Quantum Information Processing & Condensed Matter Physics with Cold (Neutral) Atoms

UNIVERSITY OF INNSBRUCK

AUSTRIAN ACADEMY OF SCIENCES

a tour …

- cold atoms in optical lattices
  – AMO Hubbard-ology for beginners
- applications
  – quantum info and condensed matter

Peter Zoller

SFB
Coherent Control of Quantum Systems

€U networks

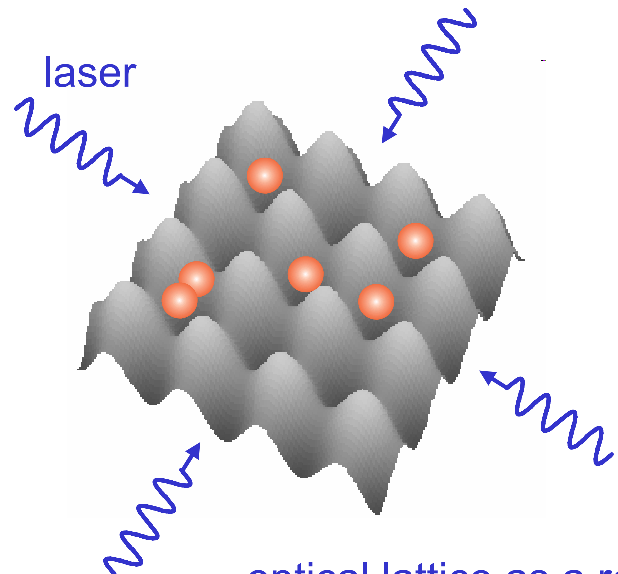
a sneak preview:

cold atoms in optical lattices

Cold atoms in optical lattices

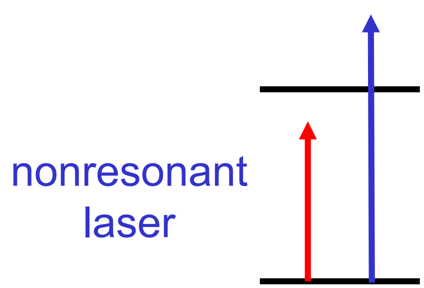
- Loading cold bosonic or fermionic atoms into an optical lattice

- Atomic Hubbard models with *controllable* parameters
  - bose / fermi atoms
  - spin models

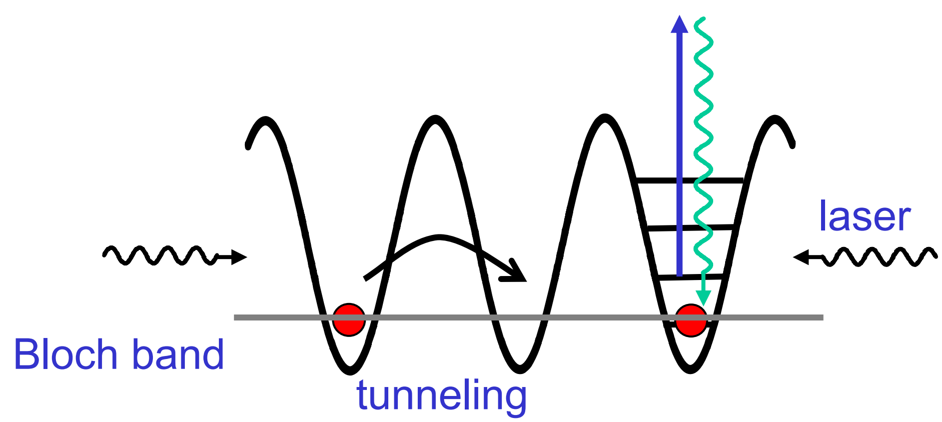


optical lattice as a regular array of microtraps for atoms

- Optical lattice



AC Stark shift



optical lattice as array of microtraps

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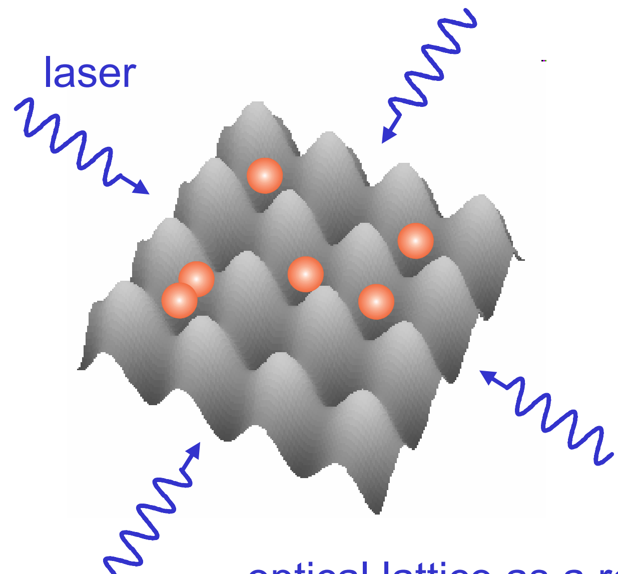

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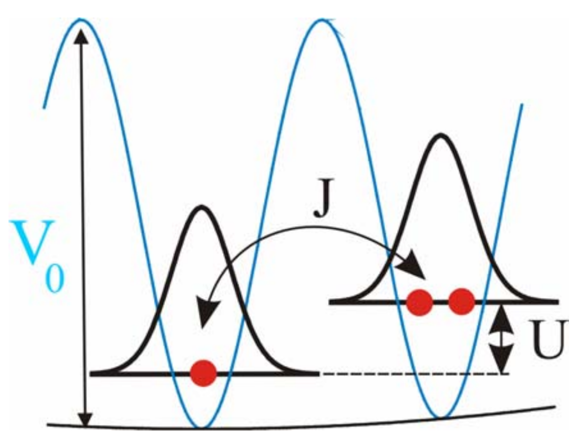

- Example: Bose Hubbard model

$$H = -\sum_{\alpha \neq \beta} J_{\alpha\beta} b_\alpha^\dagger b_\beta + \frac{1}{2} U \sum_\alpha b_\alpha^\dagger b_\alpha^\dagger b_\alpha b_\alpha$$

kinetic energy: hopping

interaction: onsite repulsion

$$[b_\alpha, b_\beta^\dagger] = \delta_{\alpha\beta}$$ bosons

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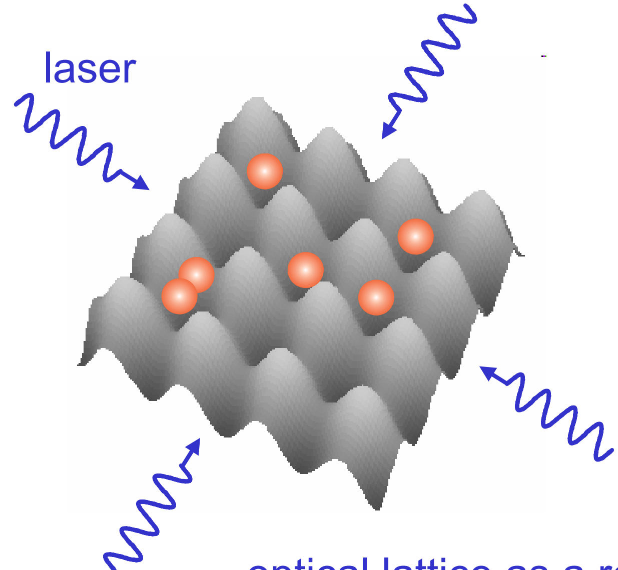

optical lattice as a regular array of microtraps for atoms

- atomic physics: features
  - ⇨ – microscopic understanding of Hamiltonian
  - ⇨ – controlling & "engineering" interactions / Hamiltonian
  - …

"engineering Hubbard Hamiltonians"

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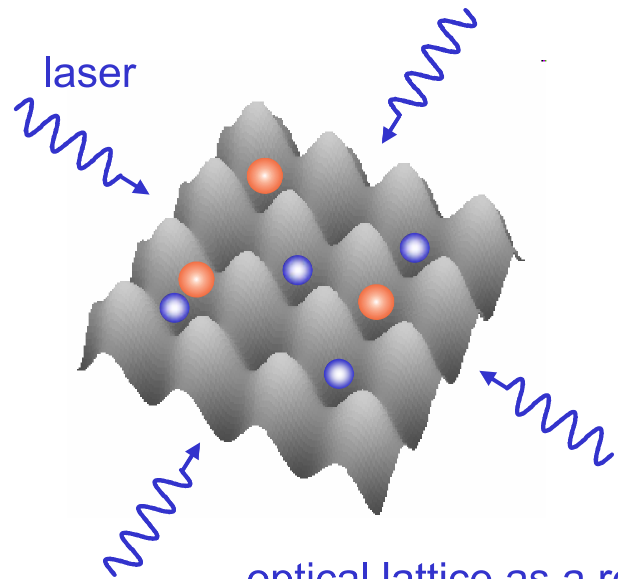
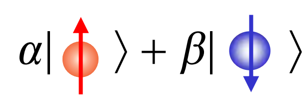
  – controlling & "engineering" interactions / Hamiltonian
  – adding internal degrees of freedom

"engineering Hubbard Hamiltonians"



optical lattice as a regular array of microtraps for atoms



filling the lattice with "qubits"

Cold atoms in optical lattices

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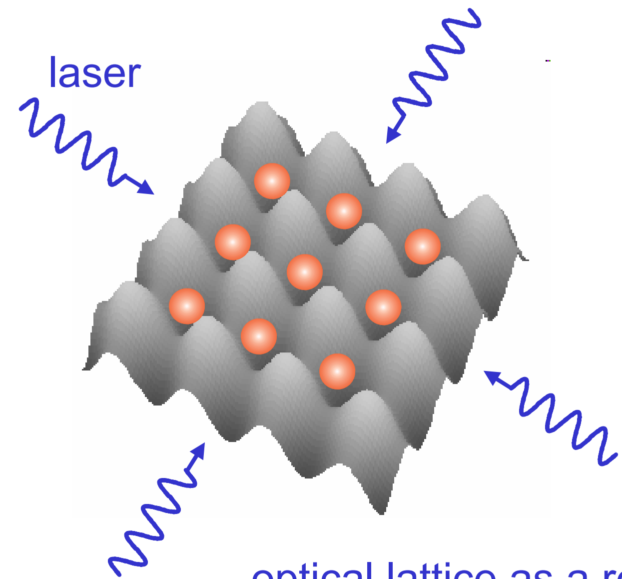
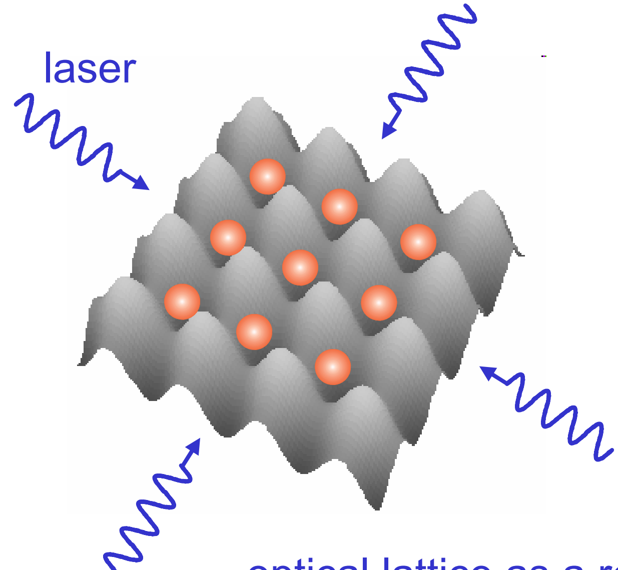


optical lattice as a regular array of microtraps for atoms

- additional aspects / features
  – controlled lattice loading, e.g. unit filling
  – measurement
  – …
  – adding phonons …
  – adding (controlled) dissipation …

Cold atoms in optical lattices

- Loading cold bosonic or fermionic atoms into an optical lattice



optical lattice as a regular array of microtraps for atoms

- Atomic Hubbard models with *controllable* parameters
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- atomic physics: features
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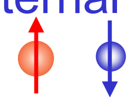
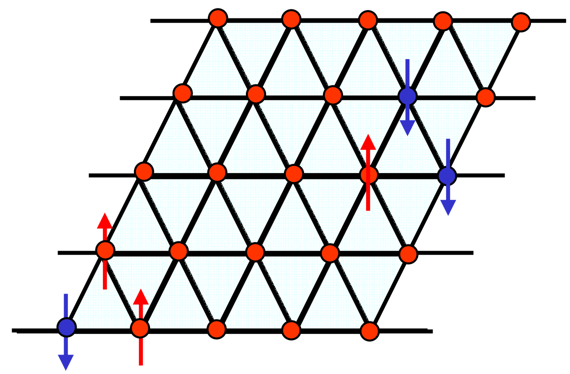


- additional aspects / features
  - controlled lattice loading, e.g. unit filling
  - measurement
  - …
  - adding phonons …
  - adding (controlled) dissipation …

"Atomic Hubbard toolbox"

Why? … condensed matter physics & quantum information

- condensed matter physics
  - strongly correlated systems: high Tc etc.
  - time dependent, e.g. quantum phase transitions
  - …
  - exotic quantum phases (?)

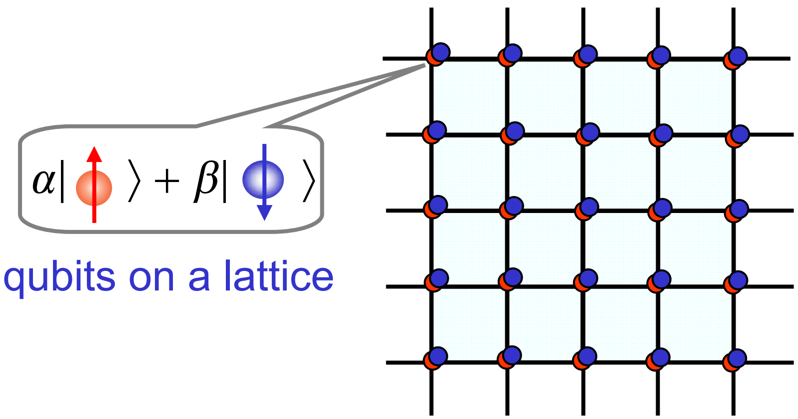


- quantum information processing
  - new quantum computing scenarios, e.g. "one way quantum computer"

"quantum simulator„

-analogue simulators

- digital simulators



qubits on a lattice

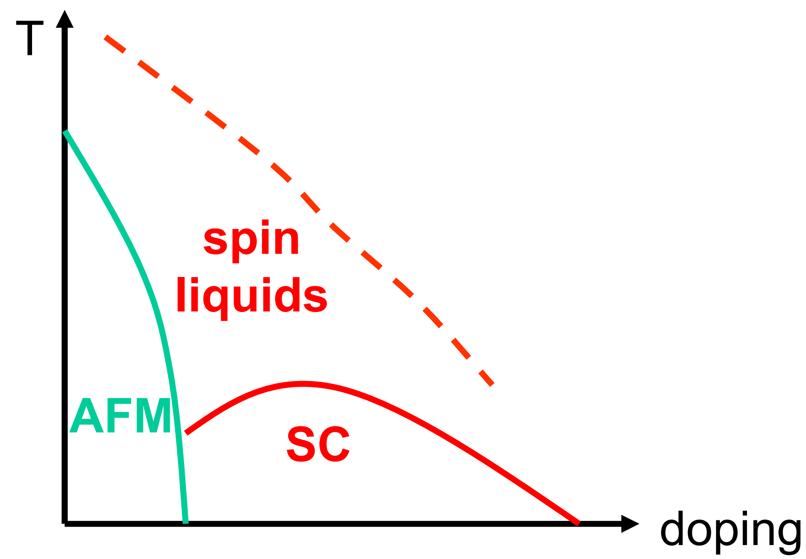
entangling qubits via "Ising"
(cluster state)

- experiments [Bloch et al. 2001, T. Esslinger et al., … J. Denschlag et al.]
  - Superfluid-Mott Insulator Quantum Phase Transition
  - spin dependent lattice & entanglement
  - molecules …

1. “Analogue simulation” of cond mat Hubbard models

- We can build Hubbard models directly to *simulate* condensed matter models with controllable parameters
- Example:
  high T_c superconductivity

$$H = -t \sum_{i,j,\sigma} (c_{i\sigma}^{+} c_{j\sigma} + c_{j\sigma}^{+} c_{i\sigma}) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



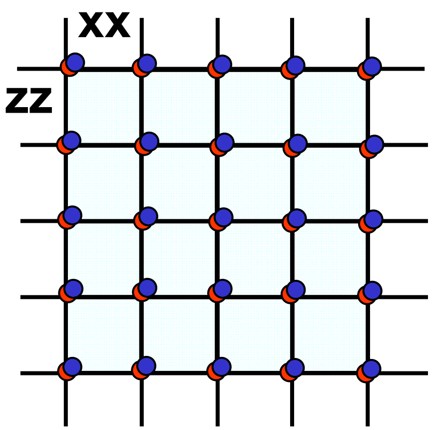
- Solving open problems … where theory does not give an answer:
  - ground state; strange quantum phase & properties
  - exotic excitations

2. … and Lattice Spin Models: topological quantum computing

- exotic spin models …

Examples:

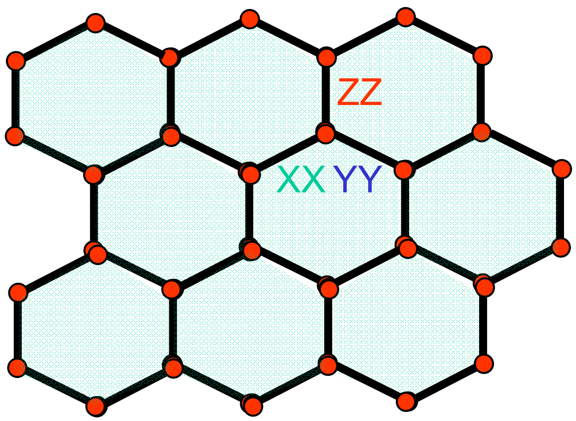
Doucot, Feigelman, Ioffe et al.



$$H^{(\mathrm{I})}_{\mathrm{spin}} = \sum_{i=1}^{\ell-1} \sum_{j=1}^{\ell-1} J(\sigma^z_{i,j}\sigma^z_{i,j+1} + \cos\zeta\, \sigma^x_{i,j}\sigma^x_{i+1,j})$$

protected quantum memory:
degenerate ground states as qubits

Kitaev



$$H^{(\mathrm{II})}_{\mathrm{spin}} = J_\perp \sum_{x\text{–links}} \sigma^x_j \sigma^x_k + J_\perp \sum_{y\text{–links}} \sigma^y_j \sigma^y_k + J_z \sum_{z\text{–links}} \sigma^z_j \sigma^z_k$$

3. Digital Quantum Simulator – an example

- Idea: build a Hamiltonian as a time-averaged effective Hamiltonian from one and two qubit gates

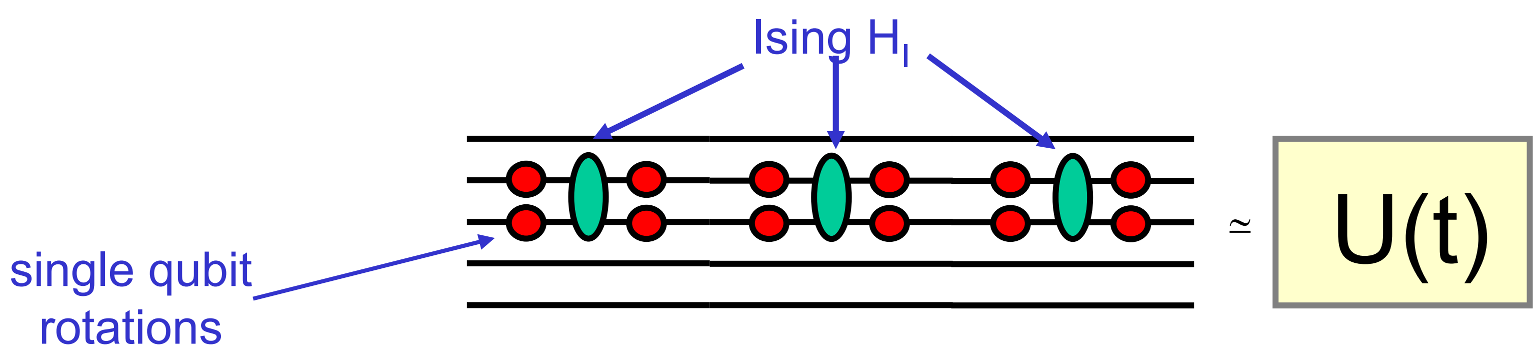
- Example: given Ising:

we will show that we can do this “easily” with atoms

$$H = -\frac{J}{2} \sum_{\langle a,b \rangle} \sigma_z^{(a)} \otimes \sigma_z^{(b)}$$

