DECOHERENCE AND QUANTUM-CLASSICAL TRANSITION IN QUANTUM INFORMATION

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Lecture 1: Decoherence and the quantum origin of the classical world

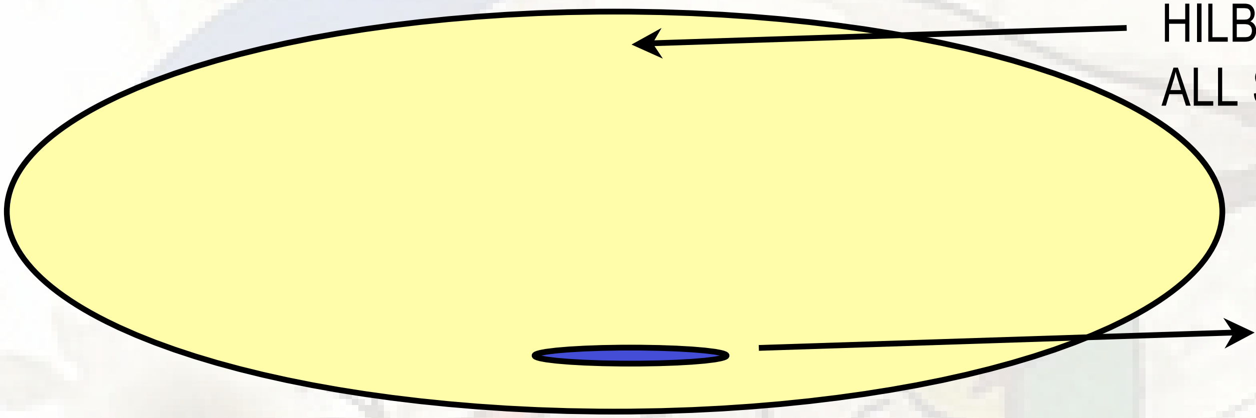
Lecture 2: Decoherence and quantum information processing, models and examples

- Decoherence, an overview
- Decoherence for classically chaotic systems. Why is it interesting, why is it different.
- Decoherence from complex environments
- Using qubits to learn about environmental properties (power of one qubit: use a single qubit to learn about properties of many)

Colaborations with: W. Zurek (LANL), M. Saraceno (CNEA), D. Mazzitelli (UBA), D. Monteoliva, C. Miquel (UBA), P. Bianucci (UBA, UT), L. Davila (UEA, UK), C. Lopez (UBA, MIT), A. Roncaglia (UBA, LANL), J. Anglin (MIT), R. Laflamme (IQC), S. Fernandez-Vidal (UAB), F. Cucchietti (LANL),

DECOHERENCE: AN OVERVIEW (I)

• DECOHERENCE AND THE QUANTUM-CLASSICAL TRANSITION:



• HOW TO EXPLAIN THE ORIGIN OF A CLASSICAL WORLD FROM A QUANTUM SUBSTRATE?: WHY IS IT THAT SOME SYSTEMS ARE ALWAYS FOUND IN "CLASSICAL STATES"?

• DECOHERENCE PARADIGM: CLASSICALITY IS AN EMERGENT PROPERTY

  • DYNAMICAL SUPRESSION OF QUANTUM SUPERPOSITIONS, EMERGENCE OF PREFERRED SET OF (POINTER) STATES

  • INDUCED ON SUBSYSTEMS DUE TO THE INTERACTION WITH THE ENVIRONMENT

DECOHERENCE: AN OVERVIEW (II)

- THE BASIC PHYICAL IDEA BEHIND DECOHERENCE IS VERY SIMPLE
  - SYSTEM-ENVIRONMENT INTERACTION CREATES CORRELATIONS
  - DECOHERENCE ARISES WHEN A RECORD OF THE STATE OF THE STATE OF THE SYSTEM IS IMPRINTED IN THE ENVIRONMENT.

SIMPLE EXAMPLE: DECOHERENCE IN A DOUBLE SLIT EXPERIMENT
(SYSTEM=CHARGE, ENVIRONMENT= E-M FIELD)

$$|\Psi(t)\rangle = \left(\alpha|\varphi_1(t)\rangle \otimes |\varepsilon_1(t)\rangle + \beta|\varphi_2(t)\rangle \otimes |\varepsilon_2(t)\rangle\right)$$

$$|\Psi(0)\rangle = \left(\alpha|\varphi_1\rangle + \beta|\varphi_2\rangle\right) \otimes |\varepsilon_0\rangle$$

INTERACTION WITH ENVIRONMENT INDUCES DECAY OF FRINGE VISIBILITY: DECOHERENCE

$$\Pr ob(x) = |\alpha|^2 |\varphi_1(x)|^2 + |\beta|^2 |\varphi_2(x)|^2 + 2\mathrm{Re}\left(\alpha\beta^* \varphi_1(x)\varphi_2^*(x)\langle\varepsilon_2(t)|\varepsilon_1(t)\rangle\right)$$

DECOHERENCE: AN OVERVIEW (III)

$$\rho = Tr_E\left(\left|\Psi(t)\right\rangle\left\langle\Psi(t)\right|\right)$$

INTERACTION WITH ENVIRONMENT INDUCES DECAY OF OFF DIAGONAL ELEMENTS OF DENSITY MATRIX IN A SPECIFIC BASIS

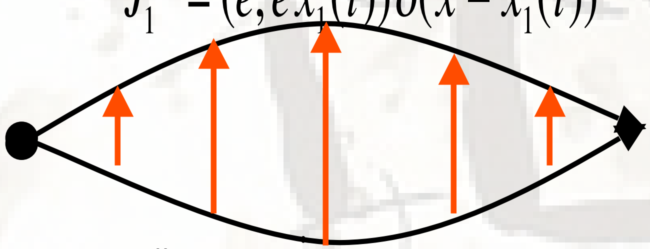
$$= |\alpha|^2 |\varphi_1(t)\rangle\langle\varphi_1(t)| + |\beta|^2 |\varphi_2(t)\rangle\langle\varphi_2(t)| + \alpha\beta^* \langle\varepsilon_2(t)|\varepsilon_1(t)\rangle |\varphi_1(t)\rangle\langle\varphi_2(t)| + \alpha^*\beta\langle\varepsilon_1(t)|\varepsilon_2(t)\rangle |\varphi_2(t)\rangle\langle\varphi_1(t)|$$

- ISN'T THIS TOO SIMPLE? (HOW MUCH CAN WE BUY WITH THIS SIMPLE IDEA?)
- THE IMPORTANT QUESTIONS: HOW IMPORTANT IS THIS PROCESS FOR PHYSICALLY RELEVANT CASES (HOW MUCH DECOHERENCE? ON WHAT TIMESCALE? WHAT ARE THE POINTER STATES, ETC).

HOW TO COMPUTE THE OVERLAP $\langle\varepsilon_2(t)|\varepsilon_1(t)\rangle$?

A SIMPLE (EXACT) RESULT:

$$J_1^\mu = (e, e\dot{\vec{x}}_1(t))\,\delta(\vec{x} - \vec{x}_1(t))$$



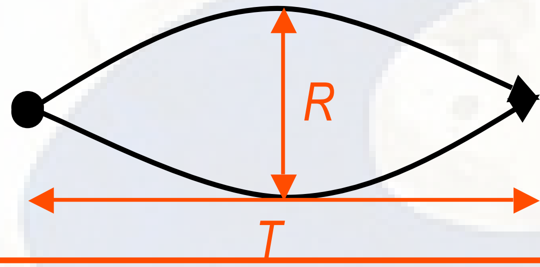
$$J_2^\mu = (e, e\dot{\vec{x}}_2(t))\,\delta(\vec{x} - \vec{x}_2(t))$$

$$\Delta J^\mu = J_1^\mu - J_2^\mu \Rightarrow \left|\langle\varepsilon_2(t)|\varepsilon_1(t)\rangle\right|^2 = 1 - P$$

P= Probability that there is at least one photon emited from the source ΔJ^μ (which is a fictitious time varying dipole)

DECOHERENCE: AN OVERVIEW (IV)

• HOW IMPORTANT IS THIS EFFECT? NOT ALWAYS STRONG!



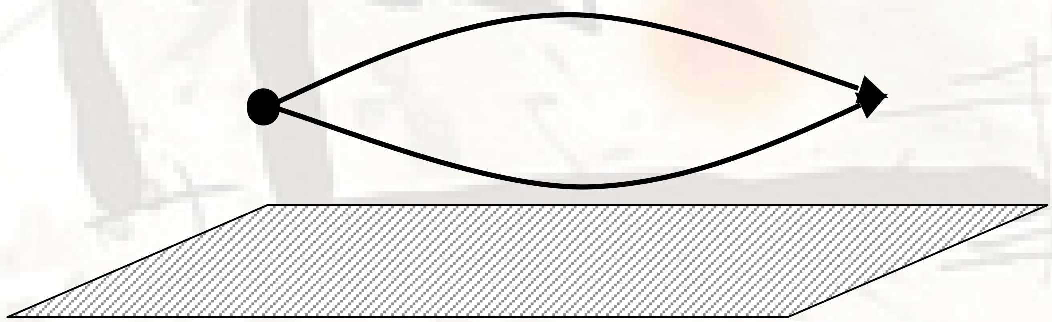
$$\Rightarrow \left|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle\right| \cong \exp\left(-\alpha\beta^2 O(1)\right) \quad \Rightarrow \quad SMALL...$$

$$\alpha \cong 1/137 \quad \beta = R/Tc \quad |\varepsilon(0)\rangle = |vacuum\rangle$$

Charges and dipoles: decoherence due to interaction with e.m. field in vacuum and relation with Casimir effect, see "Decoherence and recoherence near a conducting plate", F.D. Mazzitelli, J.P. Paz and A. Villanueva, quant-ph/0307004, Phys. Rev. A 68, 062106 (2003).

• INTERESTING: THE EFFECT IS SENSITIVE TO THE BOUNDARY CONDITIONS (THAT AFFECT THE SPACE OF STATES OF THE E.M. FIELD)

•QUIZ: CAN YOU GUESS WHAT HAPPENS WITH $\left|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle\right|$?

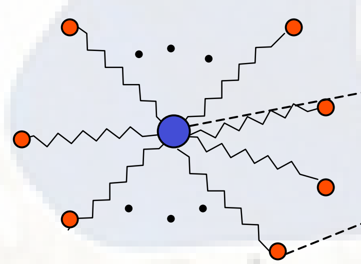


DOUBLE SLIT NEAR A PERFECT CONDUCTOR (DO WE GET MORE DECOHERENCE?)

