Solid State NMR Quantum Information Processing

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Institut Henri Poincaré, Février 2006

QIP with solid state NMR

Could we implement a multi-round protocol of quantum error correction?

Need to go to solid state
Advantages:

- **Stronger couplings (dipolar)**
- **Slower decoherence**
- **Higher polarization**

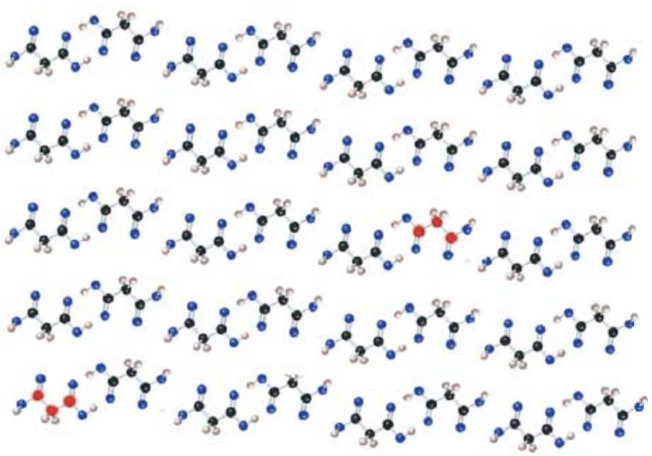
How to reach higher polarization?

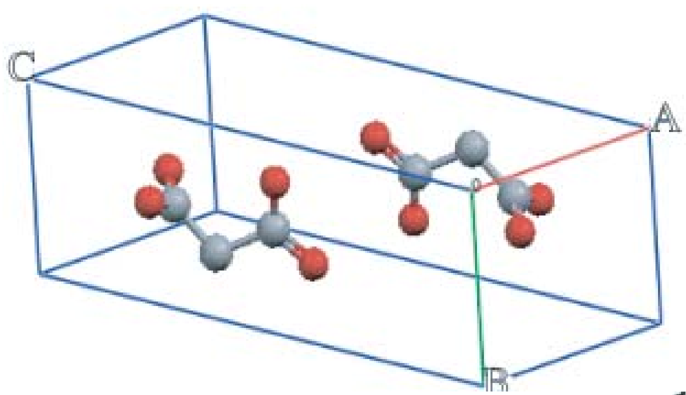
Cooling techniques:

- **Lowering Temperature**
- **Increasing Energy Gap**
- **Polarization Transfer**
- **Optical Pumping**
- **Dynamic Nuclear Polarization**
- **Para-Hydrogen (Jones et al.)**
- **Algorithmic Cooling**