- … we want to simulate the Heisenberg model:

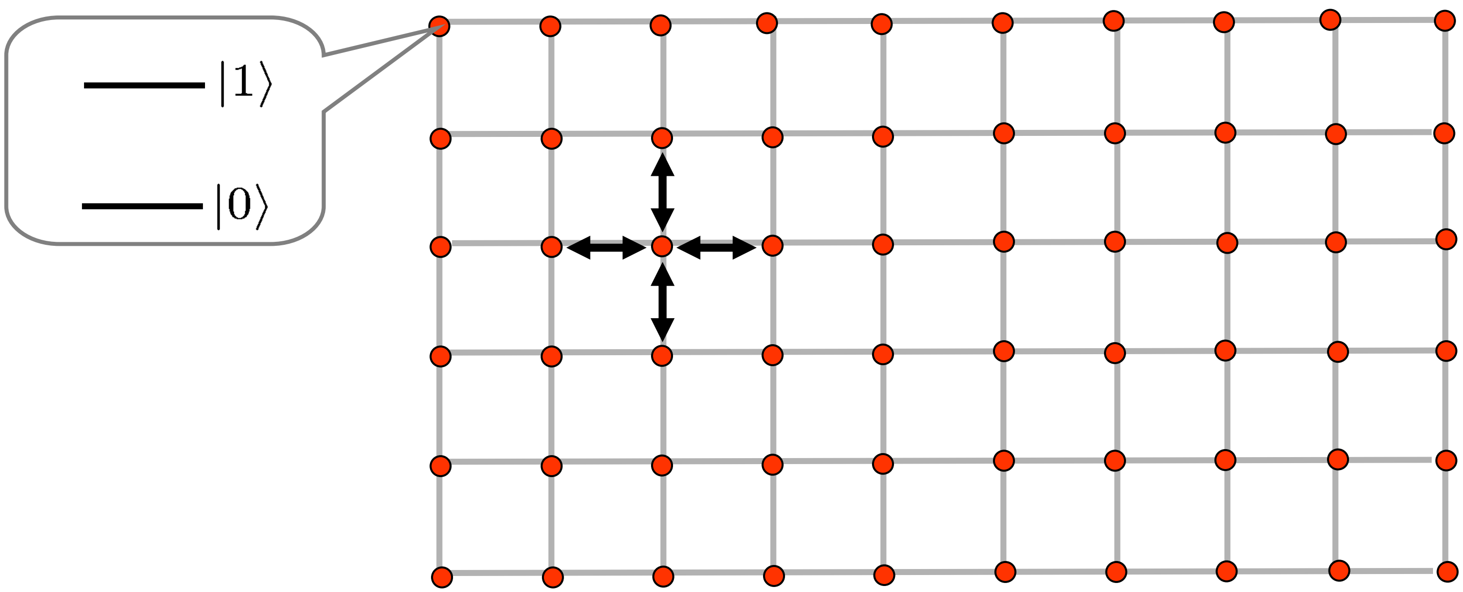
$$H = -\frac{J}{2} \sum_{\langle a,b \rangle} \left(\sigma_x^{(a)} \otimes \sigma_x^{(b)} + \sigma_y^{(a)} \otimes \sigma_y^{(b)} + \sigma_z^{(a)} \otimes \sigma_z^{(b)} \right)$$



Raussendorf & Briegel, PRL 2001

4. Measurement based quantum computing

- Cluster state as a quantum resource: exploit quantum correlations
- Information processing via measurement of single qubits on a lattice



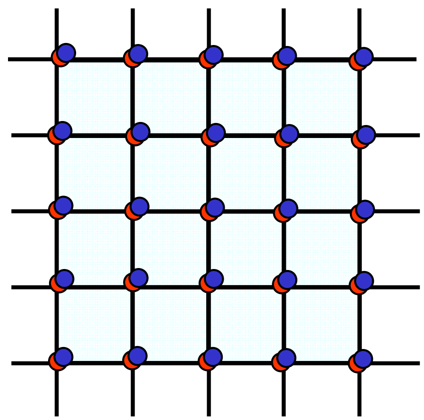
2D cluster state

How to create a cluster state?

- prepare quantum memory in product state $|s_x\rangle = \otimes_{a\in C} |+\rangle_a$
  with each qubit in state $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_z + |1\rangle_z)$

- switch on Ising interaction

$H_{\text{int}} = -\frac{1}{4}\hbar g(t)\sum_{<a,b>} \sigma_z^{(a)} \otimes \sigma_z^{(b)}$ for a given time $\varphi(t) = \int_0^\tau g(t)dt = \pi$



2D cluster state

$$|\phi\rangle_c = \exp(i\int_0^t \tfrac{1}{4}\hbar g(t)dt\sum_{<a,b>} \sigma_z^{(a)} \otimes \sigma_z^{(b)} dt)\;\; (\otimes_{a\in C} |+\rangle_a)$$

This can be implemented efficiently with a spin-dependent optical lattice.

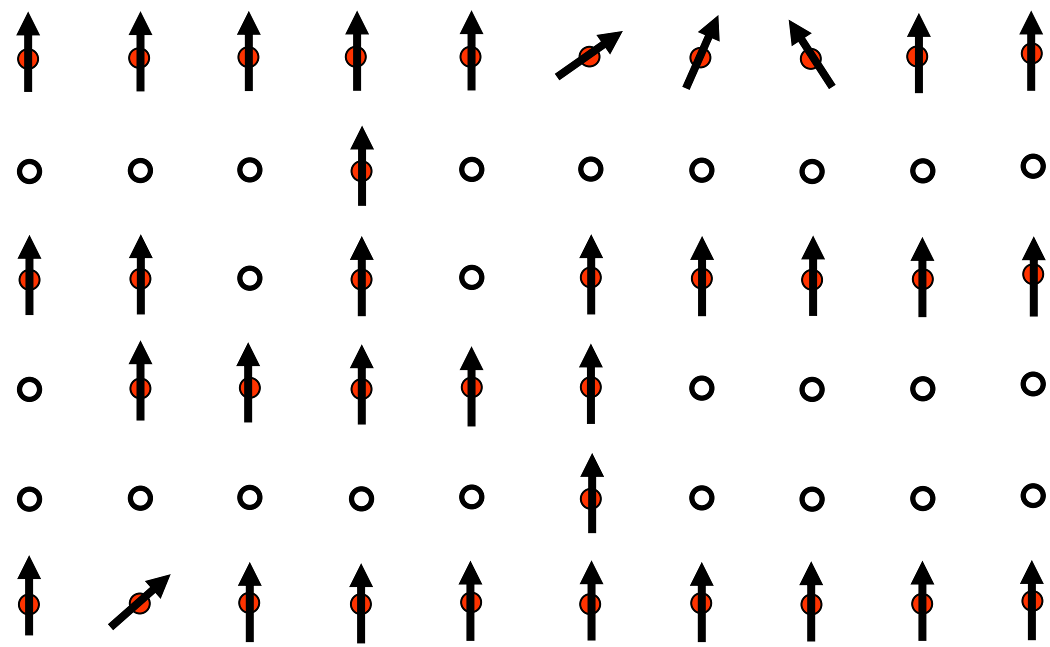
Measurement based quantum computing

- Cluster state as a quantum resource: exploit quantum correlations
- Information processing via measurement of single qubits on a lattice

○ $\boldsymbol{\sigma_z}$ **measurement**

↑ σ_x measurement

↗ in x-y plane



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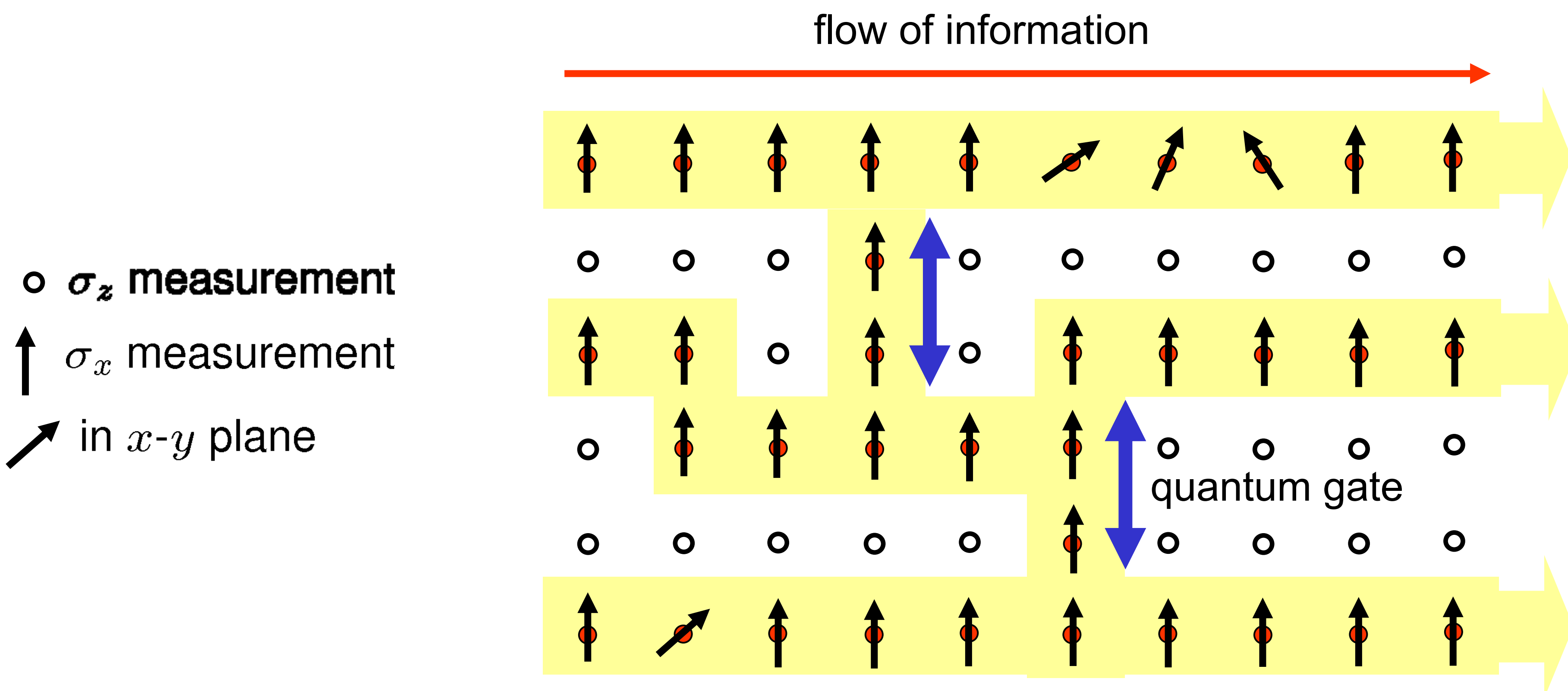
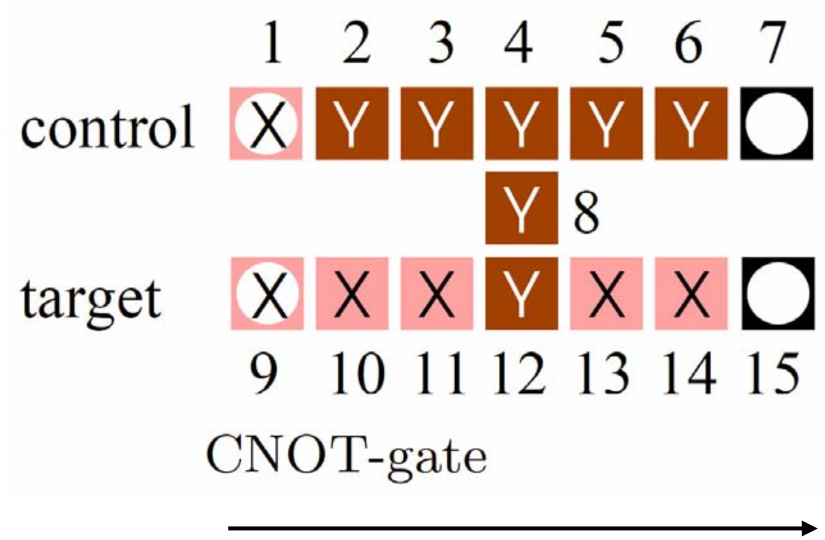
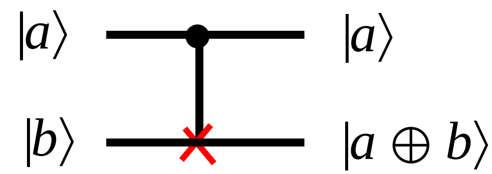


Quantum Gates



flow of information



Controlled-NOT

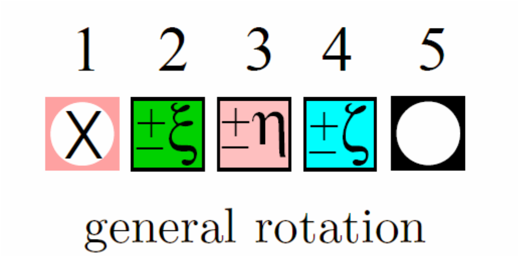
truth table CNOT:

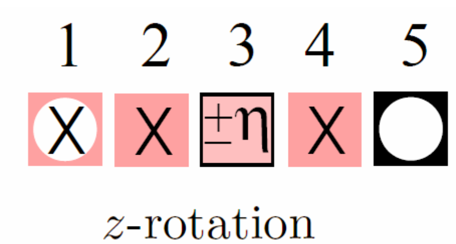
$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$

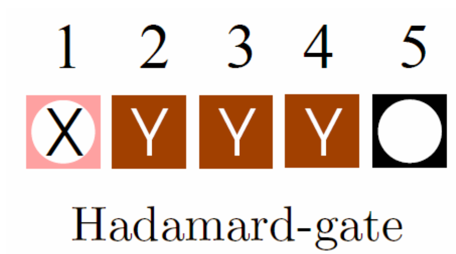
$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$

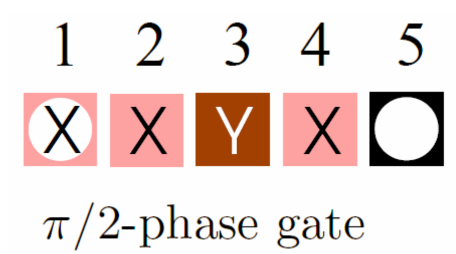
$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$

$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$

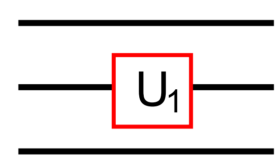








$\hat{U}_1$ = rotation of a single qubit

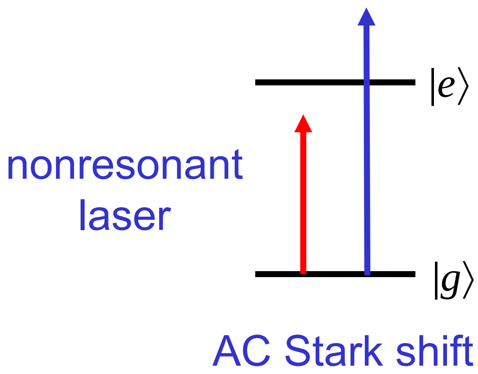


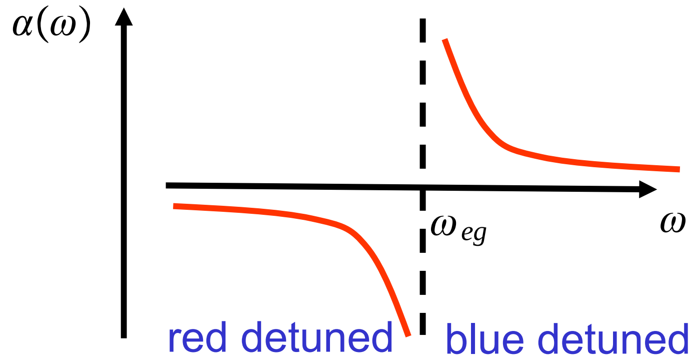
AMO Hubbard "toolbox"

- Optical lattices
  - basic ideas, properties & special topics
- Hubbard models
  - naive derivation & microscopic picture, spin models, validity
- Lattice loading & Measurements
- Time-dependent aspects

1. Optical lattices: basics

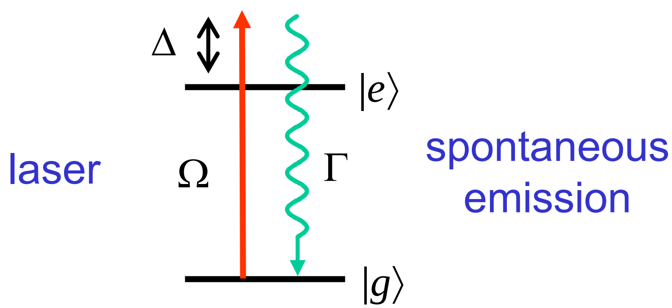
- AC Stark shift





$$\Delta E = \alpha(\omega) I \qquad (I \sim |\mathcal{E}|^2)$$

- decoherence: spontaneous emission



$$\Delta E_g = \frac{1}{4}\frac{\Omega^2}{\Delta - \frac{1}{2} i\Gamma} = \delta E_g - i \frac{1}{2}\gamma_g$$

$$\frac{\text{good}}{\text{bad}} = \frac{\delta E_g}{\gamma_g} \sim \frac{|\Delta|}{\Gamma} \gg 1$$

typical off-resonant lattice : $\gamma \sim \text{sec}^{-1}$

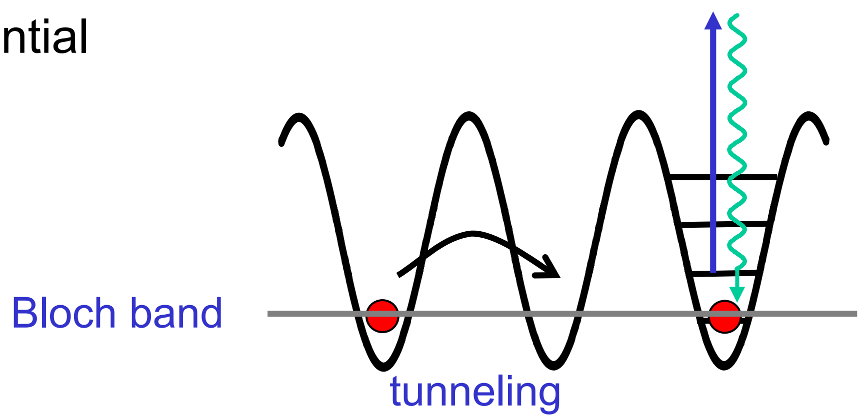
In a blue detuned lattice this can be strongly suppressed

