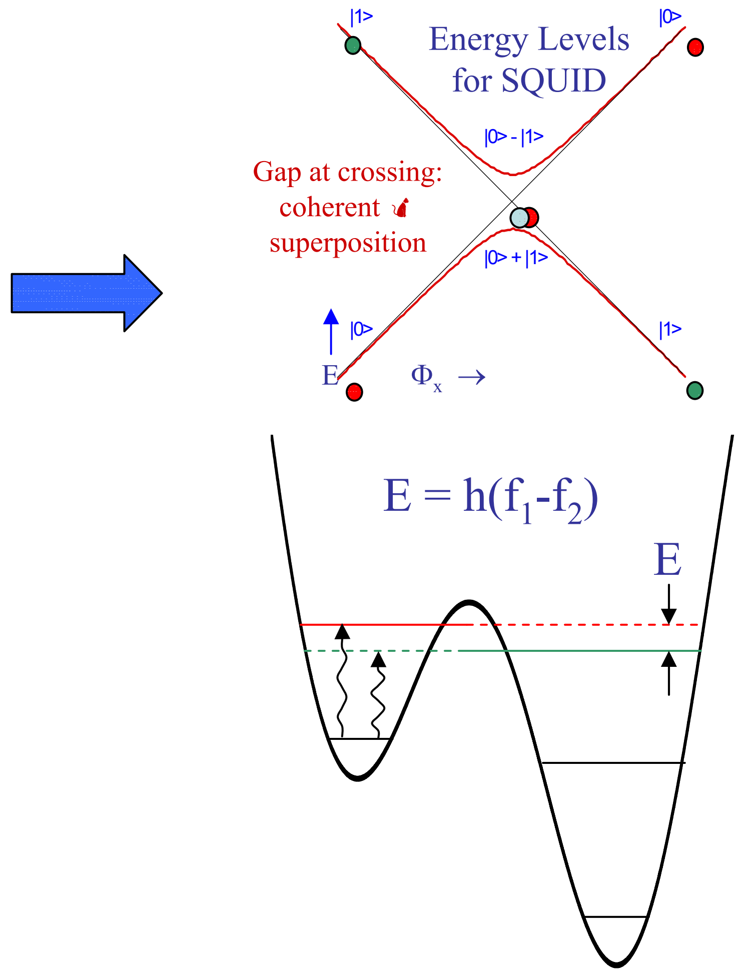
$$\Phi_x \cong \Phi_0/2, \quad \Phi_0{}^2/2L \approx E_J.$$



Observation of Coherent Superposition of Macroscopic Flux States

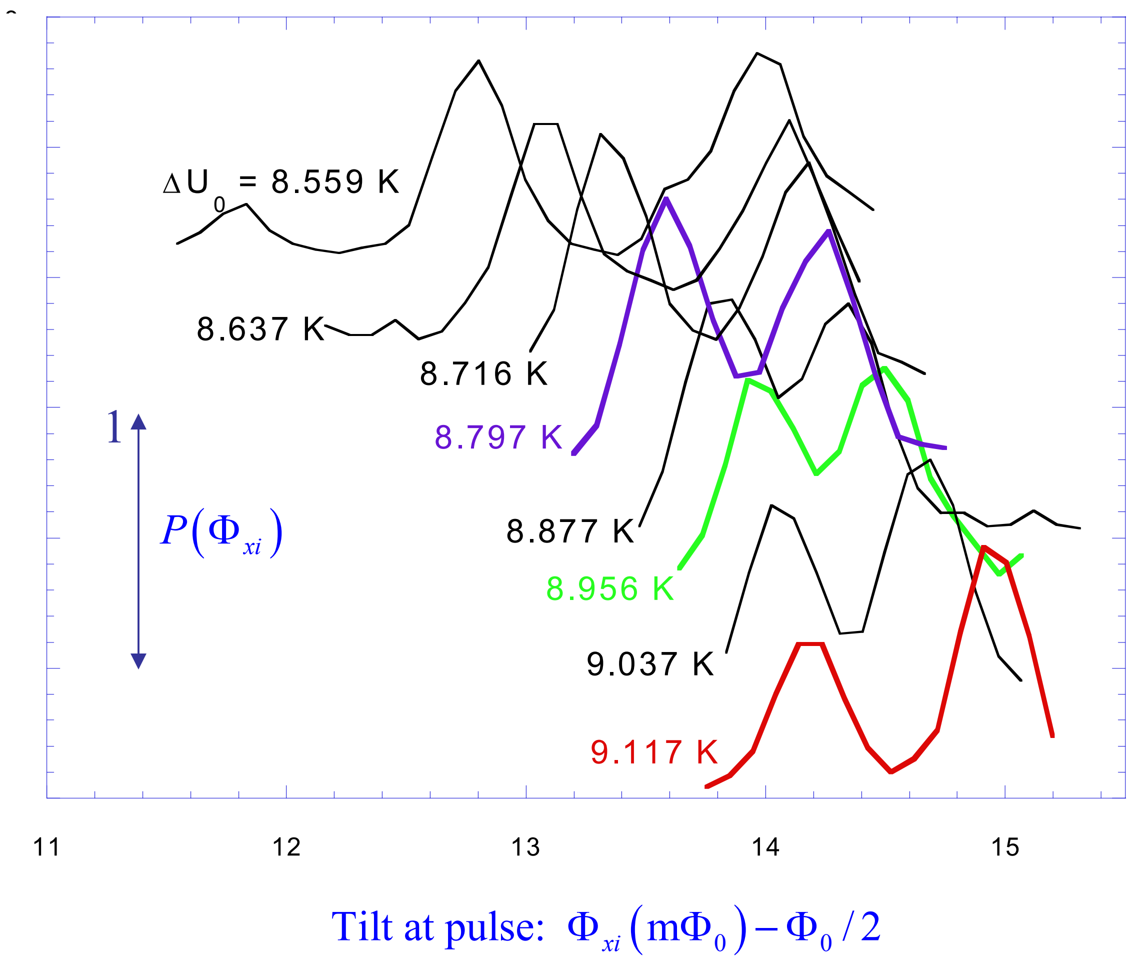
•SQUIDs ``like'' integer flux quanta, Φ_0





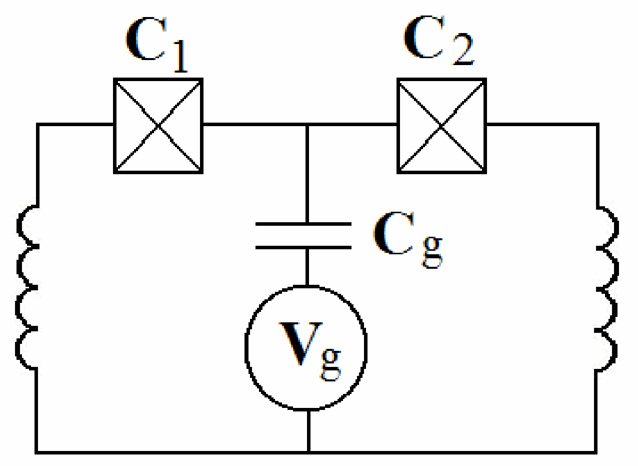
J. Friedman *et al.*, Nature, **406**, 43 (2000).

Resonances Near Anti-crossing for 96 GHz Radiation



Charge/flux qubits

- Bloch transistor in a SQUID loop can serve as a qubit that combines charge and flux dynamics. Advantage of this design is that it provides direct access to more than one dynamic variables of the qubit.



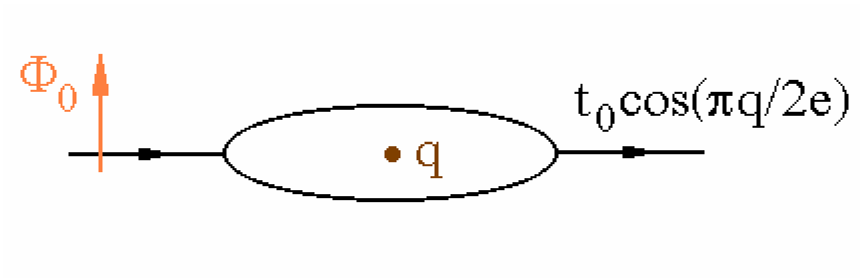
- In the <u>charge regime</u>, $E_J \ll E_C$, $L \to 0$, σ_x corresponds to the current in the SQUID loop,

$$I = I_c/2 \sin(\pi\Phi/\Phi_0)\, \sigma_x.$$

- In the <u>flux regime</u>, $E_J \gg E_C$, $L \approx E_J/\Phi_0^2$, σ_x corresponds to voltage V on the middle electrode,

$$V = (\pi\Delta_0/2e)\sin(\pi q/2e)\, \sigma_x.$$

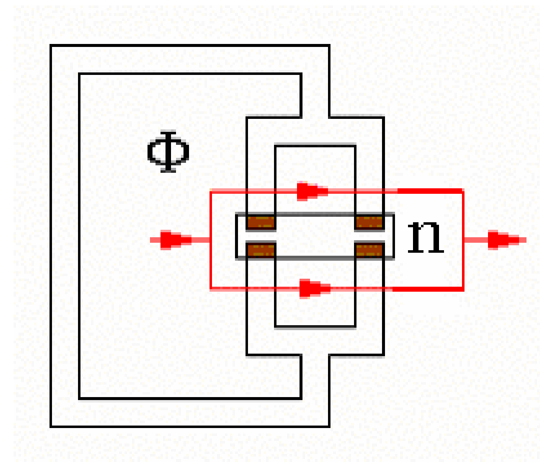
Aharonov-Casher effect



In generic *superconducting arrays* A.-C. effect manifests itself through
- periodic oscillations of conductance with the period 2e in gate-induced charge,
- voltage quantization,

and can not be distinguished qualitatively from the *Coulomb blockade*.

This problem is avoided in the charge/flux qubit, where A.-C. effect leads specifically to suppression of the flux tunneling amplitude at certain gate charge



$$\Delta = \Delta_0 \cos(\pi q / 2e)$$

J.R. Friedman and D.V.A. , PRL **88**, 050403 (2002).

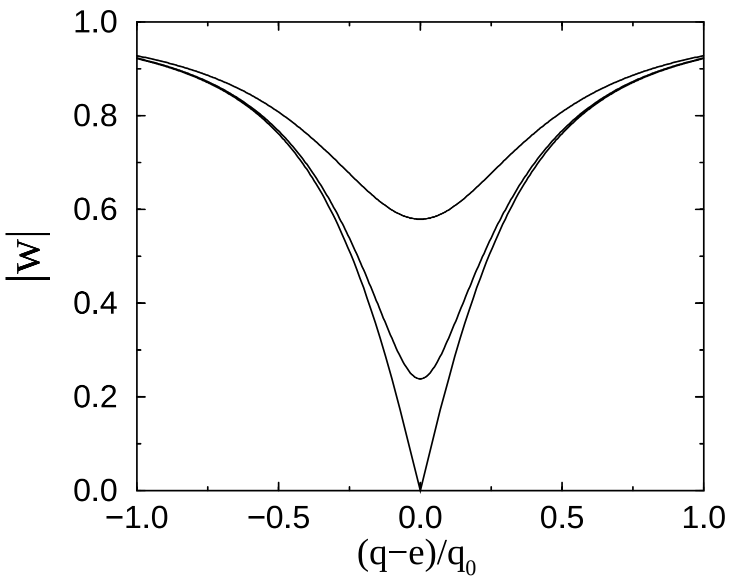
Quantitatively, the Bloch transistor in the SQUID loop exhibits coupled charge and flux dynamics:

$$H = (\Phi-\Phi_e)^2/2L + Q^2/2C_e$$

$$+(2en - q)^2/2C \;\; -2E_J\cos\Theta \cos(\pi\Phi/\Phi_0),$$

$$[n,\Theta] = i, \quad [Q, \Phi] = i\hbar$$

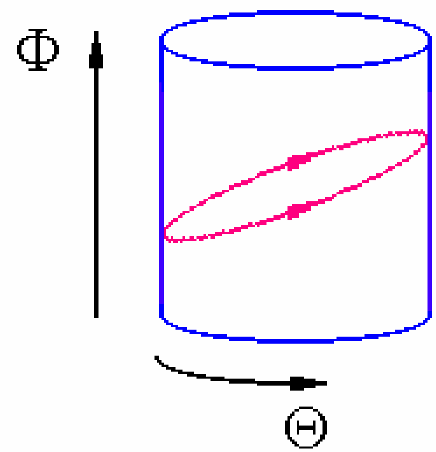
$E_J << E_C$



Imaginary-time Lagrangian:

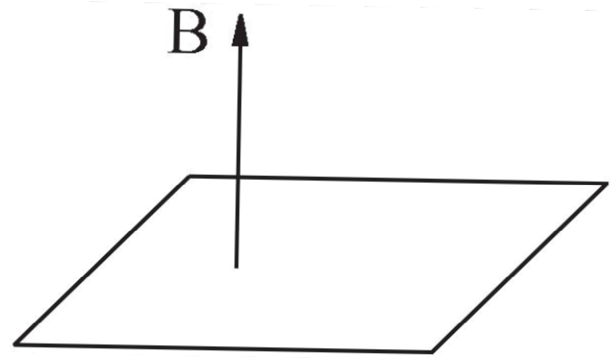
$$L = H - i(\hbar q/2e)d\Theta/d\tau\ ,$$

For $E_J >> E_C$, ``topological'' term in the Lagrangian, evaluated along the two tunneling paths $\Theta \rightarrow \Theta \pm\pi$, gives rise to the A.-C. phase and modulation of the tunneling amplitude.