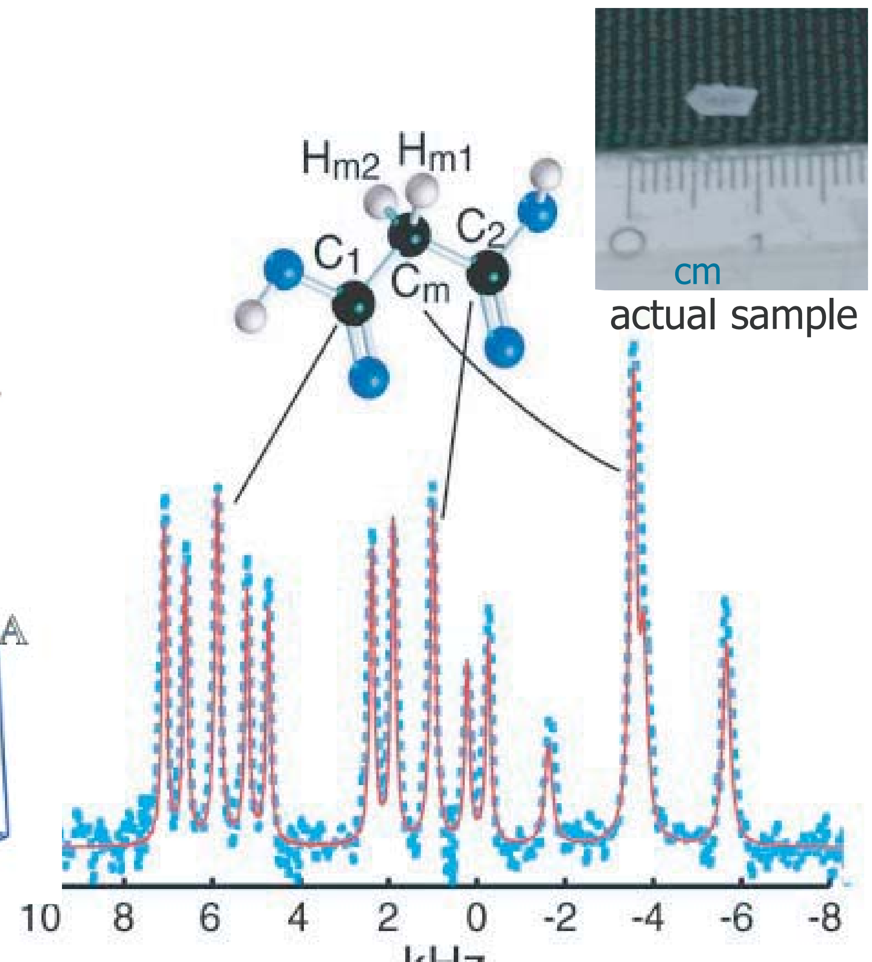
Molecule and Characterization

Malonic acid





P-1 space group:
magnetic equivalence



actual sample

^{13}C NMR spectrum

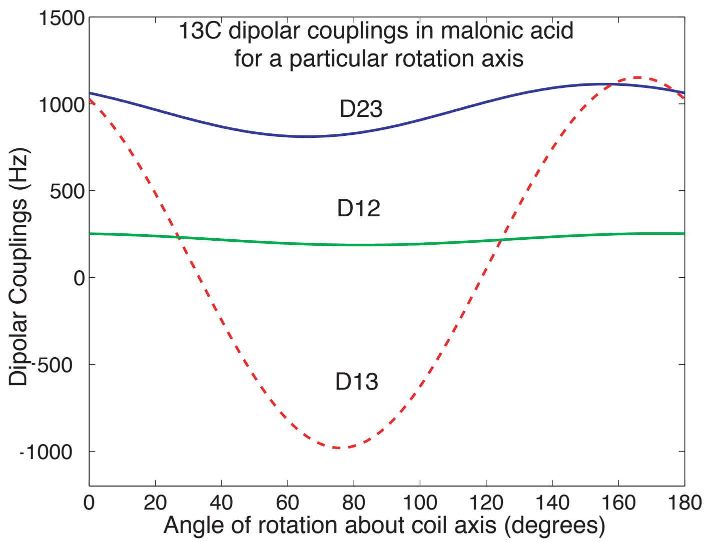
Molecule and Characterization

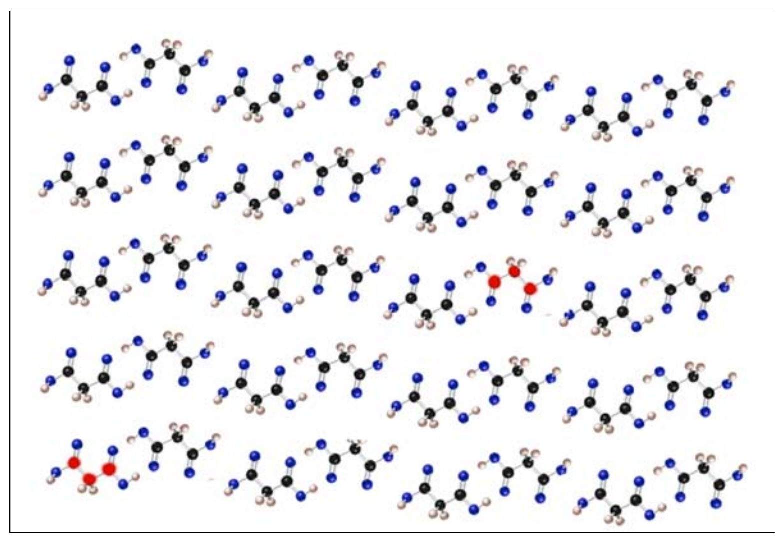
Baugh [2]

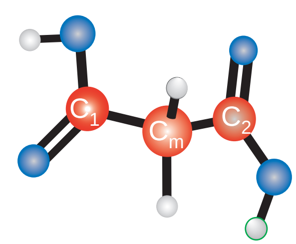
$$\mathcal{H}(t) = \mathcal{H}_C + \mathcal{H}_H + \mathcal{H}_{CH} + \mathcal{H}_{RF}(t),$$
$$\mathcal{H}_C = \mathcal{H}_{CZ} + \mathcal{H}_{CD}^{intra} + \mathcal{H}_{CD}^{inter},$$

$$\mathcal{H}_{CZ} = \sum_{j=1}^{3} \frac{\nu_j}{2} Z^j \quad ; \; \mathcal{H}_{CD}^{intra} = \sum_{m<n\leq 3} \frac{d_{mn}}{4}(2Z^m Z^n - Y^m Y^n - X^m X^n),$$

$$d_{mn} = \gamma_C^2 \hbar \frac{1 - 3cos^2(\theta_{mn})}{2r_{mn}^3},$$





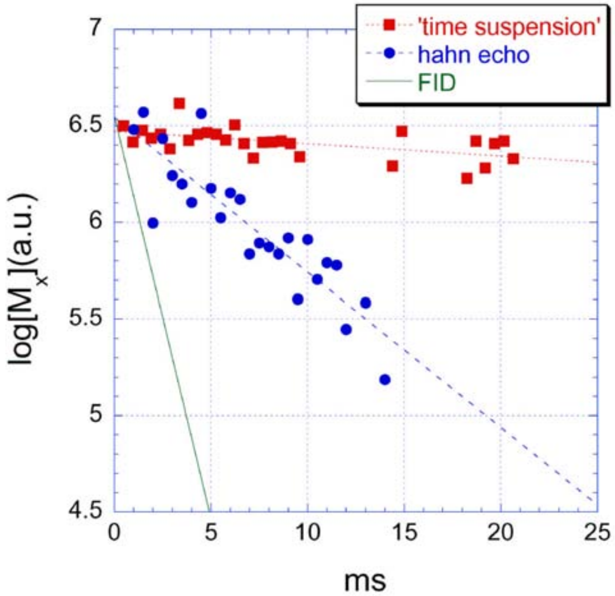


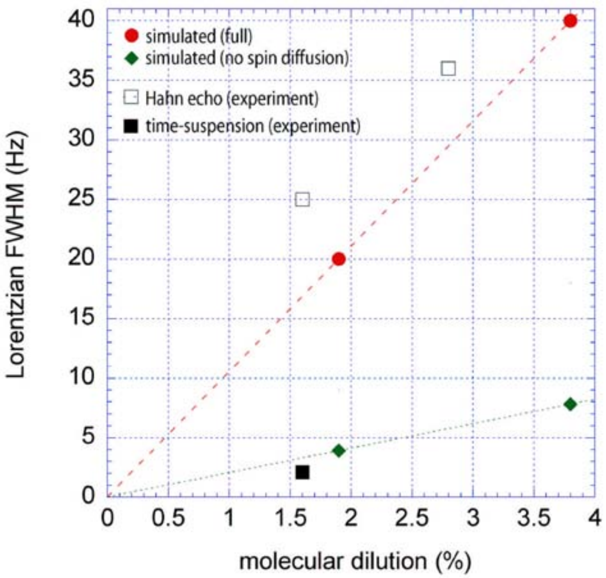
	C_1	C_2	C_m	H_{m1}	H_{m2}	T_2^*(ms)	T_1(s)
C_1	5.893	0.227	0.935	−1.5	2.0	2.4	160
C_2		1.057	1.070	1.4	1.0	2.0	325
C_m			−3.445	−18.7	−0.9	1.5	315

