DECOHERENCE AND QUANTUM-CLASSICAL TRANSITION IN QUANTUM INFORMATION

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Lecture 1: Decoherence and the quantum origin of the classical world

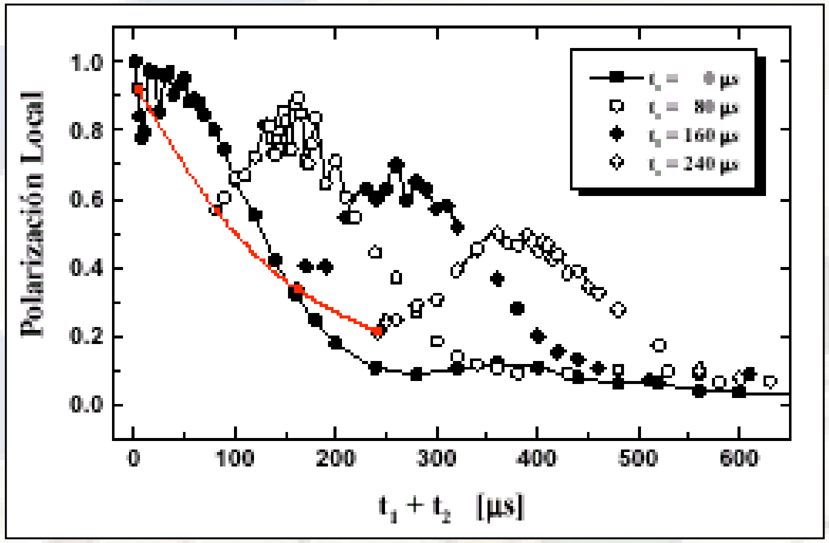
Lecture 2: Decoherence and quantum information processing, models and examples

- Decoherence, an overview
- Decoherence for classically chaotic systems. Why is it interesting, why is it different.
- Decoherence from complex environments
- Using qubits to learn about environmental properties (power of one qubit: use a single qubit to learn about properties of many)

Colaborations with: W. Zurek (LANL), M. Saraceno (CNEA), D. Mazzitelli (UBA), D. Monteoliva, C. Miquel (UBA), P. Bianucci (UBA, UT), L. Davila (UEA, UK), C. Lopez (UBA, MIT), A. Roncaglia (UBA, LANL), J. Anglin (MIT), R. Laflamme (IQC), S. Fernandez-Vidal (UAB), F. Cucchietti (LANL),

DECOHERENCE FROM COMPLEX ENVIRONMENTS (I)

EXPERIMENTS ALWAYS BRING NEW SURPRISES: Polarization echo in NMR (solid state) decays as a Gaussian with a width independent of the coupling strength with the environment!



P. R. Levstein, G. Usaj and H. M. Pastawski, J. Chem. Phys. 108, 2718 (1998)

G. Usaj, H. M. Pastawski P. R. Levstein, Mol. Phys.95, 1229 (1998)

H. M. Pastawski, P. R. Levstein, G. Usaj, J. Raya and J. Hirschinger, Physica A 283, 166 (2000)

Can we understand this?

THE QUESTION:

Are there physical situations where the decay is Gaussian with a width which becomes “universal”? (i.e., independent of the coupling strength, above a certain threshold)

THE ANSWER: Yes, and we can develop a simple model for them

DECOHERENCE FROM COMPLEX ENVIRONMENTS (II)

ATTEMPTS TO EXPLAIN POLARIZATION DECAY ("DECAY OF LOSCHMIDT ECHO"): SYSTEM & ENVIRONMENT MAY BE "CHAOTIC"

POLARIZATION DECAY HAS VARIOUS REGIMES:

$$M(t) = a\exp(-\Gamma t) + b\exp(-\lambda t)$$

FERMI GOLDEN RULE (FGR) REGIME: PERTURBATION DEPENDENT RATE

$$\Gamma \propto \Delta^2$$

LYAPUNOV REGIME: PERTURBATION INDEPENDENT RATE

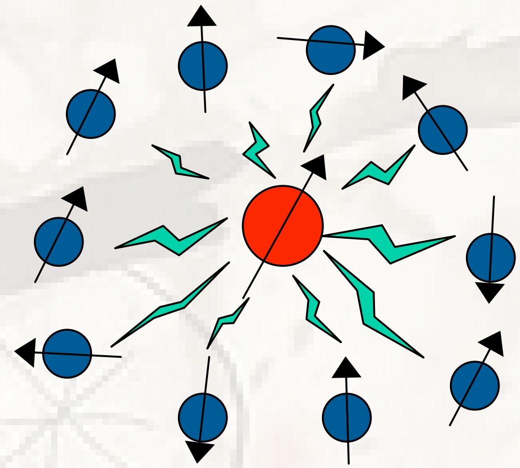
$\lambda =$ LYAPUNOV EXPONENT

BUT: DECAY IS EXPONENTIAL

R. Jalabert and H. Pastawski, PRL 86, 2490 (2001), F. Cucchietti et al, PRE 65 045206 (2002), F. Cucchietti, D. Dalvit, J.P.P., W. Zurek; PRL 91, 210403 (2003)

DECOHERENCE FROM COMPLEX ENVIRONMENTS (III)

Here: Describe and analyze a simple model where decoherence is not only Gaussian but also displays universality (independence of coupling strength above a threshold). Model: critical spin environment



Gaussian decoherence from spin environments

F. Cucchietti, J.P.P. & W.H. Zurek; Phys Rev A 72, 052113 (2005)

$$H = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \lambda_0 |0\rangle\langle 0| \sum_{i=1}^{N} \sigma_i^x + \lambda_1 |1\rangle\langle 1| \sum_{i=1}^{N} \sigma_i^x$$

DECOHERENCE: OVERLAP BETWEEN TWO STATES OF THE ENVIRONMENT OBTAINED BY EVOLVING WITH TWO DIFFERENT HAMILTONIANS

$$M(t) = \left|\langle \varepsilon(0)| \exp(iH_1 t/\hbar) \exp(-iH_0 t/\hbar) |\varepsilon(0)\rangle\right|^2$$

$$|\varepsilon_0(t)\rangle \approx \exp(-iH_0 t/\hbar)|\varepsilon(0)\rangle$$

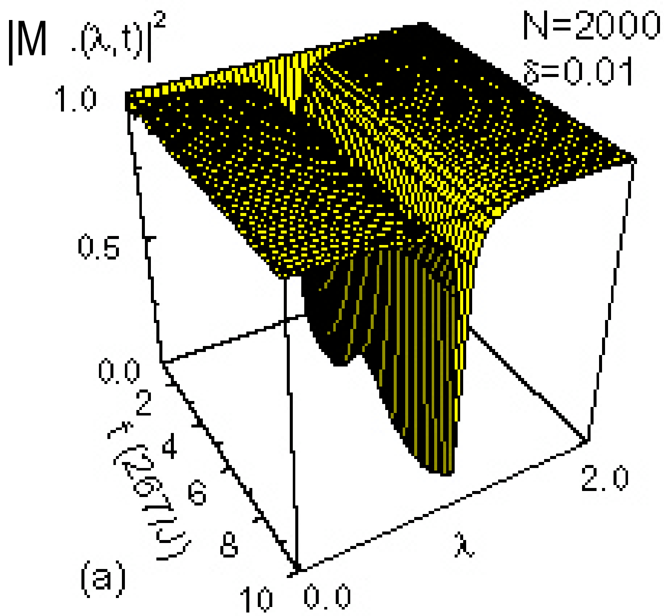
$$|\varepsilon_1(t)\rangle \approx \exp(-iH_1 t/\hbar)|\varepsilon(0)\rangle$$

$$H_l = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \lambda_l \sum_{i=1}^{N} \sigma_i^x$$

DECOHERENCE FROM COMPLEX ENVIRONMENTS (IV)

Results 1: A critical environment is very efficient in producing decoherence

see also H.T. Quan et al, quant-ph/0509007



$$H_l = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \lambda_l \sum_{i=1}^{N} \sigma_i^x$$

$$\lambda_{0,1} = \lambda \pm \delta; \quad \left|\varepsilon(0)\right\rangle = \left|ground\right\rangle_0$$

When the environment is critical decoherence is very strong (other wise it is moderate).

How does decoherence depends on the coupling strength?

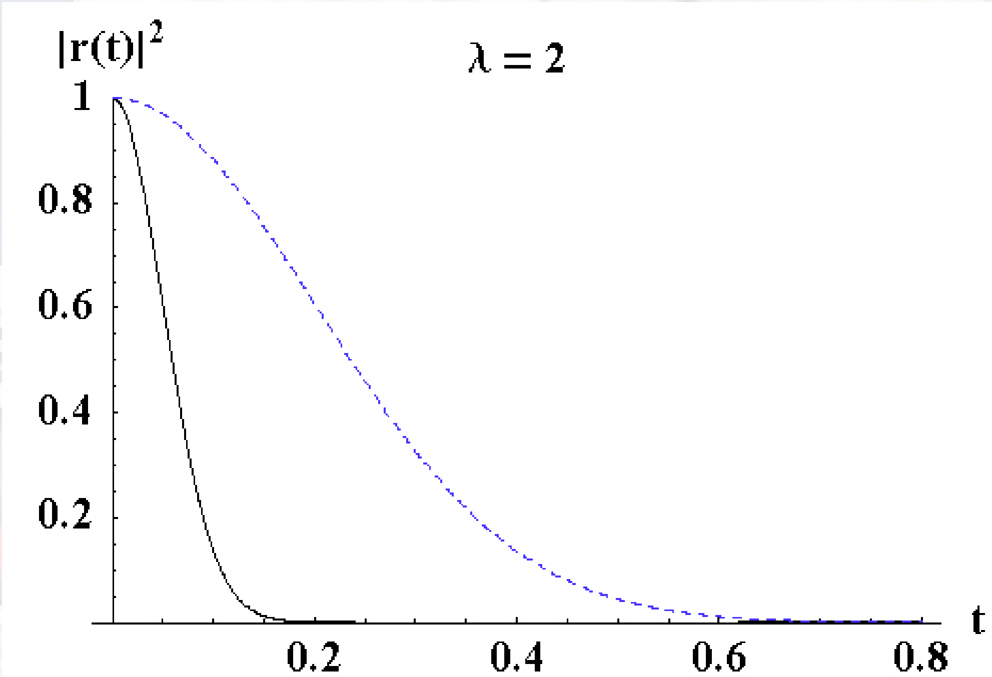
$$H_l = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \lambda_l \sum_{i=1}^{N} \sigma_i^x$$

$$\lambda_0 = 0, \quad \lambda_1 = \lambda \quad \left|\varepsilon(0)\right\rangle = \left|ground\right\rangle_0$$

DECOHERENCE FROM COMPLEX ENVIRONMENTS (V)

Results 2: A critical environment produces “universal” decoherence

F. Cucchietti, S. Fernandez-Vidal and J.P.P. (2006) to be posted



To a high degree of accuracy the following formula applies:

$$|r(t)|^2 = \exp\left(-Nt^2/2\right)\cos^{2N}(\lambda t)$$

DECOHERENCE FROM COMPLEX ENVIRONMENTS (VI)

Why? *F. Cucchietti, S. Fernandez-Vidal and J.P.P. (2006)*

$$|r(t)|^2 = \left|\langle\varepsilon(0)|\exp(iH_1 t/\hbar)\exp(-iH_0 t/\hbar)|\varepsilon(0)\rangle\right|^2 = \left|\langle\varepsilon(0)|\exp(iH_1 t/\hbar)|\varepsilon(0)\rangle\right|^2$$