Fractional Quantum Hall Effect (FQHE)

Wavefunction



$$z = x + iy, \quad \overline{A} = (B/2)\{-y, x\}.$$

$$H = \frac{1}{2m}(\overline{p} - e\overline{A})^2 =$$

$$= \frac{\hbar^2}{2m}[-\nabla^2 + (2ei/\hbar)\overline{A}\,\overline{\nabla} + (eA/\hbar)^2] = \left[z \to (eB/4\hbar)^{1/2} z;\ \omega_C \equiv eB/m\right]$$

$$= (\hbar\omega_C/2)[-\frac{\partial^2}{\partial z^* \partial z} + z^* \frac{\partial}{\partial z^*} - z\frac{\partial}{\partial z} + |z|^2].$$

If one takes $\psi(z, z^*) = \chi(z, z^*)\exp(-|z|^2),$

$$H\chi = (\hbar\omega_C/2)[-\frac{\partial^2 \chi}{\partial z^* \partial z} + 2z^* \frac{\partial \chi}{\partial z^*} + \chi] = E\chi,$$

i.e., for any $\chi(z, z^*) = f(z), \quad E = \hbar\omega_C/2.$

Fully filled lowest Landau level (antisymmetrized w.f-n with all states up to $f(z)=z^N$ occupied)

$$\psi_g(z_1,\dots,z_N)=\prod_{i<j}(z_i-z_j)\exp(-\sum_j |z_j|^2).$$

Laughlin's wavefunction:

$$\psi_g(z_1,\dots,z_N)=\prod_{i<j}(z_i-z_j)^{2m+1}\exp(-\sum_j |z_j|^2).$$

The filling factor: $\nu=1/(2m+1).$

Quasiparticle excitations are quasiholes and quasiparticles.
Quasihole:

$$\psi(\{z_j\},z_0)=\prod_j (z_j-z_0)\cdot\psi_g(\{z_j\}).$$

Fractional statistics of quasiparticles

Geometric phase in cyclic adiabatic evolution

$$H[z(t)]\psi(t) = E(t)\psi(t), \quad \psi(T) = \psi(0)e^{-i\phi+i\gamma},$$

$$\phi = -(i/\hbar)\int_0^T E(t)dt, \quad \gamma = i\oint\langle\psi\,|\,\nabla_z\psi\rangle dz.$$

For a quasihole rotating around

- the origin:

$$\gamma = i\oint dz_0\langle\psi(\{z_j\},z_0)\,|\,\nabla_{z_0}\psi(\{z_j\},z_0)\rangle = i\oint dz_0\int ds\rho(z)\nabla_{z_0}\ln(z-z_0) =$$

$$= -2\pi\nu\Phi/\Phi_0'.$$

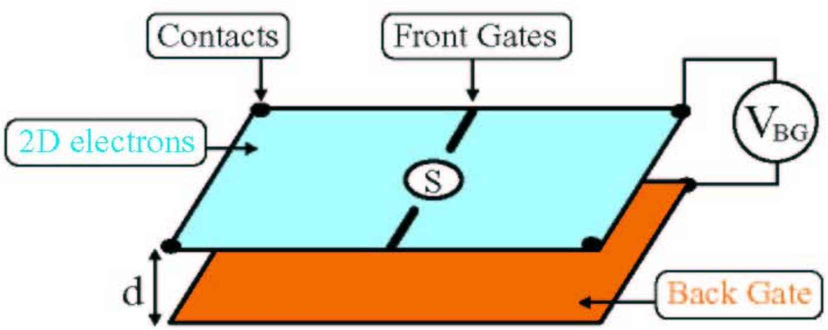
This is manifestation of the fractional quasihole charge νe.

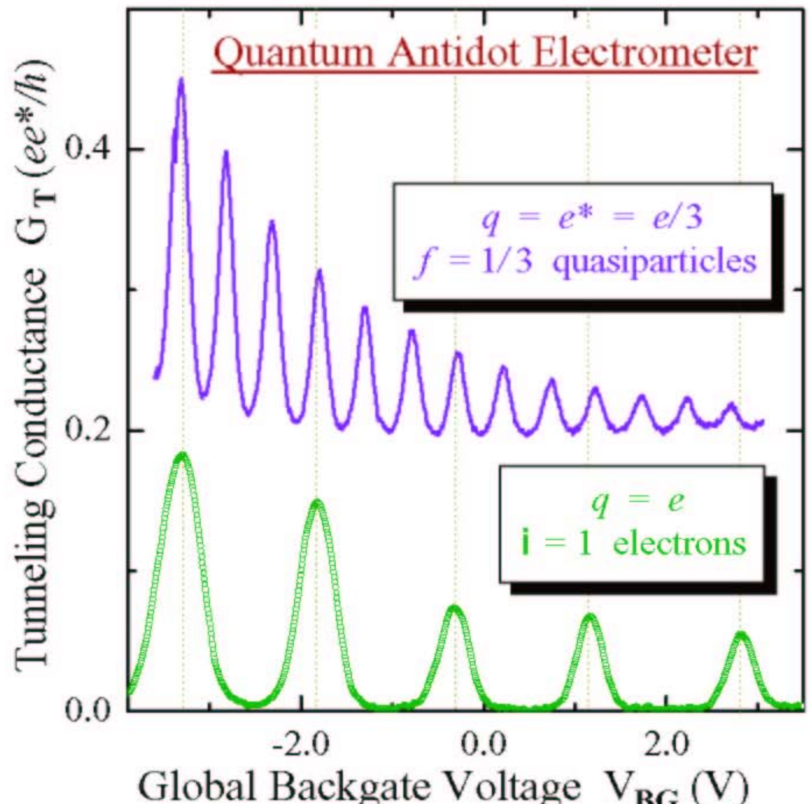
- another quasihole : $\gamma = 2\pi\nu.$ (fractional statistics)

D. Arovas *et al.*, Phys. Rev. Lett. **53**, 722 (1984).

Fractional Charge of Laughlin Quasiparticles

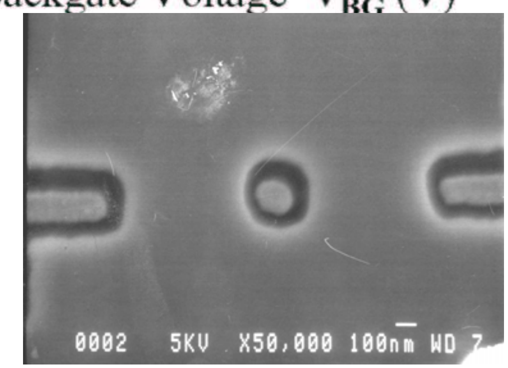
- back gate: $E_\perp = \dfrac{V_{BG}}{d}$
- induced surface charge: $\delta\sigma = \varepsilon\varepsilon_0 E_\perp$
- one particle per ΔV_{BG}
- particle charge : $q = \varepsilon\varepsilon_0 S \,\Delta E_\perp$



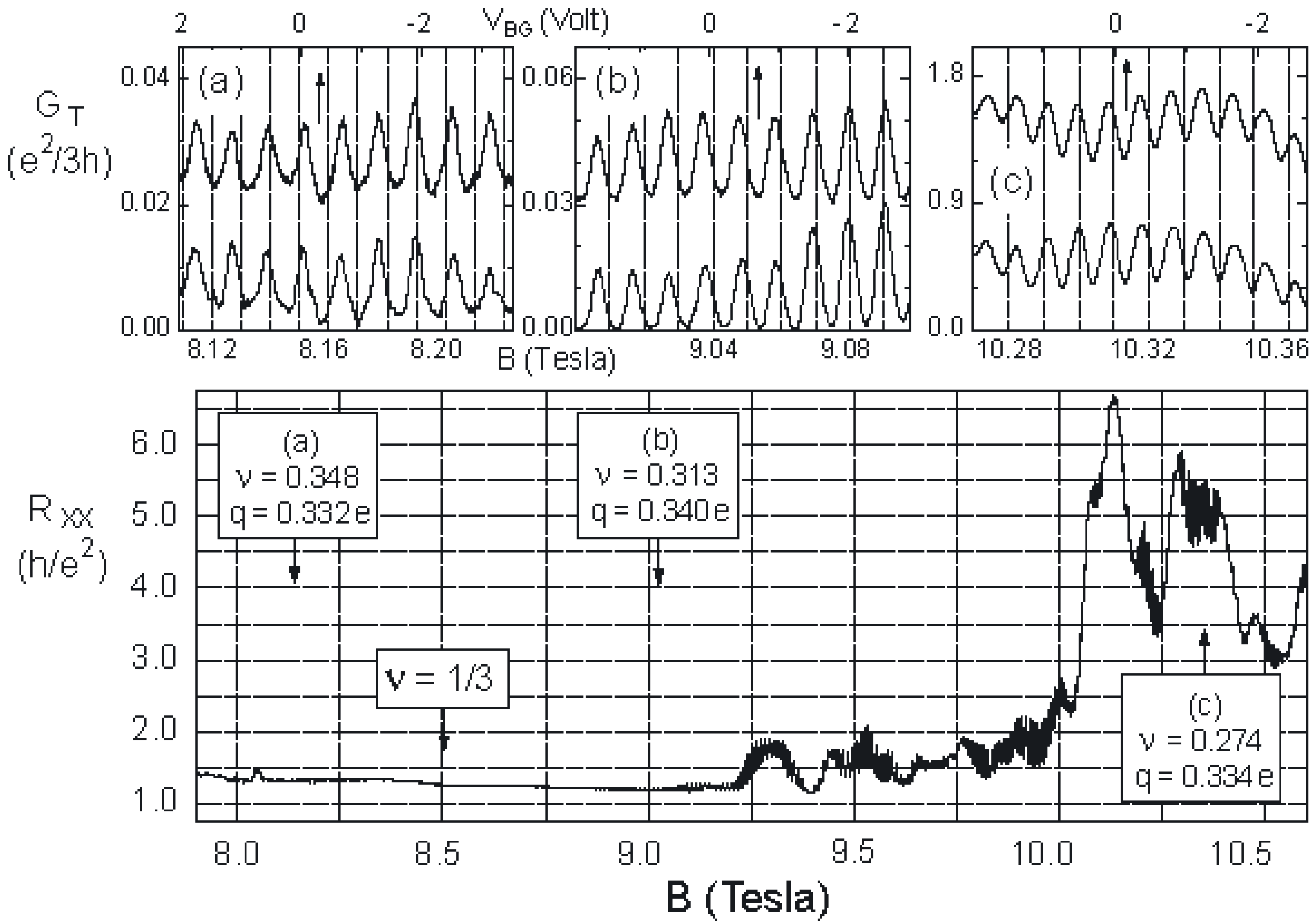


$$q = \frac{\varepsilon\varepsilon_0 \Phi_0'}{d}\,\frac{\Delta V_{BG}}{\Delta B}.$$