A MODEL: QUANTUM BROWNIAN MOTION (I)

Quantum Brownian Motion (QBM): Paradigmatic model for a quantum open system

(realistic in many, but not all, cases: Caldeira-Leggett, etc)



System: Particle (harmonic oscillator)

Environment: Collection of harmonic oscillators

Interaction: bilinear

$$H = H_S + H_E + H_{\text{int}}, \quad H_S = \frac{p^2}{2m} + V_0(x), \quad H_E = \sum_n \left(\frac{p_n^{\,2}}{2m_n} + \frac{m_n \omega_n^{\,2}}{2} q_n^{\,2} \right), \quad H_{\text{int}} = \sum_n \lambda_n q_n x,$$

Our aim: Study evolution of the state of the system

'State of the system': Reduced density matrix $\rho_S = Tr_E(\rho_{SE})$

Asumption (standard): Uncorrelated initial state $t = 0$ $\rho_{SE}(0) = \rho_S(0) \otimes \rho_E(0)$

TWO "PARAMETERS": 1) INITIAL STATE OF ENVIRONMENT (TEMPERATURE T), 2) SPECTRAL DENSITY OF ENVIRONMENT

$$J(\omega) = \sum_n \frac{\lambda_n^{\,2}}{2 m_n \omega_n} \delta(\omega - \omega_n)$$

PROBLEM IS EXACTLY SOLVABLE!. USEFUL TOOL: EXACT MASTER EQUATION (EVOLUTION EQUATION FOR THE REDUCED DENSITY MATRIX); B.L. Hu, J.P. Paz and Y. Zhang, Phys. Rev. D42, 3243 (1992)

A MODEL: QUANTUM BROWNIAN MOTION (II)

GENERAL FORM OF THE MASTER EQUATION (VALID FOR ALL VALUES OF INITIAL TEMPERATURE OF ENVIRONMENT AND FOR ALL SPECTRAL DENSITIES)

$$\dot{\rho} = -i\left[H_R + \frac{m}{2}\delta\omega^2(t)x^2, \rho\right] - i\gamma(t)\left[x,\{p,\rho\}\right] - D(t)\left[x,[x,\rho]\right] - f(t)\left[x,[p,\rho]\right]$$

Time dependent coefficients are determined by spectral density and initial temperature

$$\dot{\rho} = -i\Big[H_R + \underbrace{\frac{m}{2}\delta\omega^2(t)x^2}_{\text{Dressing (renormalization)}}, \rho\Big] - \underbrace{i\gamma(t)\left[x,\{p,\rho\}\right]}_{\text{Damping (relaxation)}} - \underbrace{D(t)\left[x,[x,\rho]\right]}_{\text{Diffusion (Decoherence)}} - \underbrace{f(t)\left[x,[p,\rho]\right]}_{\text{Anomalous Diffusion}}$$

- Frequency renormalization and damping coefficients: only depende on spectral density

$$\delta\omega^2(t) \approx -2\int_0^t dt'\cos(\Omega t')\eta(t') \quad \gamma(t) \approx \frac{1}{\Omega}\int_0^t dt'\sin(\Omega t')\eta(t') \quad \eta(t) = \int_0^\infty d\omega \sin(\omega t)J(\omega)$$

- Diffusion coefficients (D(t) and f(t)) depend on spectral density and temperature

$$D(t) \approx \int_0^t dt'\cos(\Omega t')\nu(t') \quad f(t) \approx -\frac{1}{\Omega}\int_0^t dt'\sin(\Omega t')\nu(t') \quad \nu(t) = \int_0^\infty d\omega \cos(\omega t)\coth(\frac{\omega}{kT})J(\omega)$$

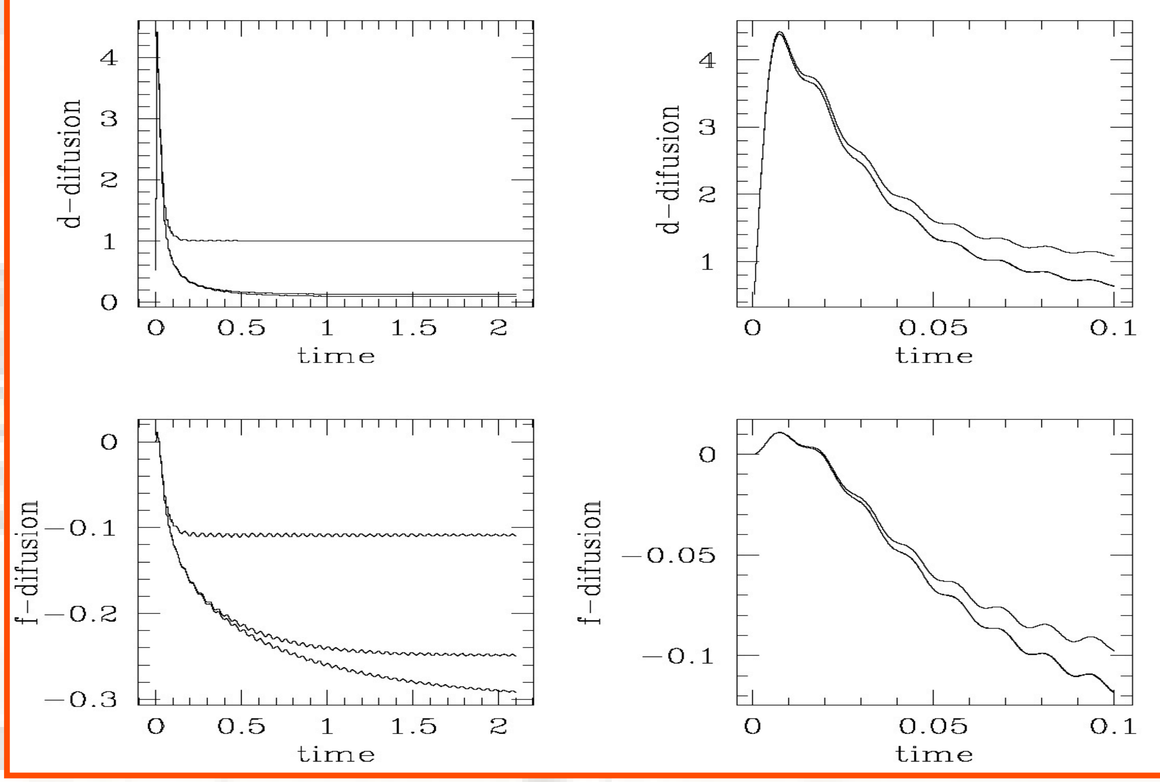
A MODEL: QUANTUM BROWNIAN MOTION (III)

$$J(\omega) = 2m\gamma\omega \quad (\omega \le \Lambda)$$

Ohmic environment

• Frequency renormalization and damping coefficients rapidly approach asymptotic values (in a timescale fixed by the high frequency cutoff)

• Diffusion coefficients (D(t) and f(t)) have initial transient and approach temperature-dependent asymptotic values



A MODEL: QUANTUM BROWNIAN MOTION (IV)

Ohmic environment in a high temperature initial state

$$\left.\begin{array}{l} J(\omega) = 2m\gamma\omega \quad (\omega \leq \Gamma), \\ k_B T >> \hbar\Omega \end{array}\right\} \quad \gamma(t) \to \gamma, \quad D(t) \to 2m\gamma k_B T, \quad f(t) \to 0$$

$$\dot{\rho} = -i\Big[H_R + \underbrace{\frac{m}{2}\delta\omega^2(t)x^2, \rho}_{\text{Dressing (renormalization)}}\Big] - \underbrace{i\gamma(t)[x,\{p,\rho\}]}_{\text{Damping (relaxation)}} - \underbrace{D(t)[x,[x,\rho]]}_{\text{Diffusion (Decoherence)}} - \underbrace{f(t)[x,[p,\rho]]}_{\text{Anomalous Diffusion}}$$

Approximate master equation (ohmic, high temperature)

$$\dot{\rho} = -i[H_R, \rho] - i\gamma[x,\{p,\rho\}] - D[x,[x,\rho]]$$

Use this to investigate:

1) What is the decoherence timescale?,

2) What are the pointer states?

DECOHERENCE IN QUANTUM BROWNIAN MOTION (V)

DECOHERENCE IN QUANTUM BROWNIAN MOTION: MAIN RESULTS ARE BETTER SEEN REPRESENTING THE STATE IN PHASE SPACE VIA WIGNER FUNCTIONS

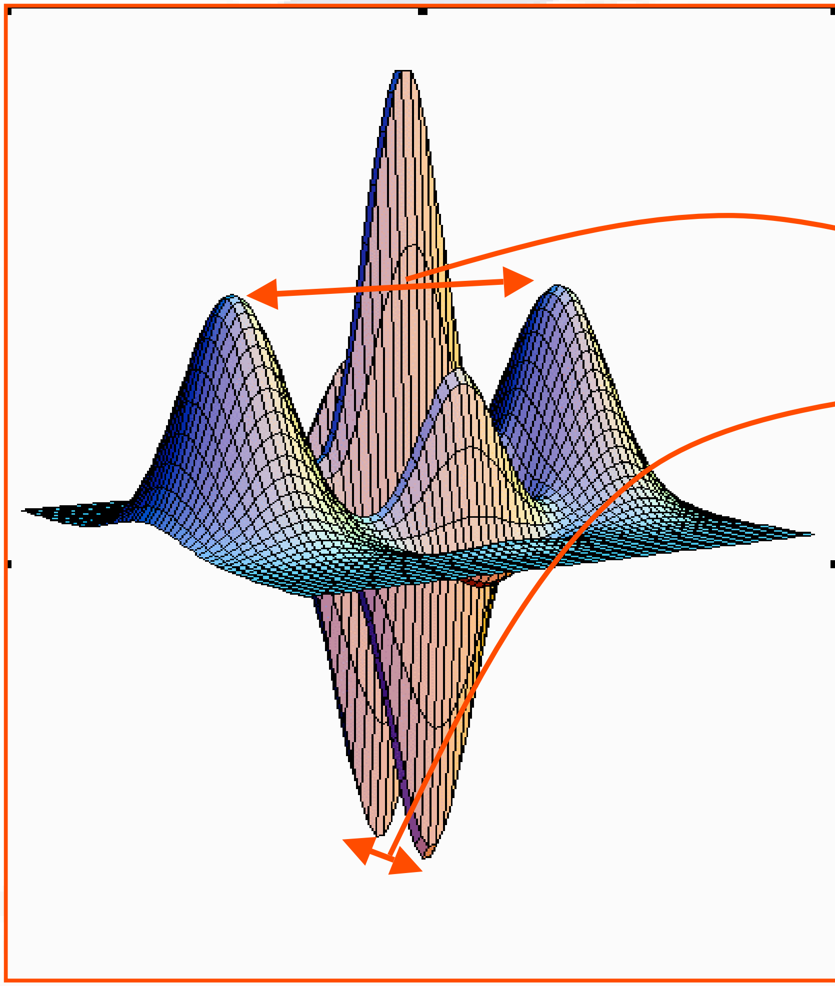
$$W(x,p) = \int \frac{dy}{2\pi\hbar}\, e^{ipy/\hbar} \langle x - y/2 | \rho | x + y/2 \rangle$$