Jordan-Wigner + Bogolubov: Diagonalize both Hamiltonians

$$H_1 = \sum_k \varepsilon^{(A)}{}_k\left(A_k{}^+ A_k - 1/2\right) \qquad H_0 = \sum_k \varepsilon^{(B)}{}_k\left(B_k{}^+ B_k - 1/2\right)$$

Creation and anihillation operators can be related (vacuum states too)

$$A_k = \cos(\varphi_k)B_k - i\sin(\varphi_k)B_{-k}{}^+ \qquad |0\rangle_0 = \prod_k\left(i\cos(\varphi_k) + \sin(\varphi_k)A_k{}^+ A_{-k}{}^+\right)|0\rangle_1$$

Creation and anihillation operators can be related (vacuum states too)

$$r(t) = \prod_k\left(\cos^2(\varphi_k)\, e^{i\varepsilon_k{}^{(A)}t} + \sin^2(\varphi_k)e^{-i\varepsilon_k{}^{(A)}t}\right)$$

$$\varepsilon_k{}^{(A)} = \sqrt{1+\lambda^2 - 2\lambda\cos 2\pi k/N}, \quad 2\varphi_k = \theta_k(\lambda) - \theta_k(0), \quad tg\theta_k(\lambda) = \frac{\sin 2\pi k/N}{\cos 2\pi k/N - \lambda}$$

Angles are uniformly distributed, energies are spread in the interval $(|\lambda-1|, |\lambda=1|)$

$$|r(t)|^2 = \exp\left(-Nt^2/2\right)\cos^{2N}(\lambda t)$$

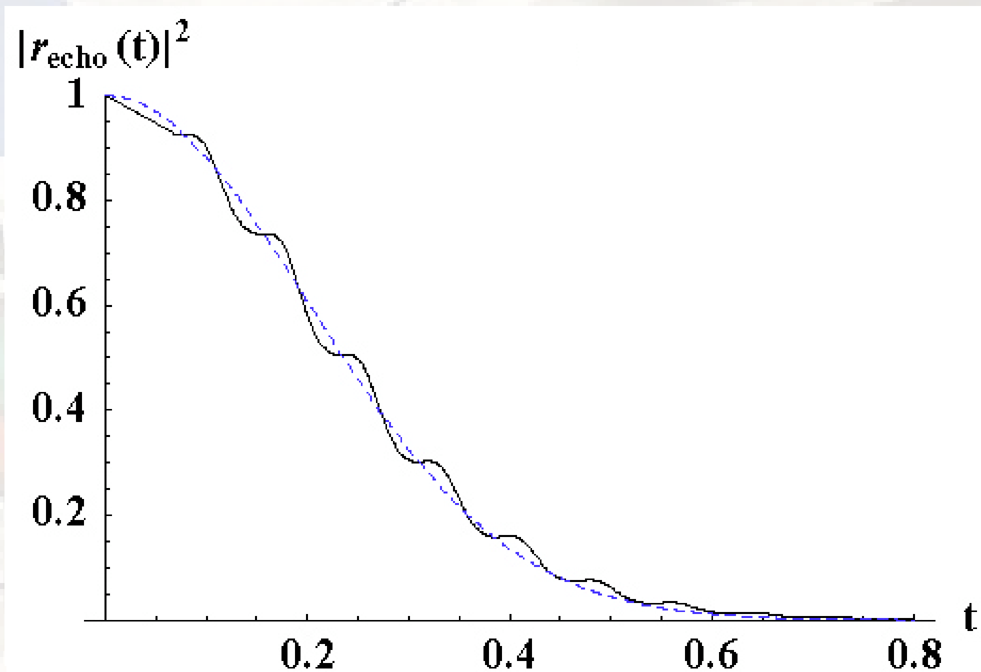
$$\Delta\varepsilon_k{}^{(A)} \qquad \bar{\varepsilon}_k{}^{(A)}$$

DECOHERENCE FROM COMPLEX ENVIRONMENTS (VII)

Results 3: Oscillations dissapear in an spin-echo experiment

t=0 t=T/2 t=T

$$H_l = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + \lambda_l \sum_{i=1}^{N} \sigma_i^x \qquad\qquad H_l = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z - \lambda_l \sum_{i=1}^{N} \sigma_i^x$$

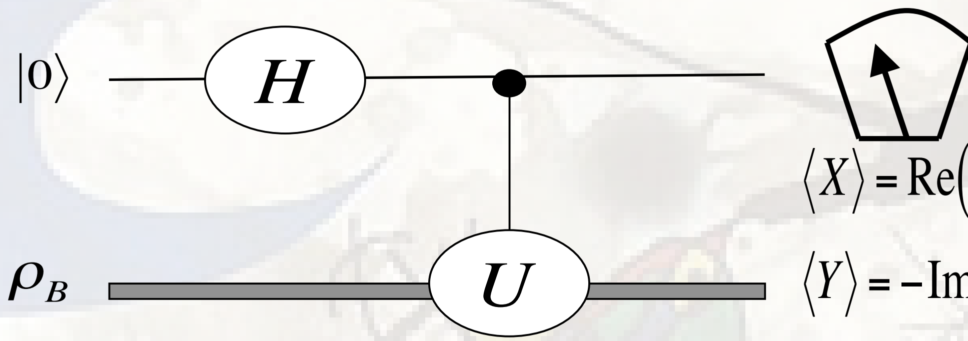


To a high degree of accuracy the following formula applies:

$$|r(t)|^2 = \exp\left(-Nt^2/2\right)\left(1 + O(1/N)\sin\lambda t/\lambda\right)$$

THE POWER OF A SINGLE QUBIT

SIMPLE SCHEME TO USE ONE QUBIT TO LEARN ABOUT PROPERTIES OF A MORE COMPLEX SYSTEM (CAN BE USED BOTH FOR TOMOGRAPHY AND SPECTROSCOPY)



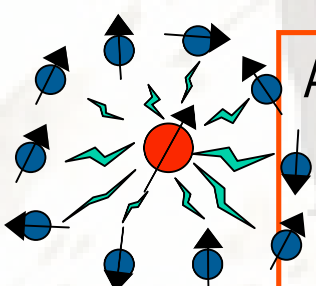
$$\langle X\rangle = \mathrm{Re}\left(Tr_B\left(\rho_B U\right)\right)$$

$$\langle Y\rangle = -\mathrm{Im}\left(Tr_B\left(\rho_B U\right)\right)$$

Know	ρ	: Use circuit to learn about	U	(“spectrometer”)
Know	U	: Use circuit to learn about	ρ	(“tomographer”)

E. Knill & R. Laflamme, Phys. Rev. Lett. 81, 5672 (1998)

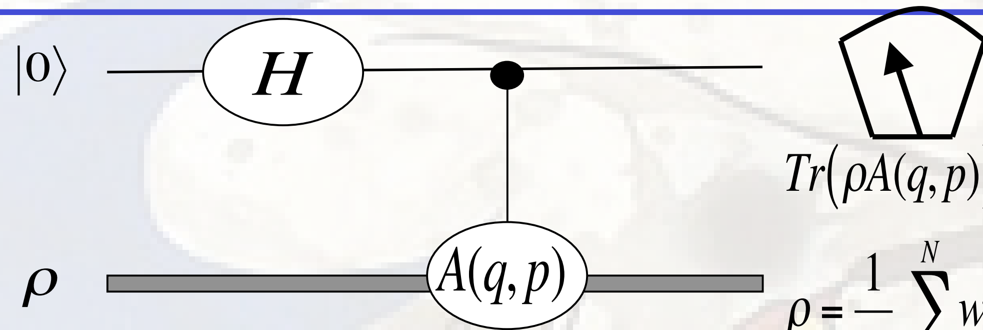
C. Miquel, J.P.P., M. Saraceno, E. Knill, R. Laflamme, C. Negrevergne, Nature 418, 59 (2002)



A new ‘spectroscopic’ application for this scheme: Measure universal features (i.e. Gaussian decay with ‘constant’ width) in the decay of quantum coherence in one qubit: an indicator of a quantum phase transition in the environment

QUANTUM CIRCUITS AND QUANTUM ALGORITHMS

Tomographic algorithm: Choose a basis A(q,p) to expand the state

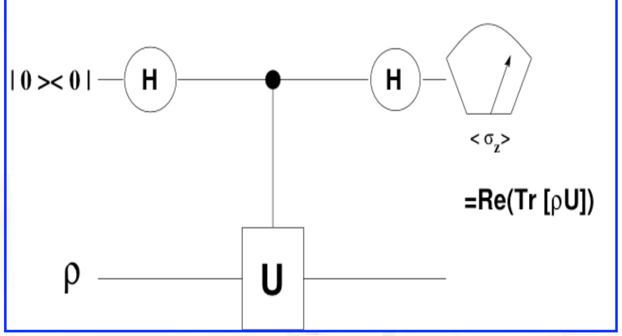


$$Tr\big(\rho A(q,p)\big) = w(q,p)$$

$$\rho = \frac{1}{N}\sum_{q,p=1}^{N} w(q,p)\, A(q,p)$$

Spectroscopic algorithm: Determine properties of U (NxN matrix) by measuring traces. Choose

Resources(DQC1): One qubit in a pure state, log(N) qubits in a maximally mixed state



$$Tr\left(\frac{U}{N}\right) \approx \begin{cases} 1/N & \textit{Chaotic} \\ 1/\sqrt{N} & \textit{Regular} \end{cases}$$

The quantum algorithm requires O(N) repetitions while classical methods seem to require O(N^2) Quadratic speedup! But exponential speedup is possible to estimate average fidelity under perturbations

David Poulin, Raymond Laflamme, G.J. Milburn, J.P.P,
Physical Review A, (2003), v68, 022302

SUMMARY

- DECOHERENCE IS A CRUCIAL INGREDIENT TO UNDERSTAND THE EMERGENCE OF CLASSICALITY
- DECOHERENCE IS THE ENEMY TO DEFEAT TO ACHIEVE QUANTUM INFORMATION PROCESSING.
- TO IMPLEMENT QUANTUM ERROR CORRECTION TOOLS WE MUST HAVE A VERY GOOD CHARACTERIZATION OF ERRORS AND DECOHERENCE IN THE DEVICES.
- SOME SYSTEMS EXHIBITS SOME UNIVERSAL FEATURES WHEN THEY DECOHERE , I.E. LYAPUNOV REGIME (INDEPENDENCE OF SYSTEM-ENVIRONMENT COUPLING)
- CRITICAL ENVIRONMENTS ARE HIGHLY EFFICIENT IN INDUCING DECOHERENCE CRITICAL ENVIRONMENT MAY INDUCE UNIVERSAL DECOHERENCE
- USE A SINGLE QUBIT AS AN INDICATOR OF A QUANTUM PHASE TRANSITION? (MAYBE)

Lecture 1: Decoherence and the quantum origin of the classical world (Pointer states, timescales, chaos).