V.J. Goldman and B. Su, 1995

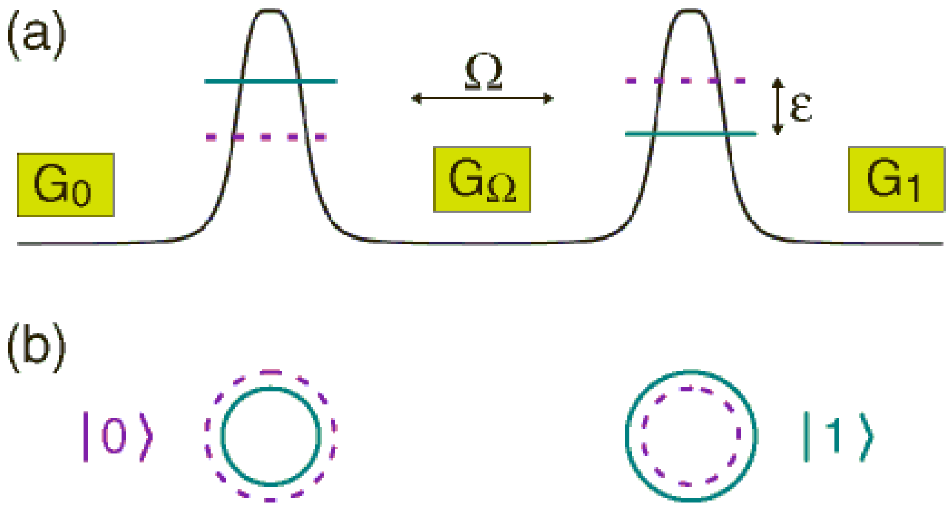


Invariance of the quasiparticle charge



V.J. Goldman *et al*., PRB 64, 085319 (01).

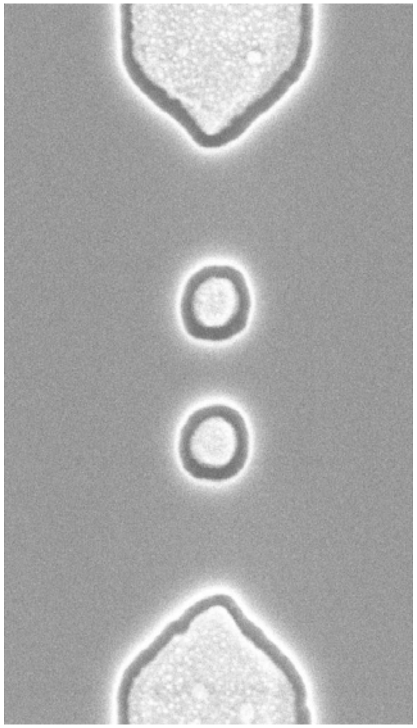
Quantum antidot qubits



G_j- control gates

Ω- inter-QAD tunnel amplitude

ε - quasiparticle "localization" energy

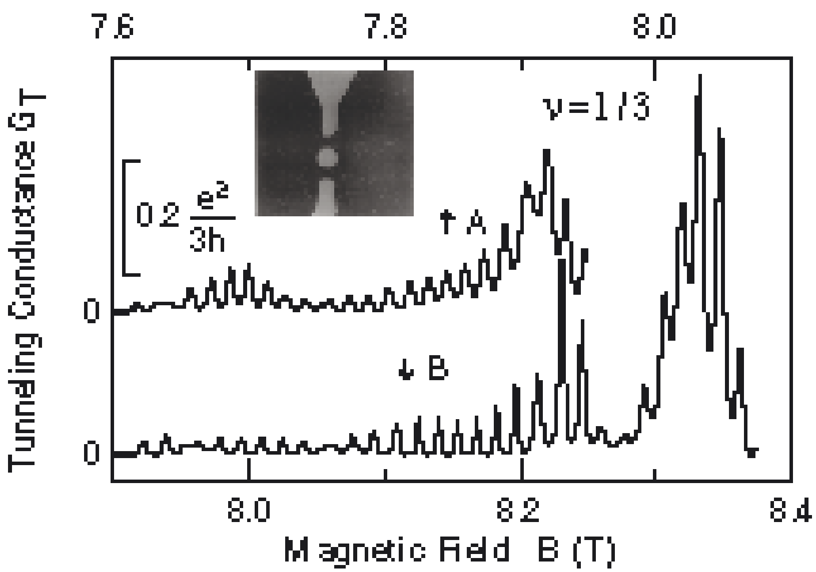


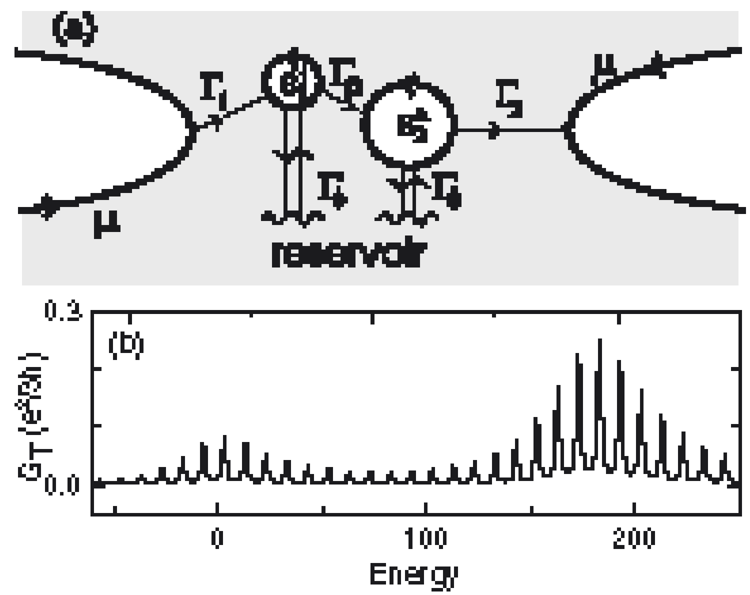
- Information encoded in the position of a quasiparticle in the system of two antidots.
- Adiabatic level-crossing dynamics can be used to transfer quasiparticles between the antidots.
- Conditions of operation: $T \ll \Omega, \varepsilon \ll \Delta E$, ΔE – quasiparticle excitation gap

(D.V.A. and V.J. Goldman, 2001)

Antidot transport of the FQHE quasiparticles

Antidot ``molecule'' demonstrates coherent quasiparticle transport in the double-antidot system

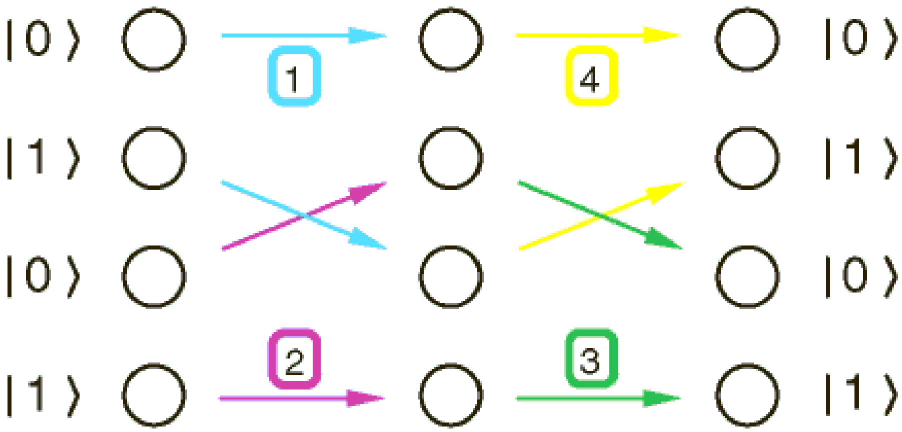




I.J.Maasilta and V.J. Goldman, PRL 84, 1776 (00).

Antidot gates

Controlled-phase gate:



Transformation matrix:

$$P = \mathrm{diag}\,[\,1,\,1,\,1,\,e^{2\pi i/3}\,]\,.$$

Controlled-NOT gate can be constructed of two P-gates combined with single-qubit transformations:

$$C = S(\pi/2)\,(U_-)^\dagger\,S(-\pi/3)\,P\,U_-\,U_+\,S(-\pi/3)\,P\,(U_+)^\dagger\,.$$

where

$$U_\pm = [\mathbf{1}]_1 \otimes [\exp\{-i\varphi(\sigma_x \pm \sigma_y)/2^{1/2}\}]_2\,,\ \cos(2\varphi) = 1/3^{1/2},$$

$$S(\alpha)=[\exp\{i\alpha\sigma_z\}]_1 \otimes [\mathbf{1}]_2$$

Edge states

For large n, the states $\psi(z) = z^n e^{-|z|^2}$ are well-localized at $|z| \approx [n/2]^{1/2}$. If there is a potential $V(|z|)$, they acquire velocity $v = E/B, \quad E = -\nabla V(|z|).$ Under the restriction $|z_j| \approx R$, the wavefunction of Landau level reduces to that of 1D electrons:

$$\psi(\{z_j\}) = \prod_{i<j} (e^{ik_i x_i} - e^{ik_j x_j}); \quad x = \varphi R, \quad k_j = n_j / R.$$

In the fractional regime: $\psi(\{z_j\}) = \prod_{i<j} (e^{ik_i x_i} - e^{ik_j x_j})^{2m+1}.$

Bosonization. Introduce displacement field φ(x), so that:

$$\rho(x) = (\sqrt{\nu}/2\pi)\partial\varphi/\partial x.$$

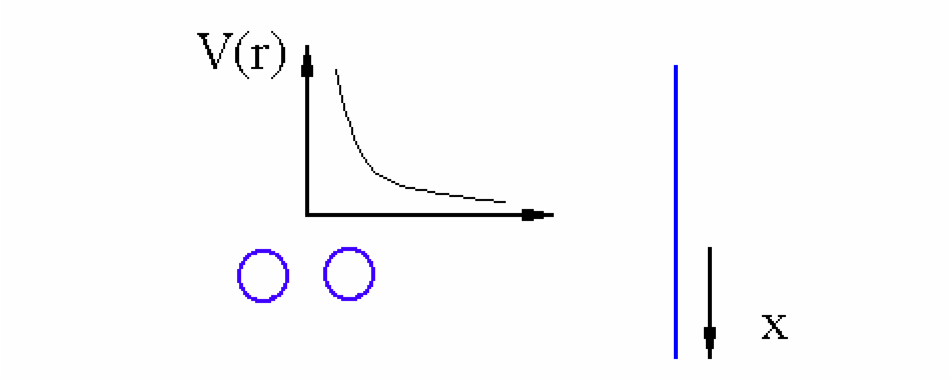
Then $H_0 = (\hbar v/4\pi) \int dx(\partial\varphi/\partial x)^2, \quad [\varphi(x), \varphi(x')] = i\pi \,\mathrm{sign}(x-x'),$

and electron and quasiparticle operators are

$$\psi_e = (2\pi\alpha)^{-1/2}\, \zeta\, e^{i\varphi/\sqrt{\nu}},$$

$$\psi_{qp} = (2\pi\alpha)^{-1/2}\, \xi\, e^{i\sqrt{\nu}\,\varphi}.$$

Edge-state decoherence



As discussed above, the edge supports chiral low-energy plasmon modes propagating with velocity v. Quasiparticle oscillating between the antidots creates the potential V(x) along the edge that excites plasmon modes:

$$H = H_0 + V, \quad V = e\sigma_z \int dx \rho(x) V(x).$$

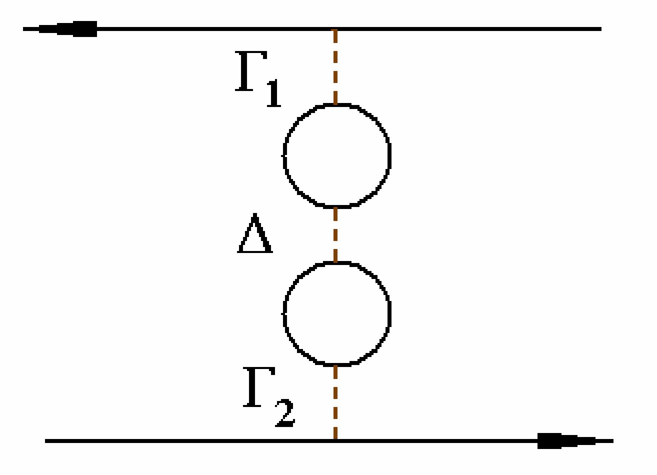
Coupling to the edge creates energy relaxation:

$$\Gamma(\Omega) = (e^2 \nu \Omega / 2\pi\hbar^2) \left| \int du V(u) e^{i\Omega u} \right|^2.$$