- standing wave laser configuration

laser → ← laser

$$\vec{E} = \vec{\epsilon}\mathcal{E}_0 \sin kz e^{-i\omega t} + \text{c.c.}$$

- optical potential



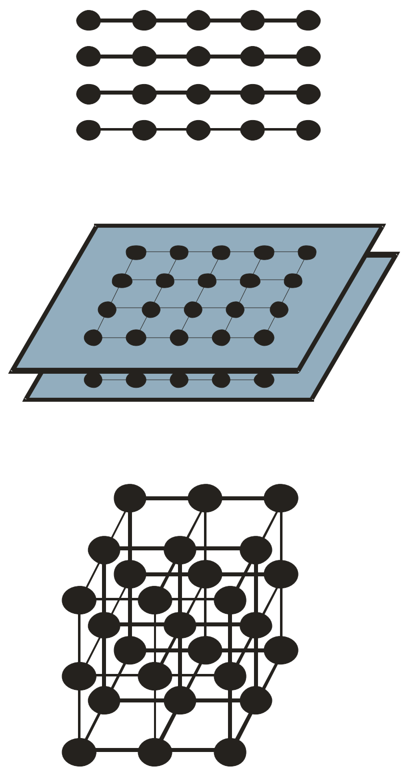
optical lattice as array of microtraps

$$V(x) = V_0 \sin^2 kx \qquad \left(k = \frac{2\pi}{\lambda}\right)$$

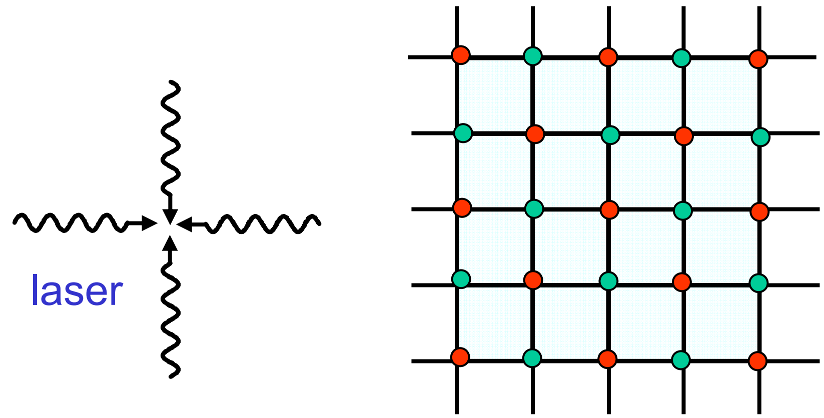
- Schrödinger equation for center of mass motion of atom

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\psi(x,t)$$

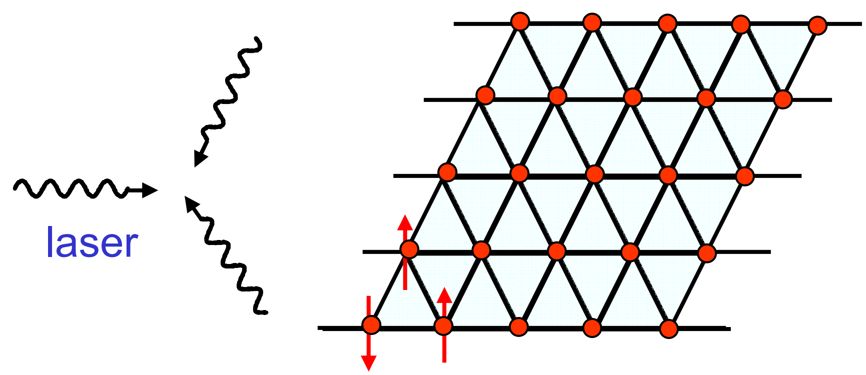
- 1D, 2D and 3D



- lattice configurations



square lattice



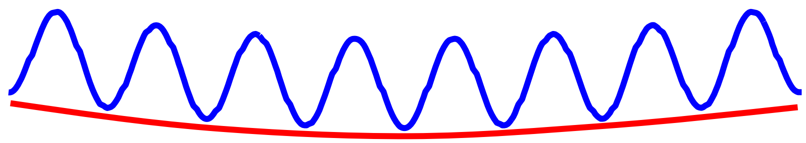
triangular lattice

Remarks:

✓ optical potentials generated by lattice beams with different frequencies add up incoherently

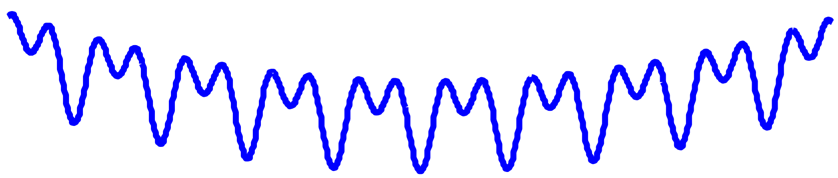
✓ interferometric stability (!?)

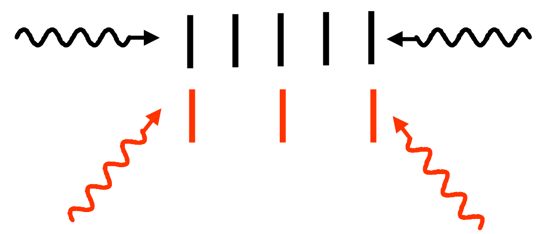
- harmonic background potential (e.g. laser focus, magnetic trap)



undo by inverse harmonic potential e.g. magnetic field

- superlattice





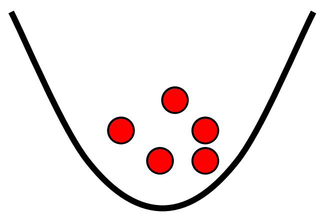
- random potentials
  - add more lasers from random direction or speckle pattern

- Single atom coherent dynamcis studied …
  - Wannier-Bloch
  - quantum chaos: kicked systems

Jaksch et al 1998

2. Bose Hubbard in optical lattice: naïve derivation

- dilute bose gas



$$H = \int \psi^\dagger(\vec{x}) \left(-\frac{\hbar^2}{2m}\nabla^2 + V_T(\vec{x}) \right) \psi(\vec{x})\, d^3x$$

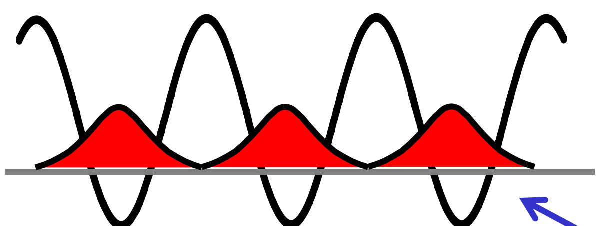
$$+\frac{1}{2} g \int \psi^\dagger(\vec{x})\psi^\dagger(\vec{x})\psi(\vec{x})\psi(\vec{x})\, d^3x$$

collisions

- validity: dilute gas, $a_s \ll a_0 < \lambda/2$

$$g = \frac{4\pi a_s \hbar^2}{m}$$ scattering length

- optical lattice



$$\psi(\vec{x}) = \sum_\alpha w(\vec{x} - \vec{x}_\alpha)\, b_\alpha$$

Wannier functions

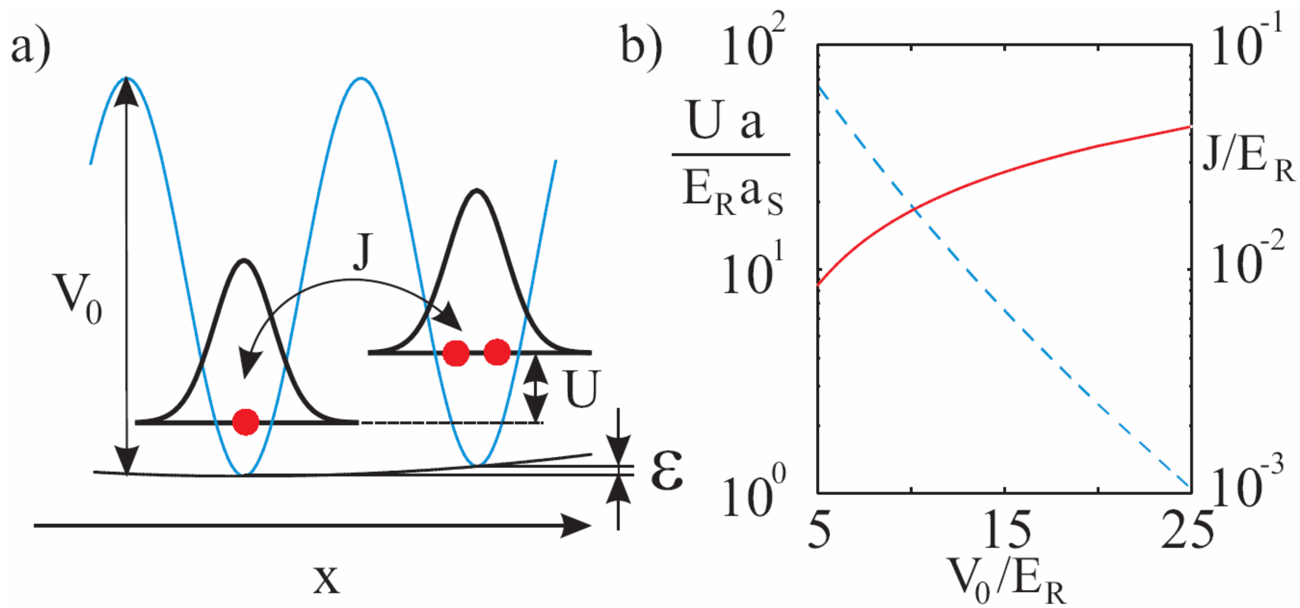
- Hubbard model

$$H = -J\sum_{<i,j>} b_i^\dagger b_j + \tfrac{1}{2}U\sum_i b_i^\dagger b_i^\dagger b_i b_i + \sum_i \epsilon_i b_i^\dagger b_i$$

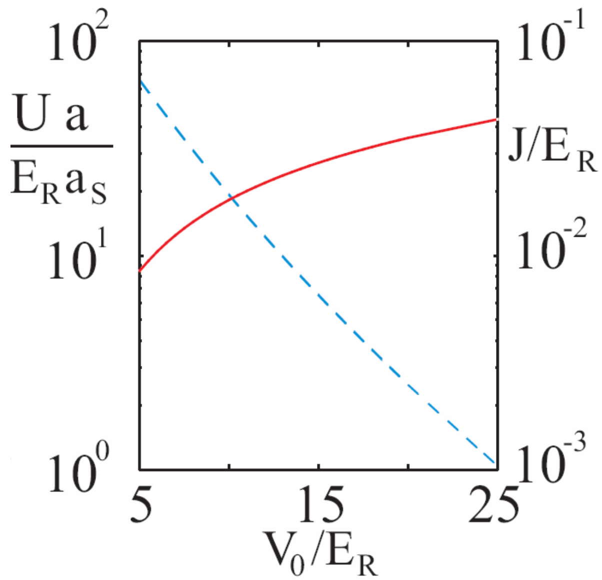
kinetic engery: hopping

interaction: onsite repulsion

$$U = g\int |w(\vec{x})|^4 d^3x$$



- feature: (time dep) tunability from weakly to strongly interacting gas
- validity …



- parameters approx. SF-Mott transition:

  recoil energy $E_R = \hbar^2 k^2/2m,\ V_0 \sim 9E_R$

  Na: $E_R = 25$ KHz, $J \sim 1$ KHz, $U \sim 10$ KHz,

  Rb: $E_R = 3.8$ KHz ...

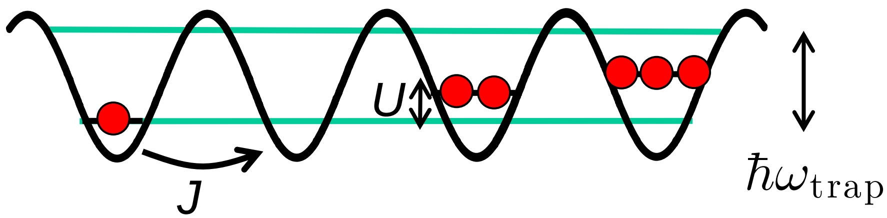
- validity:

  $a_s \ll a_0 < \lambda/2,\ U \ll \hbar\omega_{\text{Bloch}}$ and $T \sim 0 :\quad kT \ll J, U \ll \hbar\omega_{\text{Bloch}}$,

  density $n \sim 10^{14} - 10^{15}\text{cm}^{-3}$ (three particle loss)

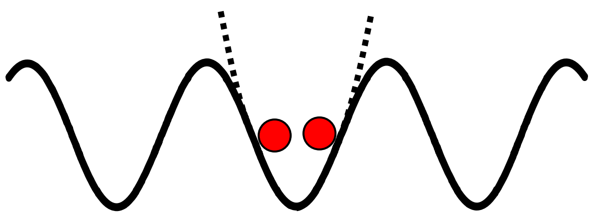
Hubbard model: microscopic picture

- Hubbard



✓ solve in n=1,2,3,… particle sector
✓ connect by tunneling (e.g in a tight binding approx)

- n=2 atoms on one lattice site: molecule

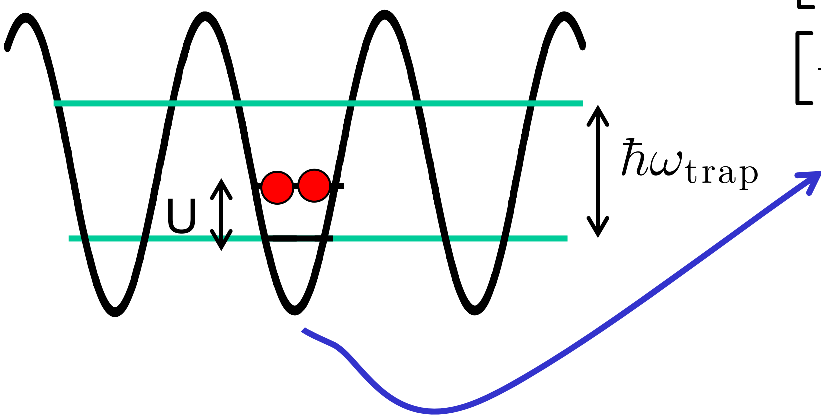


✓ molecular problem with added optical potential

- n=3 atoms on one lattice site: … e.g. Efimov-type problem
- [n>n_{max}~3 killed by three body etc. loss]

Julienne et al.

- two atoms on one site

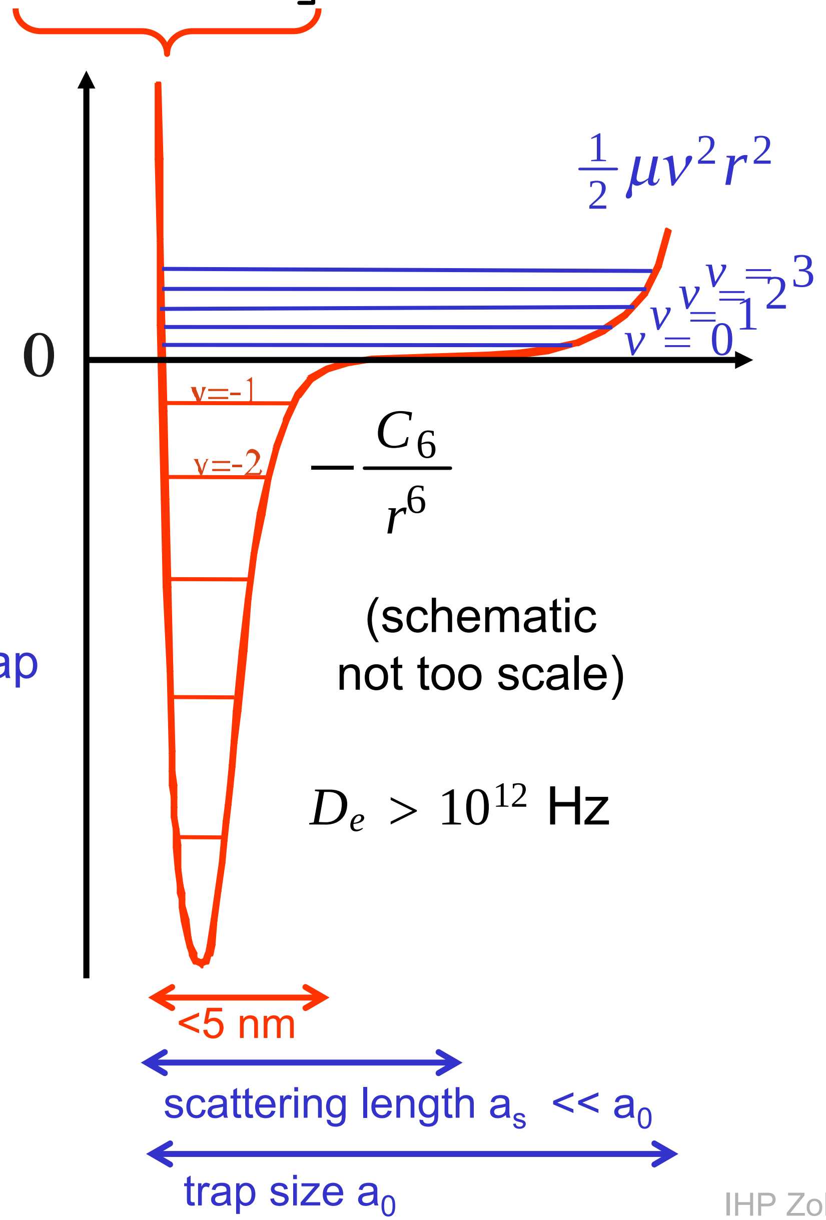


harmonic approximation

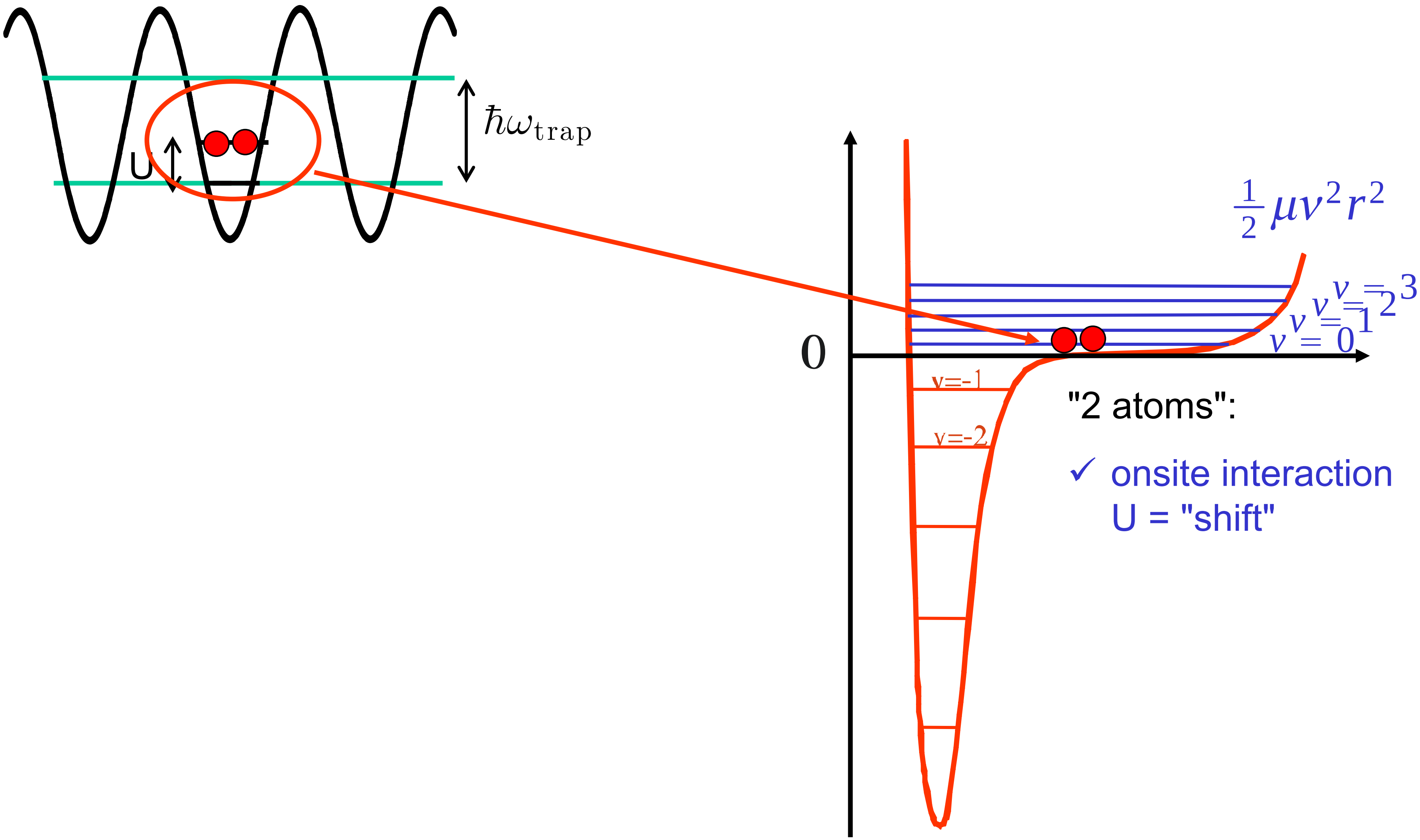
$$\left[-\frac{\hbar^2}{2(2m)}\nabla_R^2 + \frac{1}{2}(2m)\nu^2 R^2\right]\psi_{cm}(R) = E_{cm}\psi_{cm}(R)$$

$$\left[-\frac{\hbar^2}{2\mu}\nabla_r^2 + \frac{1}{2}\mu\nu^2 r^2 + V(r)\right]\psi(r) = E\psi(r)$$