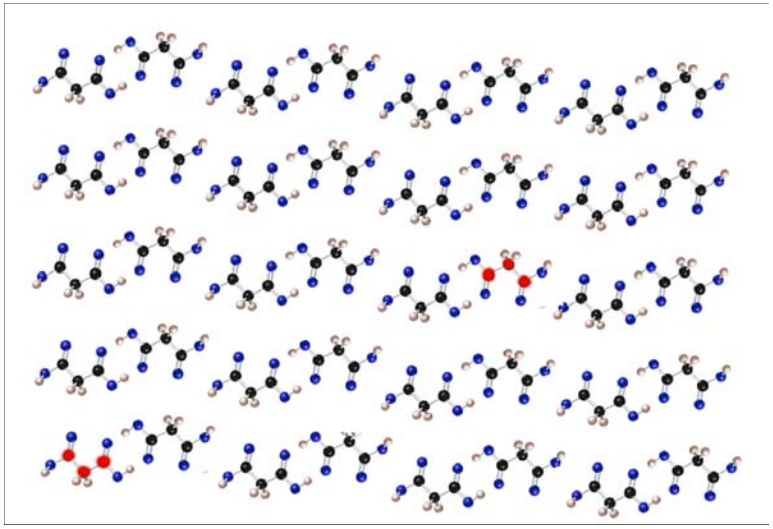
Decoherence time

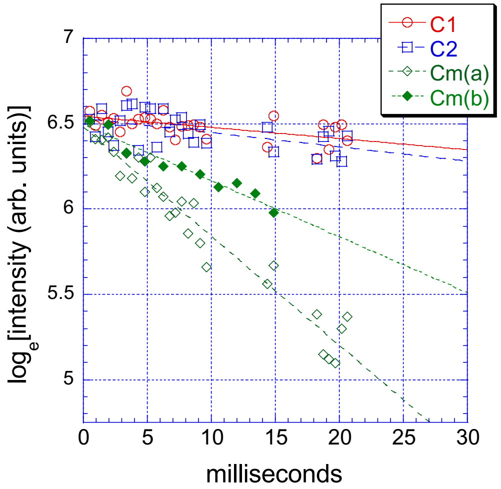
Baugh [2]

The large part of the decoherence is due to dipolar coupling with nearby spins. We can decoupled H and increase dilution of the labelled molecules.









Control of Strongly Coupled Systems

In solid state $t_{\text{one qubit gate}} \sim t_{\text{two qubit gate}}$, so how do we control the system?

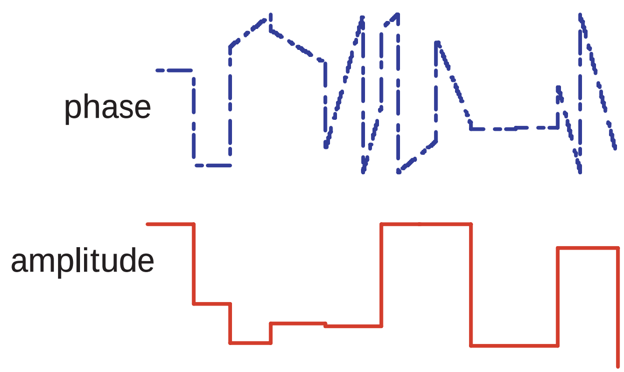
Answer: Construct a modualted RF waveform that generates the desired evolution [3], i.e minimize F by modifying $H_{rf}(t)$ such that:

$$F = \sum_{\mu} p_{\mu} |\mathrm{Tr}[U_{\mathrm{des}}^{\dagger} U_{\mathrm{cal}}^{\mu}]/N|^2$$

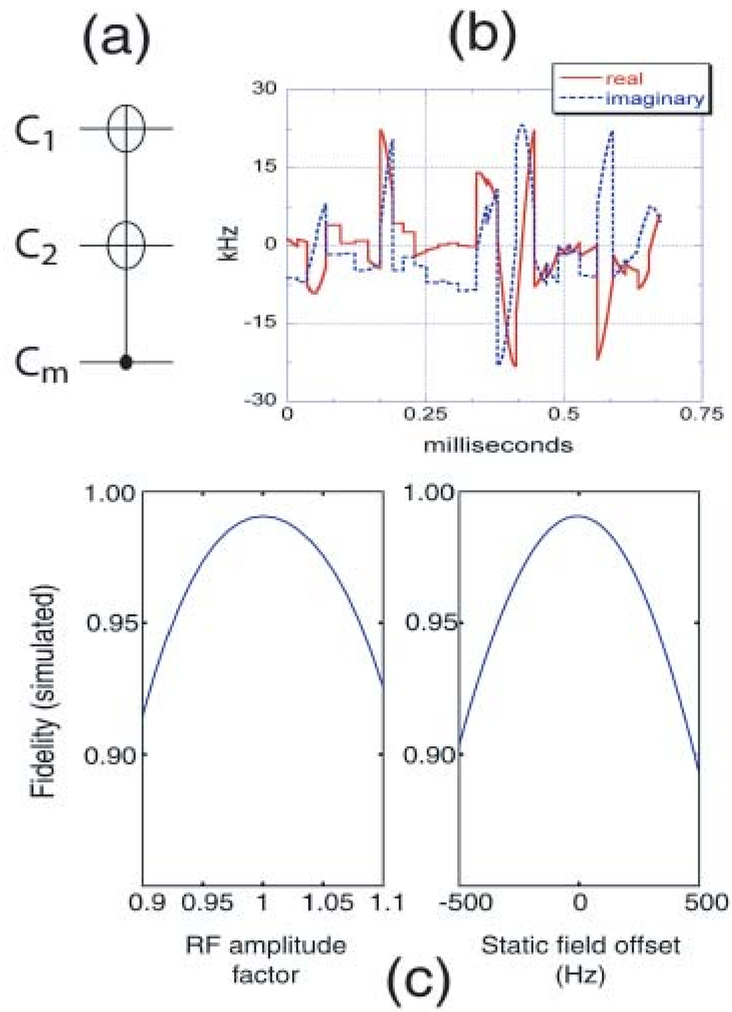
with

$$U_{cal}^{\mu} = U(t) = e^{-i \int_0^t (H_{B_o}^{\mu} + H_{int} + H_{rf}^{\mu}(t))dt}$$

using simplex methods.