For the simplest geometry of the qubit/edge system:

$$\Gamma(\Omega) = (d/L)^2 (\varepsilon^2 / 4\varepsilon\varepsilon_0 \hbar v)^2 (\Omega \nu^3 / 2\pi\hbar) e^{-2L\Omega/\hbar v}.$$

Rate of the antidot-edge tunneling



$H_{tun} = - \Sigma_j [u_j \chi_j^\dagger \psi_j + h.c.]$

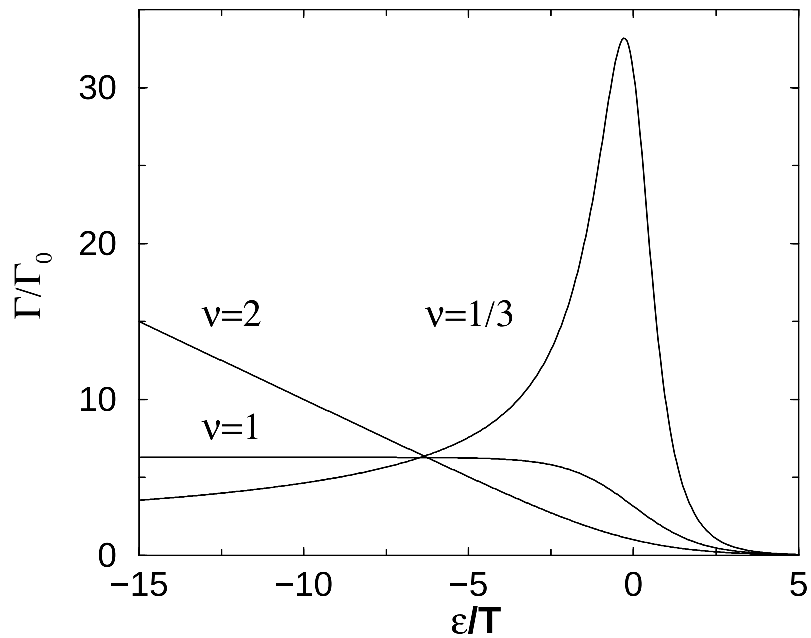
edges: $\psi_j = (2\pi\alpha)^{-1/2} \xi_j e^{i\sqrt{\nu}\phi_j}$,

$[\phi_l(x), \phi_k(y)] = i\pi\delta_{lk} \, \mathrm{sign}(x-y)$.

antidots: hard-core anions χ_j

$$\Gamma_j(\varepsilon) = \frac{2^\nu u_j^2 e^{\varepsilon/2T}}{\Gamma(\nu)\omega_c} \left(\frac{2\pi T}{\omega_c}\right)^{\nu-1} \times$$

$$\times \left| \Gamma\left(\frac{\nu}{2} + \frac{i\varepsilon}{2\pi T}\right)\right|^2 .$$

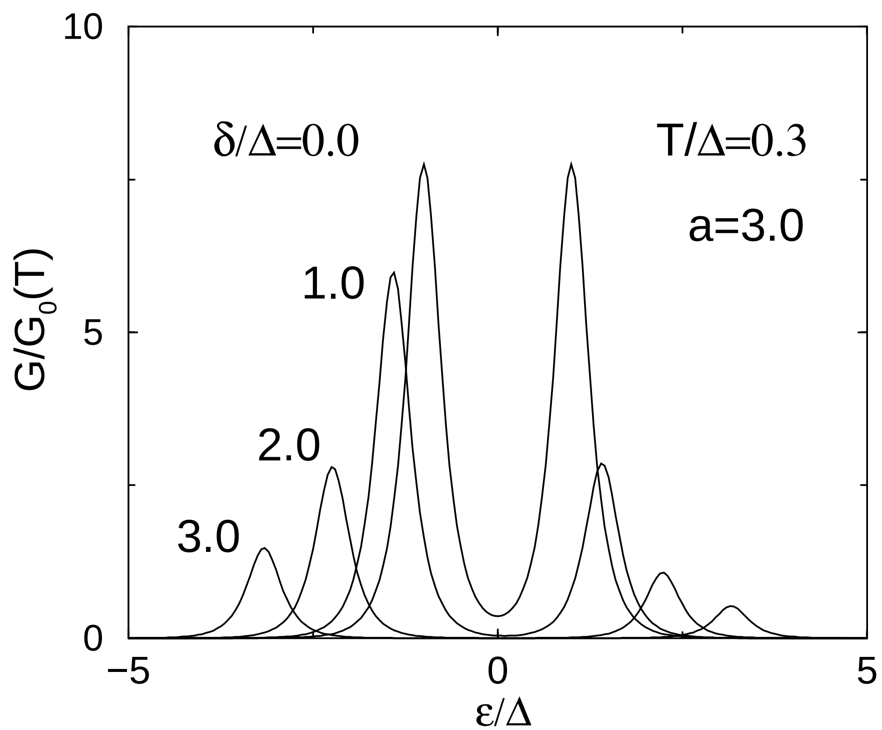


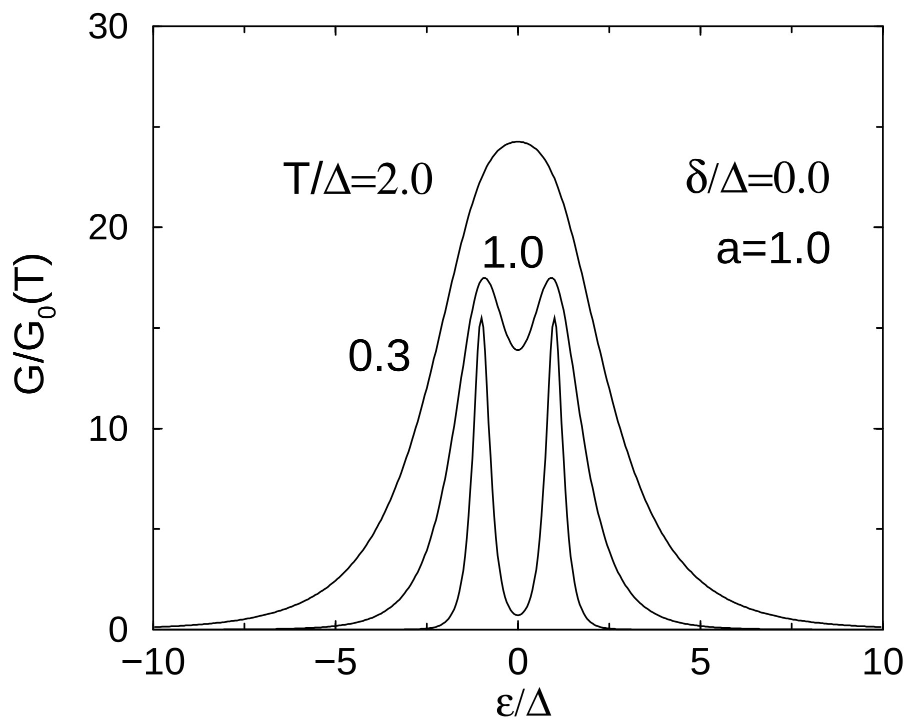
Coherent quasiparticle transport in antidot qubits

Two qubit quasiparticle states: $\varepsilon_{\pm} = \varepsilon \pm \Omega, \quad \Omega \equiv (\delta^2 + \Delta^2)^{1/2}.$

Conductance in the coherent regime, $\Gamma_j << \Delta, T \quad (\gamma_j \equiv u_j^2 / \omega_c):$

$$G = \frac{e^2 \gamma_1 \gamma_2}{2^{2-\nu}\Gamma(\nu) T} \left(\frac{2\pi T}{\omega_c} \right)^{\nu-1} \frac{\Delta^2}{\Omega^2} \sum_{\pm} \left| \Gamma\left(\frac{\nu}{2} + \frac{i\varepsilon_{\pm}}{2\pi T} \right) \right|^2 \frac{\cosh^{-1}(\varepsilon_{\pm}/2T)}{(1 \pm \delta/\Omega)\gamma_1 + (1 \mp \delta/\Omega)\gamma_2}.$$





Decoherence in Two Coupled Qubits

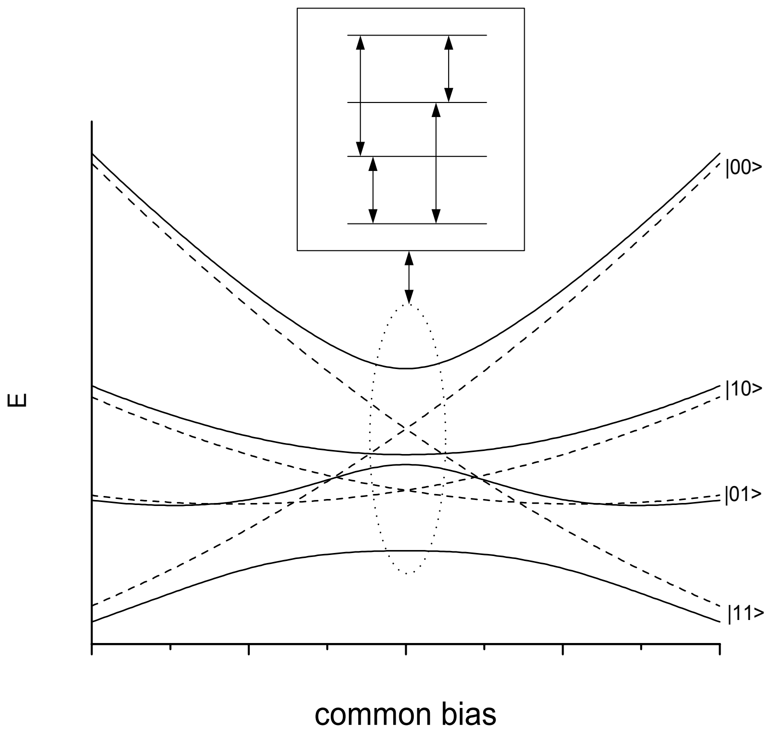
Motivation: Dynamics of coupled qubit system should probe
- the degree of correlation of environmental decoherence forces at different qubits.
- relation between one- and two-qubit decoherence rates.