- PROPERTIES:
  - ⇨ W(x,p) is real
  - ⇨ Use it to compute inner products as: $\int dx\, dp\, W_1(x,p)\, W_2(x,p) = \frac{1}{2\pi\hbar} Tr(\rho_1\, \rho_2)$
  - ⇨ Integral along lines give all marginal distributions:

$$\int_{ax+bp=c} dx\, dp\, W(x,p) = \mathrm{Probability}(aX + bP = c)$$

DECOHERENCE IN QUANTUM BROWNIAN MOTION (VI)

HOW DOES THE WIGNER FUNCTION OF A QUANTUM STATE LOOK LIKE?:
SUPERPOSITION OF TWO GAUSSIAN STATES



$Distance \quad L$

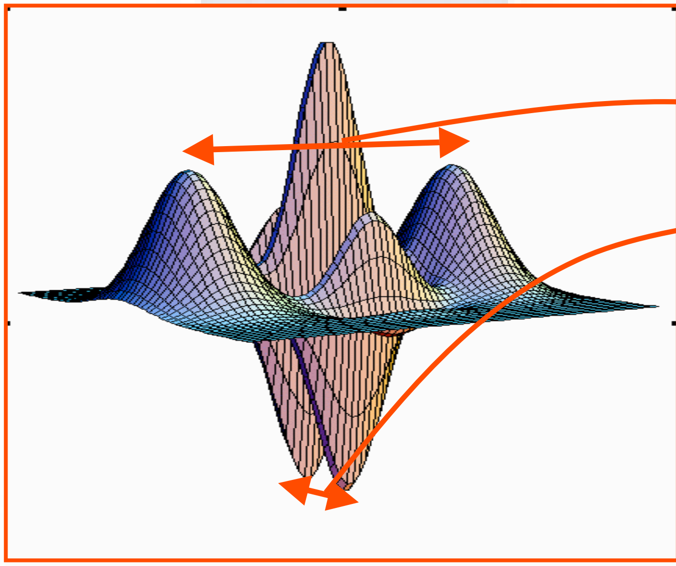
$Wavelength \quad \lambda_p = \frac{\hbar}{L}$

OSCILLATIONS IN WIGNER FUNCTION: THE SIGNAL OF QUANTUM INTERFERENCE. HOW DOES DECOHERENCE AFFECTS THIS STATE?

DECOHERENCE IN QUANTUM BROWNIAN MOTION (VII)

MASTER EQUATION CAN BE REWRITTEN FOR THE WIGNER FUNCTION: IT HAS THE FORM OF A FOKER-PLANCK EQUATION

$$\dot{W} = \{H_0, W\}_{MB} + D\partial^2{}_{pp} W + \cdots$$



Distance L

Wavelength $\lambda_p = \frac{\hbar}{L}$

$$\dot{W} = \{H_0, W\}_{MB} + D\partial^2{}_{pp} W + \cdots$$

$$W_{osc} \approx A(t)\cos(k_p p) \Rightarrow A(t) \approx \exp(-\Gamma t)$$

$$\Gamma = D k_p{}^2$$

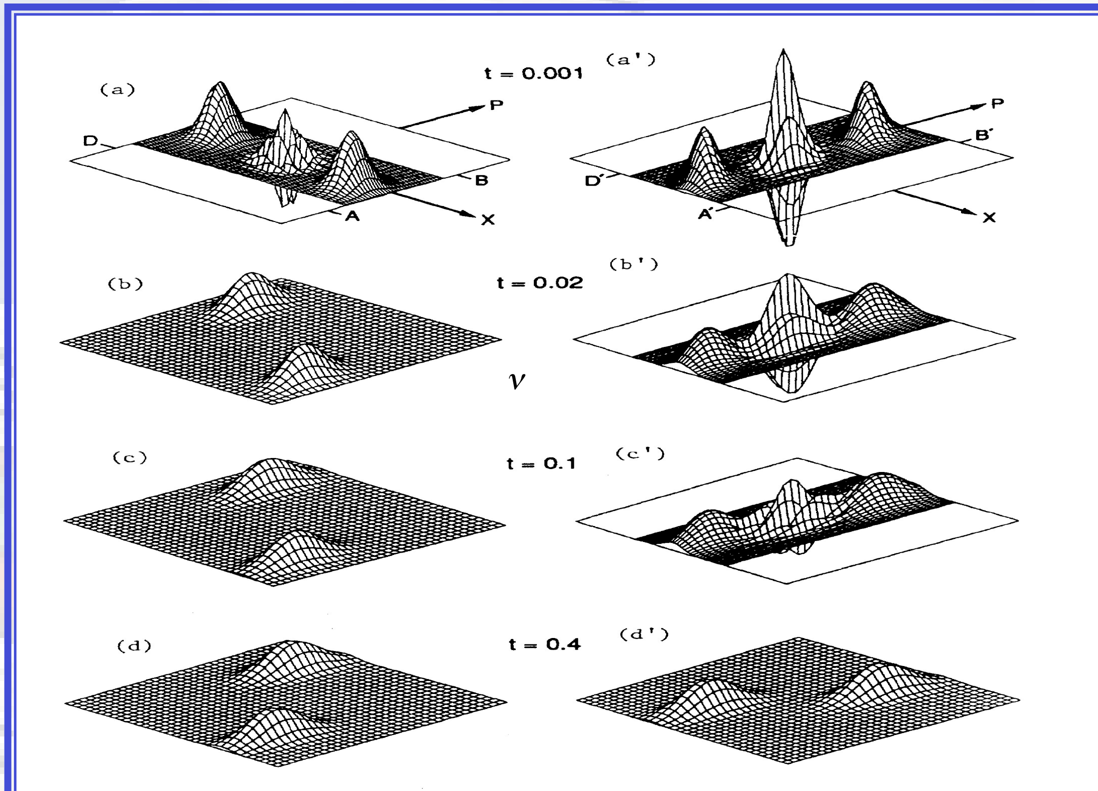
$$\Gamma = D L^2/\hbar^2, \qquad D = 2m\gamma k_B T, \qquad \lambda_{DB} = \hbar/\sqrt{2m\, k_B T}$$

$$\Rightarrow \Gamma = \gamma\left(L/\lambda_{DB}\right)^2 \approx 10^{40}\,\gamma, \quad m = 1gr, T = 300K, L = 1cm$$

DECOHERENCE RATE:
MUCH LARGER THAN
RELAXATION RATE

POINTER STATES, DECOHERENCE TIMESCALE (I)

EVOLUTION OF WIGNER FUNCTION: NOT ALL STATES ARE AFFECTED IN THE SAME WAY!



NOTICE: NOT ALL STATES ARE AFFECTED BY THE ENVIRONMENT IN THE SAME WAY (SOME SUPERPOSITIONS LAST LONGER THAN OTHERS)

POINTER STATES, DECOHERENCE TIMESCALE (II)

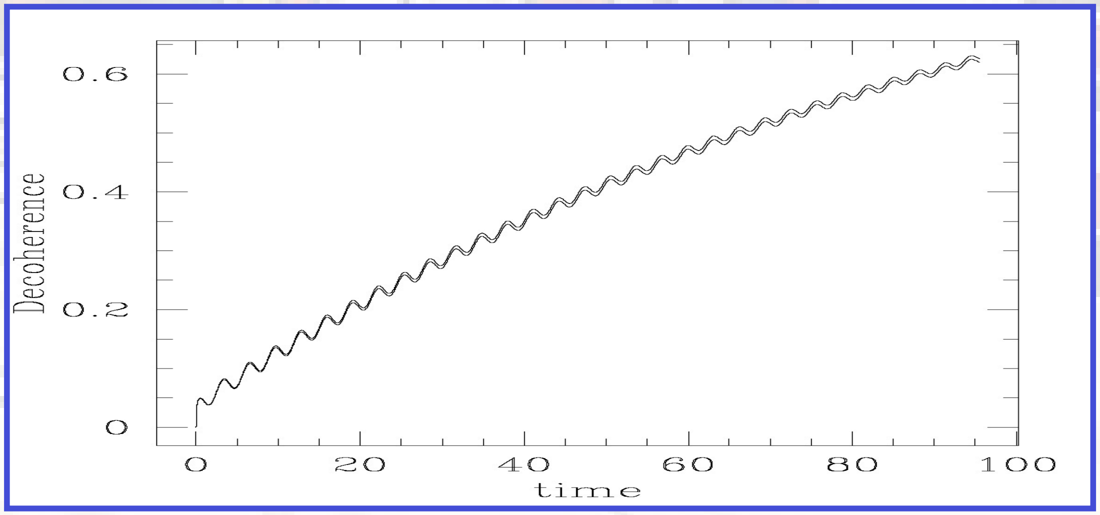
WARNING: DECOHERENCE TIMESCALE OBTAINED IN THE HIGH TEMPERATURE LIMIT IS ONLY AN APPROXIMATION!

$$\dot{W} = \{H_0, W\}_{MB} + D\partial^2{}_{pp} W + \cdots$$

$$W_{osc} \approx A(t)\cos(k_p p) \Rightarrow A(t) \approx \exp(-\Gamma t)$$

$$\Gamma = D k_p{}^2$$

EVOLUTION OF FRINGE VISIBILITY FACTOR IN AN ENVIRONMENT AT ZERO TEMPERATURE FOR QUANTUM BROWNIAN MOTION (NON-EXPONENTIAL DECAY: CAN BE UNDERSTOOD FROM TIME-DEPENDENCE OF COEFFICIENTS OF MASTER EQUATION)



POINTER STATES, DECOHERENCE TIMESCALE (III)