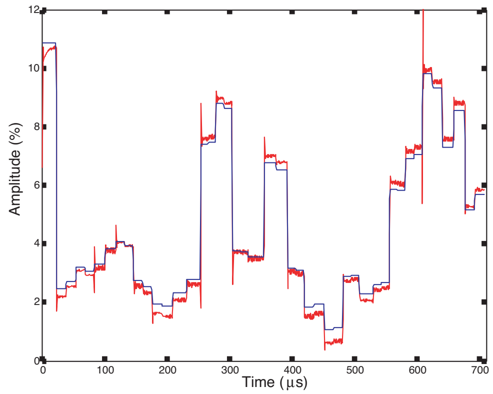
Example: Control-Not-Not

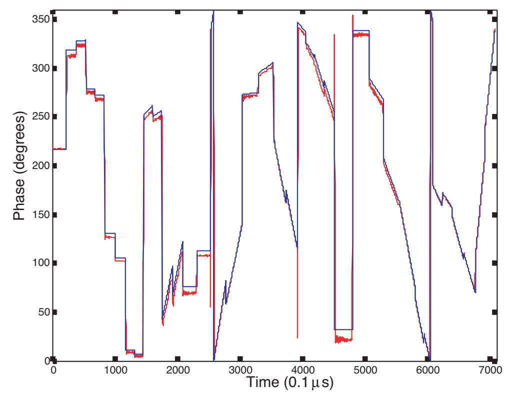


Fidelity is 98%, average RF amplitude is 9.4 KHz (magnitude of ^{13}C Hamiltonian is 7.3KHz), fidelity$>$ 90% over 1KHz range.

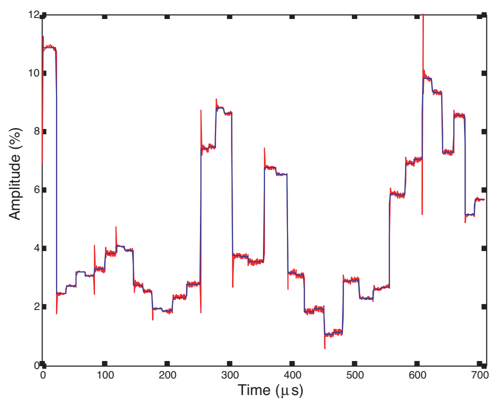
Feedback from the coil

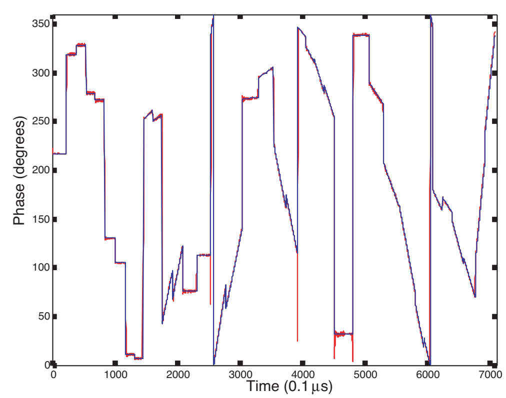
Before feedback:





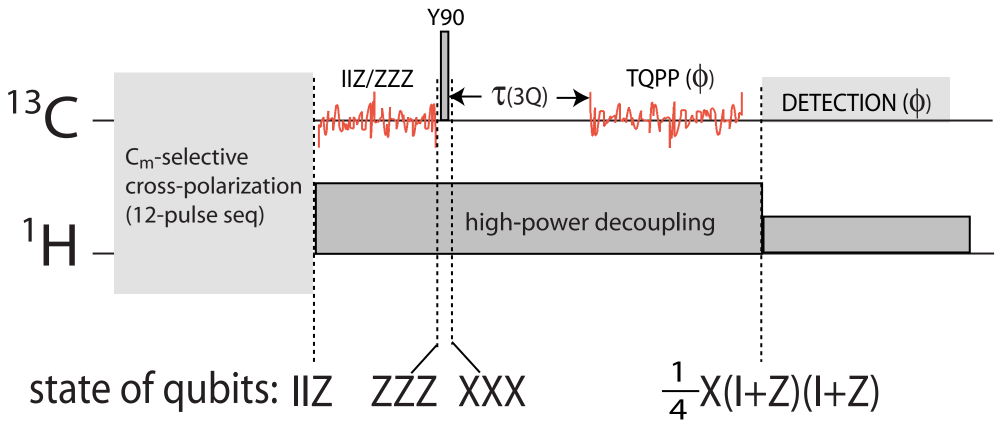
After feedback:

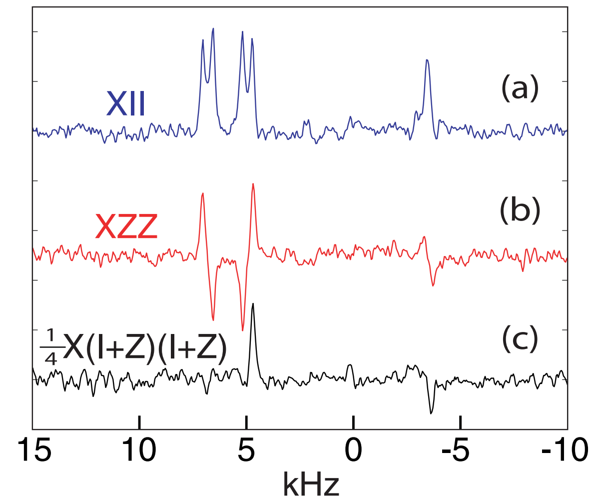


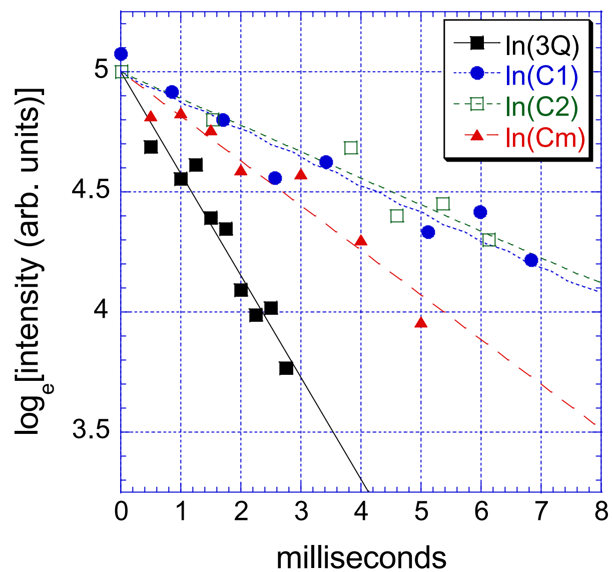


Decay of the 1 and 3 coherence

Baugh [2]