Hamiltonian at ``co-resonance'' is:

$$H = \nu\sigma_z^{(1)}\sigma_z^{(2)} + \sum_{j=1,2}(\Delta_j\sigma_x^{(j)} + \xi_j\sigma_z^{(j)}),$$

with eigenenergies:

$$\{-\Omega, -\varepsilon, \varepsilon, \Omega\}, \quad \Omega = [(\Delta_1+\Delta_2)^2 + \nu^2]^{1/2},$$
$$\varepsilon = [(\Delta_1-\Delta_2)^2 + \nu^2]^{1/2}.$$



We use the standard evolution equation for the system density matrix in the case of weak decoherence:

$$\dot{\rho} = -\int_{-\infty}^{t}\langle[V(t),[V(\tau),\rho]]\rangle d\tau, \qquad V \equiv \sum_{j=1,2}\xi_j\sigma_z^{(j)}.$$

Generic effects of weak decoherence

For weak decoherence, the energy basis plays special role: density matrix is diagonal in it in the stationary regime. For generic system with only accidental resonances in the energy spectrum, weak decoherence has universal effects in the energy basis.

Diagonal terms:

$$\dot{\rho}_{ii} = \text{``Golden rule'' transitions with rates } \Gamma_{jk}.$$

Off-diagonal terms:

pure dephasing natural linewidth

$$\dot{\rho}_{ij} = -\rho_{ij}\left[\ldots S(0)(V_{ii}-V_{jj})^2 + \sum_k (\Gamma_{ik}+\Gamma_{jk})/2\right] +$$
$$+\ \text{resonant transfer of coherences}$$

Symmetric qubits (the same Δ's and single-qubit decoherence rates):

$$\psi(0) = \left|00\right\rangle$$

$$p_1(t) = \frac{1}{2} - \frac{1}{4}\left[(1+\frac{\nu}{\Omega})e^{-\Gamma_{13}t}\cos(\omega_- t) + (1-\frac{\nu}{\Omega})e^{-\Gamma_{32}t}\cos(\omega_+ t) \right], \qquad \omega_\pm = \Omega \pm \nu,$$

$$\Gamma_{13} = 2\gamma(\omega_+)(1-\frac{\nu}{\Omega}) + [\gamma(\omega_-)+\gamma_c(\omega_-)](1+\frac{\nu}{\Omega}), \quad \Gamma_{32} = [\gamma(\omega_+)+\gamma_c(\omega_+)](1-\frac{\nu}{\Omega}).$$

$$\psi(0) = \left|10\right\rangle$$

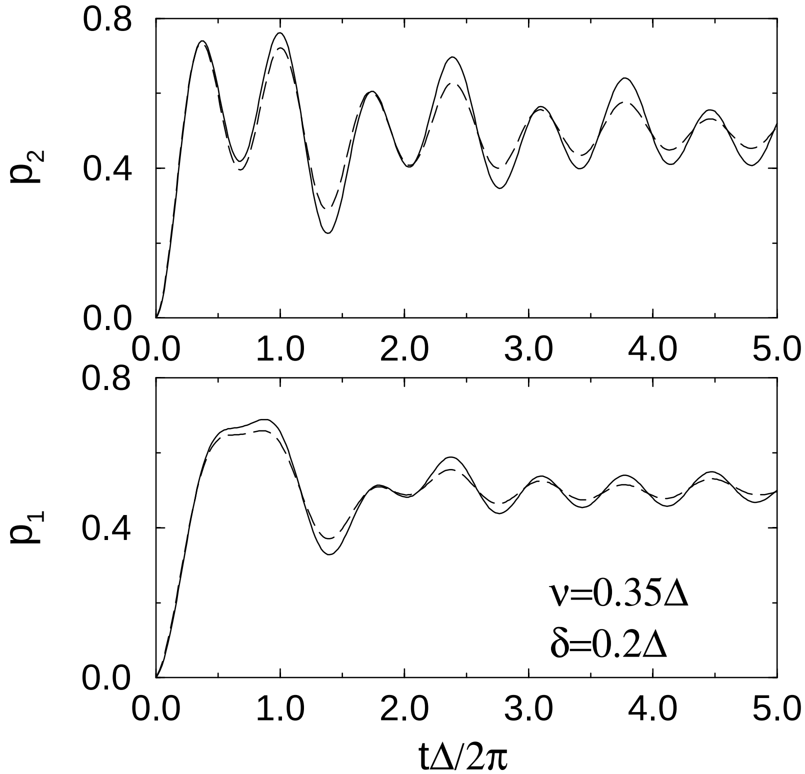
$$p_1(t) = \frac{1}{2} + \frac{1}{4}\left[(1+\frac{\nu}{\Omega})e^{-\Gamma_{42}t}\cos(\omega_- t) + (1-\frac{\nu}{\Omega})e^{-\Gamma_{14}t}\cos(\omega_+ t) \right],$$

$$\Gamma_{14} = 2\gamma(\omega_-)(1+\frac{\nu}{\Omega}) + [\gamma(\omega_+)-\gamma_c(\omega_+)](1-\frac{\nu}{\Omega}), \quad \Gamma_{42} = [\gamma(\omega_-)-\gamma_c(\omega_-)](1+\frac{\nu}{\Omega}).$$

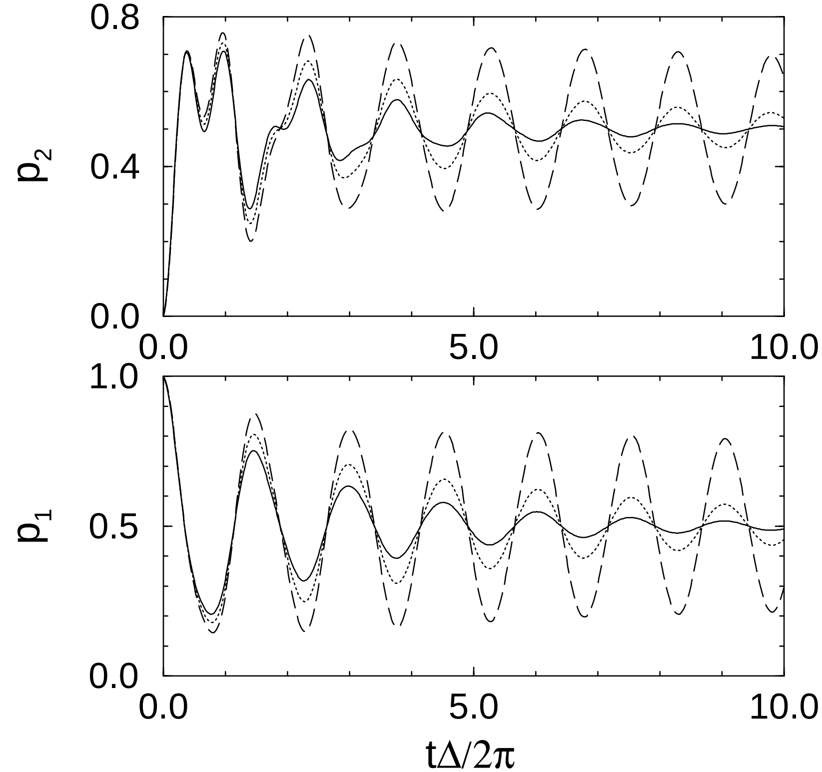
Conclusion: For qubit oscillations excited into the ``*decoherence-free'' subspace* {|10>,|01>} (insensitive to decoherence for symmetric coupling to environment in the case of stationary qubits, *H=0*) the rate of oscillation decay should measure directly degree of correlation of decoherence at the two qubits.

Oscillations in two qubits for different degrees of decoherence correlations:

$\psi(0) = |00\rangle$



$\psi(0) = |10\rangle$



K. Rabenstein and D.V.A. (2003).

Effects of weak noise

$$H = -\tfrac{1}{2}\left[\Delta\sigma_x + \left(\varepsilon + \mathrm{v}(\mathrm{t})\right)\sigma_z\right]$$

$$\left\langle \mathrm{v}(\mathrm{t})\mathrm{v}(\mathrm{t}')\right\rangle = \mathrm{v}_0^2 e^{\frac{-|t'-t|}{\tau}},$$

$$\mathrm{S_v}(\omega) = \frac{2\mathrm{v}_0^2\tau}{1+(\omega\tau)^2}.$$

- *transitions* are well described by the lowest-order perturbation theory

$$\Gamma = S(\Omega), \quad \Omega = \sqrt{\Delta^2 + \varepsilon^2}$$

- *"pure" dephasing:*

$$\rho(\mathrm{t}) = \mathrm{F}(\mathrm{t})\rho(0), \quad \mathrm{F}(\mathrm{t}) = \left\langle \exp\left\{ -i\int_0^t \left[\frac{\varepsilon \mathrm{v}(\mathrm{t}')}{\Omega} + \frac{\Delta^2 \mathrm{v}^2(\mathrm{t}')}{2\Omega^3}\right] d\mathrm{t}' \right\} \right\rangle$$

$$\mathrm{v}_0^2/\Delta^2 \to 0, \quad \tau\Delta \to \infty, \quad \tau\Delta\left(\mathrm{v}_0^2/\Delta^2\right) = const$$

Gaussian Noise

Transition probability:

$$p(\mathrm{v}_1,\mathrm{v}_2,\delta t)=\left[2\pi \mathrm{v}_0^2\left(1-\exp\left(-2\delta t/\tau\right)\right)\right]^{-1/2}$$
$$\exp\left\{-\left(\mathrm{v}_2-\mathrm{v}_1\exp\left(-\delta t/\tau\right)\right)^2/\left[2\mathrm{v}_0^2\left(1-\exp\left(-2\delta t/\tau\right)\right)\right]\right\}$$

Specific noise realization:

$$p_0\left(v_1\right)p\left(v_1,v_2,\delta t_1\right)p\left(v_2,v_3,\delta t_2\right)\cdots$$

Average over the noise can then be written in the form of the path integral :

$$\langle\cdots\rangle=\int d\mathrm{v}(0)d\mathrm{v}(\mathrm{t})D\mathrm{v}(\mathrm{t}')[\cdots]\times\exp\left\{-\frac{\mathrm{v}(0)^2+\mathrm{v}(\mathrm{t})^2}{4\mathrm{v}_0^2}-\frac{1}{4\mathrm{v}_0^2\tau}\int_0^t dt'\left(\tau^2\dot{\mathrm{v}}^2+\mathrm{v}^2\right)\right\}$$

Non-perturbative dephasing by Gaussian noise (I)

$$F(t) = e^{-t/2\tau}\left[\cosh z + \left((1+\lambda^2)\big/2\lambda\right)\sinh z\right]^{-1/2}$$
$$\times \exp\left\{-\left(\frac{\varepsilon v_0 \tau}{\Omega\lambda}\right)^2\left[\frac{t}{\tau} - \frac{2}{\lambda}\frac{1}{\coth(z/2)+\lambda}\right]\right\},$$

$$z = \lambda t\big/\tau, \quad \lambda = [1 + is(\Delta/\Omega)^3]^{1/2}, \quad s = 2v_0^2\tau\big/\Delta.$$

K. Rabenstein *et al.*,
JETP Lett. **79**, 646 (2004).

Non-perturbative dephasing by Gaussian noise (II)

Optimal bias point (ε=0):