Born Oppenheimer potentials including trap



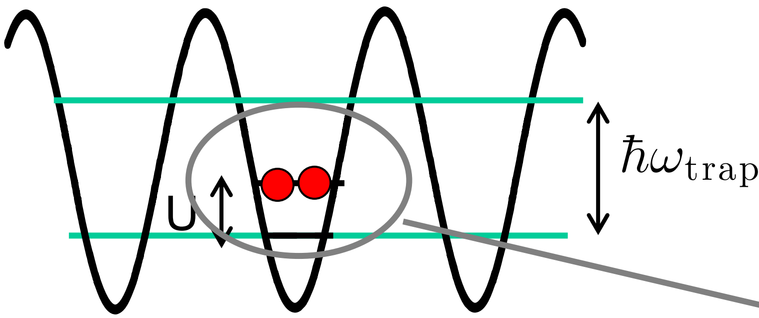
- two atoms on one site

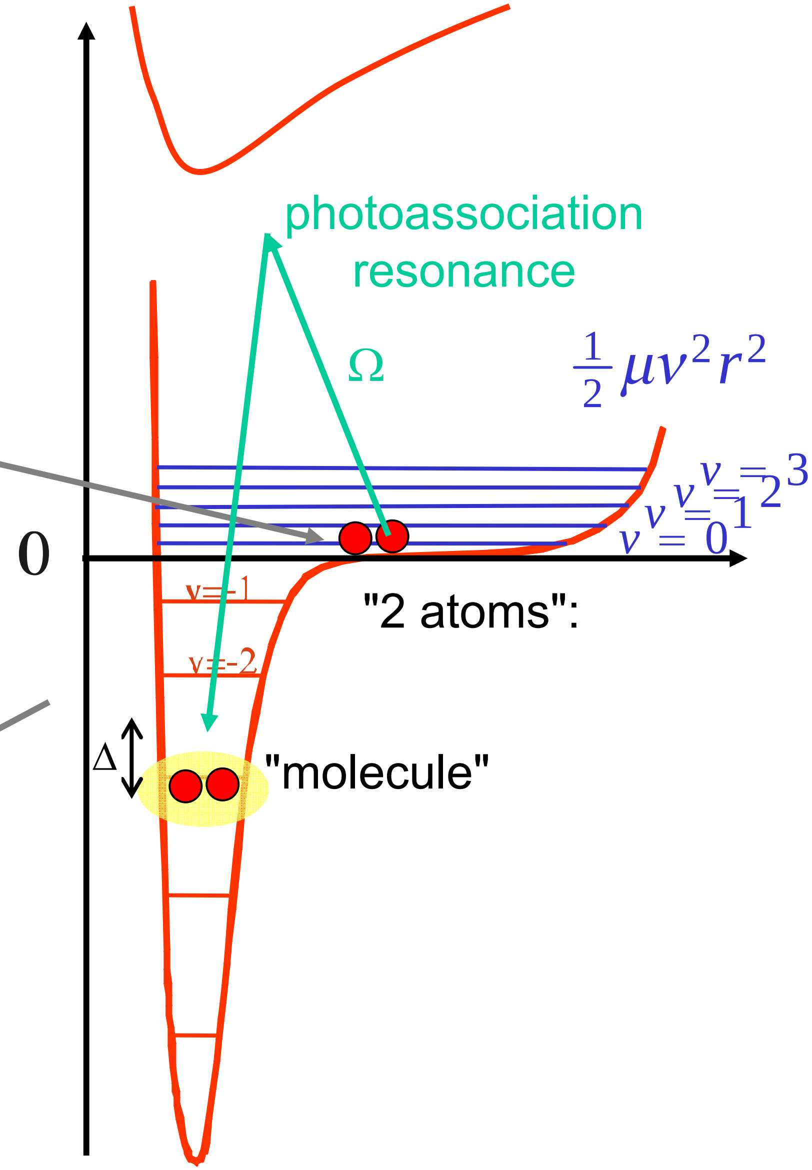


"2 atoms":

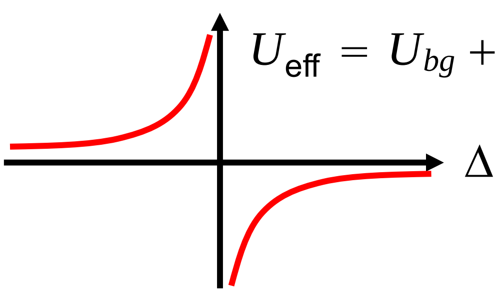
✓ onsite interaction
U = "shift"

- two atoms on one site





AC Starkshift = optical Feshbach resonance

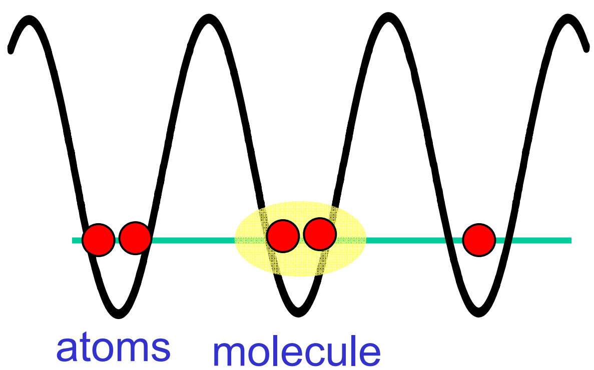


$$U_{\rm eff} = U_{bg} + \frac{1}{4}\frac{\Omega^2}{\Delta}$$

$$H = (U_{bg} + \frac{1}{4}\frac{\Omega^2}{\Delta})b^{\dagger 2}b^2$$

Hubbard model including molecules

- Hamiltonian



$$H = -J_b \sum_{<i,j>} b_i^\dagger b_j + \frac{1}{2} U_b \sum_i b_i^\dagger b_i^\dagger b_i b_i$$
$$- J_m \sum_{<i,j>} m_i^\dagger m_j + \frac{1}{2} U_m \sum_i m_i^\dagger m_i^\dagger m_i m_i - \sum_i \Delta m_i^\dagger m_i$$
$$+ \frac{1}{2} \Omega \sum_i m_i^\dagger b_i b_i + \text{h.c.}$$

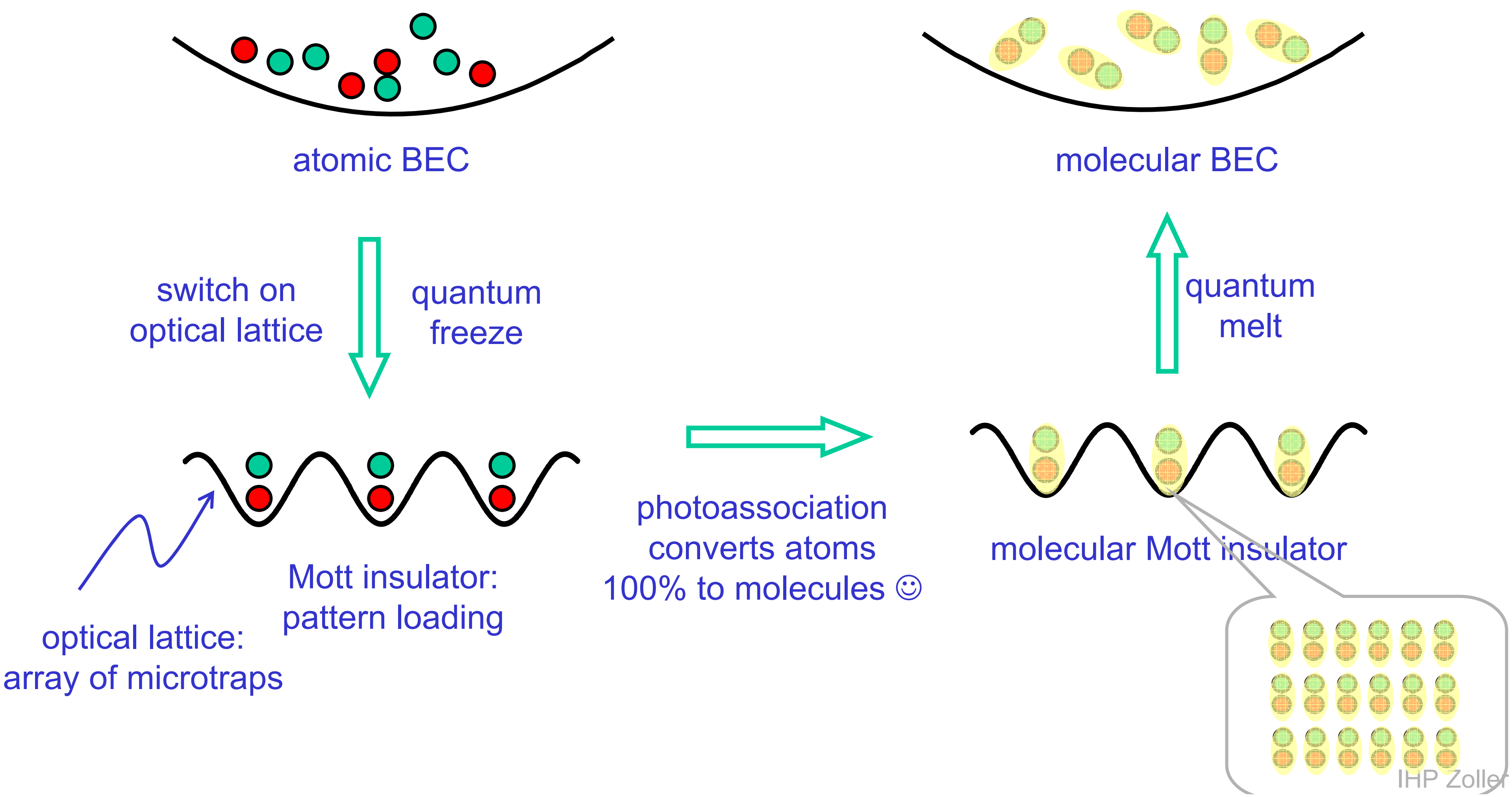
Remarks:

✓ we have derived this only for sector: 2 atoms or 1 molecule

✓ inelastic collisions / loss for >2 atoms and >1 molecules (?)

Remark: quantum phases of "composite objects"

- molecular BEC via a quantum phase transition



Remarks:

Straightforward generalization of these Hubbard *derivations* to …

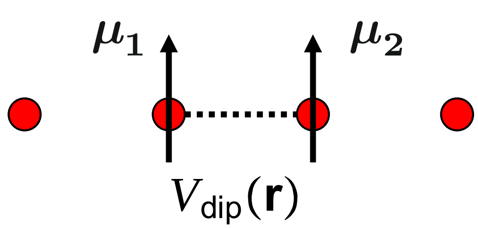
- bosons and / or fermions
- two-component mixtures of bosons / fermions
- dipolar gases / Hubbard models via heteronuclear molecules (long range dipolar forces)

Complaints:

- the time scales for tunneling are pretty long
- decoherence: spontaneous emission, laser / magnetic field fluctuations, …

Other ideas for interactions …

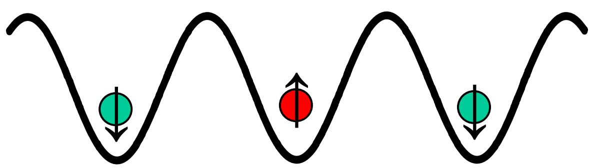
- optical dipole-dipole interactions … however loss ☹
- Rydberg-Rydberg interactions in a static electric field (huge)



e.g. Duan et al.

Spin models

- optical lattice



bosons in a Mott phase

$\langle n_{i\uparrow}\rangle + \langle n_{i\downarrow}\rangle \approx 1$

$$H = -J \sum_{<i,j>,\sigma} b_{i\sigma}^{\dagger} b_{j\sigma} + \tfrac{1}{2} U \sum_{i,\sigma} n_{i\sigma}(n_{i\sigma} - 1) + U_{\uparrow\downarrow} \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

$$\Downarrow$$

$$H = \sum_{\langle i,j\rangle} \left[\lambda_z \sigma_i^z \sigma_j^z \pm \lambda_\perp (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) \right]$$

XXZ-model

$$\lambda \sim \frac{J^2}{U}$$

pretty small ☹

$$\sigma_i^z = n_{\uparrow i} - n_{i\downarrow}$$

$$\sigma_i^x = b_{\uparrow i}^{\dagger} b_{i\downarrow} + b_{i\downarrow}^{\dagger} b_{\uparrow i}$$

$$\sigma_i^y = -i(b_{\uparrow i}^{\dagger} b_{i\downarrow} - b_{i\downarrow}^{\dagger} b_{\uparrow i})$$

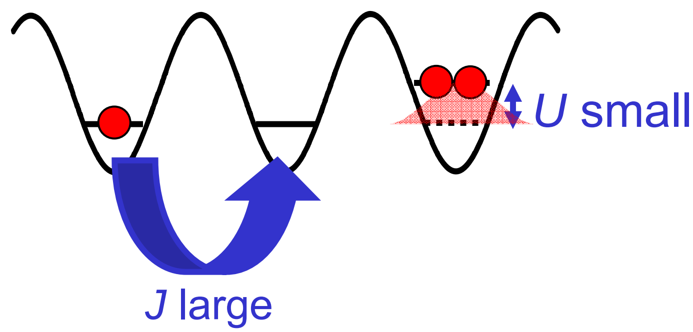
- ideas for higher order H= σ σ σ interactions …

Preparation of the lattice for quantum computing: one atom per lattice site

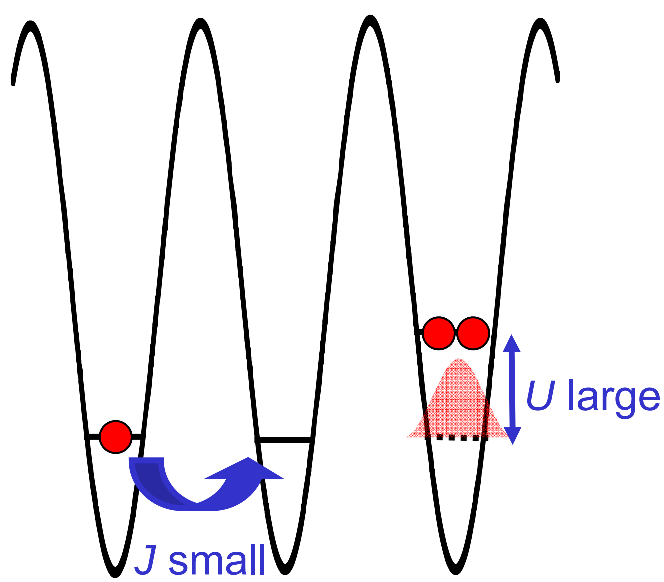
- superfluid – Mott insulator quantum phase transition in optical lattices

Laser control: kinetic vs. potential energy

- **shallow lattice** : weak laser



- **deep lattice:** intense laser



weakly interacting system:

J >> U

(kinetic energy >> interactions)

laser parameters
(time dependent)

strongly interacting system:

J << U

(kinetic energy << interactions)

Quantum phase transition

- Hamiltonian $H = H_0 + gH_1$ $[H_0, H_1] \neq 0$ no common eigenvectors

example: gas

kinetic energy (H_0)

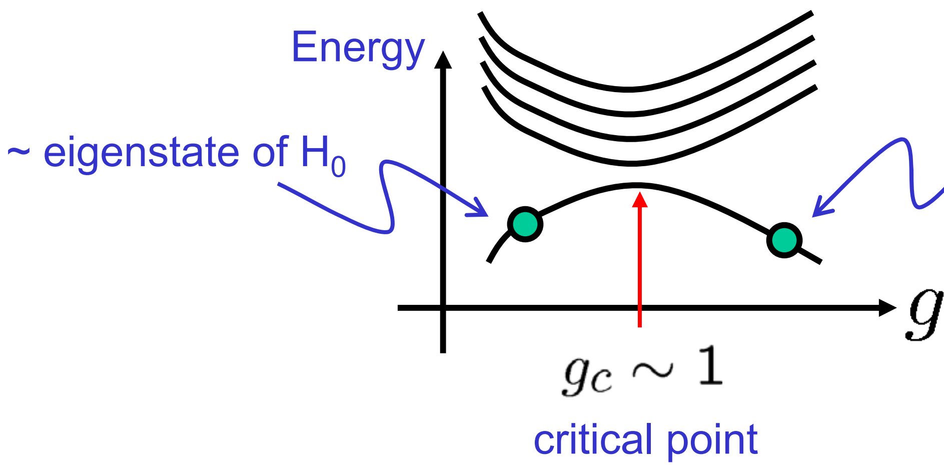
interaction energy (H_1)

g $\longrightarrow$ 0: dilute gas

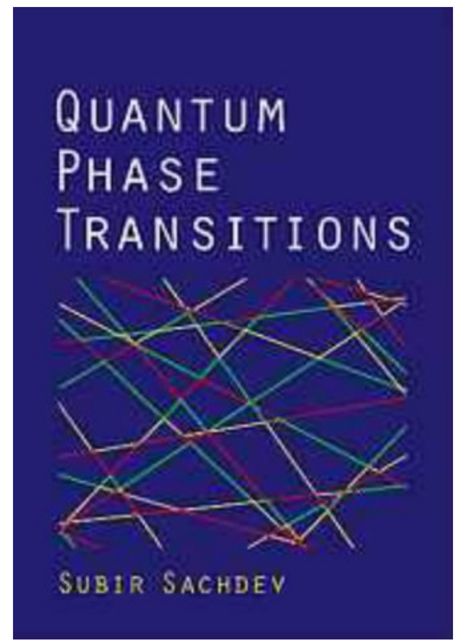
g $\longrightarrow \infty$: strongly correlated system

Quantum phase transition

- Hamiltonian $H = H_0 + gH_1 \qquad [H_0, H_1] \neq 0$ no common eigenvectors
- crossing the critical point (temperature T=0)



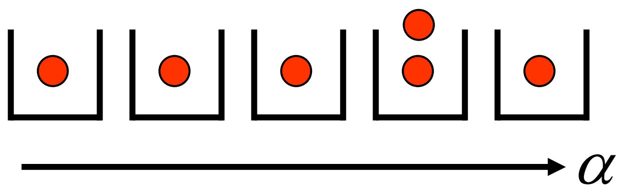
- why interesting?
  - fundamental interest
  - engineer interesting states ... e.g entangled states



BH: Fischer et al. 1989
optical lattice: Jaksch et al. 1998

Superfluid – Mott insulator quantum phase transition

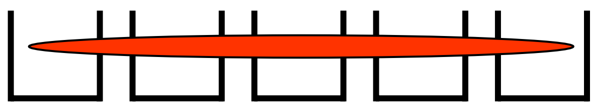
- N atoms in M lattice sites: (temperature T=0)



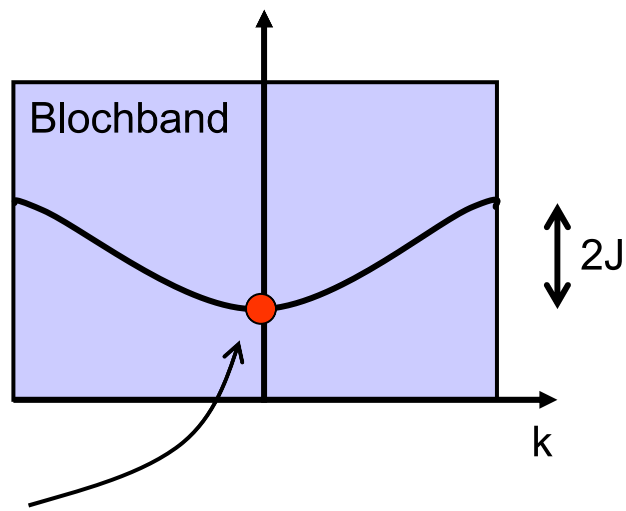
$$H = -\sum_{\alpha\neq\beta} J_{\alpha\beta} b_\alpha^\dagger b_\beta + \frac{1}{2} U \sum_\alpha b_\alpha^\dagger b_\alpha^\dagger b_\alpha b_\alpha$$

←competing interactions→

- superfluid J>>U



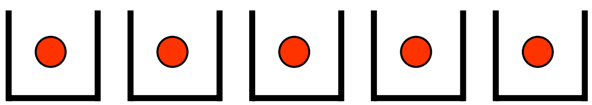
delocalized atoms: BEC



$$\left(b_{k=0}^\dagger\right)^N |\mathrm{vac}\rangle \sim \left(b_1^\dagger + \ldots + b_M^\dagger\right)^N |\mathrm{vac}\rangle$$

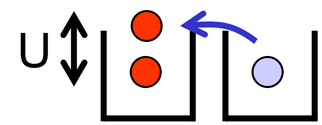
- Mott phase: J<<U

commensurate filling N=M



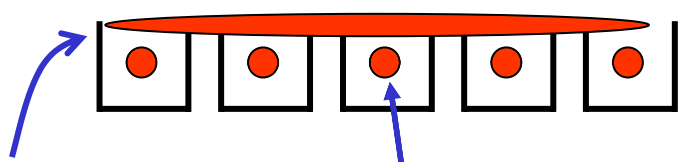
$$b_1^\dagger b_2^\dagger \ldots b_M^\dagger |\mathrm{vac}\rangle$$

excitations: gapped ~U

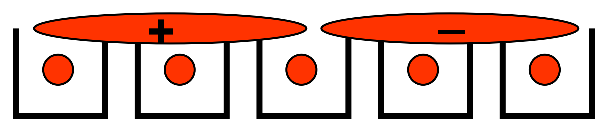


robust!