The decoherence time $T^2_{C_1} = 8.66$ms, $T^2_{C_2} = 9.07$ms, $T^2_{C_m} = 5.37$ms, $T^2_{3Q} = 2.37$ms

Algorithmic cooling

Sorensen [5], Schulman and Vazirani [4]

We have seen that we can cool a subset of spins by swapping states. For excample, with 3 spins, implementing a gate that swaps $|011\rangle \leftrightarrow |100\rangle$ will increase the order of the first spin at the expense of the last two. We could concatenate this process to reach polarization of order 1.

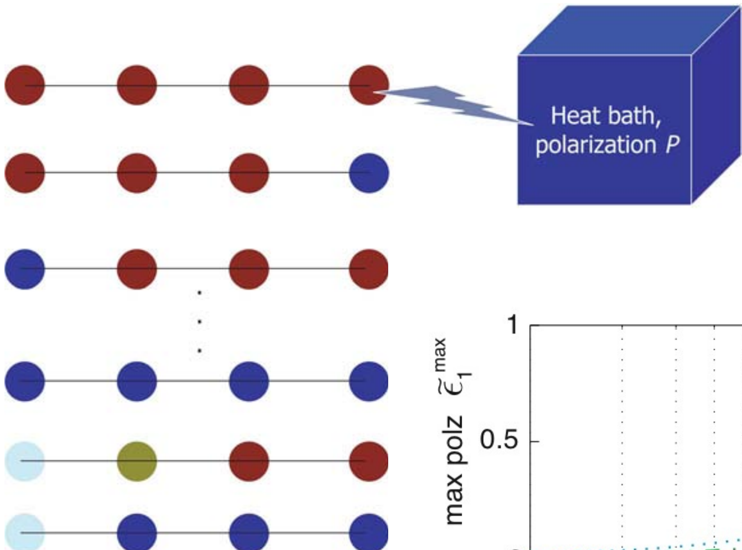
$$\rho \sim e^{-\beta H} \sim \frac{1}{2^n}(\mathbb{1} - \beta\omega(Z_1 + Z_2 + Z_3) + \dots)$$

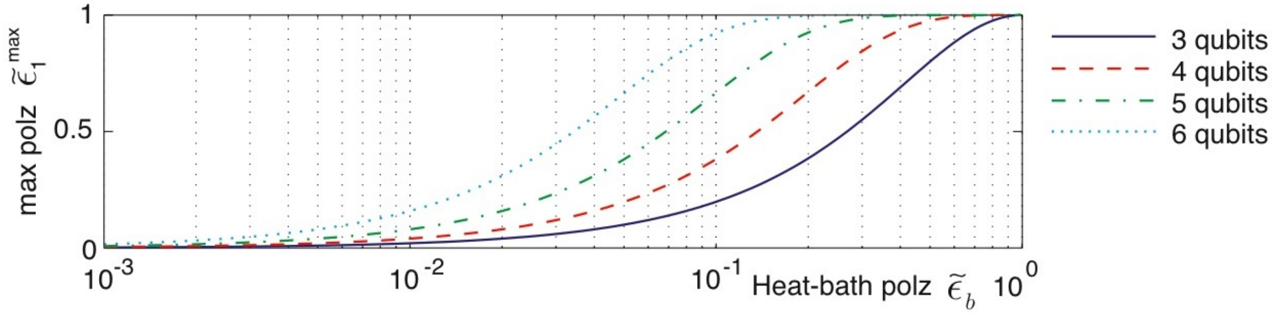
$$\rho^d_{\text{thermal}} \approx \frac{\beta\omega}{8}\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 \end{pmatrix} \Longleftrightarrow \rho^d_{\text{pol}} \approx \frac{\beta\omega}{8}\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 \end{pmatrix}$$

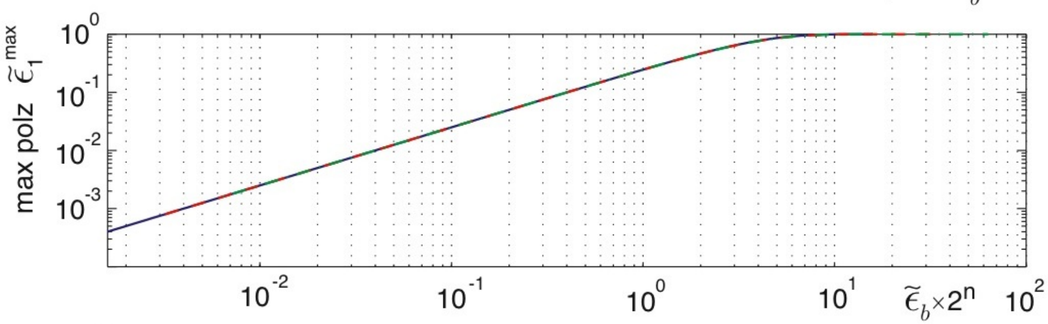
$$\bar{\rho}^d_{\text{pol}} = \text{Tr}_{2,3}\rho^d_{\text{pol}} \approx \frac{3}{4}\beta\omega\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We could concatenate this process to reach polarization of O(1), but this would take a lot of ressources ($\sim 1/\beta^2$).