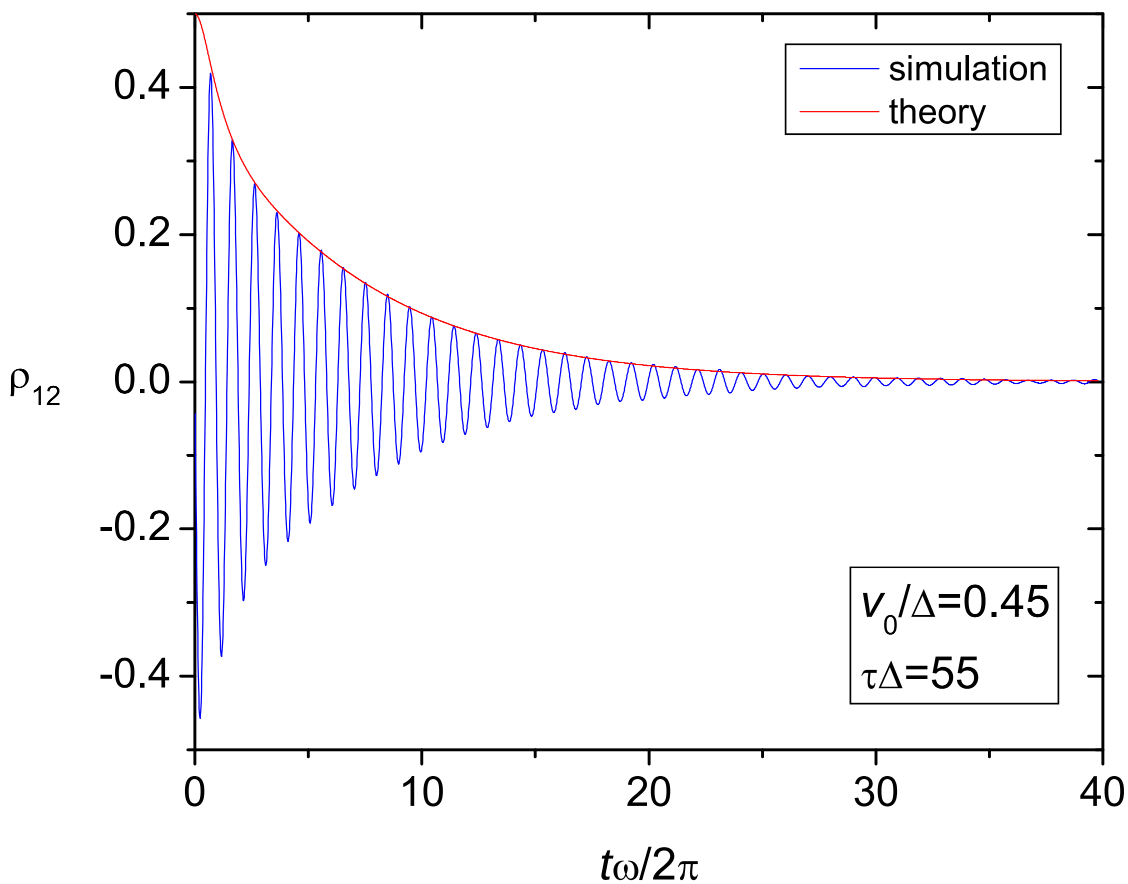
$$F(t) = \begin{cases} \dfrac{2\sqrt{\lambda}}{1+\lambda} \exp[-(\Gamma + i\delta\omega)t], & t >> \tau, \\ \left(\dfrac{1+t/\tau}{1+(1+is)t/\tau} \right)^{1/2}, & t << \tau. \end{cases}$$

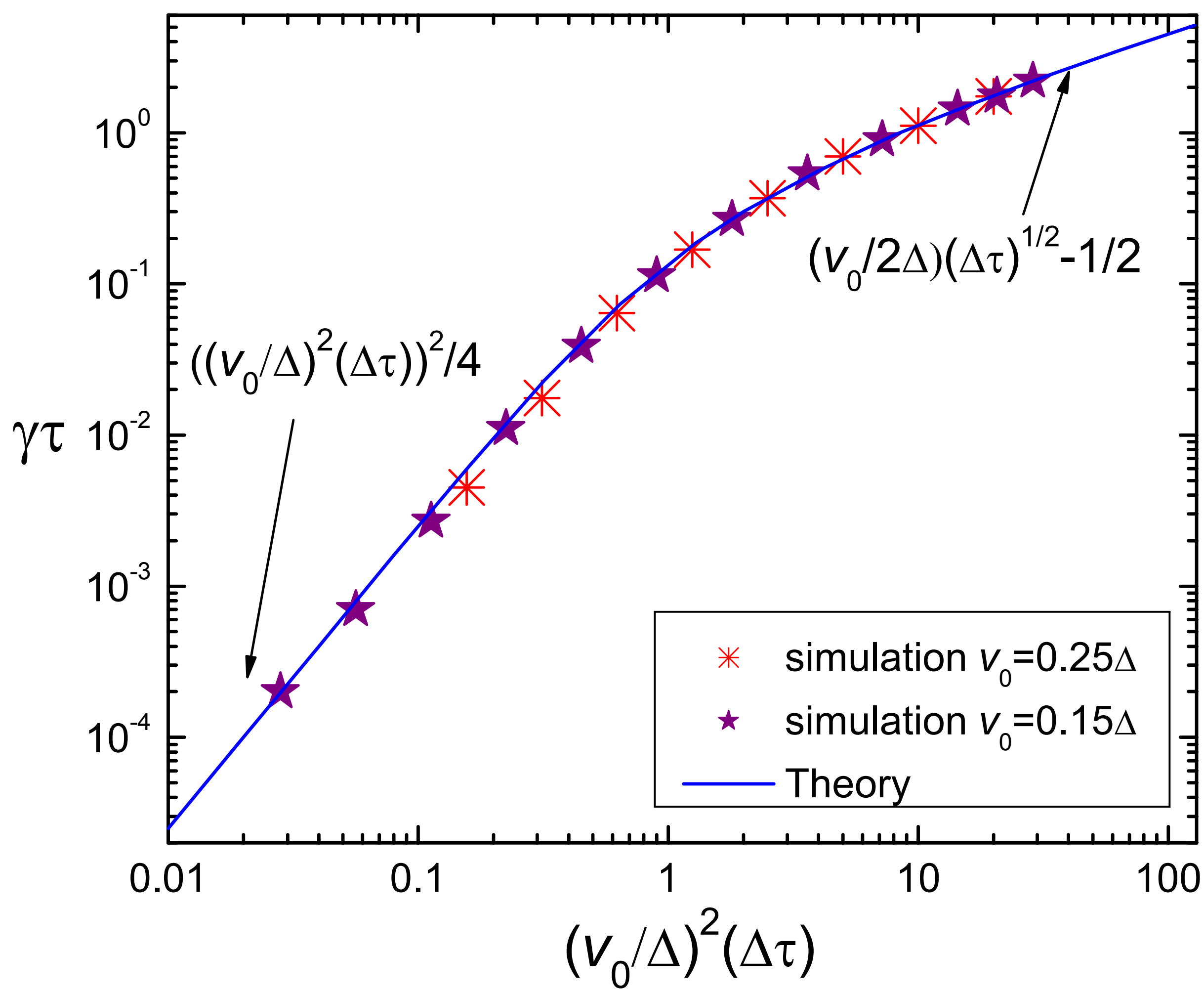
Long-time exponential decay:

$$\Gamma = \frac{1}{2\tau}\left(\left[\frac{(1+s^2)^{1/2}+1}{2} \right]^{1/2} - 1 \right) = \begin{cases} s^2/4\tau, & s << 1, \\ (\sqrt{s/2}-1)/2\tau, & s >> 1. \end{cases}$$

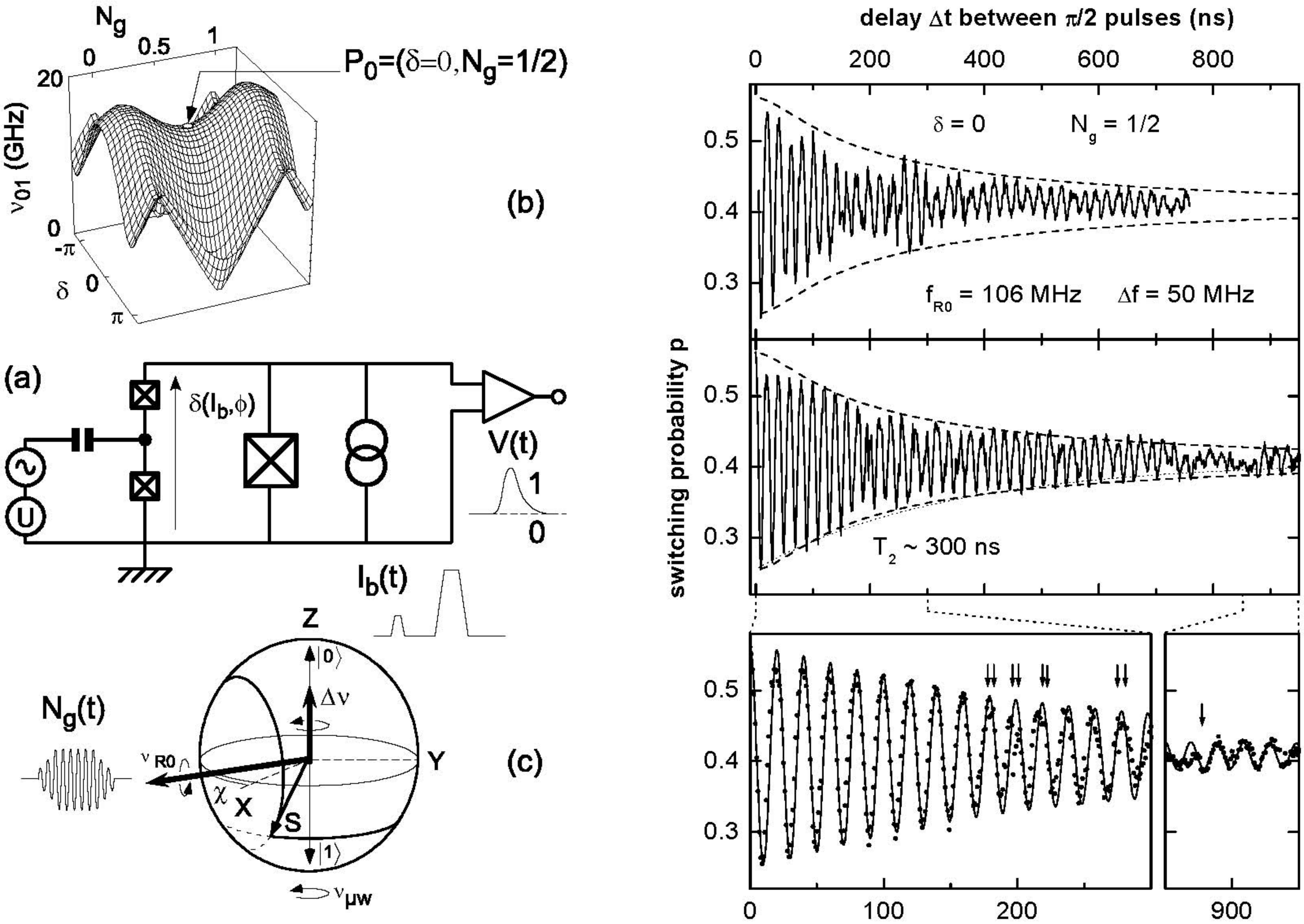
$$\delta\omega = \frac{1}{2\tau}\left[\frac{(1+s^2)^{1/2}-1}{2} \right]^{1/2} = \begin{cases} s/4\tau, & s << 1, \\ \sqrt{s/2}/2\tau, & s >> 1. \end{cases}$$

Simulations of the quantum coherent oscillations (Gaussian noise)





$\gamma\tau$
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.01
0.1
1
10
100
$(v_0/\Delta)^2(\Delta\tau)$
$((v_0/\Delta)^2(\Delta\tau))^2/4$
$(v_0/2\Delta)(\Delta\tau)^{1/2}$-1/2
simulation v_0=0.25Δ
simulation v_0=0.15Δ
Theory



G. Ithier *et al.*, (2005).

Wave-function reduction in quantum measurements (1)

Quantum mechanics:

1. ψ – amplitude of probability
2. dynamic evolution following Schrödinger equation

   1 is ``more important'' than 2

Example: Power of QC

Many algorithms based on quantum Fourier transform

$$\Psi(x) = e^{i(2\pi/N)qx} \Rightarrow \Psi(p) = \delta(p-q); \quad x, p \in 0,1,..N = 2^n.$$

Only for probability wave one immediately finds one value p=q out of 2^n possibilities.