Lecture 2: Decoherence and quantum information processing, models and examples

Lecture 2

- Evolution of quantum open systems: An approach based on quantum circuits. Models for decoherence (simple)

- Decoherence in a simple quantum algorithm (quantum walk)

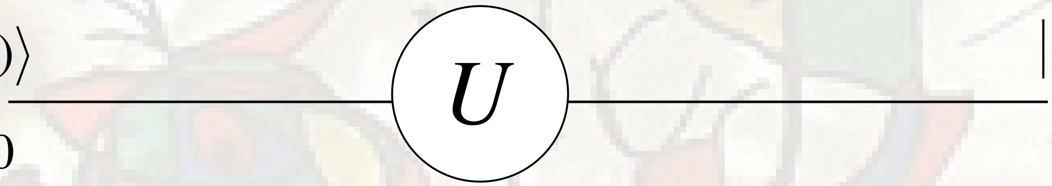
EVOLUTION OF QUANTUM OPEN SYSTEMS

QUANTUM CIRCUITS: A USEFUL TOOL TO REPRESENT EVOLUTION OF QUANTUM SYSTEMS

- A QUANTUM CIRCUIT REPRESENTS A UNITARY OPERATOR
- INPUT: INITIAL STATE OF THE SYSTEM
- OUTPUT: FINAL STATE OF THE SYSTEM

(TIMES FLOWS FROM LEFT TO RIGHT)

$|\Psi(0)\rangle$ $\quad t=0$

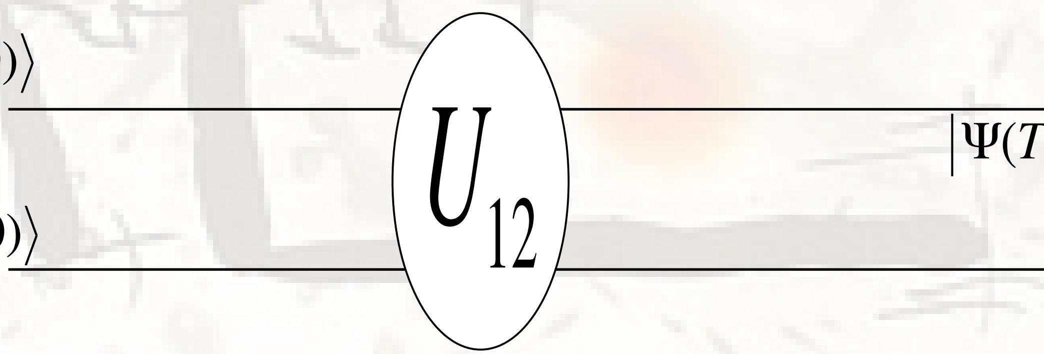


$$|\Psi(T)\rangle = U|\Psi(0)\rangle \qquad t = T$$

- WHEN A QUANTUM SYSTEM IS SPLIT IN TWO INTERACTING SUBSYSTEMS

$|\Psi_1(0)\rangle$

$|\Psi_2(0)\rangle$



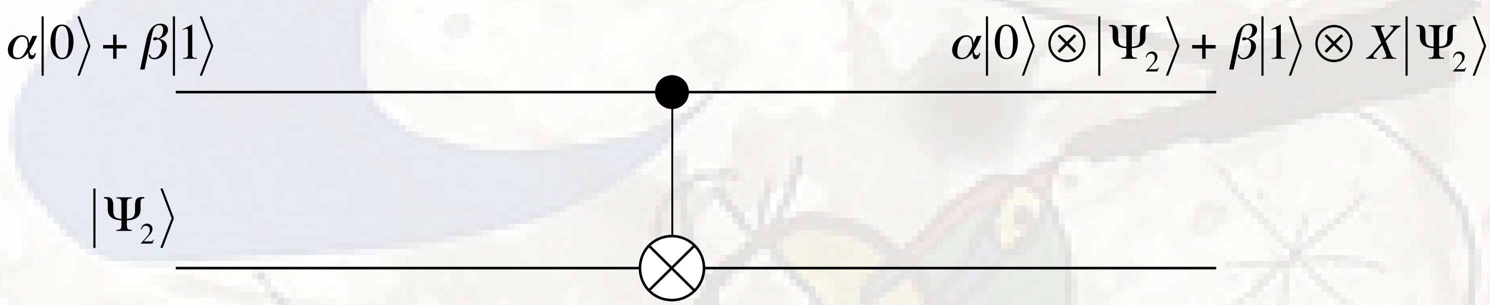
$$|\Psi(T)\rangle = U_{12}|\Psi_1(0)\rangle \otimes |\Psi_2(0)\rangle$$

$$\neq |\phi_1(0)\rangle \otimes |\phi_2(0)\rangle$$

EVOLUTION OF QUANTUM OPEN SYSTEMS

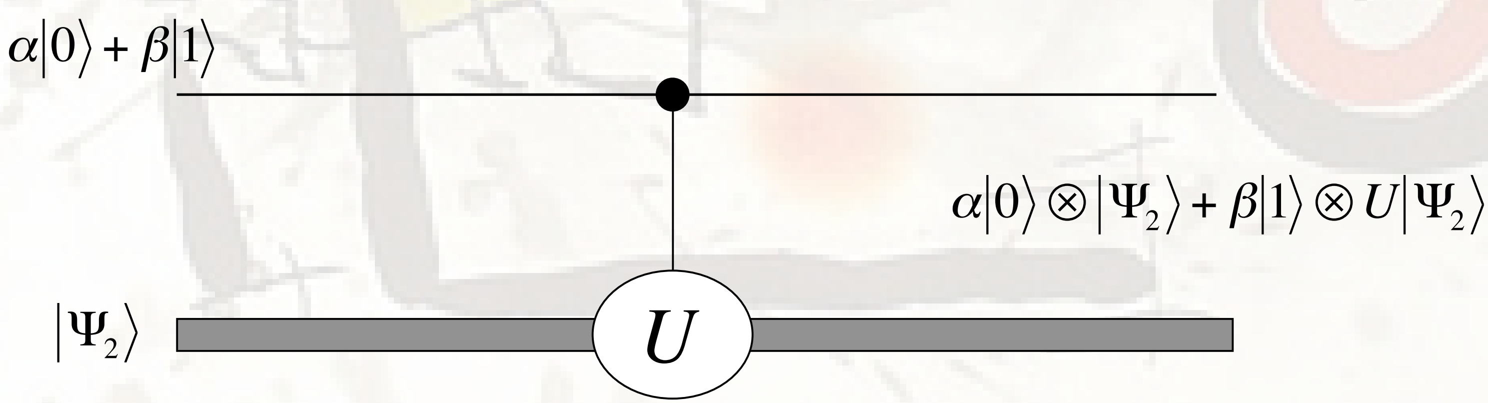
SIMPLE MODELS OF INTERACTIONS

•C-NOT INTERACTION BETWEEN TWO QUBITS



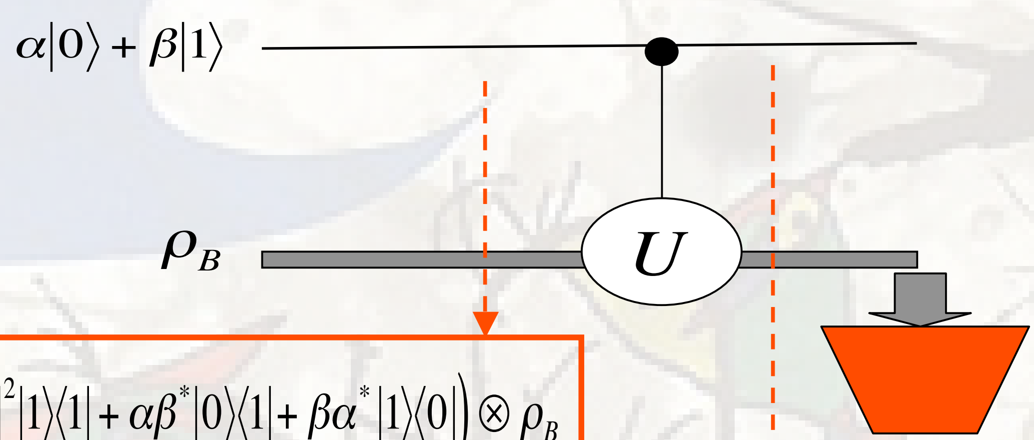
NOTATION: $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

MORE GENERAL: SECOND SUBSYSTEM MADE OUT OF MANY QUBITS (D-DIMENSIONAL HILBERT SPACE), CONTROL-U INTERACTION



EVOLUTION OF QUANTUM OPEN SYSTEMS

MORE GENERAL: SECOND SUBSYSTEM IN AN ARBITRARY (MIXED) STATE:
DESCRIBE THE COMBINED SYSTEM WITH DENSITY MATRIX



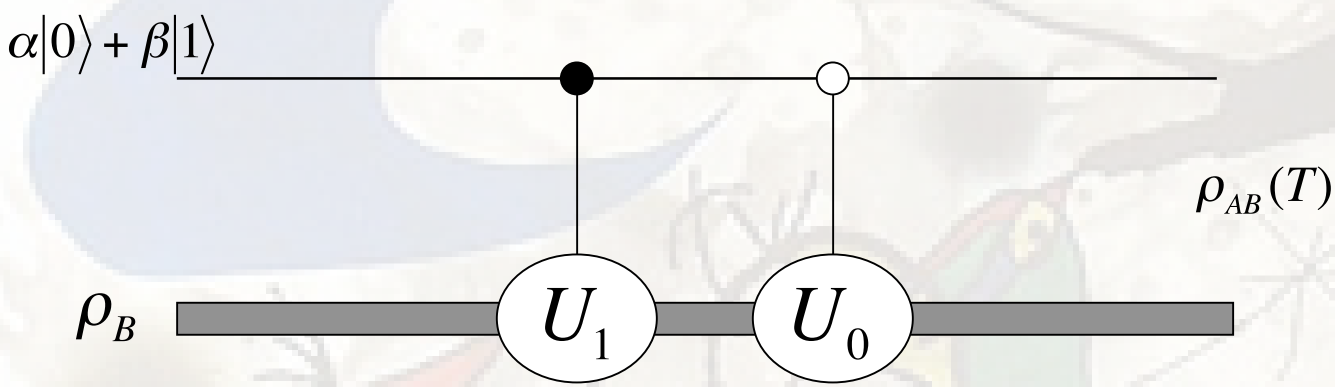
$$\rho_{AB} = \left(|\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| + \alpha\beta^*|0\rangle\langle 1| + \beta\alpha^*|1\rangle\langle 0|\right)\otimes\rho_B$$

$$\rho_{AB} = |\alpha|^2|0\rangle\langle 0|\otimes\rho_B + |\beta|^2|1\rangle\langle 1|\otimes U\rho_B U^+ + \alpha\beta^*|0\rangle\langle 1|\otimes\rho_B U^+ + \beta\alpha^*|1\rangle\langle 0|\otimes U\rho_B$$

$$\rho_A = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| + \alpha\beta^*|0\rangle\langle 1|\, Tr_B\left(\rho_B U^+\right) + \beta\alpha^*|1\rangle\langle 0|\, Tr_B\left(\rho_B U\right)$$

EVOLUTION OF QUANTUM OPEN SYSTEMS

MORE GENERAL: THE INTERACTION IS DIAGONAL IN THE {0,1} BASIS OF THE SYSTEM A



$$\rho_{AB}(T) = |\alpha|^2 |0\rangle\langle 0| \otimes U_0 \rho_B U_0^+ + |\beta|^2 |1\rangle\langle 1| \otimes U_1 \rho_B U_1^+ + \alpha\beta^* |0\rangle\langle 1| \otimes U_0 \rho_B U_1^+ + \alpha^* \beta |1\rangle\langle 0| \otimes U_1 \rho_B U_0^+$$

$$\rho_A(T) = |\alpha|^2 |0\rangle\langle 0| + |\beta|^2 |1\rangle\langle 1| + \alpha\beta^* Tr\left(U_0 \rho_B U_1^+\right) |0\rangle\langle 1| + \alpha^* \beta\, Tr\left(U_1 \rho_B U_0^+\right) |1\rangle\langle 0|$$