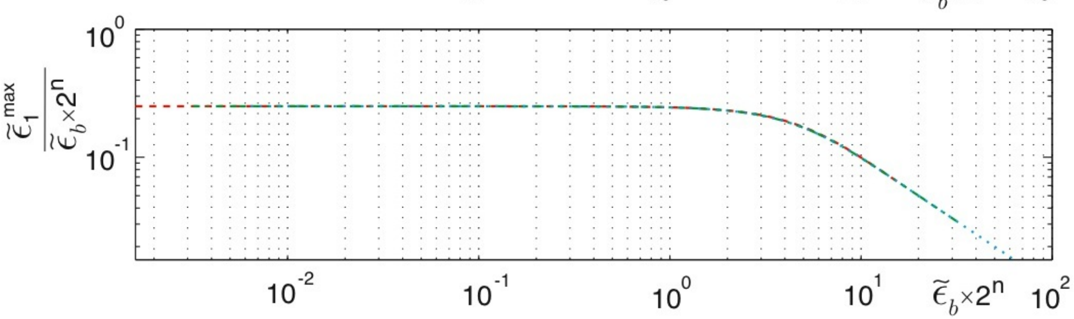
Algorithmic cooling with heat bath







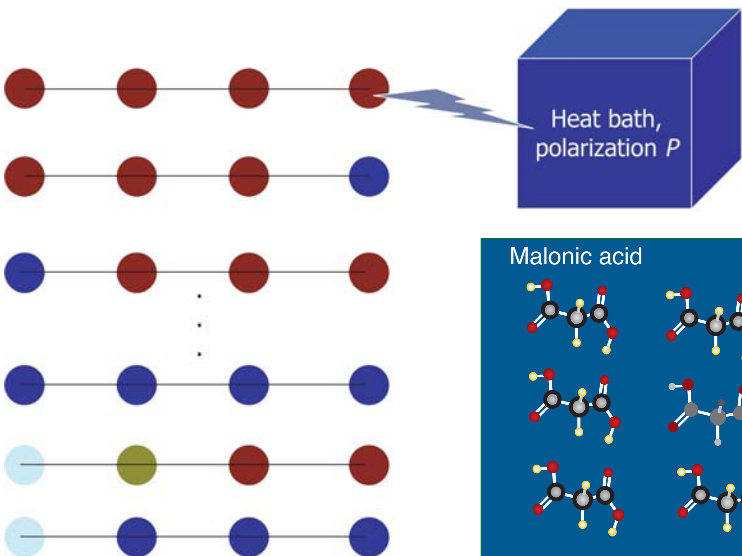
$$\tilde{\epsilon}_b \gg 2^{-n}$$
$$\rightarrow \tilde{\epsilon}_1^{max} = 1$$

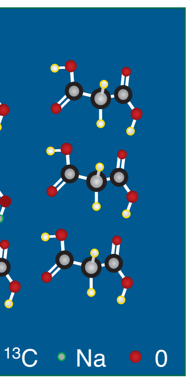


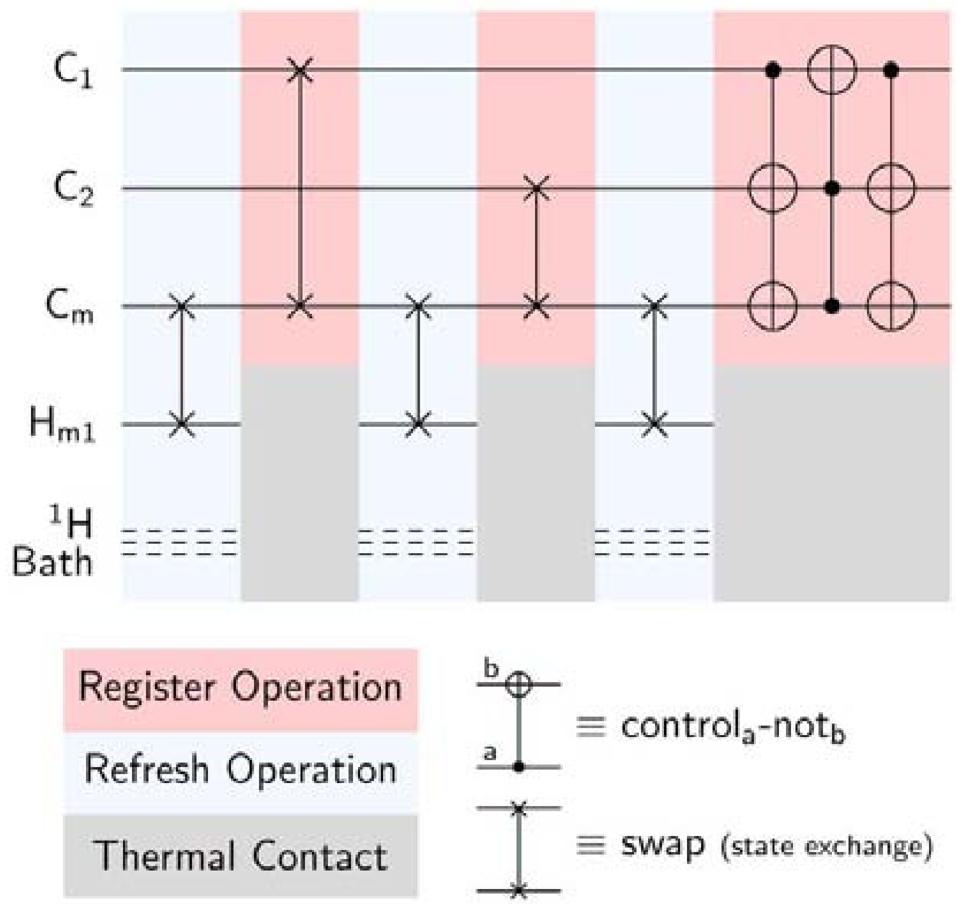
$$\tilde{\epsilon}_b < 2^{-n}$$
$$\rightarrow \tilde{\epsilon}_1^{max} = \tilde{\epsilon}_b \times 2^{n-2}$$

Algorithmic cooling with heat bath

Manipulate spins that are coupled to a heat bath. The first six steps of (Schulman, Mor and Weinstein, PRL94, 2005)