NOT ALL STATES ARE AFFECTED BY DECOHERENCE IN THE SAME WAY

QUESTION: WHAT ARE THE STATES WHICH ARE MOST ROBUST UNDER DECOHERENCE? (STATES WHICH ARE LESS SUSCEPTIBLE TO BECOME ENTANGLED WITH THE ENVIRONMENT)

POINTER STATES: STATES WHICH ARE MINIMALLY AFFECTED BY THE INTERACTION WITH THE ENVIRONMENT (MOST ROBUST STATES OF THE SYSTEM)

AN OPERATIONAL DEFINITION OF POINTER STATES:

"PREDICTABILITY SIEVE"

$$|\Psi(0)\rangle\langle\Psi(0)| \xrightarrow{\quad t \quad} \rho(t)$$

Initial state of the system (pure)

State of system at time t (mixed)

Information initially 'stored' in the system flows into correlations with the environment

$$S_{VN}(t) = -Tr\left(\rho(t)\ln\left(\rho(t)\right)\right), \quad \xi(t) = Tr\left(\rho^2(t)\right)$$

Measure information loss by entropy growth (or purity decay)

POINTER STATES, DECOHERENCE TIMESCALE (IV)

$$|\Psi(0)\rangle\langle\Psi(0)| \longrightarrow \rho(t)$$

Measure degradation of system's state with entropy (von Neuman) or purity decay

$$S_{VN}(t) = -Tr\left(\rho(t)\ln\left(\rho(t)\right)\right), \quad \xi(t) = Tr\left(\rho^2(t)\right)$$

These quantities depend on time AND on the initial state

PREDICTABILITY SIEVE: FIND THE INITIAL STATES SUCH THAT THESE QUANTITIES ARE MINIMIZED (FOR A DYNAMICAL RANGE OF TIMES)

PREDICTABILITY SIEVE IN A PHYSICALLY INTERESTING CASE?

ANALIZE QUANTUM BROWNIAN MOTION

USE MASTER EQUATION TO ESTIMATE PURITY DECAY OR ENTROPY GROWTH

$$\dot{\rho} = -i\left[H_R + \frac{m}{2}\delta\omega^2(t)x^2, \rho\right] - i\gamma(t)[x,\{p,\rho\}] - D(t)[x,[x,\rho]] - f(t)[x,[p,\rho]]$$

$$\dot{\xi} = 2Tr\left(\dot{\rho}\rho\right) = 2\gamma\xi + 2DTr\left([x,\rho]^2\right) + 2fTr\left([x,\rho][p,\rho]\right)$$

POINTER STATES, DECOHERENCE TIMESCALE (V)

A SIMPLE SOLUTION FROM THE PREDICTABILITY SIEVE CRITERION

$$\dot{\zeta} = 2Tr(\dot{\rho}\rho) = 2\gamma\zeta + 2DTr\left([x,\rho]^2\right) + 2fTr([x,\rho][p,\rho])$$

Approximations I: Neglect friction, use asymptotic form of coefficients, assume initial state is pure and apply perturbation theory:

$$\zeta(T) - \zeta(0) = 2D\int_0^T dt\, Tr\left([x(t),\rho]^2\right)$$

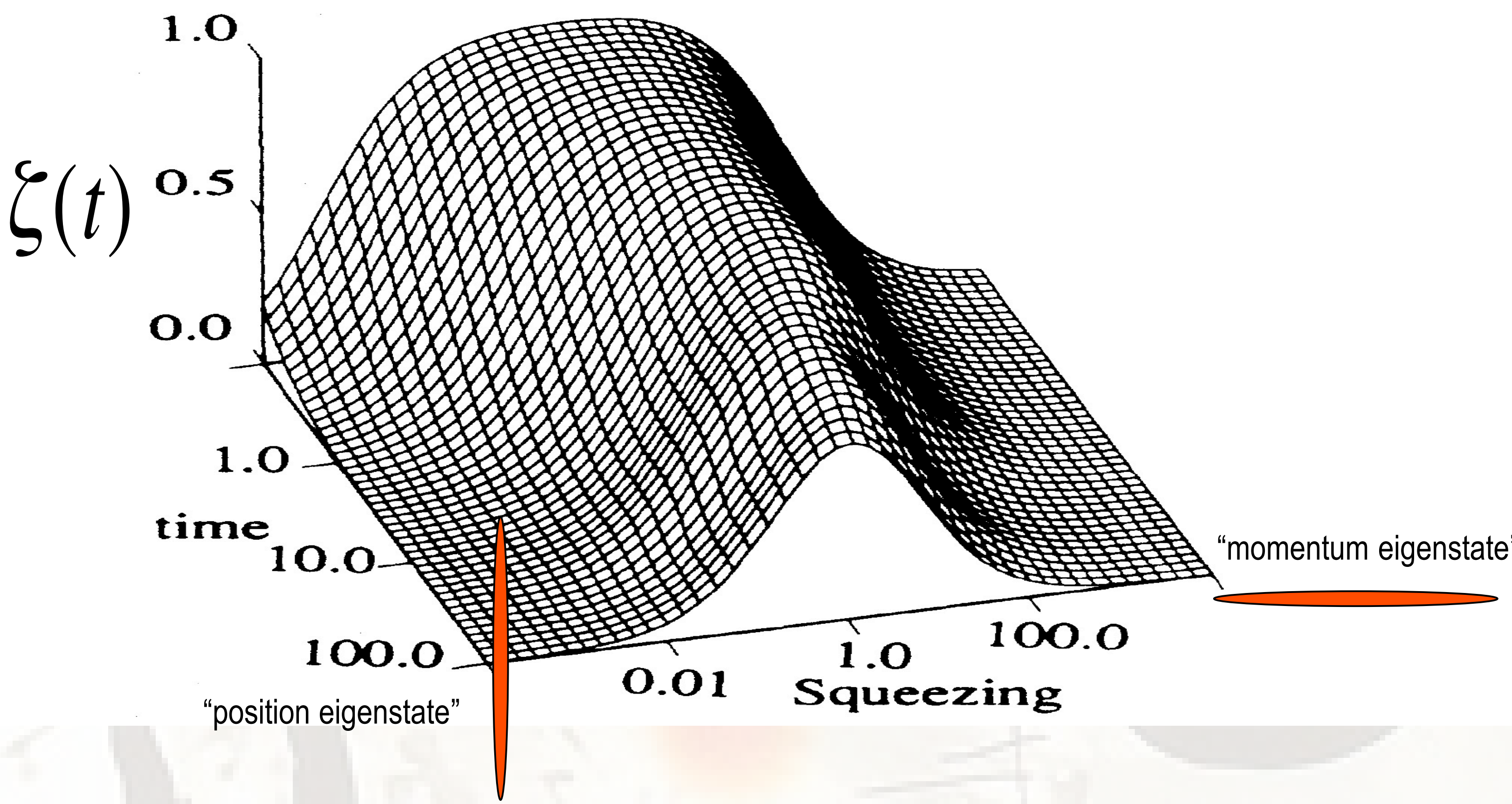
Approximations II: State remains approximately pure, average over oscillation period

$$\zeta(T) = \zeta(0) - 2D\left(\Delta x^2 + \frac{1}{m^2\omega^2}\Delta p^2\right)$$

Minimize over initial state: Pointer states for QBM are minimally uncertainty coherent states!
W.Zurek, J.P.P & S. Habib, PRL 70, 1187 (1993)

POINTER STATES, DECOHERENCE TIMESCALE (V)

A SIMPLE SOLUTION FROM THE PREDICTABILITY SIEVE CRITERION



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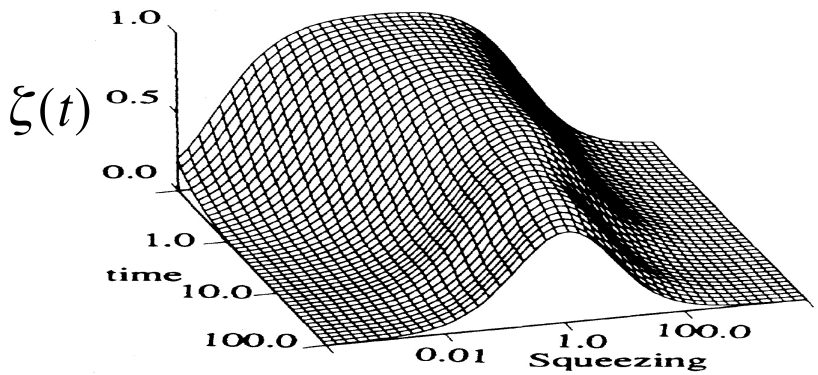
W.Zurek, J.P.P & S. Habib, PRL 70, 1187 (1993)

POINTER STATES, DECOHERENCE TIMESCALE (VI)

WARNING: DIFFERENT POINTER STATES IN DIFFERENT REGIMES!

1) Dynamical regime (QBM): Pointer basis results from interplay between system and environment

2) Static regime (Quantum Measurement): System's evolution is negligible, Pointer basis is determined by the interaction Hamiltonian (position in QBM)



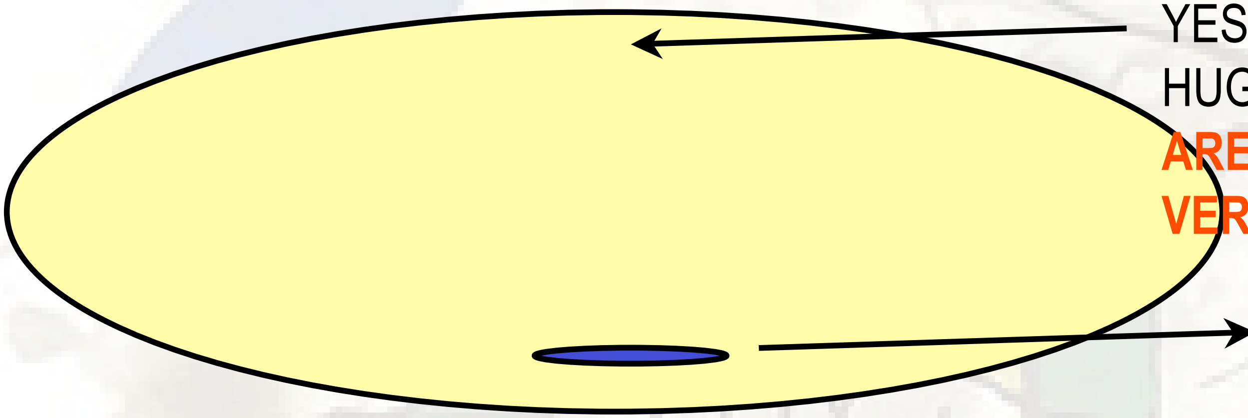
3) "Quantum" regime: The evolution of the environment is very "slow" (adiabatic environment): Pointer states are eigenstates of the Hamiltonian of the system! The environment only "learns" about properties of system which are non-vanishing when averaged in time. J.P. Paz & W.Zurek, PRL 82, 5181 (1999)