EXERCISE: SHOW THAT THIS IS A DESCRIPTION OF A (SPIN BATH) MODEL WITH HAMILTONIAN

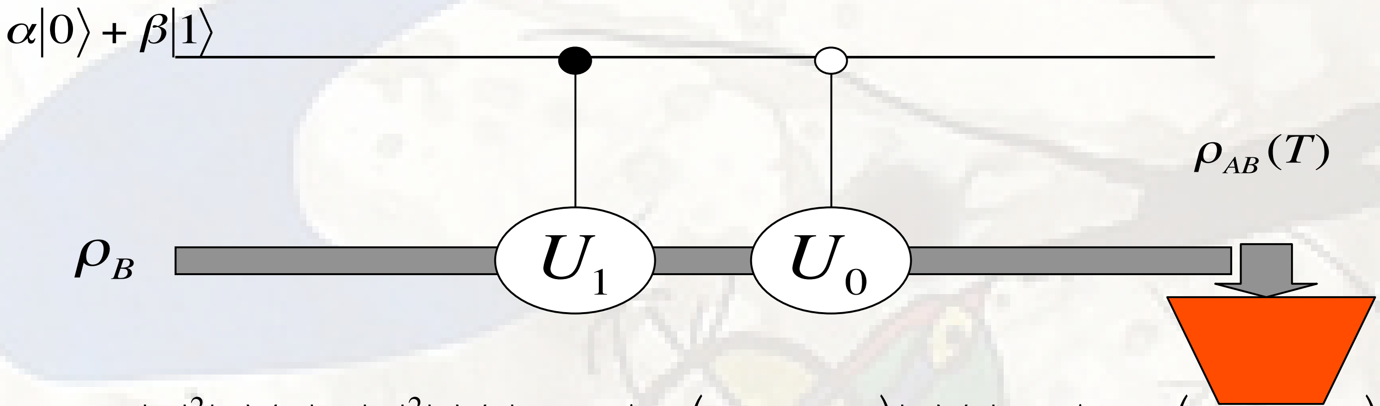
$$H = Z_A \otimes \sum_k g_k Z_k \qquad U_0 = \exp\left(-i\, t \sum_k g_k Z_k\right) = U_1^+$$

$$r(t) = Tr_B\left(U_0 \rho_B U_1^+\right) = Tr_B\left(U_0^{\,2} \rho_B\right) = \prod_k \left(\rho_{k_{00}} e^{2ig_k t} + \rho_{k_{11}} e^{-2ig_k t}\right)$$

OFF-DIAGONAL TERMS ARE SUPRESSED BY THIS FACTOR. POINTER STATES EMERGE

EVOLUTION OF QUANTUM OPEN SYSTEMS

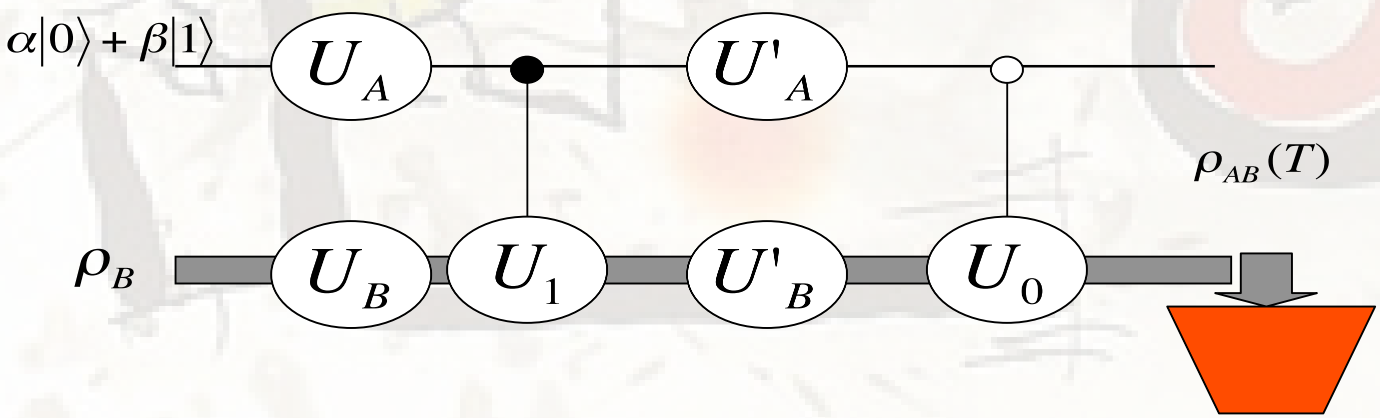
IS THIS A GOOD MODEL OF DECOHERENCE?



$$\rho_A(T) = |\alpha|^2 |0\rangle\langle 0| + |\beta|^2 |1\rangle\langle 1| + \alpha\beta^* Tr\left(U_0 \rho_B U_1^{+}\right)|0\rangle\langle 1| + \alpha^* \beta\, Tr\left(U_1 \rho_B U_0^{+}\right)|1\rangle\langle 0|$$

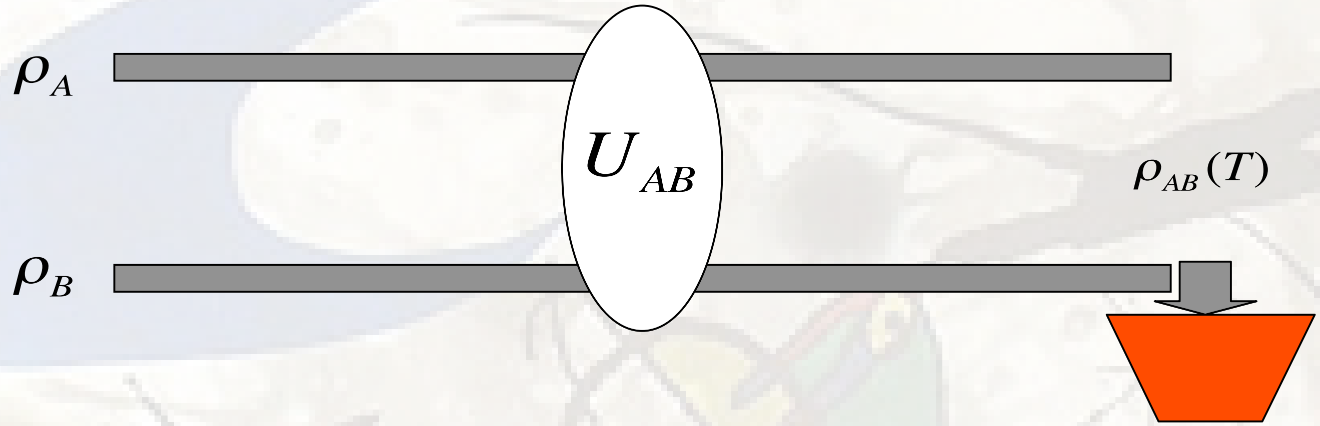
- SUPRESSION OF OFF-DIAGONAL TERMS BY A FACTOR $r(t) = Tr_B\left(U_1^{+} U_0 \rho_B\right)$
- PREFERRED STATES ARE EIGENSTATES OF INTERACTION

MORE GENERAL: INCLUDE 'SELF-EVOLUTION' OF SYSTEM AND ENVIRONMENT



EVOLUTION OF QUANTUM OPEN SYSTEMS

MOST GENERAL EVOLUTION OF TWO INTERACTING SUBSYSTEMS



REDUCED DENSITY MATRIX OF SUB-SYSTEM A

$$\rho_A(T) = Tr\left(U_{AB}\,\rho_A(0)\otimes\rho_B(0)\,U_{AB}^{\;+}\right)$$

$$\rho_A(T) = \sum_b A_b\ \rho_A(0)\ A_b^{\;+}, \quad \sum_b A_b^{\;+} A_b = I,$$

$$A_b = \langle \phi_b | U_{AB} | \Psi_B(0) \rangle$$

Valid if initial state of B is pure

KRAUSS REPRESENTATION OF EVOLUTION OF REDUCED DENSITY MATRIX

(general)

EVOLUTION OF QUANTUM OPEN SYSTEMS

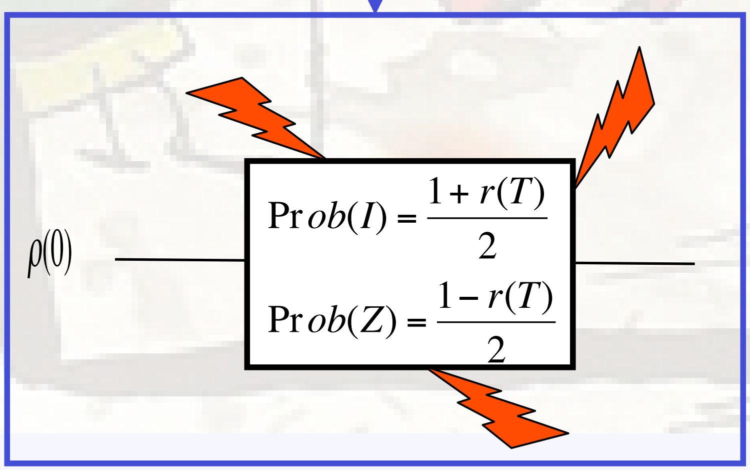
EXAMPLE: CONSIDER A DECOHERENCE MECHANISM THAT SUPRESSES OFF DIAGONAL TERMS IN {0,1} BASIS:

$$\rho_A(T) = |\alpha|^2 |0\rangle\langle 0| + |\beta|^2 |1\rangle\langle 1| + \alpha\beta^* r(T)|0\rangle\langle 1| + \alpha^*\beta\, r(T)|1\rangle\langle 0|$$

$$r(T) \leq 1, \quad r(T) \in \Re$$

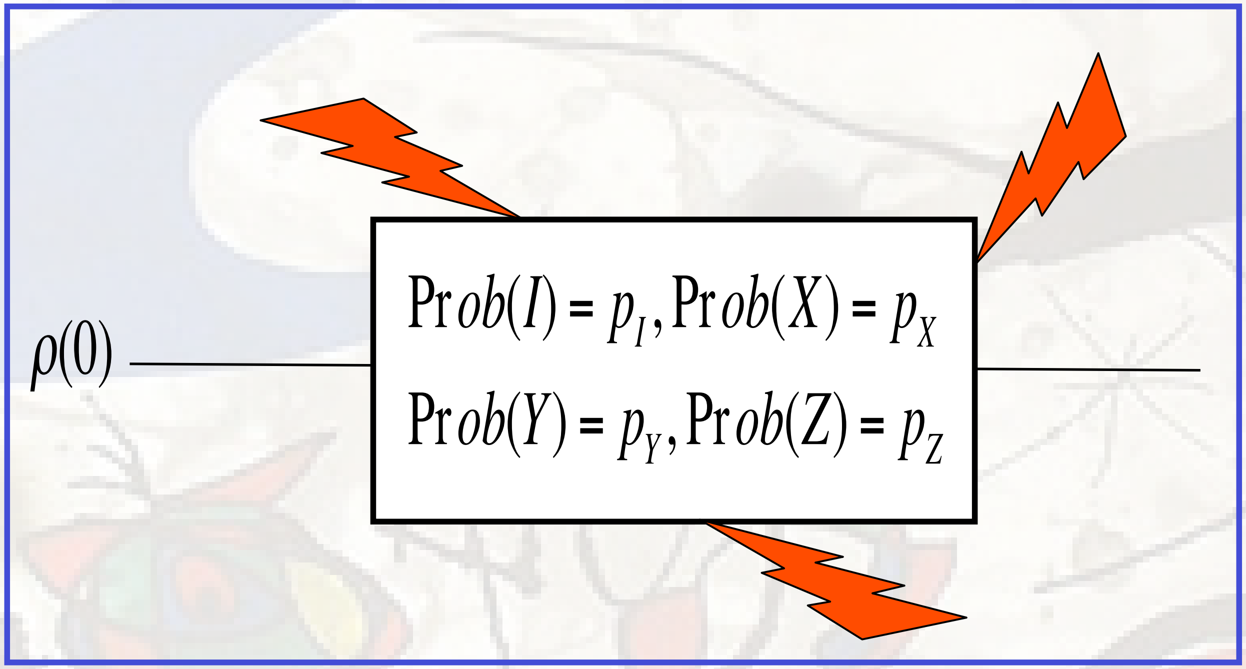
KRAUS REPRESENTATION (PHASE DAMPING CHANNEL)

$$\boxed{\rho_A(T) = \frac{1+r(T)}{2}\rho_A(0) + \frac{1-r(T)}{2} Z\rho_A(0)Z}$$



EVOLUTION OF QUANTUM OPEN SYSTEMS

MORE GENERAL NOISY CHANNEL



$$\rho_A(T) = p_I \rho_A(0) + p_X X \rho_A(0) X + p_Y Y \rho_A(0) Y + p_Z Z \rho_A(0) Z$$