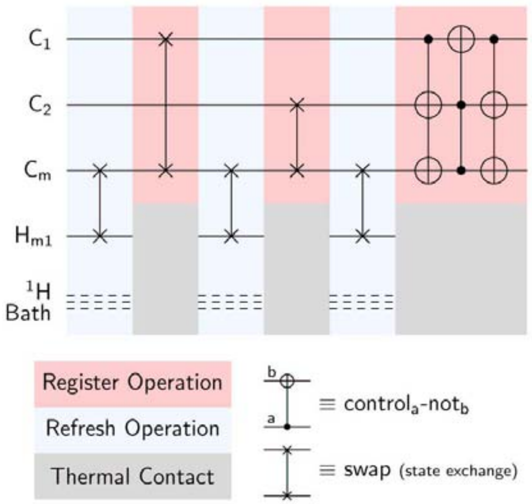


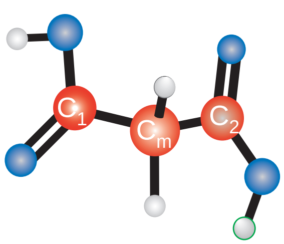


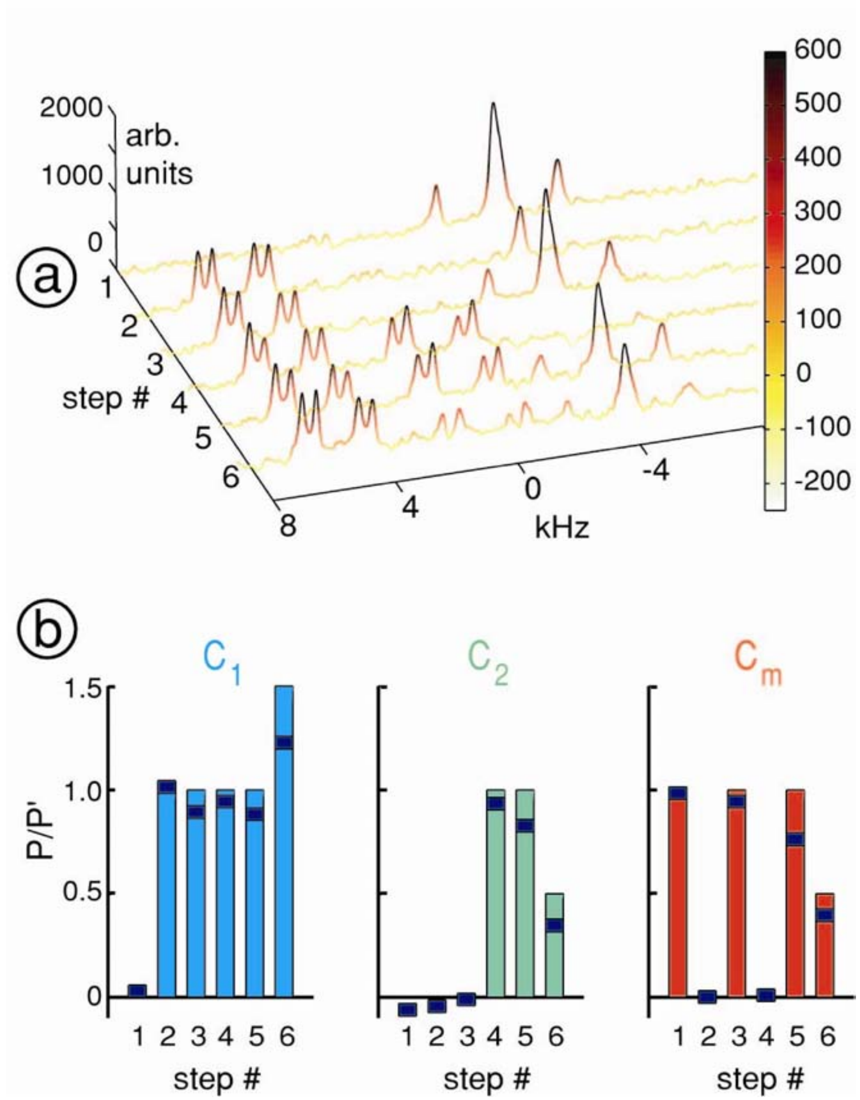


Experimental results

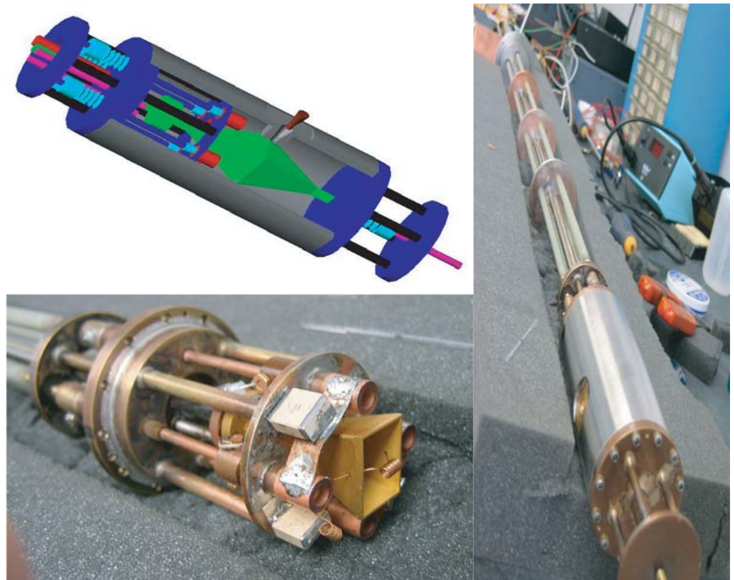
Baugh et al. Nature 438, 470, 2005 [1]