Wave-function reduction in quantum measurements (2)

``Textbook'' description:

$$\Psi_S \Psi_D(0) = (\sum_j a_j |j\rangle)\Psi_D(0) \Rightarrow \sum_j a_j |j\rangle \Psi_D^{(j)}(t) \Rightarrow |j_0\rangle \Psi_D^{(j_0)}(t)$$

with probability $|a_{j_0}|^2$

More subtle (and correct) view:

$$|\Psi_S\rangle \Rightarrow E_\lambda |\Psi_S\rangle / \langle \Psi_S | E_\lambda^+ E_\lambda | \Psi_S \rangle^{1/2}, \qquad \sum_\lambda E_\lambda^+ E_\lambda = 1.$$

Explicitly:

$$|\Psi_S\rangle = \sum_j a_j |j\rangle \Rightarrow |\widetilde{\Psi}_S\rangle = \sum_j a_j t_j |j\rangle / \left[\sum_j |a_j t_j|^2\right]^{1/2}.$$

This procedure is known as:

- Lüders postulate (G. Lüders, 1951)
- the moral aspect of quantum mechanics
  (E.P. Wigner, 1964; J.S. Bell and M. Naugenberg, 1966)
- POVM (A. Peres, ``Quantum theory'', 1993)

Was applied to solid state qubits:

A.N. Korotkov, PRB 1999; D.V.A. et al., PRB 2006.

Used indirectly through ``temporal'' Bell inequalities:

A. Garg and A.J. Leggett, 1985

A.N. Korotkov and D.V.A., 2001

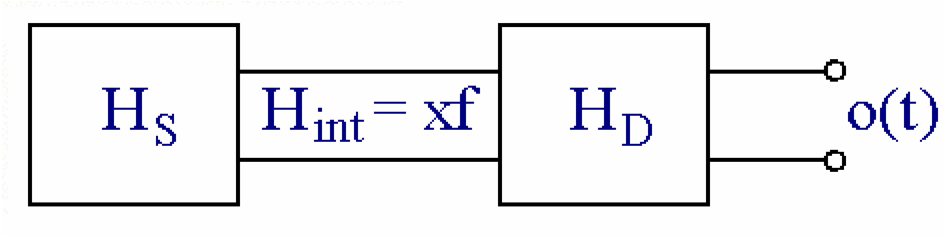
R. Ruskov, A.N. Korotkov, and A. Mizel, 2005

A.N. Jordan, A.N. Korotkov, and M. Buttiker, 2005

Linear quantum measurements

Linear-response theory enables one to develop quantitative description of the quantum measurement process with an arbitrary detector provided that it satisfies some general conditions:

- the detector/system coupling is weak so that the detector's response is linear;
- the detector is in the stationary state;
- the response is instantaneous.



$$H = H_S + H_D + xf$$

D.V.A., cond-mat/00044364, cond-mat/0301524.
S.Pilgram and M. Büttiker, PRL 89, 200401 (2002).
A.A. Clerk, S.M. Girvin, and A.D.Stone, cond-mat/0211001.

Information/back-action trade-off in quantum measurements

Dynamics of the measurement process consists of information acquisition by the detector and back-action dephasing of the measured system. The trade-off between them has the simplest form for measurements of the static system with $H_S=0$. If $x|j\rangle=x_j|j\rangle$, we have for the *back-action dephasing*:

$$\rho_{jj'}(t)=\rho_{jj'}(0)e^{-\Gamma_d t}, \qquad \Gamma_d=\pi(x_j-x_{j'})^2 S_f/\hbar^2 .$$

Information acquisition by the detector is the process of distinguishing different levels of the output signal $\langle o\rangle=\lambda x_j$ in the presence of output noise S_q. The signal level (and the corresponding eigenstates of x) can be distinguished on the time scale given by the by the measurement time τ_m:

$$\tau_m=8\pi S_q/[\lambda(x_j-x_{j'})]^2, \qquad \tau_m\Gamma_d=8(\pi/\hbar\lambda)^2 S_q S_f \geq 1/2.$$

FDT analog for quantum measurements

$$\hbar|\lambda| \leq 4\pi[S_f S_q - (\mathrm{Re}\, S_{fq})^2]^{1/2},$$

where λ is the linear response coefficient of the detector, S_f and S_q are the low-frequency spectral densities of the, respectively, back-action and output noise, $\mathrm{Re}S_{fq}$ is the classical part of their cross-correlator.

As we see, this inequality characterizes the efficiency of the trade-off between the information acquisition by the detector and back-action dephasing of the measured system. The detector that satisfies this inequality as equality is ``ideal'' or ``quantum-limited''.

Charge detector based on quantum point contact (QPC)

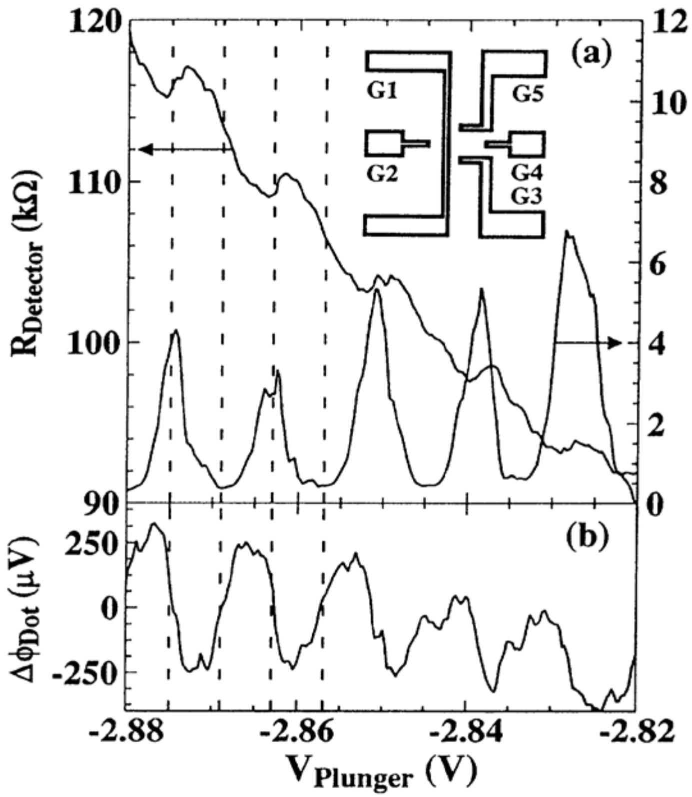
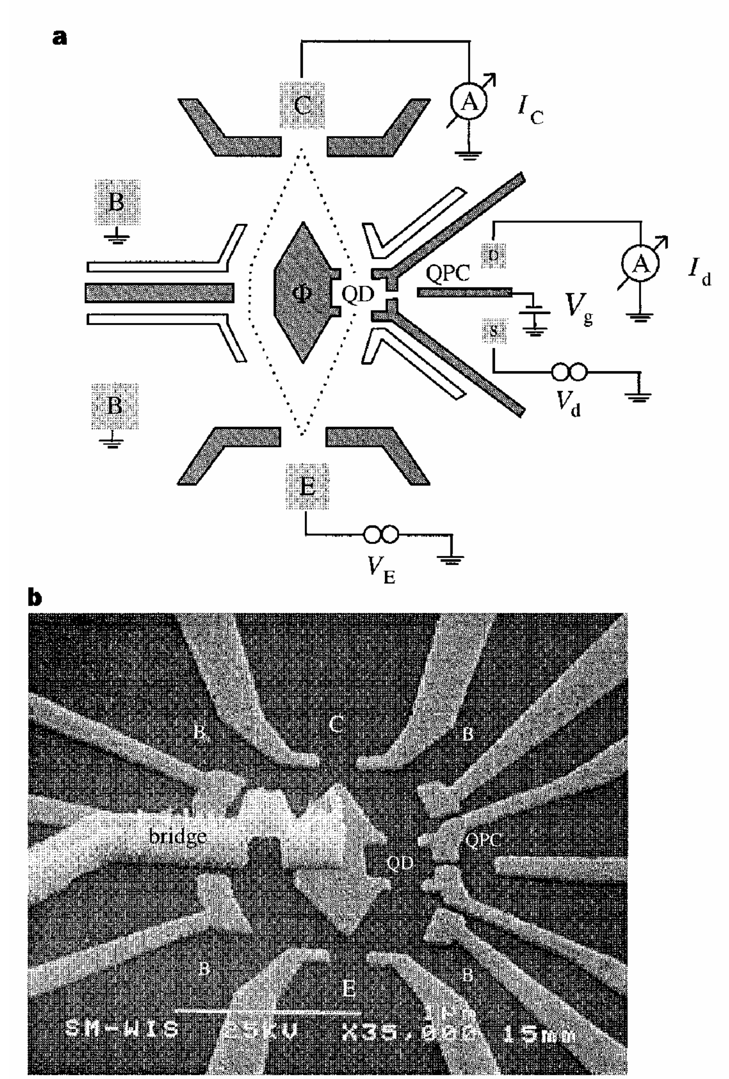


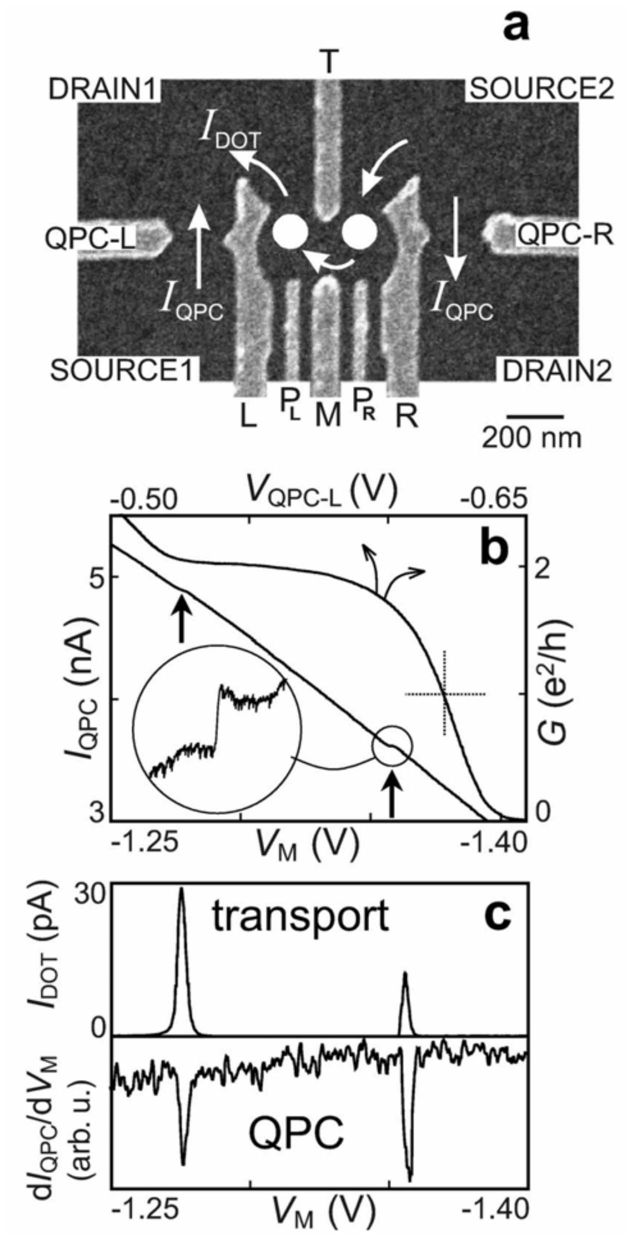
FIG. 1. (a) CB oscillations of conductance vs gate voltage through the dot, together with the resistance of the split gate detector circuit. (b) The change in dot potential calculated from the detector resistance. The overall negative slope is an artifact of the calibration procedure. Inset: a schematic diagram of the gate structure.