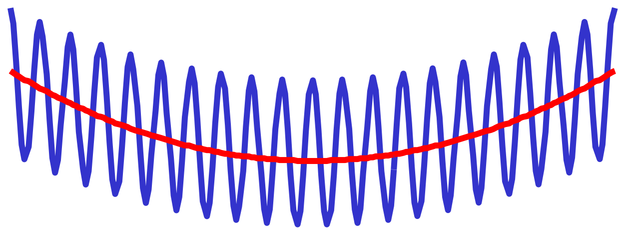
incommensurate filling N / M



excitations: ~J

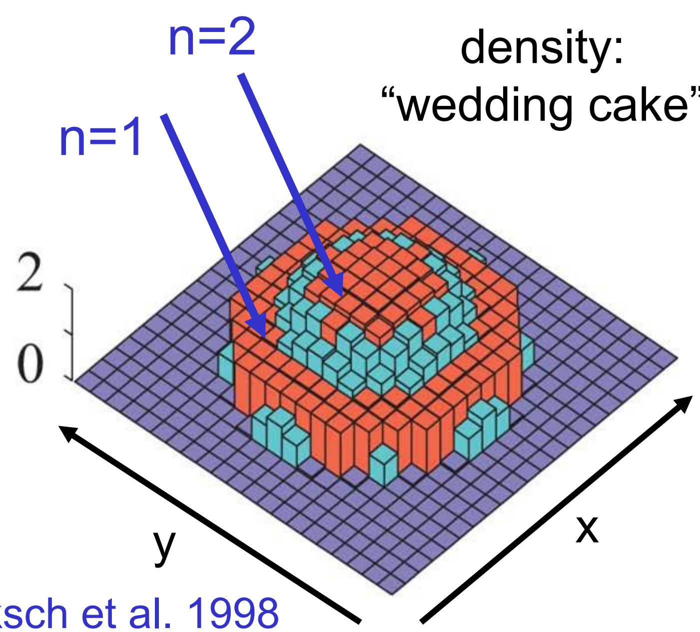


Superfluid and Mott in 2D trap



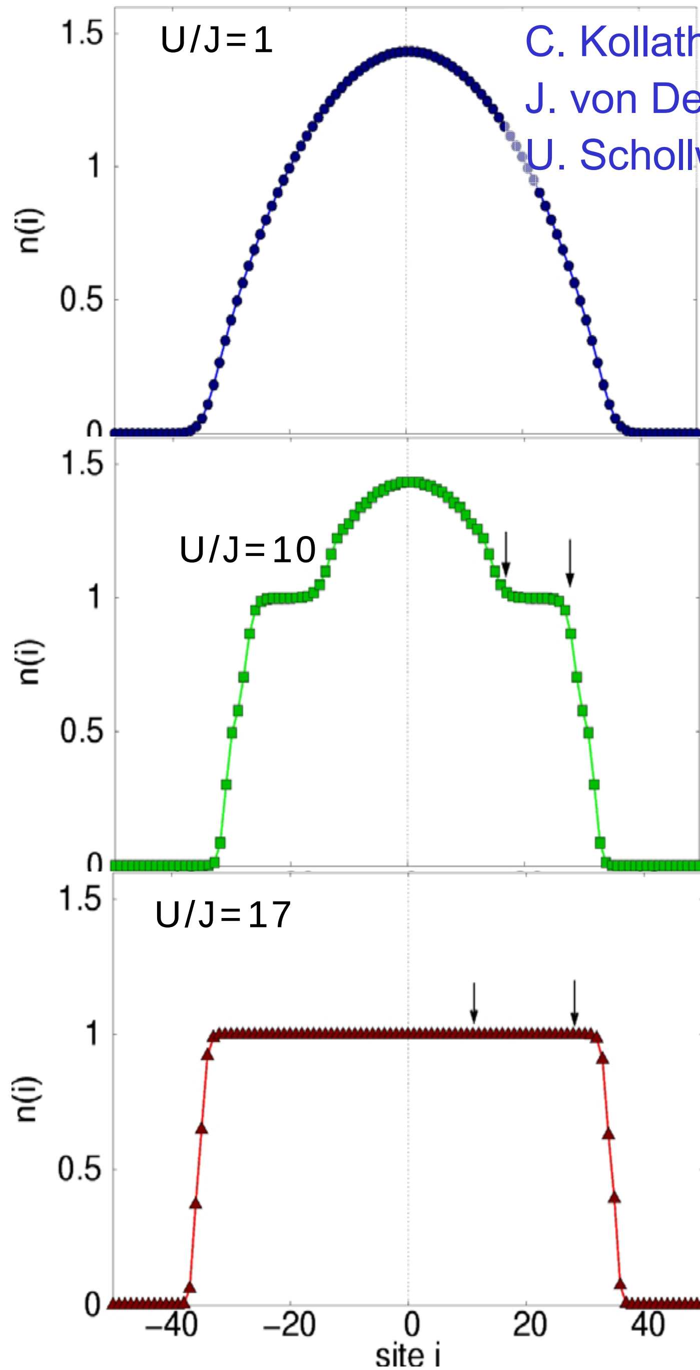
We achieve loading with exactly 1,2,… atoms!

density:
“wedding cake”



Jaksch et al. 1998

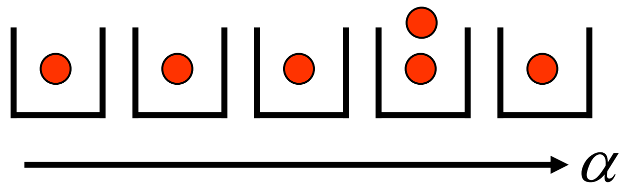
1D “exact” calculations



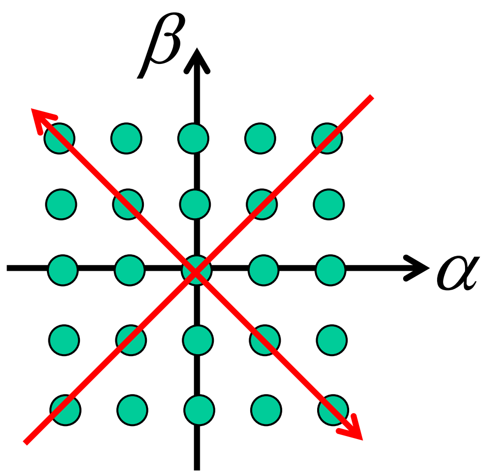
Zoller

Signatures of the Superfluid – Mott insulator transition

- spatial correlation function $\langle b_\alpha^\dagger b_\beta \rangle$



superfluid

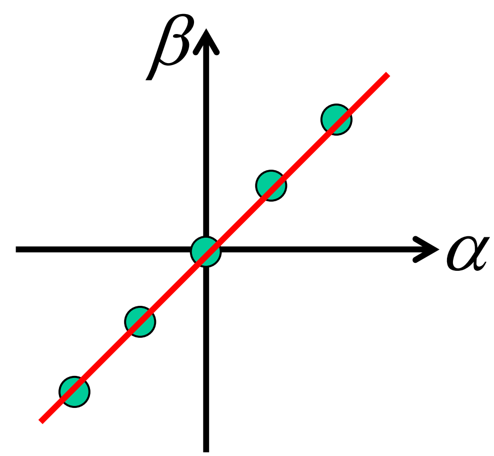


off-diagonal long range order:

interference

$$\langle b_\alpha^\dagger b_\beta \rangle \approx \psi_\alpha^* \psi_\beta$$

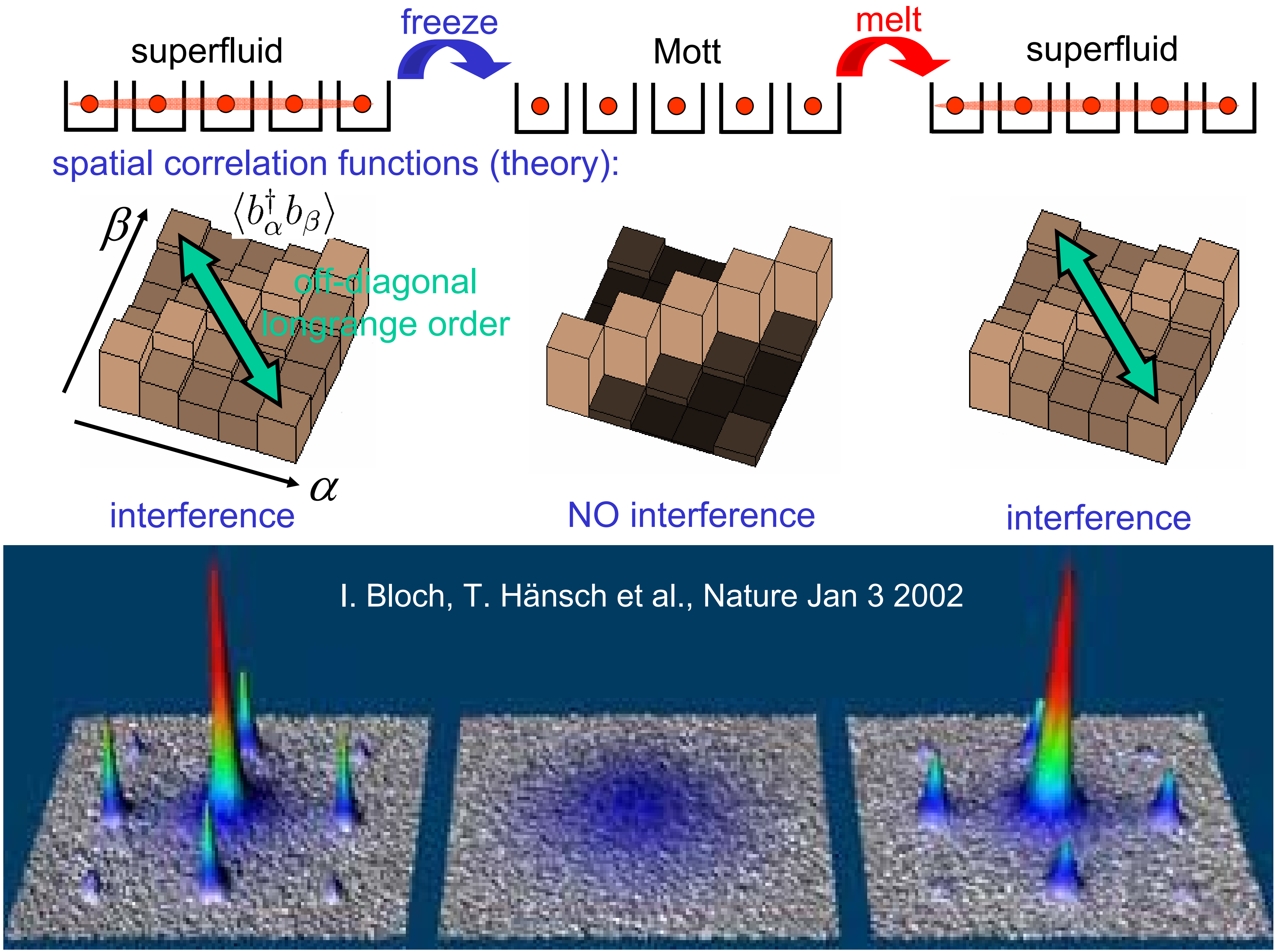
Mott



no interference

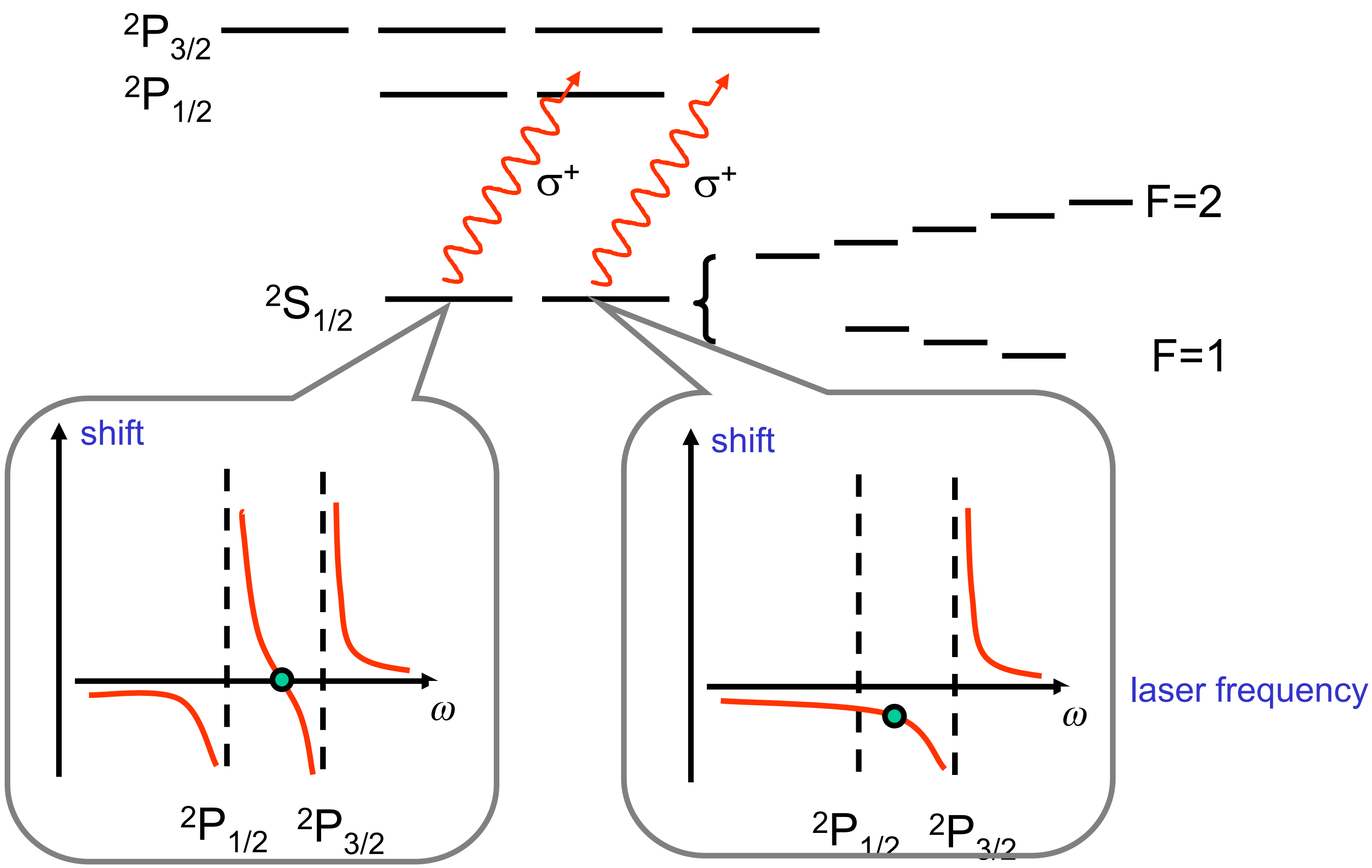
$$\langle b_\alpha^\dagger b_\beta \rangle \approx n_\alpha \delta_{\alpha\beta}$$

exp signature

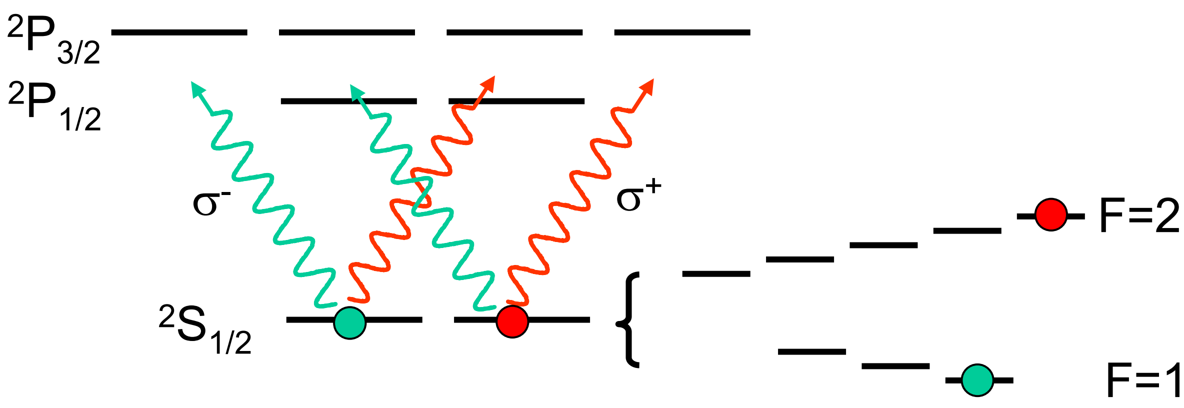

freeze
melt
superfluid
Mott
superfluid
spatial correlation functions (theory):
$\langle b_\alpha^\dagger b_\beta \rangle$
β
α
off-diagonal
longrange order
interference
NO interference
interference
I. Bloch, T. Hänsch et al., Nature Jan 3 2002

3. Optical Lattices … continued

- multiple ground states & spin-dependent lattices



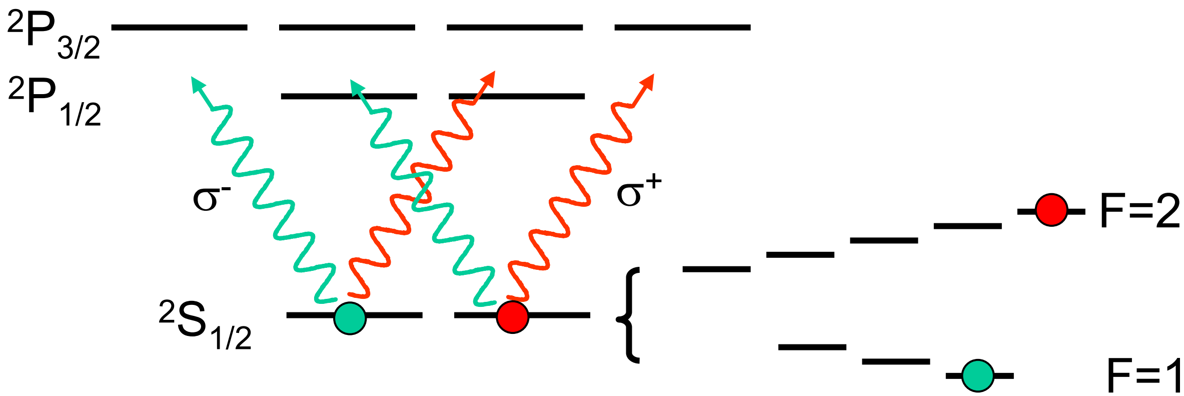
- multiple ground states & spin-dependent lattices