TAILOR MADE POINTER STATES? J.P. Paz, Nature 412, 869 (2001)

DECOHERENCE: AN OVERVIEW

SUMMARY: SOME BASIC POINTS ON DECOHERENCE

• DECOHERENCE AND THE QUANTUM-CLASSICAL TRANSITION:



YES: HILBERT SPACE IS HUGE, **BUT MOST STATES ARE UNSTABLE!! (DECAY VERY FAST INTO MIXTURES)**

CLASSICAL STATES: A (VERY!) SMALL SUBSET. **THEY ARE THE POINTER STATES OF THE SYSTEM DYNAMICALLY CHOSEN BY THE ENVIRONMENT**

•POINTER STATES: W.Zurek, S. Habib & J.P. Paz, PRL 70, 1187 (1993), J.P. Paz & W. Zurek, PRL 82, 5181 (1999)

•TIMESCALES: J.P. Paz, S. Habib & W. Zurek, PRD 47, 488 (1993), J. Anglin, J.P. Paz & W. Zurek, PRA 55, 4041 (1997)

•CONTROLLED DECOHERENCE EXPERIMENTS: Zeillinger et al (Vienna) PRL 90 160401 (2003), Haroche et al (ENS) PRL 77, 4887 (1997), Wineland et al (NIST), Nature 403, 269 (2000).

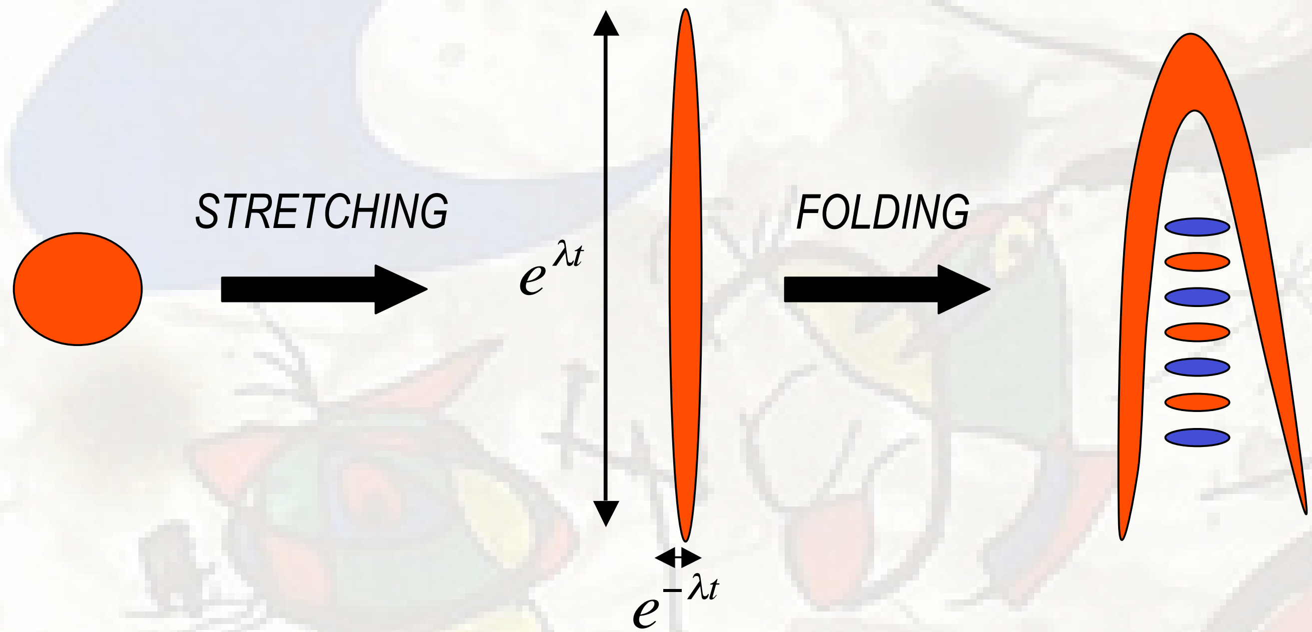
DECOHERENCE: AN OVERVIEW (VIII)

LAST DECADE: MANY QUESTIONS ON DECOHERENCE WERE ADDRESSED AND ANSWERED

- NATURE OF POINTER STATES: QUANTUM SUPERPOSITIONS DECAY INTO MIXTURES WHEN QUANTUM INTERFERENCE IS SUPRESSED. WHAT ARE THE STATES SELECTED BY THE INTERACTION? POINTER STATES: THE MOST STABLE STATES OF THE SYSTEM, DYNAMICALLY SELECTED BY THE ENVIRONMENT: W.Zurek, S. Habib & J.P. Paz, PRL 70, 1187 (1993), J.P. Paz & W. Zurek, PRL 82, 5181 (1999)
- TIMESCALES: HOW FAST DOES DECOHERENCE OCCURS? J.P. Paz, S. Habib & W. Zurek, PRD 47, 488 (1993), J. Anglin, J.P. Paz & W. Zurek, PRA 55, 4041 (1997)
- DECOHERENCE FOR CLASSICALLY CHAOTIC SYSTEMS: W. Zurek & J.P. Paz, PRL 72, 2508 (1994), D. Monteoliva & J.P. Paz, PRL 85, 3373 (2000).
- CONTROLLED DECOHERENCE EXPERIMENTS: S. Haroche et al (ENS) PRL 77, 4887 (1997), D. Wineland et al (NIST), Nature 403, 269 (2000), A. Zeillinger et al (Vienna) PRL 90 160401 (2003),
- ENVIRONMENT ENGENEERING: J.P. Paz, Nature 412, 869 (2001)

DECOHERENCE FOR CLASSICALLY CHAOTIC SYSTEMS (I)

SYSTEMS WITH CLASSICALLY CHAOTIC HAMILTONIANS ARE VERY EFFICIENT IN GENERATING “SCHRODINGER CAT” STATES



OSCILLATIONS APPEAR BECAUSE THE EVOLUTION EQUATION FOR WIGNER FUNCTION DIFFERS FROM THAT OF A CLASSICAL DISTRIBUTION

$$\dot{W} = \left\{H_0, W\right\}_{MB} = \left\{H_0, W\right\}_{PB} + \sum_{n\geq 1} \frac{(-1)^n \hbar^{2n}}{2^{2n}(2n+1)!} \partial_x^{\,2n+1} V \partial_p^{\,2n+1} W$$

DECOHERENCE FOR CLASSICALLY CHAOTIC SYSTEMS (II)

A CASE STUDY: HARMONICALLY DRIVEN QUARTIC DOUBLE WELL

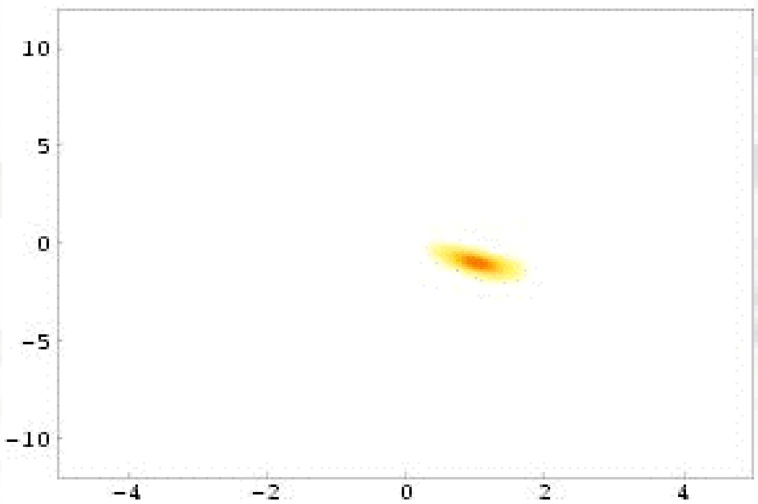
$$V(x) = -a\,x^2 + b\,x^4 + cx\cos(\omega\,t)$$

DECOHERENCE FOR CLASSICALLY CHAOTIC SYSTEMS (II)

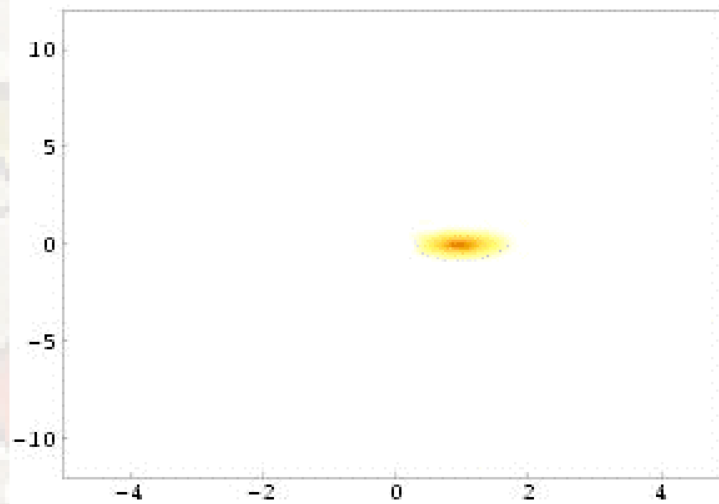
A CASE STUDY: HARMONICALLY DRIVEN QUARTIC DOUBLE WELL

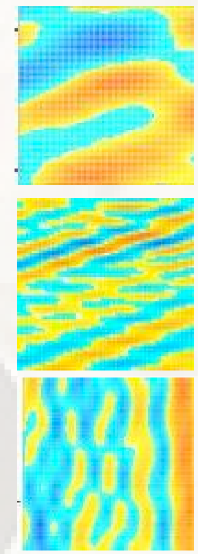
$$V(x) = -a\,x^2 + b\,x^4 + cx\cos(\omega\,t)$$