STUDY HOW THE QUANTUM STATE IS DEGRADED BY A NOISY CHANNEL

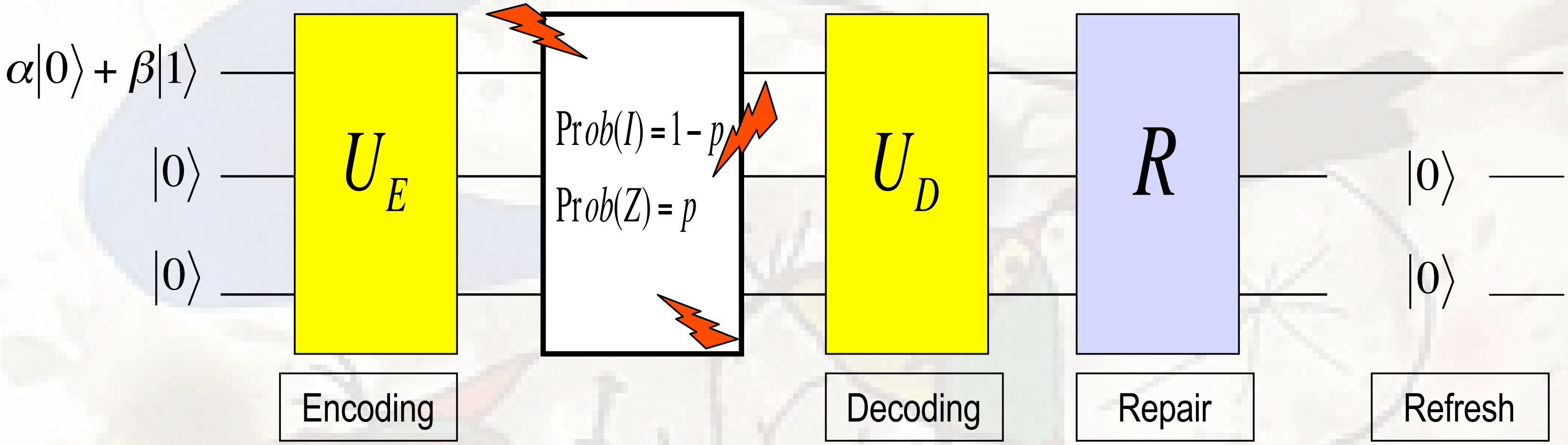
FIDELITY DECAY: $F = Tr\left(\rho_A(T)\rho_A(0)\right) = 1 - 4p|\alpha\beta|^2 \quad (p_I = 1-p, p_X = 0, p_Y = 0, p_Z = p)$

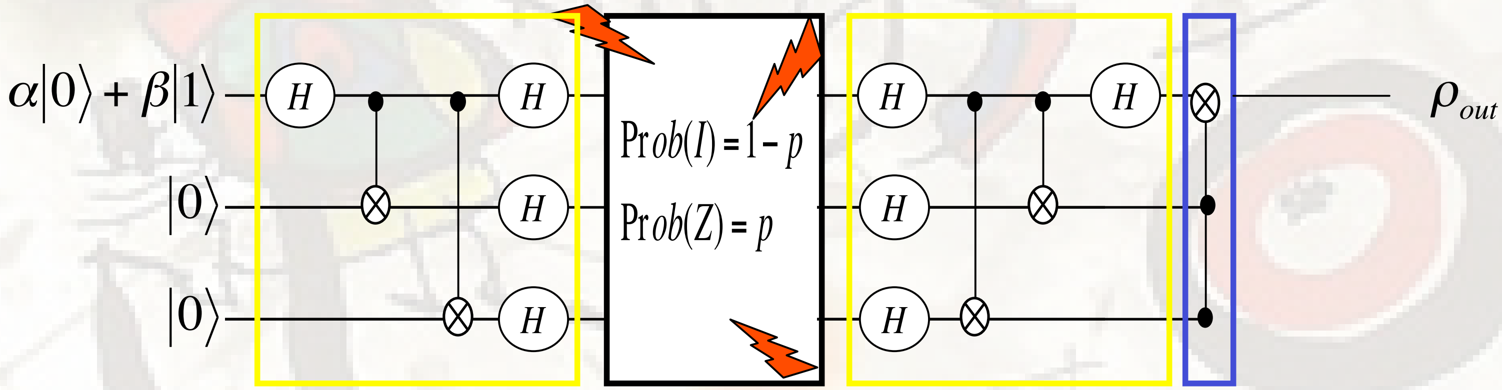
PURITY DECAY: $\zeta = Tr\left(\rho^2{}_A(T)\right) = 1 - 8p(1-p)|\alpha\beta|^4$

- DECAY DEPENDS ON THE STATE
- DECAY IS LINEAR IN p

HOW TO FIGHT AGAINST DECOHERENCE

ERROR CORRECTION CAN BE USED TO PROTECT QUANTUM INFORMATION





$$\rho_{out} = \rho_{in} + O(p^2), \quad F = Tr(\rho_{in}\rho_{in}) = 1 - O(p^2)$$

FIDELITY GOES FROM LINEAR TO QUADRATIC IN p!

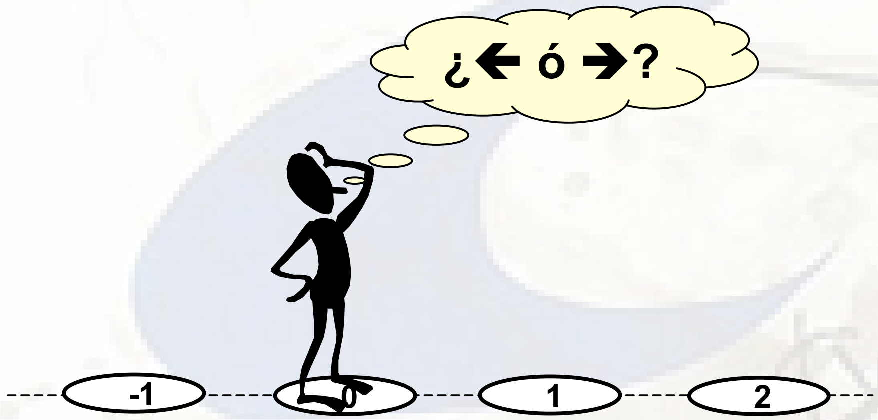
SUMMARY

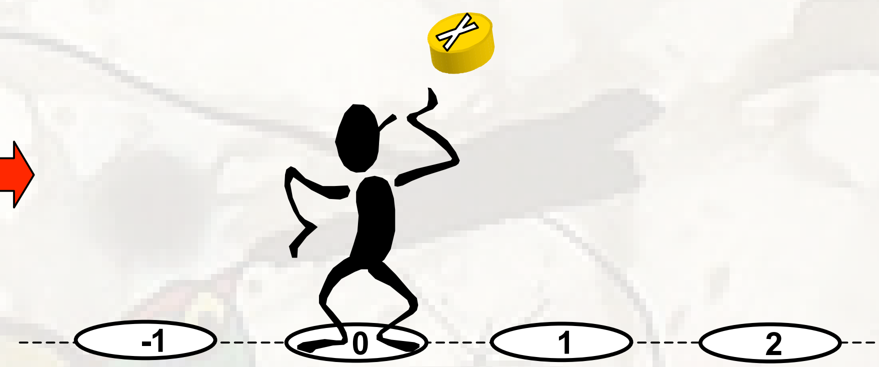
- QUANTUM CIRCUITS ARE USEFUL TOOLS TO DESCRIBE QUANTUM EVOLUTION
- SIMPLE QUANTUM CIRCUITS DESCRIBE SIMPLE MODELS OF DECOHERENCE
- MOST GENERAL EVOLUTION OF A QUANTUM OPEN SYSTEM CAN BE WRITTEN IN KRAUSS FORM (Exercise: Think about this!)

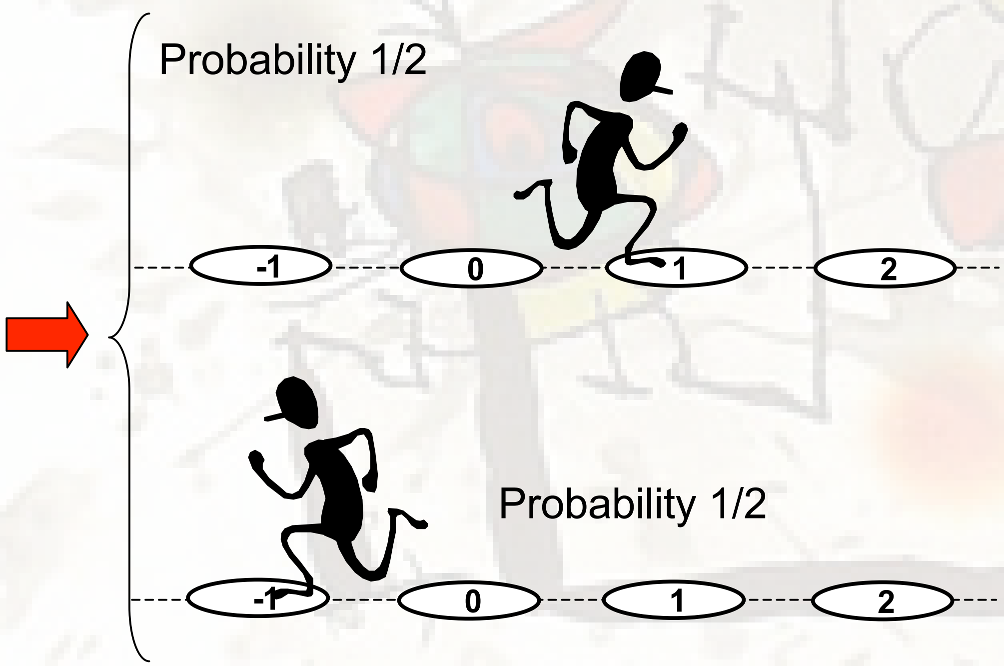
$$\rho_A(T) = \sum_b A_b\ \rho_A(0)\ A_b^{+}, \quad \sum_b A_b^{+} A_b = I$$