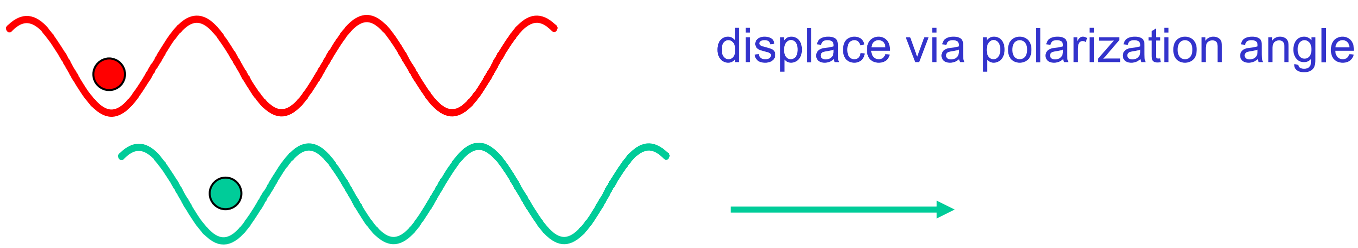


$$\vec{E} \sim \vec{\epsilon}_{\theta/2} e^{ikz-i\omega t} + \vec{\epsilon}_{-\theta/2} e^{-ikz-i\omega t}$$

$$\sim \vec{\epsilon}_{\sigma^+} \cos(kz - \theta/2) + \vec{\epsilon}_{\sigma^-} \sin(kz + \theta/2)$$

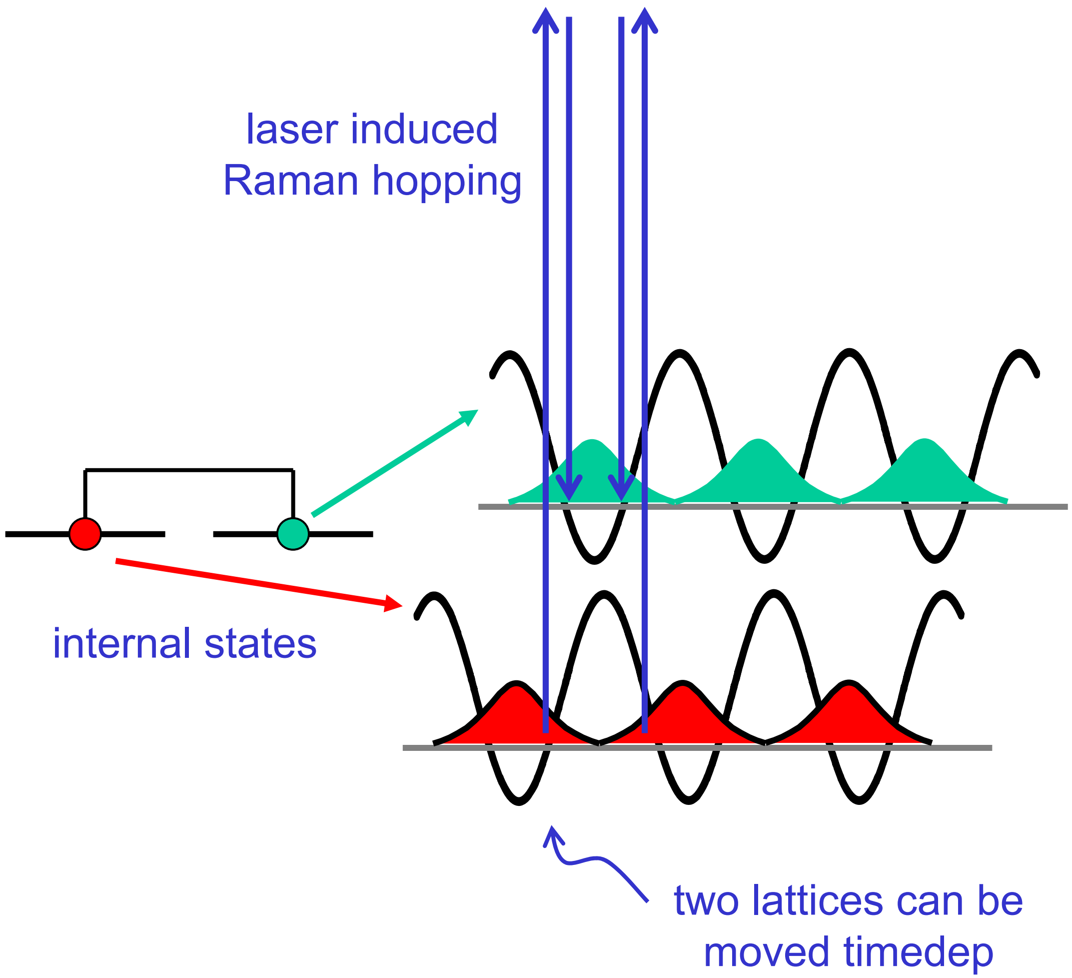
- multiple ground states & spin-dependent lattices



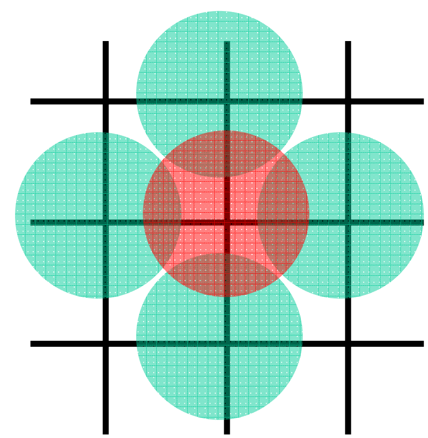


Two component Hubbard models

- hopping via Raman transitions



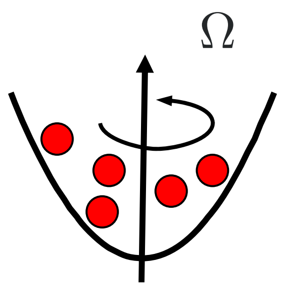
- nearest neighbor interaction



overlapping
wavefunctions
+
laser induced
Raman hopping

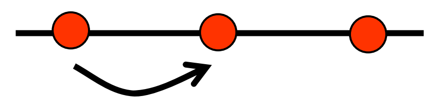
Adding “magnetic fields”

- effective magnetic field via rotation

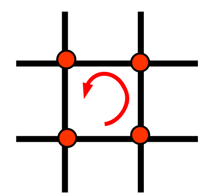


see: fractional quantum Hall effect

- effective magnetic field via lattice design



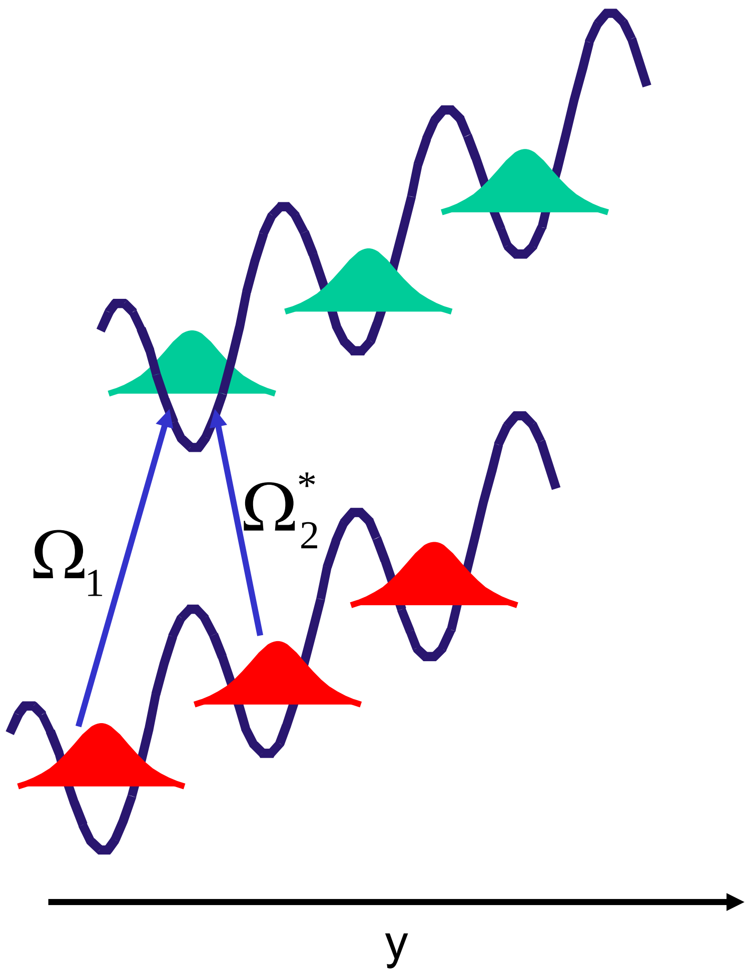
$$J_{\alpha\beta} \longrightarrow J_{\alpha\beta}\, e^{ie\int_\alpha^\beta \vec{A}\cdot d\vec{l}}$$



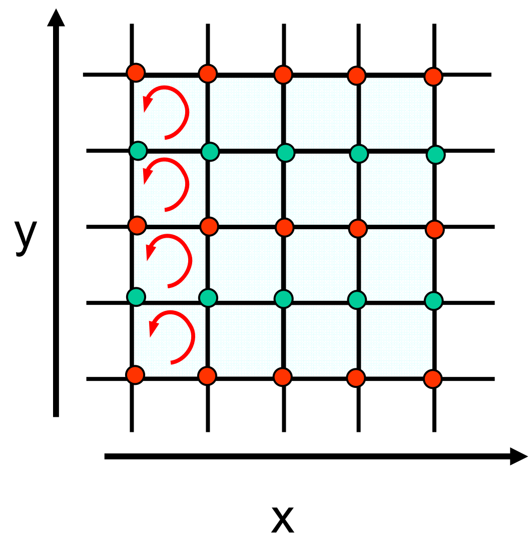
$$e^{ie\oint \vec{A}\cdot d\vec{l}} = e^{i\phi/\phi_0} \equiv e^{i\alpha\pi}$$

accumulate phase when walking around a plaquette

- effective magnetic fields: configuration



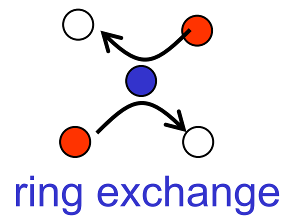
spatially varying phases
of the Raman lasers



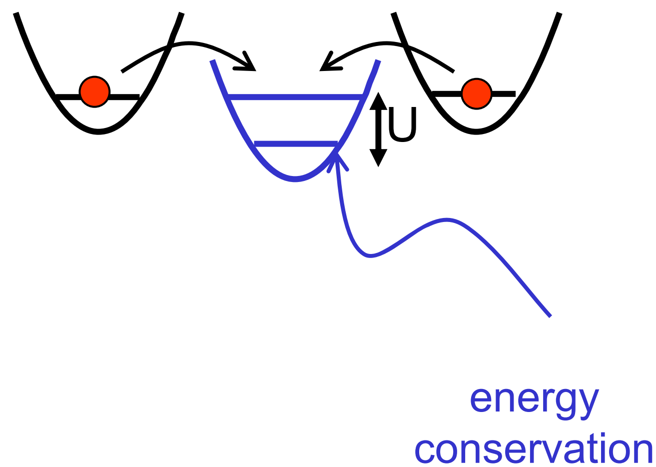
equivalent to a homogeneous
magnetic field

HP Büchler et al., PRL2005

- two particle hopping



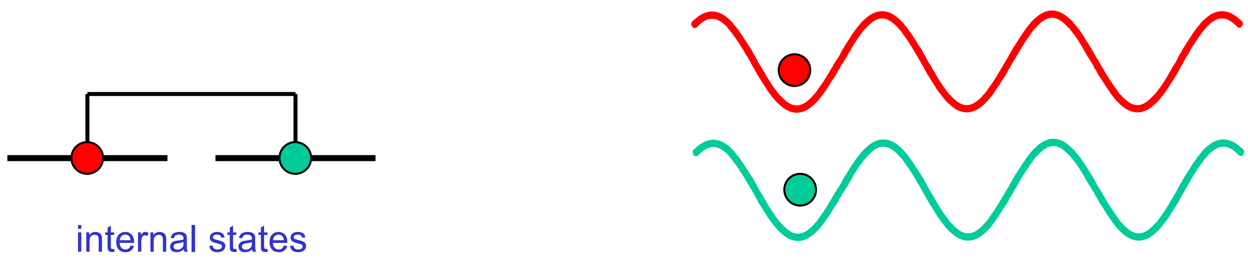
+ …



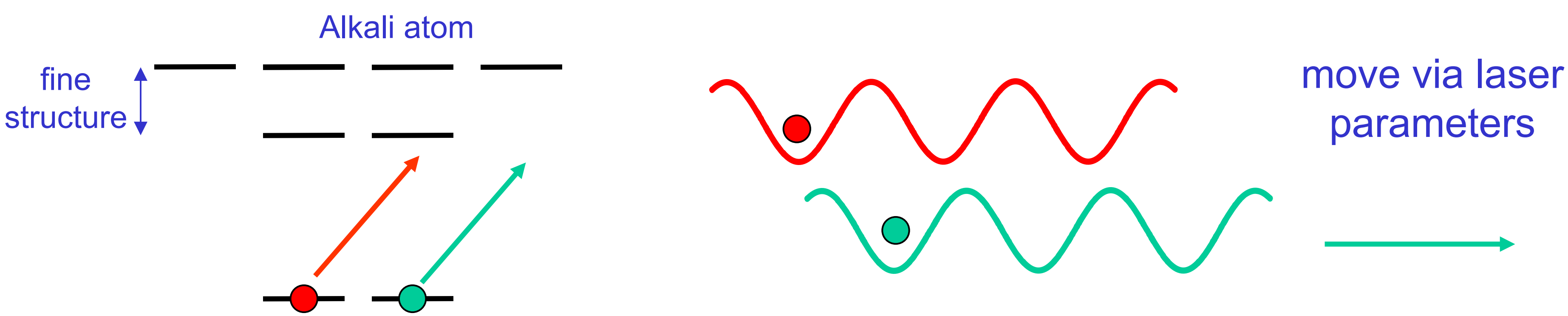
4. Coherent manipulations & entangling atoms

Spin-dependent lattices

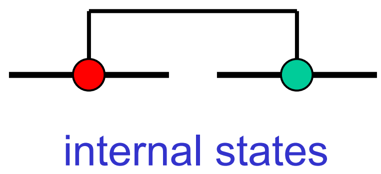
- trapping potential depends on the internal state

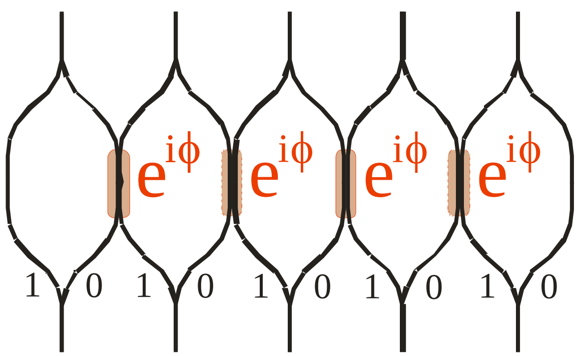


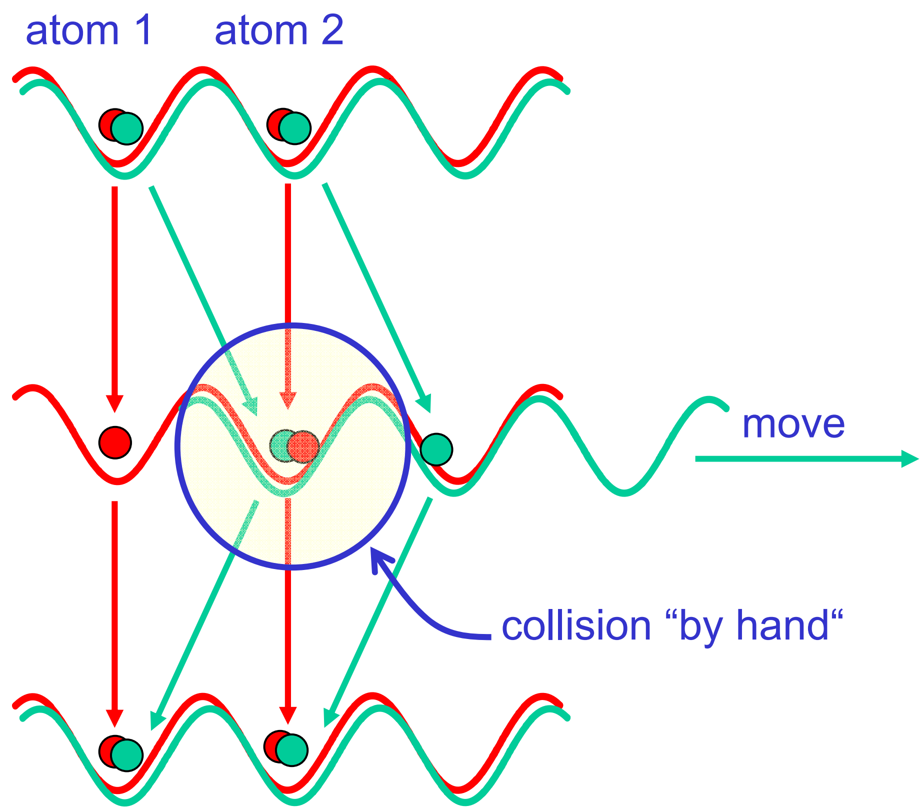
- we can move one potential relative to the other, and thus transport the component in one internal state



- interactions by moving the lattice + colliding the atoms “by hand”







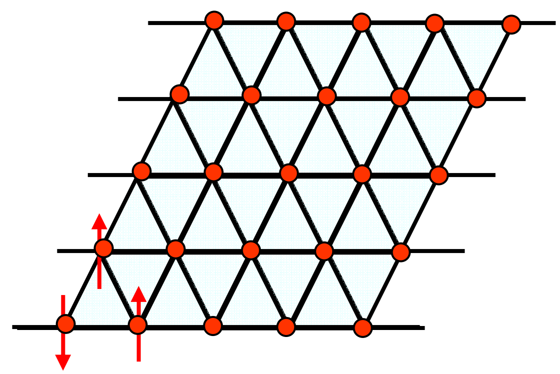
- Ising type interaction (as the building block of the UQS)

$$H = -\frac{J}{2}\sum_{\langle a,b\rangle} \sigma_z^{(a)} \otimes \sigma_z^{(b)}$$

nearest neighbor, next to nearest neighbor ....

Universal Quantum Simulator
(specialized quantum computing)

- Feynman: simulating a quantum system with a classical computer is hard
- Example: condensed matter
  - spin models
  - Hubbard models



$$|\psi\rangle = \sum_{\tilde{\sigma}} c_{\tilde{\sigma}} |\sigma_1 \sigma_2 \ldots \sigma_N\rangle$$

2^N complex coefficients

- Feynman's Universal Quantum Simulator (UQS):

Feynman, Lloyd, ...