CLASSICAL



QUANTUM





WIGNER FUNCTION DEVELOPS OSCILLATORY STRUCTURE AT SUB-PLANCK SCALES

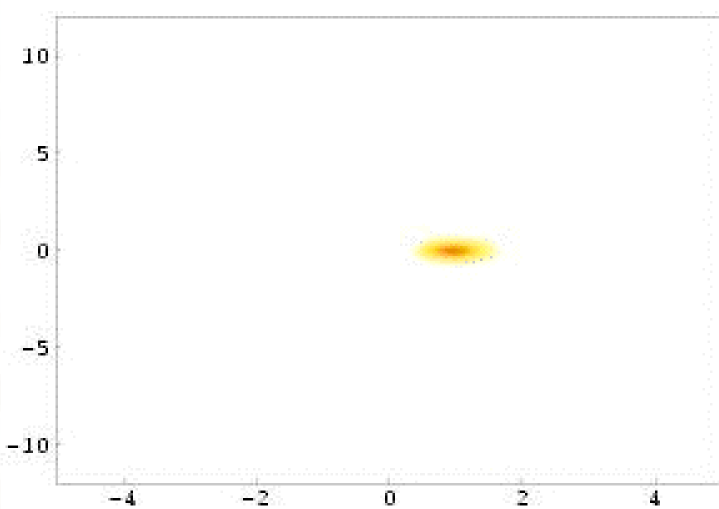
$$\lambda_X \approx \hbar / P, \quad \lambda_P \approx \hbar / L \Rightarrow \quad A \approx \lambda_X \lambda_P \approx \hbar \frac{\hbar}{LP} << \hbar$$

W.H. Zurek, Nature 412, 712 (2001); D. Monteoliva & J.P. Paz, PRE 64, 056238 (2002)

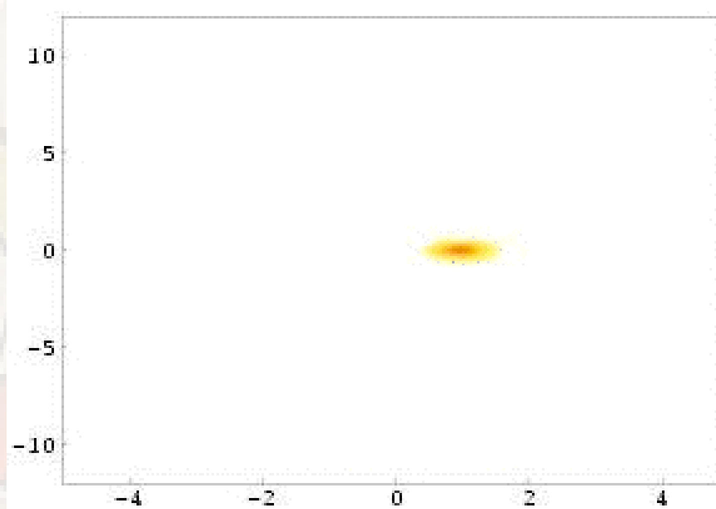
DECOHERENCE FOR CLASSICALLY CHAOTIC SYSTEMS (III)

DECOHERENCE DESTROYS THE FRINGES THAT ARE DYNAMICALLY PRODUCED

CLASSICAL + NOISE



QUANTUM + DECOHERENCE



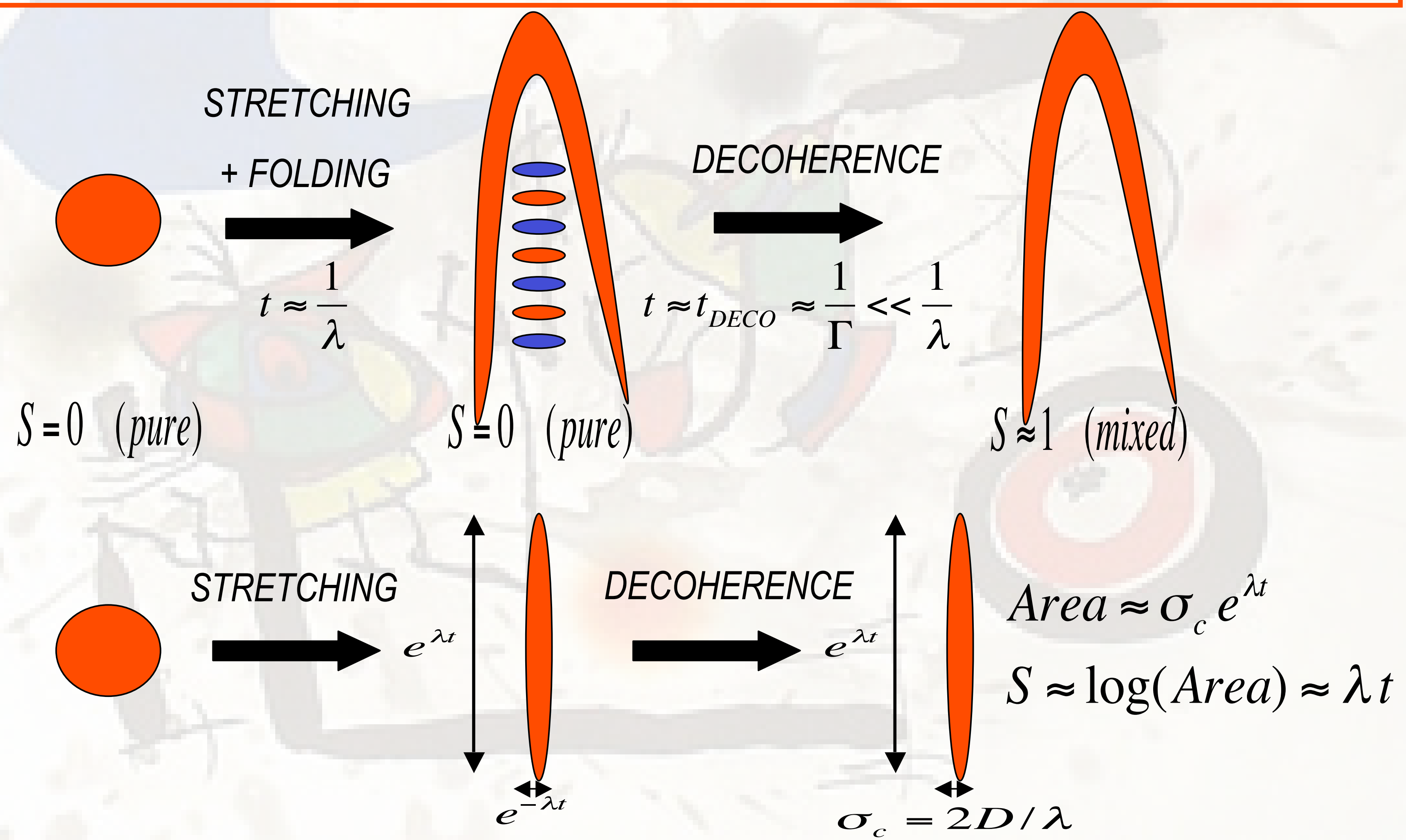
DECOHERENCE IMPLIES INFORMATION TRANSFER INTO CORRELATIONS BETWEEN SYSTEM AND ENVIRONMENT. WHAT IS THE RATE?:

Lyapunov regime: Above a certain threshold, rate of entropy production fixed by the Lyapunov exponent of the system (W. Zurek & J.P.P., 1994)

DECOHERENCE FOR CLASSICALLY CHAOTIC SYSTEMS (IV)

INTUITIVE EXPLANATION: WHY IS THERE A REGIME OF ENTROPY GROWTH FIXED BY THE LYAPUNOV EXPONENT?

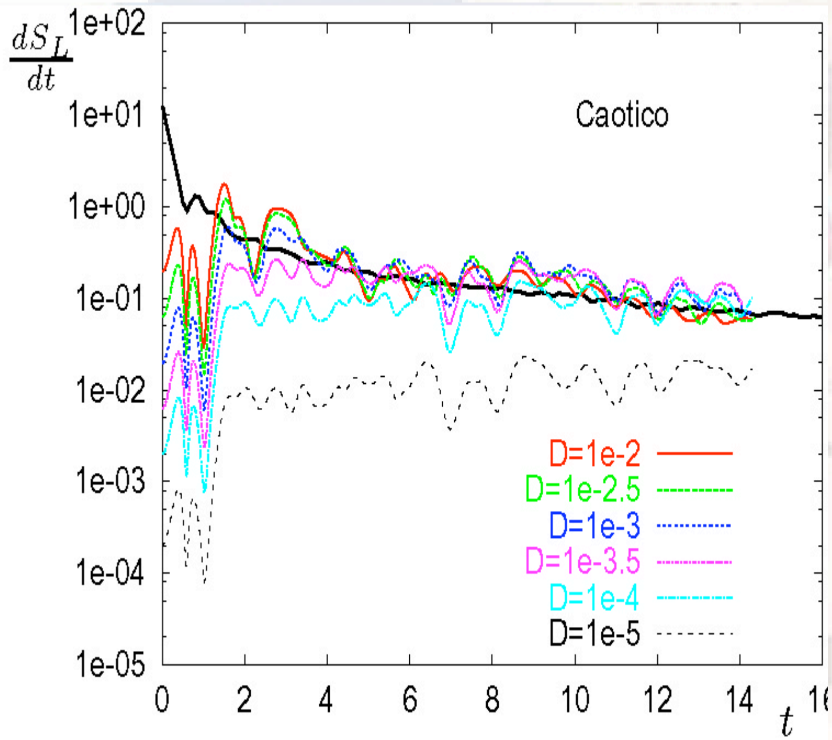
$$S_{VN} = -Tr\left(\rho \log(\rho)\right), \quad S_{LIN} = -\log\left(Tr\left(\rho^2\right)\right), \quad Tr\left(\rho^2\right) = 2\pi\hbar \int dx\, dp\, W^2(x,p)$$