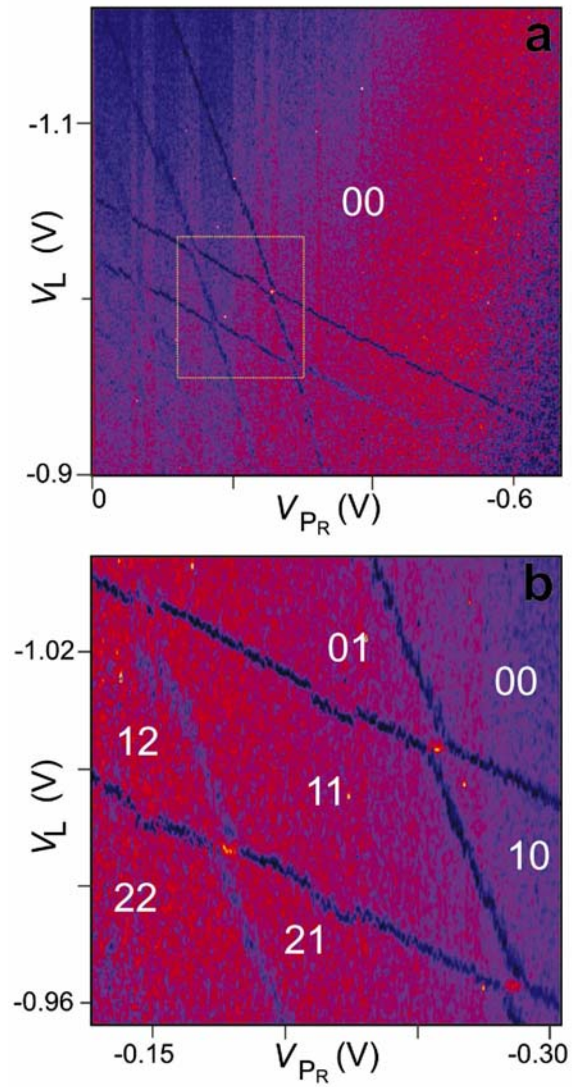


M. Field *et al*., PRL **70**, 1311 (1993).

E. Buks *et al*., Nature **391**, 871 (1999).

Quantum-dot qubits





J.M. Elzerman *et al.*, PRB **767**, 161308 (2003).

Counting statistics and detector properties of quantum point contacts

``Back-action'' dephasing rate

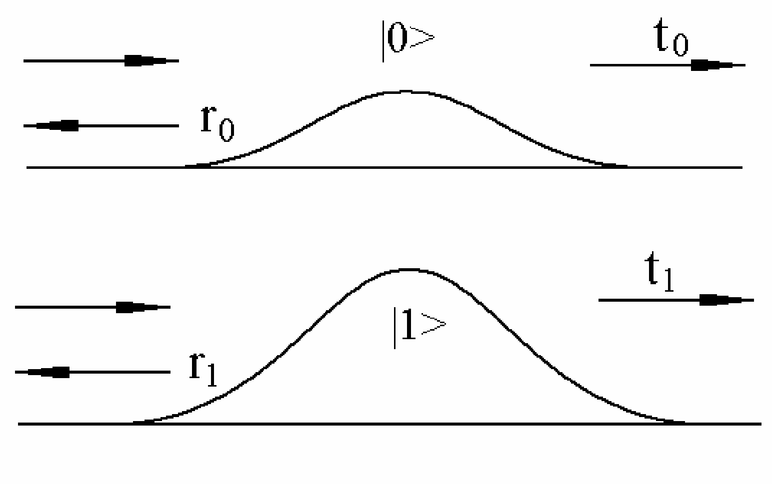
Individual scattering events:

$$(\alpha|0\rangle + \beta|1\rangle) \otimes |in\rangle \rightarrow$$
$$\alpha|0\rangle \otimes (t_0|R\rangle + r_0|L\rangle) + \beta|1\rangle \otimes (t_1|R\rangle + r_1|L\rangle)$$

$$\rho_{in} \rightarrow \rho_{out} \qquad \rho_{out}^{(qubit)} = Tr_{\{R,L\}}\rho_{out}$$

$$\rho_{11}^{(qubit)} = |\alpha|^2 = const$$

$$\rho_{12}^{(qubit)} = \alpha^*\beta \rightarrow \alpha^*\beta(t_0^* t_1 + r_0^* r_1) \le \alpha^*\beta$$



Total rate:

$$T = 0 \qquad \Gamma_d = -(eV/2\pi\hbar)\ln|t_1^* t_0 + r_1^* r_0|.$$

$$T \neq 0 \qquad \Gamma_d = \int (d\varepsilon/2\pi\hbar)\ln\left[\det(1 - f(\varepsilon) + f(\varepsilon)S_0^{-1}S_1)\right]$$

D.V. A. and E.V. Sukhorukov,
PRL **95,** 12680 (2005).

Information acquisition rate

The rate of growth of the confidence level in distinguishing probability distributions $P_j(n)$ of the transferred charge in different qubit states.

Quantitatively, using Renyi entropy:

$$W = -(1/t)\ln \sum_n [P_0(n)P_1(n)]^{1/2}.$$

$$W = -(eV/2\pi\hbar)\ln\left(\sqrt{D_0 D_1} + \sqrt{R_0 R_1}\right) \leq \Gamma_d .$$

This gives theT=0 condition of the QPC being quantum-limited detector: no information in the phases of the scattering amplitudes:

$$\phi_0 = \phi_1, \qquad \phi_j = \arg(t_j / r_j).$$

Conditional evolution

State reduction for individual scattering outcomes:

$$a_j \to t_j a_j \Big/ \left(\sum_j D_j |a_j|^2\right)^{1/2}; \qquad a_j \to r_j a_j \Big/ \left(\sum_j R_j |a_j|^2\right)^{1/2}.$$

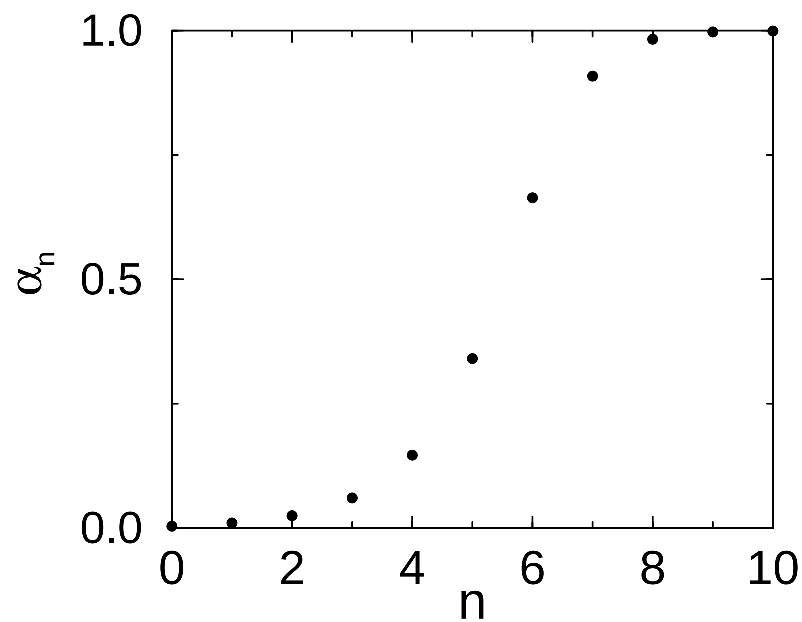
For *n* observed transmissions out of *N* scattering attempts:

$$a_j \to t_j^n r_j^{(N-n)} a_j \left(\sum_j D_j^n R_j^{(N-n)} |a_j|^2\right)^{-1/2}.$$

Example of a qubit measurement

$$|\psi_0\rangle = \left(|0\rangle + |1\rangle\right)\Big/\sqrt{2} \to |\psi_1\rangle =$$
$$= \alpha_n |0\rangle + \beta_n |1\rangle.$$

$$\alpha_n = [w_0^{(n)} / (w_0^{(n)} + w_1^{(n)})]^{1/2},$$
$$w_j^{(n)} = D_j^n R_j^{(N-n)}.$$

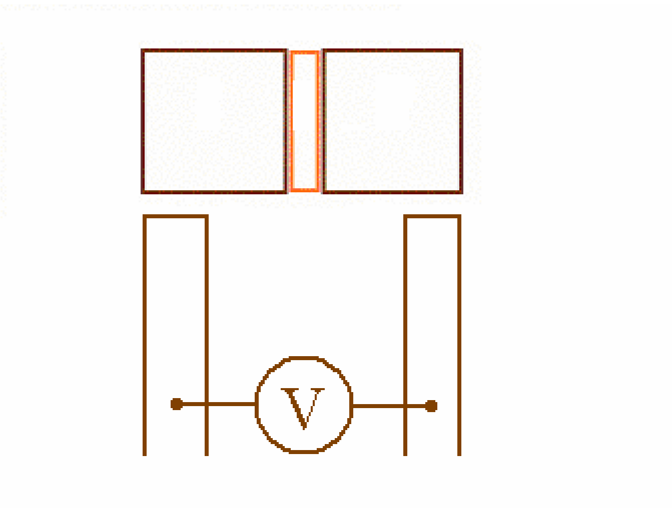


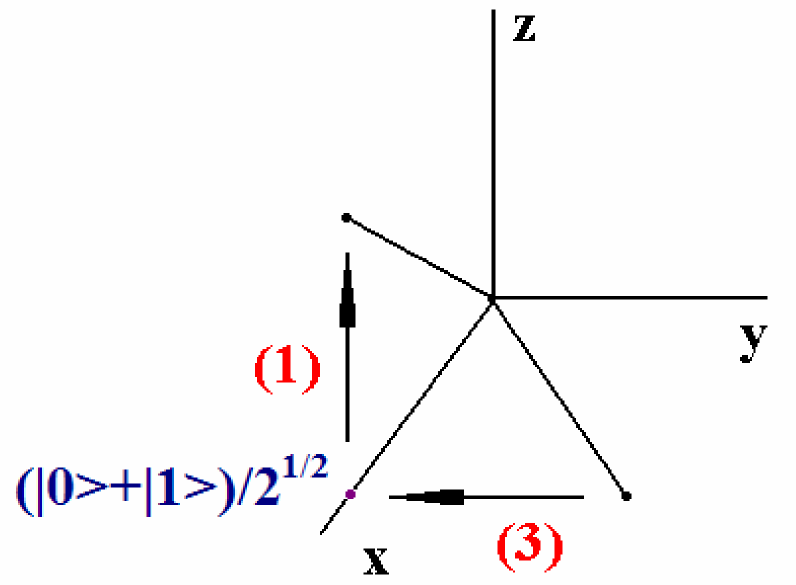
State reduction in a mesoscopic qubit

One can devise a simple transformation cycle demonstrating that in the state reduction process the charge or flux is being transferred between the qubit states even if the tunneling amplitude is vanishing.

Start with the state $\sigma_x=1$. The step (1) of the cycle:

$$|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2} \quad \rightarrow \quad |\psi\rangle = (\sqrt{w_0^{(n)}}|0\rangle + \sqrt{w_1^{(n)}}|1\rangle)/[w_0^{(n)} + w_1^{(n)}]^{1/2} .$$





The sequence of next two pulses: creating tunneling for a period of time; and changing the phase between the states 0,1:

(2) $\int \Delta(t)dt/\hbar = \pi/4,$ (3) $\int \varepsilon(t)dt/\hbar = \pi/4 - \tan^{-1}\sqrt{w_1^{(n)}/w_0^{(n)}},$

returns the qubit to the initial σ_x=1 state, i.e. we have a closed cycle (1,2,3) which involves tunneling and transfer of charge at Δ=0. For any classical charge state, there would be a probability p to find the state σ_x=-1,

$$p = \min\{w_0^{(n)}, w_1^{(n)}\}/(w_0^{(n)} + w_1^{(n)}).$$

Thus, if the observed probability of the state σ_x=-1 is less than this value,

$$p_{obs} < p,$$

the observation proves that the charge or flux is transferred through the suppressed barrier in the transformation cycle.