UQS = controlled quantum device which efficiently reproduces the dynamics of any other many-particle quantum system (with short range interactions)

UQS: a simple example

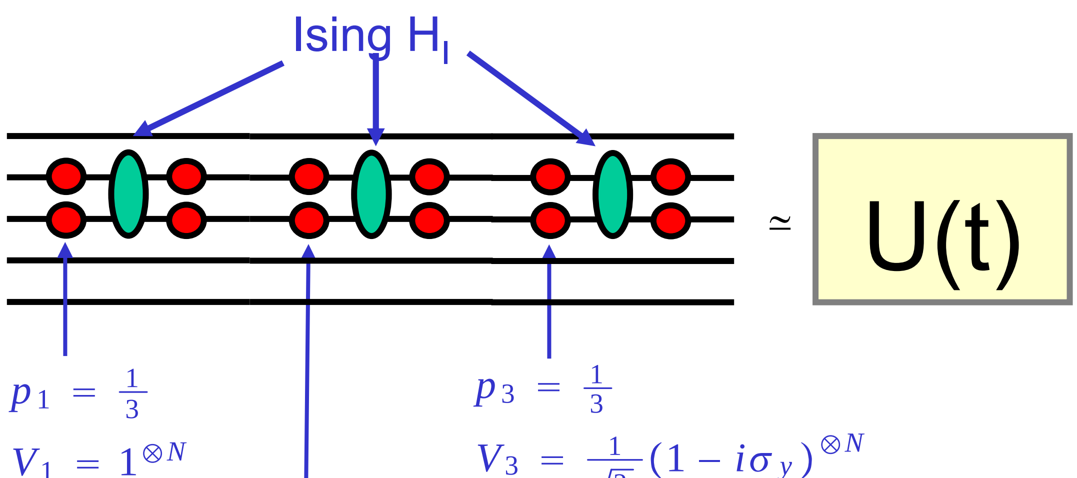
realized by movable optical lattice

- given Ising:

$$H = -\frac{J}{2}\sum_{\langle a,b\rangle} \sigma_z^{(a)} \otimes \sigma_z^{(b)}$$

- simulate Heisenberg model:

$$H = -\frac{J}{2}\sum_{\langle a,b\rangle}\left(\sigma_x^{(a)} \otimes \sigma_x^{(b)} + \sigma_y^{(a)} \otimes \sigma_y^{(b)} + \sigma_z^{(a)} \otimes \sigma_z^{(b)}\right)$$



$p_1 = \frac{1}{3}$, $V_1 = 1^{\otimes N}$

$p_2 = \frac{1}{3}$, $V_2 = \frac{1}{\sqrt{2}}(1 - i\sigma_x)^{\otimes N}$

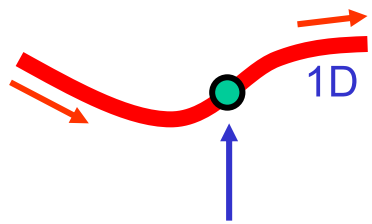
$p_3 = \frac{1}{3}$, $V_3 = \frac{1}{\sqrt{2}}(1 - i\sigma_y)^{\otimes N}$

- adding: magnetic fields, random potentials etc.

A. Micheli
A. Daley
D. Jaksch
PZ

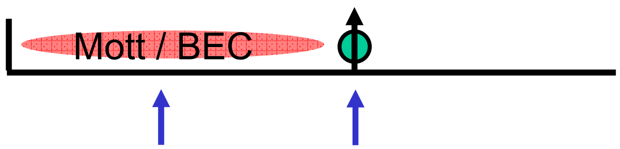
PRL Oct 2004

5. Atomic Quantum Switch / Amplifier / Read Out



Mesoscopic AMO

- "single atom quantum" mirrors for 1D quantum gas



bosonic atoms

✓ interacting

single atom as a …

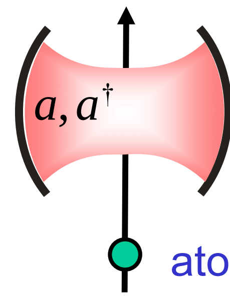
✓ mirror

✓ qubit = spin-1/2

- Questions …
  - how?
  - (time dependent) dynamics?
  - quantum gate / switch
  - qubit read out

Quantum Optics

- Cavities and CQED



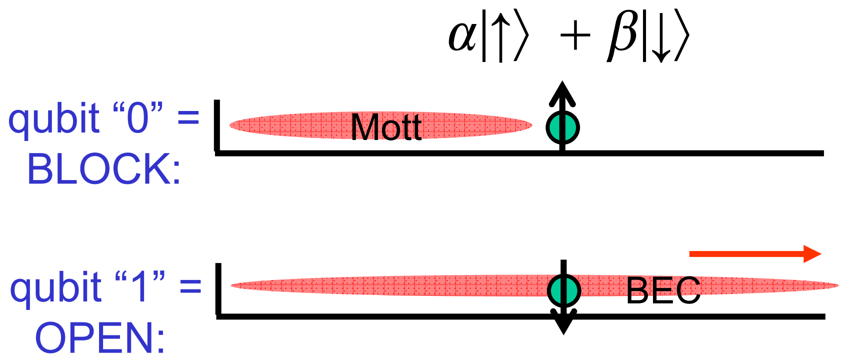
cavity mode: bosonic

atom: spin-1/2

- Jaynes-Cummings model / quantum state engineering
- dissipative / open quantum system / measurement: e.g.QND
- experimental realization:
  - microwave (Haroche, …)
  - optical (Kimble, Rempe, Feld, …)

Mesoscopic AMO

- "single atom quantum" mirrors for 1D quantum gas



$\alpha|\uparrow\rangle|\text{Mott}\rangle + \beta|\downarrow\rangle|\text{BEC}\rangle$

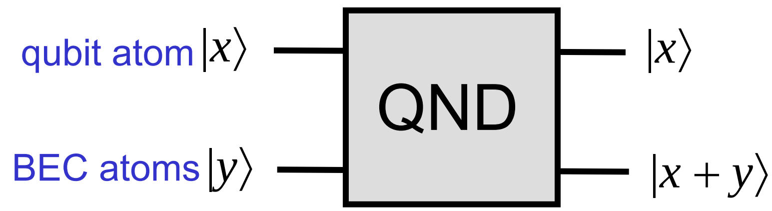
✓"amplify" spin

✓ entangled quantum phases

✓qubit read out

Quantum Computing

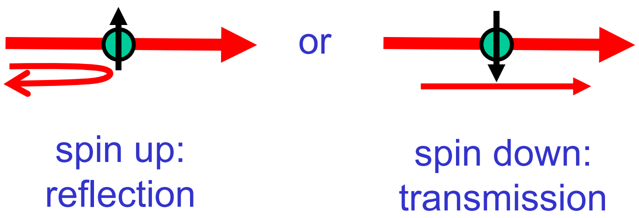
- quantum gate / switch / quantum nondemolition



$$\left(\sum_x c_x|x\rangle\right) \otimes |y\rangle \rightarrow \left(\sum_x c_x|x\rangle \otimes |x+y\rangle\right)$$

Mesoscopic AMO

- atomic qubit = spin

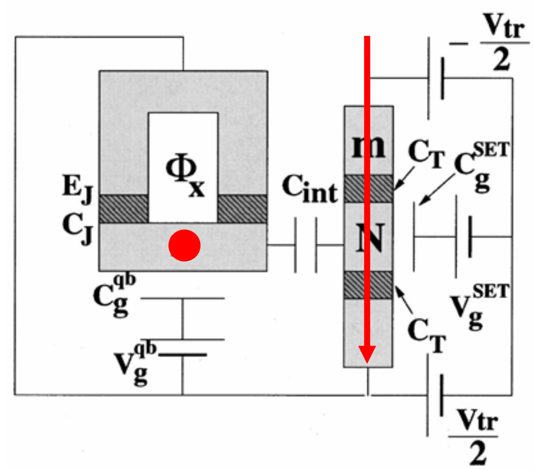


"coherent spintronics"

"single atom transistor"

Mesoscopic CMP

- transport through quantum dot / Coulomb blockade

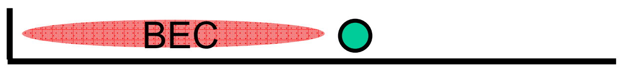


"single electron transistor"

(e.g. read out of a charge qubit)

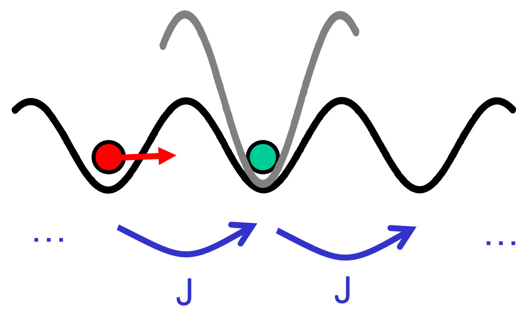
Mesoscopic AMO

- "single atom quantum" mirrors for 1D quantum gas



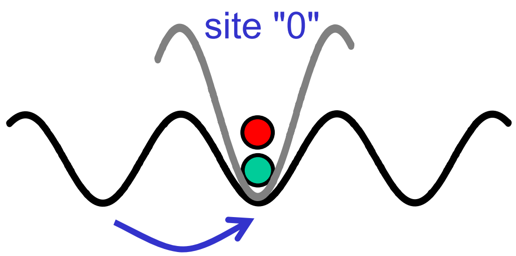
Physical Realization

- realization: 1D optical lattice



How?

- collisional interactions



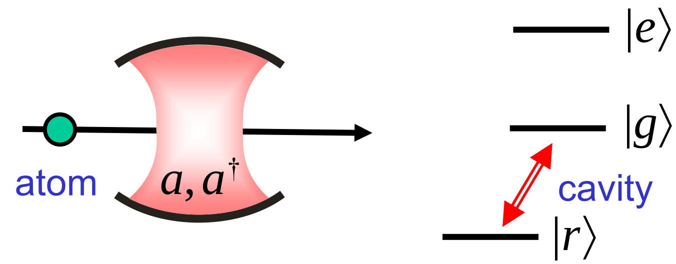
density-density

$$H = U_{ab} a_0^\dagger a_0 b_0^\dagger b_0$$

+ spin

$$H \sim a_0^\dagger a_0 \hat{\sigma}_z$$

- compare: Cavity QED



$$H \sim a^\dagger a |g\rangle\langle g| \sim a^\dagger a \hat{\sigma}_z$$

AC-Starkshift

$$[a^\dagger a, H] = 0$$

QND photon number

- **goal:** large interactions
- **answer:** Feshbach or photo association resonances
  - large scattering length
  - **better: infinite scattering length** ⟷

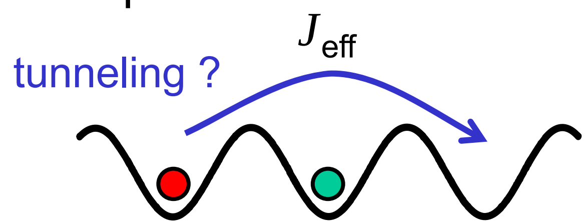
new physics:

- EIT-type quantum interference to kill transport

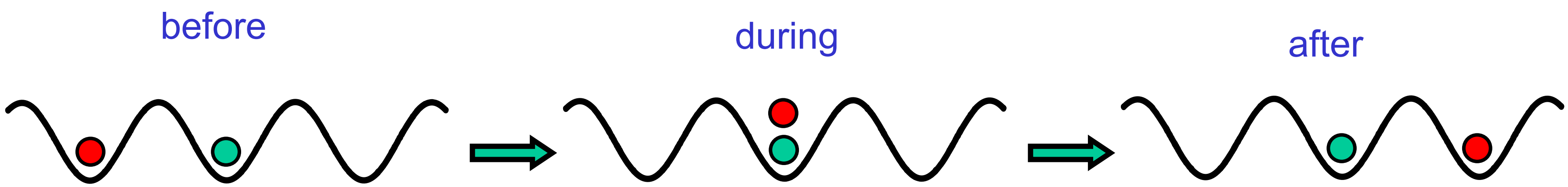
Atomic Switch by Quantum Interference

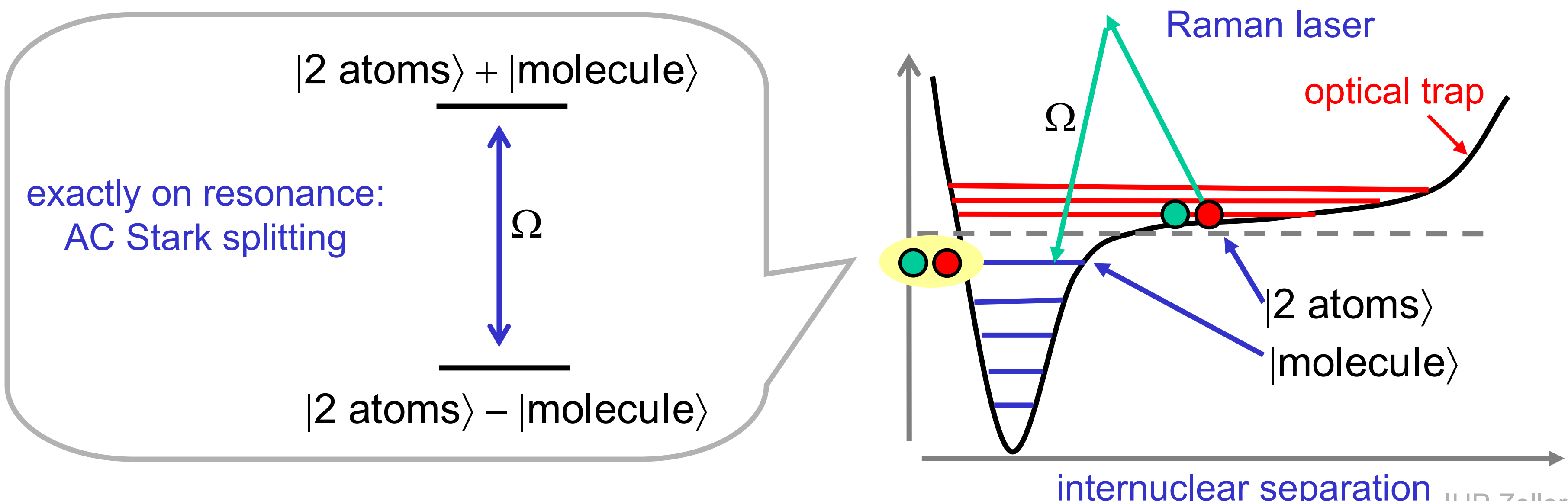
Micheli et al.

- tunneling through the quantum dot?

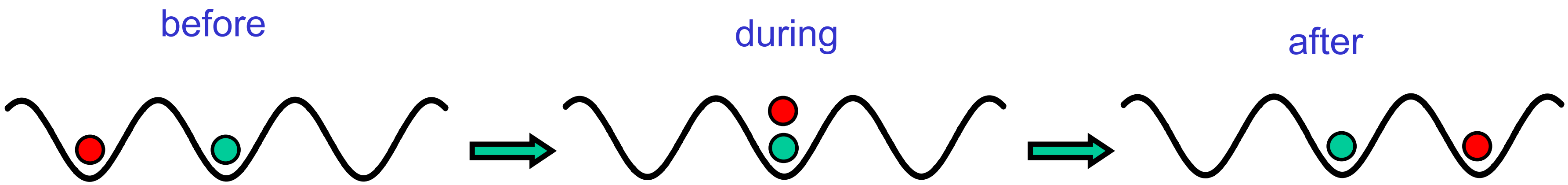


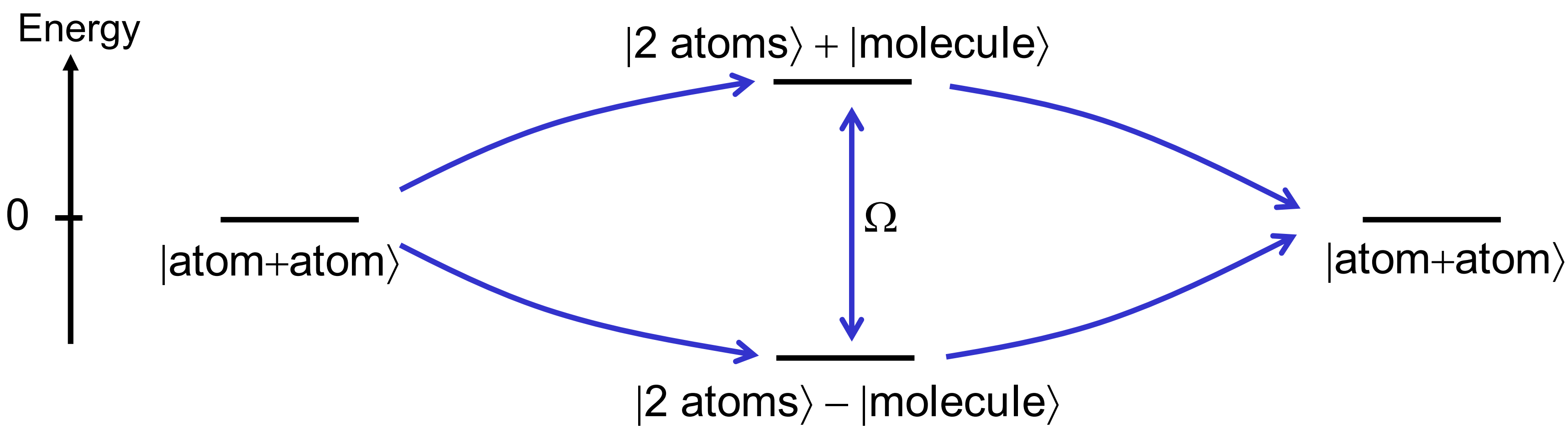
- process in an optical lattice





- blocking by quantum interference

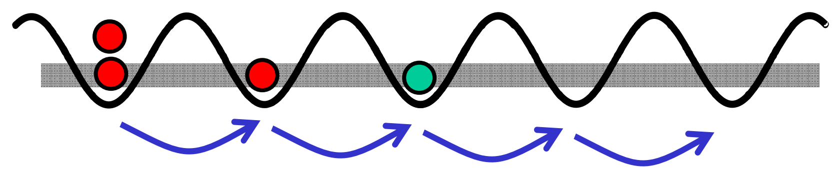




$$J_{\text{eff}} \sim J\frac{1}{E-\frac{1}{2}\Omega}J + J\frac{1}{E+\frac{1}{2}\Omega}J \overset{!}{=} 0 \quad (\text{for } E = 0)$$

we kill the transport by quantum interference:
"infinite repulsion" (EIT)

Time dependent many body dynamics

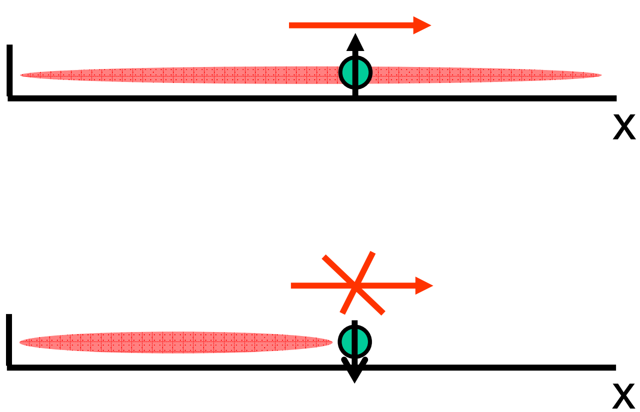


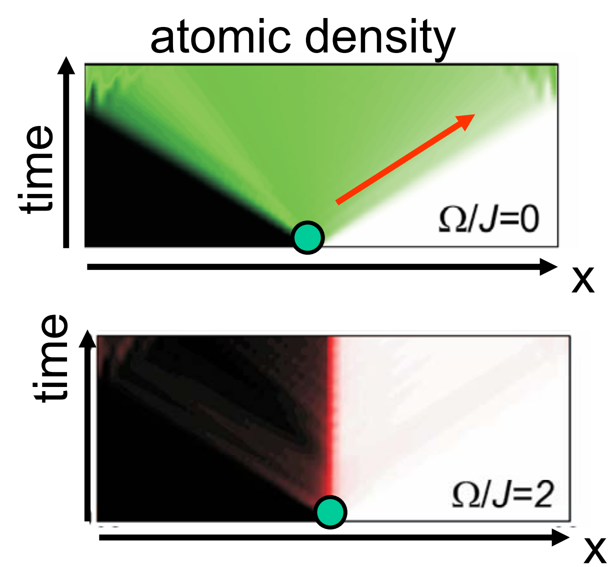
- [Exact] Solution of *time dependent* many body Schrödinger equation for up to ~ 100 atoms
  - numerical: time dep DMRG-type method G. Vidal, PRL 2003
  - semianalytical: hard core bose gas

- numerical results for N ~ 30 atoms on 61 lattice sites

switch open

switch closed



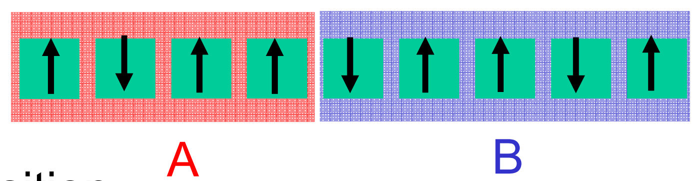


- current-voltage characteristics for transport …

Exact numerical time dependent many body in 1D

method: G. Vidal, PRL 91, 147902 (2003)