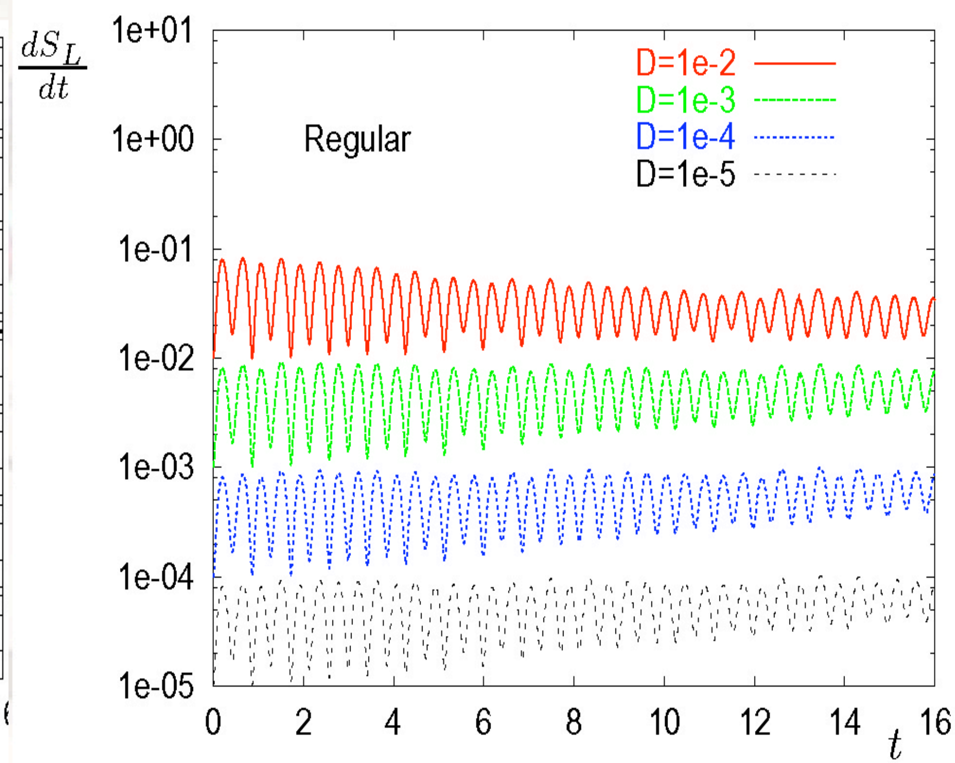


DECOHERENCE FOR CLASSICALLY CHAOTIC SYSTEMS (V)

NUMERICAL EVIDENCE IS RATHER STRONG

(DRIVEN DOUBLE WELL, D. Monteoliva and J.P.P., Phys. Rev. E (2005))

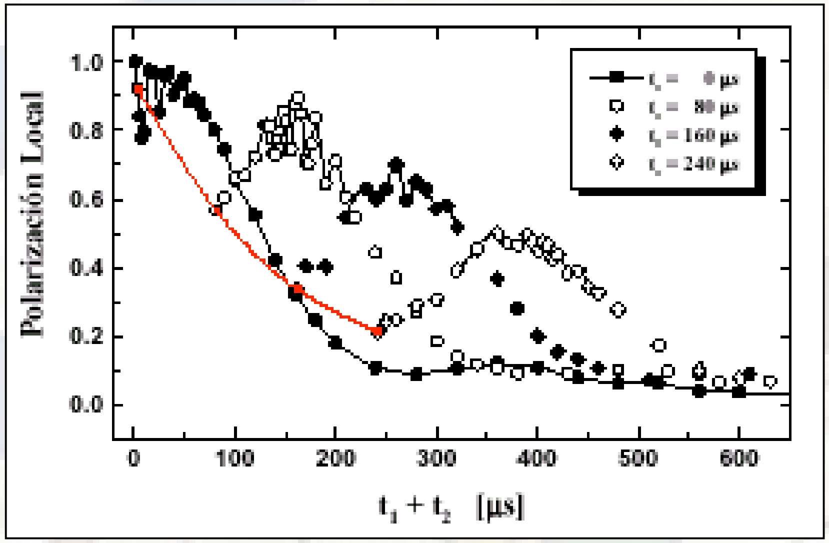




LYAPUNOV REGIME EXISTS: DECOHERENCE RATE BECOMES INDEPENDENT OF THE COUPLING STRENGTH ABOVE SOME THRESHOLD

DECOHERENCE FROM COMPLEX ENVIRONMENTS (I)

EXPERIMENTS ALWAYS BRING NEW SURPRISES: Polarization echo in NMR (solid state) decays as a Gaussian with a width independent of the coupling strength with the environment!



P. R. Levstein, G. Usaj and H. M. Pastawski, J. Chem. Phys. 108, 2718 (1998)

G. Usaj, H. M. Pastawski P. R. Levstein, Mol. Phys.95, 1229 (1998)

H. M. Pastawski, P. R. Levstein, G. Usaj, J. Raya and J. Hirschinger, Physica A 283, 166 (2000)

Can we understand this?

THE QUESTION:

Are there physical situations where the decay is Gaussian with a width which becomes “universal”? (i.e., independent of the coupling strength, above a certain threshold)

THE ANSWER: Yes, and we can develop a simple model for them

DECOHERENCE FROM COMPLEX ENVIRONMENTS (II)

ATTEMPTS TO EXPLAIN POLARIZATION DECAY (“DECAY OF LOSCHMIDT ECHO”): SYSTEM & ENVIRONMENT MAY BE “CHAOTIC”

POLARIZATION DECAY HAS VARIOUS REGIMES:

$$M(t) = a\exp(-\Gamma\, t) + b\exp(-\lambda\, t)$$

FERMI GOLDEN RULE (FGR) REGIME: PERTURBATION DEPENDENT RATE $\Gamma \propto \Delta^2$

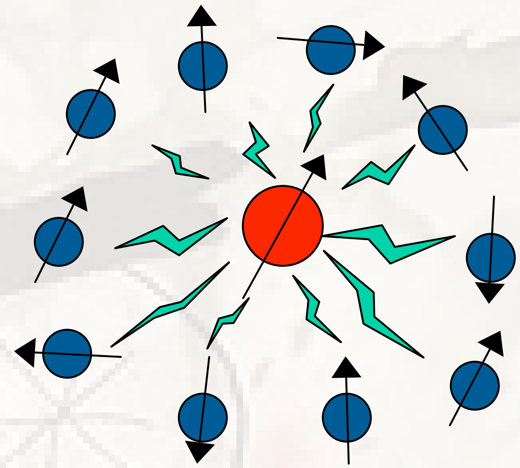
LYAPUNOV REGIME: PERTURBATION INDEPENDENT RATE $\lambda =$ LYAPUNOV EXPONENT

BUT: DECAY IS EXPONENTIAL

R. Jalabert and H. Pastawski, PRL 86, 2490 (2001), F. Cucchietti et al, PRE 65 045206 (2002), F. Cucchietti, D. Dalvit, J.P.P., W. Zurek; PRL 91, 210403 (2003)

DECOHERENCE FROM COMPLEX ENVIRONMENTS (III)

Here: Describe and analyze a simple model where decoherence is not only Gaussian but also displays universality (independence of coupling strength above a threshold). Model: critical spin environment



Gaussian decoherence from spin environments

F. Cucchietti, J.P.P. & W.H. Zurek; Phys Rev A 72, 052113 (2005)

$$H = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \lambda_0 |0\rangle\langle 0| \sum_{i=1}^{N} \sigma_i^x + \lambda_1 |1\rangle\langle 1| \sum_{i=1}^{N} \sigma_i^x$$

DECOHERENCE: OVERLAP BETWEEN TWO STATES OF THE ENVIRONMENT OBTAINED BY EVOLVING WITH TWO DIFFERENT HAMILTONIANS

$$M(t) = \left|\langle \varepsilon(0)| \exp(iH_1 t/\hbar) \exp(-iH_0 t/\hbar) |\varepsilon(0)\rangle\right|^2$$

$$|\varepsilon_0(t)\rangle \approx \exp(-iH_0 t/\hbar)|\varepsilon(0)\rangle$$

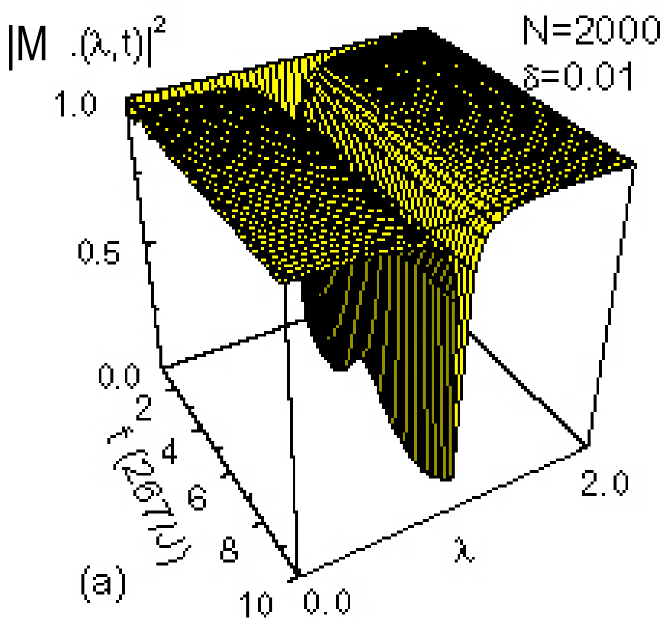
$$|\varepsilon_1(t)\rangle \approx \exp(-iH_1 t/\hbar)|\varepsilon(0)\rangle$$

$$H_l = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \lambda_l \sum_{i=1}^{N} \sigma_i^x$$

DECOHERENCE FROM COMPLEX ENVIRONMENTS (IV)

Results 1: A critical environment is very efficient in producing decoherence

see also H.T. Quan et al, quant-ph/0509007



$$H_l = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \lambda_l \sum_{i=1}^{N} \sigma_i^x$$

$$\lambda_{0,1} = \lambda \pm \delta; \quad |\varepsilon(0)\rangle = |ground\rangle_0$$

When the environment is critical decoherence is very strong (other wise it is moderate).

How does decoherence depends on the coupling strength?