EXAMPLE: DECOHERENCE IN A QUANTUM WALK

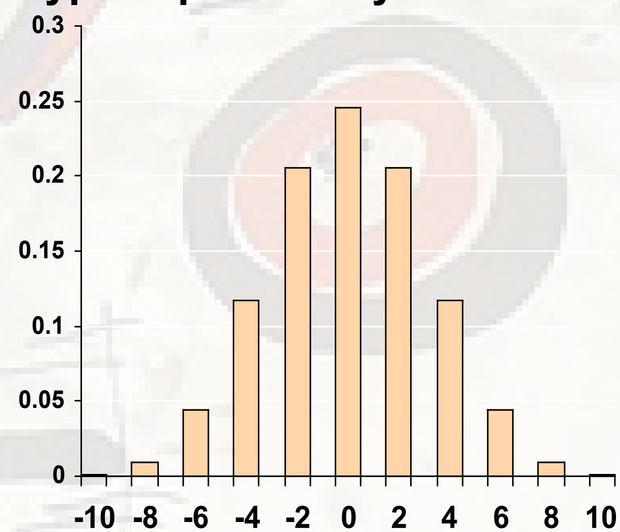
1. Classical random walk





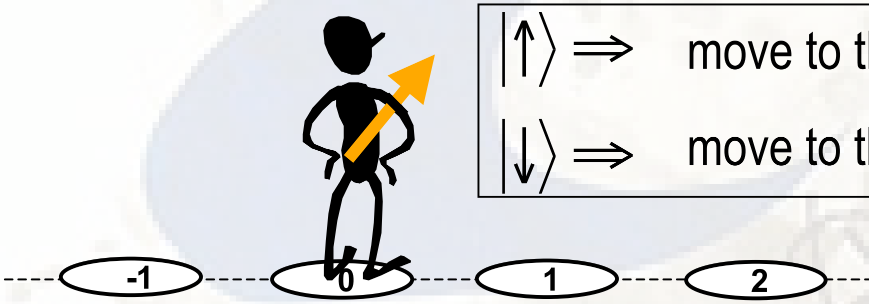


Typical probability distribution



EXAMPLE: DECOHERENCE IN A QUANTUM WALK

2. Quantum walk algorithm: A quantum coin (spin 1/2) and a quantum walker (moving in a ring with N sites)*: It could be a useful "subroutine"*



$|\uparrow\rangle \Rightarrow$ move to the right

$|\downarrow\rangle \Rightarrow$ move to the left

Quantum walk "algorithm":

- Initial state:

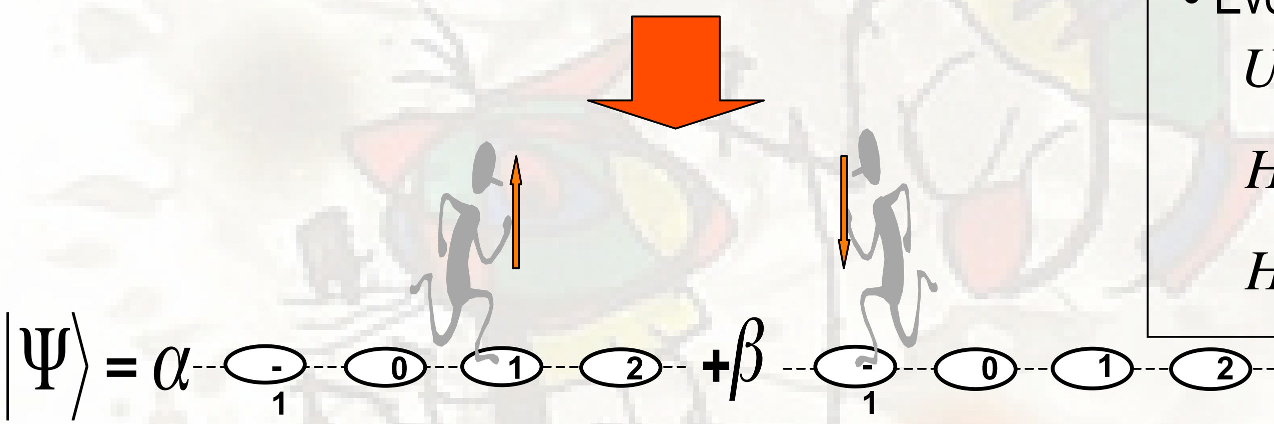
$$|\Psi\rangle = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |\varphi\rangle$$

- Evolution operator: $U^Z\ H$

$$U|n\rangle = |n+1\rangle, \quad \mathrm{mod}(N)$$

$$H|\uparrow\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$$

$$H|\downarrow\rangle = (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2}$$



The coin (spin) and the walker become entangled.
The state of the walker after t-iterations is:

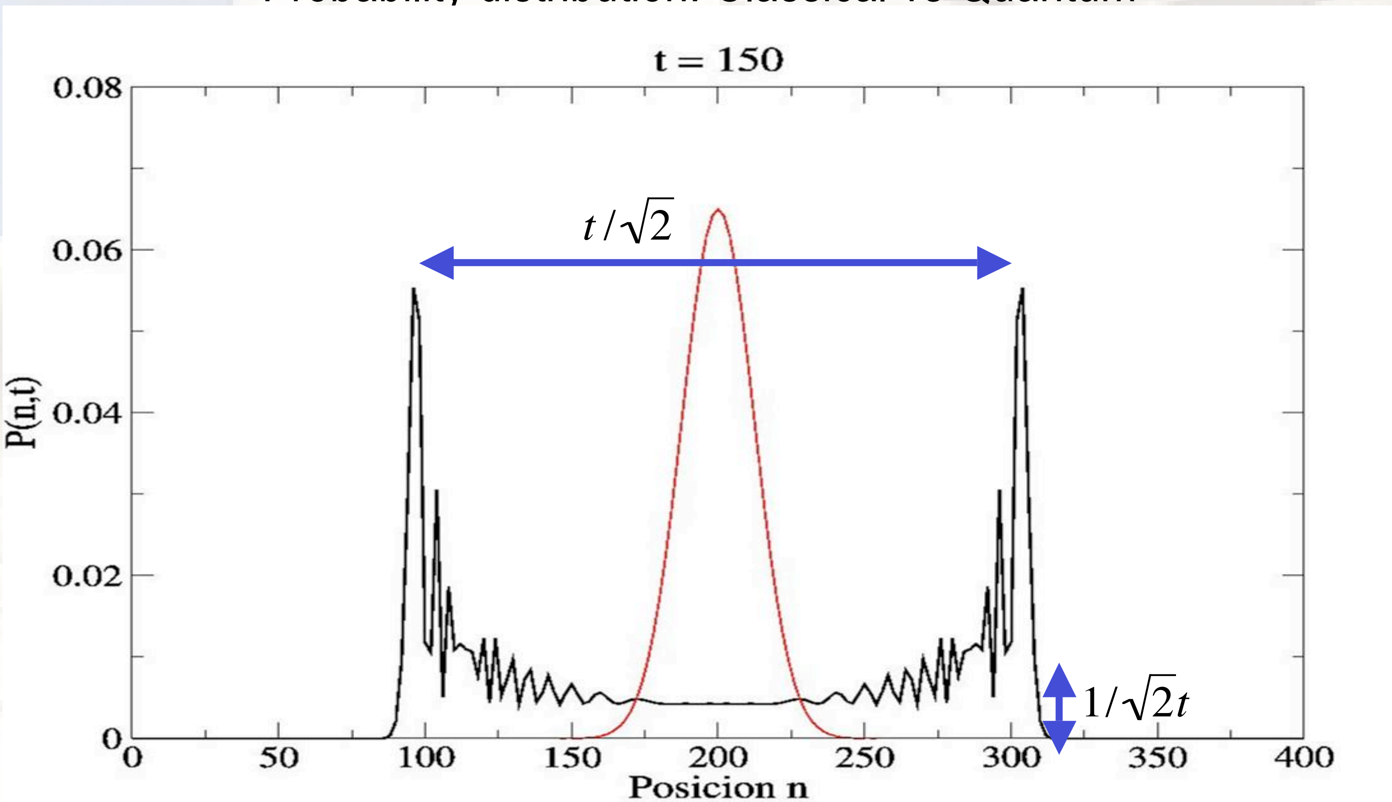
$$\rho_w(t) = Tr_C\left(U^Z H \cdots U^Z H U^Z H\, \rho_w(0) \otimes \rho_c(0) H U^{-Z} H U^{-Z} \cdots H U^{-Z}\right)$$

EXAMPLE: DECOHERENCE IN A QUANTUM WALK

Classical and quantum walks have rather different properties:

Initial state: “Impartial spin”, localized walker $|\Psi\rangle = (|\uparrow\rangle + i|\downarrow\rangle) \otimes |n = 200\rangle$

Probability distribution: Classical vs Quantum



Quantum walker spreads faster than classical!

Reason?: Quantum interference

EXAMPLE: DECOHERENCE IN A QUANTUM WALK

NOTE

Quantum walks on graphs have been proposed as potentially useful quantum subroutines

Review: J. Kempe, Contemp Phys 44, 307 (2003)

Proposed in: D. Aharonov, A. Ambainis, J. Kempe, U. Vazirani, Proc 33. ACM STOC-2001, 50-59

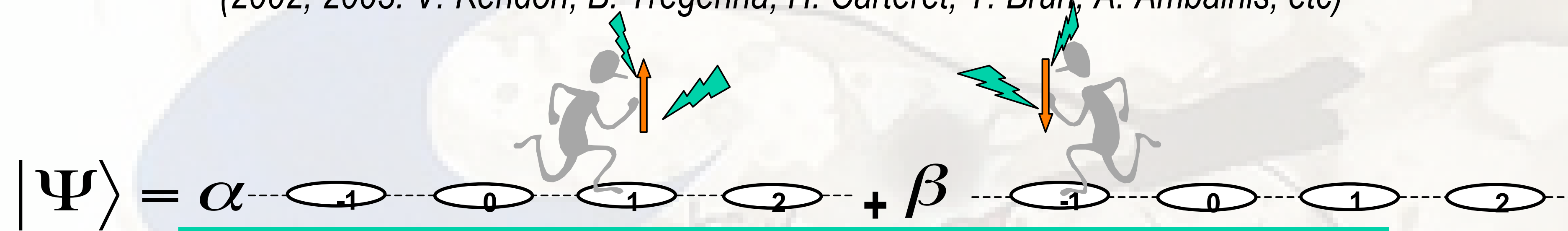
There are very few algorithms that use quantum walks as a central piece:
* N. Shenvi, J. Kempe & B. Whaley, PRA 67 052307 (2003) (DISCRETE);
* A. M. Childs et al, Proc 35 ACM STOC-2003, 59-68 (CONTINUOUS)

Key to the potential advantadge of quantum walks?: Use the quantum nature of the walk, that allows for faster spreading over the graph (this enables, for example, exponentially faster hiting times)

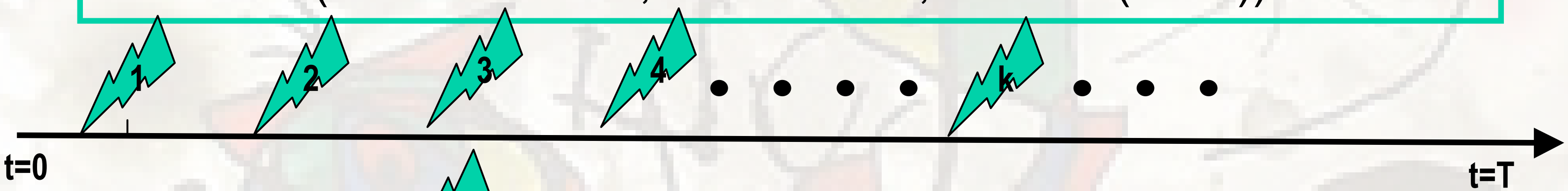
EXAMPLE: DECOHERENCE IN A QUANTUM WALK

What happens if the coin (or walker) interacts with an environment?