- efficient classical simulation of slightly entangled quantum computations
- measure of entanglement: e.g. spin system



Schmidt decomposition

$$|\Psi\rangle = \sum_{\alpha=1}^{\chi_A} \lambda_\alpha |\phi_\alpha^{[A]}\rangle |\phi_\alpha^{[B]}\rangle \quad \in \mathcal{H}_d^{\otimes n}$$

Schmidt coefficients $\qquad |\phi_\alpha^{[A]}\rangle$ is eigenvector of reduced density matrix $\rho^{[A]}$

rank χ_A of $\rho^{[A]}$ is a natural measure of entanglement:

$$\chi = \max_{\text{possible partitions A}} \chi_A \qquad\qquad E_\chi = \log_2(\chi_A)$$

property: $0 \leq E_\chi \leq \frac{1}{2} n \log_2 d$ and $E = 0$ for product state

- computational basis:

$$|\Psi\rangle = \sum_{i_1} \ldots \sum_{i_n} c_{i_1,i_2,\ldots,i_n} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_n\rangle$$

- particular decomposition of coefficients:

$$c_{i_1,i_2,\ldots,i_n} = \underbrace{\sum_{\alpha_1\ldots\alpha_n} \Gamma^{[1]i_1}_{\alpha_1} \lambda_{\alpha_1} \Gamma^{[2]i_2}_{\alpha_1\alpha_2} \lambda_{\alpha_2} \Gamma^{[3]i_3}_{\alpha_2\alpha_3} \ldots \Gamma^{[n]i_n}_{\alpha_{n-1}}}$$

n tensors $\Gamma^{[1]}, \ldots, \Gamma^{[n]}$ $\qquad i = 0, 1, \ldots, d-1$

$n-1$ vectors λ_α $\qquad \alpha = 1, \ldots, \chi$

2^n coefficients $c \Leftrightarrow (2\chi^2 + \chi)n$

generic state $|\Psi\rangle$

if $E \sim n$, then decomposition uninteresting ($\mathcal{O}(ne^n)$)

if $E \sim \log n$, then poly(n) parameters

- propagation of the time dependent Schrödinger equation: propagate Γ and λ

In our case of the Hubbard model: $\chi \approx 8$

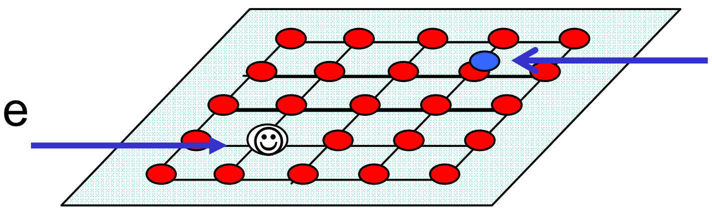
we can integrate the time dependent Schrödinger equation exactly for up to a few hundred (!?) particles (D. Jaksch & A. Daley)

6. Dissipative Dynamics in Lattices: Cooling etc.

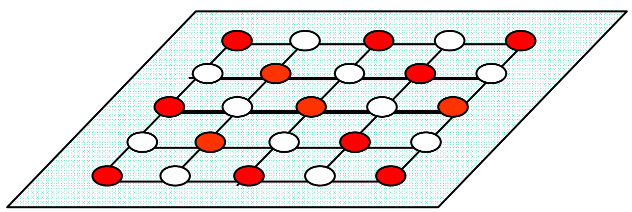
Rabl et al. PRL 2003

Healing defects & Pattern loading

- Getting rid of the last defects …?
  - cooling
  - filtering



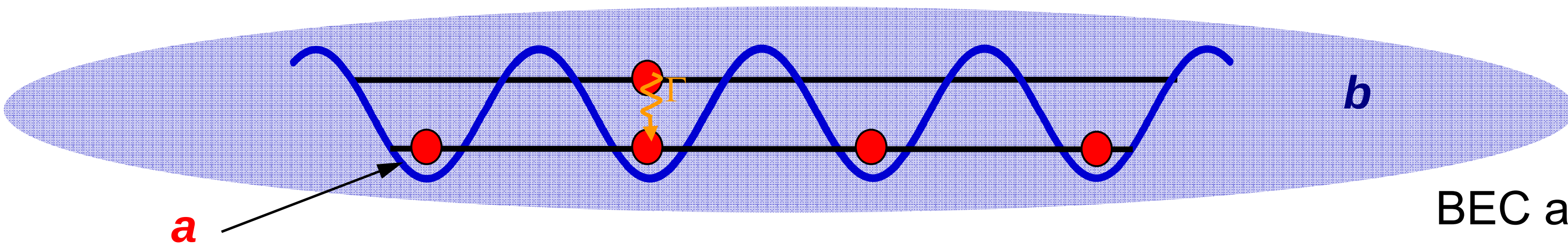
- loading spatial patterns



- fidelity of loading 1:10^4 or 10^5

Cooling via Superfluid Immersion

- We immerse the lattice system in an external BEC



BEC as reservoir

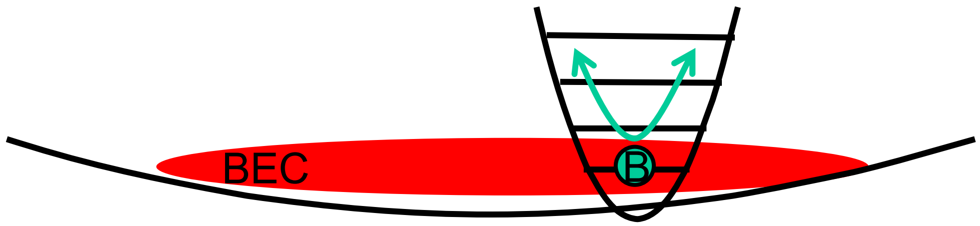
- Bose condensate as a reservoir
- atoms “a” see the lattice, bose reservoir atoms “b” see no lattice
- Cooling via (*laser assisted*) collisional interactions

Remarks:

- We will be able to cool to temperatures lower than the reservoir
- Analogy with laser cooling: photon -> phonon

Cooling via Superfluid Immersion

- Model



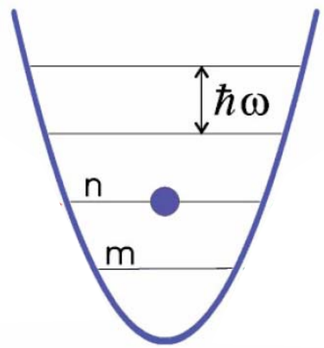
- A-B collisions
- BEC as T~0 reservoir to cool qubit motion

Remarks:

- We will be able to cool to temperatures *lower* than the reservoir
- Cooling *within* a Bloch band
- Analogy with laser cooling: photon -> phonon

A. J. Daley, P. O. Fedichev, and P. Zoller, Phys. Rev. A 69, 022306 (2004)

Cooling model …



- Atom in a Harmonic trap interacting with a superfluid reservoir
- Density-Density interaction:

$$\hat{H}_{\rm int} = g_{ab}\int \delta\hat{\rho}(\mathbf{r})\,\delta\hat{\rho}_{\rm atom}(\mathbf{r})d^3\mathbf{r} = g_{ab}\int \delta\hat{\rho}(\mathbf{r})\,\delta(\mathbf{r}-\hat{\mathbf{r}})d^3\mathbf{r}$$

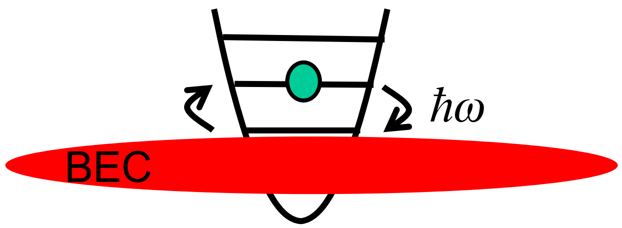
- Energy dissipated as Bogoliubov excitations:

$$\delta\hat{\rho} = \sqrt{\frac{\rho_0}{V}}\sum_{\mathbf{q}}\left[(u_{\mathbf{q}}+v_{\mathbf{q}})\hat{b}_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}} + (u_{\mathbf{q}}+v_{\mathbf{q}})\hat{b}^{\dagger}_{\mathbf{q}}e^{-i\mathbf{q}\cdot\mathbf{r}}\right]$$

- Master Equation ! occupation probabilities pm:

$$\dot{p}_m = \sum_{n>m} F_{n\to m}p_n - \sum_{n'<m} F_{m\to n'}p_m + \sum_n H_{n,m}(p_n - p_m)$$

- cooling process: excitation of phonons in the superfluid



$$\dot{p}_n = \ldots$$

master equation occupation:
SF as a reservoir

- supersonic regime: sound velocity *u* << velocity of trapped atom

$$\frac{\omega\tau_{1\to 0}}{2\pi} \sim 1.2\times 10^{-2} \times \frac{1}{\rho_0 a_{ab}^3}\frac{a_{ab}}{l_0}$$

cooling time ~ 10 oscillator cycles

Rubidium BEC:

$\rho_0 \sim 10^{14}\ \mathrm{cm}^{-3}$ $a_{bb} \sim 100 a_0$

$m_b u^2/(2\hbar) = 2\pi \times 3.7 \times 10^2\ \mathrm{s}^{-1}$

$\omega \sim 2\pi \times 10^5\ \mathrm{s}^{-1}$

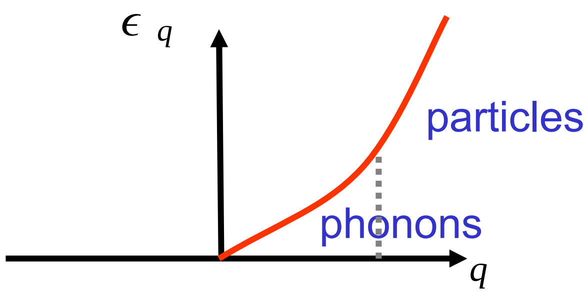
$$\dot{\varepsilon}(n) \approx -\frac{g_{ab}^2 \rho_0 m^{3/2}}{\pi\hbar^4\sqrt{2}}\alpha[\varepsilon(n)]^{3/2}$$

energy dissipation

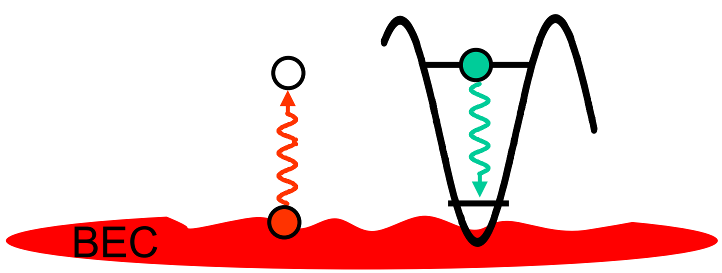
BEC as T=0 phonon reservoir

- Phonon reservoir

phonons = bosonic excitations



- Sympathetic cooling: collision

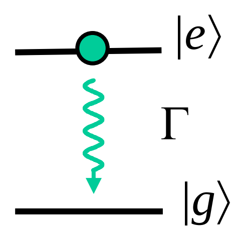


Radiation field: photon reservoir

- Radiation field / vacuum modes

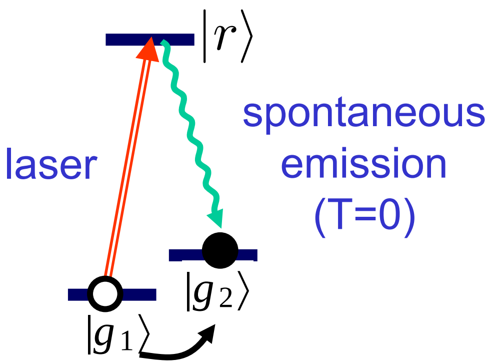
$$|\mathrm{vac}\rangle$$

- Spontaneous emission



A reminder …

- optical pumping



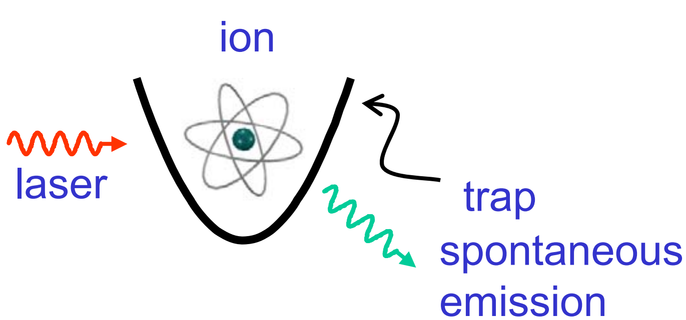
- Remark 1:

$$\rho_{sys} \otimes \rho_{env} \rightarrow |\psi\rangle_{sys}\langle\psi| \otimes \rho'_{env}$$

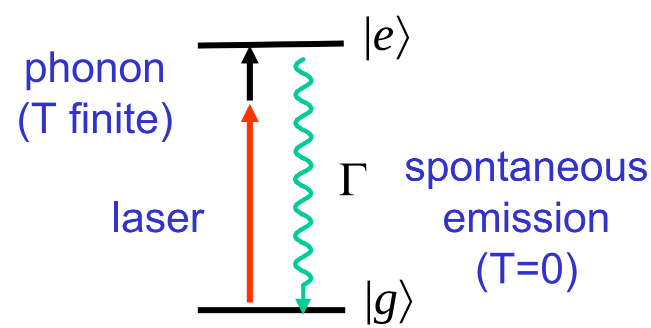
ρ_{sys} ↑ system mixed state

$|\psi\rangle_{sys}\langle\psi|$ ↑ system pure state

- laser cooling



- Remark 2:



"low frequency / high-temperature" system

⇓

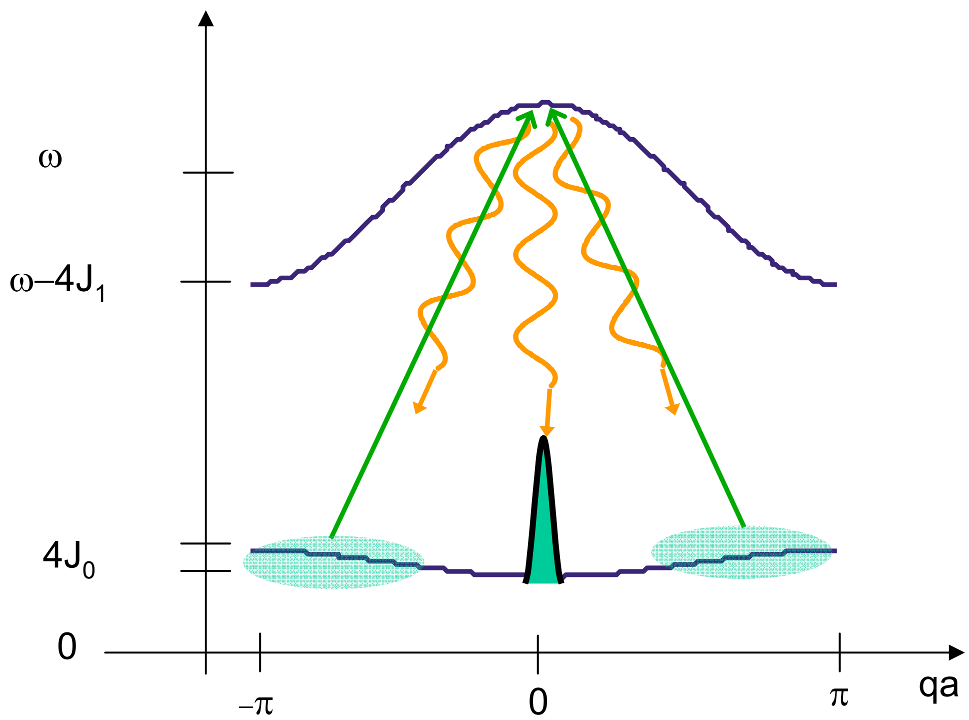
dissipative "high-frequency" / T=0 system

Photon dissipation / spontaneous emission not useful for lattice loading. We need a different reservoir.

Cooling within the Bloch Band

Ideas taken from laser cooling below the recoil limit (Chu & Kasevich; Aspect, Cohen-Tannoudji)

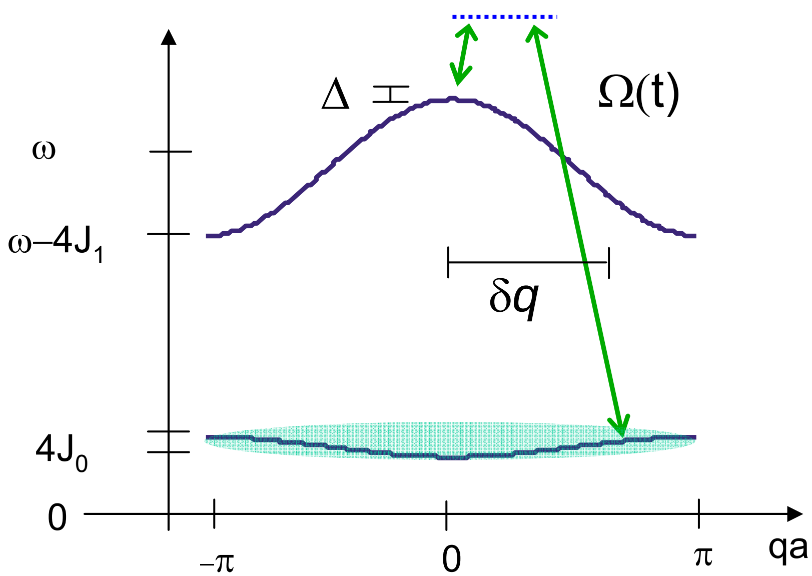
- Question: Can we cool dilute/ weakly interacting atoms in the lattice within a Bloch band?
- Idea: Combine Raman excitation to a higher band with cooling via an external reservoir gas.

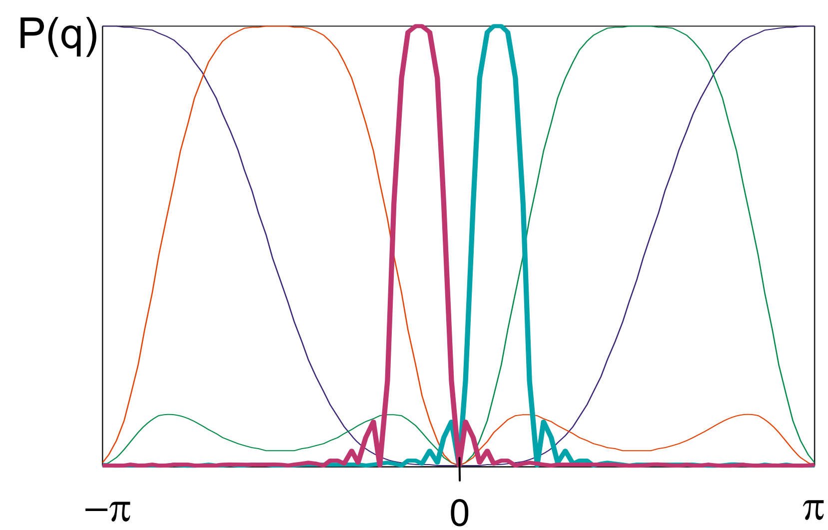


A. Griessner, A. J Daley, D. Jaksch, and P. Zoller, in preparation

Bloch Band Raman Cooling: Pulses

- We can use a series of time dependent pulses to engineer the excitation profile, via Raman detuning, Δ, wavenumber difference δq, and pulse shape [$\Omega(t)$].
- A sequence of pulses can then excite all atoms but a few near q ~ 0.





Bloch Band Raman Cooling: Discussion

- Features:
  - the atoms in the lowest Bloch band move frictionless in the BEC (subsonic = superfluidity), i.e. a BEC temperature larger than the atom temperature will not heat
  - by laser assisted transfer to the higher Bloch band the emitted “phonon“ is in the particle excitation branch, i.e. supersonic, and cooling is possible
  - Thus we are able to cool to temperatures lower than the temperature of the reservoir BEC.
- Typical Values: $T_f \sim 10^{-2} J$ in time Θ = 30ms for 87Rb!!