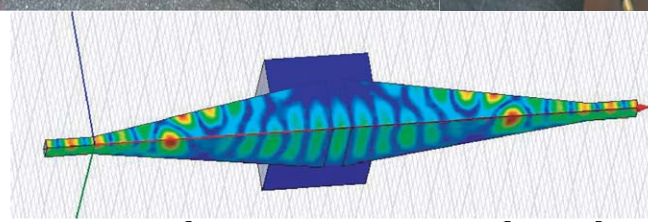


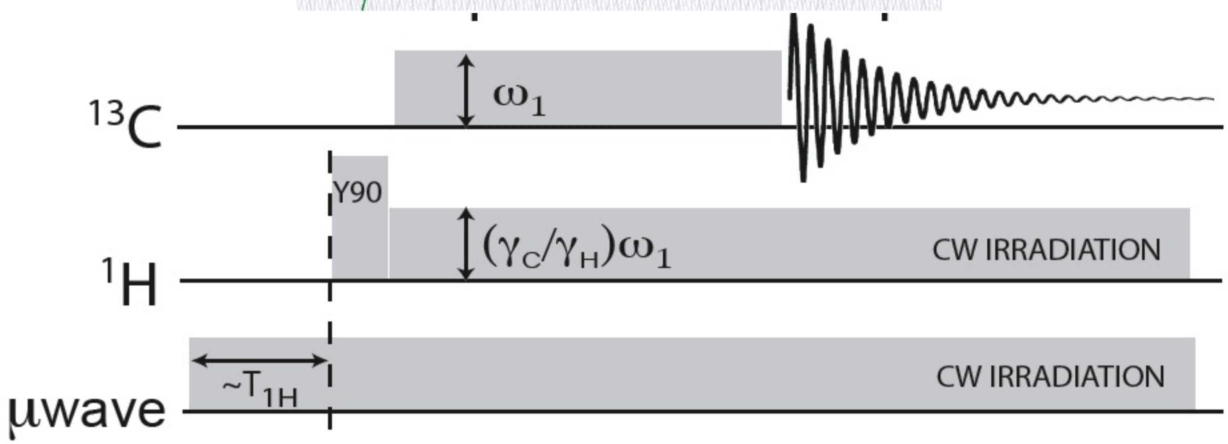




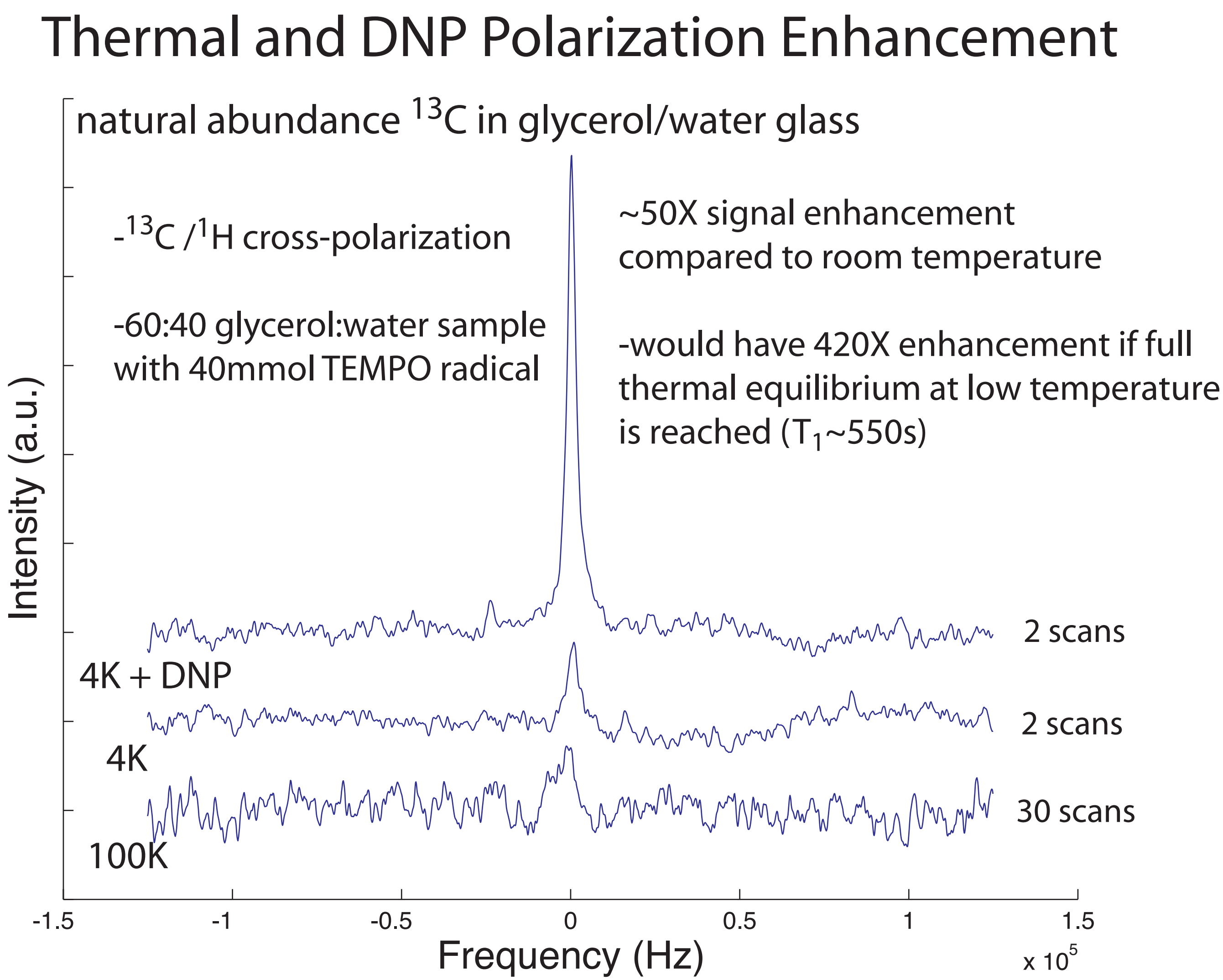
Next step: DNP







DNP results on Tempo



References

[1] J. Baugh, O. Moussa, C. Ryan, A. Nayak, and R. Laflamme. A spin-based heat engine: Experimental implementation of heat-bath algorithmic cooling. Nature, 438:470, 2005.

[2] J. Baugh, O. Moussa, C. A. Ryan, R. Laflamme, C. Ramanathan, T. F. Havel, and D. G.Cory. A solid-state nmr three-qubit homonuclear system for quantum information processing: control and characterization. Physical Review B, 73:022305, 2005.

[3] U. Haeberlen and J. S. Waugh. Coherent averaging effect in magnetic resonance. Phys. Rev., 175:453–467, 1968.

[4] L. J. Schulman and U. Vazirani. Scalable NMR quantum computation. In Proceedings of the 31th Annual ACM Symposium on the Theory of Computation (STOC), pages 322–329, El Paso, Texas, 1998. ACM Press.

[5] O. W. Sörensen. Polarization transfer experiments in high-resolution NMR spectroscopy. Prog. Nucl. Mag. Res. Spect., 21:503–569, 1989.