(2002, 2003: V. Kendon, B. Tregenna, H. Carteret, T. Brun, A. Ambainis, etc)



Simple model to simulate coupling to a spin environment (NMR)
(G.Teklemarian, et al PRA67, 062316 (2003))



k $= e^{-i\varepsilon_k \hat{\sigma}_y}$, ε_k random variables

$$\rho_w(t) = \boxed{\int d\vec{\varepsilon}\, P(\vec{\varepsilon})}\, Tr_C\left(U^Z H \boxed{e^{-i\varepsilon_{t-1}Y}} \cdots \boxed{e^{-i\varepsilon_1 Y}}\, U^Z H \rho_w(0) \otimes \rho_c(0) H U^{-Z} \boxed{e^{i\varepsilon_1 Y}} \cdots \boxed{e^{i\varepsilon_{t-1}Y}}\, H U^{-z}\right)$$

$$\int d\vec{\varepsilon}\, P(\vec{\varepsilon}) = \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} d\varepsilon_1 \cdots \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} d\varepsilon_t,$$

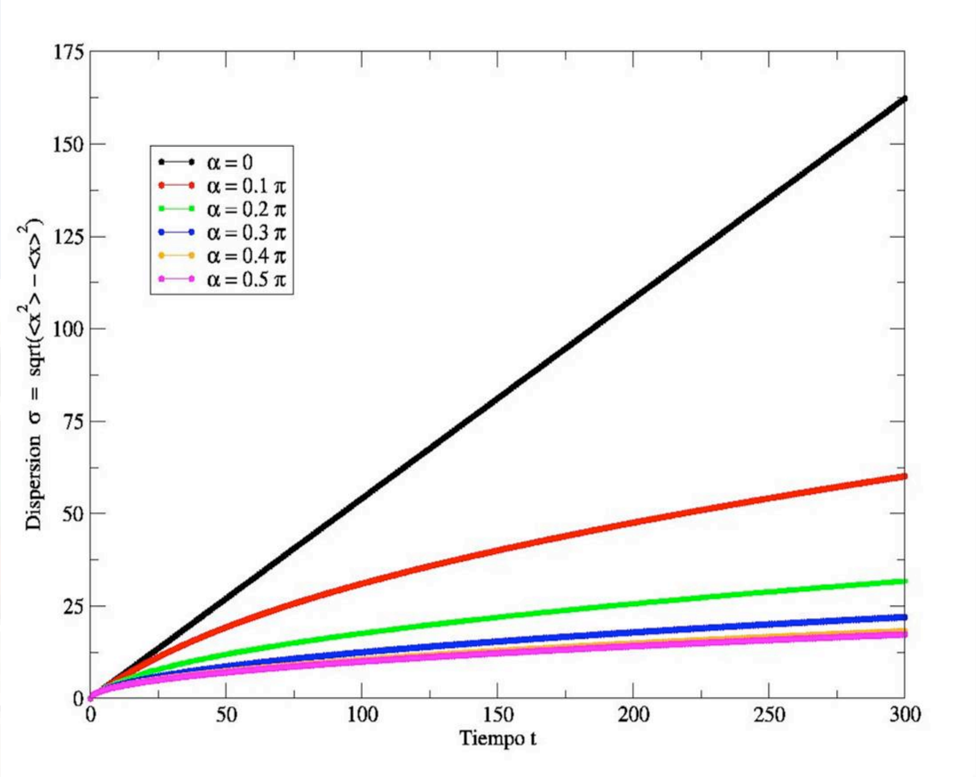
α = coupling strength

Feature: model can be exactly solved (analytic solutions available @ C.Lopez & J.P. Paz, PRA 68, 052305 (2003)

EXAMPLE: DECOHERENCE IN A QUANTUM WALK

RESULTS: Decoherence and quantum-classical transition for quantum walks

Fixe time, vary system-environment coupling strength



EXAMPLE: DECOHERENCE IN A QUANTUM WALK

Simple interpretation

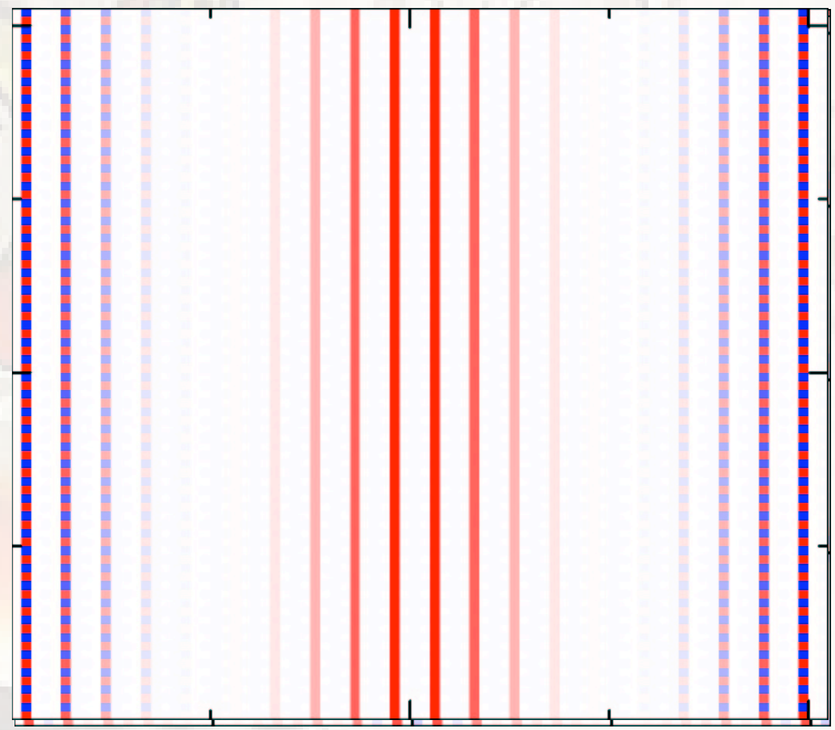
Exact formula valid in the case of full decoherence $(\alpha = \pi)$

$$\rho_w(t) = \frac{1}{2^t} \sum_{\alpha_1,\cdots\alpha_T=-1,1} Tr_C\left(U^{\alpha_t Z} H \cdots U^{\alpha_1 Z} H \rho_w(0) \otimes \rho_c(0) H U^{-\alpha_t Z} \cdots H U^{-\alpha_t Z}\right)$$

Quantum walk with decoherence in the coin looks like random walk.

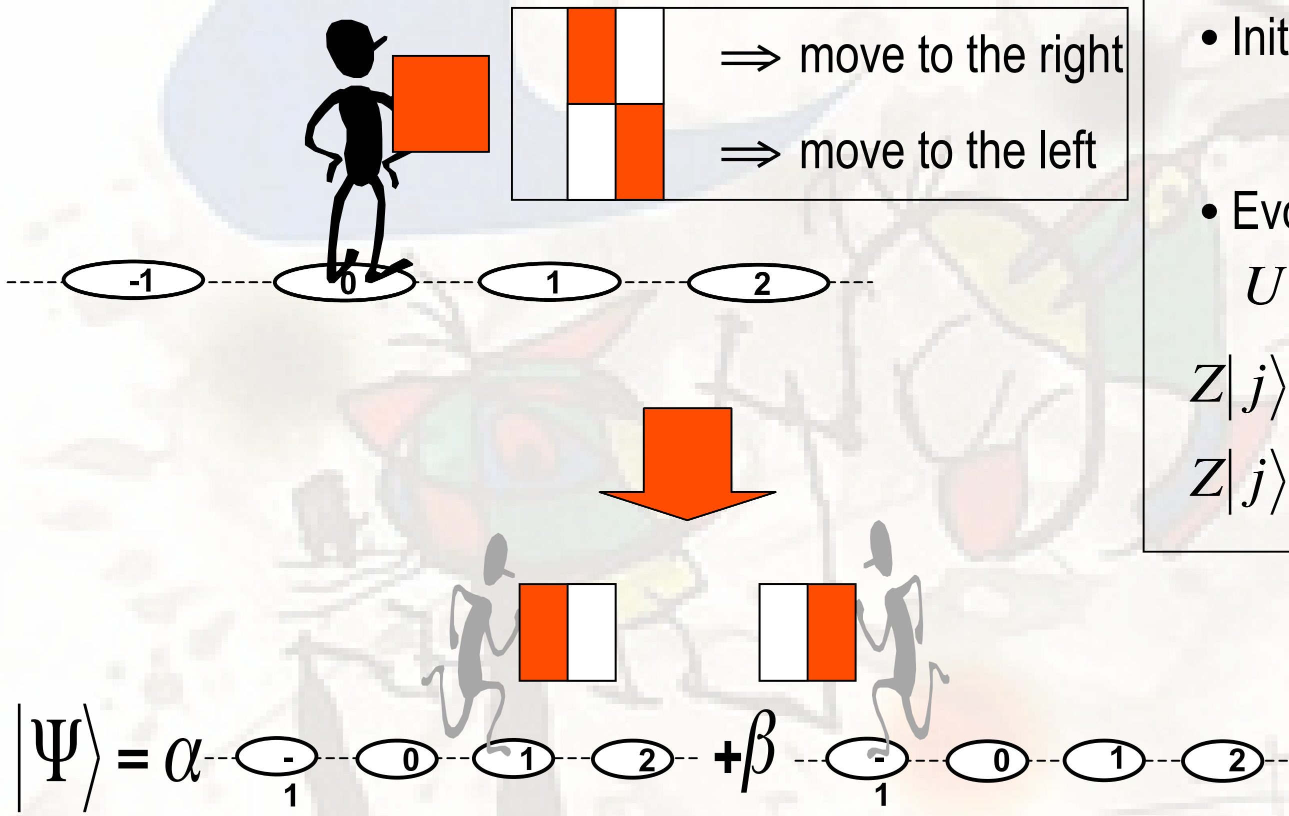
Effect: Diffusion along the position direction in phase space.

Lesson I from decoherence studies:
a) Environment couples to spin.
b) Spin couples with walker via U (displacement operator).
d) U is diagonal in momentum.
Then: momentum states are pointer states!



EFFECT OF A CHAOTIC ENVIRONMENT: AN EXAMPLE

Use a quantum coin with a D dimensional Hilbert space



Generalized quantum walk "algorithm":

- Initial state:

$$|\Psi\rangle = \sum_{j=1}^{D} c_j |j\rangle \otimes |n\rangle$$

- Evolution operator: $U^Z \; B$

$$U|n\rangle = |n+1\rangle, \quad \mathrm{mod}(N)$$

$$Z|j\rangle = |j\rangle \quad 1 \le j \le D/2$$

$$Z|j\rangle = -|j\rangle \quad D/2+1 \le j < D$$

EFFECT OF A CHAOTIC ENVIRONMENT: AN EXAMPLE

Generalized quantum walk is represented by a simple quantum circuit:

Generalized quantum walk "algorithm":

- Initial state:

$$|\Psi\rangle = \sum_{j=1}^{D} c_j |j\rangle \otimes |n\rangle$$

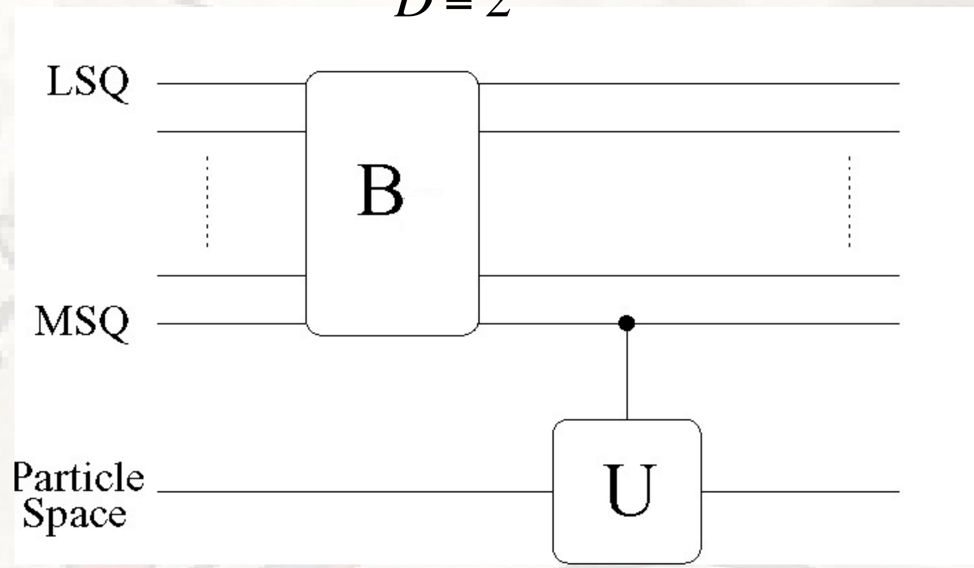
- Evolution operator: $U^Z \; B$

$$U|n\rangle = |n+1\rangle, \quad \mathrm{mod}(N)$$

$$Z|j\rangle = |j\rangle \quad 1 \le j \le D/2$$

$$Z|j\rangle = -|j\rangle \quad D/2+1 \le j < D$$

$D = 2^K$



Analogous to a quantum walk with an ordinary coin (spin 1/2) coupled to an environment: Operator B defines the interaction between quantum coin and environment

We studied differences in behavior between "regular" and "chaotic" B's. Paradigmatic example: B=Quantum Baker Map (L. Erman, J.P.P. & M. Saraceno: SEE POSTER, ASK LEO!)