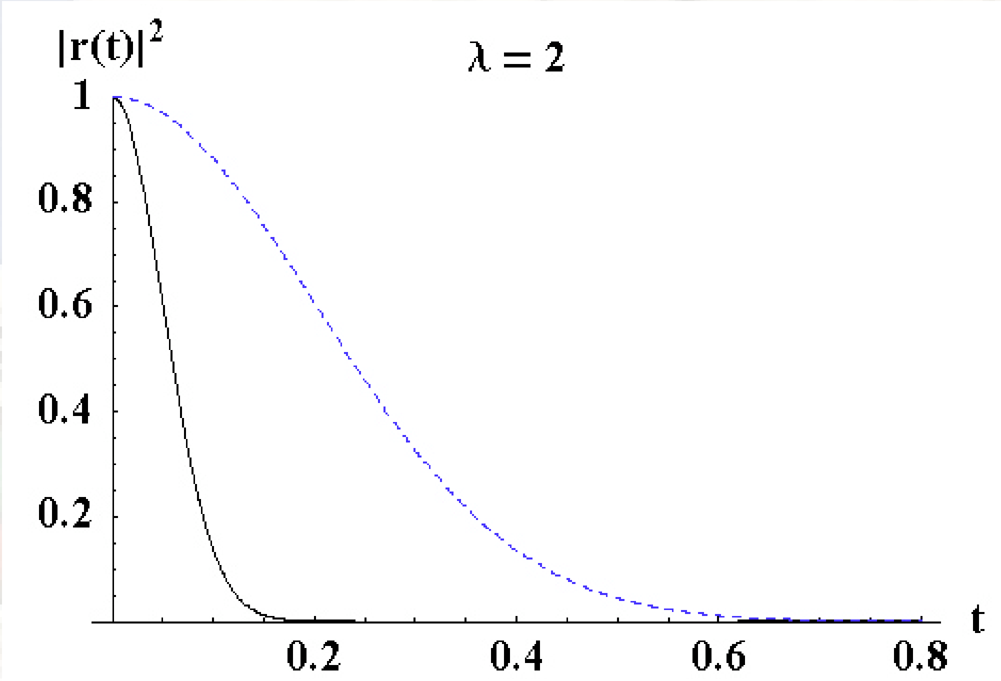
$$H_l = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \lambda_l \sum_{i=1}^{N} \sigma_i^x$$

$$\lambda_0 = 0, \quad \lambda_1 = \lambda \quad |\varepsilon(0)\rangle = |ground\rangle_0$$

DECOHERENCE FROM COMPLEX ENVIRONMENTS (V)

Results 2: A critical environment produces "universal" decoherence

F. Cucchietti, S. Fernandez-Vidal and J.P.P. (2006) to be posted



To a high degree of accuracy the following formula applies:

$$\left|r(t)\right|^2 = \exp\left(-Nt^2/2\right)\cos^{2N}\left(\lambda t\right)$$

DECOHERENCE FROM COMPLEX ENVIRONMENTS (VI)

Why? *F. Cucchietti, S. Fernandez-Vidal and J.P.P. (2006)*

$$|r(t)|^2 = \left|\langle \varepsilon(0)|\exp(iH_1 t/\hbar)\exp(-iH_0 t/\hbar)|\varepsilon(0)\rangle\right|^2 = \left|\langle \varepsilon(0)|\exp(iH_1 t/\hbar)|\varepsilon(0)\rangle\right|^2$$

Jordan-Wigner + Bogolubov: Diagonalize both Hamiltonians

$$H_1 = \sum_k \varepsilon^{(A)}{}_k \left(A_k{}^+ A_k - 1/2\right) \qquad H_0 = \sum_k \varepsilon^{(B)}{}_k \left(B_k{}^+ B_k - 1/2\right)$$

Creation and anihillation operators can be related (vacuum states too)

$$A_k = \cos(\varphi_k) B_k - i\sin(\varphi_k) B_{-k}{}^+ \qquad |0\rangle_0 = \prod_k \left(i\cos(\varphi_k) + \sin(\varphi_k) A_k{}^+ A_{-k}{}^+\right)|0\rangle_1$$

Creation and anihillation operators can be related (vacuum states too)

$$r(t) = \prod_k \left(\cos^2(\varphi_k)\, e^{i\varepsilon_k{}^{(A)} t} + \sin^2(\varphi_k) e^{-i\varepsilon_k{}^{(A)} t}\right)$$

$$\varepsilon_k{}^{(A)} = \sqrt{1 + \lambda^2 - 2\lambda\cos 2\pi k/N}, \quad 2\varphi_k = \theta_k(\lambda) - \theta_k(0), \quad tg\theta_k(\lambda) = \frac{\sin 2\pi k/N}{\cos 2\pi k/N - \lambda}$$

Angles are uniformly distributed, energies are spread in the interval $(|\lambda - 1|, |\lambda = 1|)$

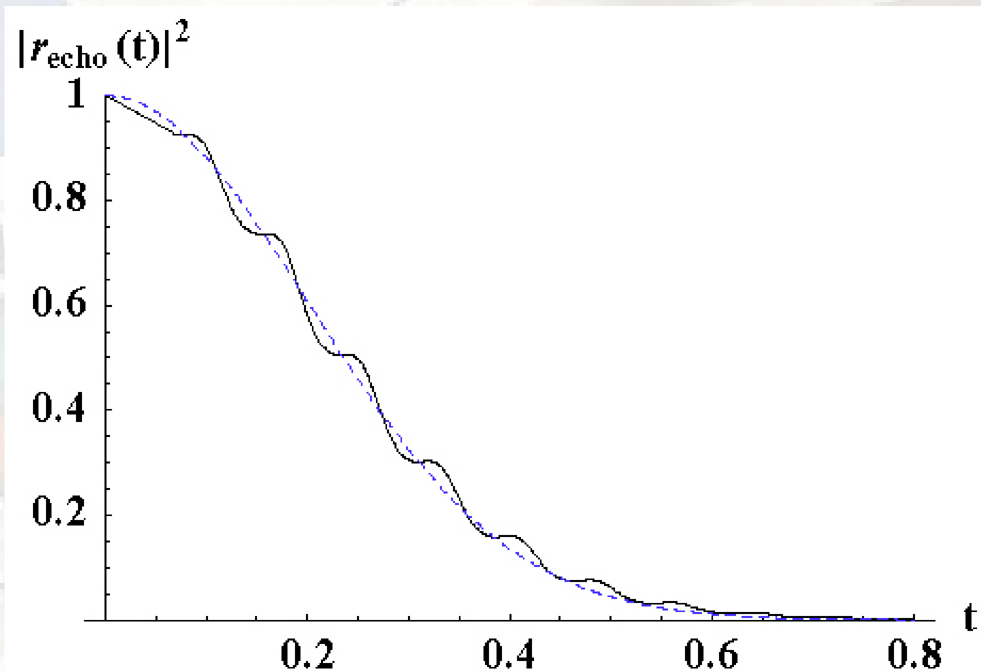
$$\Delta\varepsilon_k{}^{(A)} \leftarrow \quad |r(t)|^2 = \exp\left(-Nt^2/2\right)\cos^{2N}(\lambda t) \quad \rightarrow \bar{\varepsilon}_k{}^{(A)}$$

DECOHERENCE FROM COMPLEX ENVIRONMENTS (VII)

Results 3: Oscillations dissapear in an spin-echo experiment

t=0 — **t=T/2** — **t=T**

$$H_l = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \lambda_l \sum_{i=1}^{N} \sigma_i^x \qquad\qquad H_l = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z - \lambda_l \sum_{i=1}^{N} \sigma_i^x$$

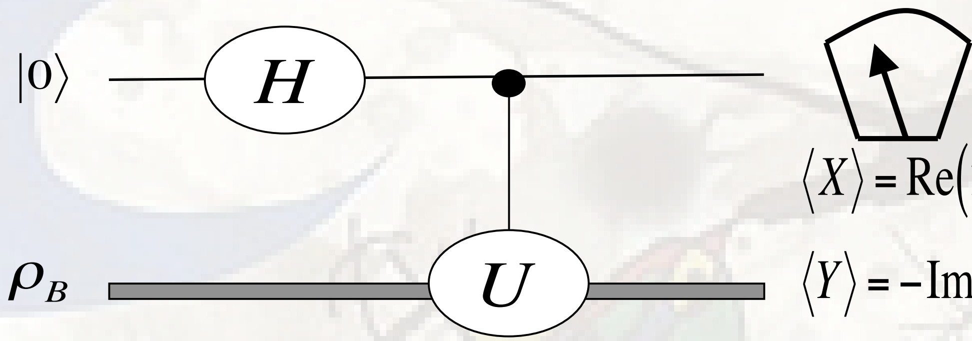


To a high degree of accuracy the following formula applies:

$$|r(t)|^2 = \exp\left(-Nt^2/2\right)\left(1 + O(1/N)\sin\lambda t/\lambda\right)$$

THE POWER OF A SINGLE QUBIT

SIMPLE SCHEME TO USE ONE QUBIT TO LEARN ABOUT PROPERTIES OF A MORE COMPLEX SYSTEM (CAN BE USED BOTH FOR TOMOGRAPHY AND SPECTROSCOPY)



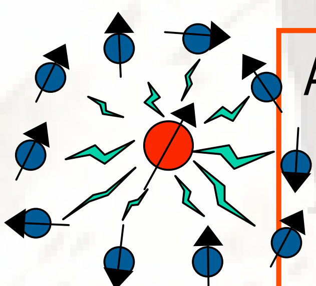
$$\langle X\rangle = \mathrm{Re}\left(Tr_B\left(\rho_B U\right)\right)$$

$$\langle Y\rangle = -\mathrm{Im}\left(Tr_B\left(\rho_B U\right)\right)$$

Know	ρ	: Use circuit to learn about	U	(“spectrometer”)
Know	U	: Use circuit to learn about	ρ	(“tomographer”)

E. Knill & R. Laflamme, Phys. Rev. Lett. 81, 5672 (1998)

C. Miquel, J.P.P., M. Saraceno, E. Knill, R. Laflamme, C. Negrevergne, Nature 418, 59 (2002)



A new ‘spectroscopic’ application for this scheme: Measure universal features (i.e. Gaussian decay with ‘constant’ width) in the decay of quantum coherence in one qubit: an indicator of a quantum phase transition in the environment

SUMMARY

• DECOHERENCE IS A CRUCIAL INGREDIENT TO UNDERSTAND THE EMERGENCE OF CLASSICALITY

• DECOHERENCE IS THE ENEMY TO DEFEAT TO ACHIEVE QUANTUM INFORMATION PROCESSING.

• TO IMPLEMENT QUANTUM ERROR CORRECTION TOOLS WE MUST HAVE A VERY GOOD CHARACTERIZATION OF ERRORS AND DECOHERENCE IN THE DEVICES.

• SOME SYSTEMS EXHIBITS SOME UNIVERSAL FEATURES WHEN THEY DECOHERE , I.E. LYAPUNOV REGIME (INDEPENDENCE OF SYSTEM-ENVIRONMENT COUPLING)

• CRITICAL ENVIRONMENTS ARE HIGHLY EFFICIENT IN INDUCING DECOHERENCE CRITICAL ENVIRONMENT MAY INDUCE UNIVERSAL DECOHERENCE

• USE A SINGLE QUBIT AS AN INDICATOR OF A QUANTUM PHASE TRANSITION? (MAYBE)