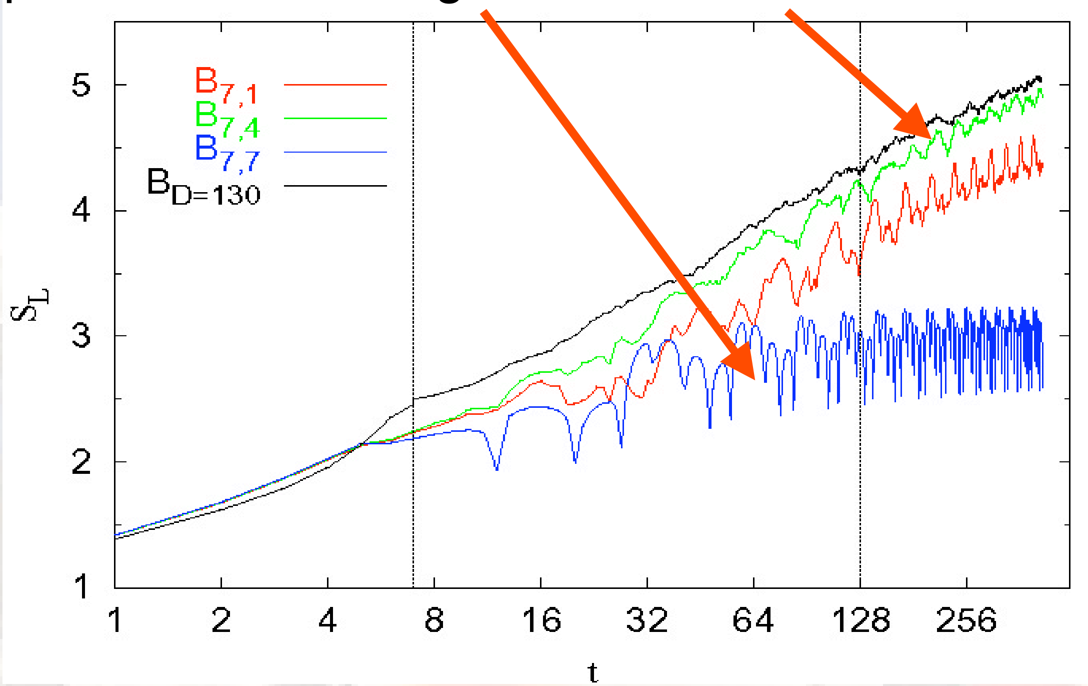
EFFECT OF A CHAOTIC ENVIRONMENT: AN EXAMPLE

Known: Chaotic environments are more efficient in inducing decoherence

L. Erman, J.P.P. & M. Saraceno: Phys Rev A (2005) to appear)

Entropy production from regular and chaotic environments

Entropy computed from the reduced density matrix of the system

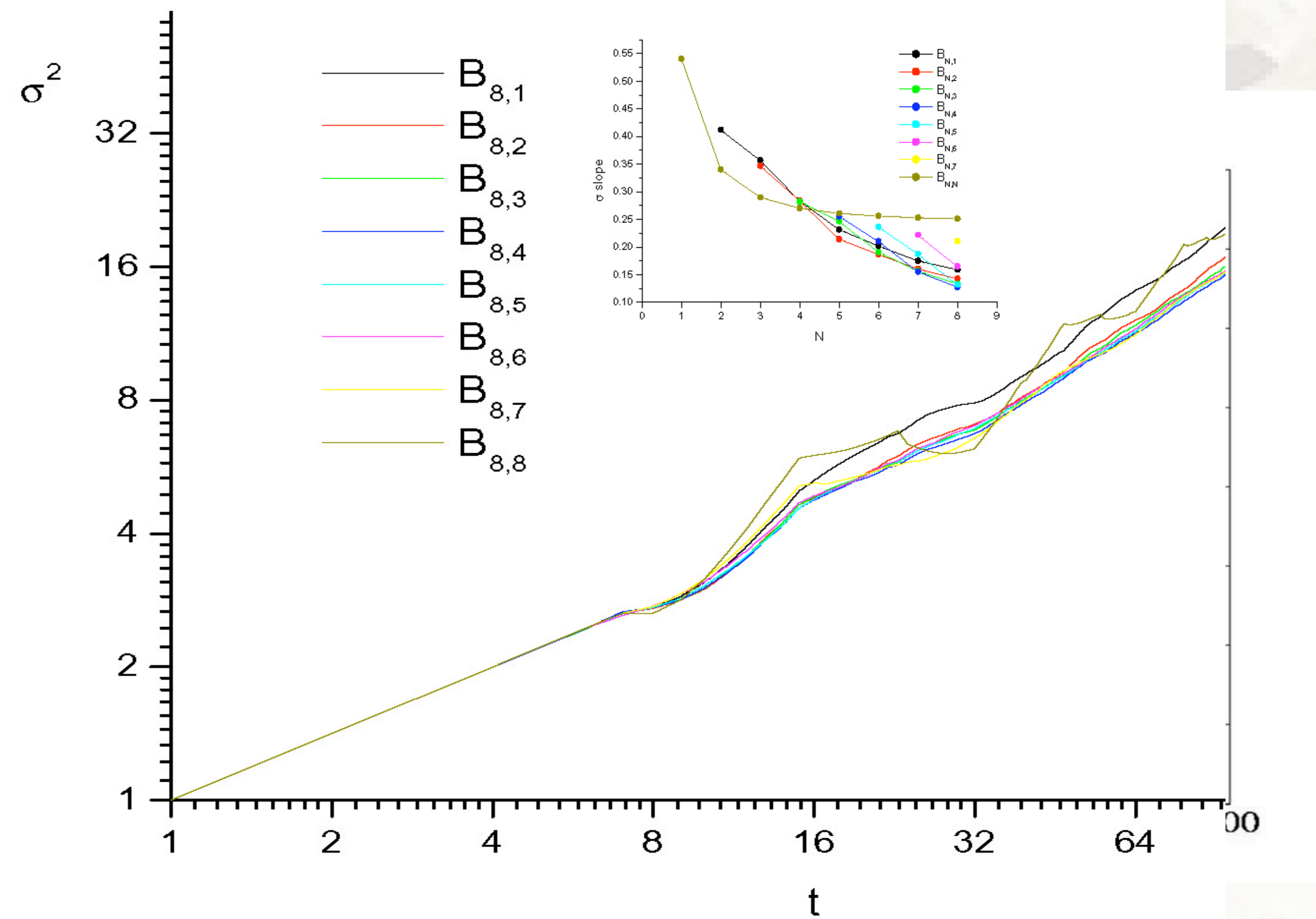


Regular case: small subset of Hilbert space is explored (log(D)-dimensional)

Chaotic case: entire (D-dimensional) Hilbert space is explored.

EFFECT OF A CHAOTIC ENVIRONMENT: AN EXAMPLE

Quantum to classical transition: Coupling to environment induces classical behavior (in the dispersion of walker, for example)



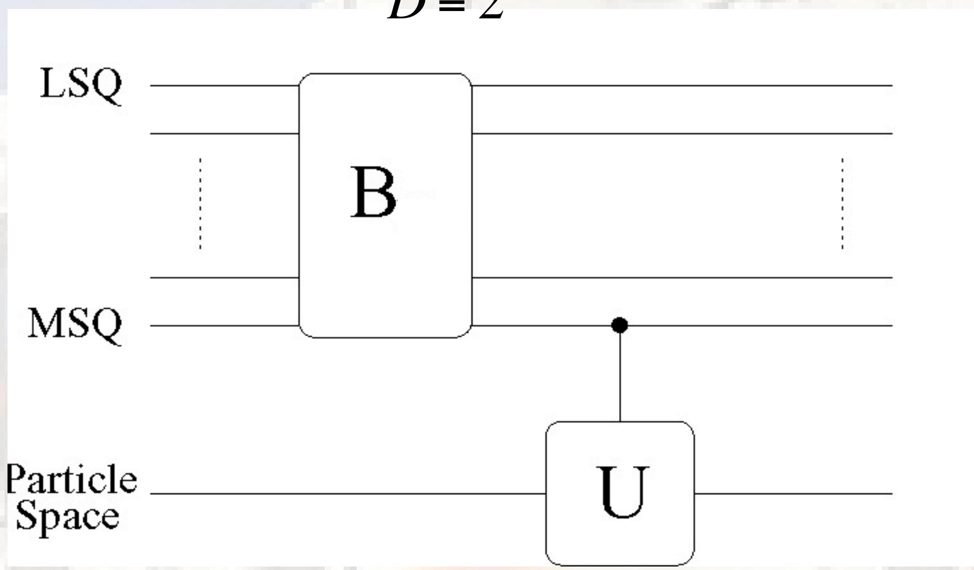
Finite dimensional environment cannot induce classical behavior for all times. D-dependence is very different for chaotic and regular environments

EFFECT OF A CHAOTIC ENVIRONMENT: AN EXAMPLE

Is there a simple way to understand why is the chaotic environment more efficient than the regular one?

YES

$$D = 2^K$$



Chaotic evolution creates more entanglement in the internal space than regular one (exponentially more). Each “quasimomentum” state of the particle is entangled with an orthogonal state of the environment

DECOHERENCE

Decoherence induced by a chaotic enviroment: A quantum walker with a complex coin
L. Ermann, J. P. Paz, and M. Saraceno; Phys. Rev. A 73, 012302 (2006)

Phase-space approach to the study of decoherence in quantum walks
C. C. López and J. P. Paz; Phys. Rev. A 68, 052305 (2003)

- DECOHERENCE IS AN ESENTIAL INGREDIENT TO UNDERSTAND THE QUANTUM CLASSICAL TRANSITION
- IT IS "THE" ENEMY FOR QUANTUM INFORMATION PROCESSING
- FIGHT AGAINST IT BY USING A COMBINATION OF TECHNIQUES:

  * PROTECT YOUR QUANTUM INFORMATION BY ENCODING IT IN DECOHERENCE FREE (POINTER) SUBSPACES,

  * USE ERROR CORRECTION CODES

- THIS REQUIRES DETAILED KNOWLEDGE OF THE REAL (PHYSICAL) ERRORS BUT IS POSSIBLE IN PRINCIPLE